HOUSTON HIGH SCHOOL

2026-2027 Pre-Term Work

Course Name	**AP Calculus AB/BC**
Teacher Name and Contact Email	**Jerica Ross** **Jerica.ross@gmsdk12.org**
Pre-Term Work Assignment Description	**Prerequisite skills for AP Calculus**
Purpose of Pre-Term Assignment	**The purpose of this pre-term assignment is to ensure that students have mastered the essential skills needed to be successful in AP Calculus. The concepts and skills included in this assignment provide the mathematical foundation for the course and are necessary for students to engage successfully with advanced topics throughout the year.** **The skills covered in the pre-term work will not be re-taught in AP Calculus. Students are expected to enter the course with a strong understanding and mastery of these mathematical concepts. Completing the assignment carefully and independently will help students identify areas that may need additional review before the course begins.**
Assignment Due Date	**First day of class**
How will this assignment be submitted to the teacher?	**Printed- physical copy turned in on the first day of class.**
How will this assignment work be assessed (include rubric, if applicable)?	**The Pre-Term Work will be assessed in two ways:** **1. Assignment Grade: Completion of the Pre-Term Work will count as an assignment grade and is due on the first day of class. Late credit deductions will apply to assignments submitted after the due date.** **2. Quiz Grade: A quiz covering the concepts and skills included in the Pre-Term Work will be administered on the second day of class. Students are expected to demonstrate mastery of these concepts, as they will not be re-taught in AP Calculus.**
Additional Resources:	1.1 Calculus (Version #2) - 1.1 Limits Graphically 1.2 Calculus (Version #2) - 1.2 Limits Analytically

AP Calculus AB/BC

Dear Students,

There are certain skills that I expect incoming students to have that they do not necessarily recall when the school year begins. Since the AP curriculum is set, we do not have time to slow down or review superfluous topics during the school year. These topics are covered in the AP Calculus Pre-Term Work. Review the topics in the packet to be sure you are comfortable with all of the topics. AP Calculus students are required to complete the pre-term work.

I have a few recommendations on how to get the most out of this packet. First, try to start it around the middle of the summer and pace yourself. If you start right away, the material and techniques will not be fresh in your mind for the beginning of the year. If you wait too long, you will not get everything done and will be at a disadvantage to start the year. Make no mistake, this is a lot of work and should be taken seriously. For second-term students, start right away and pace yourself.

Second, use any and all outside resources you need. The video should be your primary resource. I am not expecting you to understand and master everything right away. Use tutors, the internet, work together with classmates. However, try to complete most of the work on your own. Keep in mind we will have a Quiz on this material the first week of school. This is required material for the AP Exam, so copying another's work does not help you in the slightest and is an Honor Code violation.

Third, understand that I am not trying to ruin your break by assigning this work. If anything, I am trying to make your next school year as painless as possible by making sure we have plenty of time in class to complete all units and have time to review before the AP Exam. This course is rigorous and challenging, and the more you can get out in front of it, the better off you will be.

Have a great break and I look forward to seeing you soon!

AP Calculus Pre-Term Work

A student entering into AP Calculus is expected to:

- Keep an organized notebook
- Complete every homework assignment to the best of one's ability
- Be an active learner by following along and taking detailed notes
- Ask questions and participate in class on a daily basis
- Seek help outside of class if needed
- Respectfully work with classmates and the teacher
- Work with and without a calculator

The following assignment should be completed over the break. All work must be neatly shown in order to receive full credit, and it should be in pencil. Outside resources (notes, books, tutors, internet sources, etc…) may be used to help with completion of the assignment. However, a calculator SHOULD NOT be used (unless a question specifically states a calculator is required). This assignment is due on the first day of class. It will be graded and the grade will be recorded in the gradebook. The assignment topics will be quickly reviewed over the first two days and a quiz will be given.

This is a lengthy assignment. Be sure to pace yourself and LEARN as you go. The following link will provide video notes for each section. The packet consists of a Notes page (to be completed with the video) as well as Practice and Test Prep.

VIDEO NOTES LINK:
1.1 Calculus (Version #2) - 1.1 Limits Graphically
1.2 Calculus (Version #2) - 1.2 Limits Analytically

Things to Know for Calculus

TRIGONOMETRY

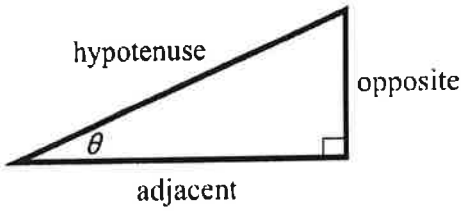
Trig Functions

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Reciprocal Functions

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}}$$

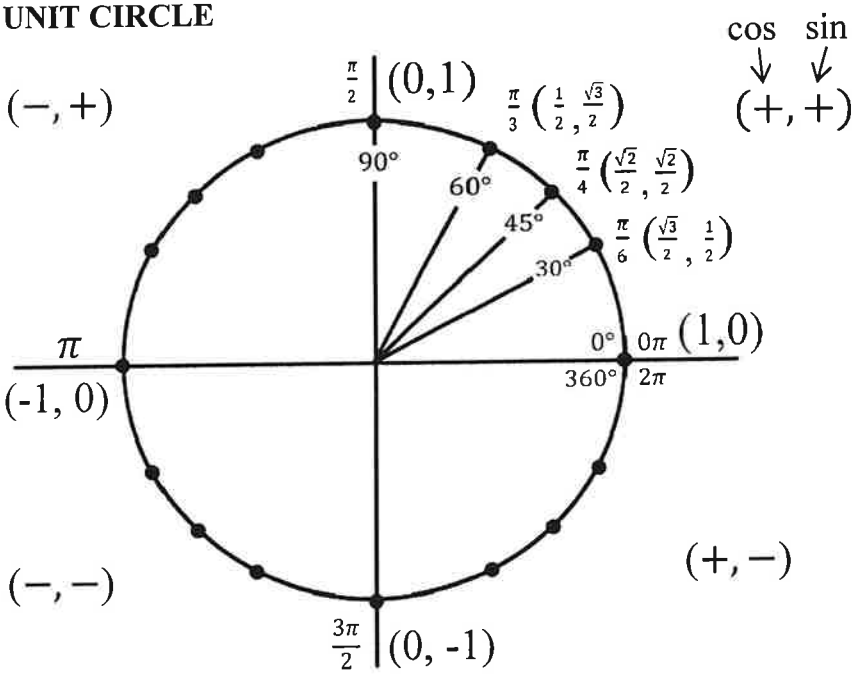
SOH-CAH-TOA



TEST ONLY USES RADIANS!

