Entering AP AB Calculus Summer Homework



Name

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Post Precalculus/Honors Entering AP AB Calculus Summer Homework

Welcome to Advanced Placement Calculus AB. Before you begin this Fall, you need to review some of your Algebra 2, Geometry, and Pre-Calculus skills. The goal of this packet to review material you learned in previous math courses. These skills are foundational to your success in AP Calculus. Therefore, take some time to complete the following problems. **Calculators should not be used.** These problems are designed for you to review the algebra, geometric, trigonometric, and pre-calculus concepts and processes that will provide the foundation for your work in AP Calculus. Remember, question marks do not answer the problem. If you are struggling, please feel free to use online resources to help you.

Each problem should be completed in the space provided. If more space is needed, please use lined paper that you will submit as well. **All work leading to your answers MUST be shown neatly**. If you need more space, please use a separate sheet of paper with the topic and problems labelled. It is strongly encouraged that you complete this over several weeks and highlight areas in which you may have questions. However, you need to attempt every problem. Question marks do not answer the problem and leaving answers blank is not acceptable. Remember, this assignment is your first introduction of your work to me.

Throughout the summer, you must also complete the posted Delta Math which will help you review the Unit Cricle.

Topic 1: Fractional & Negative Exponents

Simplify using only positive exponents.

1) $-3x^{-3}$

2) $-5\left(\frac{3}{2}\right)(4-9x)^{-\frac{1}{2}}(-9)$

3) $2\left(\frac{2}{2-x}\right)\left(\frac{-2}{(2-x)^2}\right)$

4) $\left(16x^2y\right)^{\frac{3}{4}}$

5) $-\frac{x^{-\frac{1}{2}}}{2}\sin\sqrt{x}$

6) $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

7) $-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left(\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right)$

8) $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$

9) $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

Topic 2: Domain

Find domain of the following functions:

1) $y = \dfrac{3x-2}{4x+1}$

2) $y = \dfrac{x^2-4}{2x+4}$

3) $y = \dfrac{x^2-5x-6}{x^2-3x-18}$

4) $y = \dfrac{2^{2-x}}{x}$

5) $y = \sqrt{x-3} - \sqrt{x+3}$

6) $y = \dfrac{\sqrt{2x-9}}{2x+9}$

7) $y = \dfrac{x^2+8x+12}{\sqrt[4]{x+5}}$

8) $y = \sqrt{x^2-5x-14}$

9) $y = \dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

10) $y = \log(2x-12)$

11) $y = \sqrt{\tan x}$

12) $y = \dfrac{x}{\cos x}$

Topic 3: Linear Equations

1. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.
   a. $m=-7, (-3,-7)$ b. $m=\frac{-1}{2}, (2,-8)$ c. $m=\frac{2}{3}, \left(-6,\frac{1}{3}\right)$

2. Find the equation of the line in slope-intercept form, passing through the following points.
   a. $(-3,6)$ and $(-1,2)$ b. $(-7,1)$ and $(3,-4)$ c. $\left(-2,\frac{2}{3}\right)$ and $\left(\frac{1}{2},1\right)$

3. Write equations of the line through the given point a) parallel and b) normal to the given line.
   a. $(5,-3),\ x+y=4$ b. $(-6,2),\ 5x+2y=7$ c. $(-3,-4),\ y=-2$

4. Find an equation of the line containing $(4,-2)$ and parallel to the line containing $(-1,4)$ and $(2,3)$. Put your answer in general form.

5. Find k if the lines $3x-5y=9$ and $2x+ky=11$ are a) parallel and b) perpendicular.

Topic 4: Solving Inequalities (quadratic)

Solve the following by factoring and making appropriate sign charts.

1) $x^2 - 16 > 0$

2) $x^2 + 6x - 16 > 0$

3) $x^2 - 3x \geq 10$

4) $2x^2 + 4x \leq 3$

5) $x^3 + 4x^2 - x \geq 4$

6) $2\sin^2 x \geq \sin x \quad 0 \leq x \leq 2\pi$

Topic 5: Special Factorization

Factor completely

1) x^3+8

2) x^3-8

3) $27x^3-125y^3$

4) x^4+11x^2-80

5) $ac+cd-ab-bd$

6) $2x^2+50y^2-20xy$

7) $x^2+12x+36-9y^2$

8) $x^3-xy^2+x^2y-y^3$

9) $(x-3)^2(2x+1)^3+(x-3)^3(2x+1)^2$

Topic 7: Factor Theorem (*p* over *q* method/synthetic division)

Use the *p* over *q* method and synthetic division to factor the polynomial $P(x)$. Then solve $P(x) = 0$.

