Calculus Summer Review

Summer 2026

Dear students,

To prepare you for next year's Calculus class, the Math Department requires that you complete a summer review assignment. This review will refresh your skills and prepare you for our Calculus course.

Solve each problem, showing all your work. You are expected to spend a minimum of 2 hours on this assignment, but it may require as many as 4 hours if you are not familiar with the material. ALL problems need to be done with all work shown, neat, and stapled. You may print out the summer review and complete the problems on the document itself, or complete the problems on binder paper. The completed assignment is due the first day of class. We will quickly review the assignment on that day, so bring any questions you may have to the first class. We will be having an assessment on this material in the second class, so I would encourage you to review your work the day before classes start to have it fresh in your mind if you completed the assignment early in the summer. All problems on this review should be material that you have already mastered. If there is anything that does not look familiar, please make sure you get the proper help during the summer.

If you have any questions, you may reach out to the Math Department Chair, Mrs. Brand, at: cbrand@moreaucatholic.org

See you in August!

Special thanks to our friends Bryan Passwater, Speedway HS, Speedway, Indiana for authoring this activity and to Vernon "Ted" Gott, retired teacher from Harwood, MD for providing detailed solutions. Further thanks to Jor-dan Lyerly from Fellowship Christian Academy for the final form o f these packets.

SKILLS NEEDED FOR CALCULUS

I. Algebra:

A. Exponents* *(Operations with integer, fractional and negative exponents)*

B. Factoring* *(GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)*

C. Rationalizing* *(numerator and denominator)*

D. Solving Algebraic Equations and Inequalities* *(linear, quadratic, rational, radical, and absolute value)*

II. Graphing and Functions

A. Lines* *(intercepts, slopes, write equations using point-slope and slope intercept, parallel, perpendicular, distance and midpoint formulas)*

B. Functions* *(definition, notation, domain, range, inverse, composition)*

C. Basic Shape and Transformations* *(absolute value, rational, root, higher order curves, logarithms, natural log, exponential, trigonometric, piece-wise, and inverse functions)*

III. Geometry

A. Pythagorean Theorem

B. Area Formulas *(circles, polygons, surface area of solids)*

C. Volume Formulas

D. Similar Triangles

IV. Logarithmic and Exponential Functions

A. Simplify Expressions* *(using laws of logarithms and exponents)*

B. Solve Logarithmic and Exponential Equations* *(include ln as well as log)*

C. Sketch Graphs*

D. Inverses*

V. Trigonometry

A. Unit Circle* *(definition of functions, angles in radians and degrees)*

B. Use Pythagorean Identities and Formulas to Simplify Expressions and Prove Identities

C. Solve Equations*

D. Inverse Trigonometric Functions*

E. Right Triangle Trigonometry

F. Graphs*

* A solid working foundation in these areas is critical

Formulas and Identities

Trigonometric Identities:

Reciprocal Identities:

$$sinx = \frac{1}{cscx} \qquad cosx = \frac{1}{secx} \qquad tanx = \frac{1}{cotx}$$

$$cscx = \frac{1}{sinx} \qquad secx = \frac{1}{cosx} \qquad cotx = \frac{1}{tanx}$$

Quotient Identities:

$$tanx = \frac{sinx}{cosx} \qquad cotx = \frac{cosx}{sinx}$$

Pythagorean Identities:

$$sin^2x + cos^2x = 1 \qquad 1 + tan^2x = sec^2x \qquad 1 + cot^2x = csc^2x$$

Geometric Formulas:

Area of a Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Area of a Triangle: $A = \frac{1}{2}bh$

