

AP Calculus BC Summer Packet

*If you prefer **NOT** to show work on the packet, show your numbered/labeled work neatly.*



Welcome to Calc BC, the equivalent of college Calc I & II. Calculus is my favorite math class 😊 because it explores a wide variety of concepts and draws on skills from many other math classes (“Oh, that’s why we learned that in…”). I hope you enjoy it, too, and become more curious about math as a result of taking Calc BC.

What will it take to succeed?

- Remember the **unit circle** (know how to find sin, cos, tan, csc, sec, and cot of any angle)
- Remember concepts and skills learned in the **first two chapters of Calculus**
- Have solid **algebra** and **trig skills**

To make sure you’re ready, you’ll need to do a few things over the summer.

- Complete the **first page of the packet soon** so you know what needs to be **memorized**.
- Complete the remaining pages now while your skills are fresh **OR** late in July so that your skills are fresh at the start of next year (or print a second copy from https://www.brownsburg.k12.in.us/bhs_summer_homework and do both). **Your packet (with work shown) is due on the first day of school.** Also **review the first page** for material that needs to be memorized.
- **Check your answers** to the packet using the QR code.
- Begin **learning the flashcards** at the back of this packet. (If given the front, you should be able to write the back.) You will receive a sturdier copy of flashcards on the first day. The third day of class, you’ll have a flashcard quiz.

The first day of class, we’ll address questions from the review packet and clarify misconceptions from the first two chapters.

If you have problems completing the assignment, ask a classmate or message me on StudentSquare, Schoology, or email. I look forward to working through Calc BC with you next school year!

Mrs. Mendenhall (emendenhall@brownsburg.k12.in.us)

answers! ↷



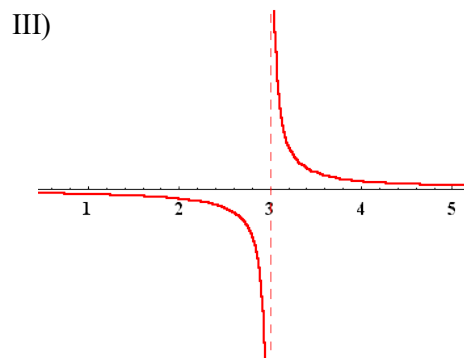
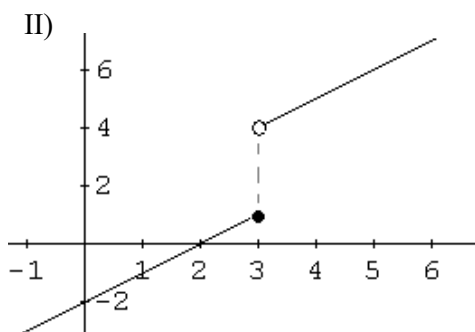
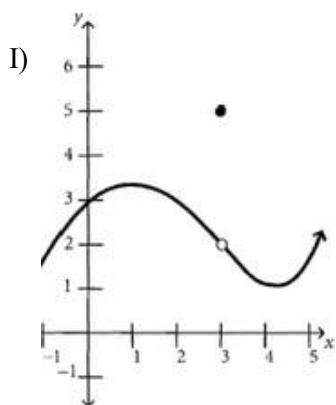
***Forgot any? Check the flashcards at the end of the packet.**

Complete each question, including work or explanations as directed. Do NOT use a calculator, except when specified.

Calculus, Chapter 1 – do now or later

Limits – Graphically (1.2 & 1.4)

4. For which of the following does $\lim_{x \rightarrow 3} f(x)$ exist? Explain why or why not.

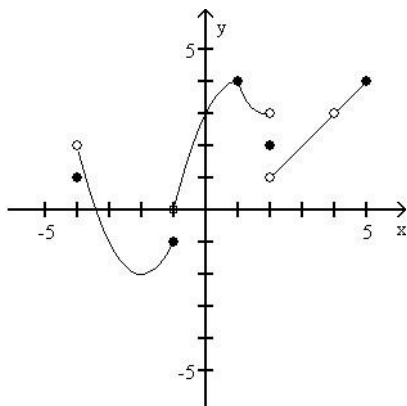


I)

II)

III)

5. Use the graph of $y = g(x)$ to find the following values.



a) $\lim_{x \rightarrow 1} g(x) =$

e) $\lim_{x \rightarrow 2} g(x) =$

b) $\lim_{x \rightarrow 2^+} g(x) =$

f) $\lim_{x \rightarrow -1^-} g(x) =$

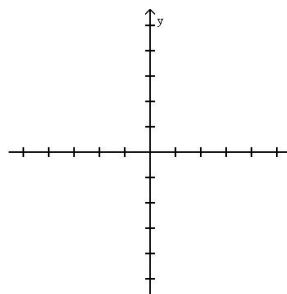
c) $\lim_{x \rightarrow 2^-} g(x) =$

g) $\lim_{x \rightarrow 4} g(x) =$

d) $g(2) =$

h) $\lim_{x \rightarrow -1^+} g(x) =$

6. Draw the graph of a function for which $\lim_{x \rightarrow 1} f(x)$ does not exist.



Limits – Numerically from Tables (1.2)

7. The table below gives the values of three functions, f , g , and h near $x = 0$. Based on the values given, for which of the functions does it appear that the limit as x approaches zero is 3? List all that apply.