1) $P(x) = x^3 + 4x^2 + x - 6$

2) $P(x) = x^3 + 5x^2 - 2x - 24$

3) $P(x) = x^3 - 6x^2 + 3x + 10$

4) $P(x) = x^3 + 2x^2 - 19x - 20$

5) $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

6) $P(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$

Topic 8: Even and Odd Functions

Show work to determine if the relation is even, odd, or neither

1) $f(x) = 2x^2 - 7$

2) $f(x) = -4x^3 - 2x$

3) $f(x) = 4x^2 - 4x + 4$

4) $f(x) = x + \frac{1}{x}$

5) $f|x| = |x| - x^2 + 1$

6) $5x^2 - 6y = 1$

7) $y = e^x - \frac{1}{e^x}$

8) $3y^3 = 4x^3 + 1$

9) $3x = |y|$

Topic 9: Solving Quadratic Equations and Quadratic Formula

Solve each equation

1) $7x^2 - 3x = 0$

2) $4x(x-2) - 5x(x-1) = 2$

3) $x^2 + 6x + 4 = 0$

4) $2x^2 - 3x + 3 = 0$

5) $2x^2 - (x+2)(x-3) = 12$

6) $x + \frac{1}{x} = \frac{13}{6}$

7) $x^4 - 9x^2 + 8 = 0$

8) $x - 10\sqrt{x} + 9 = 0$

9) $\frac{1}{x^2} - \frac{1}{x} = 6$

Topic 10: Asymptotes

For each function, find the equation of both the vertical asymptote(s) and horizontal asymptotes (if they exist). Label your work.

1) $y = \dfrac{x}{x-3}$

2) $y = \dfrac{x+4}{x^2-1}$

3) $y = \dfrac{x+4}{x^2+1}$

4) $y = \dfrac{x^2-2x+1}{x^2-3x-4}$

5) $y = \dfrac{x^2-9}{x^3+3x^2-18x}$

6) $y = \dfrac{2x^2+6x}{x^3-3x^2-4x}$

7) $y = \dfrac{x^2-x-6}{x^3-x^2+x-6}$

8) $y = \dfrac{2x^3}{x^3-1}$

9) $y = \dfrac{\sqrt{x}}{2x^2-10}$

Topic 11: Complex Fractions

Simplify the following

1) $\dfrac{x}{x-\frac{1}{2}}$

2) $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-2}$

3) $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

4) $\dfrac{\frac{3}{x}-\frac{4}{y}}{\frac{4}{x}-\frac{3}{y}}$

5) $\dfrac{1-\frac{2}{3x}}{x-\frac{4}{9x}}$

6) $\dfrac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$

7) $\dfrac{x^{-3}-x}{x^{-2}-1}$

8) $\dfrac{\frac{x}{1-x}+\frac{1+x}{x}}{\frac{1-x}{x}+\frac{x}{1+x}}$

9) $\dfrac{\frac{4}{x-5}+\frac{2}{x+2}}{\frac{2x}{x^2-3x-10}+3}$

Topic 12: Composition of Functions

If $f(x)=x^2$, $g(x)=2x-1$, and $h(x)=2^x$, find the following:

1) $f(g(2))$

2) $g(f(2))$

3) $f(h(-1))$

4) $h(f(-1))$

5) $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

6) $f(g(x))$

7) $g(f(x))$

8) $g(g(x))$

9) $f(h(x))$

Use the table of values to evaluate each expression.