Area of an Equilateral Triangle: $A = \frac{\sqrt{3}}{4}s^2$

Area of a Circle: $A = \pi r^2$

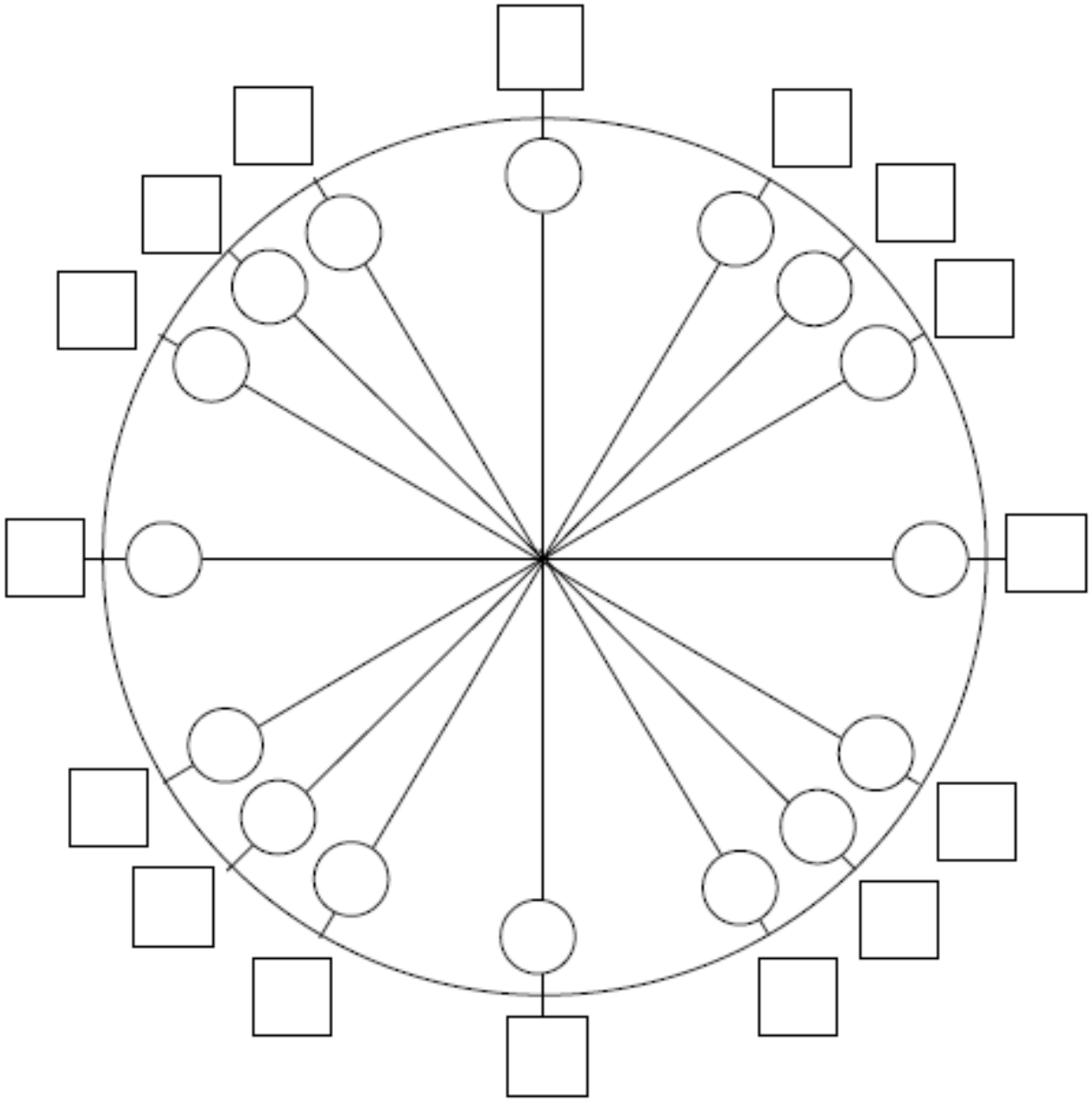
Circumference of a Circle: $C = 2\pi r$ or $C = d\pi$

Volume of a Cylinder: $V = \pi r^2 h$

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Volume of a Right Circular Cone: $V = \frac{1}{3}\pi r^2 h$

UNIT CIRCLE



Place **degree** measures in the **circles**

Place **radian** measures in the **squares**

Place $(\boldsymbol{cos\theta}, \boldsymbol{sin\theta})$ in parenthesis **outside the square**

Place $\boldsymbol{tan\theta}$ **outside the parenthesis**

Calculus Problems

Work the following problems on your own paper. Show all necessary work. Number each problem and stay organized throughout.