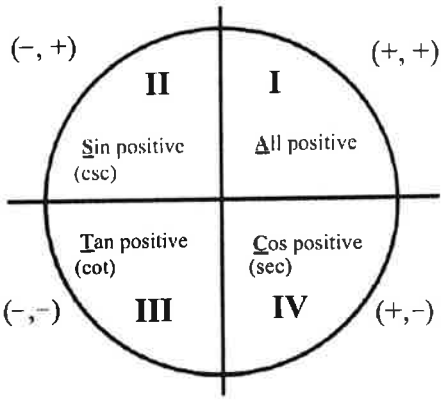
Must know trig values of special angles $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ using Unit Circle or Special Right Triangles.

UNIT CIRCLE



To help remember the signs in each quadrant

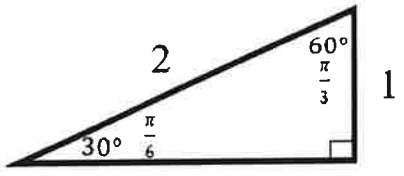
All Students Take Calculus



SPECIAL RIGHT TRIANGLES

$30° - 60° - 90°$ Triangles

Which are $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ Triangles

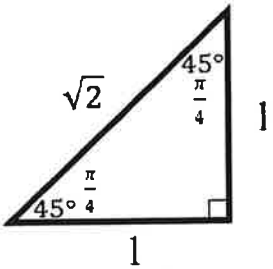


Find $\tan\left(\frac{\pi}{6}\right)$

$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$ simplify to $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$45° - 45° - 90°$ Triangles

Which are $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ Triangles



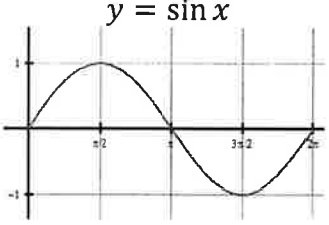
Find $\sin\left(\frac{\pi}{4}\right)$

$\sin\left(\frac{\pi}{4}\right) = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$ simplify to $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

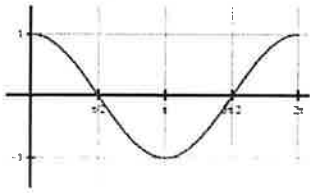
mathcentral.com

Graphs of trig functions

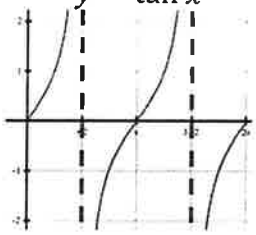
$y = \sin x$



$y = \cos x$



$y = \tan x$



Inverse Trig Function

$\sin^{-1}\theta$ is the same as $\arcsin\theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$ means what angle has a sine value of $\frac{\sqrt{3}}{2}$

that means $\theta = \frac{\pi}{3} \pm 2\pi n$ *or* $\frac{2\pi}{3} \pm 2\pi n$

Since θ has infinite answers then it isn't a function. Bummer. To make it a function we define inverses like:

sin/csc and tan/cot use quadrant I and IV for inverses
cos/sec use quadrant I and II for inverses

So… $\theta = \frac{\pi}{3}$ because it is in the first quadrant

Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity. $\mathbf{\sin^2 x + \cos^2 x = 1}$

Subtract $\sin^2 x$ to get $\cos^2 x = 1 - \sin^2 x$ or subtract $\cos^2 x$ to get $\sin^2 x = 1 - \cos^2 x$

Divide by $\sin^2 x$ to get $1 + \cot^2 x = \csc^2 x$ or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

GEOMETRY

FORMULAS

AREA

Triangle $= \frac{1}{2}bh$

Circle $= \pi r^2$

Trapezoid $= \frac{1}{2}(b_1 + b_2)h$

CIRCUMFERENCE

Circle $= 2\pi r$

SURFACE AREA

Sphere $= 4\pi r^2$

LATERAL AREA

Cylinder $= 2\pi rh$

VOLUME

Sphere $= \frac{4}{3}\pi r^3$

Cylinder $= \pi r^2 h$

Cone $= \frac{1}{3}\pi r^2 h$

Prism $= Bh$

Pyramid $= \frac{1}{3}Bh$

B is the area of the base

DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ALGEBRA

Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form
(slope-intercept Form)

$$y = mx + b$$

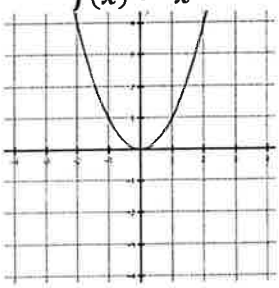
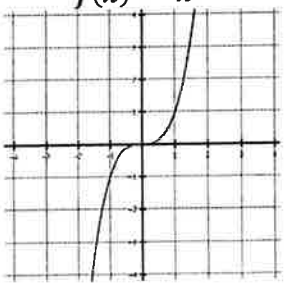
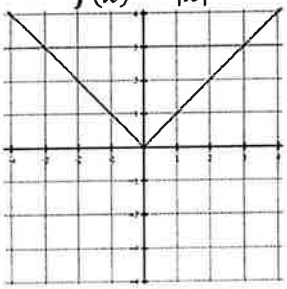
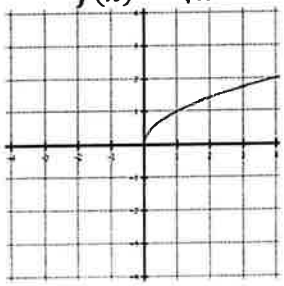
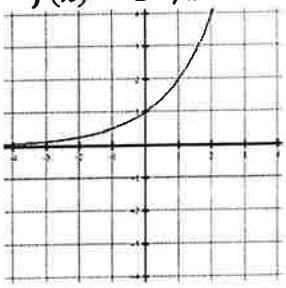
Point Slope Form

