Summer Packet in Preparation for Integrated Math 3 Regular
This outline serves as a guide for the development of a comprehensive summer packet designed to reinforce and build upon key concepts from Integrated Math II Honors, preparing students for the challenges of Integrated Math III Honors.

Section 1: Numeracy – Monomials, Radicals, Complex

Section 2: Algebraic Concepts
- Review of Quadratic Functions - forms, graphing
- Quadratic Expressions
    - Operations with quadratic functions
    - Factoring techniques (GCF, trinomials, difference of squares)
- Quadratic Expressions and Equations
    - Simplifying quadratic expressions
    - Multiplication and division of quadratic expressions
    - Solving quadratic equations

Section 3: Functions and Their Properties
- Function Properties, Notation and Evaluation
- Types of Functions - Linear, quadratic, exponential, and piecewise functions
- Inverses Functions - Finding and Verifying
- Transformations of Functions - Translations, reflections, and rotations

Section 3: Geometry and Measurement
- Pythagorean Theorem
- Perimeter and Area of geometric shapes
- Coordinate Geometry - Distance, midpoint, slope formulas
- Trigonometry Basics - sine, cosine, and tangent, Right triangle trigonometry

Section 4: Statistics and Probability
- Basic probability principles
- Compound events and probability

Section 5. Problem Solving Practice and Application

Section 1. Numeracy – Monomials, Radicals, Complex

1. Simplify the following expressions.

a. $5x-3x^2+15+11x^2-21x+8$

$8x^2-16x+23$

b. $\frac{2}{3}x+\frac{4}{5}y-\frac{1}{2}x+3y$

$\frac{4}{6}x+\frac{4}{5}y-\frac{3}{6}x+\frac{15}{5}y$

$\frac{1}{6}x+\frac{19}{5}y$

c. $2x(3w+4x)+5w(w-2x)+5w^2-2x^2$

$6xw+8x^2+5w^2-10xw+5w^2-2x^2$

$10w^2-4xw+6x^2$

d. $(3x-1)(4x+5)-(x+3)(x-2)$

$12x^2+11x-5-x^2-1x+6$

$11x^2+10x-1$

2. Simplify the following expressions.

a. $\sqrt{25a^3b^2}$

$\sqrt{25a^2b^2}\sqrt{a}$

$5ab\sqrt{a}$

b. $\sqrt{48x^3y^7z^5}$

$\sqrt{16x^2y^6z^4}\sqrt{3xyz}$

$4xy^3z^2\sqrt{3xyz}$

c. $8\sqrt{7}-\sqrt{7}$

$7\sqrt{7}$

d. $\sqrt{18}-\sqrt{54}+2\sqrt{50}$

$\sqrt{9\cdot2}-\sqrt{9\cdot6}+2\sqrt{25\cdot2}$

$3\sqrt{2}-3\sqrt{6}+10\sqrt{2}$

$13\sqrt{2}-3\sqrt{6}$

e. $(\sqrt{3}-\sqrt{2})(\sqrt{3}+2\sqrt{2})$

$\sqrt{9}-\sqrt{6}+2\sqrt{6}-2\sqrt{4}$

$3-\sqrt{6}+2\sqrt{6}-4$

$-1+\sqrt{6}$

g. $\left(2\sqrt{3}+4\right)\left(4\sqrt{3}+2\right)$

$8\sqrt{9}+4\sqrt{3}+16\sqrt{3}+8$

$24+20\sqrt{3}+8$

$32+20\sqrt{3}$

3. Simplify each, rationalizing the denominator for your final answer:

a. $\frac{12}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$\frac{12\sqrt{3}}{3}$

$4\sqrt{3}$

b. $\frac{4}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$\frac{4\sqrt{2}}{6}$

$\frac{2\sqrt{2}}{3}$

c. $\frac{\sqrt{10}}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}}$

$\frac{\sqrt{50}}{5}$

$\frac{5\sqrt{2}}{5}$

$\sqrt{2}$

4. Solve the following radical equations:

a. $6\sqrt{3k+5}=24$

$(\sqrt{3k+5})^2=(4)^2$

$3k+5=16$

$3k=11$

$k=11/3$

b. $4\sqrt{x-2}=x+1$

$16(x-2)=x^2+2x+1$

$16x-32=x^2+2x+1$

$0=x^2-14x+33$

$0=(x-11)(x-3)$

$x=11,3$

c. $\sqrt{b+1}=b-1$

$b+1=b^2-2b+1$

$b^2-b=0$

$b(b-1)=0$

$b=0,1$

no soln.

