

IB HL Honors Precalculus
Prerequisite Review

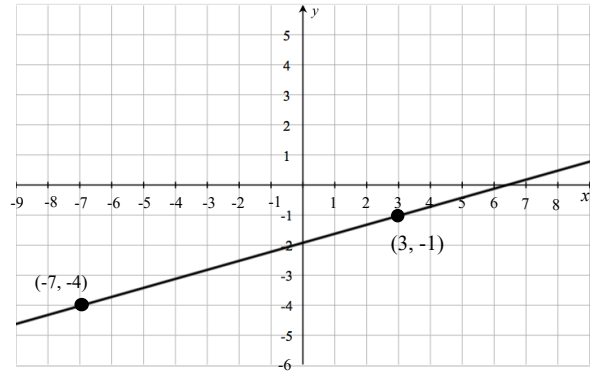
Name: _____

For each problem you must show all work or explain your thinking to receive full credit (guess and check is not a viable option). Please check solutions on the last page of this document. This review assignment will be collected on Canvas on the first full day of school as extra credit towards the Foundational Assignments category in the first Quarter.

A. Linear:

1. Write the equation of the line that passes through the points $(2, 3)$ and $(-1, 8)$. Write your final answer in point-slope form.

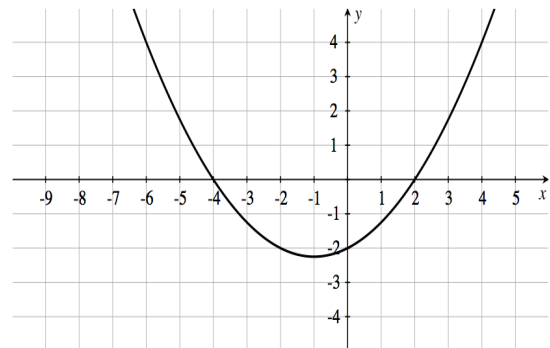
2. Write the equation of the linear function below. Write your final answer in slope-intercept form.



B. Quadratics:

1. Solve each equation below by the given method. Simplify all roots, rationalize all denominators, and write all non-real solutions in standard $a + bi$ form.
 - a. Factoring: $6x^2 + 11x = 10$
 - b. Roots: $-3(x + 1)^2 - 4 = 5$
 - c. Quadratic Formula: $x^2 + 2x + 5 = 0$
 - d. Complete the Square: $2x^2 - 4x - 7 = 0$

2. Write the equation of the quadratic function shown below. Write your final answer in standard form.

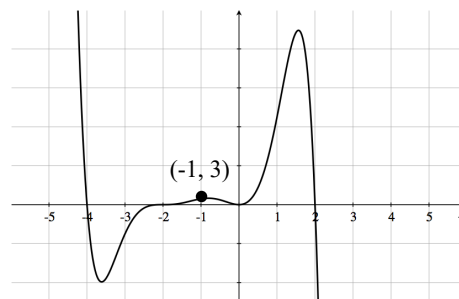


C. *Exponentials and Logarithms:*

1. Given that $\log_b 2 = m$, $\log_b 3 = n$ and $\log_b 5 = p$, then write each of the following in terms of m , n , p and/or maybe a constant.
 - a. $\log_b(60)$.
 - b. $\log_b(0.27)$.
 - c. $\log_3(b)$.
 - d. $\log(0.4)$.
2. Solve each equation below. Answers should be exact where possible; otherwise round to 0.001.
 - a. $16^{2x} = 8^{x+2}$
 - b. $\log_7(3x - 2) = 2$
 - c. $11^{2x-1} = 200$
 - d. $\log_6 x + \log_6(5 - x) = 1$
3. Convert $y = 15 \cdot 2^x$ to the *continuous model* $y = Pe^{rx}$. Round r to 0.0001.
4. Convert $y = 32 \cdot e^{0.15x}$ to the *standard model* $y = ab^x$. Round b to 0.0001.
5. The population y of Funkytown x years after the year 2000 is modeled by the equation $y = 12053 \cdot 1.0345^x$.
 - a. What was the population of Funkytown in the year 2000?
 - b. Is the population of Funkytown increasing or decreasing? By what percent?
 - c. Write an equation for the population of Funkytown as a *continuous model* $y = Pe^{rx}$. Round r to 0.0001.
 - d. Based on your equation from (c), by what continuous percent is the population of Funkytown changing?

