IB Math Analysis and Approaches HL (Year 1) Course 5022 HL

Desert Mountain High School Math Department 5032 MYP

SUMMER WORK PACKET 5232 SL

Honors/AP/IB level math courses at Desert Mountain
are for students who are enthusiastic learners of mathematics and whose work ethic is of the highest standard. These students are expected to arrive "ready to go" on the first day of school.

The attached packet is designed to help you **review** concepts with which you should already be familiar. It is recommended that you complete some of the problems from the packet at the beginning of the summer when the concepts are still fresh, and then complete the remainder of the problems near the beginning of the school year. The content in this packet will be assessed after your teacher has reviewed its contents and you have had an opportunity to ask questions. Understanding this material is foundational to your success in this math course and should be taken seriously. The problems will be reviewed and collected after we have gone over any questions you have.

Some suggestions for the presentation and completion of mathematics assignments at DMHS are listed below. If you adhere the these guidelines with your summer work, you will be ready to meet the expectations of your mathematics teacher during the school year.

- Use notebook paper or plain white paper if extra paper is needed
- All work should be neat and legible
- Neatly place completed work in a notebook, small binder or folder with brads
- Use pencil, erase completely when needed
- Work the problems in order and clearly indicate section and problem numbers if working on a separate sheet of paper
- Begin new sections on a new piece of paper

I look forward to a fun, rigorous year ahead of learning different ways to approach mathematics.

Mrs. Yoesle

The following review problems come from the textbook we will be using and is from the first chapter titled Algebra and function basics. We will discuss some of the new IB language that you will encounter the first week of school. GDC is the abbreviation for your calculator; Graphing display calculator.

Lesson 1.1-Equations and formulae

Exercise 1.1

1. Solve for the indicated variable in each formula.

(a) $m(h - x) = n$, solve for x

(b) $v = \sqrt{ab - t}$, solve for a

(c) $A = \frac{h}{2}(b_1 + b_2)$, solve for b_1

(d) $A = \frac{1}{2}r^2\theta$, solve for r

(e) $\frac{f}{g} = \frac{h}{k}$, solve for k

(f) $at = x - bt$, solve for t

(g) $V = \frac{1}{3}\pi r^3 h$, solve for r

(h) $F = \frac{g}{m_1 k + m_2 k}$, solve for k

2. Find the equation of the line that passes through the two given points. Write the line in slope-intercept form ($y = mx + c$), if possible.

(a) $(-9, 1)$ and $(3, -7)$

(b) $(3, -4)$ and $(10, -4)$

(c) $(-12, -9)$ and $(4, 11)$

(d) $\left(\frac{7}{3}, -\frac{1}{2}\right)$ and $\left(\frac{7}{3}, \frac{5}{2}\right)$

(e) Find the equation of the line that passes through the point $(7, -17)$ and is parallel to the line with equation $4x + y - 3 = 0$. Write the line in slope-intercept form ($y = mx + c$).

(f) Find the equation of the line that passes through the point $\left(-5, \frac{11}{2}\right)$ and is perpendicular to the line with equation $2x - 5y - 35 = 0$. Write the line in slope-intercept form ($y = mx + c$).

3. Find the exact distance between each pair of points and then find the midpoint of the line segment joining the two points.

(a) $(-4, 10)$ and $(4, -5)$

(b) $(-1, 2)$ and $(5, 4)$

(c) $\left(\frac{1}{2}, 1\right)$ and $\left(-\frac{5}{2}, \frac{4}{3}\right)$

(d) $(12, 2)$ and $(-10, 9)$

4. Find the value(s) of k so that the distance between the points is 5 units.

(a) $(5, -1)$ and $(k, 2)$

(b) $(-2, -7)$ and $(1, k)$

5. Show that the given points form the vertices of the indicated polygon.

(a) Right-angled triangle: $(4, 0)$, $(2, 1)$ and $(-1, -5)$

(b) Isosceles triangle: $(1, -3)$, $(3, 2)$ and $(-2, 4)$

(c) Parallelogram: $(0, 1)$, $(3, 7)$, $(4, 4)$ and $(1, -2)$

6. Use the elimination method to solve each system of linear equations.

(a) $x + 3y = 8$, $x - 2y = 3$ **(b)** $x - 6y = 1$, $3x + 2y = 13$ **(c)** $6x + 3y = 6$, $5x + 4y = -1$

