NAME_____ Preparing for ALGEBRA II - SUMMER REVIEW PACKET

To maintain a high quality program, students entering Algebra II are expected to remember the basics of the mathematics taught in their Algebra I course. In order to review the basic concepts prior to taking Algebra II, the mathematics department has prepared this review packet. For each algebra topic addressed, this packet contains several review examples with online tutorials followed by problems for the student to practice.

Since this material is designed as review, you are responsible for completing this packet on your own without using a calculator. The answer key will be available so that you are able to check your work. If you have trouble with any of these problems, you might find www.khanacademy.org

I. Order of Operations (PEMDAS)

- **P**arenthesis and other grouping symbols.
- **E**xponential expressions.
- **M**ultiplication & **D**ivision (Whichever comes first)
- **A**ddition & **S**ubtraction.

Simplify each numerical expression.

1) $6 + 2 \times 8 - 12 + 9 \div 3$

2) $25 - (2^3 + 5 \times 2 - 3)$

3) $\dfrac{-2 \cdot (-30) + 0.5 \cdot 20}{4^2 - 6}$

4) $\dfrac{15 - [8 - (2 + 5)]}{18 - 5^2}$

II. Evaluating Algebraic Expressions

To evaluate an algebraic expression:

- **Substitute the given value(s) of the variable(s).**
- **Use order of operations to find the value of the resulting numerical expression.**

Evaluate.

1) $x\left(\frac{y}{2}+3z^2\right)-2x$ *if* $x=\frac{1}{2}, y=4, z=-2$

2) $12a - 4a^2 + 7a^3$ if $a = -3$

3) $\frac{-b+\sqrt{b^2-4ac}}{2a}$ *if* $a=1, b=-4, c=-21$

4) $1.2(3)^x$ if $x = 3$

5) $\frac{3(x+y)-2(x-y)}{5x+y}$ if $x = 3$ and $y = 4$

6) $2\left(\frac{1}{3}\right)^x$ *if* $x=2$

7) If $k \odot n = k^3 - 3$ then evaluate $7 \odot 5$

III. Simplifying Radicals

An expression under a radical sign is in simplest radical form when:

1) there is no integer under the radical sign with a perfect square factor,
2) there are no fractions under the radical sign,
3) there are no radicals in the denominator

Express the following in simplest radical form.

1) $\sqrt{50}$ 2) $\sqrt{24}$ 3) $\sqrt{192}$ 4) $\sqrt{169}$ 5) $\sqrt{147}$

6) $\sqrt{\frac{13}{49}}$

7) $\sqrt{\frac{6}{27}}$

8) $\frac{3}{\sqrt{6}}$

9) $4\sqrt{5} - 2\sqrt{45}$

10) $\frac{1}{5-\sqrt{2}}$

IV. Properties of Exponents

PROPERTY		EXAMPLE
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$x^4 \cdot x^2 = x^6$
Power of a Power	$(a^m)^n = a^{m \cdot n}$	$(x^4)^2 = x^8$
Power of a Product	$(ab)^m = a^m b^m$	$(2x)^3 = 8x^3$
Negative Power	$a^{-n} = \frac{1}{a^n}$ $(a \neq 0)$	$x^{-3} = 1/x^3$
Zero Power	$a^0 = 1$ $(a \neq 0)$	$4^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$	$\frac{x^3}{x^2} = x$
Power of Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $(b \neq 0)$	$\left(\frac{x}{y}\right)^3 = x^3/y^3$

Simplify each expression. Answers should be written using only positive exponents.

1) $g^5 \cdot g^{11}$ _____

2) $(b^6)^3$ _____

3) w^{-7} _____

4) $\frac{y^{12}}{y^8}$ _____

5) $(3x^7)(-5x^3)$ _____

6) $(-4a^5b^0c)^2$ _____

7) $\frac{-15x^7}{25x^9}$ _____

8) $\left(\frac{4x^9}{12x^4}\right)^3$ _____

V. Solving Linear Equations

To solve linear equations, first simplify both sides of the equation. If the equation contains fractions, multiply the equation by the LCD to clear the equation of fractions. Use the addition and subtraction properties of equality to get variables on one side and constants on the other side of the equal sign. Use the multiplication and division properties of equality to solve for the variable. Express all answers as fractions in lowest terms.

