

AP Physics C

Unit 0: INTRODUCTION AND MATHEMATICAL CONCEPTS AND PHYSICS

This unit introduces you to the basic mathematical tools for doing physics. You will study *units* and converting between units, the trigonometric relationships of *sine*, *cosine*, and *tangent*, and understanding basic graphing techniques and applications.

Important Terms

accuracy: the degree of agreement between a measured value and the true value

base SI (System International) units: the fundamental units of meters for length, kilograms for mass, and seconds for time used in the metric system

cosine: ratio of the length of the adjacent side to an angle in a right triangle to the length of the hypotenuse

derived unit: any unit, such as m/s, in the metric system which is not fundamental and is a combination of meters, kilograms, and/or seconds

precision: the degree of exactness with which a quantity is measured

scientific method: a planned, orderly procedure applied to solving a problem to produce facts and accurate evidence to support a conclusion

sine: ratio of the length of the opposite side to an angle in a right triangle to the length of the hypotenuse

significant figures: the numbers in the given value of a physical quantity obtained by actual measurement, including the first estimated number

standard: a fixed quantity to which all measurements are compared.

tangent: ratio of the length of the opposite side to an angle in a right triangle to the length of the adjacent side

Equations and Symbols

Rectangle $A = bh$

Triangle $A = \frac{1}{2}bh$

Circle $A = \pi r^2$ and $C = 2\pi r$

Parallelepiped $V = lwh$

Cylinder $V = \pi r^2 l$ and $S = 2\pi r l + 2\pi r^2$

Sphere $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

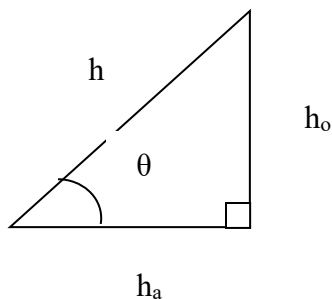
Right Triangle

$$h_o^2 + h_a^2 = h^2$$

$$\sin \theta = \frac{h_o}{h}$$

$$\cos \theta = \frac{h_a}{h}$$

$$\tan \theta = \frac{h_o}{h_a}$$



where

A = area

b = base

h = height

C = circumference

V = volume

l = length

w = width

S = surface area

r = radius

θ = angle

$\sin$ = sine

$\cos$ = cosine

$\tan$ = tangent

The Role of Units in Problem Solving

All physical quantities have units so that we can communicate their measurement. In the metric system, the base units are called SI units. The base SI units for the fundamental quantities of *mass*, *length*, and *time* are the *kilogram*, *meter*, and *second*, respectively. **Only SI units are used on the AP Physics C exam.** Any unit which is a combination of these fundamental units is called a derived unit. An example of a derived unit would be *meters/second* or *kilometers/hour*, which are both units for speed. Sometimes we will need to convert from one unit to another.

SYSTEM INTERNATIONAL: International System or (SI)

- SI is based on a decimal system.
- the units are based on the number 10.
- was first used in France about 1790.

For the AP test, it is also important to know what instrument is used for a measurement (lab based questions)

LENGTH	-----	meter (m)	(meter stick)
VOLUME	-----	liter (l)	(graduated cylinder)
MASS	-----	kilogram (kg)	(balance)
TEMPERATURE	-----	degrees Kelvin (K)	(Celsius thermometer)
TIME	-----	seconds (s)	(stopwatch)

** the AP test does not ask about a micrometer or a Vernier caliper, but for understanding significant figures it is easy to compare these three instruments:

A meter stick measure	.00 cm
A vernier caliper measures	.000 cm
A micrometer measure	.0000 cm

Therefore the micrometer is the most precise. The Vernier and micrometer can only measure small things, but the meterstick is the best overall because it can measure anything, large or small.

-a fundamental unit is one of the standard units (ex: m, l, s)

-a derived units is two or more standard units(ex: m², m³, g/cm³)

-prefix makes larger and smaller units (PREFIX)(UNIT)

example: mm= millimeter

Example 1 Convert 80.0 km/h to m/s.

$$80.0 \frac{km}{h} \left(\frac{1000m}{1km} \right) \left(\frac{1h}{3600s} \right) = 22.2 \frac{m}{s}$$

Note that we multiplied 80.0 km/h by two quotients which were each equal to one, so we didn't change the value of the speed, we only expressed it in different units.