(d) $8x - 12y = 4$, $-2x + 3y = 2$ **(e)** $5x + 7y = 9$, $-11x - 5y = 1$

7. Use the substitution method to solve each system of linear equations.

(a) $2x + y = 1$, $3x + 2y = 3$ **(b)** $3x - 2y = 7$, $5x - y = -7$

(c) $2x + 8y = -6$, $-5x - 20y = 15$ **(d)** $\frac{x}{5} + \frac{y}{2} = 8$, $x + y = 20$

(e) $2x - y = -2$, $4x + y = 5$ **(f)** $0.4x + 0.3y = 1$, $0.25x + 0.1y = -0.25$

8. Use your GDC to solve each system of two linear equations.

(a) $3x + 2y = 9$, $7x + 11y = 2$ **(b)** $3.62x - 5.88y = -10.11$, $0.08x - 0.02y = 0.92$ **(c)** $2x - 3y = 4$, $5x + 2y = 1$

9. Solve each system

(a) $\begin{cases} 4x - y + z = -5 \\ 2x + 2y + 3z = 10 \\ 5x - 2y + 6z = 1 \end{cases}$ **(b)** $\begin{cases} 4x - 2y + 3z = -2 \\ 2x + 2y + 5z = 16 \\ 8x - 5y - 2z = 4 \end{cases}$

(c) $\begin{cases} 5x - 3y + 2z = 2 \\ 2x + 2y - 3z = 3 \\ x - 7y + 8z = -4 \end{cases}$ **(d)** $\begin{cases} 3x - 2y + z = -29 \\ -4x + y - 3z = 37 \\ x - 5y + z = -24 \end{cases}$

10. Use your GDC to solve each system of three linear equations. If the system has infinite solutions, simply write 'infinite solutions'.

(a) $\begin{cases} x - 3y - 2z = 8 \\ -2x + 7y + 3z = -19 \\ x - y - 3z = 3 \end{cases}$ **(b)** $\begin{cases} 2x + 3y + 5z = 4 \\ 3x + 5y + 9z = 7 \\ 5x + 9y + 17z = 1 \end{cases}$

(c) $\begin{cases} -x + 4y - 2z = 12 \\ 2x - 9y + 5z = -25 \\ -x + 5y - 4z = 10 \end{cases}$ **(d)** $\begin{cases} 2x + 3y + 5z = 4 \\ 3x + 5y + 9z = 7 \\ 5x + 9y + 17z = 13 \end{cases}$

11. Find the value(s) of k such that the following system of equations has no solution.

$x + y + (k - 1) = 2$

$kx - z = -3$

$6x + 2y - 3z = 1$

LESSON 1.2-Definition of a function

Exercise 1.2

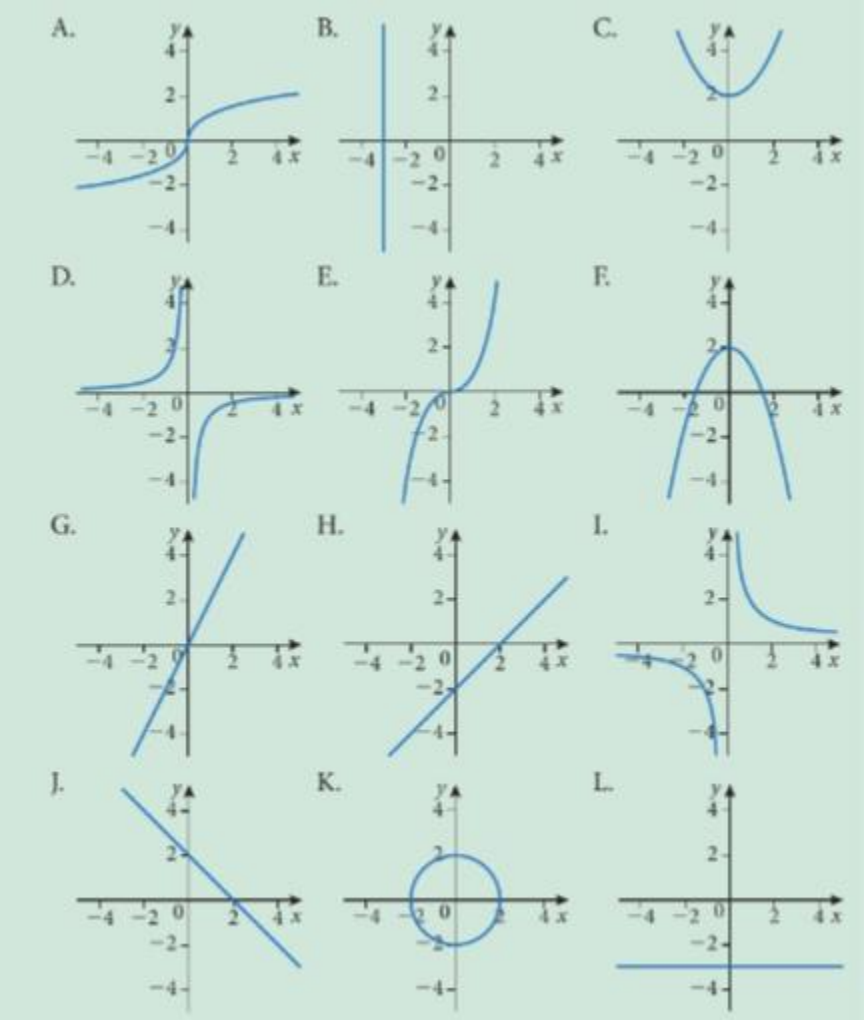
1. **(i)** Match each equation to one of the graphs.

