Entering AP BC Calculus Summer Homework



Name

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AP Calculus BC Summer Packet 2026

Name_____

This summer packet requires you to review many areas of mathematics. It has multiple parts. Please print off each part and follow the instructions for where to write your work. Expect to spend quite a few hours on this assignment. You are welcome to use old notes, books, or the internet (try Paul's online notes or Khan Academy) to look up information that you may have forgotten. If you have difficulties with any of these topics, I encourage you to be very proactive in getting help at the beginning of the year, as we will not be spending much time on these topics in class.

Please note that the TI-Nspire CX is the recommended calculator for this course.

Part 1: Mixed Topics. NO CALCULATOR should be used unless otherwise specified. Please thoroughly complete each problem in order on notebook paper.

1. **Simplify each of the following.**

a. $\dfrac{\frac{25}{a}-a}{5+a}$

b. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

c. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

2. a. $\dfrac{\frac{x}{2}}{\frac{y}{4}} =$

b. $h \div \dfrac{x+h}{h} =$

c. $\dfrac{\sqrt{x-2}+\frac{5}{\sqrt{x-2}}}{x-2} =$

3. Solve for y': $xy' + y = 1 + y'$

4. Find the average rate of change on the indicated intervals:

   a. $f(x)=x^3-2x;\ [0,4]$

   b. $f(x)=3\sqrt{x};\ [4,\ 25]$

5. A car travels 360 miles in a period of 180 minutes. Find the average velocity of the car in miles per hour over this time period.

6. In 1984, the Fizzy Cola Co. sold 23 million gallons of soda. By 2003, the company was selling 127 million gallons of soda. What is the average rate of change in number of gallons of soda per year?

7. Solve for x:

   a. $4x^2-21x=18$

   b. $2x^2-3x+3=0$

   c. $x^4-9x^2+8=0$

8. Write as a single fraction with the denominator in factored form:

$$\frac{7x^2+5x}{x^2+1}-\frac{5x}{x^2-6}=$$

9. Determine if the function is *even, odd, or neither.*

   a. $f(x)=2x^2-7$

   b. $f(x)=-4x^3-2x$

   c. $f(x)=4x^2-4x+4$

10. **Find the equation of the straight line that passes through the point (2, 4) and is**
    a. **Parallel to $2x - 3y - 8=0$**

    b. **Perpendicular to $2x - 3y - 8 = 0$**

11. **Find the domain for each function:**

    a. $f(x)=\dfrac{1}{4x^2-21x-18}$

    b. $g(x)=\sqrt{x^2-5x-14}$

    c. $h(x)=\dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

    d. $k(x)=\ln(2x-12)$

12. **Find** $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$ *if* $f(x)=8x^2+1$

13. **Given** $f(x)=\dfrac{1}{x}$, *Find*: $\dfrac{f(x+h)-f(x)}{h}$

14. *Given* $f(x)=x-3$ *and* $g(x)=\sqrt{x}$, *find the following*

    a. f(g(x))

    b. g(f(x))

    c. f(f(x))

15. Given $f(x)=\frac{1}{x-5}$ and $g(x)=x^2-5$, find the following:

a. f(g(7))

b. g(f(v))

c. g(g(x))

16. Simplify using only positive exponents. Do not rationalize the denominator.

a. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

b. $\left(\frac{1}{x^{-2}}+\frac{2}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{\frac{1}{2}}$

17. If $f(x)=x^2-1$, describe in words what the following would do to the graph of f(x).

a. f(x) - 4 b. f(x - 4) c. –f(x + 2)

d. 5f(x)+3 e. f(-x) f. |f(x)|

18. Expand the logarithms

a. $\ln\left[x^2(y-2)^3\right]$

b. $\ln\left[\frac{ex^3}{y^2z}\right]$

c. $\ln\left[\frac{4(x-2)^2}{3\sqrt[3]{y+2}}\right]$

19. Condense into one logarithm

a. $\ln x+3\ln y-\frac{1}{2}\ln z$

b. $2[\ln(x-1)-3\ln y]-4\ln z$

c. $\frac{1}{3}\ln x-2\ln(x+2)+4\ln(y+2)$

20. Divide using long division: $\frac{2x^4+5x^3-11x^2+14x-5}{2x-1}$

21. Finish the factoring

a. $2\sqrt{x}+6x^{\frac{3}{2}}=2\sqrt{x}(\ ?\ )$

b. $\sqrt{x^2+1}-\frac{x^2}{\sqrt{x^2+1}}=\frac{1}{\sqrt{x^2+1}}(\ ?\ )$

c. $(2x+1)^{\frac{3}{2}}x^{\frac{1}{2}}+(2x+1)^{\frac{5}{2}}x^{-\frac{1}{2}}=(2x+1)^{\frac{3}{2}}x^{-\frac{1}{2}}(\quad ?\quad)$

22. A 7 foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression in terms of x, for the distance from the foot of the ladder to the base of the wall.

