Operations with Integers

Adding Integers

- Negative + Negative: Add the absolute values of the two numbers and make the answer negative.

  ex: $-5 + (-9) \rightarrow 5 + 9 = 14 \rightarrow$ answer: (-14)

- Negative + Positive (or Positive + Negative): Subtract the absolute values of the two numbers (larger minus smaller) and take the sign of the number with the greater absolute value.

  ex: $-7 + 12 \rightarrow 12 - 7 = 5 \rightarrow 12 > 7$, so answer is positive $\rightarrow$ answer: (5)

  ex: $6 + (-9) \rightarrow 9 - 6 = 3 \rightarrow 9 > 6$, so answer is negative $\rightarrow$ answer: (-3)

Subtracting Integers

- Keep the first number the same, change the subtraction sign to an addition sign, and change the sign of the second number. Then use the integer addition rules.

  ex: $-3 - 9 \rightarrow -3 + (-9) = (-12)$

  ex: $15 - (-8) \rightarrow 15 + 8 = (23)$

  ex: $-6 - (-4) \rightarrow -6 + 4 = (-2)$

Multiplying & Dividing Integers

Ignore the signs and multiply or divide as usual. Then determine the sign of the answer using the following rules:

- Negative · or ÷ Negative = Positive
- Negative · or ÷ Positive (or Positive · or ÷ Negative) = Negative

  ex: $-3 \cdot (-5) \rightarrow 3 \cdot 5 = 15 \rightarrow$ neg · neg = pos $\rightarrow$ answer: (15)

  ex: $48 \div (-6) \rightarrow 48 \div 6 = 8 \rightarrow$ pos ÷ neg = neg $\rightarrow$ answer: (-8)

Order of Operations

Parentheses
Exponents
Multiplication & Division (left to right)
Addition & Subtraction (left to right)

Find the sum or difference.

1. $-80 + 77$
2. $77 + 160$
3. $-64 + (-33)$
4. $104 - (-92)$
5. $-105 - (-122)$
6. $185 - (-154)$
7. $-53 - (-59)$
8. $-6 + (-35)$
9. $15 - (-26) - (-39)$
10. $-93 + 191 + (-179)$
11. $18 + (-34) + 52$
12. $-50 - (-93) + (-17)$

Find the product or quotient.

13. $60 \div 12$
14. $-194 \div (-2)$
15. $88 \cdot (-2)$
16. $-12 \cdot 10$
17. $-10 \cdot (-11)$
18. $90 \div (-6)$
19. $3 \cdot (-59)$
20. $-7 \cdot (-2)$
21. $-28 \div (-88) \cdot (-22)$
22. $-56 \cdot 140 \div (-80)$
23. $108 \div (-11) \cdot (-11)$
24. $-84 \cdot (-17) \div 42$

Evaluate the numerical expression. (Be sure to use the order of operations!)

25. $-78 + (-2) \cdot (-56)$
26. $-65 + 6 \div (-3) + 40$
27. $-94 - (84 - 10)$
28. $43 + (-23) - (-57)$
29. $-15 - (-11) + 5 \cdot (-4)$
30. $-26 - (-64) + (-93)$
31. $-84 \div 4 + (-20)$
32. $-56 + (-50) + (-10) \cdot (-9)$

Operations with Rational Numbers

Adding & Subtracting Rational Numbers

Determine whether you should add or subtract using integer rules. Then add or subtract.

- Decimals: Line up the decimal points. Then add or subtract and bring the decimal point down. Use integer rules to determine the sign of the answer.

ex: $-9.8 + 6.24 \longrightarrow$ neg + pos: subtract $\longrightarrow$ $\begin{array}{r} 9.80 \\ -\ 6.24 \\ \hline 3.56 \end{array} \longrightarrow$ answer: -3.56

- Fractions/Mixed Numbers: Find a common denominator and then add or subtract. Borrow or convert an improper fraction answer, if necessary. Use integer rules to determine the sign of the answer.

ex: $5\frac{3}{4} - \left(-3\frac{7}{8}\right) \longrightarrow 5\frac{3}{4}\ \ 3\frac{7}{8} \longrightarrow$ pos + pos: add $\longrightarrow$ $\begin{array}{r} 5\frac{3}{4} = \frac{6}{8} \\ +\ 3\frac{7}{8} = \frac{7}{8} \\ \hline 8\ \ \frac{13}{8} \end{array} \longrightarrow$ answer: $9\frac{5}{8}$

Multiplying & Dividing Rational Numbers

Determine the sign of the answer using integer rules. Then multiply or divide.