Dimensional Analysis

Often you will need to be able to determine the validity of equations by analyzing the dimensions of the quantities involved. (example: you can't add apples and oranges)

You can't add 1m + 1 m/s (the units are different)

** one represents a distance while the other represents a speed, they are two fundamentally different quantities*

-dimensional analysis is a way of carrying your units through and canceling them in order to check your algebra and also for converting units

Example 2 Verify that the equation below is valid by using dimensional analysis.

$$F\Delta t = m(v_f - v_0)$$

where F is force measured in Newton's, t is time in seconds, m is mass in kg, and v is speed in m/s.

We will show later that a *Newton* = $\frac{kg \cdot m}{s^2}$. Thus the units for each side of the equation can be written as

$$\left(\frac{kg\ m}{s^2}\right)s = kg\left(\frac{m}{s} - \frac{m}{s}\right)$$

$$\frac{kg\ m}{s} = \frac{kg\ m}{s}$$

The unit $\frac{kg\ m}{s}$ is a unit of momentum or impulse, and will be discussed later.

Conversion of Units

FACTOR-LABEL METHOD FOR CONVERTING UNITS: WHEN THERE IS MORE THAN ONE BASIC UNIT, THIS PROCEDURE (used for units that are fractions)

Example: If you want to convert from $5 \frac{km}{hr}$ to $\frac{m}{s}$

1. Write the unit you are starting with, then write a blank fraction in parentheses $5 \frac{m}{s} \left(\quad \right)$
2. Whatever is in the numerator, put the same thing in the denominator and in the numerator put what you are converting to and the equivalence (*for example, there are 1000 m in 1 km*)
3. Cancel the units top and bottom, multiply out

$$5 \frac{km}{hr} \left(\frac{1000m}{1km} \right) \left(\frac{1hr}{60min} \right) \left(\frac{1min}{60s} \right) = 1.38 \frac{m}{s}$$

Example: The micrometer ($1\mu m$) is often called the micron. (a) How many microns make up 1.0 km? (b) What fraction equals $1.0\ \mu m$? (c) How many microns are in 1.0 yd?

(a) Since $1\ km = 1 \times 10^3\ m$ and $1\ m = 1 \times 10^6\ \mu m$,

$$1km = 10^3\ m = (10^3\ m)(10^6\ \mu m/m) = 10^9\ \mu m.$$

The given measurement is 1.0 km (two significant figures), which implies our result should be written as $1.0 \times 10^9\ \mu m$.

(b) We calculate the number of microns in 1 centimeter. Since $1\ cm = 10^{-2}\ m$, $1cm = 10^{-2}\ m = (10^{-2}m)(10^6\ \mu m/m) = 10^4\ m$.

We conclude that the fraction of one centimeter equal to 1.0 m is 1.0×10^{-4} .

(c) Since $1\ yd = (3\ ft)(0.3048\ m/ft) = 0.9144\ m$,

$$1.0\ yd = (0.91m)(10^6\ \mu m/m) = 9.1 \times 10^5\ \mu m.$$

Estimates and Order-Of-Magnitude Calculations

Order of magnitude is expressed as a power of ten

1. Express the number in scientific notation, with the multiplier of the power of ten between 1 and 10 and a unit.
2. If the multiplier is less than 3.162 (the square root of ten), the order of magnitude of the number is the power of ten in the scientific notation. If the multiplier is greater than 3.162, the order of magnitude is one larger than the power of ten in the scientific notation.

PROBLEM: The world's largest ball of string is about 2 m in radius. To the nearest order of magnitude, what is the total length, L, of the string of the ball?

SETUP: Assume that the ball is a sphere of radius 2 m. In order to get a simple estimate, assume that the cross section of the string is a square with a side edge of 4 mm. This overestimate will account for the loosely packed string with air gaps.

CALCULATE: The total volume of the string is roughly the volume of the sphere. Therefore,

Math Review (On the multiple choice part of the test there is no calculators!!!)

You have to practice doing simple calculations!
For example, g will be approximated to $\frac{4(2m)^3}{(4 \times 10^{-3}m)^2} = 2 \times 10^6\ m = 3\ km$

SCIENTIFIC NOTATION: USING POWERS OF TEN TO REPRESENT VERY LARGE AND VERY SMALL NUMBERS (show significant figures more easily)

****RULE:** ONLY ONE INTERGER TO THE LEFT OF THE DECIMAL POINT IN THE NUMBER IN FRONT OF THE POWER OF TEN.

6.7 X 10⁶ is read "six point seven times ten to the sixth power"

EXAMPLES:

1. 6 000 000 = 6×10^6
2. 6 700 000 = 6.7×10^6
3. 0.003 = 3×10^{-3}
4. 0.00035 = 3.5×10^{-4}