I. Algebra

A. Exponents:

Simplify the expression; your answer should only contain positive exponents.

1) $\dfrac{\left(8x^3yz\right)^{\frac{1}{3}}(2x)^3}{4x^{\frac{1}{3}}\left(yz^{\frac{2}{3}}\right)^{-1}}$

B. Factor Completely:

2) $9x^2+3x-3xy-y$ (hint: use grouping)

3) $64x^6-1$

4) $42x^4+35x^2-28$

5) $15x^{\frac{5}{2}}-2x^{\frac{3}{2}}-24x^{\frac{1}{2}}$ (hint: factor a GCF of $x^{\frac{1}{2}}$ fist)

6) $x^{-1}-3x^{-2}+2x^{-3}$ (hint: factor GCF of x^{-3} first)

C. Rationalizing Denominator / Numerators:

7) $\frac{3-x}{1-\sqrt{x-2}}$

8) $\frac{\sqrt{x+1}+1}{x}$

D. Simplify the rational expression:

9) $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

E. Solve: *You may use your graphing calculator to check your solutions.*

10) $(x-3)^2>4$

11) $\frac{x+5}{x-3}\leq 0$

12) $3x^3-14x^2-5x\leq 0$

13) $x<\frac{1}{x}$

14) $\frac{x^2-9}{x+1}\geq 0$

15) $\frac{1}{x-1}+\frac{4}{x-6}>0$

16) $x^2<4$

17) $|2x+1|<\frac{1}{4}$

F. Solve the System. *Solve the system algebraically and then check the solution by graphing each function and using your calculator to find points of intersection.*

18) $x-y+1=0$

$y-x^2=-5$

19) $x^2-4x+3=y$

$-x^2+6x-9=y$

II. Graphing and Functions

A. Linear graphs:

20) Passes through the point $(2,-1)$ and has the slope $-\frac{1}{3}$

21) Passes through the point $(4,-3)$ and is perpendicular to $3x+2y=4$

22) Passes through the point $(-1,-2)$ and is parallel to $y=\frac{3}{5}x-1$

B. Functions: *Find the domain of the following.*

23) $f(x)=\frac{3}{x-2}$ 24) $g(x)=\log(x-3)$

25) $h(x)=\sqrt{2x-3}$ 26) $w(x)=\frac{\sqrt{x-1}}{x^2-1}$

27) Given f(x) below, sketch the graph over the domain $[-3,3]$.

$$s(x)=\begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x-2 & \text{if } x < -1 \end{cases}$$

Find the composition/inverses as indicated below.

Let $f(x)=x^2+3x-2$ $g(x)=4x-3$ $h(x)=\ln x$ $w(x)=\sqrt{x-4}$

28) $g^{-1}(x)$ 29) $h^{-1}(x)$ 30) $w^{-1}(x)$, for $x \geq 4$ 31) $f(g(x))$ 32) $h\big(g\big(f(1)\big)\big)$

33) Does $y=3x^2-9$ have an inverse function? Explain your answer.

Let $f(x)=2x$ $g(x)=-x$ $h(x)=4$

34) $(f \circ g)(x)$ 35) $(f \circ g \circ h)(x)$

36) Let $s(x)=\sqrt{4-x}$ and $t(x)=x^2$, find the domain and range of $(s \circ t)(x)$.

C. Basic Shapes of Curves:

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

37) $y=\sqrt{x}$ 38) $y=\ln x$ 39) $y=\frac{1}{x}$ 40) $y=|x-2|$ 41) $y=\frac{1}{x-2}$ 42) $\frac{x}{x^2-4}$

43) $y=e^{-x}$ 44) $f(x)=\begin{cases} \sqrt{25-x^2} & \text{if } x < 0 \\ \frac{x^2-25}{x-5} & \text{if } x \geq 0,\ x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$

III. Logarithmic and Exponential Functions

A. Simplifying Expressions: Non-calculator

45) $log_4\left(\frac{1}{16}\right)$ 46) $3log_3 3 - \frac{3}{4}log_3 81 + \frac{1}{3}log_3\left(\frac{1}{27}\right)$ 47) $log_9 27$

