AP Calc: Summary for Chapter P (Pre-calculus Review) SUMMER REVIEW PACKET

Key Concepts:

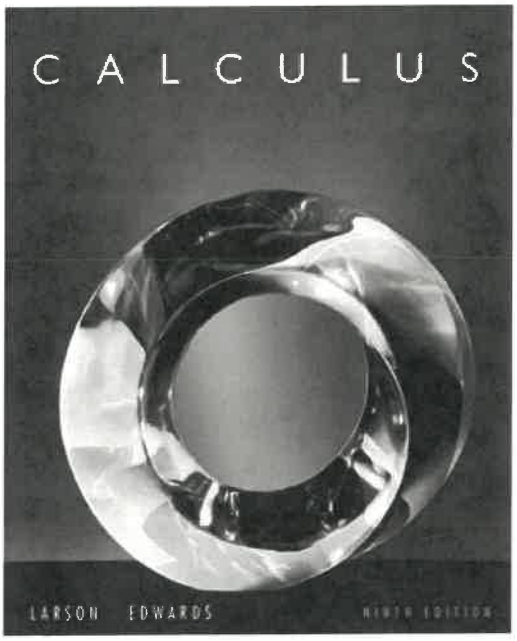
- Identify intercepts, symmetric properties, intersections, graphs and other characteristics of equations
- Review linear models and slope
- Evaluate and graph functions and their transformations and identify their domain and ranges
- Identify and manipulate quadratic, cubic, square root, absolute value, rational, trigonometric functions

Assignments:

- Packet Due: First week back from break.
- P.1 Graphs and Models pg 8 (19 – 39odd, 47 – 61odd, 65, 71 – 73tech only, 82justify)
- P.2 Linear Models and Rates of Change pg16 (19, 21, 23 – 45odd, 53 – 57odd, 60justify, 61, 67, 71, 73)
- P.3 Functions and their Graphs pg 27 (5, 9–15odd,19,21,23, 27, 30, 35, 37, 55, 58–65, 69–76, 75, 85,88)
- Trig/Function Review Worksheet. - Underlined problems
- 1.1 A Preview of Calculus (ALL)

Extra Practice:

- Algebra Review problems -



Odd answers to the problems can be found on Calc chat for Larson Calculus 9e
http://www.calcchat.com/book/Calculus-9e/

Additional intro calc topics from Pre/Calc: (answer keys for these are at the end of the packet)

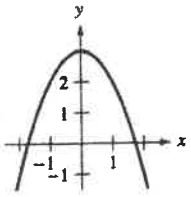
- Limits
- Derivatives

P.1 Exercises

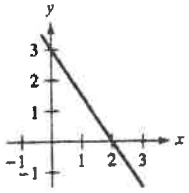
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–4, match the equation with its graph. [The graphs are labeled (a), (b), (c), and (d).]

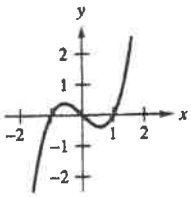
(a)



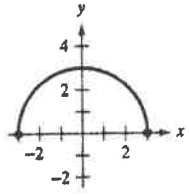
(b)



(c)



(d)



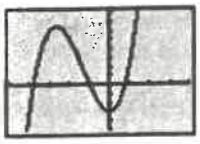
1. $y = -\frac{3}{2}x + 3$
2. $y = \sqrt{9 - x^2}$
3. $y = 3 - x^2$
4. $y = x^3 - x$

In Exercises 5–14, sketch the graph of the equation by point plotting.

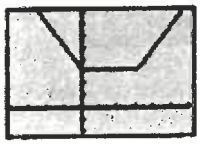
5. $y = \frac{1}{2}x + 2$
6. $y = 5 - 2x$
7. $y = 4 - x^2$
8. $y = (x - 3)^2$
9. $y = |x + 2|$
10. $y = |x| - 1$
11. $y = \sqrt{x} - 6$
12. $y = \sqrt{x + 2}$
13. $y = \frac{3}{x}$
14. $y = \frac{1}{x + 2}$

In Exercises 15 and 16, describe the viewing window that yields the figure.

15. $y = x^3 + 4x^2 - 3$



16. $y = |x| + |x - 16|$



In Exercises 17 and 18, use a graphing utility to graph the equation. Move the cursor along the curve to approximate the unknown coordinate of each solution point accurate to two decimal places.

17. $y = \sqrt{5 - x}$ (a) $(2, y)$ (b) $(x, 3)$
18. $y = x^5 - 5x$ (a) $(-0.5, y)$ (b) $(x, -4)$

In Exercises 19–28, find any intercepts.

19. $y = 2x - 5$
20. $y = 4x^2 + 3$
21. $y = x^2 + x - 2$
22. $y^2 = x^3 - 4x$
23. $y = x\sqrt{16 - x^2}$
24. $y = (x - 1)\sqrt{x^2 + 1}$
25. $y = \frac{2 - \sqrt{x}}{5x}$
26. $y = \frac{x^2 + 3x}{(3x + 1)^2}$
27. $x^2y - x^2 + 4y = 0$
28. $y = 2x - \sqrt{x^2 + 1}$

In Exercises 29–40, test for symmetry with respect to each axis and to the origin.

29. $y = x^2 - 6$
30. $y = x^2 - x$
31. $y^2 = x^3 - 8x$
32. $y = x^3 + x$
33. $xy = 4$
34. $xy^2 = -10$
35. $y = 4 - \sqrt{x + 3}$
36. $xy - \sqrt{4 - x^2} = 0$
37. $y = \frac{x}{x^2 + 1}$
38. $y = \frac{x^2}{x^2 + 1}$
39. $y = |x^3 + x|$
40. $|y| - x = 3$

In Exercises 41–58, sketch the graph of the equation. Identify any intercepts and test for symmetry.

41. $y = 2 - 3x$
42. $y = -\frac{3}{2}x + 6$
43. $y = \frac{1}{2}x - 4$
44. $y = \frac{2}{3}x + 1$
45. $y = 9 - x^2$
46. $y = x^2 + 3$
47. $y = (x + 3)^2$
48. $y = 2x^2 + x$
49. $y = x^3 + 2$
50. $y = x^3 - 4x$
51. $y = x\sqrt{x + 5}$
52. $y = \sqrt{25 - x^2}$
53. $x = y^3$
54. $x = y^2 - 4$
55. $y = \frac{8}{x}$
56. $y = \frac{10}{x^2 + 1}$
57. $y = 6 - |x|$
58. $y = |6 - x|$

In Exercises 59–62, use a graphing utility to graph the equation. Identify any intercepts and test for symmetry.

59. $y^2 - x = 9$
60. $x^2 + 4y^2 = 4$
61. $x + 3y^2 = 6$
62. $3x - 4y^2 = 8$

In Exercises 63–70, find the points of intersection of the graphs of the equations.

63. $x + y = 8$
$4x - y = 7$
64. $3x - 2y = -4$
$4x + 2y = -10$
65. $x^2 + y = 6$
$x + y = 4$
66. $x = 3 - y^2$
$y = x - 1$

The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.

67. $x^2 + y^2 = 5$
$x - y = 1$

68. $x^2 + y^2 = 25$
$-3x + y = 15$

69. $y = x^3$
$y = x$

70. $y = x^3 - 4x$
$y = -(x + 2)$

In Exercises 71–74, use a graphing utility to find the points of intersection of the graphs. Check your results analytically.

71. $y = x^3 - 2x^2 + x - 1$
$y = -x^2 + 3x - 1$

72. $y = x^4 - 2x^2 + 1$
$y = 1 - x^2$

73. $y = \sqrt{x + 6}$
$y = \sqrt{-x^2 - 4x}$

74. $y = -|2x - 3| + 6$
$y = 6 - x$

75. ***Modeling Data*** The table shows the Consumer Price Index (CPI) for selected years. *(Source: Bureau of Labor Statistics)*

Year	1975	1980	1985	1990	1995	2000	2005
CPI	53.8	82.4	107.6	130.7	152.4	172.2	195.3

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at^2 + bt + c$ for the data. In the model, y represents the CPI and t represents the year, with $t = 5$ corresponding to 1975.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the CPI for the year 2010.

76. ***Modeling Data*** The table shows the numbers of cellular phone subscribers (in millions) in the United States for selected years. *(Source: Cellular Telecommunications and Internet Association)*

Year	1990	1993	1996	1999	2002	2005
Number	5	16	44	86	141	208

(a) Use the regression capabilities of a graphing utility to find a mathematical model of the form $y = at^2 + bt + c$ for the data. In the model, y represents the number of subscribers and t represents the year, with $t = 0$ corresponding to 1990.

(b) Use a graphing utility to plot the data and graph the model. Compare the data with the model.

(c) Use the model to predict the number of cellular phone subscribers in the United States in the year 2015.

77. ***Break-Even Point*** Find the sales necessary to break even ($R = C$) if the cost C of producing x units is

$C = 5.5\sqrt{x} + 10{,}000$ Cost equation

and the revenue R from selling x units is

$R = 3.29x.$ Revenue equation

78. ***Copper Wire*** The resistance y in ohms of 1000 feet of solid copper wire at 77°F can be approximated by the model

$$y = \frac{10{,}770}{x^2} - 0.37, \quad 5 \le x \le 100$$

where x is the diameter of the wire in mils (0.001 in.). Use a graphing utility to graph the model. If the diameter of the wire is doubled, the resistance is changed by about what factor?

WRITING ABOUT CONCEPTS

In Exercises 79 and 80, write an equation whose graph has the indicated property. (There may be more than one correct answer.)

79. The graph has intercepts at $x = -4$, $x = 3$, and $x = 8$.

80. The graph has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

81. (a) Prove that if a graph is symmetric with respect to the x-axis and to the y-axis, then it is symmetric with respect to the origin. Give an example to show that the converse is not true.

(b) Prove that if a graph is symmetric with respect to one axis and to the origin, then it is symmetric with respect to the other axis.

CAPSTONE

82. Match the equation or equations with the given characteristic.

(i) $y = 3x^3 - 3x$ (ii) $y = (x + 3)^2$ (iii) $y = 3x - 3$
(iv) $y = \sqrt[3]{x}$ (v) $y = 3x^2 + 3$ (vi) $y = \sqrt{x + 3}$

(a) Symmetric with respect to the y-axis

(b) Three x-intercepts

(c) Symmetric with respect to the x-axis

(d) $(-2, 1)$ is a point on the graph

(e) Symmetric with respect to the origin

(f) Graph passes through the origin

True or False? **In Exercises 83–86, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

83. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the x-axis, then $(4, -5)$ is also a point on the graph.

84. If $(-4, -5)$ is a point on a graph that is symmetric with respect to the y-axis, then $(4, -5)$ is also a point on the graph.

85. If $b^2 - 4ac > 0$ and $a \neq 0$, then the graph of $y = ax^2 + bx + c$ has two x-intercepts.

86. If $b^2 - 4ac = 0$ and $a \neq 0$, then the graph of $y = ax^2 + bx + c$ has only one x-intercept.

In Exercises 87 and 88, find an equation of the graph that consists of all points (x, y) having the given distance from the origin. (For a review of the Distance Formula, see Appendix C.)

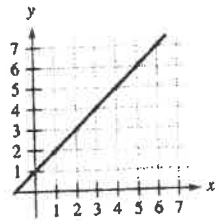
87. The distance from the origin is twice the distance from $(0, 3)$.

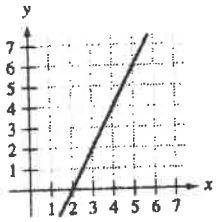
88. The distance from the origin is $K (K \neq 1)$ times the distance from $(2, 0)$.

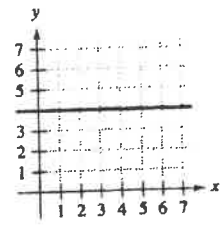
P.2 Exercises

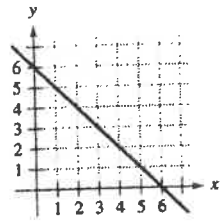
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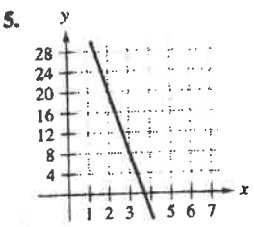
In Exercises 1–6, estimate the slope of the line from its graph. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

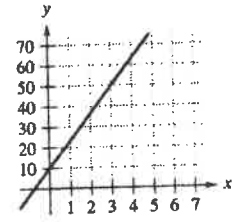
1. 