ADDING AND SUBTRACTING WITH LIKE EXPONENTS

example: $5 \times 10^3 + 2 \times 10^3 = 7 \times 10^3$

ADDING AND SUBTRACTING WITH UNLIKE EXPONENTS

example: $4.0 \times 10^4 + 5 \times 10^3 = 4.0 \times 10^4 + .5 \times 10^4 = 4.5 \times 10^4$

MULTIPLICATION OF NUMBERS EXPRESSED IN SCIENTIFIC NOTATION

1. ADD THE EXPONENTS TO OBTAIN THE EXPOENENT OF THE POWER OF TEN IN THE ANSWER

example: $2 \times (3 \times 10^4) = 6 \times 10^4$

DIVISION OF NUMBERS EXPRESSED IN SCIENTIFIC NOTATION

example: $\frac{7.5 \times 10^5}{3.0 \times 10^2} = \frac{7.5}{3.0} \times \frac{10^5}{10^2} = 2.5 \times 10^{5-2} = 2.5 \times 10^3$

Significant Figures

-when reading an instrument, read the instrument to it's *smallest division*.

-estimate within a part of the division, the figures you write down for the measurements are called *significant digits*.

-found by counting the doubtful digit and all the digits to the left of it up to and including the last digit that is not zero.

1. DIGITS OTHER THAN ZERO ARE ALWAYS SIGNIFICANT (1-9)
2. ANY FINAL ZERO OR ZEROS USED AFTER THE DECIMAL POINT ARE SIGNIFICANT (5.60)
3. ZEROS BETWEEN TWO OTHER SIGNIFICANT DIGITS ARE ALWAYS SIGNIFICANT (2002)
4. ZEROS USED SOLELY FOR SPACING ARE NOT SIGNIFICANT (.007)

****hint:** put it in scientific notation to make it easier

examples:

- | | |
|------------------|---|
| (a) 75 m → 2 | (d) $1.87 \times 10^6 \rightarrow 3$ |
| (b) 75.00 → 4 | (e) $1.008 \times 10^8 \rightarrow 4$ |
| (c) 0.007060 → 4 | (f) $1.20 \times 10^{-4} \rightarrow 3$ |

ACCURACY: THE DEGREE OF AGREEMENT BETWEEN A MEASURED VALUE AND THE TRU VALUE (ex: the accepted value of π to six digits is 3.14159)

-suppose you calculate the value of π to six digits as 3.14176 (only the first four digits agree with the accepted value). So, your value is accurate to only four places.

PRECISION: THE DEGREE OF EXACTNESS WITH WHICH A QUANTITY IS MEASURED (precision need not be an indication of accuracy)

-your value of π is just as precise as the accepted value because both value contain six digits!

SCIENTIFIC METHOD:

1. IDENTIFY THE PROBLEM TO BE SOLVED (in order to identify and understand a problem, it must be limited to a single, clearly defined problem).
2. GATHER EVIDENCE (in order to determine whether or not an idea has merit, a scientist searches through all available reference materials to see what has been done in the past to solve that problem).
3. MAKE A HYPOTHESIS (a trial solution to a problem is a hypothesis. It is also called an “educated guess” because it is based on what is learned as the scientist gathers evidence about the problem. Several hypotheses may be tested and rejected before the problem is solved).
4. TEST THE HYPOTHESIS (scientists establish a set of procedures to test the hypothesis)
5. OBSERVE AND RECORD (ACCURATE AND UP TO DATE RECORDS SHOULD BE KEPT OF ALL OBSERVATIONS. Careful measurements and the accurate presentation of data prevent the omission of important details)

THEORY: AN EXPLANATION BACKED BY RESULTS FROM REPEATED TESTS, EXPERIEMENTS, OR OBSERVATION. (*examples: atomic theory, the theory of evolution, the theory of relativity, and the quantum theory*)
-most scientific theories are changed or replaced many times as a result of new data.

LAW: A SCIENTIFIC RULE OF NATURE THAT DESCRIBES THE BEHAVIOR OF SOMETHING IN NATURE, BUT DOESN'T EXPLAIN WHY IT WILL HAPPEN IN A GIVEN SITUATION (not why, but what).

(*examples: Newton's Law of Motion, Boyle's Law of gases, the law of gravity, the laws of thermodynamics, the law of conservation of mass and energy, etc.*)

-can sometimes be expressed in terms of a single mathematical equation

*The biggest difference between a theory and a law is that a theory is much more complex and dynamic. A law governs a single action, whereas a theory explains an entire group of related phenomena

GRAPHING

Purpose: The purpose of making graphs is to state in a visual manner statistics which could be confusing in written form.