(ii) State whether or not the equation represents any of the functions shown. Justify your answer. Assume that x is the independent variable and y is the dependent variable.

(a) $y = 2x$ **(b)** $y = -3$ **(c)** $x - y = 2$

(d) $x^2 + y^2 = 4$ **(e)** $y = 2 - x$ **(f)** $y = x^2 + 2$

(g) $y^3 = x$ **(h)** $y = \frac{2}{x}$ **(i)** $x^2 + y = 2$



2. Express the area, A, of a circle as a function of its circumference, C.
3. Express the area, A, of an equilateral triangle as a function of the length, ℓ, of each of its sides.
4. A rectangular swimming pool with dimensions 12 metres by 18 metres is surrounded by a pavement of uniform width x metres. Find the area of the pavement, A, as a function of x.
5. In a right-angled isosceles triangle, the two equal sides have length x units and the hypotenuse has length h units. Write h as a function of x.
6. The pressure P (measured in kilopascals, kPa) for a particular sample of gas is directly proportional to the temperature T (measured in degrees kelvin, K) and inversely proportional to the volume V (measured in litres, L). With k representing the constant of proportionality, this relationship can be written in the form of the equation $P = k\frac{T}{V}$

(a) Find the constant of proportionality, k, if 150 L of gas exerts a pressure of 23.5 kPa at a temperature of 375 K.

(b) Using the value of k from part **(a)** and assuming that the temperature is held constant at 375 K, write the volume V as a function of pressure P for this sample of gas.

7. In physics, Hooke's law states that the force F (measured in newtons, N) needed to extend a spring by x units beyond its natural length is directly proportional to the extension x. Assume that the constant of proportionality is k (known as the spring constant for a particular spring).
   (a) Write F as a function of x.
   (b) A spring has a natural length of 12 cm and a force of 25 N stretches the spring to a length of 16 cm. Work out the spring constant k.
   (c) What force is needed to stretch the spring to a length of 18 cm?

8. Find the domain of each of the following functions.
   (a) $\{(-6.2, -7), (-1.5, -2), (0.7, 0), (3.2, 3), (3.8, 3)\}$
   (b) Surface area of a sphere: $S = 4\pi r^2$
   (c) $f(x) = \frac{2}{5}x - 7$ **(d)** $h: x \mapsto x^2 - 4$ **(e)** $g(t) = \sqrt{3 - t}$
   (f) $h(t) = \sqrt[3]{t}$ **(g)** $f: x \mapsto \frac{6}{x^2 - 9}$ **(h)** $f(x) = \sqrt[3]{\frac{1}{x^2} - 1}$

9. Do all linear equations represent a function? Explain.

10. Consider the function $h(x) = \sqrt{x - 4}$
   (a) Find: **(i)** $h(21)$ **(ii)** $h(53)$ **(iii)** $h(4)$
   (b) Find the values of x for which h is undefined.
   (c) State the domain and range of h.