$\emptyset$

5. Demonstrate how to evaluate the following expression without a calculator.

a. $5!$

$5\cdot4\cdot3\cdot2\cdot1$

$20(6)$

120

b. $\frac{6!}{4!}$

$\frac{6\cdot5\cdot4!}{4!}$

30

c. $\frac{10!}{8!2!}$

$\frac{5\cdot10\cdot9\cdot8!}{8!\,2\cdot1}$

45

d. $\frac{5!}{2!(3!)}$

$\frac{5\cdot4\cdot3!}{2\cdot1\cdot3!}$

10

6. Simplify each of the monomial expressions below.

Consider the following: $x^a\cdot x^b=x^{a+b}$, $(x^a)^b=x^{ab}$, $\frac{x^a}{x^b}=x^{a-b}$, $x^{-1}=\frac{1}{x}$.

a. $\left(-x^4y^7\right)^5$

$-x^{20}y^{35}$

b. $(2x)^3(5x^2)$

$(8x^3)(5x^2)$

$40x^5$

c. $\frac{57k^7d^3}{19k^{-3}d^5}$

$\frac{3k^{10}}{d^2}$

7. Simplify each of the following:

i

-1

$-i$

1

a. i^2

-1

b. $5-7i+3(2-i)$

$5-7i+6-3i$

$11-10i$

c. $-2(2-3i)(9+4i)$

$-2(18-27i+8i-12i^2)$

$-2(30-19i)$

$-60+38i$

Section 2. Algebraic Concepts

1. Completely factor the following expressions.

a. $3x^2 - 5x - 22$

$(3x-11)(x+2)$

b. $\frac{1}{9}m^2 - 4$

$\left(\frac{1}{3}m+2\right)\left(\frac{1}{3}m-2\right)$

c. $2x^2 - 4x - 126$

$2(x^2-2x-63)$

$2(x-9)(x+7)$

2. Solve by factoring:

a. $x^2 + 7x + 10 = 0$

$(x+2)(x+5)=0$

$x=-2,-5$

b. $2x^2 + 7x = 9$

$2x^2+7x-9=0$

$(2x+9)(x-1)=0$

$x=-9/2, 1$

3. Solve by using the quadratic formula:

a. $x^2 = 8 - 10x$

$x^2+10x-8=0$

$a=1$, $b=10$, $c=-8$

$x=\frac{-10\pm\sqrt{100-4(-8)}}{2}$

$x=\frac{-10\pm\sqrt{132}}{2}$

$x=(-10\pm 2\sqrt{33})/2$

$x=-5\pm\sqrt{33}$

b. $x^2 = 7x$

$x^2-7x=0$

$a=1$, $b=-7$, $c=0$

$x=\frac{7\pm\sqrt{49-4(1)(0)}}{2}$

$x=\frac{7\pm\sqrt{49}}{2}$

$x=\frac{7\pm 7}{2}$

$x=7, 0$

4. Solve by using the square root method:

a. $16x^2 - 64 = 0$

$(4x+8)(4x-8)=0$

$x=-8/4, 8/4$

$x=-2, 2$

b. $2(x+5)^2 + 3 = 0$

$(x+5)^2=-3/2$

$x=-5\pm i\sqrt{\frac{3}{2}}$

$x=-5\pm i\frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

5. Solve by completing the square:

a. $x^2 = 72 - 2x$

$x^2 + 2x + 1 = 72 + 1$

$(x+1)^2 = 73$

$x = -1 \pm \sqrt{73}$

b. $5x^2 - 45x + 90 = 0$

$5(x^2 - 9x + (\frac{9}{2})^2) = -90 + 101.25$

$5(x - \frac{9}{2})^2 = 11.25$