D. *Polynomials*:

1. Is $(x + 1)$ a factor of $x^{900} - 3x^{450} + 2x^{225} + 4$? Justify.
2. The polynomial function $r(x)$ at right is of least possible degree and has zeroes at -4, -2, 0, and 2, and also passes through the point $(-1, 3)$. Find the equation for $r(x)$ in factored form. (*Hint*: you will need to find the lead coefficient.)
3. Determine the quotient and remainder when . . .
 - a. $4x^3 - 2x^2 + x - 3$ is divided by $2x^2 - x + 3$ using Long Division.
 - b. $3x^3 - 4x^2 - 2x - 1$ is divided by $3x - 1$ using Synthetic Division.
You may (want to) check your answer using Long Division.



E. *Systems:*

1. Solve the system for x and y .

$$\mathbf{Ax} + \mathbf{By} = \mathbf{C}$$

$$\mathbf{Dx} + \mathbf{Ey} = \mathbf{F}$$

Note: your values for x and y should be in terms of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, $\mathbf{E}$ and/or $\mathbf{F}$.

2. Solve the system for x , y , and z .

$$2x + 3y - z = 4$$

$$3x - y + 2z = 5$$

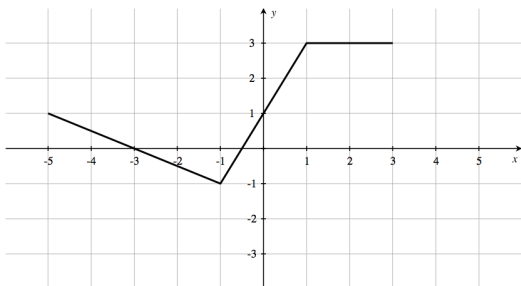
$$x - 4y + 3z = 1$$

Write your final answer as an ordered triple (x, y, z) .

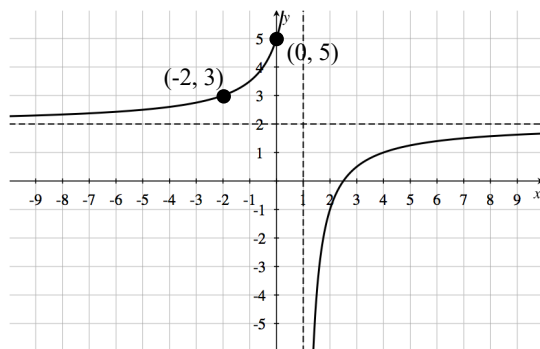
F. *Transformations:*

1. Below is the graph of $y = f(x)$, which is the image of the pre-image function $p(x)$. The function f is defined as: $\frac{f(x) - 3}{2} = p\left(\frac{x - 1}{4}\right)$.

- Write the mapping notation for this transformation.
- Define the points on p from which the given points, $(-5, 1)$, $(-1, -1)$, $(1, 3)$, and $(3, 3)$, on f come.



2. Determine the equation of the graph shown below. Show how you determined your answer.



G. *Sequences and Series:*

1. An amphitheater is set up so that there are 10 seats in the first row, 13 seats in the second row, 16 in the third row, 19 in the fourth row, etc., until there are 85 seats in the last row of the amphitheater.
 - a. How many rows are in the amphitheater?
 - b. How many total seats are in the amphitheater?
2. A sequence $\{g_n\}$ is defined recursively as: $g_1 = 72$

$$g_n = -\frac{2}{3} \cdot g_{n-1} \quad \text{for } n \geq 2$$

- a. Determine the values of g_2 , g_3 , and g_4 .
- b. Write an explicit formula for g_n .
- c. Determine an **exact** value for $S_{10} = \sum_{k=1}^{10} g_k$.