2. 

3. 

4. 

5. 

6. 

In Exercises 7 and 8, sketch the lines through the point with the indicated slopes. Make the sketches on the same set of coordinate axes.

	Point	*Slopes*			
7.	(3, 4)	(a) 1	(b) -2	(c) $-\frac{3}{2}$	(d) Undefined
8.	$(-2, 5)$	(a) 3	(b) -3	(c) $\frac{1}{3}$	(d) 0

In Exercises 9–14, plot the pair of points and find the slope of the line passing through them.

9. $(3, -4), (5, 2)$
10. $(1, 1), (-2, 7)$
11. $(4, 6), (4, 1)$
12. $(3, -5), (5, -5)$
13. $\left(-\frac{1}{2}, \frac{2}{3}\right), \left(-\frac{3}{4}, \frac{1}{6}\right)$
14. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

In Exercises 15–18, use the point on the line and the slope of the line to find three additional points that the line passes through. (There is more than one correct answer.)

	Point	*Slope*		*Point*	*Slope*
15.	(6, 2)	$m = 0$	16.	$(-4, 3)$	m is undefined.
17.	(1, 7)	$m = -3$	18.	$(-2, -2)$	$m = 2$

19. ***Conveyor Design*** A moving conveyor is built to rise 1 meter for each 3 meters of horizontal change.
   (a) Find the slope of the conveyor.
   (b) Suppose the conveyor runs between two floors in a factory. Find the length of the conveyor if the vertical distance between floors is 10 feet.

20. ***Rate of Change*** Each of the following is the slope of a line representing daily revenue y in terms of time x in days. Use the slope to interpret any change in daily revenue for a one-day increase in time.
   (a) $m = 800$ (b) $m = 250$ (c) $m = 0$

21. ***Modeling Data*** The table shows the populations y (in millions) of the United States for 2000 through 2005. The variable t represents the time in years, with $t = 0$ corresponding to 2000. *(Source: U.S. Bureau of the Census)*

t	0	1	2	3	4	5
y	282.4	285.3	288.2	291.1	293.9	296.6

   (a) Plot the data by hand and connect adjacent points with a line segment.
   (b) Use the slope of each line segment to determine the year when the population increased least rapidly.

22. ***Modeling Data*** The table shows the rate r (in miles per hour) that a vehicle is traveling after t seconds.

t	5	10	15	20	25	30
r	57	74	85	84	61	43

   (a) Plot the data by hand and connect adjacent points with a line segment.
   (b) Use the slope of each line segment to determine the interval when the vehicle's rate changed most rapidly. How did the rate change?

In Exercises 23–28, find the slope and the y-intercept (if possible) of the line.

23. $y = 4x - 3$
24. $-x + y = 1$
25. $x + 5y = 20$
26. $6x - 5y = 15$
27. $x = 4$
28. $y = -1$

In Exercises 29–34, find an equation of the line that passes through the point and has the indicated slope. Sketch the line.

	Point	*Slope*		*Point*	*Slope*
29.	(0, 3)	$m = \frac{3}{4}$	30.	$(-5, -2)$	m is undefined.
31.	(0, 0)	$m = \frac{2}{3}$	32.	(0, 4)	$m = 0$
33.	(3, -2)	$m = 3$	34.	$(-2, 4)$	$m = -\frac{3}{5}$

In Exercises 35–44, find an equation of the line that passes through the points, and sketch the line.

35. $(0, 0), (4, 8)$
36. $(0, 0), (-1, 5)$
37. $(2, 1), (0, -3)$
38. $(-2, -2), (1, 7)$
39. $(2, 8), (5, 0)$
40. $(-3, 6), (1, 2)$
41. $(6, 3), (6, 8)$
42. $(1, -2), (3, -2)$
43. $\left(\frac{1}{2}, \frac{2}{3}\right), \left(0, \frac{3}{4}\right)$
44. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

45. Find an equation of the vertical line with x-intercept at 3.

46. Show that the line with intercepts $(a, 0)$ and $(0, b)$ has the following equation.

$$\frac{x}{a} + \frac{y}{b} = 1, \quad a \neq 0, b \neq 0$$

In Exercises 47–50, use the result of Exercise 46 to write an equation of the line in general form.

47. x-intercept: $(2, 0)$; y-intercept: $(0, 3)$
48. x-intercept: $\left(-\frac{2}{3}, 0\right)$; y-intercept: $(0, -2)$
49. Point on line: $(1, 2)$; x-intercept: $(a, 0)$; y-intercept: $(0, a)$; $(a \neq 0)$
50. Point on line: $(-3, 4)$; x-intercept: $(a, 0)$; y-intercept: $(0, a)$; $(a \neq 0)$

In Exercises 51–58, sketch a graph of the equation.

51. $y = -3$
52. $x = 4$
53. $y = -2x + 1$
54. $y = \frac{1}{3}x - 1$
55. $y - 2 = \frac{3}{2}(x - 1)$
56. $y - 1 = 3(x + 4)$
57. $2x - y - 3 = 0$
58. $x + 2y + 6 = 0$

59. ***Square Setting*** Use a graphing utility to graph the lines $y = 2x - 3$ and $y = -\frac{1}{2}x + 1$ in each viewing window. Compare the graphs. Do the lines appear perpendicular? Are the lines perpendicular? Explain.

(a)
```
Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -5
Ymax = 5
Yscl = 1
```

(b)
```
Xmin = -6
Xmax = 6
Xscl = 1
Ymin = -4
Ymax = 4
Yscl = 1
```

CAPSTONE

60. A line is represented by the equation $ax + by = 4$.
   (a) When is the line parallel to the x-axis?
   (b) When is the line parallel to the y-axis?
   (c) Give values for a and b such that the line has a slope of $\frac{5}{8}$.
   (d) Give values for a and b such that the line is perpendicular to $y = \frac{2}{5}x + 3$.
   (e) Give values for a and b such that the line coincides with the graph of $5x + 6y = 8$.

In Exercises 61–66, write the general forms of the equations of the lines through the point (a) parallel to the given line and (b) perpendicular to the given line.

	Point	*Line*		*Point*	*Line*
61.	$(-7, -2)$	$x = 1$	62.	$(-1, 0)$	$y = -3$
63.	$(2, 1)$	$4x - 2y = 3$	64.	$(-3, 2)$	$x + y = 7$
65.	$\left(\frac{3}{4}, \frac{7}{8}\right)$	$5x - 3y = 0$	66.	$(4, -5)$	$3x + 4y = 7$

Rate of Change **In Exercises 67–70, you are given the dollar value of a product in 2008 *and* the rate at which the value of the product is expected to change during the next 5 years. Write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 0$ represent 2000.)**

	2008 Value	*Rate*
67.	\$1850	\$250 increase per year
68.	\$156	\$4.50 increase per year
69.	\$17,200	\$1600 decrease per year
70.	\$245,000	\$5600 decrease per year

In Exercises 71 and 72, use a graphing utility to graph the parabolas and find their points of intersection. Find an equation of the line through the points of intersection and graph the line in the same viewing window.

71. $y = x^2$; $y = 4x - x^2$
72. $y = x^2 - 4x + 3$; $y = -x^2 + 2x + 3$

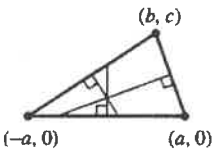
In Exercises 73 and 74, determine whether the points are collinear. (Three points are *collinear* if they lie on the same line.)

73. $(-2, 1), (-1, 0), (2, -2)$
74. $(0, 4), (7, -6), (-5, 11)$

WRITING ABOUT CONCEPTS

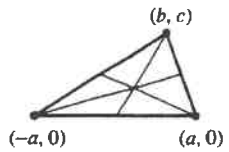
In Exercises 75–77, find the coordinates of the point of intersection of the given segments. Explain your reasoning.

75.



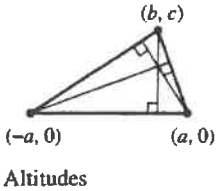
Perpendicular bisectors

76.



Medians

77.



Altitudes

78. Show that the points of intersection in Exercises 75, 76, and 77 are collinear.

P.3 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1 and 2, use the graphs of f and g to answer the following.

(a) Identify the domains and ranges of f and g.

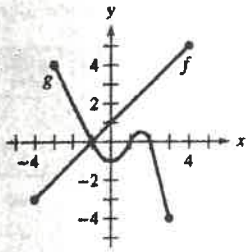
(b) Identify $f(-2)$ and $g(3)$.

(c) For what value(s) of x is $f(x) = g(x)$?

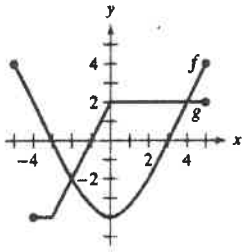
(d) Estimate the solution(s) of $f(x) = 2$.

(e) Estimate the solutions of $g(x) = 0$.

1.



2.



In Exercises 3–12, evaluate (if possible) the function at the given value(s) of the independent variable. Simplify the results.

3. $f(x) = 7x - 4$
   (a) $f(0)$
   (b) $f(-3)$
   (c) $f(b)$
   (d) $f(x - 1)$

4. $f(x) = \sqrt{x + 5}$
   (a) $f(-4)$
   (b) $f(11)$
   (c) $f(-8)$
   (d) $f(x + \Delta x)$

5. $g(x) = 5 - x^2$
   (a) $g(0)$
   (b) $g(\sqrt{5})$
   (c) $g(-2)$
   (d) $g(t - 1)$

6. $g(x) = x^2(x - 4)$
   (a) $g(4)$
   (b) $g(\frac{3}{2})$
   (c) $g(c)$
   (d) $g(t + 4)$

7. $f(x) = \cos 2x$
   (a) $f(0)$
   (b) $f(-\pi/4)$
   (c) $f(\pi/3)$

8. $f(x) = \sin x$
   (a) $f(\pi)$
   (b) $f(5\pi/4)$
   (c) $f(2\pi/3)$

9. $f(x) = x^3$

   $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

10. $f(x) = 3x - 1$

    $\dfrac{f(x) - f(1)}{x - 1}$

11. $f(x) = \dfrac{1}{\sqrt{x - 1}}$

    $\dfrac{f(x) - f(2)}{x - 2}$

12. $f(x) = x^3 - x$

    $\dfrac{f(x) - f(1)}{x - 1}$

In Exercises 13–20, find the domain and range of the function.

13. $f(x) = 4x^2$
14. $g(x) = x^2 - 5$
15. $g(x) = \sqrt{6x}$
16. $h(x) = -\sqrt{x + 3}$
17. $f(t) = \sec \dfrac{\pi t}{4}$
18. $h(t) = \cot t$
19. $f(x) = \dfrac{3}{x}$
20. $g(x) = \dfrac{2}{x - 1}$

In Exercises 21–26, find the domain of the function.

21. $f(x) = \sqrt{x} + \sqrt{1 - x}$
22. $f(x) = \sqrt{x^2 - 3x + 2}$
23. $g(x) = \dfrac{2}{1 - \cos x}$
24. $h(x) = \dfrac{1}{\sin x - \frac{1}{2}}$
25. $f(x) = \dfrac{1}{|x + 3|}$
26. $g(x) = \dfrac{1}{|x^2 - 4|}$

In Exercises 27–30, evaluate the function as indicated. Determine its domain and range.

27. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$
    (a) $f(-1)$ (b) $f(0)$ (c) $f(2)$ (d) $f(t^2 + 1)$

28. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$
    (a) $f(-2)$ (b) $f(0)$ (c) $f(1)$ (d) $f(s^2 + 2)$

29. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$
    (a) $f(-3)$ (b) $f(1)$ (c) $f(3)$ (d) $f(b^2 + 1)$

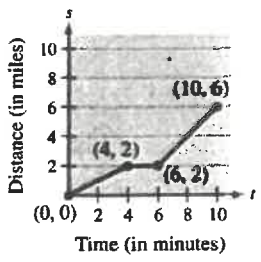
30. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$
    (a) $f(-3)$ (b) $f(0)$ (c) $f(5)$ (d) $f(10)$

In Exercises 31–38, sketch a graph of the function and find its domain and range. Use a graphing utility to verify your graph.