48) $log_{125}\left(\frac{1}{5}\right)$ 49) $log_w w^{45}$ 50) $\ln e$ 51) $\ln 1$ 52) $\ln e^2$

B. Solve Equations: Non-calculator

53) $log_6(x+3) + log_6(x+4) = 1$ 54) $\log x^2 - \log 100 = \log 1$ 55) $3^{x+1} = 15$

IV. Trigonometry:

A. Unit Circle: *Know the unit circle – radian and degree measure. Be prepared for a quiz.*

56) State the domain, range and fundamental period of each function.

a) $y = \sin x$ b) $y = \cos x$ c) $y = \tan x$

B. Solve the Equations

57) $cos^2 x = cosx + 2;\ \ 0 \le x \le 2\pi$ 58) $2\sin(2x) = \sqrt{3};\ \ 0 \le x \le 2\pi$

59) $cos^2 x + sinx + 1 = 0;\ \ 0 \le x \le \pi$

C. Inverse Trig Functions: note: $Sin^{-1}x = \text{Arcsin}\, x$

60) $\text{Arcsin}\, 1$ 61) $\text{Arcsin}\left(-\frac{\sqrt{2}}{2}\right)$ 62) $\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$ 63) $\sin\left(Arccos\left(\frac{\sqrt{3}}{2}\right)\right)$

64) State the domain and range for each: a) $\text{Arcsin}\,(x)$ b) $Arccos(x)$ c) $Arctan(x)$

D. Be able to do the following on your graphing calculator:

Be familiar with the calculator commands to find values, roots, minimums, maximums, and intersections. You may need to zoom in on areas of your graph to find the information.

Answers should be accurate to 3 decimal places. Sketch each graph.

65-68. Given the following function $f(x) = 2x^4 - 11x^3 - x^2 + 30x$.

65) Find all roots. 66) Find all local maxima. 67) Find all local minima.

Local maxima or local minima are the points on the graph where there is a highest or lowest point within an interval, such as a vertex on a parabola.

68) Find the following: $f(-1),\ f(2),\ f(0),\ f(.125)$

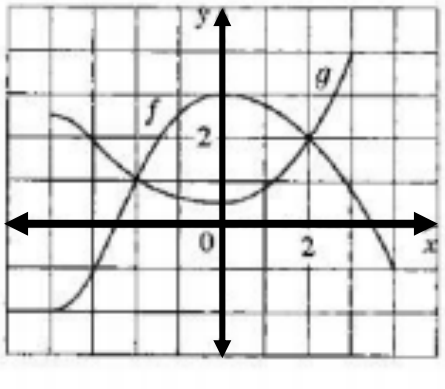
69) Graph the following two functions and find their points of intersection using the intersect command on your calculator.

$y = x^3 + 5x^2 - 7x + 2$ and $y = .2x^2 + 10$ Window: x min: -10 x max: 10 scale 1

Y min: -10 y max: 50 scale 0

V. Functions and Models

70) The graphs of f and g are given.



a) State the values of f(-4) and $g(3)$

b) For what values of x if $f(x) = g(x)$?

c) Estimate the solution of the equation $f(x) = 1$.

d) On what interval is f decreasing?

e) State the domain and range for f.

f) State the domain and range for g.

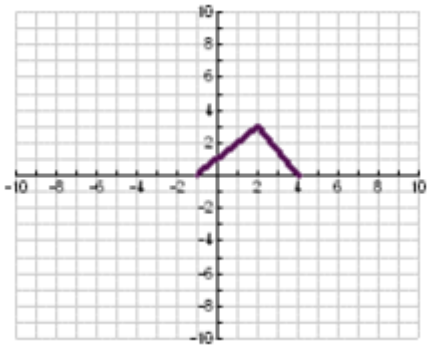
71) If $f(x) = 3x^2 - x + 2$, find $f(2),\ f(a),\ f(-a),\ f(a+1),\ 2f(a),\ f(a^2),\ [f(a)]^2$.