x	-3	-2	-1	0	1	2	3
$f(x)$	11	9	7	5	3	1	-1
$g(x)$	-8	-3	0	1	0	-3	-8

10) $f(g(1))$

11) $g(f(2))$

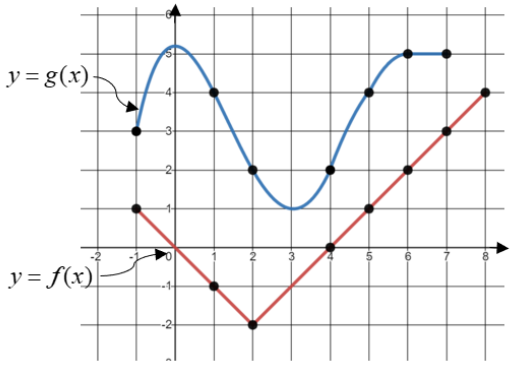
12) $f(f(3))$

Evaluate each expression using the graphs of $y=f(x)$ and $y=g(x)$

13) $g(f(-1))$

14) $f(g(4))$

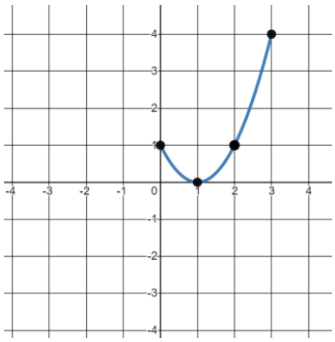
15) $g(g(2))$



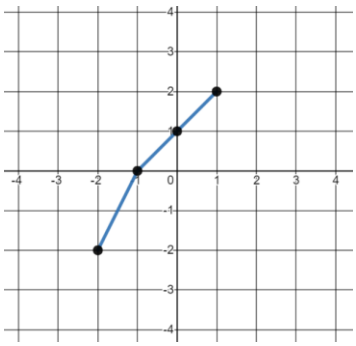
Topic 13: Inverse Functions

Draw the graph of the inverse for each of the graphs below.

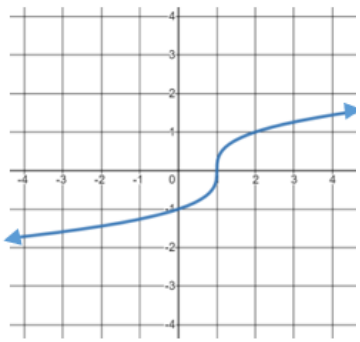
1)



2)



3)



Verify the functions f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

4) $f(x) = 3x + 4;\ \ g(x) = \frac{1}{3}(x - 4)$

5) $f(x) = x^3 - 8;\ \ g(x) = \sqrt[3]{x + 8}$

6) $f(x) = \frac{1}{x};\ \ g(x) = \frac{1}{x}$

7) $f(x) = \frac{2x + 3}{x + 4};\ \ g(x) = \frac{4x - 3}{2 - x}$

Find the inverse for each of the following. State the domain and range of f and f^{-1}.

8) $f(x) = 4x + 2$

9) $f(x) = \frac{2}{3 + x}$

10) $f(x) = x^2 + 4;\ x \geq 0$

Topic 14: Solving Rational (Fractional) Equations

Solve each equation for x

1) $\frac{2}{3}-\frac{5}{6}=\frac{1}{x}$

2) $x+\frac{6}{x}=5$

3) $\frac{x+1}{3}-\frac{x-1}{2}=1$

4) $-\frac{x-5}{x+1}=\frac{3}{5}$

5) $\frac{60}{x}-\frac{60}{x-5}=\frac{2}{x}$

6) $\frac{2}{x+5}+\frac{1}{x-5}=\frac{16}{x^2-25}$

7) $\frac{x}{x-2}+\frac{2x}{4-x^2}=\frac{5}{x+2}$

8) $\frac{x}{2x-6}-\frac{3}{x^2-6x+9}=\frac{x-2}{3x-9}$

9) $\frac{2x+3}{x-1}=\frac{10}{x^2-1}+\frac{2x-3}{x+1}$

Topic 15: Exponential and Logarithmic Expression and Equations

Answer each of the following without using a calculator. Show work. Answers should be left as exact values.

1. Find a. $\log_2 \frac{1}{4}$ b. $\log_8 4$ c. $\ln \frac{1}{\sqrt[3]{e^2}}$

d. $5^{\log_5 40}$ e. $e^{\ln 12}$ f. $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

g. $\log_2 \frac{2}{3} + \log_2 \frac{3}{32}$ h. $\log_{\frac{1}{3}} \frac{4}{3} - \log_{\frac{1}{3}} 12$ i. $\log_3 \left(\sqrt{3}\right)^5$

2. Solve a. $\log_5 (3x-8) = 2$ b. $\log_9 \left(x^2 - x + 3\right) = \frac{1}{2}$ c. $\log(x-3) + \log 5 = 2$

d. $\log_2 (x-1) + \log_2 (x+3) = 5$ e. $\log_5 (x+3) - \log_5 x = 2$ f. $\ln x^3 - \ln x^2 = \frac{1}{2}$

g. $3^{x-2} = 18$ h. $e^{3x+1} = 10$ i. $8^x = 5^{2x-1}$

Topic 16: Concepts in Trigonometry

Solve the following problems.