31. $f(x) = 4 - x$
32. $g(x) = \dfrac{4}{x}$
33. $h(x) = \sqrt{x - 6}$
34. $f(x) = \frac{1}{4}x^3 + 3$
35. $f(x) = \sqrt{9 - x^2}$
36. $f(x) = x + \sqrt{4 - x^2}$
37. $g(t) = 3 \sin \pi t$
38. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

WRITING ABOUT CONCEPTS

39. The graph of the distance that a student drives in a 10-minute trip to school is shown in the figure. Give a verbal description of characteristics of the student's drive to school.

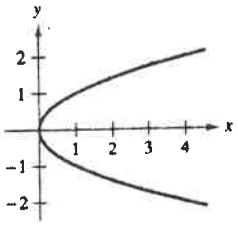


WRITING ABOUT CONCEPTS (continued)

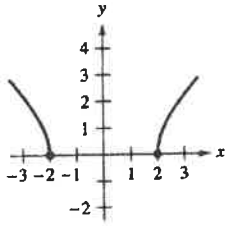
40. A student who commutes 27 miles to attend college remembers, after driving a few minutes, that a term paper that is due has been forgotten. Driving faster than usual, the student returns home, picks up the paper, and once again starts toward school. Sketch a possible graph of the student's distance from home as a function of time.

In Exercises 41–44, use the Vertical Line Test to determine whether y is a function of x. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

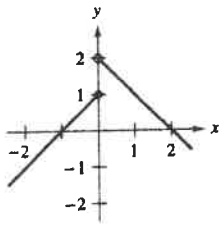
41. $x - y^2 = 0$



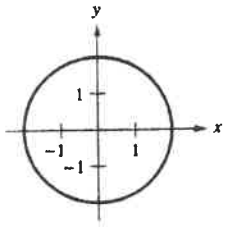
42. $\sqrt{x^2 - 4} - y = 0$



43. $y = \begin{cases} x + 1, & x \leq 0 \\ -x + 2, & x > 0 \end{cases}$



44. $x^2 + y^2 = 4$



In Exercises 45–48, determine whether y is a function of x.

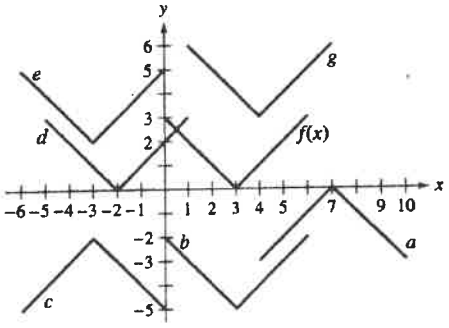
45. $x^2 + y^2 = 16$

46. $x^2 + y = 16$

47. $y^2 = x^2 - 1$

48. $x^2y - x^2 + 4y = 0$

In Exercises 49–54, use the graph of $y = f(x)$ to match the function with its graph.



49. $y = f(x + 5)$

50. $y = f(x) - 5$

51. $y = -f(-x) - 2$

52. $y = -f(x - 4)$

53. $y = f(x + 6) + 2$

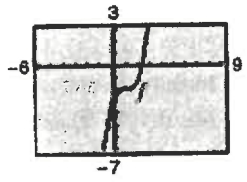
54. $y = f(x - 1) + 3$

55. Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

(a) $f(x + 3)$ (b) $f(x - 1)$

(c) $f(x) + 2$ (d) $f(x) - 4$

(e) $3f(x)$ (f) $\frac{1}{4}f(x)$

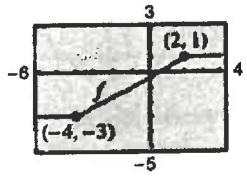


56. Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

(a) $f(x - 4)$ (b) $f(x + 2)$

(c) $f(x) + 4$ (d) $f(x) - 1$

(e) $2f(x)$ (f) $\frac{1}{2}f(x)$



57. Use the graph of $f(x) = \sqrt{x}$ to sketch the graph of each function. In each case, describe the transformation.

(a) $y = \sqrt{x} + 2$ (b) $y = -\sqrt{x}$ (c) $y = \sqrt{x - 2}$

58. Specify a sequence of transformations that will yield each graph of h from the graph of the function $f(x) = \sin x$.

(a) $h(x) = \sin\left(x + \frac{\pi}{2}\right) + 1$ (b) $h(x) = -\sin(x - 1)$

59. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, evaluate each expression.

(a) $f(g(1))$ (b) $g(f(1))$ (c) $g(f(0))$

(d) $f(g(-4))$ (e) $f(g(x))$ (f) $g(f(x))$

60. Given $f(x) = \sin x$ and $g(x) = \pi x$, evaluate each expression.

(a) $f(g(2))$ (b) $f\left(g\left(\frac{1}{2}\right)\right)$ (c) $g(f(0))$

(d) $g\left(f\left(\frac{\pi}{4}\right)\right)$ (e) $f(g(x))$ (f) $g(f(x))$

In Exercises 61–64, find the composite functions $(f \circ g)$ and $(g \circ f)$. What is the domain of each composite function? Are the two composite functions equal?

61. $f(x) = x^2$, $g(x) = \sqrt{x}$

62. $f(x) = x^2 - 1$, $g(x) = \cos x$

63. $f(x) = \frac{3}{x}$, $g(x) = x^2 - 1$

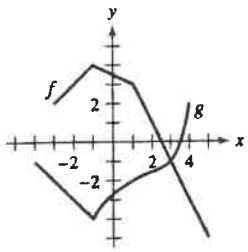
64. $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x + 2}$

65. Use the graphs of f and g to evaluate each expression. If the result is undefined, explain why.

(a) $(f \circ g)(3)$ (b) $g(f(2))$

(c) $g(f(5))$ (d) $(f \circ g)(-3)$

(e) $(g \circ f)(-1)$ (f) $f(g(-1))$



66. ***Ripples*** A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius (in feet) of the outer ripple is given by $r(t) = 0.6t$, where t is the time in seconds after the pebble strikes the water. The area of the circle is given by the function $A(r) = \pi r^2$. Find and interpret $(A \circ r)(t)$.

Think About It **In Exercises 67 and 68, $F(x) = f \circ g \circ h$. Identify functions for f, g, and h. (There are many correct answers.)**

67. $F(x) = \sqrt{2x - 2}$

68. $F(x) = -4 \sin(1 - x)$

In Exercises 69–72, determine whether the function is even, odd, or neither. Use a graphing utility to verify your result.

69. $f(x) = x^2(4 - x^2)$

70. $f(x) = \sqrt[3]{x}$

71. $f(x) = x \cos x$

72. $f(x) = \sin^2 x$

Think About It **In Exercises 73 and 74, find the coordinates of a second point on the graph of a function f if the given point is on the graph and the function is (a) even and (b) odd.**

73. $\left(-\frac{3}{2}, 4\right)$

74. $(4, 9)$

75. The graphs of f, g, and h are shown in the figure. Decide whether each function is even, odd, or neither.

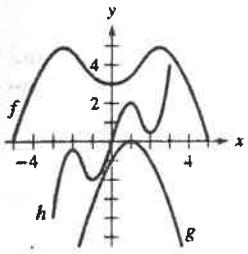


Figure for 75

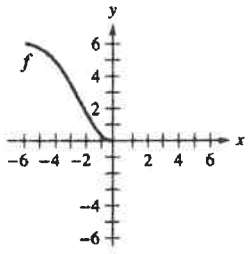


Figure for 76

76. The domain of the function f shown in the figure is $-6 \leq x \leq 6$.

(a) Complete the graph of f given that f is even.

(b) Complete the graph of f given that f is odd.

Writing Functions **In Exercises 77–80, write an equation for a function that has the given graph.**

77. Line segment connecting $(-2, 4)$ and $(0, -6)$

78. Line segment connecting $(3, 1)$ and $(5, 8)$

79. The bottom half of the parabola $x + y^2 = 0$

80. The bottom half of the circle $x^2 + y^2 = 36$

In Exercises 81–84, sketch a possible graph of the situation.

81. The speed of an airplane as a function of time during a 5-hour flight

82. The height of a baseball as a function of horizontal distance during a home run

83. The amount of a certain brand of sneaker sold by a sporting goods store as a function of the price of the sneaker

84. The value of a new car as a function of time over a period of 8 years

85. Find the value of c such that the domain of

$$f(x) = \sqrt{c - x^2}$$

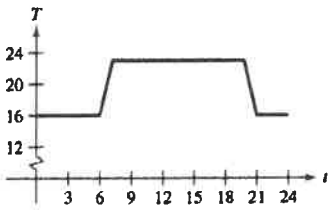
is $[-5, 5]$.

86. Find all values of c such that the domain of

$$f(x) = \frac{x + 3}{x^2 + 3cx + 6}$$

is the set of all real numbers.

87. ***Graphical Reasoning*** An electronically controlled thermostat is programmed to lower the temperature during the night automatically (see figure). The temperature T in degrees Celsius is given in terms of t, the time in hours on a 24-hour clock.



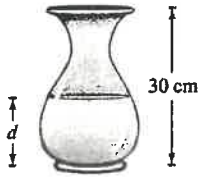
(a) Approximate $T(4)$ and $T(15)$.

(b) The thermostat is reprogrammed to produce a temperature $H(t) = T(t - 1)$. How does this change the temperature? Explain.

(c) The thermostat is reprogrammed to produce a temperature $H(t) = T(t) - 1$. How does this change the temperature? Explain.

CAPSTONE

88. Water runs into a vase of height 30 centimeters at a constant rate. The vase is full after 5 seconds. Use this information and the shape of the vase shown to answer the questions if d is the depth of the water in centimeters and t is the time in seconds (see figure).



(a) Explain why d is a function of t.

(b) Determine the domain and range of the function.

(c) Sketch a possible graph of the function.

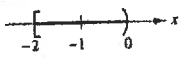
(d) Use the graph in part (c) to approximate $d(4)$. What does this represent?

UNDERLINED #s

Trig Review

EXERCISES FOR APPENDIX C.1

16. Complete the table with the appropriate interval notation, set notation, and graph on the real number line.

Interval Notation	Set Notation	Graph
		
$(-\infty, -4]$		
	$\{x: 3 \le x \le \frac{11}{2}\}$	
$(-1, 7)$		

In Exercises 25–44, solve the inequality and graph the solution on the real number line.

25. $2x - 1 \ge 0$

26. $3x + 1 \ge 2x + 2$

27. $-4 < 2x - 3 < 4$

28. $0 \le x + 3 < 5$

29. $\frac{x}{2} + \frac{x}{3} > 5$

30. $x > \frac{1}{x}$

31. $|x| < 1$

32. $\frac{x}{2} - \frac{x}{3} > 5$

33. $\left|\frac{x-3}{2}\right| \ge 5$

34. $\left|\frac{x}{2}\right| > 3$

35. $|x - a| < b,\ b > 0$

36. $|x + 2| < 5$

37. $|2x + 1| < 5$

38. $|3x + 1| \ge 4$

39. $\left|1 - \frac{2}{3}x\right| < 1$

40. $|9 - 2x| < 1$

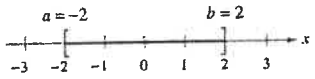
41. $x^2 \le 3 - 2x$

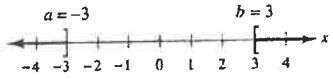
42. $x^4 - x \le 0$

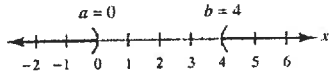
43. $x^2 + x - 1 \le 5$

44. $2x^2 + 1 < 9x - 3$

In Exercises 49–54, use absolute value notation to define the interval or pair of intervals on the real number line.

49. 

50. 

51. 

53. (a) All numbers that are at most 10 units from 12.
(b) All numbers that are at least 10 units from 12.

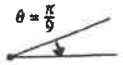
60. *Fleet Costs* A utility company has a fleet of vans. The annual operating cost C (in dollars) of each van is estimated to be

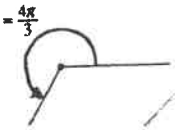
$$C = 0.32m + 2300$$

where m is measured in miles. The company wants the annual operating cost of each van to be less than \$10,000. To do this, m must be less than what value?

EXERCISES FOR APPENDIX C.3

In Exercises 3 and 4, determine two coterminal angles in radian measure (one positive and one negative) for each angle.

3. (a) $\theta = \frac{\pi}{9}$ 

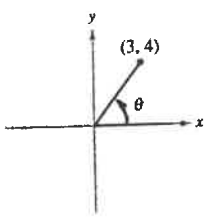
(b) $\theta = \frac{4\pi}{3}$ 

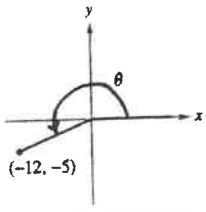
In Exercises 5 and 6, rewrite each angle in radian measure as a multiple of π and as a decimal accurate to three decimal places.