H. *Trigonometry:*

1. Let θ be an acute angle in a right triangle and let $\sin\theta = \frac{1}{5}$. Find the **exact** value of $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$ in *rationalized and simplest form*, then determine the measure of the two acute angles in the triangle (rounded to 0.01°). Draw a picture of the triangle you are using.
2. *Without a calculator* find the exact values for each of the following. Show or explain how you determined each answer.

a. $\sin(315^\circ)$

b. $\cos\left(\frac{5\pi}{6}\right)$

c. $\tan(180^\circ)$

d. $\cot\left(\frac{17\pi}{6}\right)$

e. $\sec\left(-\frac{2\pi}{3}\right)$

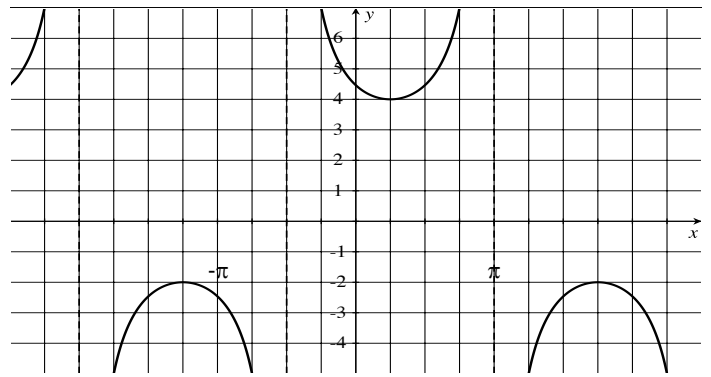
f. $\csc(120^\circ)$

H. *Trigonometry* Continued:

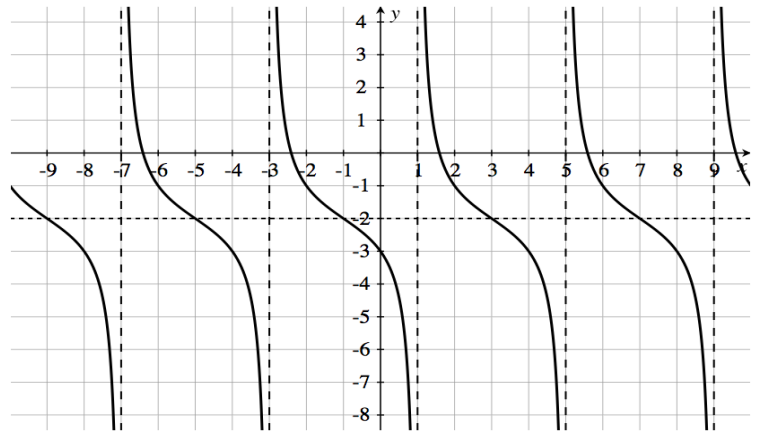
3. Given that α is an angle in standard position with $\sec\alpha = 3$ for $\frac{3\pi}{2} \leq \alpha \leq 2\pi$, then find the **exact** value of each of the following in simplest form. Show or explain how you determined your answer.
- a. $\cos(-\alpha)$ b. $\tan(\pi + \alpha)$ c. $\sin(\pi - \alpha)$ d. $\csc\left(\frac{\pi}{2} - \alpha\right)$
- e. Approximate α to the nearest 0.001.
4. The terminal side of an angle, β , in *standard position* passes through the point $(-15, -8)$ where $0^\circ \leq \beta \leq 360^\circ$.
- a. What is the *reference angle* for β (rounded to 0.1°)?
- b. What is the measure of β (rounded to 0.1°)?
- c. Determine three angles that are *co-terminal* with β – at least one angle must be negative.
- d. Find the **exact** value of $\sin\beta$, $\cos\beta$, $\tan\beta$, $\csc\beta$, $\sec\beta$, $\cot\beta$.
5. Given $\triangle TRI$ with $t = 10$, $r = 12$, and $i = 15$. Find $m\angle T$ (rounded to 0.1°).
6. Given $\triangle ABC$ with $m\angle A = 36^\circ$, $m\angle B = 65^\circ$, and $b = 17$ cm.
- a. Find a (rounded to 0.01). b. Find the area of $\triangle ABC$ (rounded to 0.001).

H. *Trigonometry Continued Again:*

7. For the function graphed at right, determine the vertical scale change, vertical translation, period, asymptotes, horizontal scale change, horizontal translation to write as a secant, and horizontal translation to write as a cosecant, then write the equation for the function in terms of secant and in terms of cosecant.



8. For the function graphed at right, determine the vertical scale change, vertical translation, period, asymptotes, horizontal scale change, horizontal translation to write as a tangent, and horizontal translation to write as a cotangent, then write the equation for the function in terms of tangent and in terms of cotangent.