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
$f(x)$	3	3	3	3	2	2	2
$g(x)$	2.971	2.987	2.997	Undefined	2.997	2.987	2.971
$h(x)$	3.018	3.008	3.002	3	3.002	3.008	3.018

Limits – Analytically from Expressions (1.3) and/or L'Hopital's Rule

8. If $a \neq 0$, then what is $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$? **Show your work.**

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

9. If $f(x) = \begin{cases} \ln x^4 & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ then find $\lim_{x \rightarrow 2} f(x)$. **Show your work.**

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

10. What is $\lim_{x \rightarrow 0} \frac{5x^5 + 12x^3}{3x^5 - 6x^3}$? **Show your work.**

(A) $-\frac{17}{3}$ (B) 0 (C) $\frac{5}{3}$ (D) -2 (E) nonexistent

Continuity (1.4)

11. What makes a function, f , continuous at $x = a$? (forgot? check the flashcards at the end of the packet)

12. Which of these functions are continuous for **all** real numbers x ? **Sketch each or explain.**

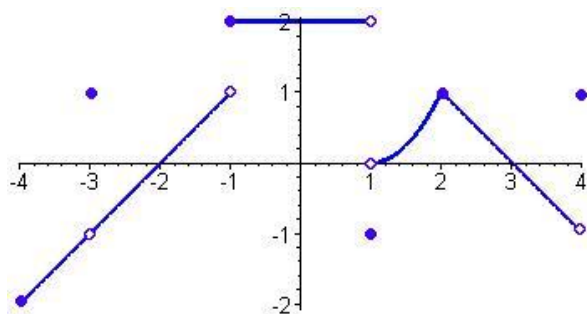
I. $y = \ln x$

II. $y = 3^x$

III. $y = \cos x$

(A) None (B) I only (C) II only (D) I and II (E) II and III

13. The graph of the function f is shown below. Identify whether each statement about f is true or false. **Explain.**



- (A) f is continuous at $x = -1$. True or false?
- (B) $x = -1$ is in the domain of f . True or false?
- (C) $\lim_{x \rightarrow -1^+} f(x)$ is equal to $\lim_{x \rightarrow -1^-} f(x)$. True or false?
- (D) $\lim_{x \rightarrow -1} f(x)$ exists. True or false?

14. Determine the values of b and c such that the function is continuous on the entire real line. **Show your work.**

$$h(x) = \begin{cases} x + 1, & 1 < x < 3 \\ x^2 + bx + c, & x \leq 1, x \geq 3 \end{cases}$$

Intermediate Value Theorem (1.4)

15. What does IVT state?

16. Suppose that f is a continuous function defined for all real numbers x and $f(-4) = 3$ and $f(-1) = -4$. If

$f(x) = 0$ for one and only one value of x , then which of the following could be x ? **Explain your answer.**

Sketching the graph may be helpful.

- (A) -7 (B) -3 (C) 0 (D) 1 (E) -4

17. Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem? **Sketch or explain.**

- (A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.
- (B) $f(3) = 15$
- (C) $f'(x) > 0$ for all x in the open interval $(2, 4)$.

Asymptotes and Limits at Infinity (1.5-1.6)

18. How do you find a vertical asymptote? (1.5)
19. How do you find a horizontal asymptote? (1.6)
20. The line $y = 5$ is a horizontal asymptote to the graph of a function. Which one? **Show work or explain.**
- (A) $y = \frac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \frac{1}{x-5}$ (D) $y = \frac{5x}{1-x}$ (E) $y = \frac{20x^2-x}{1+4x^2}$
21. The vertical line $x = 2$ is an asymptote for the graph of the function f . Which of the following statements must be false? **Explain and/or sketch a graph.**
- (A) $\lim_{x \rightarrow 2} f(x) = 0$ (B) $\lim_{x \rightarrow 2} f(x) = -\infty$ (C) $\lim_{x \rightarrow 2} f(x) = \infty$
(D) $\lim_{x \rightarrow \infty} f(x) = 2$ (E) $\lim_{x \rightarrow \infty} f(x) = \infty$
22. Find $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^4+1}}{4x^2+3}$. **Show work or explain.**
- (A) $1/3$ (B) $3/4$ (C) $3/2$ (D) $9/4$ (E) infinite