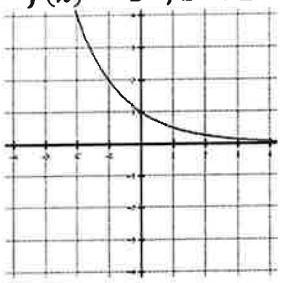
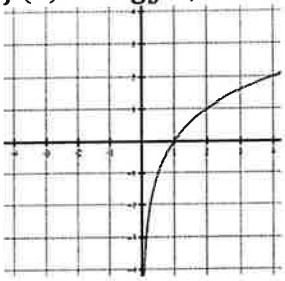
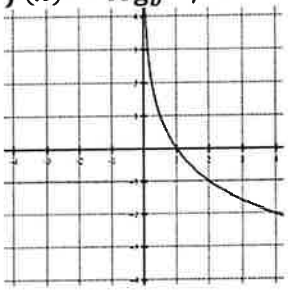
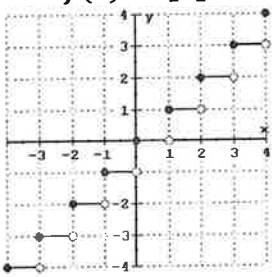
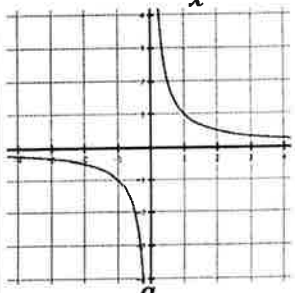
$$y - y_1 = m(x - x_1)$$

Parallel Lines
Have the same slope

Perpendicular Lines
Have the opposite reciprocal slopes

Functions

Quadratic Function	Cubic Function	Absolute Value	Square Root Function	Exponential Function
$f(x) = x^2$	$f(x) = x^3$	$f(x) = \|x\|$	$f(x) = \sqrt{x}$	$f(x) = b^x, b > 1$
				
$y = a(x-h)^2 + k$	$y = a(x-h)^3 + k$	$y = a\|x-h\| + k$	$y = a\sqrt{x-h} + k$	$y = a \cdot b^{(x-h)} + k$

Exponential Function	Logarithmic Function	Logarithmic Function	Greatest Integer	Rational Function
$f(x) = b^x, b < 1$	$f(x) = \log_b x, b > 1$	$f(x) = \log_b x, b < 1$	$f(x) = [\![x]\!]$	$f(x) = \frac{1}{x}$
				
$y = a \cdot b^{(x-h)} + k$	$y = a\log_b(x-h) + k$	$y = a\log_b(x-h) + k$	$y = a[\![x-h]\!] + k$	$y = \frac{a}{x-h} + k$

Translations

All functions move the same way!

Given the parent function $y = x^2$

Move up 4	Move down 3	Move left 2	Move right 1	Move left 2 and down 3
$y = x^2 + 4$	$y = x^2 - 3$	$y = (x+2)^2$	$y = (x-1)^2$	$y = (x+2)^2 - 3$

To flip (reflect) the function vertically $y = -x^2$
To flip (reflect) the function horizontally $y = (-x)^2$

So $f(x) = -\sqrt{x-3} + 1$ is a square root function reflected vertically, shifted right 3 and up 1

Notation

Notice open parenthesis ( ) versus closed [ ]

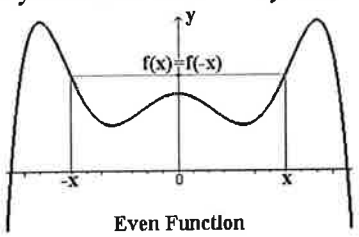
Inequality		Interval
$-3 < x \leq 5$	$\longleftrightarrow$	$(-3,5]$
$-3 \leq x \leq 5$	$\longleftrightarrow$	$[-3,5]$
$-3 < x < 5$	$\longleftrightarrow$	$(-3,5)$
$-3 \leq x < 5$	$\longleftrightarrow$	$[-3,5)$

Infinity is always open parenthesis

Inequality		Interval
$x < 3$	$\longleftrightarrow$	$(-\infty, 3)$
$x \leq 3$ or $x > 5$	$\longleftrightarrow$	$(-\infty, 3](5, \infty)$
$x \neq 3$	$\longleftrightarrow$	$(-\infty, 3)(3, \infty)$
all Real numbers	$\longleftrightarrow$	$(-\infty, \infty)$

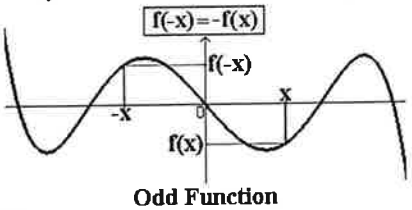
Even and Odd Functions

EVEN
$f(-x) = f(x)$
Symmetric about the y-axis



Even Function

ODD
$f(-x) = -f(x)$
Symmetric about the origin



Odd Function

Mrpreemath.com

Domain and Range

Domain = all possible x values

Range = all possible y values

Algebraically

You can't divide by zero

You can't square root a negative

$y = \sqrt{2x+5}$

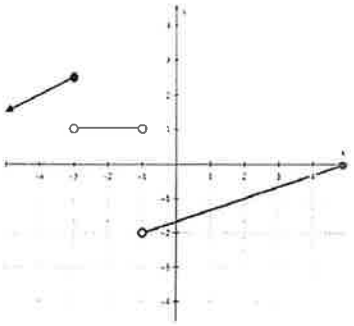
D: $[-\frac{5}{2}, \infty)$

$y = \dfrac{x^2-1}{x^2+7x+12}$

D: $(-\infty,-4)(-4,-3)(-3,\infty)$

Graphically

Just look at it



D: $(-\infty,-1)(-1,5]$

R: $(-\infty, 2.5]$

Finding zeros

Must be able to factor and use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Special products

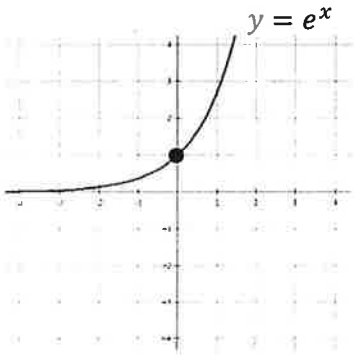
Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Exponential and Logarithmic Properties

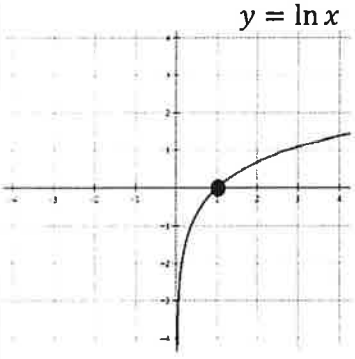
The exponential function b^x of base b is one-to-one which means it has an inverse which is called the logaritmic function of base b or logarithm of base b which is denoted $\log_b x$ which reads "the logarithm of base b of x" or "log base b of x". So…

BASE e



KNOW! $e^0 = 1$

Horizontal asymptote at $y = 0$



KNOW! $\ln 1 = 0$

Vertical asymptote at $x = 0$

$$y = \log_b x \longleftrightarrow x = b^y$$

Exponential		Logarithmic
$b^x b^y = b^{x+y}$	Product Rule	$\log_b xy = \log_b x + \log_b y$
$\frac{b^x}{b^y} = b^{x-y}$	Quotient Rule	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
$(b^x)^y = b^{xy}$	Power Rule	$\log_b x^y = y \log_b x$
$b^{-x} = \frac{1}{b^x}$		$\log_b \left(\frac{1}{x}\right) = -\log_b x$
$b^0 = 1$		$\log_b 1 = 0$
$b^1 = b$		$\log_b b = 1$
	Change of Base	$\log_b x = \frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x = \ln e$
	Common Log	$\log_{10} x = \log x$

AP - Prereq. Skills

Calculus - SUMMER PACKET

NAME:_____

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x+2) =$

3. $f(x+h) =$

Use the graph $f(x)$ to answer the following.