Examples:

a) $3(x + 5) + 4(x + 2) = 21$

$3x + 15 + 4x + 8 = 21$

$7x + 23 = 21$

$7x = -2$

$x = -\frac{2}{7}$

b) $2(5x - 4) - 10x = 6x + 3(2x - 5)$

$10x - 8 - 10x = 6x + 6x - 15$

$-8 = 12x - 15$

$7 = 12x$

$\frac{7}{12} = x$

c) $\frac{2}{3}x + 5 = 6x - \frac{3}{4}$

$12\left(\frac{2}{3}x + 5 = 6x - \frac{3}{4}\right)$

$8x + 60 = 72x - 9$

$69 = 64x$

$\frac{69}{64} = x$

Solve for the indicated variable:

1) $3n + 1 = 7n - 5$

2) $2[x + 3(x - 1)] = 18$

3) $6(y + 2) - 4 = -10$

4) $2x^2 = 50$

5) $5 + 2(k + 4) = 5(k - 3) + 10$

6) $6 + 2x(x - 3) = 2x^2$

7) $\frac{2}{3}x - 18 = \frac{x}{6}$

8) $\frac{x - 2}{3} = \frac{2x + 1}{4}$

VI. Operations With Polynomials

To add or subtract polynomials, just combine like terms.
To multiply polynomials, multiply the numerical coefficients and apply the rules for exponents.

Examples:

a) $(x^2 + 3x - 2) - (3x^2 - x + 5)$
$x^2 + 3x - 2 - 3x^2 + x - 5$
$-2x^2 + 4x - 7$

b) $3x(2x + 5)^2$
$3x(4x^2 + 20x + 25)$
$12x^3 + 60x^2 + 75x$

c) $4(5x^2 + 3x - 4) + 3(-2x^2 - 2x + 3)$
$20x^2 + 12x - 16 - 6x^2 - 6x + 9$
$14x^2 + 6x - 7$

d) $(4x - 5)(3x + 7)$
$12x^2 + 28x - 15x - 35$
$12x^2 + 13x - 35$

Perform the indicated operations and simplify:

1) $(7x^2 + 4x - 3) - (-5x^2 - 3x + 2)$

2) $(7x - 3)(3x + 7)$

3) $(4x + 5)(5x + 4)$

4) $(n^2 + 5n + 3) + (2n^2 + 8n + 8)$

5) $(5x^2 - 4) - 2(3x^2 + 8x + 4)$

6) $-2x(5x + 11)$

7) $(2m + 6)(2m + 6)$

8) $(5x - 6)^2$

VII. Factoring Polynomials

Examples:

Factoring out the GCF

a) $6x^2 + 21x$

$3x(2x + 7)$

Difference of Squares

b) $x^2 - 64$

$(x - 8)(x + 8)$

Perfect Square Trinomial

c) $x^2 - 10x + 25$

$(x - 5)^2$

Trinomial

d) $3x^2 + 7x + 2$

$(3x + 1)(x + 2)$

Trinomial

e) $2x^2 - 13x + 15$

$(2x - 3)(x - 5)$

Grouping

f) $2x^3 + x^2 + 6x + 3$

$= x^2(2x + 1) + 3(2x + 1)$

$= (2x + 1)(x^2 + 3)$

Factor Completely.

1) $16y^2 + 8y$

2) $18x^2 - 12x$

3) $6m^3 + 12m^2 - 5m - 10$

4) $6y^2 - 13y - 5$

5) $x^2 - 121$

6) $12x^2 + 23x + 10$

VIII. Solving Quadratic Equations by Factoring

Rearrange equation to be equal to zero; factor and set each factor equal to zero.

