

BC Summer work FAQ

What purpose does this summer work serve?

I have two goals in mind. First, I want to emphasize precalculus concepts that are specific to BC Calculus. Those include (but are not limited to) vectors, polar functions, and sums. Second, I am introducing some calculus concepts (the idea of a limit and the mechanical process for finding simple derivatives). As of this writing (April), I'm not certain whether you will have seen this in Precalculus or not, so I will provide necessary resources for these concepts once I have the class rosters for BC Calculus next year (that should happen in either late May or early June).

What if I have questions about the summer work?

For one, you're allowed and encouraged to work with other students and to ask for help if you'd like. Moreover, you're welcome to use your calculator or computer to graph functions, solve equations, etc. There are certain things you'll ultimately be expected to know, but for our purposes here, any use of technology is fine/encouraged as needed.

At any point during the summer, you are welcome to email me if you would like to ask about something. As well, I will be available at times during the week before school starts (days and times TBD), and you are welcome to come to office hours, lunch, after school to go over anything you'd like.

What if I don't do the summer work?

This is perhaps going to sound snarky, but...

Why are you enrolled in BC Calculus next year if the idea of doing math during the summer doesn't appeal to you?

I appreciate that by the end of the school year – after you have survived the grind of AP exams, review and final exams, as well as athletics and/or arts responsibilities – you want (need) a break. It is perfectly sound and reasonable for you to feel that way, and I encourage you to take some time for yourself. However, at a certain point, you should "miss it." This is my version of "summer reading" for math students and I hope that it gets you excited and looking forward to seeing some very cool and interesting concepts and problems next year in my class.

Also, while my intention is not to cover this in the first or second day of class, I will reference it during the second week of school. So...

Obviously, work all problems on separate sheets of paper.

The following problems were taken from the Precalculus Honors final and review materials in 2016. You may use any technology as needed, but appropriate supporting work should be shown for all problems.

Find an expression for the general term of each **geometric** sequence in 1 – 4.

1. 7, 14, 28, ...

2. $1, \frac{1}{2}, \frac{1}{4}, \dots$

3. xyz, xy, ...

4. $g_1 = 7, g_8 = 896$

Find the indicated sum in 5 – 7 (show and use appropriate formulas).

5. $\sum_{k=1}^{\infty} 2\left(\frac{1}{3}\right)^{k-1}$

6. $\sum_{k=5}^{14} 4(3)^{k-1}$

7. $1000 + 500 + 250 + \dots$

8. Find the sum of the first 20 terms of the geometric sequence whose first term is 3 and third term is 6.75. (2 answers)

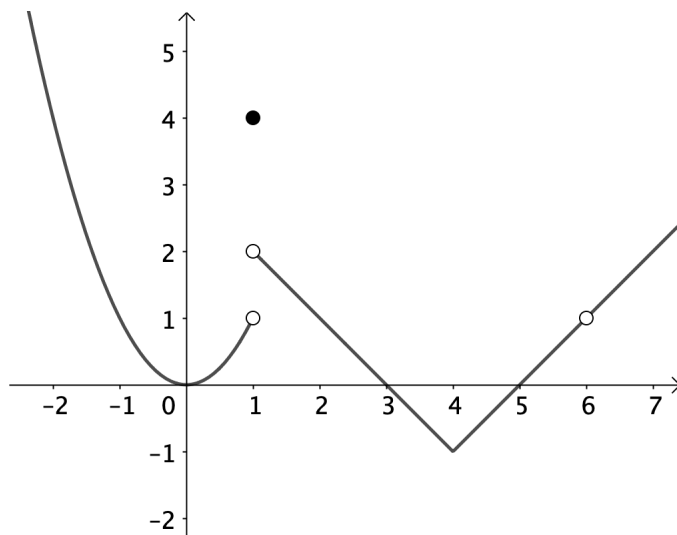
9. Suppose you drop a ball that rebounds to nine-tenths of its original height. If it is dropped from a height of 10 feet, how far will the ball travel before coming to rest?

10. Use the dot product formula to find the angle between the vectors $\langle 6, 6 \rangle$ and $\langle 5, -5\sqrt{3} \rangle$.

11. Find the groundspeed (in component form) of an airplane that flies at 200 mph with direction 105° if the wind is blowing 40 mph with direction 195° .

12. Find the cube roots of $64(\cos 216^\circ + i \sin 216^\circ)$. Express your answers in rectangular form.

Use the graph of $y = f(x)$ below for problems 13–20.



Evaluate each limit. If the limit does not exist, state why.

13. $\lim_{x \rightarrow 2} f(x)$

14. $\lim_{x \rightarrow 6} f(x)$

15. $f(-1)$

16. $f(6)$

17. $\lim_{x \rightarrow 1} f(x)$

18. $f(1)$

19. $\lim_{x \rightarrow 7} \frac{1}{f(x)}$

20. $\lim_{x \rightarrow 5} \frac{1}{f(x)}$

Using the power, sum, and difference rules, find the derivative of each function.