23. . Solve to the nearest thousandth: (Calculator Active)

a. $4^x=3$

b. $\ln x=1.09$

24. Express y in terms of x:

a. $\log y = x+2$

b. $\ln y = 2\ln x$

c. $\ln y = 4\ln x+3$

25. **Solve for x:**

a. $3^{3x+5} = 9^{2x+1}$ b. $\left(\frac{1}{9}\right)^x = 27^{2x+4}$ c. $\left(\frac{1}{6}\right)^x = 216$

26. **Evaluate the following logarithms**

a. $\log_7 7$ b. $\log_3 27$

c. $\log_2 \frac{1}{32}$ d. $\log_{25} 5$

e. $\log_9 1$ f. $\log_4 8$

g. $\ln\sqrt{e}$ h. $\ln\frac{1}{e}$

27. **Use the properties of logarithms to evaluate the following**

a. $\log_2 2^5$ b. $\ln e^3$ c. $\log_2 8^3$ d. $\log_3 \sqrt[5]{9}$

e. $2^{\log_2 10}$ f. $e^{\ln 8}$ g. $9\ln e^2$ h. $\log_9 9^3$

i. $\log_{10} 25 + \log_{10} 4$ j. $\log_2 40 - \log_2 5$ k. $\log_2\left(\sqrt{2}\right)^5$

Part 2: Trig Review. NO CALCULATOR should be used unless specified.
Please thoroughly complete each problem in order on notebook paper.

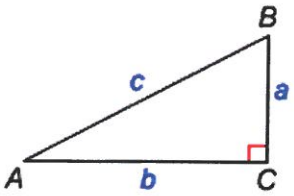
1. Let θ be an acute angle of a right triangle. Find the values of the other five trigonometric functions of θ.

$\tan\theta = \frac{7}{10}$

2. If θ is an acute angle of a right triangle and $\cos\theta = \frac{40}{41}$, what is $\tan\theta$?

Express your answer as a reduced, improper fraction, if necessary.

3. Solve $\triangle ABC$. Round your answer to nearest hundredth, if necessary.



$B = 25^\circ$

$a = 15$

4. Evaluate the expression in radians.

$$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

5. Evaluate the expression in radians.

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

6. Solve the equation $\sin\theta = -0.5$ where $180^\circ < \theta < 270^\circ$.

7. What is the general solution of $\sin x = \tan x \cdot \sin x$?

8. $\cos\theta = \frac{4}{5}$ and $270° < \theta < 360°$

   Find $\sin 2\theta$

9. Verify the identity: $\frac{1-\tan^2 x}{1+\tan^2 x} = 1 - 2\sin^2 x$

10. Write an equation of the graph of $y = \sec x$ translated up 3 units and right 2 units.

11. Graph $y = -2\sin(3x) + 1$.

12. Graph $y = 4\cos(2x - \pi)$.

13. Graph y = cot(x)

14. Graph y = arctanx and give the domain and range.

15. Graph y = arcsinx and give the domain and range.
    Now go back and check your answer to question 4.

16. Graph y = arccosx and give the domain and range.
    Now go back and check your answer to question 5.

17. Solve for x: Pay attention to given domain.

    a. $2\sin^2\theta = 1 - \sin\theta$ on $[0, 2\pi]$

    b. $\sin 2\theta + \sin\theta = 0$ On $(-\infty, \infty)$

    c. $2\tan\theta - \sec^2\theta = 0$ On $(-\infty, \infty)$ and also on $[0, 2\pi]$

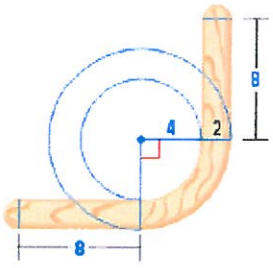
18. Solve for θ, $0 \le \pi < 2\pi$

$2+\cos^2\theta = 3\sin^2\theta$

19. Find the length of the arc intercepted by the central angle $\frac{2\pi}{5}$ in a circle with radius 4.

20. From the top of a 300-ft building, George notices a car moving towards him. If the angle of depression of the car changes from 15° to 23° during the observation, how far does the car travel? Draw a diagram and show all steps in solving the problem.

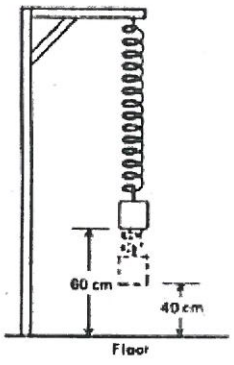
21. Find the area of the boomerang shown. The dimensions are given in inches. Give your answer in terms of π and to two decimal places.



22. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 seconds.

a. Write an equation to represent the height if the weight as a function of time.

b. Find the second time the weight reaches a height of 45cm. Answer algebraically, showing all your steps.



23. Simplify the expression to a single term. $\sin x(\tan x + \cot x)$

24. Prove the identity. $\tan^2\theta - \sin^2\theta = \tan^2\theta \cdot \sin^2\theta$

Part 3: More Mixed Review. Please answer the questions in the provided spaces in the packet.

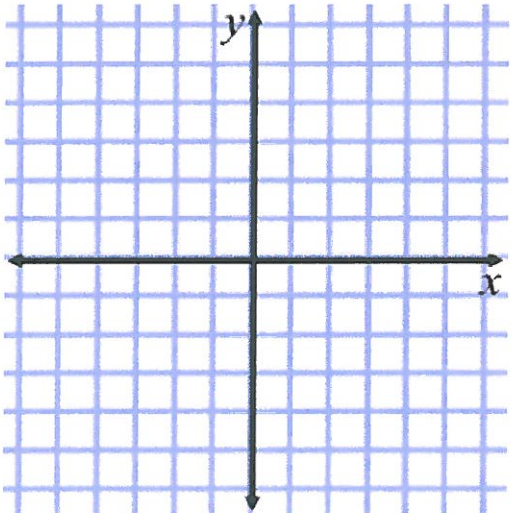
No Calculator should be used unless specified.