Calculus, Chapter 2 – do now or later

Power Rule, Differentiability, and Tangent Lines (2.1 & 2.2)

23. In general, how can you find the average rate of change of f on $[a, b]$?
24. In general, how can you find the instantaneous rate of change of f at $x = a$?
25. What two things do you need to find in order to write the equation of a tangent line?
26. What makes a function, f , differentiable at $x = a$?
27. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$? **Show work.**
- (A) $y = 7x - 3$ (B) $y = 7x + 7$ (C) $y = 7x + 11$ (D) $y = -5x - 1$ (E) $y = -5x - 5$

28. Let f be the function $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$. Is each statement about f true or false? **Explain and/or show work.**

I. f has a limit at $x = 2$. T/F?

II. f is continuous at $x = 2$. T/F?

III. f is differentiable at $x = 2$. T/F?

29. Let f be the function defined by $f(x) = \sqrt{|x - 2|}$ for all x . Which of the following statements is true? **Explain.** (HINT: sketch the graph...could you sketch without a calculator?)

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \rightarrow 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f .

30. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then what is $f'(1)$?

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

31. The graph of $y = e^{\tan x} - 2$ crosses the x -axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point? (calculator allowed)

(A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747

32. The function $P(t)$ models the population of the world, in billions of people, where t is the number of years since January 1, 2010. Which of the following is the best interpretation of the statement $P'(1) = 0.076$?

(A) On February 1, 2010, the population of the world was increasing at a rate of 0.076 billion people per year.

(B) On January 1, 2011, the population of the world was increasing at a rate of 0.076 billion people per year.

(C) On January 1, 2011, the population of the world was 0.076 billion people.

(D) From January 1, 2010 to January 1, 2011, the population of the world was increasing at an average rate of 0.076 billion people per year.

(E) When the population of the world was 1 billion people, the population of the world was increasing at a rate of 0.076 billion people per year.

Product Rule, Quotient Rule, and Trig (2.3)

33. If $y = \frac{2x+3}{3x+2}$, find $\frac{dy}{dx}$. **Show work.**

- (A) $\frac{12x+13}{(3x+2)^2}$ (B) $\frac{12x-13}{(3x+2)^2}$ (C) $\frac{5}{(3x+2)^2}$ (D) $\frac{-5}{(3x+2)^2}$ (E) $\frac{2}{3}$

34. Given that $f'(x) = 3 \sec^2 x + 2$, which of the following could be $f(x)$?

- (A) $3 \tan x$ (B) $3 \tan x + 2x$ (C) $3 \sec x + 2x$ (D) $\sec^3 x + 2x$ (E) $6 \sec^2 x \tan x$

35. Let $f(x)$ be a differentiable function. Let $g(x) = \frac{1}{f(x)}$. Use the table to find the value of $g'(2)$. **Show work.**

x	1	2	3	4
$f(x)$	-3	-8	-9	0
$f'(x)$	-5	-4	3	16

- (A) $-\frac{1}{8}$ (B) 0 (C) $\frac{1}{16}$ (D) $\frac{1}{64}$ (E) 16

36. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = x f(x)$. What is the equation of the line tangent to the graph of g at the point where $x = 2$? **Show work.**

Chain Rule and Combinations of Rules (2.4)

37. If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$

- (A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

38. If $y = x^2 \sin(2x)$, then $\frac{dy}{dx} = ?$ **Show work.**

- (A) $2x \cos 2x$ (B) $4x \cos 2x$ (C) $2x(\sin 2x + \cos 2x)$
 (D) $2x(\sin 2x - x \cos 2x)$ (E) $2x(\sin 2x + x \cos 2x)$

39. Use the following table to find $h'(1)$, given that $h(x) = f(g(x))$. **Show work.**

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

Derivatives of Inverse Functions (2.7)

47. Let $f(x) = (2x + 1)^3$ and let g be the inverse function of f . Given that $f(0) = 1$, what is the value of $g'(1)$? **Show work.**

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

Motion (ch. 2)

48. A particle moves along the x -axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time(s) t is the particle at rest? **Show work.**

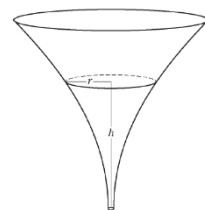
49. A particle moves along the x -axis so that its velocity at time t is given by $v(t) = -(t + 2) \sin \frac{t^2}{3}$. **You may use a graphing calculator.**

a) Find the acceleration of the particle at time $t = 5$. **Show work or concept.**

b) Find all times t in the open interval $0 < t < 4$ when the particle changes direction. **Justify your answer.**

Related Rates (2.8)

50. The inside of a funnel of height 10 inches has circular cross sections. At height h , the radius of the funnel is given by $r = \frac{3}{20} + \frac{1}{20}h^2$, where $0 \leq h \leq 10$. The units of r and h are inches.



