AP® Physics C
St. Xavier High School
Mr. D'Alessandro

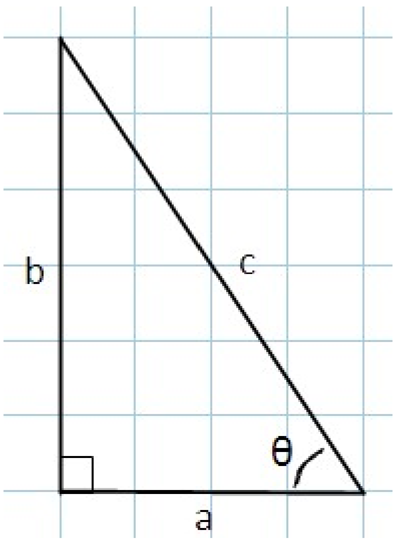
Name________________________________

Date______________________

Summer Worksheet

Trigonometry and Calculus are integral to the work of physicists. There are some basic concepts that it would be handy for you to understand, as well as some formulae that are good to have memorized. Memorize them.

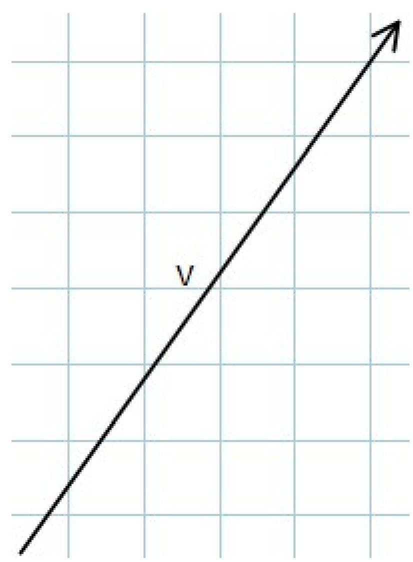
Trigonometry



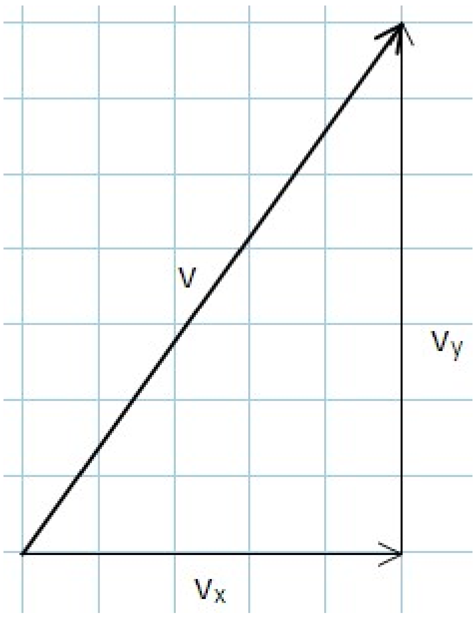
$$\sin\theta = \frac{b}{c} \qquad \cos\theta = \frac{a}{c} \qquad \tan\theta = \frac{b}{a}$$

$$a^2 + b^2 = c^2$$

These are regularly used for, among other things, vectors. Vectors are measurements with both length and direction, and can be represented by displacement arrows, like this diagram of $\vec{v}$. (Note the arrow over the "v?" That implies it is a vector.)



We may want to "resolve" the vector $\vec{v}$ into its components, or x- and y- parts...



Since the x-axis is always perpendicular to the y-axis, we can use the trigonometry we already know, if we know the angle, or can measure it.

What is the length of $\vec{v}$?________________________

What is the length of v_x? ________________________

What is the length of v_y? ________________________

What is the angle θ from $\vec{v}$ to the x-axis? ____________

How could you calculate the length of v_x if you were given the length of $\vec{v}$ and the angle θ?

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How could you calculate the length of $\vec{v}$ if you were given the lengths of v_x and v_y?

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These are used in a short-hand notation called unit-vector notation, as well as in ordered-pair notation. The unit-vector that points along the x-axis has a length of one unit and has either the symbol $\hat{\imath}$ or $\hat{x}$. The unit-vector that points along the y-axis has a length of one unit and has either the symbol $\hat{\jmath}$ or $\hat{y}$. So, a vector $\vec{v}$ with magnitude (another word for length) of $5.0\,\frac{m}{s}$ and direction 53.3° above the x-axis could be represented by...

$$\vec{v} = \left(5.0\,\frac{m}{s}, 53.3°\right) = \left(3.0\,\frac{m}{s}, 4.0\,\frac{m}{s}\right) = 3.0\,\frac{m}{s}\hat{\imath} + 4.0\,\frac{m}{s}\hat{\jmath} = 3.0\,\frac{m}{s}\hat{x} + 4.0\,\frac{m}{s}\hat{y}$$

(Calculate the x- and y-components and check!)

Calculus

Things in the universe change. So, physicists use the time-rate of change of a lot of formulas in their calculations. This is sometimes called the time-derivative. It is the slope of the graph of the function. Here are some to memorize. (Note the strange "operator" symbol of a "d" over a "dt"; it is like a fraction)

Derivatives

Constants do not change, so the derivative of a constant is zero.

Line 1 $\frac{d}{dt}(t) = 1$ $\quad$ $\frac{d}{dt}(t^2) = 2t$ $\quad$ $\frac{d}{dt}(3t^3) = 9t^2$ $\quad$ $\frac{d}{dt}(At^n) = nAt^{n-1}$

Line 2 $\frac{d}{dt}(t + 1) = 1$ $\quad$ $\frac{d}{dt}(t^2 + t + 5) = 2t + 1$ $\quad$ $\frac{d}{dt}(\sum_{i=1}^{m} A_i t^{n_i}) = \sum_{i=1}^{m} n_i A_i t^{n_i - 1}$

If y is a function of time, such that...

	$y = 2t$	$y = \sin \omega t$	$y = \cos \omega t$
So,	$\frac{dy}{dt}$ =2	$\frac{dy}{dt} = \omega \cos \omega t$	$\frac{dy}{dt} = -\omega \sin \omega t$
And,	$\frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2}{dt^2} y = 0$	$\frac{d^2}{dt^2} y = -\omega^2 \sin \omega t$	$\frac{d^2 y}{dt^2} = -\omega^2 \cos \omega t$

Take a good look at the line 1 row of equations above. Do you see the pattern? Can you get away with memorizing only one, or two of them? Explain.

Also, in words, explain the last formula in line 2. Note the similarities to line 1.

Anti-derivatives (aka "Integrals")

Physicists also have to "undo" the time-rate of change of functions. This gives the "area under the curve" of the function. It has a strange symbol, which requires a note at the end to tell you the variable with respect to which you are taking the anti-derivative.

Zero means the original function did not change, so the integral of a zero is a constant (which we represent with a "C" unless we know what it is, like an initial speed). This also means that EVERY indefinite integral ends with a constant of integration, since we can add a constant that doesn't change to any function.

$\int 1dt = \int dt = t + C$

$\int (t+2)dt = \frac{1}{2}t^2 + 2t + C$

$\int (At^n)dt = \frac{1}{n+1}At^{n+1} + C$

$\int (\sum_{i=1}^{m} A_i t^{n_i})\, dt = \sum_{i=1}^{m} \frac{1}{n_i+1} A_i t^{n_i+1} + C$

If y is a function of time, such that...

	$y = 2t$	$y = \sin \omega t$	$y = \cos \omega t$
So,	$\int ydt = t^2 + C$	$\int ydt = -\frac{1}{\omega}\cos \omega t + C$	$\int ydt = \frac{1}{\omega}\sin \omega t + C$

How could you check to make certain you performed the anti-derivative correctly?

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