11. For each function below:
   (i) find the domain and range of the function
   (ii) sketch a comprehensive graph of the function, clearly indicating any intercepts or asymptotes.
   (a) $f: x \mapsto \frac{1}{x - 5}$ **(b)** $g(x) = \frac{1}{\sqrt{x^2 - 9}}$ **(c)** $h(x) = \frac{2x - 1}{x + 2}$
   (d) $p: x \mapsto \sqrt{5 - 2x^2}$ **(e)** $f(x) = \frac{1}{x} - 4$

Lesson 1.3-Composite functions

Exercise 1.3

1. Let $f(x) = 2x$ and $g(x) = \frac{1}{x-3}, x \neq 0$

Find the value of:

(a) $(f \circ g)(5)$ **(b)** $(g \circ f)(5)$

Find the function rule (expression) for:

(c) $(f \circ g)(x)$ **(d)** $(g \circ f)(x)$

2. Let $f: x \mapsto 2x - 3$ and $g: x \mapsto 2 - x^2$

Evaluate:

(a) $(f \circ g)(0)$ **(b)** $(g \circ f)(0)$ **(c)** $(f \circ f)(4)$

(d) $(g \circ g)(-3)$ **(e)** $(f \circ g)(-1)$ **(f)** $(g \circ f)(-3)$

Find the expression for:

(g) $(f \circ g)(x)$ **(h)** $(g \circ f)(x)$ **(i)** $(f \circ f)(x)$ **(j)** $(g \circ g)(x)$

3. For each pair of functions, find $(f \circ g)(x)$ and $(g \circ f)(x)$ and state the domain for each.

(a) $f(x) = 4x - 1, g(x) = 2 + 3x$ **(b)** $f(x) = x^2 + 1, g(x) = -2x$

(c) $f(x) = \sqrt{x+1}, g(x) = 1 + x^2$ **(d)** $f(x) = \frac{2}{x+4}, g(x) = x - 1$

(e) $f(x) = 3x + 5, g(x) = \frac{x-5}{3}$ **(f)** $f(x) = 2 - x^3, g(x) = \sqrt[3]{1-x^2}$

(g) $f(x) = \frac{2x}{4-x}, g(x) = \frac{1}{x^2}$

(h) $f(x) = \frac{2}{x+3} - 3, g(x) = \frac{2}{x+3} - 3$

(i) $f(x) = \frac{x}{x-1}, g(x) = x^2 - 1$

4. Let $g(x) = \sqrt{x-1}$ and $h(x) = 10 - x^2$. Find:

(a) $(g \circ h)(x)$ and its domain and range

(b) $(h \circ g)(x)$ and its domain and range.

5. Let $f(x) = \frac{1}{x}$ and $g(x) = 10 - x^2$. Find:

(a) $(f \circ g)(x)$ and its domain and range

(b) $(g \circ f)(x)$ and its domain and range.

6. Determine functions g and h so that $f(x) = g(h(x))$

(a) $f(x) = (x+3)^2$ **(b)** $f(x) = \sqrt{x-5}$ **(c)** $f(x) = 7 - \sqrt{x}$

(d) $f(x) = \frac{1}{x+3}$ **(e)** $f(x) = 10^{x+1}$ **(f)** $f(x) = \sqrt[5]{x-9}$

(g) $f(x) = |x^2 - 9|$ **(h)** $f(x) = \frac{1}{\sqrt{x-5}}$

7. Find the domain for:

(i) the function f **(ii)** the function g **(iii)** the composite function $f \circ g$

(a) $f(x) = \sqrt{x}, g(x) = x^2 + 1$ **(b)** $f(x) = \frac{1}{x}, g(x) = x + 3$

(c) $f(x) = \frac{3}{x^2-1}, g(x) = x + 1$ **(d)** $f(x) = 2x + 3, g(x) = \frac{x}{2}$

Lesson 1.4-Inverse functions

IB notation seen in some questions that looks like this (a) $f:x \mapsto x+6$ the same as $f(x) = x + 6$

Exercise 1.4

In questions 1–4, assume that f is a one-to-one function.

1. (a) If $f(2) = -5$, then what is $f^{-1}(-5)$?
   (b) If $f^{-1}(6) = 10$, then what is $f(10)$?