5. (a) 30° (b) 150° (c) 315° (d) 120°

6. (a) −20° (b) −240° (c) −270° (d) 144°

In Exercises 11 and 12, determine all six trigonometric functions for the angle θ.

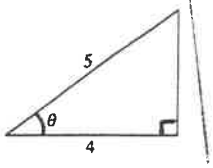
11. (a) 

(b) 

In Exercises 19–22, evaluate the sine, cosine, and tangent of each angle *without* using a calculator.

19. (a) 60°
(b) 120°
(c) $\frac{\pi}{4}$
(d) $\frac{5\pi}{4}$

17. $\cos\theta = \frac{4}{5}$
$\cot\theta =$ [illegible]



21. (a) 225°
(b) −225°
(c) $\frac{5\pi}{3}$
(d) $\frac{11\pi}{6}$

In Exercises 27–30, find two solutions of each equation. Give your answers in radians ($0 \le \theta < 2\pi$). Do not use a calculator.

29. (a) $\tan\theta = 1$
(b) $\cot\theta = -\sqrt{3}$

In Exercises 31–38, solve the equation for $\theta (0 \le \theta < 2\pi)$.

31. $2\sin^2\theta = 1$

32. $\tan^2\theta = 3$

33. $\tan^2\theta - \tan\theta = 0$

34. $2\cos^2\theta - \cos\theta = 1$

35. $\sec\theta\csc\theta = 2\csc\theta$

36. $\sin\theta = \cos\theta$

37. $\cos^2\theta + \sin\theta = 1$

38. $\cos\frac{\theta}{2} - \cos\theta = 1$

In Exercises 45–48, find the period of the function.

45. $y = 5\tan 2x$

46. $y = 7\tan 2\pi x$

47. $y = \sec 5x$

48. $y = \csc 4x$

61. $y = 1 + \cos\left(x - \frac{\pi}{2}\right)$ Graph.

1.1 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–5, decide whether the problem can be solved using precalculus or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, explain your reasoning and use a graphical or numerical approach to estimate the solution.

1. Find the distance traveled in 15 seconds by an object traveling at a constant velocity of 20 feet per second.
2. Find the distance traveled in 15 seconds by an object moving with a velocity of $v(t) = 20 + 7\cos t$ feet per second.
3. A bicyclist is riding on a path modeled by the function $f(x) = 0.04(8x - x^2)$, where x and $f(x)$ are measured in miles. Find the rate of change of elevation at $x = 2$.

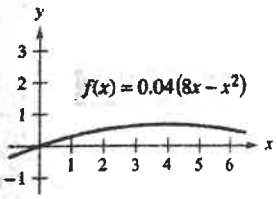


Figure for 3

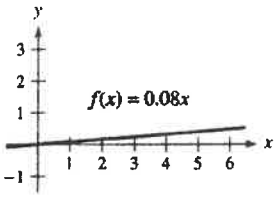
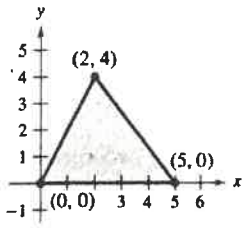


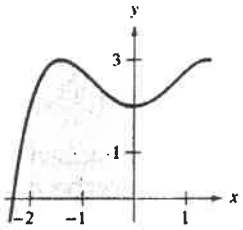
Figure for 4

4. A bicyclist is riding on a path modeled by the function $f(x) = 0.08x$, where x and $f(x)$ are measured in miles. Find the rate of change of elevation at $x = 2$.
5. Find the area of the shaded region.

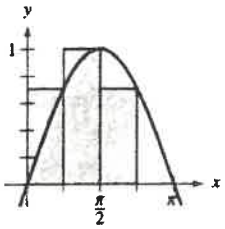
(a)

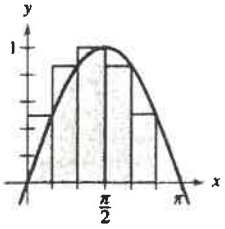


(b)

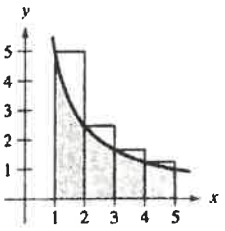


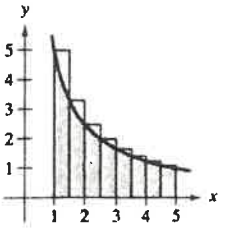
6. ***Secant Lines*** Consider the function $f(x) = \sqrt{x}$ and the point $P(4, 2)$ on the graph of f.
   (a) Graph f and the secant lines passing through $P(4, 2)$ and $Q(x, f(x))$ for x-values of 1, 3, and 5.
   (b) Find the slope of each secant line.
   (c) Use the results of part (b) to estimate the slope of the tangent line to the graph of f at $P(4, 2)$. Describe how to improve your approximation of the slope.
7. ***Secant Lines*** Consider the function $f(x) = 6x - x^2$ and the point $P(2, 8)$ on the graph of f.
   (a) Graph f and the secant lines passing through $P(2, 8)$ and $Q(x, f(x))$ for x-values of 3, 2.5, and 1.5.
   (b) Find the slope of each secant line.
   (c) Use the results of part (b) to estimate the slope of the tangent line to the graph of f at $P(2, 8)$. Describe how to improve your approximation of the slope.
8. (a) Use the rectangles in each graph to approximate the area of the region bounded by $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$.





   (b) Describe how you could continue this process to obtain a more accurate approximation of the area.
9. (a) Use the rectangles in each graph to approximate the area of the region bounded by $y = 5/x$, $y = 0$, $x = 1$, and $x = 5$.





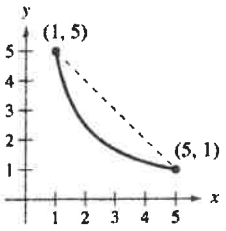
   (b) Describe how you could continue this process to obtain a more accurate approximation of the area.

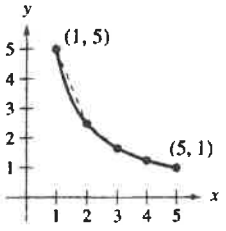
CAPSTONE

10. How would you describe the instantaneous rate of change of an automobile's position on the highway?

WRITING ABOUT CONCEPTS

11. Consider the length of the graph of $f(x) = 5/x$ from $(1, 5)$ to $(5, 1)$.





   (a) Approximate the length of the curve by finding the distance between its two endpoints, as shown in the first figure.
   (b) Approximate the length of the curve by finding the sum of the lengths of four line segments, as shown in the second figure.
   (c) Describe how you could continue this process to obtain a more accurate approximation of the length of the curve.

Algebra Review

0.1 Monomial Factors

Factor as indicated:

(a) $3x^4 + 4x^3 - x^2 = x^2(\quad)$

(b) $2\sqrt{x} + 6x^{3/2} = 2\sqrt{x}(\quad)$

(c) $e^{-x} - xe^{-x} + 2x^2e^{-x} = e^{-x}(\quad)$

(d) $x^{-1} - 2 + x = x^{-1} = x^{-1}(\quad)$

(e) $\frac{x}{2} - 6x^2 = \frac{x}{2}(\quad)$

(f) $\sin x + \tan x = \sin x(\quad)$

(g) $\frac{1}{2x^2 + 4x} = \frac{1}{2x}(\quad)$

Solution:

(a) $3x^4 + 4x^3 - x^2 = x^2(3x^2 + 4x - 1)$

(b) $2\sqrt{x} + 6x^{3/2} = 2\sqrt{x}(1 + 3x)$

(c) $e^{-x} - xe^{-x} + 2x^2e^{-x} = e^{-x}(1 - x + 2x^2)$

(d) $x^{-1} - 2 + x = x^{-1} = x^{-1}(1 - 2x + x^2)$

(e) $\frac{x}{2} - 6x^2 = \frac{x}{2}(1 - 12x)$

(f) $\sin x + \tan x = \sin x + \frac{\sin x}{\cos x} = \sin x\left(1 + \frac{1}{\cos x}\right)$

$= \sin x(1 + \sec x)$

(g) $\frac{1}{2x^2 + 4x} = \frac{1}{2x}\left(\frac{1}{x + 2}\right)$

0.2 Binomial Factors

Factor as indicated:

(a) $(x - 1)^2(x) - (x - 1) = (x - 1)(\quad)$

(b) $3(x^2 + 4)(x^2 + 1) + 6(x^2 + 4)^2 = 3(x^2 + 4)(\quad)$

(c) $\sqrt{x^2 + 1} - \frac{x^2}{\sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}}(\quad)$

(d) $(x - 3)^3(x + 2) - 2(x - 3)^2(x + 2)^2 = (x - 3)^2(x + 2)(\quad)$

(e) $(2x + 1)^{3/2}(x^{1/2}) + (2x + 1)^{5/2}(x^{-1/2}) = (2x + 1)^{3/2}(x^{-1/2})(\quad)$

Solution:

(a) $(x - 1)^2(x) - (x - 1) = (x - 1)[(x - 1)x - 1]$

$= (x - 1)(x^2 - x - 1)$

(b) $3(x^2 + 4)(x^2 + 1) + 6(x^2 + 4)^2 = 3(x^2 + 4)[(x^2 + 1) + 2(x^2 + 4)]$

$= 3(x^2 + 4)(3x^2 + 9)$

—CONTINUED—

—CONTINUED—

(c) $\sqrt{x^2+1} - \dfrac{x^2}{\sqrt{x^2+1}} = (x^2+1)^{1/2} - x^2(x^2+1)^{-1/2}$

$= (x^2+1)^{-1/2}[(x^2+1) - x^2]$

$= \dfrac{1}{\sqrt{x^2+1}}$

(d) $(x-3)^3(x+2) - 2(x-3)^2(x+2)^2 = (x-3)^2(x+2)[(x-3) - 2(x+2)]$

$= (x-3)^2(x+2)(-x-7)$

(e) $(2x+1)^{3/2}(x^{1/2}) + (2x+1)^{5/2}(x^{-1/2}) = (2x+1)^{3/2}(x^{-1/2})[x + (2x+1)]$

$= (2x+1)^{3/2}(x^{-1/2})(3x+1)$

0.3 Factoring Quadratic Expressions

Factor as indicated:

(a) $x^2 - 3x + 2 = (\quad)(\quad)$

(b) $x^2 - 9 = (\quad)(\quad)$

(c) $x^2 + 5x - 6 = (\quad)(\quad)$

(d) $x^2 + 5x + 6 = (\quad)(\quad)$

(e) $2x^2 + 5x - 3 = (\quad)(\quad)$

(f) $e^{2x} + 2 + e^{-2x} = (\quad)^2$

(g) $x^4 - 7x^2 + 12 = (\quad)(\quad)(\quad)$

(h) $1 - \sin^2 x = (\quad)(\quad)$

Solution:

(a) $x^2 - 3x + 2 = (x-2)(x-1)$

(b) $x^2 - 9 = (x+3)(x-3)$

(c) $x^2 + 5x - 6 = (x+6)(x-1)$

(d) $x^2 + 5x + 6 = (x+2)(x+3)$

(e) $2x^2 + 5x - 3 = (2x-1)(x+3)$

(f) $e^{2x} + 2 + e^{-2x} = (e^x + e^{-x})^2$

(g) $x^4 - 7x^2 + 12 = (x^2-3)(x^2-4) = (x^2-3)(x+2)(x-2)$

(h) $1 - \sin^2 x = (1+\sin x)(1-\sin x)$

0.4 Cancellation

Reduce each expression to lowest terms:

(a) $\dfrac{3x+9}{6x}$

(b) $\dfrac{x^2}{x^{1/2}}$

(c) $\dfrac{(x+1)^3(x-2) + 3(x+1)^2}{(x+1)^4}$

(d) $\dfrac{x^{1/2} - x^{1/3}}{x^{1/6}}$

(e) $\dfrac{\sqrt{x-1} + (x-1)^{3/2}}{\sqrt{x-1}}$

(f) $\dfrac{1 - (\sin x + \cos x)^2}{2\sin x}$

Solution:

(a) $\dfrac{3x+9}{6x} = \dfrac{3(x+3)}{3(2x)} = \dfrac{x+3}{2x}$

(b) $\dfrac{x^2}{x^{1/2}} = \dfrac{(x^{1/2})(x^{3/2})}{x^{1/2}} = x^{3/2}$

—CONTINUED—

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(c) $$\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4} = \frac{(x+1)^2[(x+1)(x-2)+3]}{(x+1)^4}$$

$$= \frac{x^2-x+1}{(x+1)^2}$$

(d) $$\frac{x^{1/2}-x^{1/3}}{x^{1/6}} = \frac{x^{1/6}(x^{2/6}-x^{1/6})}{x^{1/6}} = x^{1/3}-x^{1/6}$$

(e) $$\frac{\sqrt{x-1}+(x-1)^{3/2}}{\sqrt{x-1}} = \frac{\sqrt{x-1}[1+(x-1)]}{\sqrt{x-1}} = x$$

(f) $$\frac{1-(\sin x+\cos x)^2}{2\sin x} = \frac{1-(\sin^2 x+2\sin x\cos x+\cos^2 x)}{2\sin x}$$

$$= \frac{1-(\sin^2 x+\cos^2 x)-2\sin x\cos x}{2\sin x}$$

$$= \frac{1-1-2\sin x\cos x}{2\sin x} = -\cos x$$

0.5 Quadratic Formula

Equation	*Solve for*
(a) $x^2-4x-1=0$	x
(b) $2x^2+x-3=0$	x
(c) $\cos^2 x+3\cos x+2=0$	$\cos x$
(d) $x^2-xy-(1+y^2)=0$	x
(e) $x^4-4x^2+2=0$	x^2

Solution:

(a) $$x = \frac{4\pm\sqrt{16+4}}{2} = \frac{4\pm\sqrt{20}}{2} = \frac{4\pm 2\sqrt{5}}{2} = 2\pm\sqrt{5}$$

(b) $$x = \frac{-1\pm\sqrt{1+24}}{4} = \frac{-1\pm 5}{4}$$

$$x = \frac{4}{4} = 1 \quad \text{or} \quad x = -\frac{6}{4} = -\frac{3}{2}$$

(c) $$\cos x = \frac{-3\pm\sqrt{9-8}}{2} = \frac{-3\pm 1}{2}$$

$$\cos x = -\frac{2}{2} = -1 \quad \text{or} \quad \cos x = -\frac{4}{2} = -2$$

(d) $$x = \frac{y\pm\sqrt{y^2+4(1+y^2)}}{2} = \frac{y\pm\sqrt{y^2+4+4y^2}}{2}$$

$$= \frac{y\pm\sqrt{5y^2+4}}{2}$$

(e) $$x^2 = \frac{4\pm\sqrt{16-8}}{2} = \frac{4\pm\sqrt{8}}{2} = \frac{4\pm 2\sqrt{2}}{2} = 2\pm\sqrt{2}$$

0.6 Synthetic Division

Using synthetic division to factor as indicated:

(a) $x^3 - 4x^2 + 2x + 1 = (x - 1)(\quad)$

(b) $2x^3 + 5x + 7 = (x + 1)(\quad)$

(c) $x^4 - 3x^3 + x^2 + x + 2 = (x - 2)(\quad)$

(d) $4x^4 + 3x^2 - 1 = (2x - 1)(\quad)$

Solution:

(a) $x^3 - 4x^2 + 2x + 1$

$$\begin{array}{c|cccc} 1 & 1 & -4 & 2 & 1 \\ & & 1 & -3 & -1 \\ \hline & 1 & -3 & -1 & 0 \end{array}$$

$x^3 - 4x^2 + 2x + 1 = (x - 1)(x^2 - 3x - 1)$

(b) $2x^3 + 5x + 7$

$$\begin{array}{c|cccc} -1 & 2 & 0 & 5 & 7 \\ & & -2 & 2 & -7 \\ \hline & 2 & -2 & 7 & 0 \end{array}$$

$2x^3 - 5x + 7 = (x + 1)(2x^2 - 2x + 7)$

(c) $x^4 - 3x^3 + x^2 + x + 2$

$$\begin{array}{c|ccccc} 2 & 1 & -3 & 1 & 1 & 2 \\ & & 2 & -2 & -2 & -2 \\ \hline & 1 & -1 & -1 & -1 & 0 \end{array}$$

$x^4 - 3x^3 + x^2 + x + 2 = (x - 2)(x^3 - x^2 - x - 1)$

(d) $4x^4 + 3x^2 - 1$

$$\begin{array}{c|ccccc} \frac{1}{2} & 4 & 0 & 3 & 0 & -1 \\ & & 2 & 1 & 2 & 1 \\ \hline & 4 & 2 & 4 & 2 & 0 \end{array}$$

$$4x^4 + 3x^2 - 1 = \left(x - \frac{1}{2}\right)(4x^3 + 2x^2 + 4x + 2)$$
$$= (2x - 1)(2x^3 + x^2 + 2x + 1)$$

0.7 Special Products

Factor completely (into linear or irreducible quadratic factors):

(a) $x^3 - 27$

(b) $x^3 - 3x^2 + 3x - 1$

(c) $x^3 + 6x^2 + 12x + 8$

(d) $x^4 - 25$

(e) $x^4 - 8x^3 + 24x^2 - 32x + 16$

Solution:

(a) $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$

(b) $x^3 - 3x^2 + 3x - 1 = (x - 1)^3$

(c) $x^3 + 6x^2 + 12x + 8 = x^3 + 3(2)x^2 + 3(2^2)x + 2^3 = (x + 2)^3$

(d) $x^4 - 25 = (x^2 + 5)(x^2 - 5) = (x^2 + 5)\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)$

(e) $x^4 - 8x^3 + 24x^2 - 32x + 16 = x^4 - 4(2)x^3 + 6(2^2)x^2 - 4(2^3)x + 2^4 = (x - 2)^4$

0.8 Factoring by Grouping

Factor completely (into linear or irreducible quadratic factors):

(a) $x^3 + 4x^2 - 2x - 8$

(b) $x^3 + 2x^2 + 3x + 6$

(c) $5\cos^2 x - 5\sin^2 x + \sin x + \cos x$

(d) $\cos^2 x + 4\cos x + 4 - \tan^2 x$

Solution:

(a) $x^3 + 4x^2 - 2x - 8 = x^2(x + 4) - 2(x + 4)$

$= (x^2 - 2)(x + 4)$

$= \left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)(x + 4)$

—CONTINUED—

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(b) $x^3 + 2x^2 + 3x + 6 = x^2(x + 2) + 3(x + 2)$

$= (x^2 + 3)(x + 2)$

(c) $5\cos^2 x - 5\sin^2 x + \sin x + \cos x = 5(\cos^2 x - \sin^2 x) + (\sin x + \cos x)$

$= 5(\cos x - \sin x)(\cos x - \sin x) + (\cos x + \sin x)$

$= (\cos x + \sin x)[5(\cos x - \sin x) + 1]$

(d) $\cos^2 x + 4\cos x + 4 - \tan^2 x = (\cos x + 2)^2 - \tan^2 x$

$= (\cos x + 2 + \tan x)(\cos x + 2 - \tan x)$

0.9 Simplifying

Rewrite each of the following in simplest form:

(a) $\dfrac{(x - 1)(x + 3) - (x + 1)^2}{x + 1}$

(b) $\dfrac{\sqrt{x^2 + 1} - \dfrac{1}{\sqrt{x^2 + 1}}}{x^2 + 1}$

(c) $\dfrac{x^2 - 5x + 6}{x^2 - 4x + 4}$

(d) $\dfrac{1}{x + 1} - \dfrac{1}{x - 1} - \dfrac{2}{x^2 - 1}$

(e) $\dfrac{x(-2x)}{2\sqrt{1 - x^2}} + \sqrt{1 - x^2} + \dfrac{1}{\sqrt{1 - x^2}}$

Solution:

(a) $\dfrac{(x - 1)(x + 3) - (x + 1)^2}{x + 1} = \dfrac{(x^2 + 2x - 3) - (x^2 + 2x + 1)}{x + 1} = \dfrac{-4}{x + 1}$

(b) $\dfrac{\sqrt{x^2 + 1} - \dfrac{1}{\sqrt{x^2 + 1}}}{x^2 + 1} = \dfrac{\dfrac{1}{\sqrt{x^2 + 1}}(x^2 + 1 - 1)}{x^2 + 1} = \dfrac{x^2 + 1 - 1}{\sqrt{x^2 + 1}(x^2 + 1)} = \dfrac{x^2}{(x^2 + 1)^{3/2}}$

(c) $\dfrac{x^2 - 5x + 6}{x^2 - 4x + 4} = \dfrac{(x - 2)(x - 3)}{(x - 2)^2} = \dfrac{x - 3}{x - 2}$

(d) $\dfrac{1}{x + 1} - \dfrac{1}{x - 1} - \dfrac{2}{x^2 - 1} = \dfrac{(x - 1) - (x + 1) - 2}{x^2 - 1} = \dfrac{-4}{x^2 - 1}$

(e) $\dfrac{x(-2x)}{2\sqrt{1 - x^2}} + \sqrt{1 - x^2} + \dfrac{1}{\sqrt{1 - x^2}} = \dfrac{-x^2}{\sqrt{1 - x^2}} + \dfrac{1 - x^2}{\sqrt{1 - x^2}} + \dfrac{1}{\sqrt{1 - x^2}} = \dfrac{2 - 2x^2}{\sqrt{1 - x^2}}$

$= \dfrac{2(1 - x^2)}{\sqrt{1 - x^2}} = 2\sqrt{1 - x^2}$

0.10 Rationalizing

Remove the sum or difference from the denominator by multiplying the numerator and denominator by the conjugate of the denominator.

(a) $\dfrac{1}{1 - \cos x}$

(b) $\dfrac{x}{1 - \sqrt{x^2 + 1}}$

(c) $\dfrac{2}{x + \sqrt{x^2 + 1}}$

Solution:

(a) $\dfrac{1}{1 - \cos x} = \left(\dfrac{1}{1 - \cos x}\right)\left(\dfrac{1 + \cos x}{1 + \cos x}\right)$

$= \dfrac{1 + \cos x}{1 - \cos^2 x} = \dfrac{1 + \cos x}{\sin^2 x}$

—CONTINUED—

—CONTINUED—

(b) $\left(\dfrac{x}{1-\sqrt{x^2+1}}\right)\left(\dfrac{1+\sqrt{x^2+1}}{1+\sqrt{x^2+1}}\right) = \dfrac{x\left(1+\sqrt{x^2+1}\right)}{1-(x^2+1)}$

$$= \frac{x\left(1+\sqrt{x^2+1}\right)}{-x^2} = \frac{1+\sqrt{x^2+1}}{-x}$$

(c) $\left(\dfrac{2}{x+\sqrt{x^2+1}}\right)\left(\dfrac{x-\sqrt{x^2+1}}{x-\sqrt{x^2+1}}\right) = \dfrac{2\left(x-\sqrt{x^2+1}\right)}{x^2-(x^2+1)} = -2\left(x-\sqrt{x^2+1}\right)$

0.11 Algebraic Errors to Avoid

Error	*Correct form*	*Comments*
$a-(x-b) \neq a-x-b$	$a-(x-b) = a-x+b$	Change all signs when distribution negative through parentheses.
$(a+b)^2 \neq a^2+b^2$	$(a+b)^2 = a^2+2ab+b^2$	Don't forget middle term when squaring binomials.
$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) \neq \frac{1}{2}ab$	$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) = \frac{1}{4}(ab)$	1/2 occurs twice as a factor.
$\dfrac{a}{x+b} \neq \dfrac{a}{x}+\dfrac{a}{b}$	Leave as $\dfrac{a}{x+b}$	Don't add denominators when adding fractions.
$\dfrac{1}{a}+\dfrac{1}{b} \neq \dfrac{1}{a+b}$	$\dfrac{1}{a}+\dfrac{1}{b} = \dfrac{a+b}{ab}$	Use definition for adding fractions.
$\dfrac{\frac{x}{a}}{b} \neq \dfrac{bx}{a}$	$\dfrac{\frac{x}{a}}{b} = \left(\dfrac{x}{a}\right)\left(\dfrac{1}{b}\right) = \dfrac{x}{ab}$	Multiply by reciprocal of the denominator.
$\dfrac{1}{3x} \neq \dfrac{1}{3}x$	$\dfrac{1}{3x} = \dfrac{1}{3}\cdot\dfrac{1}{x}$	Use definition for multiplying fractions.
$1/x+2 \neq \dfrac{1}{x+2}$	$1/x+2 = \dfrac{1}{x}+2$	Be careful when using a slash to denote division.
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^{2\cdot 3} = x^6$	Multiply exponents when an exponential form is raised to a power.
$2x^3 \neq (2x)^3$	$2x^3 = 2(x^3)$	Exponents have priority over coefficients.
$\dfrac{1}{x^2+x^3} \neq x^{-2}+x^{-3}$	Leave as $\dfrac{1}{x^2+x^3}$	Don't shift term-by-term from denominator to numerator.
$\sqrt{5x} \neq 5\sqrt{x}$	$\sqrt{5x} = \sqrt{5}\sqrt{x}$	Radicals apply to every factor inside radical.
$\sqrt{x^2+a^2} \neq x+a$	Leave as $\sqrt{x^2+a^2}$	Don't apply radicals term-by-term.
$\dfrac{a+bx}{a} \neq 1+bx$	$\dfrac{a+bx}{a} = 1+\dfrac{b}{a}x$	Cancel common factor, *not* common terms.
$\dfrac{a+ax}{a} \neq a+x$	$\dfrac{a+ax}{a} = 1+x$	Factor *before* canceling.