9. Find **exact** solutions on the interval $[0, 2\pi)$, then write the general solution to each equation below.

a. $\sin x = \frac{\sqrt{3}}{2}$

b. $\tan x = -1$

c. $\sec x = -2$

10. As Otto Mobeel stops at a stop sign a pebble becomes lodged between the treads on his front driver's side tire, which has a diameter of 24 inches. As he drives off, the distance the pebble is from the pavement varies *sinusoidally* with the horizontal distance traveled along the road.
- What is the period of this sinusoidal graph? Explain.
 - Sketch a graph of this sinusoidal function for the height, y , of the pebble from the pavement as a function of the horizontal distance, x , traveled. Include appropriate units.
 - Determine the vertical translation, amplitude, vertical scale change, period, horizontal scale change, horizontal translation to write as a cosine, and horizontal translation to write as a sine.
 - Write an equation that models this situation in terms of cosine and in terms of sine.
 - How far above the level of the pavement will the pebble be when it has traveled a horizontal distance of 10 ft?
 - What are the first four values of x (rounded to 0.01) at which the pebble will be 15 inches above the pavement? Show or explain how you determined your answer.

Answers

- A. 1. $y - 3 = -\frac{5}{3}(x - 2)$ or $y - 8 = -\frac{5}{3}(x + 1)$ 2. $y = \frac{3}{10}x - \frac{19}{10}$
- B. 1. a. $x = \left\{-\frac{5}{2}, \frac{2}{3}\right\}$ 2. $y = \frac{1}{4}x^2 + \frac{1}{2}x - 2$
 b. $x = \{-1 - i\sqrt{3}, -1 + i\sqrt{3}\}$
 c. $x = \{-1 - 2i, -1 + 2i\}$
 d. $x = \left\{1 - \frac{3\sqrt{2}}{2}, 1 + \frac{3\sqrt{2}}{2}\right\}$
- C. 1. a. $2m + n + p$ b. $3n - (2m + 2p)$ 2. a. $x = \frac{6}{5}$ b. $x = 17$
 c. $\frac{1}{n}$ d. $\frac{m - p}{m + p}$ c. $x = \frac{1 + \log_{11} 200}{2}$ d. $x = 2, 3$
 2. $y = 15 \cdot e^{x \ln 2} \approx 15 \cdot e^{0.6931x}$ 3. $y = 32 \cdot (1.0151)^x$
 4. a. 12,053 b. increasing by 3.45% per year
 c. $y = 12053 \cdot e^{0.0339x}$ d. increasing by 3.39% continuously
- D. 1. $x + 1 = 0$ when $x = -1$, and since
 $(-1)^{900} - 3(-1)^{450} + 2(-1)^{225} + 4 = 0$,
 $(x + 1)$ is a factor of $x^{900} - 3x^{450} + 2x^{225} + 4$ 2. $r(x) = -\frac{1}{3}x^2(x + 2)^3(x + 4)(x - 2)$
 3. a. $q(x) = 2x$ and $r(x) = -5x - 3$
 b. $q(x) = x^2 - x - 1$ and $r(x) = -2$
- E. 1. $x = \frac{CE - BF}{AE - BD}$ and $y = \frac{AF - CD}{AE - BD}$ 2. $\left(-\frac{5}{11}z + \frac{19}{11}, \frac{7}{11}z + \frac{2}{11}, z\right)$ or
 $\left(-\frac{5}{7}y + \frac{13}{7}, y, \frac{11}{7}y - \frac{2}{7}\right)$ or
 $\left(x, -\frac{7}{5}x + \frac{13}{5}, -\frac{11}{5}x + \frac{19}{5}\right)$
- F. 1. a. $T(x, y) = (4x + 1, 2y + 3)$ 2. $y = -\frac{3}{x - 1} + 2$
 b. $(-1.5, -1) \rightarrow (-5, 1)$ $(-0.5, -2) \rightarrow (-1, -1)$
 $(0, 0) \rightarrow (1, 3)$ $(0.5, 0) \rightarrow (3, 3)$
- G. 1. a. 26 rows 2. a. $g_2 = -48, g_3 = 32, g_4 = -\frac{64}{3}$
 b. $S_{26} = 1,235$ b. $g_n = 72 \cdot \left(-\frac{2}{3}\right)^{n-1}$
 c. $S_{10} = \frac{92,840}{2187}$