21. $a(x) = x^3 + 2x - 4$

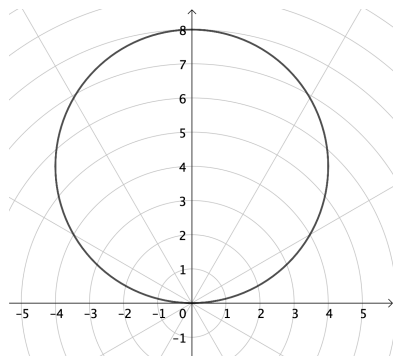
22. $b(x) = \sqrt{x} + 4$

23. $c(x) = \frac{1}{x} + \frac{2}{x^2} + 3x^3$ (hint: rewrite using negative exponents as appropriate)

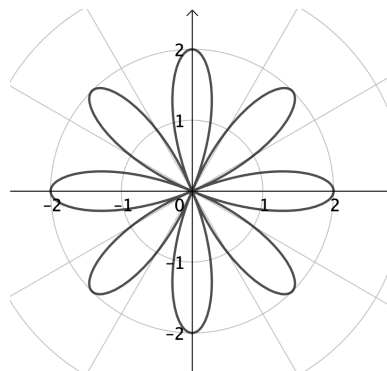
24. $d(x) = \frac{2}{3}x + 2$

Write the *polar* equation for each of the graphs shown below.

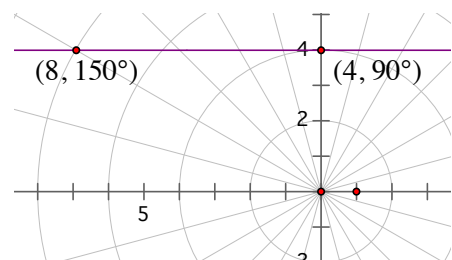
25.



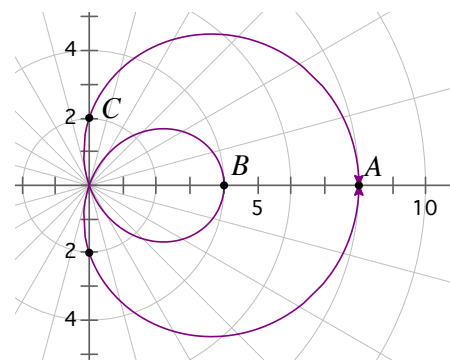
26.



27.



28.



A is $(8, 0)$, B is $(-4, 180^\circ)$ and

C is $(2, 90^\circ)$

29. Find two polar representations of the rectangular point $(-12, 8)$ such that

$0 \leq \theta < 360^\circ$.

30. Expand $(3x - y)^4$.

31. Find a formula for the sequence

$$\frac{4}{6}, \frac{9}{24}, \frac{16}{120}, \frac{25}{720}, \dots$$

32. In how many different ways can 5 boys and 5 girls line up if the lineup is to be all girls and then all boys?

For the following problems (33 – 36), make sure to show the appropriate formulas for finding the sum in order to earn full credit.

33. $21 + 27 + 33 + 39 + \dots + 345$

34. Find the sum of all positive three-digit numbers whose last digit is a 7.

35. $\sum_{k=1}^{\infty} 4 \left(\frac{3}{4} \right)^k$

36. $\sum_{n=0}^{20} 3 \left(\frac{3}{2} \right)^n$

37. Recall that the rules of craps are such:

- 1st roll - win on a 7 or 11, lose on a 2, 3, or 12.
- 1st roll - if none of 2, 3, 7, 11, or 12 - your roll becomes your "point" (goal).
- subsequent rolls - win if you roll your "point," lose if you roll a 7, roll again if neither

What is the probability that you win a game of craps with a final roll of a six?

(Hint: it is possible that you could roll infinite times and not get a six...)

38. Prove by induction that for all positive integers n

$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

For 39–40, four cards are drawn without replacement from a standard deck of cards. Assume the order of the cards isn't important.

39. What is the probability that you draw a hand with four Kings?

40. What is the probability that you draw a hand with two hearts and two spades?

Evaluate each limit or state that it does not exist.

41. $\lim_{x \rightarrow -2} \frac{2x^3 + 16}{x + 2}$

42. $\lim_{x \rightarrow 0} \frac{\cos(2x)}{2x}$

43. $\lim_{x \rightarrow 5} \left[\frac{x}{2} + 2 \right]$

44. $\lim_{x \rightarrow 1} \frac{x - 4}{|x - 4|}$

45. Write the equation of the line tangent to $f(x) = x^2 - 4x + 3$ at the point where $x = 1$.

46. Given that $f'(x) = 3x^2 - 4$ and $f(-1) = 5$, find $f(x)$.

Use $f(x)$ as defined below for problems 47 – 50.

$$f(x) = \begin{cases} -x^2 + 4 & x \leq 1 \\ \frac{1}{(x-2)^2} & 1 < x < 3 \\ x - 2 & 3 < x \end{cases}$$

Evaluate each limit. If the limit does not exist, state why.

47. $\lim_{x \rightarrow 0} f(x)$

48. $\lim_{x \rightarrow 1} f(x)$

49. $\lim_{x \rightarrow 2} f(x)$

50. $\lim_{x \rightarrow 3} f(x)$