2. (a) If $f(-1) = 13$, then what is $f^{-1}(13)$?
   (b) If $f^{-1}(b) = a$, then what is $f(a)$?

3. If $g(x) = 3x - 7$, then what is $g^{-1}(5)$?

4. If $h(x) = x^2 - 8x$, with $x \geqslant 4$, then what is $h^{-1}(-12)$?

5. For each pair of functions, show algebraically and graphically that f and g are inverse functions by:
   (i) verifying that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$
   (ii) sketching the graphs of f and g on the same set of axes with equal scales on the x-axis and y-axis.
   Use your GDC to assist in making your sketches on paper.
   (a) $f:x \mapsto x + 6$; $g:x \mapsto x - 6$
   (b) $f:x \mapsto 4x$; $g:x \mapsto \frac{x}{4}$
   (c) $f:x \mapsto 3x + 9$; $g:x \mapsto \frac{1}{3}x - 3$
   (d) $f:x \mapsto \frac{1}{x}$; $g:x \mapsto \frac{1}{x}$
   (e) $f:x \mapsto x^2 - 2, x \geqslant 0$; $g:x \mapsto \sqrt{x+2}, x \geqslant -2$
   (f) $f:x \mapsto 5 - 7x$; $g:x \mapsto \frac{5-x}{7}$
   (g) $f:x \mapsto \frac{1}{1+x}$; $g:x \mapsto \frac{1-x}{x}$
   (h) $f:x \mapsto (6-x)^{\frac{1}{2}}$; $g:x \mapsto 6 - x^2, x \geqslant 0$
   (i) $f:x \mapsto x^2 - 2x + 3, x \geqslant 1$; $g:x \mapsto 1 + \sqrt{x-2}, x \geqslant 2$
   (j) $f:x \mapsto \sqrt[3]{\frac{x+6}{2}}$; $g:x \mapsto 2x^3 - 6$

6. Find the inverse function f^{-1} and state its domain.
   (a) $f(x) = 2x - 3$
   (b) $f(x) = \frac{x+7}{4}$
   (c) $f(x) = \sqrt{x}$
   (d) $f(x) = \frac{1}{x+2}$
   (e) $f(x) = 4 - x^2, x \geqslant 0$
   (f) $f(x) = \sqrt{x-5}$
   (g) $f(x) = ax + b, a \neq 0$
   (h) $f(x) = x^2 + 2x, x \geqslant -1$
   (i) $f(x) = \frac{x^2-1}{x^2+1}, x \leqslant 0$
   (j) $f(x) = x^3 + 1$

7. Determine if f has an inverse f^{-1}. If not, restrict the domain of f in order to find an inverse function. Graph f and its inverse f^{-1} on the same set of axes.
   (a) $f(x) = \frac{2x+3}{x-1}$
   (b) $f(x) = (x-2)^2$
   (c) $f(x) = \frac{1}{x^2}$
   (d) $f(x) = 2 - x^4$

8. Use your GDC to graph the function $f(x) = \frac{2x}{1+x^2}, x \in \mathbb{R}$. Find three intervals for which f is a one-to-one function (monotonic) and hence, will have an inverse f^{-1} on the interval. The union of all three intervals is all real numbers.

9. Use the functions $g(x) = x + 3$ and $h(x) = 2x - 4$ to find the indicated value or the indicated function.

(a) $(g^{-1} \circ h^{-1})(5)$

(b) $(h^{-1} \circ g^{-1})(9)$

(c) $(g^{-1} \circ g^{-1})(2)$

(d) $(h^{-1} \circ h^{-1})(2)$

(e) $g^{-1} \circ h^{-1}$

(f) $h^{-1} \circ g^{-1}$

(g) $(g \circ h)^{-1}$

(h) $(h \circ g)^{-1}$

10. The reciprocal function, $f(x) = \frac{1}{x}$, is its own inverse (self-inverse). Show that any function in the form $f(x) = \frac{a}{x + b} - b$, $a \neq 0$ is its own inverse.

Lesson 1.5-Transformation of functions

Exercise 1.5

1. Sketch the graph of f, without a GDC or plotting points, by using your knowledge of some of the basic functions shown at the start of the Section 1.5.