4. $f(0) =$

$f(4) =$

$f(-1) =$

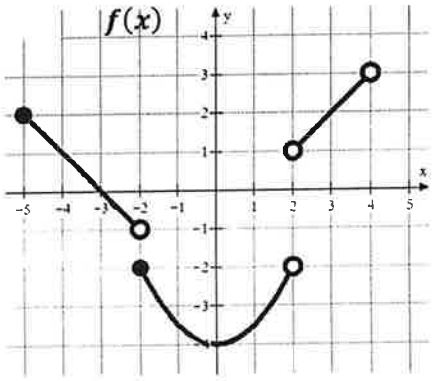
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$ when $x =$?

$f(x) = -3$ when $x =$?



Write the equation of the line meets the following conditions. Use point-slope form.

$y - y_1 = m(x - x_1)$

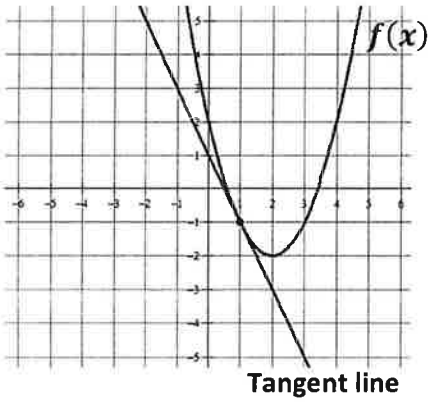
5. slope $= 3$ and $(4, -2)$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

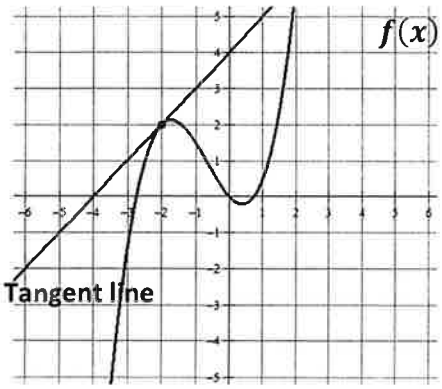
7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$

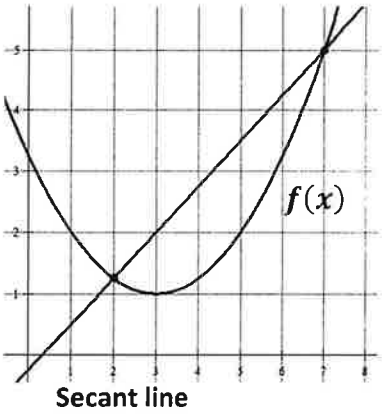


9. The line tangent to $f(x)$ at $x = -2$



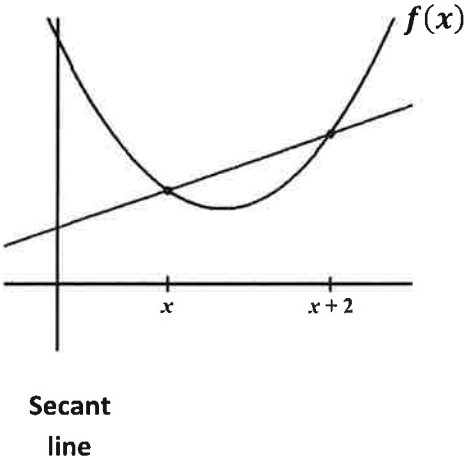
MULTIPLE CHOICE! Remember slope $= \frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?



A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$

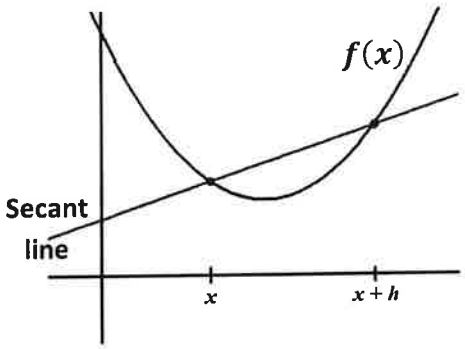
11. Which choice represents the slope of the secant line shown?



A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

12. Which choice represents the slope of the secant line shown?



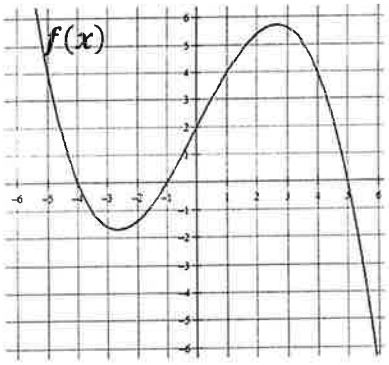
A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

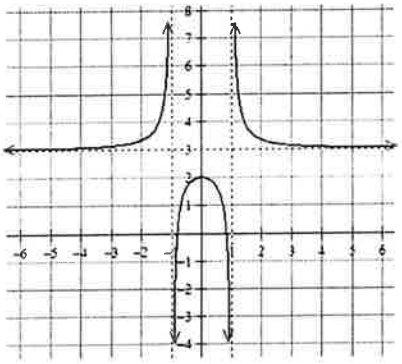
I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



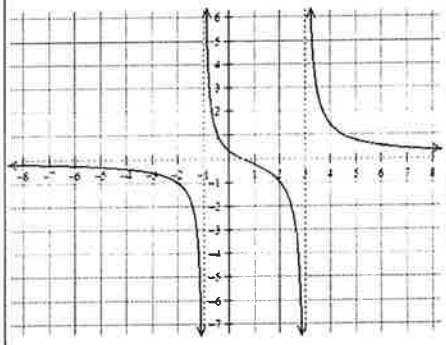
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



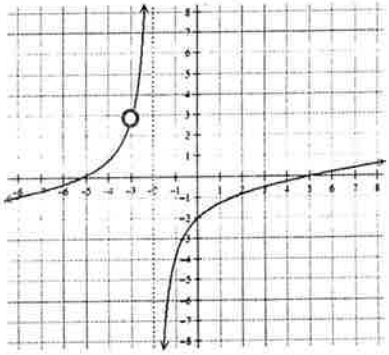
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

---

18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $f(x) = \{(3,5),(2,4),(1,7)\}$ $g(x) = \sqrt{x-3}$, then determine each of the following.
$h(x) = \{(3,2),(4,3),(1,6)\}$ $k(x) = x^2 + 5$

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f(h(3))$

84. $g(k(7))$

85. $h(3)$

86. $g(g(9))$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k(g(x))$

90. $g(f(2))$

Calculus

Write your questions and thoughts here!