- Multiplying Decimals: Ignore the decimal points. Multiply the numbers. Then count the decimal places in the problem to determine the location of the decimal point in the answer.

ex: $-9.23 \cdot (-1.1) \longrightarrow$ neg · neg = pos $\longrightarrow$ $\begin{array}{r} 9.23 \\ \times\ 1.1 \\ \hline 923 \\ 9230 \\ \hline 10153 \end{array} \longrightarrow$ answer: 10.153

- Dividing Decimals: Move the decimal in the divisor to the end of the number. Move the decimal in the dividend the same number of places and then bring it straight up in quotient.

ex: $-5.2 \div 0.2 \longrightarrow$ neg ÷ pos = neg $\longrightarrow$ $02.\overline{)\,52.}\ \ 26. \longrightarrow$ answer: -26

- Multiplying Fractions: Convert mixed numbers to improper fractions. Then cross-simplify. Multiply the numerators and multiply the denominators. Simplify if necessary.

ex: $-1\frac{3}{4} \cdot \frac{6}{14} \longrightarrow$ neg · pos = neg $\longrightarrow$ $\overset{1}{\cancel{7}}{}^{\backslash}_{2}\ \cdot\ \frac{\cancel{6}^{3}}{\cancel{14}_{2}} = \frac{3}{4} \longrightarrow$ answer: $-\frac{3}{4}$

- Dividing Fractions: Convert mixed numbers to improper fractions. Then flip the second fraction to its reciprocal and multiply the two fractions. Simplify if necessary.

ex: $-\frac{1}{2} \div \left(-\frac{3}{8}\right) \longrightarrow$ neg ÷ neg = pos $\longrightarrow$ $\frac{1}{\cancel{2}_{1}} \cdot \frac{\cancel{8}^{4}}{3} = \frac{4}{3} \longrightarrow$ answer: $1\frac{1}{3}$

Find the sum, difference, product, or quotient.

33. $38.61 + 36.841$

34. $1.755 - 1.23$

35. $0.71 \cdot 9.2$

36. $13.12 \div 0.1$

37. $3.651 - (-12.63)$

38. $-3.9 + (-7.6)$

39. $17.6 \cdot 4.3$

40. $6 \cdot (-16.7)$

41. $26.474 - 14.527$

42. $-2.1 + 3.78$

43. $-6.15 \div (-8.2)$

44. $-12.8 \cdot (-4.88)$

Find the sum, difference, product, or quotient.

45. $15\ 1/2 + 15\ 1/4$

46. $18\ 11/20 - 17\ 1/2$

47. $2\ 1/4 \cdot 1\ 4/5$

48. $3\ 1/2 \div 1\ 3/7$

49. $3\ 1/3 - 5\ 1/9$

50. $5 \cdot (-1\ 2/5)$

51. $-4\ 2/3 + (-1\ 3/4)$

52. $-\ 5/6 \div (-2\ 1/6)$

53. $9 \div (-4\ 1/2)$

54. $-18 + 3\ 4/5$

55. $-5\ 2/3 \cdot (-2\ 5/6)$

56. $-5\ 3/4 - (-3\ 7/8)$

Solving Equations

Solving One-Step Equations

- Cancel out the number on the same side of the equation as the variable by using the inverse operation. (Addition/Subtraction; Multiplication/Division). Be sure to do the same thing to both sides of the equation!