Example: $x^2 - 2x = 63$ $x^2 - 2x - 63 = (x - 9)(x + 7) = 0$ solutions: $x = 9$ and $x = -7$

1) $2x^2 + 5x = -3$

2) $20x^2 = 7 - 31x$

IX. Linear Equations in Two Variables

Examples:

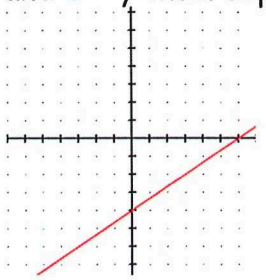
a) Find the slope of the line passing through the points (-1, 2) and (3, 5).

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} \quad \rightarrow \quad m = \frac{5-2}{3-(-1)} = \frac{3}{4}$$

b) Graph $y = 2/3\,x - 4$ with slope-intercept method.

Reminder: $y = mx + b$ is slope-intercept form where m =. slope and b = y-intercept.
Therefore, slope is 2/3 and the y-intercept is – 4.

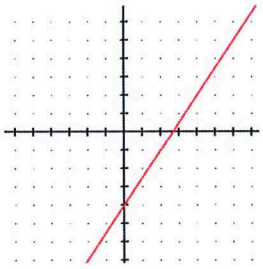
Graph accordingly. ⟶



c) Graph $3x - 2y - 8 = 0$ with slope-intercept method.

Put in Slope-Intercept form: $y = -3/2\,x + 4$

$m = 3/2$ $\quad b = -4$



d) Write the equation of the line with a slope of 3 and passing through the point (2, -1)

$y = mx + b$
$-1 = 3(2) + b$
$-7 = b$ ⟶ Equation: $y = 3x - 7$

Find the slope of the line passing through each pair of points:

1) (-3, -4) (-4, 6) 2) (-4, -6) (-4, -8) 3) (-5, 3) (-11, 3)

Write an equation, in slope-intercept form using the given information.

4) (5, 4) $m = \frac{-2}{3}$ 5) (-2, 4) $m = -3$ 6) (-6, -3) (-2, -5)

X. Solving Systems of Equations

Solve for x and y: $x = 2y + 5 \quad 3x + 7y = 2$ Using **substitution** method: $3(2y + 5) + 7y = 2$ $6y + 15 + 7y = 2$ $13y = -13$ $y = -1$ $x = 2(-1) + 5$ $x=3$ Solution: (3, -1)	Solve for x and y: $3x + 5y = 1 \quad 2x + 3y = 0$ Using **linear combination** (addition/ subtraction) method: $3(3x + 5y = 1)$ $-5(2x + 3y = 0)$ $9x + 15y = 3$ $-10x - 15y = 0$ $-1x = 3$ $x = -3$ $2(-3) + 3y = 0$ $y=2$ Solution: (-3, 2)

Solve each system of equations by either the substitution method or the linear combination (addition/ subtraction) method. Write your answer as an ordered pair.

1) $y = 2x + 4$
$-3x + y = -9$

2) $2x + 3y = 6$
$-3x + 2y = 17$

3) $x - 2y = 5$
$3x - 5y = 8$

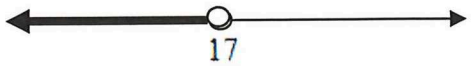
4) $3x + 7y = -1$
$6x + 7y = 0$

XI. Solving One-Variable Inequalities

Examples:

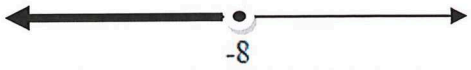
1) Solve $3(x - 5) < 4 - (2 - 2x)$.

$3x - 15 < 4 - 2 + 2x$
$3x - 15 < 2 + 2x$
$x - 15 < 2$
$x < 17$ *is the solution.*



2) Solve $5x - 12 \geq 7x + 4$.

$-2x - 12 \geq 4$
$-2x \geq 16$
$x \leq -8$ *is the solution.*



Note: Dividing both sides by -2 changed the direction of the inequality.

Absolute Value Inequalities:

$|x - 2| \geq 3$ *remove absolute value* $x - 2 \leq -3$ *or* $x - 2 \geq 3$ *solution:* $x \leq -1$ *or* $x \geq 5$

$|3x + 1| < 5$ *remove absolute value* $-5 < 3x + 1 < 5$, $-6 < 3x < 4$, *solution:* $-2 < x < \frac{4}{3}$

Practice:

Solve and Graph each inequality.