-Graphs are easier to read than tables of numbers; however, information must be accurate, depicted clearly, and effectively illustrated.

The *error* of a measurement is the amount by which the measurement differs from a known, accepted value as determined by skilled observers using high-precision equipment.

It is a measure of the accuracy of the method of measurement.

On the AP test you may be asked to draw a graph, you need to determine what to graph and what is the dependent and independent variable.

Graphing Techniques

Frequently an experiment will involve finding out how changing one quantity affects the value of another. The quantity that is deliberately varied is called the *independent variable*. The quantity that changes due to the variation in the independent variable is called *the dependent variable*. More often than not the relationship between the independent and dependent variable is not obvious from simply looking at the written data. However, if one quantity is plotted against the other, the resulting graph gives evidence of what sort of relationship, if any, exists between the variables.

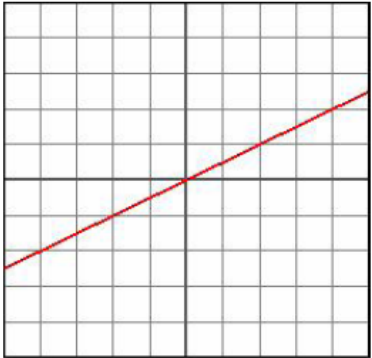
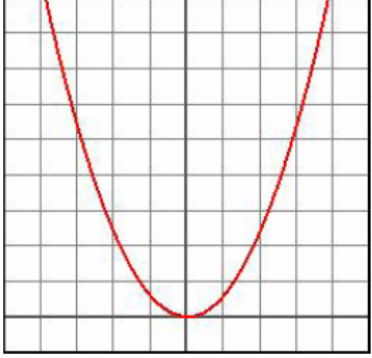
1. Identify the independent and dependent variables.
2. Choose your scale carefully. The scale for each axis can be different. Spread out the data to get a good shape.
3. Plot the independent variable on the horizontal (x) axis and the dependent variable on the (y) axis.
4. If the data points appear to lie roughly in a straight line, draw the best straight line you can with a straight edge. Never “connect the dots”. If the points do not form a straight line, draw the best smooth curve possible.

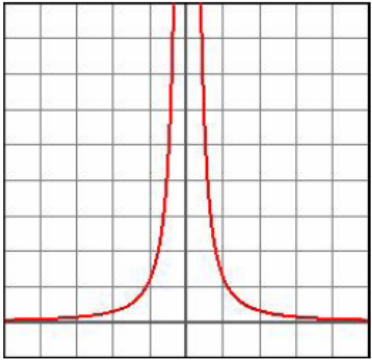
In an experiment you generally control one variable and measure the effect it has on another variable while you hold all other factors constant. For example, you might vary the force on a cart and measure its acceleration while you keep the mass of the cart constant. After the data is collected, you then make a graph of acceleration versus force using techniques for good graphing. The graph gives you a better understanding of the relationship between two variables. Sometimes you need information about a value that you have not determined experimentally.

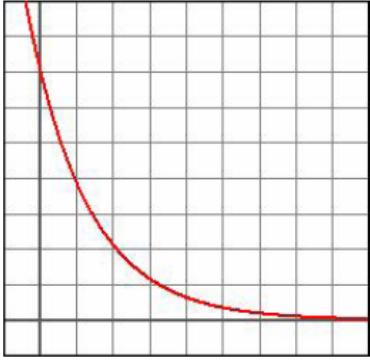
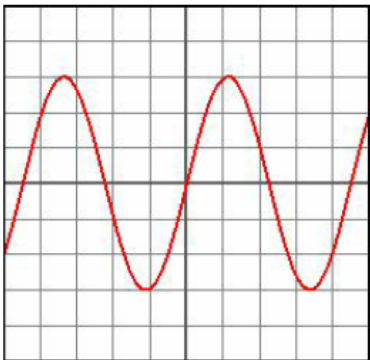
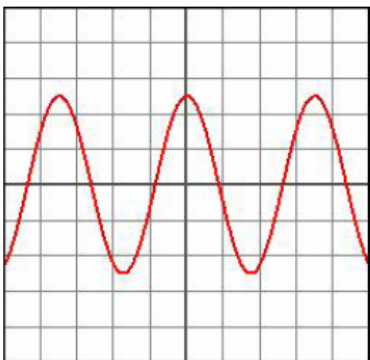
Functions for AP Physics

In the mathematical sciences there are some common types of equations and relationships. It is helpful for the student to be familiar with the form, terminology, and properties of the functions most frequently encountered. This document is meant to be a helpful overview and resource for physics students.