ex: $6x = -18 \rightarrow \frac{6x}{6} = \frac{-18}{6} \rightarrow$ answer: $x = -3$

ex: $y + 23 = -9 \rightarrow y + 23 - 23 = -9 - 23 \rightarrow$ answer: $y = -32$

ex: $\frac{h}{3} = 4 \rightarrow 3 \cdot \frac{h}{3} = 4 \cdot 3 \rightarrow$ answer: $h = 12$

ex: $w - 13 = -5 \rightarrow w - 13 + 13 = -5 + 13 \rightarrow$ answer: $w = 8$

Solving Two-Step Equations

- Undo operations using inverse operations one at a time using the order of operations in reverse. (i.e.: undo addition/subtraction before undoing multiplication/division)

ex: $7x - 4 = -32 \rightarrow 7x - 4 + 4 = -32 + 4 \rightarrow \frac{7x}{7} = \frac{-28}{7} \rightarrow$ answer: $x = -4$

ex: $\frac{j}{5} + 13 = 15 \rightarrow \frac{j}{5} + 13 - 13 = 15 - 13 \rightarrow 5 \cdot \frac{j}{5} = 2 \cdot 5 \rightarrow$ answer: $j = 10$

ex: $\frac{b + 7}{3} = -2 \rightarrow 3 \cdot \frac{b + 7}{3} = -2 \cdot 3 \rightarrow b + 7 - 7 = -6 - 7 \rightarrow$ answer: $b = -13$

Solve the one-step equation.

57. $19 + j = -34$

58. $m - 26 = 13$

59. $\frac{x}{5} = -3$

60. $12f = 216$

61. $g - (-31) = -7$

62. $\frac{h}{9} = 13$

63. $b + (-3) = -9$

64. $-4w = -280$

Solve the two-step equation.

65. $5m - 3 = 27$

66. $7 + \frac{y}{2} = -3$

67. $4 + 3r = -8$

68. $\frac{1}{2}p - 4 = 7$

69. $\frac{k+8}{3} = -2$

70. $\frac{f}{5} - (-13) = 12$

71. $-15 - \frac{g}{3} = -5$

72. $-8 + 4m = 2$

73. $-18 - \frac{3}{4}v = 3$

74. $\frac{-5+n}{4} = -1$

75. $3.5m + 0.75 = -6.25$

76. $2y + 3 = 19$

Proportions and Percent

Solving Proportions

- Set cross-products equal to each other and then solve the one-step equation for the given variable.

ex: $\frac{5}{b}=\frac{4}{10}$ → $5 \cdot 10 = 4b$ → $\frac{50}{4}=\frac{4b}{4}$ → answer: $b = 12.5$

Solving Percent Problems with Proportions

- Set up and solve a proportion as follows: $\frac{\%}{100}=\frac{\text{part}}{\text{whole}}$

ex: 25 is what percent of 500? → $\frac{x}{100}=\frac{25}{500}$ → answer: $x = 5\%$

ex: What is 15% of 88? → $\frac{15}{100}=\frac{x}{88}$ → answer: $x = 13.2$

ex: 18 is 30% of what number? → $\frac{30}{100}=\frac{18}{x}$ → answer: $x = 60$

Solving Percent Problems with Equations

- Translate the question to an equation and then solve. (Be sure to convert percents to decimals or fractions.)

ex: 20 is 40% of what number? → $20 = 0.4x$ → answer: $x = 50$

ex: 8 is what percent of 32? → $8 = 32x$ → $x = 0.25$ → answer: 25%

ex: What is 25% of 88? → $x = 0.25 \cdot 88$ → answer: $x = 22$

Real-World Percent Problems

(This is just one way of many to solve real-world percent problems)

- Tax: Find the amount of tax using a proportion or equation. Then add the tax to the original amount to find the total cost.

- Discount: Find the amount of the discount using a proportion or equation. Then subtract the amount of discount from the original price to find the sale price.

Solve the proportion.