If point P is on the terminal side of θ, find all 6 trig functions of θ. Draw a picture

1) $P(-2,4)$

2) $P\left(\sqrt{5},-2\right)$

3) If $\cos\theta = -\frac{5}{13}$, θ in quadraant II, find $\sin\theta$ and $\tan\theta$

4) If $\cot\theta = 3$, θ in quadrant III, find $\sin\theta$ and $\cos\theta$

Find the exact value of the following without a calculator:

5) $\sin^2\frac{5}{4}\pi - \cos^2\frac{5}{3}\pi$

6) $\left(6\sec\pi - 4\cot\frac{\pi}{2}\right)^2$

7) $\left(4\cos\frac{\pi}{6} - 6\sin\frac{2\pi}{3}\right)^{-2}$

Solve the following triangles (3 decimal place accuracy)

8) $A =$ $a = 21.7$
$B = 16°$ $b =$
$C = 90°$ $c =$

9) $A =$ $a = 6$ feet
$B =$ $b =$
$C = 90°$ $c = 95$ inches

Topic 17: Solving Trigonometric Equations

Solve each equation on the interval $[0, 2\pi)$

1) $\sin x = \frac{1}{2}$

2) $\cos^2 x = \cos x$

3) $2\cos x + \sqrt{3} = 0$

4) $4\sin^2 x = 1$

5) $2\sin^2 x + \sin x = 1$

6) $\cos^2 x + 2\cos x = 3$

7) $2\sin x \cos x + \sin x = 0$

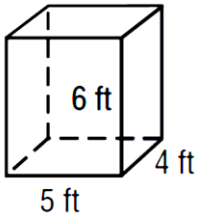
8) $8\cos^2 x - 2\cos x = 1$

9) $\sin^2 x - \cos^2 x = 0$

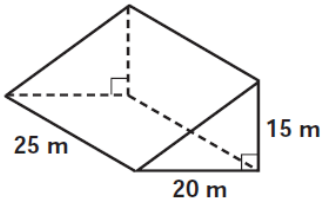
Topic 18: Geometry

For #1-8 - Find the surface area and volume of each solid. You may have to look up the formulas. 😊

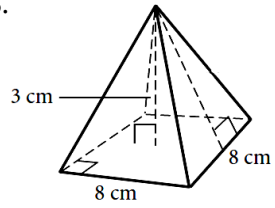
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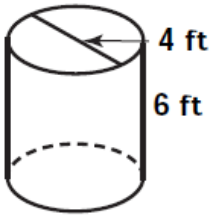
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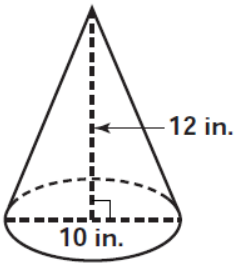
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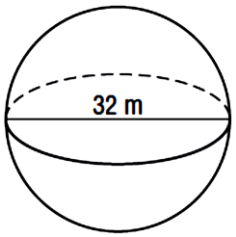
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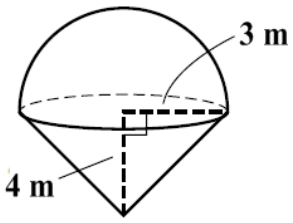
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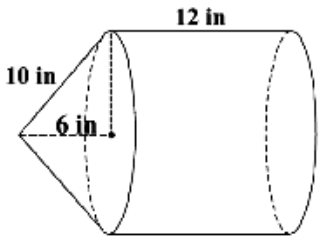
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7.



8.



9. Sylvia has just discovered that the valve on her cement truck failed during the night and that all the contents ran out to form a giant cone of hardened cement. To make an insurance claim, she needs to figure out how much cement is in the cone. The circumference of its base is 44 feet, and it is 4 feet high. Calculate the volume to nearest cubic foot.

10. You are producing 500 of these metal wedges, and you must electroplate them with a thin layer of high-conducting silver (surface area). The measurements shown below are in centimeters. Find the total cost for silver if silver plating costs $3 for every 200 square centimeters. Assume each quadrilateral is a rectangle.