(a) $f:x \mapsto x^2 - 6$

(b) $f:x \mapsto (x-6)^2$

(c) $f:x \mapsto |x| + 4$

(d) $f:x \mapsto |x+4|$

(e) $f:x \mapsto 5 + \sqrt{x-2}$

(f) $f:x \mapsto \dfrac{1}{x-3}$

(g) $f:x \mapsto \dfrac{1}{(x+5)^2} + 2$

(h) $f:x \mapsto -x^3 - 4$

(i) $f:x \mapsto -|x-1| + 6$

(j) $f:x \mapsto \sqrt{-x+3}$

(k) $f:x \mapsto 3\sqrt{x}$

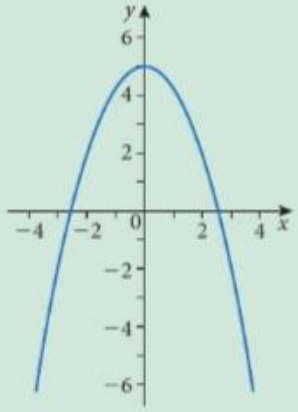
(l) $f:x \mapsto \dfrac{1}{2}x^2$

(m) $f:x \mapsto \left(\dfrac{1}{2}x\right)^2$

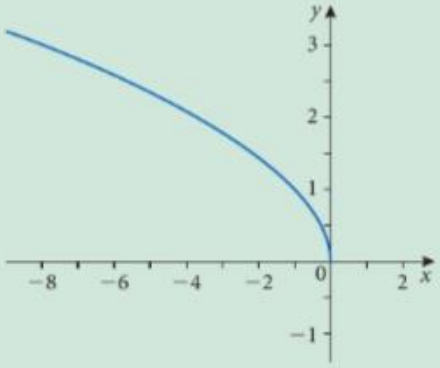
(n) $f:x \mapsto (-x)^3$

2. Write the equation for each graph.

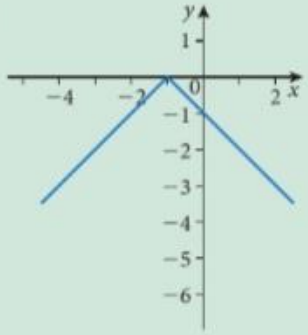
(a)



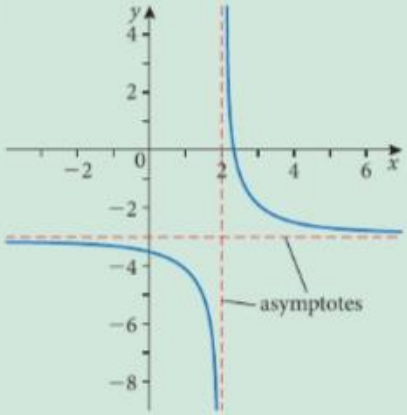
(b)



(c)



(d)



3. The graph of f is given. Sketch the graph of each transformed function.

(a) $y = f(x) - 3$

(b) $y = f(x - 3)$

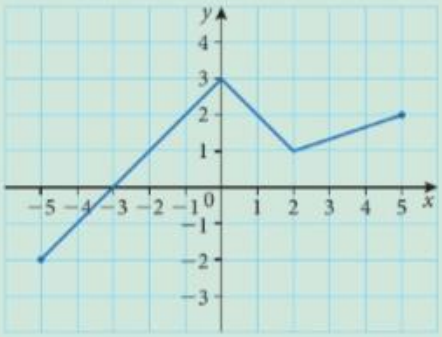
(c) $y = 2f(x)$

(d) $y = f(2x)$

(e) $y = -f(x)$

(f) $y = f(-x)$

(g) $y = 2f(x) + 4$



4. Specify a sequence of transformations to perform on the graph of $y = x^2$ to obtain the graph of the given function.

(a) $g: x \mapsto (x - 3)^2 + 5$

(b) $h: x \mapsto -x^2 + 2$

(c) $p: x \mapsto \frac{1}{2}(x + 4)^2$

(d) $f: x \mapsto [3(x - 1)]^2 - 6$

5. Without using your GDC, for each function $f(x)$, sketch the graph of:

(i) $\dfrac{1}{f(x)}$ **(ii)** $|f(x)|$ **(iii)** $f(|x|)$

Clearly label any intercepts or asymptotes.

(a) $f(x) = \frac{1}{2}x - 4$

(b) $f(x) = (x - 4)(x + 2)$

(c) $f(x) = x^3$