1.1 Limits Graphically

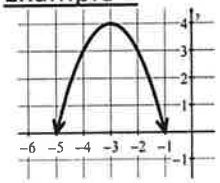
Name:_____

Notes

What is a limit?

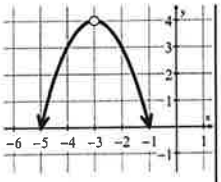
A ***limit*** is the _____ a function _____ from *both* the left and the right side of a given _____.

Example 1



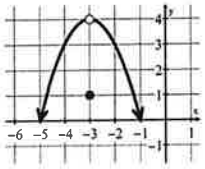
$f(-3) =$

$\lim_{x\to -3} f(x) =$



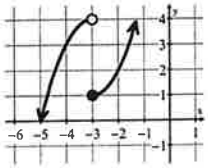
$f(-3) =$

$\lim_{x\to -3} f(x) =$



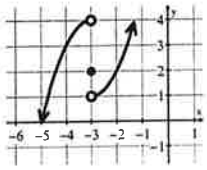
$f(-3) =$

$\lim_{x\to -3} f(x) =$



$f(-3) =$

$\lim_{x\to -3} f(x) =$



$f(-3) =$

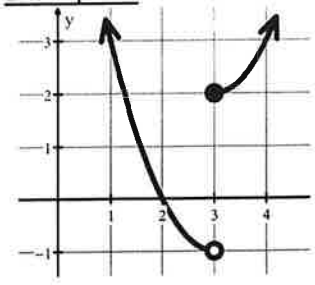
$\lim_{x\to -3} f(x) =$

Limit: (*geeky math definition for Mr. Kelly*)
Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x\to c} f(x) = R$.

What is a *one-sided* limit?

A ***one-sided limit*** is the _____ a function approaches as you approach a given _____ from either the _____ or _____ side.

Example 2



"The limit of f as x approaches 3 from the left side is -1." $\lim_{x\to 3^-} f(x) = -1$

"The limit of f as x approaches 3 from the right side is 2." $\lim_{x\to 3^+} f(x) = 2$

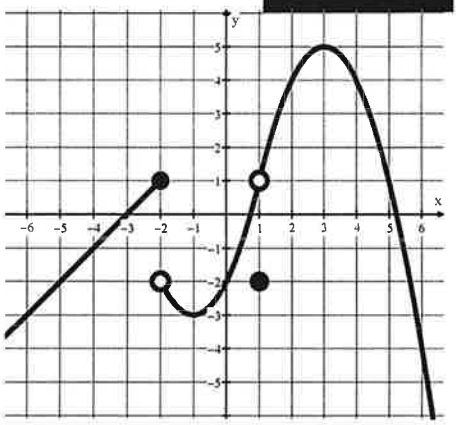
1.1 Limits Graphically

Write your questions and thoughts here!

Notes

Example 3

a. $\lim_{x\to -2^-} f(x) =$	b. $\lim_{x\to -2^+} f(x) =$	c. $\lim_{x\to -2} f(x) =$
d. $\lim_{x\to 1} f(x) =$	e. $\lim_{x\to 0} f(x) =$	f. $\lim_{x\to 3^-} f(x) =$
g. $\lim_{x\to -1} f(x) =$	h. $\lim_{x\to -3} f(x) =$	i. $f(-2) =$
j. $f(1) =$		



When does a limit not exist?

1.
2.
3.

Example 4

Sketch a graph of a function g that satisfies all of the following conditions.

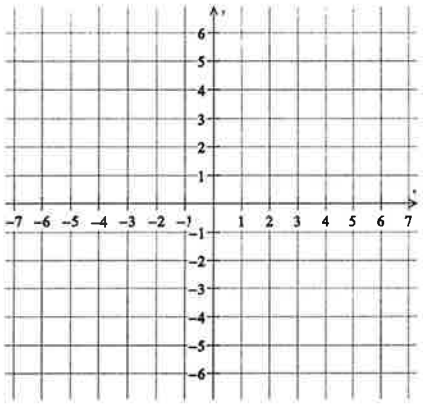
a. $g(3) = -1$

b. $\lim_{x\to 3} g(x) = 4$

c. $\lim_{x\to -2^+} g(x) = 1$

d. g is increasing on $-2 < x < 3$

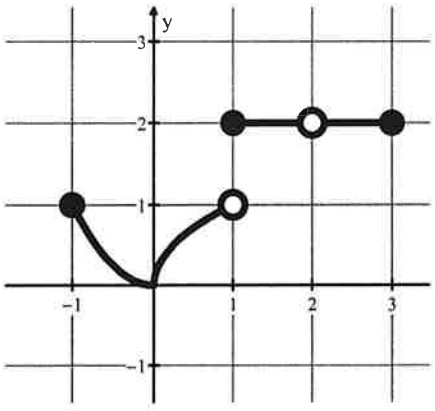
e. $\lim_{x\to -2^-} g(x) > \lim_{x\to -2^+} g(x)$



Example 5

Write **T** (true) or **F** (false) under each statement. Use the graph on the right.

a. $\lim_{x\to -1^+} f(x) = 1$	b. $\lim_{x\to 2} f(x) = 2$	c. $\lim_{x\to 1^-} f(x) = 1$
d. $\lim_{x\to 1^+} f(x) = 2$	e. $\lim_{x\to 1} f(x) =$ does not exist	
f. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$	g. $\lim_{x\to 2} f(x) =$ does not exist	



Now summarize what you learned!