72) Find the domain of each function.

a) $f(x) = \frac{x}{3x-1}$ b) $g(u) = \sqrt{u} + \sqrt{4-u}$

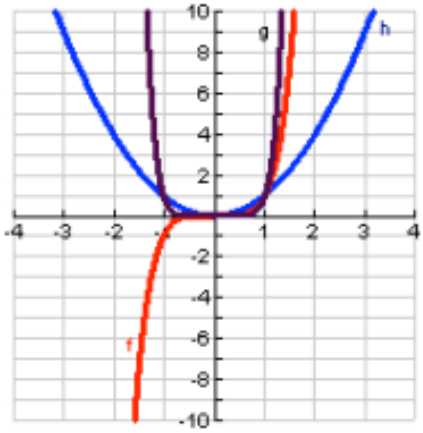
73) Find the expression for the bottom half of the parabola $x + (y-1)^2 = 0$.

74) Find the expression for the function whose graph is the given curve. (hint: piece-wise function)



75) Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator).

a) $y = x^2$ b) $y = x^5$ c) $y = x^8$

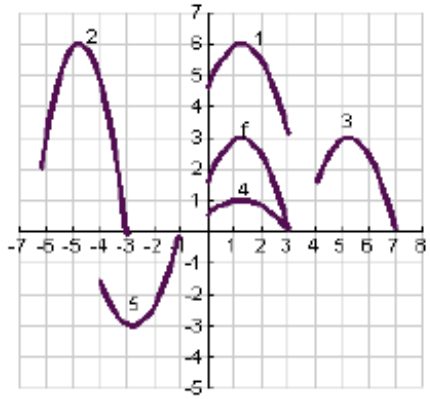


76) Suppose the graph of $y = f(x)$ is given. Write the equations for the graphs that are obtained from the graph of f (hint: use your knowledge of transformations).:

a) shift 3 units upward
b) shift 3 units downward
c) shift 3 units to the right
d) shift 3 units to the left
e) reflect about the x-axis
f) reflect about he y-axis
g) stretch vertically by a factor of 3
h) shrink vertically by a factor of 3

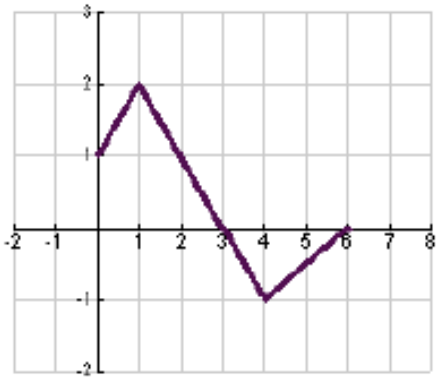
77) The graph $y = f(x)$ is given. Math each equation with its graph and given reasons for your choices.

a) $y = f(x-4)$ b) $y = f(x)+3$ c) $y = \frac{1}{3}f(x)$ d) $y = -f(x+4)$ e) $y = 2f(x+6)$



78) The graph of $y = f(x)$ is given. Use it to graph the following functions.

a) $f(2x)$ b) $f\left(\frac{1}{2}x\right)$ c) $y = f(-x)$ d)$y = -f(-x)$



79) Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ as well as their domains.

$f(x) = \sin x$ $g(x) = 1 - \sqrt{x}$

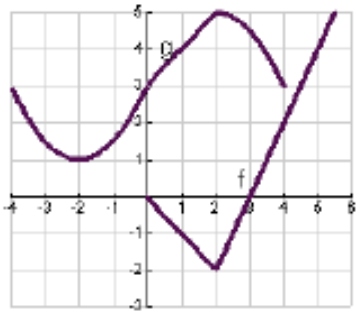
80) Express the function in the form $f \circ g$

$F(x) = (x^2 + 1)^{10}$

81) Use the given graphs of f and g to evaluate each expression, or explain why it is undefined.

a) $f\big(g(2)\big)$ b) $g\big(f(0)\big)$ c) $(f \circ g)(0)$

d) $(g \circ f)(6)$ e) $(g \circ g)(-2)$ f) $(f \circ f)(4)$



82) Graph the ellipse $4x^2 + 2y^2 = 1$ by graphing the functions whose graphs the upper and lower halves of the ellipse.