77. $\frac{h}{6} = \frac{20}{24}$

78. $\frac{5}{7} = \frac{c}{14}$

79. $\frac{6}{8} = \frac{21}{b}$

80. $\frac{30}{j} = \frac{26}{39}$

81. $\frac{5}{k} = \frac{15}{20}$

82. $\frac{32}{112} = \frac{a}{14}$

83. $\frac{16}{7} = \frac{18}{g}$

84. $\frac{w}{60} = \frac{15}{200}$

Solve the percent problem.

85. Find 15% of 85.

86. 6 is 75% of what number?

87. 40 is what percent of 320?

88. What is 20% of 45?

89. 70 is what percent of 350?

90. Find $33.\overline{3}$% of 81.

91. A $58 camera is on sale for 20% off. Find the sale price.

92. Find the total price of a $14.00 shirt including the 7% sales tax.

Geometry

Geometry Basics

- Perimeter is the distance around a polygon
- Circumference is the distance around a circle
- Area is the space inside a figure
- Volume is the capacity of a 3-dimensional figure
- Surface Area is the sum of the areas of all the faces on a 3-dimensional figure

2-Dimensional Geometry Formulas

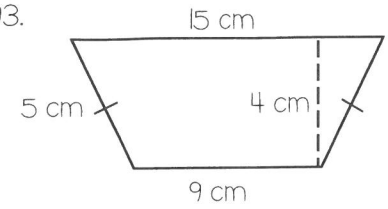
- Perimeter of Any Figure: sum of side lengths
- Circumference = $\pi \cdot$ diameter
- Area of Parallelogram = base $\cdot$ height
- Area of Triangle = $\frac{1}{2} \cdot$ base $\cdot$ height
- Area of Trapezoid = $\frac{1}{2} \cdot$ height$(\text{base}_1 + \text{base}_2)$
- Area of Circle = $\pi \cdot \text{radius}^2$

3-Dimensional Geometry Formulas

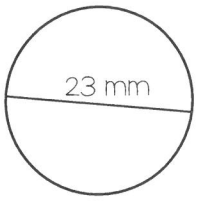
- Volume of Rectangular Prism = length $\cdot$ width $\cdot$ height
- Volume of Cylinder = $\pi \cdot \text{radius}^2 \cdot$ height
- Surface Area of Rectangular Prism = $2 \cdot$ length $\cdot$ width $+ 2 \cdot$ length $\cdot$ height $+ 2 \cdot$ height $\cdot$ width
- Surface Area of Cylinder = $2 \cdot \pi \cdot \text{radius}^2 + 2 \cdot \pi \cdot$ radius $\cdot$ height

Find the perimeter (or circumference) and area. Use 3.14 for pi.

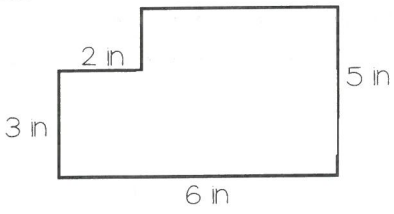
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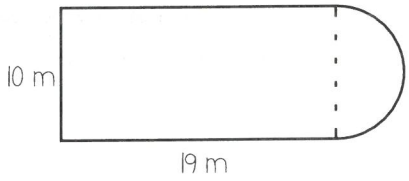
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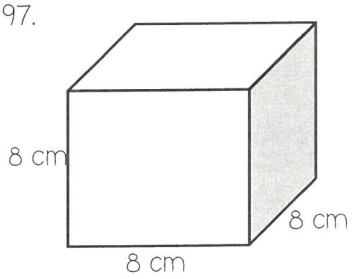


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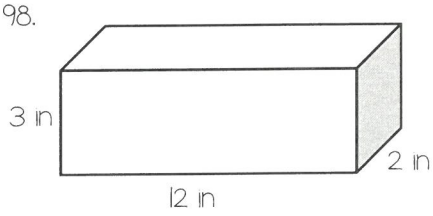


Find the surface area and volume.

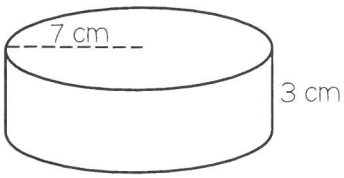
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