$(x - \frac{9}{2})^2 = \frac{9}{4}$

$x = \frac{9}{2} \pm \frac{3}{2}$

$x = 6, 3$

6. Write an equation <u>in factored form</u> with the given solutions:

a. $x = 2, x = -7$

$(x-2)(x+7) = 0$

b. $x = 6, x = \frac{3}{5}$

$(x-6)(5x-3) = 0$

c. $x = -\frac{1}{2}, x = \frac{2}{7}$

$(2x+1)(7x-2) = 0$

7. Write an equation <u>in standard form</u> with the given solutions:

a. $x = 1, x = -9$

$(x-1)(x+9) = 0$

$x^2 + 8x - 9 = 0$

b. $x = -7, x = -\frac{1}{3}$

$(x+7)(3x+1) = 0$

$3x^2 + 22x + 7 = 0$

c. $x = 2 \pm 3i\sqrt{5}$

$x - 2 = 3i\sqrt{5}$

$(x-2)^2 = (3i\sqrt{5})^2$

$x^2 - 4x + 4 = -9(5)$

$x^2 - 4x + 49 = 0$

8. Find the inverse of the function $f(x) = 5x + 9$.

$x = 5y + 9$

$\frac{x-9}{5} = y$

$f^{-1}(x) = \frac{x-9}{5}$

9. The cheerleaders at Bulls games launch T-Shirts into the crowd every time the Bulls win. The height of the T-shirt can be modeled by the function $h(x) = -16x^2 + 48x + 6$, where $h(x)$ represents the height in feet of the T-shirt after x seconds.

a. At what height was the T-shirt launched?

y-int = 6 feet

b. What is the maximum height that the T-shirt reaches?

c. When was the maximum height reached?

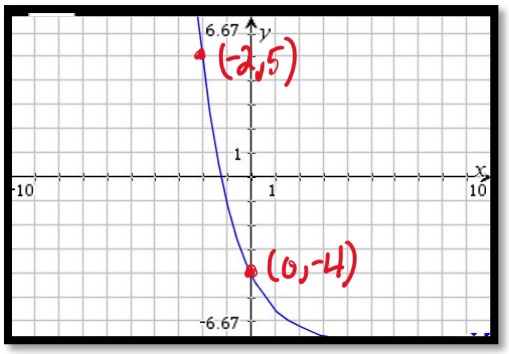
$x = \frac{-b}{2a} = \frac{48}{32} = \frac{3}{2}$ or 1.5 feet

$f(1.5) = -16(1.5)^2 + 48(1.5) + 6$

$= 37.5$ seconds

Section 3. Functions and their Properties

1. Find the rate of change of the graph below over the interval $[-2,0]$.



$\frac{5-(-4)}{-2-0}$

$\boxed{-\frac{9}{2}}$

3. Given the following functions, write the equation for the function translated 3 units up and 4 units to the right.

a. $g(x)=3(x-1)^2+7$ $g'(x)=3(x-5)^2+10$

b. $g(x)=\sqrt{x-5}$ $g'(x)=\sqrt{x-9}+3$

c. $g(x)=-4|x+11|-8$ $g'(x)=-4|x+7|-5$

5. Given the following functions, write the equation for the functions reflected over the y-axis.

a. $p(x)=-(x+3)^3-11$ $p'(x)=-(x-3)^3-11$

b. $p(x)=-3|x|+5$ $p'(x)=-3|x|+5$

c. $p(x)=2\sqrt{x+5}-3$ $p'(x)=2\sqrt{x-5}-3$

7. $f(x)=x-6$ and $g(x)=3x+2$. If $g(x)$ can be written as $a\cdot f(x)+k$, what are the values of a and k?