Review Problems: Copy the problem and SHOW your work on a separate sheet.

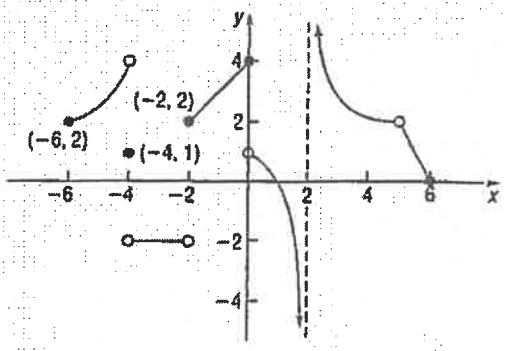
In Problems 1–22, find the limit. Use the TABLE feature of a graphing utility to verify your answer.

1. $\lim_{x\to 2}(3x^2 - 2x + 1)$
2. $\lim_{x\to 1}(-2x^3 + x + 4)$
3. $\lim_{x\to -2}(x^2 + 1)^2$
4. $\lim_{x\to -2}(x^3 + 1)^2$
5. $\lim_{x\to 3}\sqrt{x^2 + 7}$
6. $\lim_{x\to -2}\sqrt[3]{x + 10}$
7. $\lim_{x\to 1^-}\sqrt{1 - x^2}$
8. $\lim_{x\to 2^+}\sqrt{3x - 2}$
9. $\lim_{x\to 2}(5x + 6)^{3/2}$
10. $\lim_{x\to -3}(15 - 3x)^{-3/2}$
11. $\lim_{x\to -1}\dfrac{x^2 + x + 2}{x^2 - 9}$
12. $\lim_{x\to 3}\dfrac{3x + 4}{x^2 + 1}$
13. $\lim_{x\to 1}\dfrac{x - 1}{x^3 - 1}$
14. $\lim_{x\to -1}\dfrac{x^2 - 1}{x^2 + x}$
15. $\lim_{x\to -3}\dfrac{x^2 - 9}{x^2 - x - 12}$
16. $\lim_{x\to -3}\dfrac{x^2 + 2x - 3}{x^2 - 9}$
17. $\lim_{x\to -1^-}\dfrac{x^2 - 1}{x^3 - 1}$
18. $\lim_{x\to 2^-}\dfrac{x^2 - 4}{x^3 - 8}$
19. $\lim_{x\to 2}\dfrac{x^3 - 8}{x^3 - 2x^2 + 4x - 8}$
20. $\lim_{x\to 1}\dfrac{x^3 - 1}{x^3 - x^2 + 3x - 3}$
21. $\lim_{x\to 3}\dfrac{x^4 - 3x^3 + x - 3}{x^3 - 3x^2 + 2x - 6}$
22. $\lim_{x\to -1}\dfrac{x^4 + x^3 + 2x + 2}{x^3 + x^2}$

In Problems 23–30, determine whether f is continuous at c.

23. $f(x) = 3x^4 - x^2 + 2 \quad c = 5$
24. $f(x) = \dfrac{x^2 - 9}{x + 10} \quad c = 2$
25. $f(x) = \dfrac{x^2 - 4}{x + 2} \quad c = -2$
26. $f(x) = \dfrac{x^2 + 6x}{x^2 - 6x} \quad c = 0$
27. $f(x) = \begin{cases} \dfrac{x^2 - 4}{x + 2} & \text{if } x \neq -2 \\ 4 & \text{if } x = -2 \end{cases} \quad c = -2$
28. $f(x) = \begin{cases} \dfrac{x^2 + 6x}{x^2 - 6x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases} \quad c = 0$
29. $f(x) = \begin{cases} \dfrac{x^2 - 4}{x + 2} & \text{if } x \neq -2 \\ -4 & \text{if } x = -2 \end{cases} \quad c = -2$
30. $f(x) = \begin{cases} \dfrac{x^2 + 6x}{x^2 - 6x} & \text{if } x \neq 0 \\ -1 & \text{if } x = 0 \end{cases} \quad c = 0$

In Problems 31–50, use the accompanying graph of $y = f(x)$.



31. What is the domain of f?
32. What is the range of f?
33. Find the x-intercept(s), if any, of f.
34. Find the y-intercept(s), if any, of f.
35. Find $f(-6)$ and $f(-4)$.
36. Find $f(-2)$ and $f(6)$.
37. Find $\lim_{x\to -4^-} f(x)$.
38. Find $\lim_{x\to -4^+} f(x)$.
39. Find $\lim_{x\to -2^-} f(x)$.
40. Find $\lim_{x\to -2^-} f(x)$.
41. Find $\lim_{x\to -2^-} f(x)$.
42. Find $\lim_{x\to 2^+} f(x)$.
43. Does $\lim_{x\to 0} f(x)$ exist? If it does, what is it?
44. Does $\lim_{x\to -2} f(x)$ exist? If it does, what is it?
45. Is f continuous at -2?
46. Is f continuous at -4?
47. Is f continuous at 0?
48. Is f continuous at 2?
49. Is f continuous at 4?
50. Is f continuous at 5?

Limits #5A back (Review #3)

15) $\lim_{x\to -3} \frac{x^2-9}{x^2-x-12} = \frac{(x+3)(x-3)}{(x+3)(x-4)} = \frac{x-3}{x-4} = \frac{-3-3}{-3-4} = \boxed{\frac{6}{7}}$

16) $\lim_{x\to -3} \frac{x^2+2x-3}{x^2-9} = \frac{(x+3)(x-1)}{(x+3)(x-3)} = \frac{x-1}{x-3} = \frac{-3-1}{-3-3} = \frac{-4}{-6} = \boxed{\frac{2}{3}}$

17) $\lim_{x\to -1^-} \frac{x^2-1}{x^3-1} = \frac{(x+1)(x-1)}{(x-1)(x^2+x+1)} = \frac{(x+1)}{(x^2+x+1)} = \frac{-1+1}{1-1+1} = \boxed{0}$

18) $\lim_{x\to 2^+} \frac{x^2-4}{x^3-8} = \frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)} = \frac{x+2}{x^2+2x+4} = \frac{4}{4+4+4} = \frac{4}{12} = \boxed{\frac{1}{3}}$

19) $\lim_{x\to 2} \frac{x^3-8}{x^3-2x^2+4x-8} = \frac{(x-2)(x^2+2x+4)}{x^2(x-2)+4(x-2)} = \frac{(x-2)(x^2+2x+4)}{(x-2)(x^2+4)} = \frac{2^2+2(2)+4}{2^2+4} = \frac{12}{8} = \boxed{\frac{3}{2}}$

20) $\lim_{x\to 1} \frac{x^3-1}{x^3-x^2+3x-3} = \frac{(x-1)(x^2+x+1)}{x^2(x-1)+3(x-1)} = \frac{(x-1)(x^2+x+1)}{(x-1)(x^2+3)} = \frac{1+1+1}{1+3} = \boxed{\frac{3}{4}}$

21) $\lim_{x\to 3} \frac{x^4-3x^3+x-3}{x^3-3x^2+2x-6} = \frac{x^3(x-3)+1(x-3)}{x^2(x-3)+2(x-3)} = \frac{(x^3+1)(x-3)}{(x^2+2)(x-3)}$

$\frac{3^3+1}{3^2+2} = \boxed{\frac{28}{11}}$

22) $\lim_{x\to -1} \frac{x^4+x^3+2x+2}{x^3+x^2} = \frac{x^3(x+1)+2(x+1)}{x^2(x+1)} = \frac{(x^3+2)(x+1)}{x^2(x+1)} = \frac{1}{1} = \boxed{1}$

23) $3(5)^4 - 5^2 + 2 = 1852$ Continuous $f(5)$ exists

25) $\frac{x^2-4}{x+2} = \frac{(x+2)(x-2)}{(x+2)}$ not continuous hole @ $x=-2$

24) $\frac{x^2-9}{x+10} = \frac{4-9}{4+10} = \frac{-5}{14}$ Continuous $f(2)$ exists

26) $\frac{x^2+6x}{x^2-6x} \quad \frac{x(x+6)}{x(x-6)}$ not continuous hole @ $x=0$

27) $\begin{cases} \frac{x^2-4}{x+2} & x \neq -2 \\ 4 & x=-2 \end{cases}$ not continuous hole @ $x=-2$ $f(-2) \neq \lim_{x\to -2} f(x)$ *

* $\lim_{x\to -2} f(x) = -4$ $(x+2)(x-2)$ (-4)

Limits #5A back (Review #3)

1) $\lim_{x\to 2} (3x^2-2x+1) = 3(2)^2-2(2)+1 = \boxed{9}$

2) $\lim_{x\to 1} (-2x^3+x+4) = -2(1)^3+1+4 = \boxed{3}$

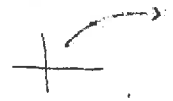
3) $\lim_{x\to 2} (x^2+1)^2 - (2^2+1)^2 = \boxed{25}$

4) $\lim_{x\to -2} (x^3+1)^2 = ((-2)^3+1)^2 = (-7)^2 = \boxed{49}$

5) $\lim_{x\to 3} \sqrt{x^2+7} = \sqrt{3^2+7} = \sqrt{16} = \boxed{4}$

6) $\lim_{x\to -2} \sqrt[3]{x+10} = \sqrt[3]{-2+10} = \boxed{2}$

7) $\lim_{x\to 1^-} \sqrt{1-x^2}$ graph  $= \boxed{0}$

8) $\lim_{x\to 2^+} \sqrt{3x-2} = \boxed{2}$ graph 

9) $\lim_{x\to 2} (5x+6)^{3/2} = 16^{3/2} = (\sqrt{16})^3 = \boxed{64}$

10) $\lim_{x\to -3} (15-3x)^{-3/2} = (15+9)^{-3/2} = \frac{1}{(24)^{3/2}} = \frac{1}{(\sqrt{24})^3} = \frac{1}{(2\sqrt{6})^3} = \frac{1}{8\cdot 6\sqrt{6}}$ (4·6)

$= \frac{1}{48\sqrt{6}} = \boxed{\frac{\sqrt{6}}{288}}$

11) $\lim_{x\to -1} \frac{x^2+x+2}{x^2-9} = \frac{1-1+2}{1-9} = \frac{2}{-8} = \boxed{\frac{-1}{4}}$

12) $\lim_{x\to 3} \frac{3x+4}{x^2+1} = \frac{9+4}{9+1} = \boxed{\frac{13}{10}}$

13) $\lim_{x\to 1} \frac{x-1}{x^3-1} = \frac{(x-1)}{(x-1)(x^2+x+1)} = \frac{1}{x^2+x+1} = \frac{1}{1+1+1} = \boxed{\frac{1}{3}}$

14) $\lim_{x\to -1} \frac{x^2-1}{x^2+x} = \frac{(x+1)(x-1)}{x(x+1)} = \frac{x-1}{x} = \frac{-1-1}{-1} = \boxed{2}$

$\frac{0}{0}$ try something else

Limits 5A back (Review #3)

$\lim_{x\to 0} \frac{x(x+6)}{x(x-6)} = \frac{-1}{5}$

28) $\begin{cases} \frac{x^2+6x}{x^2-6x} & x \neq 0 \\ 1 & x = 0 \end{cases}$ not Continuous, hole @ $x=0$ $\quad f(0) \neq \lim_{x\to 0} f(x)$

29) $\begin{cases} \frac{x^2-4}{x+2} & x \neq -2 \\ -4 & x = -2 \end{cases}$ $\lim_{x\to -2} \frac{(x+2)(x-2)}{(x+2)} = -4$ (−2−2) hole at $x=-2$ Continuous $f(-2) = \lim_{x\to -2} f(x)$

30) $\begin{cases} \frac{x^2+6x}{x^2-6x} & x \neq 0 \\ -1 & x = 0 \end{cases}$ $\lim_{x\to 0} \frac{x(x+6)}{x(x-6)} = \frac{6}{-6} = -1$ hole at $x=0$ Continuous $f(0) = \lim_{x\to 0} f(x)$

31) D $[-6, 2) \cup (2, 5) \cup (5, 6]$

32) R $(-\infty, +\infty)$

33) x-int $(6, 0)$ & $(1, 0)$

34) Y-int. $(0, 4)$

35) $f(-6) = 2 \quad f(-4) = 1$

36) $f(-2) = 2 \quad f(6) = 0$

37) $\lim_{x\to -4^-} f(x) = 4$

38) $\lim_{x\to -4^+} f(x) = -2$

39) $\lim_{x\to -2^-} f(x) = -2$

40) $\lim_{x\to -2^+} f(x) = 2$

41) $\lim_{x\to 2^-} f(x) = -\infty$

42) $\lim_{x\to 2^+} f(x) = +\infty$

43) $\lim_{x\to 0} f(x) = \text{DNE}$ Step or jump-function

44) $\lim_{x\to 2} f(x) = \text{DNE}$ approaches different values from neg & pos sides

45) f continuous @ −2 NO step

46) f cont @ −4 NO step

47) f cont @ 0 NO step

48) f cont @ 2 NO assump.