1 Graph each function, clearly marking at least 3 points on each graph. List the domain and range.

a. $f(x) = -\sqrt[3]{x} + 2$

Domain:

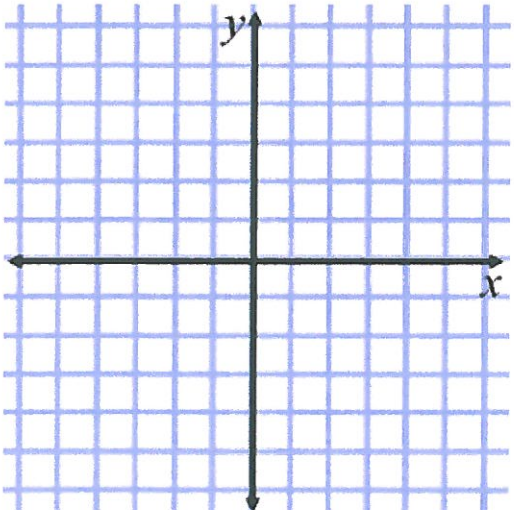
Range:



b. $f(x) = 2\sqrt{-(x-3)}$

Domain:

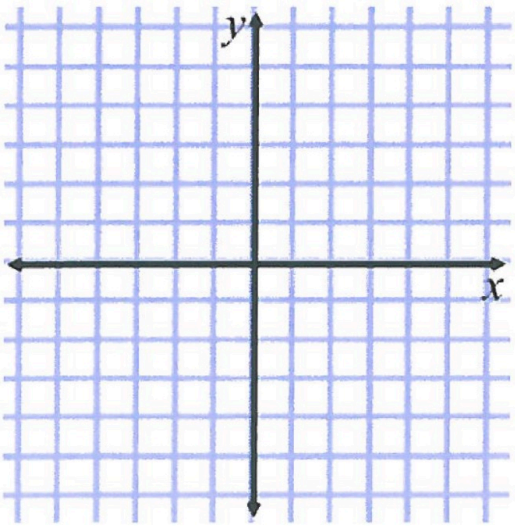
Range:



c. $f(x) = \frac{1}{x+2} - 3$

Domain:

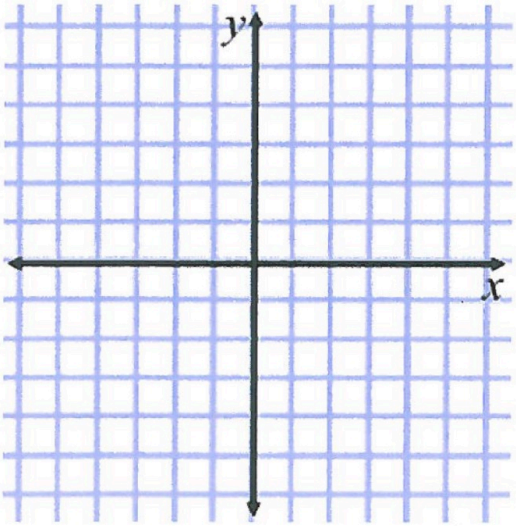
Range:



d. $f(x) = -\frac{1}{2}(x-1)^2 + 3$

Domain:

Range:

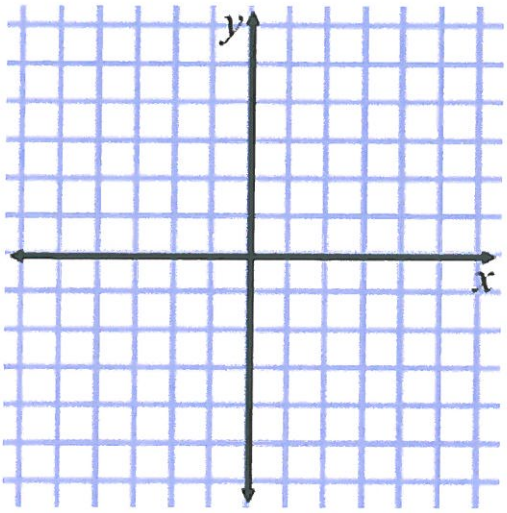


Graph each function, clearly marking at least 3 points on each graph. List the domain and range.

e. $f(x) = \frac{2}{3}|x-2| - 4$

Domain:

Range:

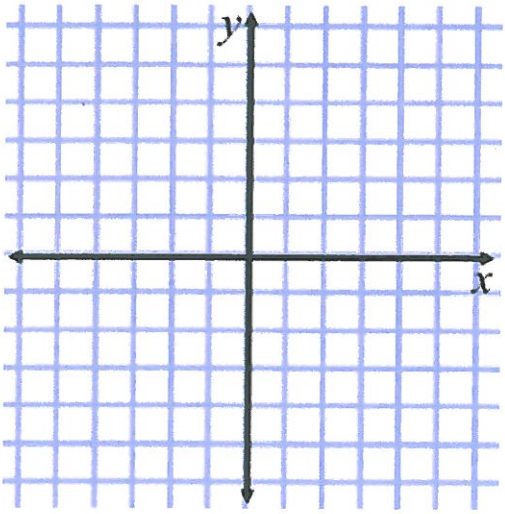


f. $f(x) = -3\cos\left(x + \frac{\pi}{4}\right) - 1$

Graph at least one full period. Label the x-axis in terms of π. Clearly mark the central axis, maxima, and minima.