$g(x)=a(x+6)+k$

$3x+2=ax+6a+k$

$3x=ax$ $\boxed{3=a}$

$2=6a+k$

$2=18+k$

$\boxed{-16=k}$

8. If $f(x)=3x^2-4x+1$ and $g(x)=-5x^2+2x-10$, find $f(2)-g(3)$

$3(2)^2-4(2)+1-(-5(3)^2+2(3)-10)$

$12-8+1+45-6+10$

$\boxed{54}$

Section 4. Geometry and Measurements

1. The formula to find the rate of change or slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Solve the equation for y_2.

$m(x_2 - x_1) = y_2 - y_1$

$y_1 + m(x_2 - x_1) = y_2$

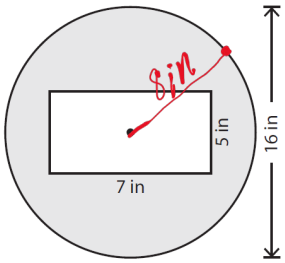
2. The distance formula for finding the distance between two points is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Solve the equation for x_1.

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\sqrt{d^2 - (y_2 - y_1)^2} = x_2 - x_1$

$x_1 = x_2 - \sqrt{d^2 - (y_2 - y_1)^2}$

3. Find the area of the shaded region:



$A_\circ - A_\square$

$\pi(64) - 75$

$(64\pi - 35)\,in^2$

4. Molly is designing a poster. The length of the poster is 6 inches longer than the width. If the poster requires $616 in^2$ of poster board, find the width w of the poster.



$lw = (l + w)w$

$w^2 + lw - lw = 0$

$(w + 28)(w - 22) = 0$

$w = -28, 22$

$w = 22$ in

factoring...

308 | 2
154 | 4
11 · 14 · 4
11 · 7 · 2 · 4
22 · 28

5. The three angles of a triangle have measures of $(3x)^0$, $(x+10)^0$, and $(2x-40)^0$. Find the measure of the smallest angle in the triangle.

$3x + x + 10 + 2x - 40 = 180$

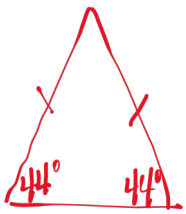
$6x - 30 = 180$

$6x = 210$

$x = 35$

30°

6. Suppose one base angle of an isosceles triangle has a measurement of 44°. What is the measure of the vertex angle?



$180 - 44 - 44$

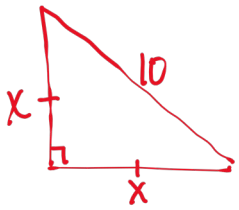
92°

7. In a 30-60-90 triangle, the shortest side is 5 cm. What is the length of the longest side?



$5\sqrt{3}$ cm

8. In a 45-45-90 triangle, if the length of the hypotenuse is 10 cm, what is the length of each leg?

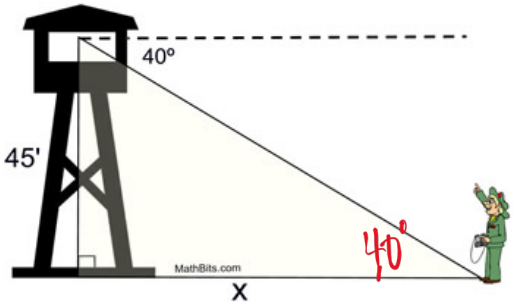


$x\sqrt{2} = 10$

$x = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = 5\sqrt{2}$ cm

9. Roger is looking up a guard tower that is 45 feet tall in the forest. The angle of depression from the top of the guard tower is 40°. To the nearest foot, determine how far is Roger from the guard tower?