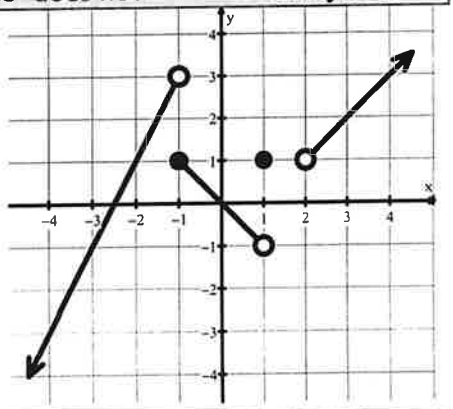
1.1 Limits Graphically

Practice

Calculus Name: _____

For 1-5, give the value of each statement. If the value does not exist, write *"does not exist"* or *"undefined."*

1.

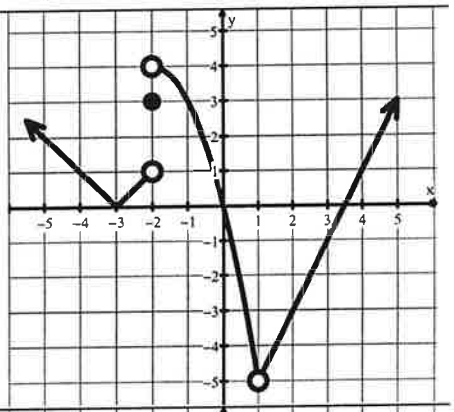


a. $\lim_{x\to -1^-} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 2^+} f(x) =$ e. $f(-1) =$ f. $f(2) =$

g. $\lim_{x\to -1^+} f(x) =$ h. $\lim_{x\to 1^-} f(x) =$ i. $\lim_{x\to 2} f(x) =$

2.

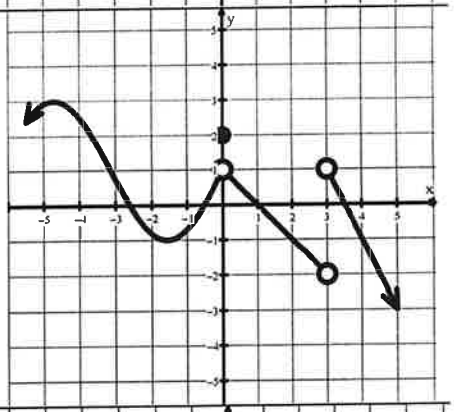


a. $\lim_{x\to -3} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 1} f(x) =$

d. $\lim_{x\to -2^+} f(x) =$ e. $f(3) =$ f. $\lim_{x\to -2^-} f(x) =$

g. $\lim_{x\to -2} f(x) =$ h. $f(-2) =$ i. $f(4) =$

3.

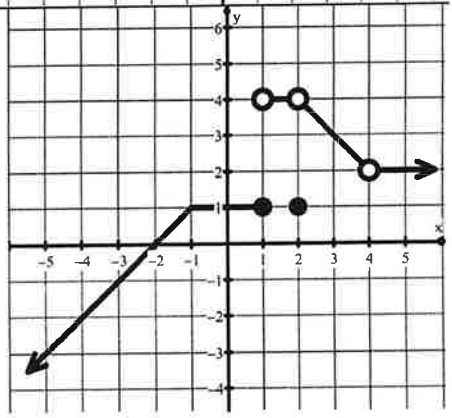


a. $\lim_{x\to 3^+} f(x) =$ b. $f(3) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 3} f(x) =$ e. $f(0) =$ f. $\lim_{x\to 3^-} f(x) =$

g. $\lim_{x\to 0^+} f(x) =$ h. $f(1) =$ i. $f(-1.6) =$

4.



a. $\lim_{x\to -1^-} f(x) =$ b. $f(2) =$ c. $\lim_{x\to 2} f(x) =$

d. $\lim_{x\to -1} f(x) =$ e. $f(4) =$ f. $\lim_{x\to 1^-} f(x) =$

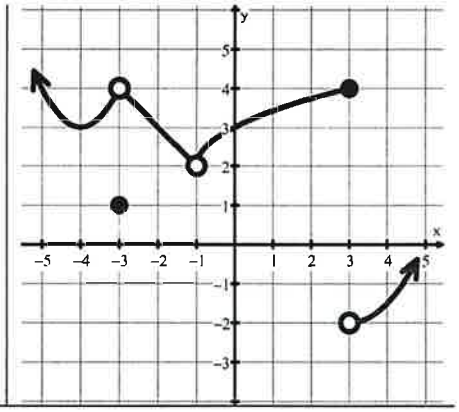
g. $\lim_{x\to -1^+} f(x) =$ h. $f(1) =$ i. $\lim_{x\to 4} f(x) =$

5.

a. $\lim_{x\to 3^-} f(x) =$ b. $f(-1) =$ c. $\lim_{x\to -3} f(x) =$

d. $\lim_{x\to -1} f(x) =$ e. $f(-3) =$ f. $\lim_{x\to 3^+} f(x) =$

g. $f(3) =$ h. $\lim_{x\to 0} f(x) =$ i. $f(-4) =$



---

6. Sketch a graph of a function f that satisfies all of the following conditions.

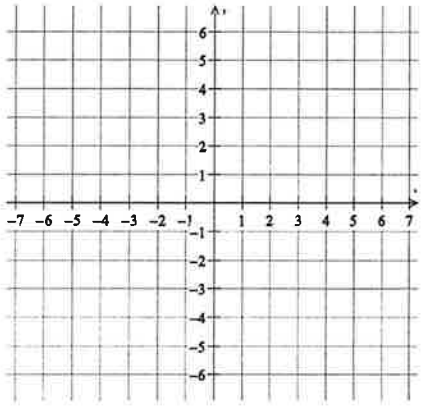
a. $f(-2) = 5$

b. $\lim_{x\to -2} f(x) = 1$

c. $\lim_{x\to 4^+} f(x) = 3$

d. f is increasing on $x < -2$

e. $\lim_{x\to 4^-} f(x) < \lim_{x\to 4^+} f(x)$



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7. Sketch a graph of a function g that satisfies all of the following conditions.