1. $3x - 9 \leq 2x + 6$

2. $-0.17x - 0.23 < 0.75 - 1.17$

3. $|x + 3| \geq 4.5$

4. $|2x - 1| < 7$

XII. Graphing Two-Variable Inequalities

GRAPHING A LINEAR INEQUALITY

To graph a linear inequality in two variables, follow these steps:

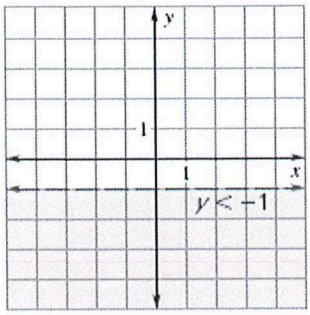
Step 1: Graph the boundary line for the inequality. Use a _dashed_ line for < or > and a _solid_ line for ≤ or ≥.

Step 2: Test a point ***not on*** the boundary line to determine whether it is a solution of the inequality. If it ***is*** a solution, shade the side containing the point. If it ***is not*** a solution, shade the other side.

Example 1

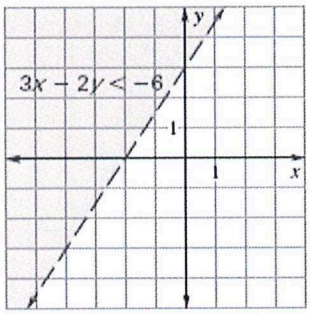
Graph a linear inequality with one variable

Graph $y < -1$ in a coordinate plane.



Solution Graph the boundary line $y = -1$. Use a _dashed_ line because the inequality symbol is <.
Test the point (0, 0). Because (0, 0) _is not_ a solution of the inequality, shade the half-plane that _does not_ contain (0, 0).

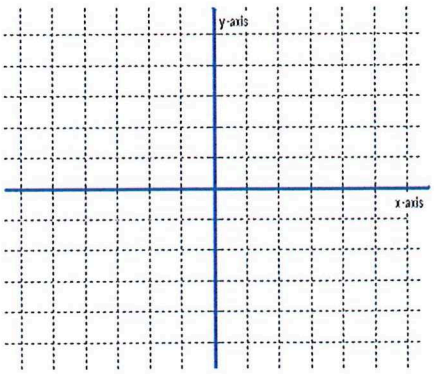
Graph $3x - 2y < -6$ in a coordinate plane.



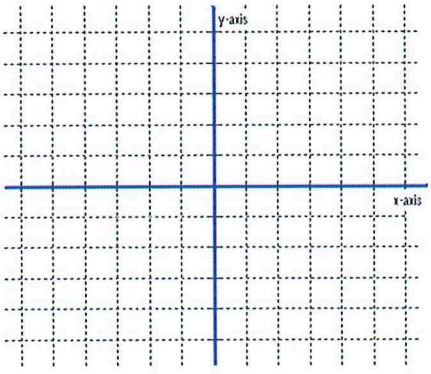
Solution Graph the boundary line $3x - 2y = -6$. Use a _dashed_ line because the inequality symbol is <.
Test the point (0, 0). Because (0, 0) _is not_ a solution of the inequality, shade the half-plane that _does not_ contain (0, 0).

Practice:
Graph each inequality.

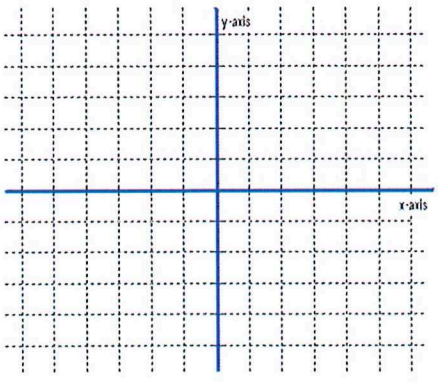
1. $y \leq -x + 2$



2. $9x + 3y > 9$



3. $5x - 2y > -12$



XIII. Fraction Operations

1. If $\frac{n}{30}$ lies between $\frac{1}{5}$ and $\frac{1}{3}$, what are all the possible values of n if n is a whole number?