83) Use your calculator to find all solutions of the equation accurate to three decimal places

$$x^3 - 9x^2 - 4 = 0$$

84) Starting with the graph of $y = e^x$, write the equation of the graph that results from

a) shifting 2 units downward
b) shifting 2 units to the right
c) reflecting about the x-axis
d) reflecting about the y-axis
e) reflecting about the x-axis and then about the y-axis

For #85-87, find the formula for the inverse of the function.

85) $f(x) = \sqrt{10 - 3x}$ 86) $f(x) = e^{x^3}$ 87) $f(x) = \ln(x + 3)$

For #88-89, find the exact value of each expression (non-calculator)

88) a) $log_2 64$ b) $log_6 \frac{1}{36}$

89) b) $log 1.25 + log 80$ b) $log_5 10 + log_5 20 - 3log_5 2$

90) Express the given quantity as a single logarithm.

$$2 \ln 4 - \ln 2$$

Answers:

1. $4x^{11/3}y^{4/3}z$ 2. $(3x + 1)(3x - y)$ 3. $(2x - 1)(4x^2 + 2x + 1)(2x + 1)(4x^2 - 2x + 1)$

4. $7(3x^2 + 4)(2x^2 - 1)$ 5. $x^{1/2}(3x - 4)(5x + 6)$

6. $x^3(x - 2)(x - 1)$ 7. $1 + \sqrt{x - 2}$ 8. $\frac{1}{\sqrt{x+1}-1}$ 9. $\frac{x^2 - x + 1}{(x + 1)^2}$

10. $x > -1$ or $x < -5$ 11. $-5 \le x < 3$ 12. $x \le -\frac{1}{3}$ or $0 \le x \le 5$ 13. $0 < x < 1$ or $x < -1$

14. $[-3, -1) \cup [3, \infty)$ 15. $x > 6$ or $1 < x < 2$

16. $-2 < x < 2$ 17. $-\frac{5}{8} < x < -\frac{3}{8}$ 18. (3, 4), (-2, -1) 19. (2, -1), (3, 0)

20. $y = -\frac{1}{3}x - \frac{1}{3}$ 21. $y = \frac{2}{3}x - \frac{17}{3}$ 22. $y = \frac{3}{5}x - \frac{7}{5}$ 23. D: $x \ne 2$ 24. $D: x > 3$

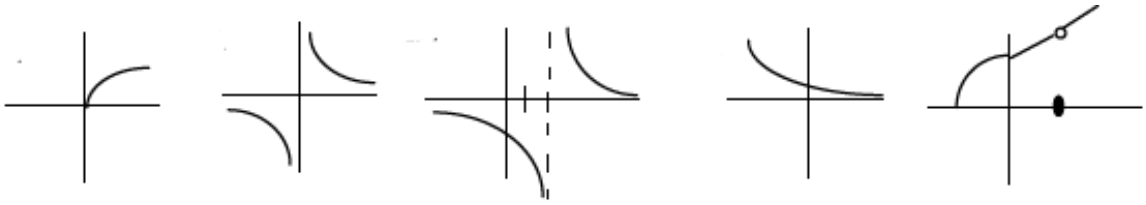
25. D: $x \ge 3/2$ 26. $x > -1$ and $x \ne 1$ 27. R: $-5 \le y < -3$ or $0 \le y \le 3$ 28. $g^{-1}(x) = \frac{x+3}{4}$

29. $h^{-1}(x) = e^x$ 30. $y = x^2 + 4$ $x \ge 0$, 31. $f(g(x)) = 16x^2 - 12x - 2$ 32. $\ln 5$

33. no, explain: this function Is not one-to-one (pass the horizontal line test)

34. $-2x$ 35. -8 36. D: $-2 \le x \le 2$ R: $0 \le y \le 2$

37. 39. 41. 43. 44.