H. 1. $\cos\theta = \frac{2\sqrt{6}}{5}$ $\tan\theta = \frac{\sqrt{6}}{12}$
 $\sec\theta = \frac{5\sqrt{6}}{12}$ $\cot\theta = 2\sqrt{6}$ $\csc\theta = 5$
 $\theta = 11.54^\circ$ $90^\circ - \theta = 78.46^\circ$

3. a. $\frac{1}{3}$ b. $-2\sqrt{2}$ c. $-\frac{2\sqrt{2}}{3}$
d. 3 e. 5.052

5. $m\angle T = 41.65^\circ$

7. vsc: ± 3 vt: 1
per: 3π hsc: $\frac{3}{2}$
ht (sec): $\frac{\pi}{4} + \frac{3\pi}{2}k$ ht (csc): $-\frac{\pi}{2} + \frac{3\pi}{2}k$

asymptotes: $x = -\frac{\pi}{2} + \frac{3\pi}{2}k$ where $k \in \mathbb{Z}$

secant: $y = 1 + 3 \cdot \sec\left(\frac{2}{3}\left(x - \frac{\pi}{4}\right)\right)$

cosecant: $y = 1 + 3 \cdot \csc\left(\frac{2}{3}\left(x + \frac{\pi}{2}\right)\right)$

9. a. $x = \left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$

GS: $x = \begin{cases} \frac{\pi}{3} + 2\pi k \\ \frac{2\pi}{3} + 2\pi k \end{cases}$ where $k \in \mathbb{Z}$

b. $x = \left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

GS: $x = \frac{3\pi}{4} + \pi k$ where $k \in \mathbb{Z}$

c. $x = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

GS: $x = \begin{cases} \frac{2\pi}{3} + 2\pi k \\ \frac{4\pi}{3} + 2\pi k \end{cases}$ where $k \in \mathbb{Z}$

2. a. $-\frac{\sqrt{2}}{2}$ b. $-\frac{\sqrt{3}}{2}$ c. 0
d. $-\sqrt{3}$ e. -2 f. $\frac{2\sqrt{3}}{3}$

4. a. 28.1° b. 208.1°
c. $-511.9^\circ, -151.9^\circ, 208.1^\circ, 568.1^\circ, 928.1^\circ$
d. $\sin\beta = -\frac{8}{17}$ $\cos\beta = -\frac{15}{17}$ $\tan\beta = \frac{8}{15}$
 $\csc\theta = -\frac{17}{8}$ $\sec\theta = -\frac{17}{15}$ $\cot\theta = \frac{15}{8}$

6. a. $a = 11.03$ cm b. Area = 92.03 cm²

8. vsc: ± 1 vt: -2
per: 4 hsc: $\frac{4}{\pi}$

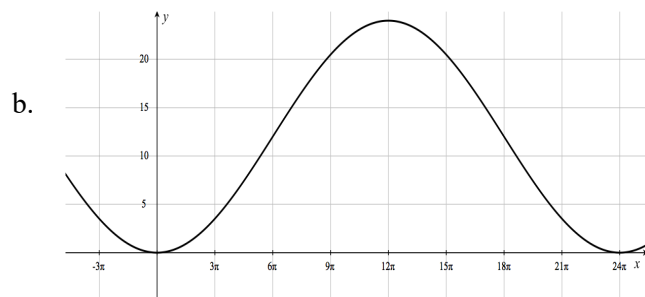
ht (tan): $-1 + 4k$ ht (cot): $1 + 4k$

asymptotes: $x = 1 + 4k$ where $k \in \mathbb{Z}$

tangent: $y = -2 - \tan\left(\frac{\pi}{4}(x+1)\right)$

cotangent: $y = -2 + \cot\left(\frac{\pi}{4}(x-1)\right)$

10. a. period = circumference of tire = 24π inches



c. vt: 12 amp: 12 vsc: ± 12

per: 24π hsc: 12

ht (cos): $12\pi + 12\pi k$ ht (sin): $6\pi + 12\pi k$

Cosine: $y = 12 + 12 \cos\left(\frac{1}{12}(x - 12\pi)\right)$

Sine: $y = 12 + 12 \sin\left(\frac{1}{12}(x - 6\pi)\right)$

d. 22.07 in

e. {21.88 in, 53.52 in, 97.28 in, 128.92 in}