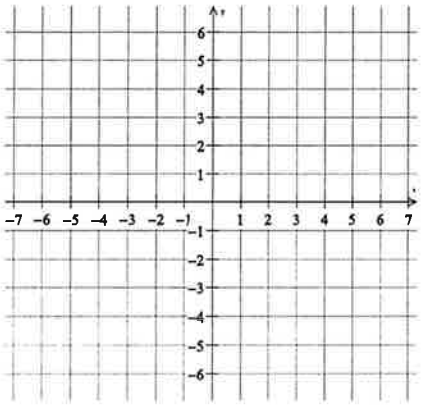
a. $g(1) = 3$

b. $\lim_{x\to 1} g(x) = -2$

c. $\lim_{x\to -3^+} g(x) = 5$

d. g is increasing only on $-5 < x < -3$ and $x > 1$

e. $\lim_{x\to -3^-} g(x) > \lim_{x\to -3^+} g(x)$



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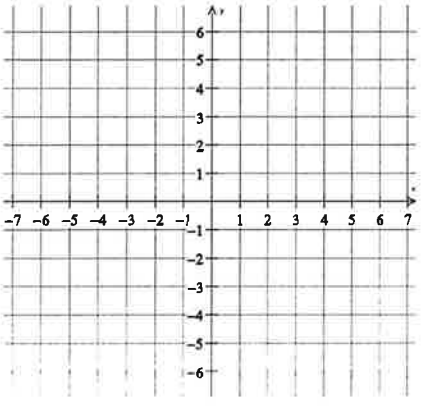
8. Sketch a graph of a function h that satisfies all of the following conditions.

a. $\lim_{x\to 3} h(x) = h(-2) = 1$

b. $h(3)$ is undefined.

c. $\lim_{x\to -2^-} h(x) < \lim_{x\to -2^+} h(x)$

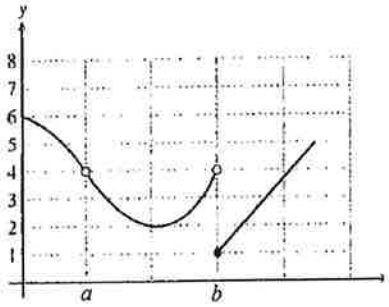
d. h is constant on $-2 < x < 3$ and decreasing everywhere else.



1.1 Limits Graphically

Test Prep

1. The graph of the function f is shown. Which of the following statements about f is true?



(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$

(B) $\lim_{x\to a} f(x) = 4$

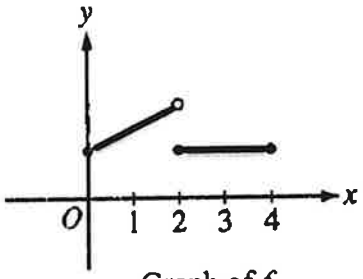
(C) $\lim_{x\to b} f(x) = 4$

(D) $\lim_{x\to b} f(x) = 1$

(E) $\lim_{x\to a} f(x)$ does not exist.

2. The figure below shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x\to 2^-} f(x)$ exists.
II. $\lim_{x\to 2^+} f(x)$ exists.
III. $\lim_{x\to 2} f(x)$ exists.



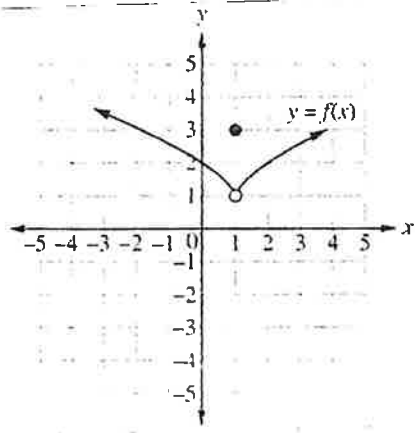
Graph of f

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

3. If $[x]$ represents the greatest integer that is less than or equal to x, then $\lim_{x\to 0^-} \frac{2}{[x]} =$

(A) -2 (B) -1 (C) 0 (D) 2 (E) the limit does not exist

4. Consider the function $y = f(x)$ shown below. Which of the following statements is true?



(A) $\lim_{x\to 1} f(x) = 3$

(B) $f(1) = 1$

(C) $f(x)$ is continuous for all x.

(D) $\lim_{x\to 1} f(x) = f(1)$

(E) None of the above

Calculus

Write your questions and thoughts here!

1.2 Limits Analytically

Name:_____

Notes

Recall: What is a limit?

Finding a limit:

1.

2.

a.

b.

3.

Direct Substitution		Factor and Cancel	
1. $\lim_{x\to -1}(x^2+2x-4)$	2. $\lim_{x\to 2}\sqrt{3x-2}$	3. $\lim_{x\to 0}\frac{4x^2-5x}{x}$	4. $\lim_{x\to -7}\frac{2x^2+13x-7}{x+7}$

Rationalize	Two variables
5. $\lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}$	6. $\lim_{h\to 0}\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$

1.2 Limits Analytically

Notes

Write your questions and thoughts here!

Piecewise defined functions and limits

$$f(x)=\begin{cases}\sqrt{11-x}, & x<-5\\ \dfrac{x+3}{5-x^2}, & x\geq -5\end{cases}$$

$$g(x)=\begin{cases}\sqrt{10-x^2}, & x<-1\\ \dfrac{26-5x^2}{7} & -1<x\leq e\\ \ln x^3, & x>e\end{cases}$$

7. $\lim_{x\to -5^-} f(x) =$

8. $\lim_{x\to -5^+} f(x) =$

9. $\lim_{x\to -5} f(x) =$

10. $\lim_{x\to -1} g(x) =$

11. $\lim_{x\to e^+} g(x) =$

12. $\lim_{x\to e} g(x) =$

Special Trig Limits:

$$\lim_{x\to 0}\frac{\sin x}{x}= \quad \text{or} \quad \lim_{x\to 0}\frac{x}{\sin x}=$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}= \quad \text{or} \quad \lim_{x\to 0}\frac{\cos x-1}{x}=$$

13. $\lim_{x\to 0}\frac{\sin 3x}{x}$

14. $\lim_{x\to 0}\frac{\tan 4x}{8x}$

15. $\lim_{x\to 0}\frac{\cos^2 x-1}{x(\cos x+1)}$

Now summarize what you learned!

1.2 Limits Analytically

Calculus

Name: _____

Practice

Evaluate each limit.