For the equations below: x , y , and t are variables, whereas all other symbols represent constants.

$y = mx$		y is directly proportional to x - passes through the origin m is the slope (or the “constant of proportionality”) $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m > 0$, line slants upward (like this one) $m < 0$, line slants downward
$y = mx + b$		x and y have a linear relationship m is the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m > 0$, line slants upward $m < 0$, line slants downward (like this one) b is the y-intercept
$y = ax^2$		y is proportional to x-squared - the curve is a parabola - vertex of parabola is on the origin a determines the amount of curvature $a > 0$, parabola opens upward (like this one) $a < 0$, parabola opens downward

$y = ax^2 + bx + c$		· x and y have a quadratic relationship · the curve is a parabola · a, b, and c are sometimes called coefficients · a determines the amount of curvature · $a > 0$, parabola opens upward · $a < 0$, parabola opens downward (like this one) · b and c determine where the vertex is located · c is the y-intercept of the parabola
$y = k\sqrt{x}$		· y is proportional to the square root of x · the curve is a parabola (half of one) · vertex of parabola is on the origin · k determines the amount of curvature
$y = \frac{k}{x}$		· y is inversely proportional to x · the curve is a hyperbola · x and y axes are asymptotes · k determines the amount of curvature
$y = \frac{k}{x^2}$		· x and y have an inverse square relationship · x and y axes are asymptotes · k determines the amount of curvature

$y = ae^{-bt}$		· y is an exponential function · a is the y-intercept · b determines the amount of curvature · the x-axis is an asymptote
$y = a(1 - e^{-bt})$		· y is an inverse exponential function · passes through the origin · the line $y = a$ is an asymptote
$y = a \sin(\omega t + \phi)$		· y is a sinusoidal function · a is the amplitude · ω is the angular frequency · ϕ is the phase angle
$y = a \cos(\omega t + \phi)$		· y is a sinusoidal function · a is the amplitude · ω is the angular frequency · ϕ is the phase angle · note: depending on the values of the phase angle these last two curves are indistinguishable and interchangeable – for both of the example graphs the phase angle is zero

Percent error: (usually more important than actual error) is found by dividing the difference between the measured value and the accepted value of a quantity by the accepted value, and then multiplying this quotient by 100%.

$$\% \text{ error} = \left| \frac{\text{accepted value} - \text{measured value}}{\text{accepted value}} \right| \times 100\%$$

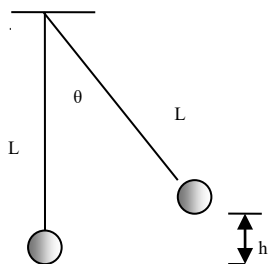
Example: Suppose the measured value of the acceleration of gravity is found to be 9.44 m/s^2 . The accepted value is 9.80 m/s^2 . The difference between these two values is $(9.8 \text{ m/s}^2) - (9.44 \text{ m/s}^2)$, or 0.36 m/s^2 .

$$\% \text{ error} = \frac{0.36 \text{ m/s}^2}{9.80 \text{ m/s}^2} \times 100\% = 3.7\%$$

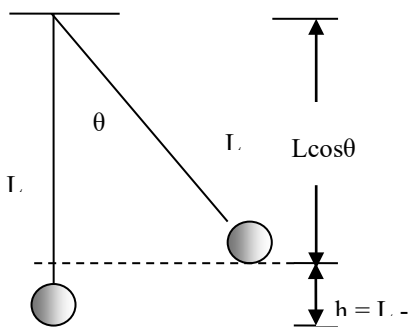
Trigonometry

Trigonometry is the study of triangles, and often right triangles. The lengths of the sides of a right triangle can be used to define some useful relationships, called the *sine*, *cosine*, and *tangent*, abbreviated *sin*, *cos*, and *tan*, respectively.

Example 3 A ball on the end of a string of length $L = 0.50 \text{ m}$ is hung from a hook in the ceiling. The ball is pulled back to an angle $\theta = 30^\circ$ from the vertical. What is the height h above the lowest point of the ball?



If we draw a horizontal line from the raised ball to form a right triangle, the portion of the vertical line above the horizontal line forms the adjacent side of the right triangle. This length would then be $L \cos \theta$.



Thus the height h would be the difference between L and $L \cos \theta$:

$$h = L - L \cos \theta = L(1 - \cos \theta)$$

For any triangle:

Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

-used to find the length of a side of any triangle (acute, obtuse, or right)

Law of sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

-used to find the angle or side of a triangle and with this equation you only use two parts at a time.