2. The product of any fraction and its reciprocal is always _____.

3. Combine the following fractions according to the given operation(s):

a) $\frac{2}{5} + \frac{2}{3} =$ b) $3\frac{2}{4} - 1\frac{1}{6} =$ c) $5 \cdot 3\frac{1}{2} =$

d) $\frac{3}{8} \div \frac{1}{4} =$ e) $3\frac{2}{3} \div 1\frac{1}{3} =$ f) $\left(2\frac{7}{8} - \frac{1}{2}\right) \div \left(\frac{1}{2}\right)^2 =$

Operations with Polynomial Fractions

Examples:

$$\frac{2x+1}{x+5} + \frac{3-5x}{x+5} = \frac{4-3x}{x+5}$$

$$\frac{3x}{x^2-4} + \frac{5}{x+2} = \frac{3x}{(x-2)(x+2)} + \frac{5}{x+2} \cdot \frac{(x-2)}{(x-2)} = \frac{8x-10}{x^2-4}$$

4. Add/subtract the polynomial fractions. Reduce answers when possible.

a) $\frac{5x}{x-y} - \frac{5y}{x-y} =$ b) $\frac{2}{x+3} + \frac{2}{x-3} =$ c). $\frac{x}{x-1} - \frac{3}{x+5} =$

Examples of multiplying and dividing. FACTORING is the key!

$$\frac{x^2-10x+9}{x^2+x-6}\cdot\frac{x^2-4}{x^2-1}=\frac{(x-9)(x-1)}{(x+3)(x-2)}\cdot\frac{(x+2)(x-2)}{(x-1)(x+1)}=\frac{(x-9)(x+2)}{(x+3)(x+1)}=\frac{x^2-7x-18}{x^2+4x+3}$$

Change division to multiplying by the reciprocal of the second fraction.

5. Multiply or divide the polynomial fractions. Reduce answers when possible.

a) $\dfrac{x^2-1}{x^2-2x-3}\cdot\dfrac{x+4}{6x-6}=$

b) $\dfrac{x+4}{6x^3-24x}\cdot\dfrac{2x^3-8x}{x^2+4x}=$

c) $\dfrac{m^2-2m}{m^2+3m-10}\div\dfrac{m-5}{m+5}$

Algebra II - Summer Review Packet KEY

I. Order of Operations

1) 13 2) 10 3) 7 4) -2

II. Evaluating Algebraic Expressions

1) 7 2) -261 3) 7 4) 32.4 5) $\frac{23}{19}$

6) $\frac{2}{9}$ 7) 132

III. Simplifying Radicals

1) $5\sqrt{2}$ 2) $2\sqrt{6}$ 3) $8\sqrt{3}$ 4) 13 5) $7\sqrt{3}$ 6) $\frac{\sqrt{13}}{7}$

7) $\frac{\sqrt{2}}{3}$ 8) $\frac{\sqrt{6}}{2}$ 9) $-2\sqrt{5}$ 10) $\frac{5+\sqrt{3}}{23}$