49) f cont @ 4 YES

50) f cont @ 5 NO hole @ 5

Differentiation Rules

General Differentiation Rules (let u and v be differentiable functions of x)

Constant Multiple Rule:

$\frac{d}{dx}[cu] = cu'$

Sum or Difference Rule:

$\frac{d}{dx}[u \pm v] = u' + v'$

Product Rule:

$\frac{d}{dx}[uv] = uv' + vu'$

Quotient Rule:

$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$

Derivatives of Algebraic Functions

Constant Rule:

$\frac{d}{dx}[c] = 0$

Simple Power Rule:

$\frac{d}{dx}[x^n] = nx^{n-1}, \frac{d}{dx}[x] = 1$

General Power Rule:

$\frac{d}{dx}[u^n] = nu^{n-1}u'$

Derivatives of Trigonometric Functions

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = sec^2 x$

$\frac{d}{dx}[\cot x] = -csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

Chain Rule: $\frac{d}{dx}[f(u)] = f'(u)u'$

Others: $\frac{d}{dx}[e^x] = e^x$ $\frac{d}{dx}[\ln x] = \frac{1}{x}$

Theorem: The Chain Rule

If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ or, equivalently, $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Using the Chain Rule: Find $\frac{dy}{dx}$ for $y = (x^2 + 1)^3$

Solution: For this function, you can consider the inside function to be $u = x^2 + 1$. By the Chain Rule, you obtain

$\frac{dy}{dx} = 3(x^2 + 1)^2(2x) = 6x(x^2 + 1)^2$

$\frac{dy}{du}$ $\frac{du}{dx}$

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Differentiation - Power, Constant, and Sum Rules

Date_____ Period____

Differentiate each function with respect to x.

1) $y = 5$

2) $f(x) = 5x^{18}$

3) $y = 4x^5 + x$

4) $f(x) = 4x^4 - 5x - 3$

5) $y = 3x^{\frac{5}{4}}$

6) $y = \frac{5}{4}x^{\frac{2}{3}}$

7) $y = -4x^{-5}$

8) $y = \frac{3}{x^3}$

9) $y = x^{\frac{2}{3}}$

10) $f(x) = -2\sqrt[4]{x}$

Kuta Software - Infinite Precalculus Name_____

Power, Constant, and Sum Rules

Date_____ Period____

Differentiate each function with respect to x.

1) $y = 6$

2) $y = -8$

3) $y = 7$

4) $y = x^5$

5) $y = 5x^9$

6) $y = -8x^4$

7) $y = 5x^7 + 4x$

8) $y = x^3 - 8x^2$

9) $y = 4x^6 - 7x^5 - 9x^3$

10) $y = 8x^7 + 5x^5 + 9x^3$

Critical thinking question:

11) a, b, and c are constants. If $y = x^{3b} + 3x^{4c}$, what is $\frac{dy}{dx}$?

Kuta Software - Infinite Calculus Name_____

Differentiation - Power, Constant, and Sum Rules

Date_____ Period____

Differentiate each function with respect to x.

1) $y = 5$

$$\frac{dy}{dx} = 0$$

2) $f(x) = 5x^{18}$

$$f'(x) = 90x^{17}$$

3) $y = 4x^5 + x$

$$\frac{dy}{dx} = 20x^4 + 1$$

4) $f(x) = 4x^4 - 5x - 3$

$$f'(x) = 16x^3 - 5$$

5) $y = 3x^{\frac{5}{4}}$

$$\frac{dy}{dx} = \frac{15x^{\frac{1}{4}}}{4}$$

6) $y = \frac{5}{4}x^{\frac{2}{3}}$

$$\frac{dy}{dx} = \frac{5}{6}x^{-\frac{1}{3}} = \frac{5}{6x^{\frac{1}{3}}}$$

7) $y = -4x^{-5}$

$$\frac{dy}{dx} = 20x^{-6} = \frac{20}{x^6}$$

8) $y = \frac{3}{x^3}$

$$\frac{dy}{dx} = -9x^{-4} = -\frac{9}{x^4}$$

9) $y = x^{\frac{2}{3}}$

$$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3x^{\frac{1}{3}}}$$

10) $f(x) = -2\sqrt[4]{x}$

$$f'(x) = -\frac{1}{2}x^{-\frac{3}{4}} = -\frac{1}{2x^{\frac{3}{4}}}$$

Kuta Software - Infinite Precalculus Name_____

Power, Constant, and Sum Rules

Date_____ Period____

Differentiate each function with respect to x.

1) $y = 6$

$$\frac{dy}{dx} = 0$$

2) $y = -8$

$$\frac{dy}{dx} = 0$$

3) $y = 7$

$$\frac{dy}{dx} = 0$$

4) $y = x^5$

$$\frac{dy}{dx} = 5x^4$$

5) $y = 5x^9$

$$\frac{dy}{dx} = 45x^8$$

6) $y = -8x^4$

$$\frac{dy}{dx} = -32x^3$$

7) $y = 5x^7 + 4x$

$$\frac{dy}{dx} = 35x^6 + 4$$

8) $y = x^3 - 8x^2$

$$\frac{dy}{dx} = 3x^2 - 16x$$

9) $y = 4x^6 - 7x^5 - 9x^3$

$$\frac{dy}{dx} = 24x^5 - 35x^4 - 27x^2$$

10) $y = 8x^7 + 5x^5 + 9x^3$

$$\frac{dy}{dx} = 56x^6 + 25x^4 + 27x^2$$

Critical thinking question:

11) a, b, and c are constants. If $y = x^{3b} + 3x^{4c}$, what is $\frac{dy}{dx}$?

$$\frac{dy}{dx} = 3bx^{3b-1} + 12cx^{4c-1}$$

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Name_____

Differentiation - Quotient Rule

Date_____ Period____

Differentiate each function with respect to x.

1) $y = \frac{2}{2x^4 - 5}$

2) $f(x) = \frac{2}{x^5 - 5}$

3) $f(x) = \frac{5}{4x^3 + 4}$

4) $y = \frac{4x^3 - 3x^2}{4x^5 - 4}$

5) $y = \frac{3x^4 + 2}{3x^3 - 2}$

6) $y = \frac{4x^5 + 2x^2}{3x^4 + 5}$

7) $y = \frac{4x^5 + x^2 + 4}{5x^2 - 2}$

8) $y = \frac{3x^4 + 5x^3 - 5}{2x^4 - 4}$

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Name_____

Differentiation - Product Rule

Date_____ Period____

Differentiate each function with respect to x.

1) $y = -x^3(3x^4 - 2)$

2) $f(x) = x^2(-3x^2 - 2)$

3) $y = (-2x^4 - 3)(-2x^2 + 1)$

4) $f(x) = (2x^4 - 3)(x^2 + 1)$

5) $f(x) = (5x^5 + 5)(-2x^5 - 3)$

6) $f(x) = (-3 + x^{-3})(-4x^3 + 3)$

7) $y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$

8) $y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$

Kuta Software - Infinite Calculus Name_____

Differentiation - Quotient Rule

Date_____ Period____

Differentiate each function with respect to x.

1) $y = \dfrac{2}{2x^4 - 5}$

$$\frac{dy}{dx} = -\frac{2 \cdot 8x^3}{(2x^4 - 5)^2} = -\frac{16x^3}{4x^8 - 20x^4 + 25}$$

2) $f(x) = \dfrac{2}{x^5 - 5}$

$$f'(x) = -\frac{2 \cdot 5x^4}{(x^5 - 5)^2} = -\frac{10x^4}{x^{10} - 10x^5 + 25}$$

3) $f(x) = \dfrac{5}{4x^3 + 4}$

$$f'(x) = -\frac{5 \cdot 12x^2}{(4x^3 + 4)^2} = -\frac{15x^2}{4x^6 + 8x^3 + 4}$$

4) $y = \dfrac{4x^3 - 3x^2}{4x^5 - 4}$

$$\frac{dy}{dx} = \frac{(4x^5 - 4)(12x^2 - 6x) - (4x^3 - 3x^2) \cdot 20x^4}{(4x^5 - 4)^2} = \frac{-8x^7 + 9x^6 - 12x^2 + 6x}{4x^{10} - 8x^5 + 4}$$

5) $y = \dfrac{3x^4 + 2}{3x^3 - 2}$

$$\frac{dy}{dx} = \frac{(3x^3 - 2) \cdot 12x^3 - (3x^4 + 2) \cdot 9x^2}{(3x^3 - 2)^2} = \frac{9x^6 - 24x^3 - 18x^2}{9x^6 - 12x^3 + 4}$$

6) $y = \dfrac{4x^5 + 2x^2}{3x^4 + 5}$

$$\frac{dy}{dx} = \frac{(3x^4 + 5)(20x^4 + 4x) - (4x^5 + 2x^2) \cdot 12x^3}{(3x^4 + 5)^2} = \frac{12x^8 - 12x^5 + 100x^4 + 20x}{9x^8 + 30x^4 + 25}$$

7) $y = \dfrac{4x^5 + x^2 + 4}{5x^2 - 2}$

$$\frac{dy}{dx} = \frac{(5x^2 - 2)(20x^4 + 2x) - (4x^5 + x^2 + 4) \cdot 10x}{(5x^2 - 2)^2} = \frac{60x^6 - 40x^4 - 44x}{25x^4 - 20x^2 + 4}$$

8) $y = \dfrac{3x^4 + 5x^3 - 5}{2x^4 - 4}$

$$\frac{dy}{dx} = \frac{(2x^4 - 4)(12x^3 + 15x^2) - (3x^4 + 5x^3 - 5) \cdot 8x^3}{(2x^4 - 4)^2} = \frac{-5x^6 - 4x^3 - 30x^2}{2x^8 - 8x^4 + 8}$$

Kuta Software - Infinite Calculus Name_____

Differentiation - Product Rule

Date_____ Period____

Differentiate each function with respect to x.