The funnel contains liquid that is draining from the bottom. At the instant when the height of liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

Show work and include units.

(A) $-2/3$ (B) $2/3$ (C) $3/10$ (D) $-3/10$

Units:

Other Limits (multiple ways to solve)

51. Find $\lim_{h \rightarrow 0} \frac{\ln(4+h) - \ln(4)}{h}$. **Show work or explain.**

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

answers! ↴



Flashcards – Chapters 1-2A (you'll receive sturdier ones in the fall)

f is continuous at $x = a$ 1	*** IVT (v.1) (Intermediate Value Theorem) *** 1	*** IVT (v.2) (Intermediate Value Theorem) *** 1
$\lim_{x \rightarrow \infty} \frac{ax + b}{cx^2 + d}$ 1	$\lim_{x \rightarrow \infty} \frac{ax^2 + b}{cx^2 + d}$ 1	$\lim_{x \rightarrow \infty} \frac{ax^2 + b}{cx + d}$ 1
horizontal asymptote 1	vertical asymptote 1	$\lim_{x \rightarrow a} f(x) = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$ 1
	start of Chapter 2	
f is differentiable at $x = a$ 2A	average velocity 2A	sign of velocity 2A
average rate of change of $f(x)$ on $[a, b]$ 2A	average acceleration 2A	sign of acceleration 2A

Flashcards – Chapters 1-2A

Since f is continuous on $[a, b]$,
by IVT, f takes on
every value between
 $f(a)$ and $f(b)$.
 $f(a) < _ < f(b)$

Since f is continuous on $[a, b]$
and $f(a) = A$ and $f(b) = B$,
then IVT guarantees that
 $f(x) = C$ in $[a, b]$
because $A < C < B$.

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

(left limit = right limit = $f(a)$)

∞ or $-\infty$
(DNE)

$$\frac{a}{c}$$

0

Indeterminate form;
simplify limit or use
l'Hopital

Denominator of f
equals zero;
factor and cancel first

Limit of f
at ∞ AND $-\infty$

+ is moving up or right
0 is stopped
- is moving down or left

Slope of position

(continuous first)

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

and

$$\lim_{x \rightarrow a^-} f'(x) = \lim_{x \rightarrow a^+} f'(x) = f'(a)$$

+ is accel. up or right
0 is constant velocity
(no acceleration)
- is accel. down or left

Slope of velocity

$$\frac{f(b) - f(a)}{b - a}$$

(slope of f)

Flashcards – Chapter 2A-2B (sturdier ones coming in class)

$\frac{d}{dx}[\sin x]$ 2A	$\frac{d}{dx}[\cos x]$ 2A	$\frac{d}{dx}[\tan x]$ 2A
$\frac{d}{dx}[\csc x]$ 2A	$\frac{d}{dx}[\sec x]$ 2A	$\frac{d}{dx}[\cot x]$ 2A
$\frac{d}{dx}[e^u]$ 2A	$\frac{d}{dx}[\ln u]$ 2A	
$\frac{d}{dx}[\arcsin u]$ AKA $\frac{d}{dx}(\sin^{-1} u)$ 2B	$\frac{d}{dx}[\arccos u]$ AKA $\frac{d}{dx}(\cos^{-1} u)$ 2B	$\frac{d}{dx}[\arctan u]$ AKA $\frac{d}{dx}(\tan^{-1} u)$ 2B
$\frac{d}{dx}[a^u]$ 2B	If f and g are inverses, then $f'(x) =$ 2B	
l'Hôpital's Rule applies to limits that are ____ 2B	l'Hôpital's Rule 2B	

Flashcards – Chapter 2A-2B

$$\sec^2 x$$

$$-\sin x$$

$$\cos x$$

$$-\csc^2 x$$

$$\sec x \tan x$$

$$-\csc x \cot x$$

$$\frac{u'}{u} \text{ or } \frac{1}{u}(u')$$

$$e^u (u')$$

$$\frac{u'}{1+u^2}$$

$$\frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{1}{g'(y)}$$

$$a^u (\ln a) (u')$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

if f and g are continuous

indeterminate:

$$\frac{0}{0} \text{ or } \frac{\pm \infty}{\pm \infty}$$