Domain:

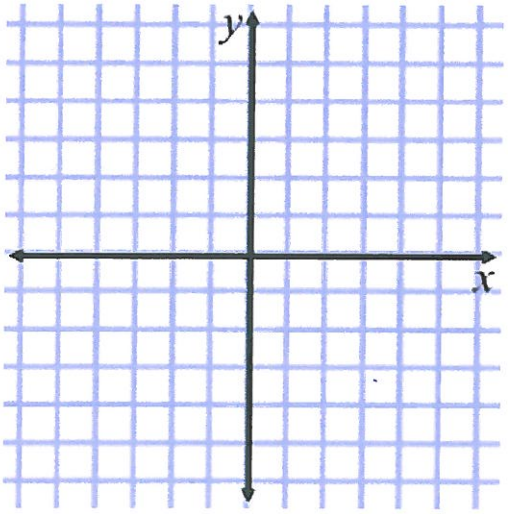
Range:



g. $f(x) = 2(0.5)^x - 4$

Domain:

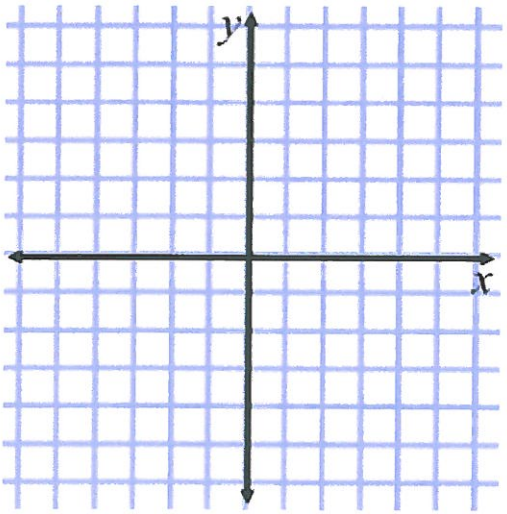
Range:



h. $f(x) = \log_2(x+3)$

Domain:

Range:



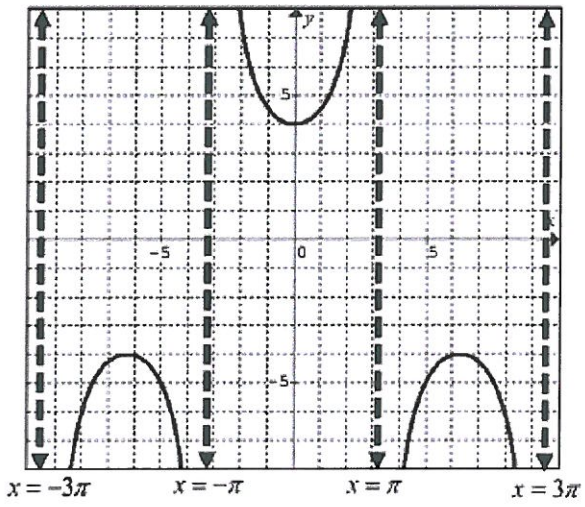
2. Write the equation for each graph. Also write the domain and range for each.

a.

Equation:

Domain:

Range:

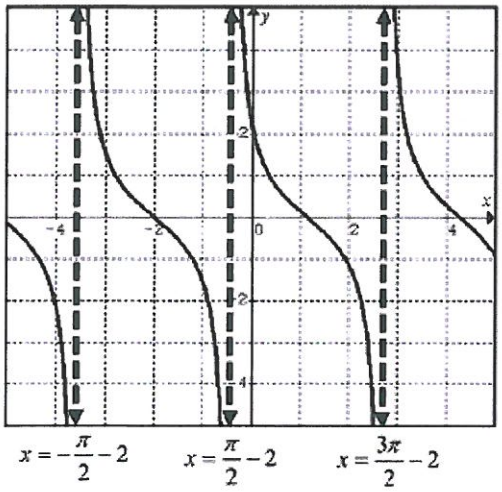


b.

Equation:

Domain:

Range:

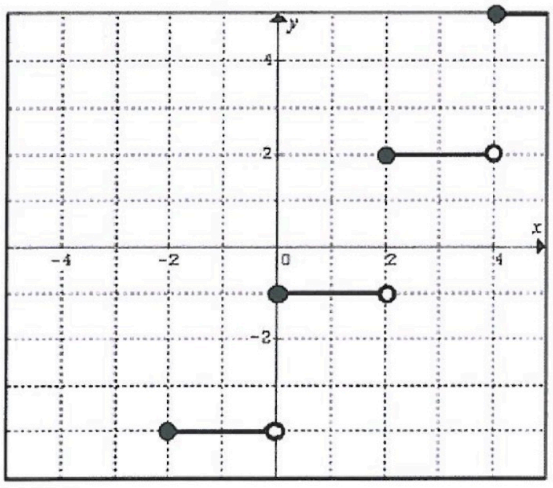


c.

Equation:

Domain:

Range:

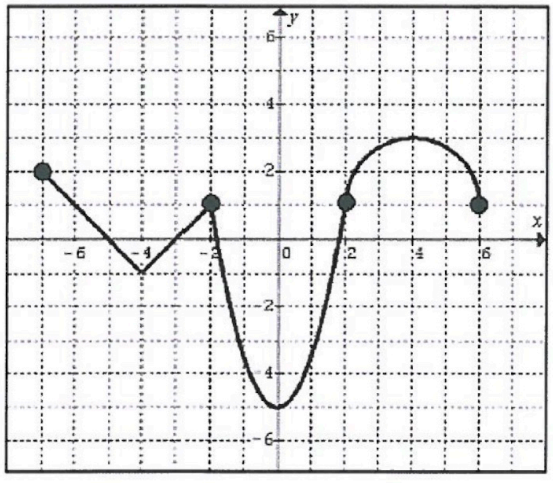


d.

Equation:

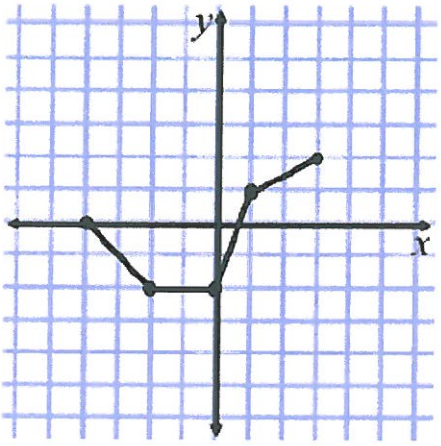
Domain:

Range:

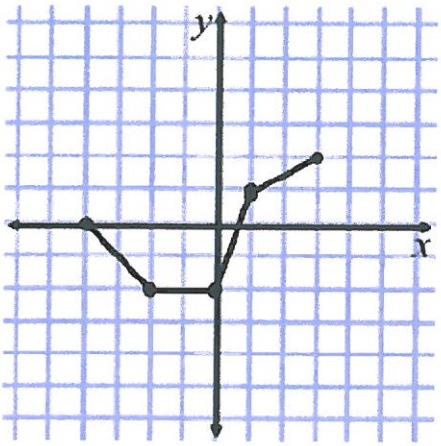


3. Use the graph of $y = f(x)$ below. Draw the graph using a colored pencil so it is easy to see.

   a. Construct the graph of g(x) =-3f(x-2).



   b. Construct the graph of $h(x) = f^{-1}(x)$



4. State whether the function is even, odd, or neither. Show the use of the even/odd function tests to support your answer.

   a. $f(x) = \cos(x) - x^4$

   b. $g(x) = \frac{|x|}{4-x^2}$

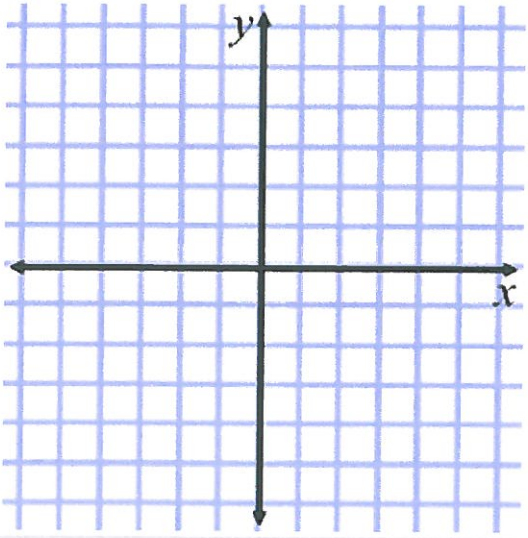
   c. $h(x) = 3x^3 - \frac{4}{x}$

5. Consider the function $f(x) = .5\sqrt{x+3}$ on the domain $x \geq -3$.

   a. Find the inverse of (x) .

   b. Graph both $f(x)$and $f^{-1}(x)$.

   c. Graph and write the equation of the line of symmetry between $f(x)$and $f^{-1}(x)$.



6. Find the inverse of $f(x) = \frac{x+1}{x-4}$.

7. Write two different sets of functions for $f(x)$ and $g(x)$ such that $(f \circ g)(x) = \frac{4}{\sqrt[3]{x+2}}$.

Set 1:	Set 2:
$f(x) =$	$f(x) =$
$g(x) =$	$g(x) =$

8. Use the composite function test to determine whether or not $f(x) = x^3 - 8$ and $g(x) = \sqrt[3]{x+2}$ are inverses of each other. Show both tests.

9. Graph the function on your calculator and state the intervals on which the function is increasing.

$$y = 2 + |x - 1|$$

10. Use your graphing calculator to find all relative maximum and relative minimum values of each function. Also state value of x at which each relative extremum occurs.

$$y = x^3 - 2x^2 + 3x - 1$$

11. Solve the equation algebraically. $\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2-4x+3}$

12. The Paniati Paper Company has been contracted to manufacture a box with no top that is to be made by removing squares of width x from the corners of a 10 inch by 15 inch piece of cardboard.

    a. Draw and label a diagram for the problem.

    b. Write an equation that models the volume of the box. Specify the domain for your equation.

    c. Determine x so that the box has a volume of 120 in^3.

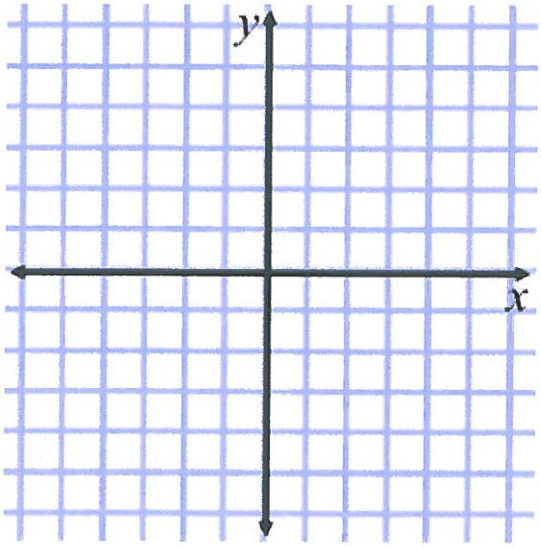
    d. What are the dimensions of the box that yield the maximum volume?

13. Consider the function $f(x) = \frac{x+3}{x^2+x-6}$.

    a. Graph the function.

    b. Identify the values of x.for.which.f(x) discontinuous.

    c. Write the equation of the horizontal asymptote. Use limits to represent the corresponding behavior.



    d. Write the equation of the vertical asymptote. Use limits to represent the corresponding behavior.

14. Write an exponential function of the form $y = Ae^{kt}$ that passes through the points (0,5) and (2,12). Show your work.

15. The population of Mathland is 314,000 and increases by 2.7% each year.
    a. Write an equation to model the population.

    b. When will the population reach 500,000? Answer algebraically.

16. You invest $1,000 at 3.25% APR, compounded monthly. How many years will it take for the investment to grow to $2,000? Answer algebraically.

17. You invest $800 and interest is compounded continuously. If your money doubled in 12 years, what interest rate did you receive? Answer algebraically.

18. Solve each equation algebraically.
    a. $\log x^2 = 2$

    b. $\log(x) - \frac{1}{2}\log(x+4) = 1$

    c. $\ln(3x-2) + \ln(x-1) = 2\ln(x)$

19. **Find the (x,y) pair for the value of the parameter:**
$x = t^2 + 3t$ **and** $y = \sqrt{t+3}$ **for t = 1.**

20. Make a table of values and sketch the curve, indicating the direction of your graph. Then eliminate the parameter. Do not use your calculator.

a. $x = 2t + 1$ and $y = t - 1$

b. $x = 2t$ and $y = t^2, -1 \le t \le 2$

c. $x = \sqrt{t+2}$ and $y = 3 - t$

d. $x = 2t$ and $y = |t - 1|$

e. $x = 2\sin t - 1$ and $y = \cos t + 2$

21. **Brian hits a baseball straight toward a 20-ft high fence that is 400 ft from home plate. The ball is hit when it is 3 ft above the ground and leaves the bat at an angle of 30°with the horizontal. Find the initial velocity needed to clear the fence. Show your work. Hint: Projectile motion.**

22. Convert each equation to polar form.

a. $5x^2+5y^2=7$ b. $y^2=3x$ c. $y=6$

23. Convert each equation to rectangular form

a. $r=\sin\theta$ b. $r^2=\sin\theta$ c. $r=5$

d. $r\cos\theta=8$ e. $\theta=\frac{\pi}{6}$

24. Graph each polar equation. Make angle boundaries of petals/loops clear.

a. $r = 2\cos\theta$

b. $r = 5 + 5\sin\theta$

c. $r = 5\sec\theta$

d. $r = 3$

e. $r = 4\sin 3\theta$

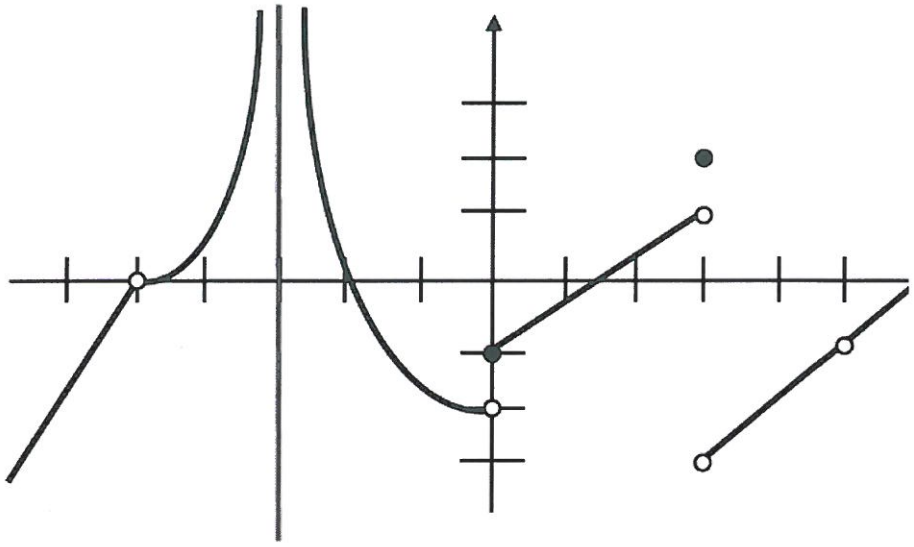
25. Find the intersection of the two polar curves. Express the solution in the form (r, θ).

a. $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$

b. $r = 2 - 3\cos\theta$ and $r = \cos\theta$

c. $r = 4\sin\theta$ and $r = 2$

26. **Answer the following questions using the graph of $f(x)$ given below.**



(a) Find $f(0)$

(b) Find $f(3)$

(c) Find $\lim_{x\to -5} f(x)$

(d) Find $\lim_{x\to 0^+} f(x)$

(e) Find $\lim_{x\to 3^-} f(x)$

(f) Find $\lim_{x\to -3^+} f(x)$

(g) List all x-values for which $f(x)$ has a removable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

(h) List all x-values for which $f(x)$ has a nonremovable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

In problems 27-32, find the limit (if it exists) using analytic methods (i.e. without using a calculator).

27. $\lim_{x\to -2}\dfrac{3x^2+21x+30}{x^3+8}$

28. $\lim_{x\to \pi/6}\dfrac{1-\cos^2 x}{4x}$

29. $\lim_{x\to 4}\dfrac{\sqrt{x-3}-1}{x-4}$

30. $\lim_{x\to 0}\dfrac{[1/(x+1)]-1}{x}$

31. $\lim_{x\to 0}\dfrac{\left[1/\sqrt{1+x}\right]-1}{x}$

32. $\lim_{x\to 6^-}\dfrac{|6x-36|}{6-x}$

33. $\lim_{\Delta x\to 0}\dfrac{\sin((\pi/6)+\Delta x)-(1/2)}{\Delta x}$

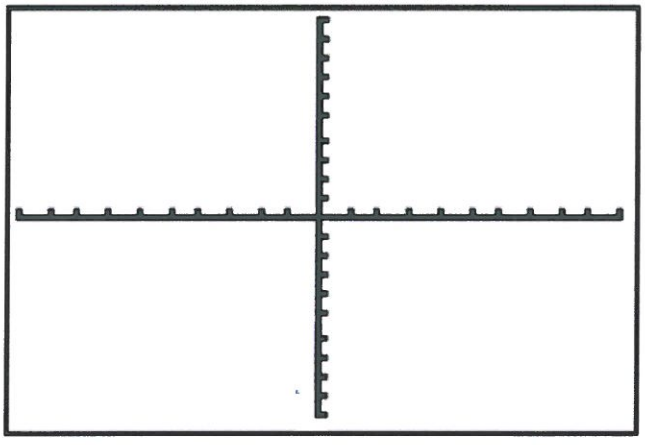
Hint: $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

34. **Suppose** $f(x)=\begin{cases}\dfrac{\sqrt{2x+1}-\sqrt{3}}{x-1}, x\geq 0\\ 4x^2+k, x<0\end{cases}$.

(a) **For what value of *k* will *f* be piecewise continuous at *x* = 0? Explain why this is true using one-sided limits. (Hint: A function is continuous at *x* = *c* if (1) *f*(*c*) exists, (2) $\lim_{x\to c} f(x)$ exists, and (3) $\lim_{x\to c} f(x)=f(c)$.)**

(b) **Using the value of *k* that you found in part (a), accurately graph *f* below. Approximate the value of $\lim_{x\to 1} f(x)$**

$\lim_{x\to 1} f(x)=$ _____

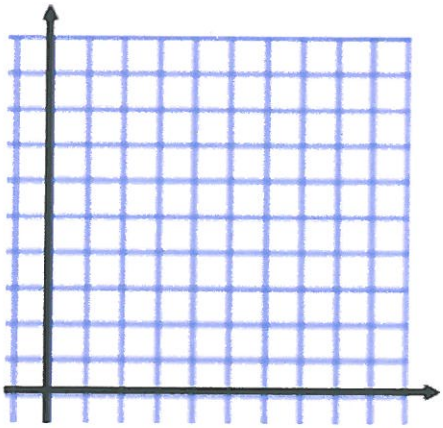


(c) **Rationalize the numerator to find $\lim_{x\to 1} f(x)$ analytically.**

35. Use the limit definition of the derivative to find the derivative of $f(x)=\frac{x+1}{x-1}$

36. Calculator active

Suppose that the position of an object can be modeled by the function $x(t) = -0.4t^2 + 3t$, where t is measured in hours and x is measured in miles.



a. **Graph the function on the interval [0,7.5].**

b. **Calculate the average rate of change of the object from t = 0 to t = 6. Show your work. Include units on your answer.**

c. **Draw the secant line on the graph that corresponds to the average rate of change of the object from t = 0 to t = 6. Show/explain how this is connected.**

d. **Calculate the instantaneous rate of change at t = 2.Include units on your answer.**

e. **Draw the tangent line on the graph that corresponds to the instantaneous rate of change at t = 2. Show/explain how this is connected.**

Part 4: You must know the following properties/identities. They are assumed knowledge in an AP Calculus class. Study them!

Properties of Logarithms

Let a be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, the following properties are true.

	Logarithm with Base a	*Natural Logarithm*
1. Product Property:	$\log_a(uv) = \log_a u + \log_a v$	$\ln(uv) = \ln u + \ln v$
2. Quotient Property:	$\log_a \frac{u}{v} = \log_a u - \log_a v$	$\ln \frac{u}{v} = \ln u - \ln v$
3. Power Property:	$\log_a u^n = n \log_a u$	$\ln u^n = n \ln u$

Also:

$\log_b 1 = 0.$

$\log_b b = 1.$

$\log_b b^2 = 2.$

$\log_b b^x = x.$

$b^{\log_b x} = x.$

Trigonometry: List of most commonly used identities in AP Calc

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

It is expected that you know the domain/range of the following inverse functions:

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

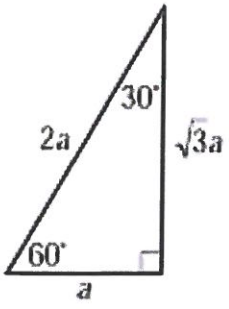
Alternate Notation

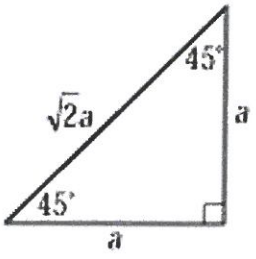
$$\sin^{-1} x = \arcsin x$$

$$\cos^{-1} x = \arccos x$$

$$\tan^{-1} x = \arctan x$$

Special Triangles





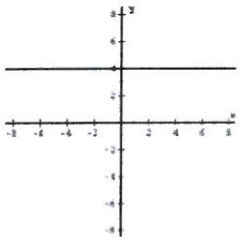
Of course, don't forget about these special triangles. You must know your trig ratios of these angles in every quadrant!

Also know your trig ratios of quadrant boundary angles such as

$$0, \quad \frac{\pi}{2}, \quad \pi, \quad \frac{3\pi}{2}, \quad 2\pi$$

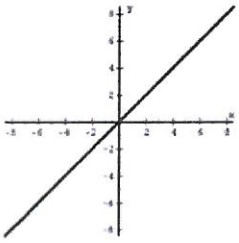
Families of functions: Please know all the key info of the following parent functions:

PARENT FUNCTIONS



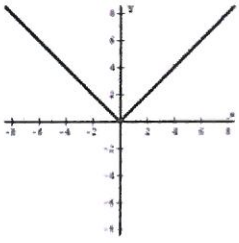
$f(x) = a$

Constant



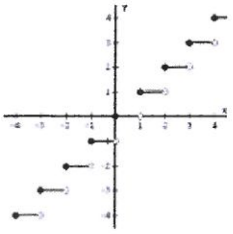
$f(x) = x$

Linear



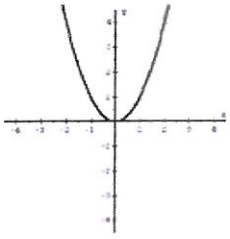
$f(x) = |x|$

Absolute Value



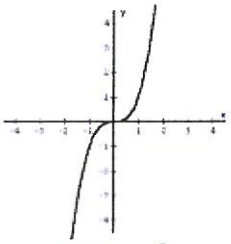
$f(x) = \text{int}(x) = [x]$

Greatest Integer



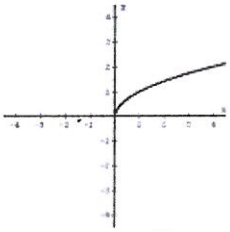
$f(x) = x^2$

Quadratic



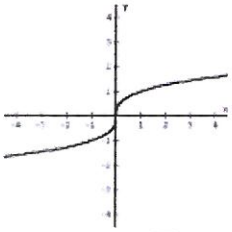
$f(x) = x^3$

Cubic



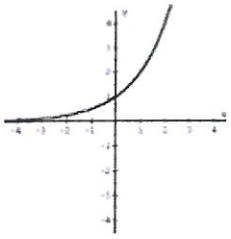
$f(x) = \sqrt{x}$

Square Root



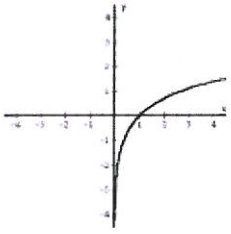
$f(x) = \sqrt[3]{x}$

Cube Root



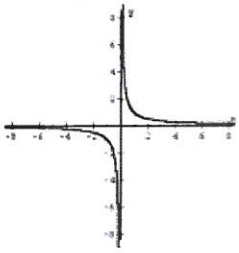
$f(x) = a^x$

Exponential



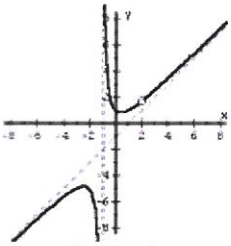
$f(x) = \log_a x$

Logarithmic



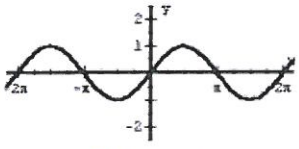
$f(x) = \dfrac{1}{x}$

Reciprocal

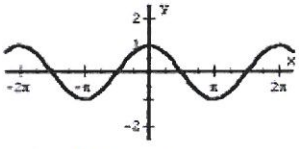


$f(x) = \dfrac{(x^2+1)(x-2)}{(x+1)(x-2)}$

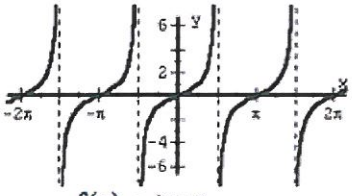
Rational



$f(x) = \sin x$



$f(x) = \cos x$



$f(x) = \tan x$

Trigonometric Functions