1) $y = -x^3(3x^4 - 2)$

$$\frac{dy}{dx} = -x^3 \cdot 12x^3 + (3x^4 - 2) \cdot -3x^2 = -21x^6 + 6x^2$$

2) $f(x) = x^2(-3x^2 - 2)$

$$f'(x) = x^2 \cdot -6x + (-3x^2 - 2) \cdot 2x = -12x^3 - 4x$$

3) $y = (-2x^4 - 3)(-2x^2 + 1)$

$$\frac{dy}{dx} = (-2x^4 - 3) \cdot -4x + (-2x^2 + 1) \cdot -8x^3 = 24x^5 - 8x^3 + 12x$$

4) $f(x) = (2x^4 - 3)(x^2 + 1)$

$$f'(x) = (2x^4 - 3) \cdot 2x + (x^2 + 1) \cdot 8x^3 = 12x^5 + 8x^3 - 6x$$

5) $f(x) = (5x^5 + 5)(-2x^5 - 3)$

$$f'(x) = (5x^5 + 5) \cdot -10x^4 + (-2x^5 - 3) \cdot 25x^4 = -100x^9 - 125x^4$$

6) $f(x) = (-3 + x^{-3})(-4x^3 + 3)$

$$f'(x) = (-3 + x^{-3}) \cdot -12x^2 + (-4x^3 + 3) \cdot -3x^{-4} = 36x^2 - \frac{9}{x^4}$$

7) $y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$

$$\frac{dy}{dx} = (-2x^4 + 5x^2 + 4) \cdot -6x + (-3x^2 + 2)(-8x^3 + 10x) = 36x^5 - 76x^3 - 4x$$

8) $y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$

$$\frac{dy}{dx} = (x^4 + 3)(-20x^4 + 20x^3) + (-4x^5 + 5x^4 + 5) \cdot 4x^3 = -36x^8 + 40x^7 - 60x^4 + 80x^3$$

Handout - Derivative - Chain Rule & $Sin(x)$, $Cos(x)$, e^x, $\ln(x)$

Power-Chain Rule a, b are constants.

Function	Derivative	
$y = a \cdot x^n$	$\frac{dy}{dx} = a \cdot n \cdot x^{n-1}$	Power Rule
$y = a \cdot u^n$	$\frac{dy}{dx} = a \cdot n \cdot u^{n-1} \cdot \frac{du}{dx}$	**Power-Chain Rule**

Exercises

Find the derivatives of the expressions

a) $6(9x+26)^9$

b) $6(8x+29)^8$

c) $5\left(x^2+3x+23\right)^9$

d) $5\left(7x^2+2x+22\right)^6$

e) $7\left(2x^2+5x+28\right)^{25/3}$

f) $8\left(5x^2+7x+23\right)^{16/3}$

g) $4\sqrt{4x^2+3x+26}$

h) $6\sqrt{2x^2+5x+27}$

i) $\frac{9}{\sqrt{3x^2+6x+29}}$

j) $\frac{9}{\sqrt{5x^2+6x+25}}$

Answers a) $486(9x+26)^8$; b) $384(8x+29)^7$;
c) $45(2x+3)\left(x^2+3x+23\right)^8$; d) $30(14x+2)\left(7x^2+2x+22\right)^5$;
e) $\frac{175}{3}(4x+5)\left(2x^2+5x+28\right)^{22/3}$; f) $\frac{128}{3}(10x+7)\left(5x^2+7x+23\right)^{13/3}$;
g) $\frac{2(8x+3)}{\sqrt{4x^2+3x+26}}$; h) $\frac{3(4x+5)}{\sqrt{2x^2+5x+27}}$;
i) $-\frac{9(6x+6)}{2(3x^2+6x+29)^{3/2}}$; j) $-\frac{9(10x+6)}{2(5x^2+6x+25)^{3/2}}$;

Sine and Cosine - Chain Rules a, b are constants.

Function	Derivative	
$y = \sin(x)$	$\frac{dy}{dx} = \cos(x)$	Sine Rule
$y = \cos(x)$	$\frac{dy}{dx} = -\sin(x)$	Cosine Rule
$y = a \cdot \sin(u)$	$\frac{dy}{dx} = a \cdot \cos(u) \cdot \frac{du}{dx}$	**Chain-Sine Rule**
$y = a \cdot \cos(u)$	$\frac{dy}{dx} = -a \cdot \sin(u) \cdot \frac{du}{dx}$	**Chain-Cosine Rule**

Exercises

Find the derivatives of the expressions

a) $7\cos\left(2x^2+5x+9\right)$

b) $8\cos\left(5x^3+5x^2+2\right)$

c) $5\cos\left(x^3+3x^2+1\right)$

d) $7\cos\left(7x^2+1\right)$

e) $\left(6\sin\left(4x^4\right)+4\cos(3x)\right)^3$

f) $\left(7\sin\left(9x^2\right)+5\cos\left(6x^5\right)\right)^4$

g) $\left(8\sin\left(6x^4\right)+\cos\left(8x^4\right)\right)^3$

h) $\left(4\sin\left(8x^4\right)+\cos\left(8x^2\right)\right)^4$

i) $\left(7\sin(4x)+6\cos\left(6x^4\right)\right)^{\frac{5}{2}}$

j) $\left(7\sin(5x)+9\cos\left(4x^3\right)\right)^{\frac{1}{2}}$

Answers a) $-7(4x+5)\sin\left(2x^2+5x+9\right)$; b) $-8\left(15x^2+10x\right)\sin\left(5x^3+5x^2+2\right)$;
c) $-5\left(3x^2+6x\right)\sin\left(x^3+3x^2+1\right)$; d) $-98x\sin\left(7x^2+1\right)$;
e) $3\cdot\left(96x^3\cos\left(4x^4\right)-12\sin(3x)\right)\cdot\left(6\sin\left(4x^4\right)+4\cos(3x)\right)^2$;
f) $4\cdot\left(126x\cos\left(9x^2\right)-150x^4\sin\left(6x^5\right)\right)\cdot\left(7\sin\left(9x^2\right)+5\cos\left(6x^5\right)\right)^3$;
g) $3\cdot\left(192x^3\cos\left(6x^4\right)-32x^3\sin\left(8x^4\right)\right)\cdot\left(8\sin\left(6x^4\right)+\cos\left(8x^4\right)\right)^2$;
h) $4\cdot\left(128x^3\cos\left(8x^4\right)-16x\sin\left(8x^2\right)\right)\cdot\left(4\sin\left(8x^4\right)+\cos\left(8x^2\right)\right)^3$;
i) $\frac{5}{2}\cdot\left(28\cos(4x)-144x^3\sin\left(6x^4\right)\right)\cdot\left(7\sin(4x)+6\cos\left(6x^4\right)\right)^{\frac{3}{2}}$;
j) $\frac{1}{2}\cdot\left(35\cos(5x)-108x^2\sin\left(4x^3\right)\right)\cdot\left(7\sin(5x)+9\cos\left(4x^3\right)\right)^{-\frac{1}{2}}$;

Exponent and Logarithmic - Chain Rules a,b are constants.

Function	Derivative	
$y = e^x$	$\frac{dy}{dx} = e^x$	Exponential Function Rule
$y = \ln(x)$	$\frac{dy}{dx} = \frac{1}{x}$	Logarithmic Function Rule
$y = a \cdot e^u$	$\frac{dy}{dx} = a \cdot e^u \cdot \frac{du}{dx}$	**Chain-Exponent Rule**
$y = a \cdot \ln(u)$	$\frac{dy}{dx} = \frac{a}{u} \cdot \frac{du}{dx}$	**Chain-Log Rule**

Exercises

Find the derivatives of the expressions

a) $3e^{5x+27}$

b) $2e^{2x+22}$

c) $4\ln(2x+21)$

d) $4\ln(8x+25)$

e) $7e^{6x^2+6x+29}$

f) $4e^{6x^2+9x+28}$

g) $9\ln\left(3x^2+4x+21\right)$

h) $3\ln\left(3x^2+8x+21\right)$

i) $-e^{4x^{5/2}+4x^{7/3}}$

j) $2\ln\left(2x^{5/2}+5x^{4/3}\right)$

Answers a) $15e^{5x+27}$; b) $4e^{2x+22}$; c) $\frac{8}{2x+21}$; d) $\frac{32}{8x+25}$;
e) $7\cdot(12x+6)\cdot e^{6x^2+6x+29}$; f) $4\cdot(12x+9)\cdot e^{6x^2+9x+28}$;
g) $\frac{9(6x+4)}{3x^2+4x+21}$; h) $\frac{3(6x+8)}{3x^2+8x+21}$;
i) $-1\cdot\left(10x^{3/2}+\frac{28x^{4/3}}{3}\right)\cdot e^{4x^{5/2}+4x^{7/3}}$; j) $2\cdot\left(5x^{3/2}+\frac{20\sqrt[3]{x}}{3}\right)\cdot\frac{1}{2x^{5/2}+5x^{4/3}}$;

Exercises

Find the derivatives of the expressions

a) $5\cos(4x) + 3e^{3x+1} - 2\sqrt[5]{2x-1}$

b) $4\sin(3x) + 2e^{x+1} + 4\sqrt[5]{2x-1}$

c) $2\cos(4x) + 3\ln(4x+1) + 5(4x-1)^{2/3}$

d) $5\sin(3x) + 5e^{x-1} - \frac{4}{(x-1)^{5/8}}$

e) $\left(\sin(4x) + 2e^{4x} - \frac{5}{\sqrt[3]{x}}\right)^3$

f) $\left(5\sin(4x) + 2\ln(3x) + \frac{6}{\sqrt{x}}\right)^3$

g) $\left(4\sin(2x) + e^{5x} + \frac{4}{\sqrt[5]{x}}\right)^{\frac{1}{2}}$

h) $\left(\sin(4x) + 2\ln(5x) + 4\sqrt[7]{x}\right)^{-\frac{1}{2}}$

Answers a) $-20\sin(4x) + 9e^{3x+1} - \frac{4}{5(2x-1)^{4/5}}$; b) $12\cos(3x) + 2e^{x+1} + \frac{8}{5(2x-1)^{4/5}}$;

c) $-8\sin(4x) + \frac{12}{4x+1} + \frac{40}{3\sqrt[3]{4x-1}}$; d) $15\cos(3x) + 5e^{x-1} + \frac{5}{2(x-1)^{13/8}}$;

e) $3 \cdot \left(4\cos(4x) + 8e^{4x} + \frac{5}{3x^{4/3}}\right) \cdot \left(\sin(4x) + 2e^{4x} - \frac{5}{\sqrt[3]{x}}\right)^2$;

f) $3 \cdot \left(20\cos(4x) + \frac{2}{x} - \frac{3}{x^{5/2}}\right) \cdot \left(5\sin(4x) + 2\ln(3x) + \frac{6}{\sqrt{x}}\right)^2$;

g) $\frac{1}{2} \cdot \left(8\cos(2x) + 5e^{5x} - \frac{4}{5x^{6/5}}\right) \cdot \left(4\sin(2x) + e^{5x} + \frac{4}{\sqrt[5]{x}}\right)^{-\frac{1}{2}}$;

h) $-\frac{1}{2} \cdot \left(4\cos(4x) + \frac{2}{x} + \frac{4}{7x^{6/7}}\right) \cdot \left(\sin(4x) + 2\ln(5x) + 4\sqrt[7]{x}\right)^{-\frac{3}{2}}$;

Practice - Chain Rule

P1. Find the derivative of $y = 7\left(4x^2 + 7x + 22\right)^8$

P2. Find the derivative of $y = \frac{8}{\sqrt{x^2 + 13x + 25}}$

P3. Find the derivative of $y = 6(8x-7)^9 + \frac{2}{(2x+9)^4}$

P4. Find the derivative of $y = 2\left(3x^2 + 6\right)^{2/15} + \frac{5}{\left(8x^3 - 1\right)^{19/2}}$

P5. Find the derivative of $y = 4e^{6x^5 + 9x + 27}$

P6. Find the derivative of $y = 7\ln\left(9x^5 + 3x + 25\right)$

P7. Find the derivative of $y = \frac{1}{\sqrt{2\sin\left(6x^4\right) + 2\cos\left(9x^3\right)}}$

P8. Find the derivative of $y = 5\sin(5x) + 5\ln(3x+1) + 7(x-1)^7$

ANSWERS: P1) $56(8x+7)\left(4x^2 + 7x + 22\right)^7$ P2) $-\frac{4(2x+13)}{(x^2+13x+25)^{3/2}}$ P3) $432(8x-7)^8 - \frac{16}{(2x+9)^5}$

P4) $\frac{8x}{5(3x^2+6)^{13/15}} - \frac{1140x^2}{(8x^3-1)^{21/2}}$ P5) $4 \cdot (30x^4 + 9) \cdot e^{6x^5+9x+27}$ P6) $\frac{7(45x^4+3)}{9x^5+3x+25}$

P7) $-\frac{48x^3\cos\left(6x^4\right) - 54x^2\sin\left(9x^3\right)}{2(2\sin(6x^4)+2\cos(9x^3))^{3/2}}$

P8) $25\cos(5x) + \frac{15}{3x+1} + 49(x-1)^6$