$\tan 40^\circ = \frac{45'}{x}$

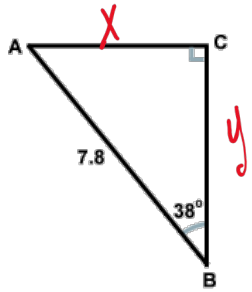
$x \cdot \tan 40 = 45'$

$x = \frac{45'}{\tan 40}$

$x \approx 53.62$

54 feet

10. Find the length of both AC and BC.



$\sin 38^\circ = \frac{x}{7.8}$

$7.8(\sin 38) = x$

$4.802 \approx x = AC$

$\cos 38 = \frac{y}{7.8}$

$7.8(\cos 38) = y$

$6.146 \approx y = BC$

Section 5 – Statistics and Probability

1. Multiple Choice: For which of the following sets of numbers is the mean greater that the median?

a. {4,5,6,7,8} mean: 6 median: 6

b. {6,7,8,9,15} 8 9

c. {4,6,6,6,8} 6 6

d. {2,6,6,6,6} 6 5.2

← order is important.

2. Identify each situation or scenario is a Combination or a Permutation.

a. A committee of 3 students is to be formed from a group of 10 students. P

b. The arrangement of 5 books on a shelf. P

c. Selecting 2 team captains from a basketball team of 8 players. C

d. Creating a password using 4 different letters from the alphabet (A-Z). C

3. Use what you know about Permutations and Combinations to determine the following.

a. How many different committees of 4 students can be formed from a class of 15 students?

$_{15}C_4 = 1365$ ways

b. In how many different ways can 3 students be chosen and arranged in a line from a group of 10 students?

$_{10}P_3 = 120$ ways

c. If a contest has 5 prizes and there are 12 participants, in how many ways can the prizes be awarded to the participants?

* prizes all the same. $_{12}C_5 = 792$ ways

d. How many ways can you select and arrange 4 books from a shelf containing 7 different books?

$_7P_4 = 840$ ways.

Section 6. Problem Solving Practice and Application

1. The function $g(x) = 160 + 1.5x$ models the weight gain of a basketball player as he starts a workout program where g is the weight in pounds after x weeks.

a. Explain the meaning of 160 in the context of this problem.

- the starting weight of the player

b. Explain the meaning of 1.5 in the context of this problem.

- the rate of weight gain each week (1.5 lbs/week)

c. Evaluate $g(6)$ and explain its meaning.

- amount of weigh at the end of 6 weeks

$g(6) = 160 + 1.5(6)$

$= 169$ lbs

2. Lucille has a total of 23 dimes and pennies. The value of her coins is $1.85. How many dimes does she have? How many pennies does she have?

$d + p = 23$

$.1d + .01p = 1.85$ } solve system

$d = 15$ dimes

$p = 5$ pennies

3. Solve the following system of equations $\begin{cases} y - x = 1 \\ y = x^2 - 5 \end{cases}$.

$x = (y-1)$

$y = (y-1)^2 - 5$

$y = y^2 - 2y + 1 - 5$

$0 = y^2 - 3y - 4$

$0 = (y-4)(y+1)$

$y = 4, -1$

$x = y - 1$

$x_1 = 4 - 1 = 3$

$x_2 = -1 - 1 = -2$

$(3, 4)$

$(-2, -1)$

4. Suppose you buy 3 shirts and 2 pairs of slacks on sale at a clothing store for $72. The next day, a friends buys 2 shirts and 4 pairs of slacks for $96. If the shirts you each bought were all the same price and the slacks were also all the same price, then what was the total cost of two shirts and two pairs of slacks?

$-2(3s + 2p = 72)$

$2s + 4p = 96$

$-6s - 4p = -144$

$2s + 4p = 96$

$-4s = -48$

$s = 12$

$2(12) + 4p = 96$

$4p = 72$

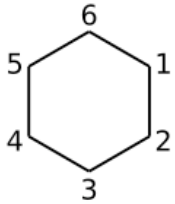
$p = 18$

$2s + 2p$

$24 + 36$

$60

5. Jim cuts a board in the shape of a regular hexagon and pound a nail at each vertex. How many rubber bands will he need to stretch a rubber band across every possible pair of nails?



i.e. how many diagonals?

$d = \frac{n(n-3)}{2}$

$d = \frac{6(3)}{2}$

9 rubber band