45. -2 46. –1 47. 3/2 48. – 1/3 49. 45 50. 1 51. 0 52. 2 53. -1

54. x = 10, -10 55. $\frac{\log 15}{\log 3}$ - 1 56. a) D: all reals, R: $-1 \le x \le 1$, 2π

b) D: all reals, R: $-1 \le x \le 1$, 2π c) D: $x \ne \pi/2$ R: all reals, π

57. π 58. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$ 59. $\frac{3\pi}{2}$ 60. $\frac{\pi}{2}$

61. $-\frac{\pi}{4}$ 62. $\frac{\pi}{6}$ 63. $\frac{1}{2}$ 64. Arcsin(x) D: [-1, 1] Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$,

Arc cos (x) D: [-1, 1]. R: $[0, \pi]$ Arctan (x) D: all reals R: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

65. -1.5, 0, 2, 5 66. rel max. (1.07, 20.1) 67. rel. min (-.89, -18.48), (3.94, -88)

68. f(-1) = -18 f(2)=0 f(0)=0 f(0.125)=3.7133 69. 3 points of intersection - one is (-5.77, 16.66)

69. Do not use trace to find points. Use CALC commands.

70. (a) -2,4 (b) -2 and 2 (c) x= -3 and 4 (d) [0,4] (e) d: [-4,4] r:[-2,3] (f) d: [-4,3] r:[0.5,4]

71. f(2)=12, f(-2) = 16, f(a) = 2a^2 -a +2,
f(-a)=3a^2+a+2, f(a+1)=3a^2+5a+4, 2f(a)=6a^2-2a+4, f(2a)=12a^2-2a+2, f(a^2)=3a^4 -a^2 +2,
[f(a)]^2=9a^4 -6a^3+13a^2-4a+4, f(a+h)=3a^2 +6ah+3h^2 -a-h+2

72. (a) $(-\infty, 1/3) \cup (1/3, \infty)$ (b) [0,4] 73. $f(x) = 1 - \sqrt{-x}$ (domain: $x \le 0$)

74. $f(x)=\begin{cases} x+1 & \text{if } -1\le x\le 2 \\ -\frac{3}{2}x+6 & \text{if } 2<x\le 4 \end{cases}$

75. (a) matches with h (b) matches with f and (c) matches with g.

76. (a) y = f(x)+3 (b) y = f(x) -3 (c) y = f(x-3) (d) y = f(x+3) (e) y = -f(x) (f) y = f(-x) (g) y = 3f(x) (h) y = 1/3 f(x) 113. (a) graph 3 (b) graph 1 (c)graph 4 (d) graph 5 (e) graph 2 77. (a) shrink horizontally by a factor of 2 (b) stretch horizontally by a factor of 2 (c) reflect the graph of f about the y-axis (d) reflect the graph of f about the y-axis, then about the x-axis

79. (f+g)(x) =x^3+5x^2-1 d: all real numbers (f-g)(x)= x^3-x^2+1 d: all reals (fg)(x)=3x^5+6x^4-x^3-2s^2 d: all reals (f/g)(x)=(x^3 +2x^2)/(3x^2 -1) d: x cannot Equal $\pm\frac{1}{\sqrt{3}}$ 79. $(f\circ g)(x)=\sin(1-\sqrt{x})$ $d:[0,\infty)$ $(g\circ f)(x)=1-\sqrt{\sin x}$ $d:[0,\pi],[2\pi,3\pi]$ *etc*

$(f\circ f)(x)=\sin(\sin x)$ $d:(-\infty,\infty)$ $(g\circ g)(x)=1-\sqrt{1-\sqrt{x}}$ $d:[0,1]$ 80. g(x)=x^2 +1 and F(x)=x^10 81. (a) 4 (b) 3 (c)0 (d) not defined (e) 4 (f) -2 82. graph y = +sqr((1-4x^2)/2) and -sqr((1-4x^2)/2) 83. about 9.05 84. (a) y = e^x -2 (b) y = e^(x-2) (c) y = -e^x (d) y = e^(-x) (e) y = - e^(-x) 85. $f^{-1}(x)=-\frac{1}{3}x^2+\frac{10}{3}$ d: [0,∞) 86. $f^{-1}(x)=\sqrt[3]{\ln x}$ 87. $f^{-1}(x)=e^x-3$

88 (a) 6 (b) -2 89. (a) 2 (b) 2 90. ln8