IV. Properties of Exponents

1) g^{16} 2) b^{18} 3) $\frac{1}{w^7}$ 4) y^4 5) $-15x^{10}$

6) $16b^{10}c^2$ 7) $\frac{-3}{5x^2}$ 8) $\frac{x^{15}}{27}$

V. Solving Linear Equations

1) $n = 3/2$ 2) $x = 3$ 3) $y = -3$ 4) $x = \pm 5$

5) $k = 6$ 6) $x = 1$ 7) $x = 36$ 8) $x = \frac{-11}{2}$

VI. Operations with Polynomials

1) $12x^2 + 7x - 5$ 2) $21x^2 + 40x - 21$ 3) $20x^2 + 41x + 20$

4) $3n^2 + 13n + 11$ 5) $-x^2 - 16x - 12$ 6) $-10x^2 - 22x$

7) $4m^2 + 24m + 36$ 8) $25x^2 - 60x + 36$

VII. Factoring Polynomials

1) $8y(2y+1)$ 2) $6x(3x-2)$ 3) $(m+2)(6m^2-5)$

4) $(3y+1)(2y-5)$ 5) $(x-11)(x+11)$ 6) $(4x+5)(3x+2)$

VIII. Solving Quadratic Equations by Factoring

1) $x = -\frac{3}{2}$ or $x = 1$ 2) $x = \frac{1}{5}$ or $x = \frac{-7}{4}$

IX. Linear Equations in Two Variables

1) $m = -10$ 2) slope is undefined $\left(\frac{-2}{0}\right)$ 3) $m = 0$

4) $y = -\frac{2}{3}x + \frac{22}{3}$ 5) $y = -3x - 2$ 6) $y = -2x - 9$

X. Solving Systems of Equations

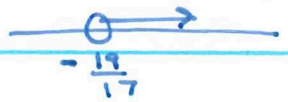
1) $(13, 30)$ 2) $(-3, 4)$ 3) $(-9, -7)$ 4) $\left(\frac{1}{3}, \frac{-2}{7}\right)$

XI. Solving One-variable Inequalities

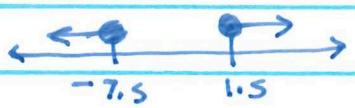
1) $x \leq 15$



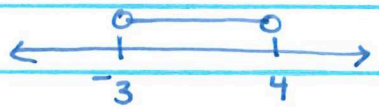
2) $x > \frac{-19}{17}$



3) $x \leq -7.5$ or $x \geq 1.5$

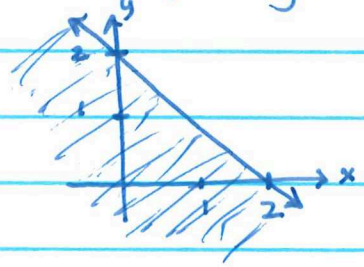


4) $-3 < x < 4$

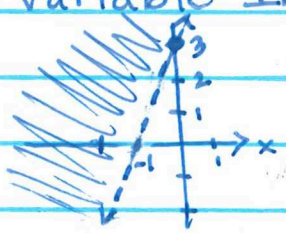


XII. Graphing Two-Variable Inequalities

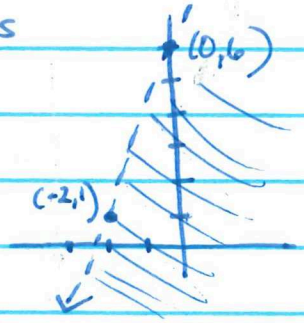
1)



2)



3)



XIII Fraction Operations

1) 7, 8, 9 2) 1

3) a) $\frac{16}{15}$ b) $2\frac{1}{3}$ or $\frac{7}{3}$ c) $\frac{35}{2}$ or $17\frac{1}{2}$

d) $\frac{3}{2}$ or $1\frac{1}{2}$ e) $\frac{11}{4}$ or $2\frac{3}{4}$ f) $\frac{19}{2}$ or $9\frac{1}{2}$

4) a) $\frac{5x-5y}{x-y} = \frac{5(x-y)}{x-y} = \boxed{5}$ b) $\frac{2(x-3)+2(x+3)}{(x+3)(x-3)} = \boxed{\frac{4x}{x^2-9}}$

c) $\frac{x(x+5)-3(x-1)}{(x-1)(x+5)} = \boxed{\frac{x^2+2x+3}{x^2+4x-5}}$

5) a) $\frac{(x-1)(x+1)}{(x-3)(x+1)} \cdot \frac{(x+4)}{6(x-1)} = \frac{x+4}{6(x-3)} = \boxed{\frac{x+4}{6x-18}}$

b) $\frac{(x+4)}{6x(x^2-4)} \cdot \frac{2x(x^2-4)}{x(x+4)} = \boxed{\frac{1}{3x}}$

c) $\frac{m(m-2)}{(m+5)(m-2)} \cdot \frac{(m+5)}{(m-5)} = \boxed{\frac{m}{m-5}}$