1. $\lim_{x\to 2}(x - x^2)$	2. $\lim_{x\to 5}(x+1)^2$	3. $\lim_{x\to 1}\frac{x^2-5x}{x-1}$	4. $\lim_{x\to 1}\frac{x^2+x-30}{x-1}$	5. $\lim_{x\to 0}\frac{3x}{\sin x}$
6. $\lim_{x\to 0}\frac{\sin(2x)}{3x}$	7. $\lim_{x\to 0}\frac{\sqrt{x+7}-\sqrt{7}}{x}$	8. $\lim_{x\to 7}\frac{\sqrt{x+9}-4}{x-7}$	9. $\lim_{x\to -2}(3x^2 - x + 1)$	10. $\lim_{x\to 3}(2x^2 + 5x - 6)$
11. $\lim_{x\to -7}\frac{2x^3+11x^2-21x}{x^2+7x}$	12. $\lim_{x\to 8}\frac{x^2+2x-80}{x-8}$	13. $\lim_{x\to 0}\frac{\sqrt{x+9}-3}{x}$	14. $\lim_{x\to 0}\frac{\sqrt{x+11}-\sqrt{11}}{x}$	15. $\lim_{x\to 5}\sqrt{4x-9}$

16. $\lim_{x\to -1}\sqrt{3-x}$

17. $\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

18. $\lim_{h\to 0}\frac{5\sqrt{x+h}-5\sqrt{x}}{h}$

19. $\lim_{x\to \frac{1}{3}}\frac{6x^2+13x-5}{3x-1}$

20. $\lim_{x\to 0}\frac{7x^2+x}{x}$

21. $\lim_{x\to 2}\frac{\sqrt{5x-6}}{x}$

22. $\lim_{x\to \frac{\pi}{2}}\tan\left(\frac{x}{2}\right)$

23. $\lim_{x\to 1} 3$

24. $\lim_{x\to -3} 14$

25. $\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}$

26. $\lim_{x\to 0}\frac{\frac{1}{(x+2)^2}-\frac{1}{4}}{x}$

27. $\lim_{x\to 0}(-2)$

28. $\lim_{x\to 1}\frac{\sqrt{x+5}+\sqrt{6}}{x}$

29. $\lim_{x\to 0} \frac{x^2+2x-8}{x-4}$

30. $\lim_{x\to -2} \frac{x^2-4x-10}{x}$

31. $\lim_{x\to 0} \frac{3x^2+5x}{x}$

32. $\lim_{x\to 4} \frac{5x^2-21x+4}{x-4}$

33. $\lim_{x\to \frac{1}{2}} \frac{1-x-2x^2}{2x-1}$

34. $\lim_{x\to \pi} \cos x$

35. $\lim_{x\to \frac{\pi}{8}} \sin(4x)$

36. $\lim_{x\to 2} \frac{x^2+6x-16}{2-x}$

37. $\lim_{x\to 5} \frac{2x^2-17x+35}{5-x}$

38. $\lim_{x\to 0} \frac{(1-\cos^2 x)\sin x}{x^2}$

39. $\lim_{h\to 0} \frac{(x+h)^2+6(x+h)-(x^2+6x)}{h}$

40. $\lim_{h\to 0} \frac{4(x+h)^2-5(x+h)-2-(4x^2-5x-2)}{h}$

On the AP exam, there will be questions where you must find the hole of a function. This is basically finding the limit as you approach the discontinuity. The problems on the exam will be worded differently, but if you can recognize what to do, they are not that difficult.

The following functions have a removable discontinuity (hole). If we fill in this hole to make the function continuous, what is the coordinate point to fill in?

41. $\frac{x^2-x-12}{x-4}$

42. $\frac{x^2+7x}{2x}$

43. $\frac{2x-1}{2x^2+x-1}$

44. $\frac{3x^2+13x+4}{x+4}$

Using the following piecewise functions, find the given values.

$$g(x)=\begin{cases}\sqrt{5-x}, & x<-4\\ x^2-5, & -4\le x<2\\ x-3, & x\ge 2\end{cases}$$

$\lim_{x\to 2^-} g(x) =$

$\lim_{x\to -4^+} g(x) =$

$g(2) =$

$\lim_{x\to -4^-} g(x) =$

$\lim_{x\to 2^+} g(x) =$

$\lim_{x\to 2} g(x) =$

$\lim_{x\to -4} g(x) =$

$g(-4) =$

$$h(x)=\begin{cases}-|x|, & x\le -5\\ 20-x^2, & -5<x\le 3\\ 4x-1, & x>3\end{cases}$$

$\lim_{x\to -5^+} h(x) =$

$\lim_{x\to -5} h(x) =$

$h(3) =$

$\lim_{x\to -5^-} h(x) =$

$\lim_{x\to 3^+} h(x) =$

$\lim_{x\to 3} h(x) =$

$h(-5) =$

$\lim_{x\to 3^-} h(x) =$

$$w(\theta)=\begin{cases}\sin\theta, & \theta\le\pi\\ \cos\theta, & \pi<\theta<2\pi\\ \tan\theta, & \theta>2\pi\end{cases}$$

$\lim_{x\to \pi^-} w(\theta) =$

$w(\pi) =$

$\lim_{x\to \pi^+} w(\theta) =$

$\lim_{x\to 2\pi^-} w(\theta) =$

$\lim_{x\to \pi} w(\theta) =$

$\lim_{x\to 2\pi^+} w(\theta) =$

$\lim_{x\to 2\pi} w(\theta) =$

$w(2\pi) =$

1.2 Limits Analytically

Test Prep

1. $\lim_{x\to -1} \cos(\pi x) =$

(A) π (B) 1 (C) 0 (D) -1 (E) The limit does not exist

2. If $f(x) = \begin{cases} \ln 3x, & 0 < x \leq 3 \\ x \ln 3, & 3 < x \leq 4 \end{cases}$, then $\lim_{x \to 3} f(x)$ is

(A) $\ln 9$ (B) $\ln 27$ (C) $3 \ln 3$ (D) $3 + \ln 3$ (E) nonexistent

3. Evaluate $\lim_{x \to 1} \frac{\ln x}{3x}$ is

(A) 0 (B) $\frac{3}{e}$ (C) e (D) 3 (E) The limit does not exist.

4. $\lim_{x \to 0} 4 \frac{\sin x \cos x - \sin x}{x^2}$ is

(A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefined

5. $\lim_{x \to a} \frac{x^2 - 2ax + a^2}{x - a} =$

(A) $-\infty$ (B) a (C) 0 (D) ∞ (E) The limit does not exist.

6. Let $f(x) = \begin{cases} \frac{\sin x}{x}, & x \geq 0 \\ \cos x, & x < 0 \end{cases}$ Which of the following statements about $f(x)$ is true?

I. $\lim_{x \to 0^+} f(x) = 1$

II. $\lim_{x \to 0^-} f(x) = 1$

III. $\lim_{x \to 0} f(x) = 1$

(A) None of these statements are true. (B) I only (C) II only (D) I and II only (E) I, II, and III

7. $\lim_{x \to 0} \left(\frac{3x^2 + 5\cos x - 5}{2x} \right) =$

(A) 0 (B) $\frac{5}{2}$ (C) 3 (D) 4 (E) Does not exist

8. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent