AP Calculus AB and BC

Overview Information

Title of Course: AP Calculus AB and BC	
Course Author(s): Math Department	Schools where the course will be taught: Archie Williams, Redwood and Tamalpais High Schools
Length of Course: AB-1 year BC-1 year	Subject Area and Discipline: Math
Grade Levels: 11th/12th	Is this course an integrated course? No
Is this course being submitted for possible UC honors designation? No	Are you seeking UC approval? If so, in what area (A-G)? Yes - area C
Prerequisites (required): • For Calculus AB: C or better in Precalculus or Honors Precalculus or AP Precalculus • For Calculus BC: B or better in Precalculus or B- or better in Honors Precalculus or AP Precalculus	Co-requisites (required or recommended): none
Course prerequisites for **AP Calculus** (either AB or BC) exist to ensure that students are adequately prepared for the advanced mathematical concepts in the course. Students will be expected to build on prior knowledge from the content areas of Algebra, Geometry, Trigonometry, and Precalculus. Students all need to be prepared for the rigor and pace of the course since this is an Advanced Placement course, it is taught at a college level. We want to ensure students are not overwhelmed in the course and do not need to spend time relearning foundational concepts. Finally, prerequisites ensure students have conceptual readiness for the course. Students will need to be able to think abstractly. Calculus AB is a year long course and includes Units 1-6. Calculus. Calculus BC is a year long course and covers Units 1-10 and moves at a faster pace in order to cover more outcomes.	
Check all that apply: ✅ UC A-G course ❏ Graduation Requirement (specify the requirement this course meets) ✅ Elective ✅ Honors/AP ❏ CTE	

Introduction to the Course

Course Overview: AP Calculus AB and AP Calculus BC focus on students' understanding of calculus concepts and provide experience with methods and applications. Through the use of big ideas of calculus (e.g., modeling change, approximation and limits, and analysis of functions), each course becomes a cohesive whole, rather than a collection of unrelated topics. Both courses require students to use definitions and theorems to build arguments and justify conclusions. The courses feature a multi-representational approach to calculus, with concepts, results, and problems expressed graphically, numerically, analytically, and verbally. Exploring connections among these representations builds understanding of how calculus applies limits to develop important ideas, definitions, formulas, and theorems. A sustained emphasis on clear communication of methods, reasoning, justifications, and conclusions is essential. Teachers and students should regularly use technology to reinforce relationships among functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

AP Calculus AB and BC engage students in relevant, real-world learning by applying calculus concepts to diverse, authentic contexts such as population modeling, environmental change, and economic forecasting—promoting critical thinking through multiple perspectives. Inclusive teaching strategies affirm and reflect the identities and experiences of marginalized groups by incorporating culturally responsive examples and encouraging diverse problem-solving approaches. The courses support the TUHSD graduate profile by fostering analytical thinking, collaboration, and ethical decision-making, while integrating media literacy through the critical evaluation of digital tools, data sources, and simulations. Students also learn responsible online behavior by using technology purposefully, citing sources appropriately, and engaging respectfully in collaborative digital spaces.

The following units will be guided by the 4 Mathematical Practices as outlined in the Framework for College Board AP Calculus AB and BC Course and Exam Description (page 21). Alternatively go to the end of this course of study document.

All of the information in this Course of Study: (College Board. (2020). AP Calculus AB and BC Course and Exam Description
https://apcentral.collegeboard.org/pdf/ap-calculus-ab-and-bc-course-and-exam-description.pdf

Unit 1 Title: ***Unit 1: Limits and Continuity***

Unit 1 Summary:
Students should be prepared to evaluate or estimate limits presented graphically, numerically, analytically, or verbally. To avoid missed opportunities to earn points on the AP Exam, students should consistently practice using correct mathematical notation and presenting setups and appropriately rounded answers when using a calculator. From the first unit onward, emphasize the importance of hypotheses for theorems. Explore why each hypothesis is needed in order to ensure that the conclusion follows. Students should establish the practice of explicitly verifying that a theorem's hypotheses are satisfied before applying the theorem.

Mathematical information may be organized or presented graphically, numerically, analytically, or verbally. Mathematicians must be able to communicate effectively in all of these contexts and transition seamlessly from one representation to another. Limits lay the groundwork for students' ongoing development of skills associated with taking what is presented in a table, an equation, or a sentence and translating that information into a graph (or vice versa). Help students explicitly practice matching

different representations that show the same information, focusing on building their comfort level with translating analytical and verbal representations. This will be instrumental to their development of proficiency in this practice. The use of graphing calculators to help students explore these connections is strongly encouraged. Mathematicians also explain reasoning and justify conclusions using definitions, theorems, and tests. A common student misunderstanding is that they don't need to write relevant given information before drawing the conclusion of a theorem. In this unit, students should be given explicit instruction and time to practice "connecting the dots" by first demonstrating that all conditions or hypotheses have been met and then drawing the conclusion. **Essential Questions:** ● Can change occur at an instant? ● How does knowing the value of a limit, or that a limit does not exist, help you to make sense of interesting features of functions and their graphs? ● How do we close loopholes so that a conclusion about a function is always true? **Building Mathematical Practices:** Mathematical information may be organized or presented graphically, numerically, analytically, or verbally. Mathematicians must be able to communicate effectively in all of these contexts and transition seamlessly from one representation to another. Limits lay the groundwork for students' ongoing development of skills associated with taking what is presented in a table, an equation, or a sentence and translating that information into a graph (or vice versa). Help students explicitly practice matching different representations that show the same information, focusing on building their comfort level with translating analytical and verbal representations. This will be instrumental to their development of proficiency in this practice. The use of graphing calculators to help students explore these connections is strongly encouraged. Mathematicians also explain reasoning and justify conclusions using definitions, theorems, and tests. A common student misunderstanding is that they don't need to write relevant given information before drawing the conclusion of a theorem. In Unit 1, students should be given explicit instruction and time to practice "connecting the dots" by first demonstrating that all conditions or hypotheses have been met and then drawing the conclusion.
Unit 1 Outcomes:

Enduring Understanding	Topic	Suggested Skills
CHA-1	**1.1 Introducing Calculus: Can Change Occur at an Instant?**	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
LIM-1	**1.2 Defining Limits and Using Limit Notation**	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
	1.3 Estimating Limit Values from Graphs	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
	1.4 Estimating Limit Values from Tables	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
	1.5 Determining Limits Using Algebraic Properties of Limits	1.E Apply appropriate mathematical rules or procedures, with and without technology.
	1.6 Determining Limits Using Algebraic Manipulation	1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
	1.7 Selecting Procedures for Determining Limits	1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
	1.8 Determining Limits Using the Squeeze Theorem	3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
	1.9 Connecting Multiple Representations of Limits	Identify a re-expression of mathematical information presented in a given representation.

LIM-2	**1.10 Exploring Types of Discontinuities**	3.B Identify an appropriate mathematical definition, theorem, or test to apply.
	1.11 Defining Continuity at a Point	3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
	1.12 Confirming Continuity over an Interval	1.E Apply appropriate mathematical rules or procedures, with and without technology.
	1.13 Removing Discontinuities	1.E Apply appropriate mathematical rules or procedures, with and without technology.
	1.14 Connecting Infinite Limits and Vertical Asymptotes	3.D Apply an appropriate mathematical definition, theorem, or test.
	1.15 Connecting Limits at Infinity and Horizontal Asymptotes	2.D Identify how mathematical characteristics or properties of functions are related in different representations.
FUN-1	**1.16 Working with the Intermediate Value Theorem (IVT)**	3.E Provide reasons or rationales for solutions or conclusions.

Sample Unit 1 Assignments:

Students will evaluate limits as x-approaches a number using a graph, a calculator, or limit properties. Students will practice evaluating limits using analytical methods, for both limits to infinity and limits to a number. Students will evaluate limits analytically without a calculator; use the definition of continuity, and will use limits to identify asymptotes.

Sample Assignment 1
Sample Assignment 2
Sample Assignment 3
AP Classroom: Personal Progress Check 1

- Multiple-choice: 45 questions
- Free-response: 3 questions (partial)

Sample Unit 1 Assessment:

Students will be given the opportunity to show their knowledge by evaluating limits graphically, analytically, and numerically both with and without the use of a graphing technology. Students will demonstrate their understanding of continuity by applying the definition of continuity in proofs and justification of discontinuity. Students will use limits to describe and identify asymptotes. Students will apply the Intermediate Value Theorem in application. Throughout the assessment, students will be expected to show clear mathematical reasoning and proper notation, as required on the AP Exam. The assessments will include multiple-choice and free-response questions designed to reflect the structure and rigor of AP-style problems.

Sample Assessment Unit 1

Unit Title: ***Unit 2: Differentiation: Definition and Fundamental Properties***

Unit 2 Summary:
Mathematicians know that a solution will only be as good as the procedure used to find it and that the difference between being correct and incorrect can often be traced to an arithmetic or procedural error. In other words, mathematicians know that the details matter. Students often find it difficult to apply mathematical procedures—including the rules of differentiation—with precision and accuracy. For example, students may drop important notation, such as a parenthesis, or misapply the product rule by taking the derivative of each factor separately and then multiplying those together. The content of Unit 2 is a foundational entry point for practicing the skill of applying mathematical procedures and learning to self-correct before common mistakes occur.

This is also an opportunity to revisit and reinforce the practice of connecting representations, as students will be seeing derivatives presented in analytical, numerical, graphical, and verbal representations. Students can practice by extracting information about the original function, f, from a graphical representation of f'. This can help prevent misunderstandings when examining the graph of a derivative (such as misinterpreting it as the graph of the original function instead).

Essential Questions:

- How can a state determine the rate of change in high school graduates at a particular level of public investment in education (in graduates per dollar) based on a model for the number of graduates as a function of the state's education budget?
- Why do mathematical properties and rules for simplifying and evaluating limits apply to differentiation?
- If you knew that the rate of change in high school graduates at a particular level of public investment in education (in graduates per dollar) was a positive number, what might that tell you about the number of graduates at that level of investment?

Building Mathematical Practices: Mathematicians know that a solution will only be as good as the procedure used to find it and that the difference between being correct and incorrect can often be traced to an arithmetic or procedural error. In other words, mathematicians know that the details matter. Students often find it difficult to apply mathematical procedures—including the rules of differentiation—with precision and accuracy. For example, students may drop important notation, such as a parenthesis, or misapply the product rule by taking the derivative of each factor separately and then multiplying those together. The content of Unit 2 is a foundational entry point for practicing the skill of applying mathematical procedures and learning to self-correct before common mistakes occur. This is also an opportunity to revisit and reinforce the practice of connecting representations, as students will be seeing derivatives presented in analytical, numerical, graphical, and verbal representations. Students can practice by extracting information about the original function, f, from a graphical representation of f'. This can help prevent misunderstandings when examining the graph of a derivative (such as misinterpreting it as the graph of the original function instead).

Unit Outcomes:

Topic	Suggested Skills
2.1 Defining Average and Instantaneous Rates of Change at a Point	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
2.2 Defining the Derivative of a Function and Using Derivative Notation	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems. 4.C Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).
2.3 Estimating Derivatives of a Function at a Point	1.E Apply appropriate mathematical rules or procedures, with and without technology.
2.4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist	3.E Provide reasons or rationales for solutions and conclusions.
2.5 Applying the Power Rule	1.E Apply appropriate mathematical rules or procedures, with and without technology.
2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple	1.E Apply appropriate mathematical rules or procedures, with and without technology.
2.7 Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$	1.E Apply appropriate mathematical rules or procedures, with and without technology.
2.8 The Product Rule	1.E Apply appropriate mathematical rules or procedures, with and without technology.
2.9 The Quotient Rule	1.E Apply appropriate mathematical rules or procedures, with and without technology.
2.10 Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.

Sample Unit 2 Assignments:

Students will apply key concepts such as average rate of change, instantaneous rate of change, and tangent lines using both algebraic and limit-based approaches. The first assignment involves partner work to encourage collaboration and deepen conceptual understanding through discussion and shared problem-solving. In the second assignment, students will independently practice all of the derivative rules, reinforcing their procedural fluency and preparing them for more complex applications of differentiation.

Sample Assignment 1 Unit 2
Sample Assignment 2 Unit 2

AP Classroom: Personal Progress Check 2
- Multiple-choice 30 questions
- Free-response: 3 questions (partial)

Sample Unit 2 Assessments:

Students will complete a combination of multiple-choice and "free-response" questions that require them to calculate average and instantaneous rates of change, interpret the meaning of derivatives in context, and use limit definitions to find the derivative. They will also be assessed on their ability to apply all major derivative rules—including the power, product, quotient, and chain rules—in both pure and applied problems. Emphasis is placed on clear mathematical communication, correct use of notation, and the ability to justify reasoning, ensuring students are developing both procedural fluency and conceptual insight.

Sample Assessment 1 Unit 2
Sample Assessment 2 Unit 2

Unit Title: ***Unit 3: Differentiation: Composite, Implicit, and Inverse Functions***

Unit 3 Summary:
Identifying composite and implicit functions is a key differentiation skill. Students must recognize functions embedded in functions and be able to decompose composite functions into their "outer" and "inner" component functions. Misapplying the chain rule by forgetting to also differentiate the "inner" function or misidentifying the "inner" function are common errors. Provide sample responses that demonstrate these errors to help students be mindful of them in their own work. Reinforcing the chain rule structure sets the stage for Unit 6, when students learn the inverse of this process.

Students should continue to practice using correct notation and applying procedures accurately. Checking one another's work, reviewing sample responses (with and without errors), and using technology to check calculations develop these skills. Emphasize that taking higher-order derivatives mirrors familiar differentiation processes (i.e., "function is to first derivative as first derivative is to second derivative"). Use questioning techniques such as, "What does this mean?" to help students develop a more solid conceptual understanding of higher-order differentiation.

Essential Question:
- If pressure experienced by a diver is a function of depth and depth is a function of time, how might we find the rate of change in pressure with respect to time?

Building Mathematical Practices: Identifying composite and implicit functions is a key differentiation skill. Students must recognize functions embedded in functions and be able to decompose composite functions into their "outer" and "inner" component functions. Misapplying the chain rule by forgetting to also differentiate the "inner" function or misidentifying the "inner" function are common errors. Provide sample responses that demonstrate these errors to help students be mindful of them in their own work. Reinforcing the chain rule structure sets the stage for Unit 6, when students learn the inverse of this process. Students should continue to practice using correct notation and applying procedures accurately.

Checking one another's work, reviewing sample responses (with and without errors), and using technology to check calculations develop these skills. Emphasize that taking higher-order derivatives mirrors familiar differentiation processes (i.e., "function is to first derivative as first derivative is to second derivative"). Use questioning techniques such as, "What does this mean?" to help students develop a more solid conceptual understanding of higher-order differentiation

Unit 3 Outcomes:

Topic	Suggested Skills
3.1 The Chain Rule	1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
3.2 Implicit Differentiation	1.E Apply appropriate mathematical rules or procedures, with and without technology.
3.3 Differentiating Inverse Functions	3.G Confirm that solutions are accurate and appropriate.
3.4 Differentiating Inverse Trigonometric Functions	1.E Apply appropriate mathematical rules or procedures, with and without technology.
3.5 Selecting Procedures for Calculating Derivatives	1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
3.6 Calculating Higher-Order Derivatives	1.E Apply appropriate mathematical rules or procedures, with and without technology.

Sample Unit 3 Assignments:

Students complete a variety of assignments designed to build fluency with differentiation techniques and deepen their understanding of how derivatives are used in mathematical and real-world contexts. Students will learn to apply the chain rule in combination with the power, product, and quotient rules to differentiate a wide range of functions, including polynomial, rational, exponential, logarithmic, and trigonometric expressions. Students will also work on problems involving implicit differentiation, giving them the tools to find derivatives of equations where variables are not explicitly solved for. Students will practice the derivatives of inverse functions and higher order derivatives. Students will regularly justify their reasoning using correct mathematical notation, a key skill for success on the AP Exam. These assignments prepare students not only to perform derivative calculations but also to explain and apply their understanding in written form.

Sample Assignment 1 Unit 3
Sample Assignment 2 Unit 3
Sample Assignment 3 Unit 3

AP Classroom: Personal Progress Check 3

- Multiple-choice: 15 questions
- Free-response: 3 questions

Sample Unit 3 Assessments:

Students will demonstrate their mastery of the rules and techniques of differentiation. Students will be able to differentiate functions using the power rule, product rule, quotient rule, and chain rule. They will apply these techniques to a variety of functions, including polynomial, exponential, logarithmic, and trigonometric functions. Students will also use implicit differentiation to find derivatives when variables are not explicitly solved for and apply the derivative of inverse functions when appropriate. Students will interpret the meaning of a derivative in context, such as slope or rate of change, and use derivatives to analyze motion, including problems involving velocity and acceleration. They will also compute and interpret higher-order derivatives. Throughout the assessment, students will be expected to show clear mathematical reasoning and proper notation, as required on the AP Exam. The assessments will include multiple-choice and free-response questions designed to reflect the structure and rigor of AP-style problems.

Sample Assessment 1 Unit 3
Sample Assessment 2 Unit 3

Unit Title: ***Unit 4: Contextual Applications of Differentiation***

Unit 4 Summary:
Students will begin applying concepts from Units 2 and 3 to scenarios encountered in the world. Students often struggle to translate these verbal scenarios into the mathematical procedures necessary to answer the question. To solve these problems, students will need explicit instruction and intentional practice identifying key information, determining which procedure applies to the scenario presented (i.e., that "rates of change" indicate differentiation), stating what is changing and how, using correct units, and explaining what their answer means in the context of the scenario. Provide scenarios with different contexts but similar procedures so students begin to recognize and apply the reasoning behind those problem-solving decisions, rather than grasping at rules haphazardly.

Students must also be able to explain how an approximated value relates to the value it's intended to approximate. Students may not understand why they would use a tangent line approximation (i.e., linearization) rather than simply evaluating a function. Expose them to scenarios where an exact function value can't be calculated, and then ask them to determine whether a particular approximation is an overestimate or an underestimate of the function

Essential Questions:

- How are problems about position, velocity, and acceleration of a particle in motion over time structurally similar to problems about the volume of a rising balloon over an interval of heights, the population of London over the 14th century, or the metabolism of a dose of medicine over time?
- Since certain indeterminate forms seem to actually approach a limit, how can we determine that limit, provided it exists?

Building Mathematical Practices: Students will begin applying concepts from Units 2 and 3 to scenarios encountered in the world. Students often struggle to translate these verbal scenarios into the mathematical procedures necessary to answer the question. To solve these problems, students will need explicit instruction and intentional practice identifying key information, determining which procedure applies to the scenario presented (i.e., that "rates of change" indicate differentiation), stating what is changing and how, using correct units, and explaining what their answer means in the context of the scenario. Provide scenarios with different contexts but similar procedures so students begin to recognize and apply the reasoning behind those problem-solving decisions, rather than grasping at rules haphazardly. Students must also be able to explain how an approximated value relates to the value it's intended to approximate. Students may not understand why they would use a tangent line approximation (i.e., linearization) rather than simply evaluating a function. Expose them to scenarios where an exact function value can't be calculated, and then ask them to determine whether a particular approximation is an overestimate or an underestimate of the function

Unit 4 Outcomes:

Topic	Suggested Skills
4.1 Interpreting the Meaning of the Derivative in Context	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.
4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration	1.E Apply appropriate mathematical rules or procedures, with and without technology.
4.3 Rates of Change in Applied Contexts Other Than Motion	2.A Identify common underlying structures in problems involving different contextual situations.
4.4 Introduction to Related Rates	1.E Apply appropriate mathematical rules or procedures, with and without technology.
4.5 Solving Related Rates Problems	3.F Explain the meaning of mathematical solutions in context.
4.6 Approximating Values of a Function Using Local Linearity and Linearization	1.F Explain how an approximated value relates to the actual value.
4.7 Using L'Hospital's Rule for Determining Limits of Indeterminate Forms	3.D Apply an appropriate mathematical definition, theorem, or test.

Sample Unit 4 Assignments:

Students will begin applying concepts from Units 2 and 3 to scenarios encountered in the world. Students will practice identifying key information, determining which procedure applies to the scenario presented stating what is changing and how, using correct units, and explaining what their answer means in the context of the scenario. Students will analyze the motion of a particle or object moving along a straight path, using position functions to understand how its velocity and acceleration change over time.

Sample Assignment 1 Unit 4.
Sample Assignment 2 Unit 4
Sample Assignment 3 Unit 4

AP Classroom: Personal Progress Check 4
- Multiple-choice: 15 questions
- Free-response: 3 questions

Sample Unit 4 Assessments:

Students will analyze motion along a straight line, including determining position, velocity, and acceleration functions, use the sign of velocity and acceleration to determine when an object is speeding up, slowing down, or changing direction, calculate and interpret displacement and total distance traveled over an interval. Students will apply the Mean Value Theorem for derivatives in a real-world setting. Students will solve related rates problems by identifying changing quantities and using the chain rule appropriately, apply derivatives to solve optimization problems, including defining variables, writing equations, and justifying solutions.

Sample Assessment 1 Unit 4
Sample Assessment 2 Unit 4

Unit Title: ***Unit 5: Analytical Applications of Differentiation***

Unit 5 Summary:
The underlying processes of finding critical points and extrema are the foundation for the justifications students will write in this unit. Students should use calculators to graph a function and its derivatives to explore the related features of these graphs and confirm the results of their calculations.

Students often struggle with misinterpreting the characteristics of the graph of a derivative as though they are characteristics of the original function. Or, they use nonspecific language that conflates different functions (e.g., "it" rather than "f"). To prevent ongoing misconceptions, hold students accountable for extreme precision by having them practice matching graphs of functions to their derivatives and requiring them to explain their reasons to a peer.

Students also tend to rely on insufficient evidence or descriptions in their justifications, stating, for example, that "the graph of f is increasing because it's going up." This happens especially when examining derivative graphs on a calculator. Model calculus-based justifications (i.e., reasoning based on analysis of a derivative) both in discussion and in writing. Give students repeated opportunities to practice writing and revising their own justifications based on teacher feedback and feedback from their peers.

Essential Questions:
- How might the Mean Value Theorem be used to justify a conclusion that you were speeding at some point on a certain stretch of highway, even without knowing the exact time you were speeding?
- What additional information is included in a sound mathematical argument about optimization that a simple description of an equivalent answer lacks?

Building Mathematical Practices: The underlying processes of finding critical points and extrema are the foundation for the justifications students will write in this unit. Students should use calculators to graph a function and its derivatives to explore the related features of these graphs and confirm the results of their calculations. Students often struggle with misinterpreting the characteristics of the graph of a derivative as though they are characteristics of the original function. Or, they use nonspecific language that conflates different functions (e.g., "it" rather than "f"). To prevent ongoing misconceptions, hold students accountable for extreme precision by having them practice matching graphs of functions to their derivatives and requiring them to explain their reasons to a peer. Students also tend to rely on insufficient evidence or descriptions in their justifications, stating, for example, that "the graph of f is increasing because it's going up." This happens especially when examining derivative graphs on a calculator. Model calculus-based justifications (i.e., reasoning based on analysis of a derivative) both in discussion and in writing. Give students repeated opportunities to practice writing and revising their own justifications based on teacher feedback and feedback from their peers.

Unit 5 Outcomes:

Topic	Suggested Skills
5.1 Using the Mean Value Theorem	3.E Provide reasons or rationales for solutions and conclusions.
5.2 Extreme Value Theorem, Global Versus Local Extrema, and Critical Points	3.E Provide reasons or rationales for solutions and conclusions.
5.3 Determining Intervals on Which a Function is Increasing or Decreasing	2.E Describe the relationships among different representations of functions and their derivatives
5.4 Using the First Derivative Test to Determine Relative (Local) Extrema	3.D Apply an appropriate mathematical definition, theorem, or test.
5.5 Using the Candidates Test to Determine Absolute (Global) Extrema	1.E Apply appropriate mathematical rules or procedures, with and without technology.
5.6 Determining Concavity of Functions over Their Domains	2.E Describe the relationships among different representations of functions and their derivatives.
5.7 Using the Second Derivative Test to Determine Extrema	3.D Apply an appropriate mathematical definition, theorem, or test.
5.8 Sketching Graphs of Functions and Their Derivatives	2.D Identify how mathematical characteristics or properties of functions are related in different representations.
5.9 Connecting a Function, Its First Derivative, and Its Second Derivative	2.D Identify how mathematical characteristics or properties of functions are related in different representations.

5.10 Introduction to Optimization Problems	2.A Identify common underlying structures in problems involving different contextual situations.
5.11 Solving Optimization Problems	3.F Explain the meaning of mathematical solutions in context.
5.12 Exploring Behaviors of Implicit Relations	1.E Apply appropriate mathematical rules or procedures, with and without technology. 3.E Provide reasons or rationales for solutions and conclusions.

Sample Unit 5 Assignments:

Students will complete assignments that build their understanding of the relationship between a function and its derivatives. Students will analyze functions using the first and second derivatives to find critical points, identify intervals of increase and decrease, determine relative extrema, and examine concavity. Students will apply, justify, and make correct conclusions for the Mean Value Theorem, the First Derivative Test, and the Second Derivative Tests. Students will interpret graphical behavior from analytical information. Students will sketch the graphs of functions and their derivatives to reinforce conceptual connections. Additionally, students will work through real-world optimization problems requiring them to construct equations, find maxima or minima, and justify their solutions. The unit will also include tasks involving implicit differentiation, helping students handle more complex relationships between variables.

Sample Assignment 1 Unit 5
Sample Assignment 2 Unit 5
Sample Assignment 3 Unit 5

AP Classroom: Personal Progress Check 5

- Multiple-choice: 35 questions
- Free-response: 3 questions

Sample Unit 5 Assessment:

Students will analyze the behavior of functions using the first and second derivatives. Students will be expected to identify intervals where a function is increasing or decreasing, determine relative extrema using the First Derivative Test, and analyze concavity and inflection points using the Second Derivative Test. Students will interpret and justify extreme values and points of inflection based on graphical or analytical information. Students will apply the mean value theorem. Students will use given extrema, intervals of increasing, decreasing, and concavity to sketch the function. Students will use the graph of a function to sketch the graph of its derivative. Students will solve optimization problems involving area and perimeter, applying calculus concepts to find maximum or minimum values under given constraints and justifying their solutions using appropriate derivative tests.

Sample Assessment 1 Unit 5

Unit Title: ***Unit 6: Integration and Accumulation of Change***

Unit 6 Summary:
Students often struggle with the relationship between differentiation and integration. They think that integration is simply differentiation in reverse order. However, to apply the rules of integration correctly, students must think more strategically, taking into consideration how the “pieces” fit together. Students

will need explicit guidance for choosing an appropriate antidifferentiation strategy that's based on the underlying patterns in different categories of integrands (e.g., using u-substitution when they recognize that the integrand is a factor of the derivative of a composite function or using integration by parts for an integrand, udv, that is related to a term in the derivative of the product uv bc only).

Students also struggle with relating a symbolic limit of a Riemann sum to that limit expressed as a definite integral, because of the complexity of the expressions. To help students feel more comfortable working with these expressions, use explicit strategies, such as helping students to break complex expressions into familiar components, or matching expressions for a definite integral with the limit of a Riemann sum, and vice versa.

Essential Questions:

- Given information about a rate of population growth over time, how can we determine how much the population changed over a given interval of time?
- If compounding more often increases the amount in an account with a given rate of return and term, why doesn't compounding continuously result in an infinite account balance, all other things being equal?
- How is integrating to find areas related to differentiating to find slopes?

Building Mathematical Practices: Students often struggle with the relationship between differentiation and integration. They think that integration is simply differentiation in reverse order. However, to apply the rules of integration correctly, students must think more strategically, taking into consideration how the "pieces" fit together. Students will need explicit guidance for choosing an appropriate antidifferentiation strategy that's based on the underlying patterns in different categories of integrands (e.g., using u-substitution when they recognize that the integrand is a factor of the derivative of a composite function or using integration by parts for an integrand, udv, that is related to a term in the derivative of the product uv bc only). Students also struggle with relating a symbolic limit of a Riemann sum to that limit expressed as a definite integral, because of the complexity of the expressions. To help students feel more comfortable working with these expressions, use explicit strategies, such as helping students to break complex expressions into familiar components, or matching expressions for a definite integral with the limit of a Riemann sum, and vice versa.

Unit 6 Outcomes:

Topic	Suggested Skills
6.1 Exploring Accumulations of Change	4.B Use appropriate units of measure.
6.2 Approximating Areas with Riemann Sums	1.F Explain how an approximated value relates to the actual value.
6.3 Riemann Sums, Summation Notation, and Definite Integral Notation	2.C Identify a re-expression of mathematical information presented in a given representation.
6.4 The Fundamental Theorem of Calculus and Accumulation Functions	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.

6.5 Interpreting the Behavior of Accumulation Functions Involving Area	**2.D** Identify how mathematical characteristics or properties of functions are related in different representations.
6.6 Applying Properties of Definite Integrals	**3.D** Apply an appropriate mathematical definition, theorem, or test.
6.7 The Fundamental Theorem of Calculus and Definite Integrals	**3.D** Apply an appropriate mathematical definition, theorem, or test.
6.8 Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation	**4.C** Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).
6.9 Integrating Using Substitution	**1.E** Apply appropriate mathematical rules or procedures, with and without technology.
6.10 Integrating Functions Using Long Division and Completing the Square	**1.E** Apply appropriate mathematical rules or procedures, with and without technology.

Topic	Suggested Skills
6.11 Integrating Using Integration by Parts BC ONLY	**1.E** Apply appropriate mathematical rules or procedures, with and without technology.
6.12 Integrating Using Linear Partial Fractions BC ONLY	**1.E** Apply appropriate mathematical rules or procedures, with and without technology.
6.13 Evaluating Improper Integrals BC ONLY	**1.E** Apply appropriate mathematical rules or procedures, with and without technology.
6.14 Selecting Techniques for Antidifferentiation	**1.C** Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).

Sample Unit 6 Assignments:

Students will develop their understanding of definite integrals and the concept of accumulation. Through practice problems, graphical analysis, and real-world modeling tasks, students learn to interpret integrals as the accumulation of quantities over an interval. Assignments focus on estimating area under a curve using Riemann sums, evaluating definite integrals using geometry and algebraic rules, and understanding integrals as limits of Riemann sums. Students also work with functions represented graphically, numerically, and analytically to reinforce connections between different representations of integrals. Additionally, students complete tasks that help them understand and apply the Fundamental Theorem of Calculus, which links derivatives and integrals conceptually and computationally. They use this theorem to evaluate definite integrals and analyze accumulation functions. Students will work on choosing an appropriate antidifferentiation strategy that's based on

the underlying patterns in different categories of integrands or using integration by parts for an integrand, udv, that is related to a term in the derivative of the product uv BC only
 	
Sample Assignment 1 Unit 6
Sample Assignment 2 Unit 6
Sample Assignment 3 Unit 6
Sample Assignment 4 Unit 6 **BC Only**
 	
AP Classroom: Personal Progress Check 6
• Multiple-choice:
  ○ 25 questions (AB)
  ○ 35 questions (BC)
• Free-response: 3 questions

Sample Unit 6 Assessments: Students will demonstrate their ability to interpret definite integrals as the accumulation of a quantity over an interval and apply this understanding in a variety of contexts. Students will estimate areas under curves using Riemann sums (left, right, and midpoint) and evaluate definite integrals using both geometric reasoning and algebraic rules. Students will be assessed on their ability to interpret and compute integrals as limits of Riemann sums, reinforcing the foundational concept of accumulation. Students will also be expected to apply the Fundamental Theorem of Calculus to evaluate definite integrals and to analyze accumulation functions graphically and algebraically. Assessments may include interpreting function behavior based on graphs of derivatives or accumulation functions. In BC-level content, students will demonstrate the ability to identify appropriate antidifferentiation strategies based on the structure of an integrand and apply integration by parts for expressions of the form *u dv*, particularly when the integrand corresponds to the derivative of a product *uv*. **Sample Assessment 1 Unit 6** **Sample Assessment 2 Unit 6** **Sample Assessment BC Unit 6**

Unit Title: ***Unit 7: Differential Equations***
Unit 7 Summary: In this unit, students will translate mathematical information from one representation to another by matching equations and slope fields, rewriting verbal statements as differential equations, and sketching slope fields that match their symbolic representations. Provide students with explicit guidance on how to select an appropriate graphing technique. As students practice Euler's method, encourage them to transfer skills using tangent line approximations, rather than simply memorizing an algorithm **BC only.** Because the problems in this unit model real-world scenarios, help students to develop proficiency in transferring the mathematical procedures they've learned in "x's and y's" to equivalent environments with variable names other than x, y, and t. Using differentiation to confirm that solutions to differential equations are accurate and appropriate also helps students to develop an understanding of what it means to say that an equation is a solution to a differential equation. **Essential Question:** • How can we derive a model for the number of computers, C, infected by a virus, given a model for how fast the computers are being infected, dC/dt, at a particular time? **Building Mathematical Practices:** In this unit, students will translate mathematical information from

one representation to another by matching equations and slope fields, rewriting verbal statements as differential equations, and sketching slope fields that match their symbolic representations. Provide students with explicit guidance on how to select an appropriate graphing technique. As students practice Euler's method, encourage them to transfer skills using tangent line approximations, rather than simply memorizing an algorithm bc only. Because the problems in this unit model real-world scenarios, help students to develop proficiency in transferring the mathematical procedures they've learned in "x's and y's" to equivalent environments with variable names other than x, y, and t. Using differentiation to confirm that solutions to differential equations are accurate and appropriate also helps students to develop an understanding of what it means to say that an equation is a solution to a differential equation.

Unit 7 Outcomes:

Topic	Suggested Skills
7.1 Modeling Situations with Differential Equations	2.C Identify a re-expression of mathematical information presented in a given representation.
7.2 Verifying Solutions for Differential Equations	3.G Confirm that solutions are accurate and appropriate.
7.3 Sketching Slope Fields	2.C Identify a re-expression of mathematical information presented in a given representation.
7.4 Reasoning Using Slope Fields	4.D Use appropriate graphing techniques.
7.5 Approximating Solutions Using Euler's Method BC ONLY	1.E Apply appropriate mathematical rules or procedures, with and without technology.
7.6 Finding General Solutions Using Separation of Variables	1.E Apply appropriate mathematical rules or procedures, with and without technology.
7.7 Finding Particular Solutions Using Initial Conditions and Separation of Variables	1.E Apply appropriate mathematical rules or procedures, with and without technology.
7.8 Exponential Models with Differential Equations	3.G Confirm that solutions are accurate and appropriate.
7.9 Logistic Models with Differential Equations BC ONLY	3.F Explain the meaning of mathematical solutions in context.

Sample Unit 7 Assignments: BC Only

Students will apply appropriate techniques to find general and particular solutions. Assignments will include solving initial value problems, interpreting slope fields, and analyzing exponential growth and decay models. Additionally, students will explore applications such as population dynamics, mixing problems, and motion, reinforcing how differential equations describe changing systems in various contexts. BC Calculus students will also engage with logistic growth models to understand how populations grow with limiting factors. Students in BC Calculus will also use Euler's Method to approximate solutions without the use of a calculator. These tasks will develop students' ability to connect mathematical solutions to practical situations.

Sample Assignment 1 Unit 7
Sample Assignment 2 Unit 7
Sample Assignment 3 Unit 7
Sample Assignment 4 Unit 7

AP Classroom: Personal Progress Check 7

- Multiple-choice:
  - 15 questions (AB)
  - 20 questions (BC)
- Free-response: 3 questions

Sample Unit 7 Assessments:

Students will be expected to demonstrate their ability to solve and analyze differential equations in various forms. They will need to accurately solve separable differential equations, including initial value problems, and interpret slope fields to understand the behavior of solutions. Students will apply these skills to real-world scenarios. BC Calculus students will additionally be required to work with logistic growth models, showing understanding of how limiting factors affect population growth and use Euler's method to approximate solutions to differential equations when exact answers are difficult to find.. Throughout the assessment, students must clearly set up differential equations based on given situations, solve them using appropriate methods, and explain the meaning of their solutions in context.

Sample Assessment 1 Unit 7
Sample Assessment 2 Unit 7-BC Topics

Unit Title: ***Unit 8: Applications of Integration*** **BC Only**

Unit 8 Summary: As in Unit 4, students will need to practice interpreting verbal scenarios, extracting relevant mathematical information, selecting an appropriate procedure, and then applying that procedure correctly and interpreting their solution in the context of the problem. Now that students have been exposed to application problems involving both differentiation and antidifferentiation, some may struggle to determine which procedure is applicable. Walk students through different types of scenarios and explain the underlying reasons why some situations call for differentiation while others call for integration.

This unit also involves geometric applications of integration. When using the disc and washer methods, focusing on orientation (i.e., horizontal or vertical) will help students determine whether the "thickness" is with respect to x or y. Students should practice solving variations on these calculus-based geometry problems until they can decide which variable to integrate with respect to without prompting. Relating graphical representations to symbolic representations, such as Riemann sums and definite integrals, develops these skills and helps students to master the content.

Essential Questions:

- How is finding the number of visitors to a museum over an interval of time based on information about the rate of entry similar to finding the area of a region between a curve and the x-axis?

Building Mathematical Practices: As in Unit 4, students will need to practice interpreting verbal scenarios, extracting relevant mathematical information, selecting an appropriate procedure, and then applying that procedure correctly and interpreting their solution in the context of the problem. Now that students have been exposed to application problems involving both differentiation and antidifferentiation, some may struggle to determine which procedure is applicable. Walk students through different types of scenarios and explain the underlying reasons why some situations call for differentiation while others call for integration. This unit also involves geometric applications of integration. When using the disc and washer methods, focusing on orientation (i.e., horizontal or vertical) will help students determine whether the "thickness" is with respect to x or y. Students should practice solving variations on these calculus-based geometry problems until they can decide which variable to integrate with respect to without prompting. Relating graphical representations to symbolic representations, such as Riemann sums and definite integrals, develops these skills and helps students to master the content.

Unit 8 Outcomes:

Topic	Suggested Skills
8.1 Finding the Average Value of a Function on an Interval	1.E Apply appropriate mathematical rules or procedures, with and without technology.
8.2 Connecting Position, Velocity, and Acceleration of Functions Using Integrals	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.
8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts	3.D Apply an appropriate mathematical definition, theorem, or test.

8.4 Finding the Area Between Curves Expressed as Functions of *x*	4.C Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).
8.5 Finding the Area Between Curves Expressed as Functions of *y*	1.E Apply appropriate mathematical rules or procedures, with and without technology.
8.6 Finding the Area Between Curves That Intersect at More Than Two Points	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
8.7 Volumes with Cross Sections: Squares and Rectangles	3.D Apply an appropriate mathematical definition, theorem, or test.
8.8 Volumes with Cross Sections: Triangles and Semicircles	3.D Apply an appropriate mathematical definition, theorem, or test.
8.9 Volume with Disc Method: Revolving Around the *x*- or *y*-Axis	3.D Apply an appropriate mathematical definition, theorem, or test.
8.10 Volume with Disc Method: Revolving Around Other Axes	2.D Identify how mathematical characteristics or properties of functions are related in different representations.
8.11 Volume with Washer Method: Revolving Around the *x*- or *y*-Axis	4.E Apply appropriate rounding procedures.
8.12 Volume with Washer Method: Revolving Around Other Axes	2.D Identify how mathematical characteristics or properties of functions are related in different representations.
8.13 The Arc Length of a Smooth, Planar Curve and Distance Traveled BC ONLY	3.D Apply an appropriate mathematical definition, theorem, or test.

Sample Unit 8 Assignments:

Students will find the area between curves and calculate volumes using both the disk/washer and known cross-section methods. Students will complete AP Free Response Questions from past exams that focus on these topics, allowing them to apply their understanding in a format aligned with the AP test. For BC Calculus students, assignments will also include tasks involving arc length and distance traveled along curves.

Sample Assignment 1 Unit 8
Sample Assignment 2 Unit 8
Sample Assignment 3 Unit 8
Sample Assignment 4 Unit 8

AP Classroom: Personal Progress Check 8

- Multiple-choice: ~30 questions
- Free-response: 3 questions

Sample Unit 8 Assessments:

Students will demonstrate their ability to find the area between curves and calculate volumes of solids formed by revolving regions around an axis. These tasks will require them to accurately determine points of intersection to identify the correct regions for integration. They will apply disk and washer methods, to compute volumes. Additionally, students in BC Calculus will be assessed on their understanding of arc length, requiring them to set up and evaluate integrals that measure the length of a curve over a specified interval. Overall, these assessments will test students' skills in setting up integrals based on geometric interpretation and solving them to find precise area, volume, and length values.

Sample Assessment 1 Unit 8
Sample Assessment 2 Unit 8
Sample Assessment 3 Unit 8- BC Topics

Unit Title: ***Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions BC ONLY***

Unit 9 Summary: **BC ONLY**
As students transition to parametric and vector-valued functions, they'll need to practice previously learned concepts and skills to reinforce the new procedures and representations they're learning in Unit 9. As with particle motion on a line, students learning to handle motion in the plane will need to practice interpreting which procedure is needed for different scenarios (differentiation or integration) and solving for speed, velocity, distance traveled, or initial position. Reinforce the importance of precise notation, particularly regarding the variable of differentiation, as well as correct application of the chain rule. Leibniz notation helps students to remember how to find the derivative of y with respect to x for coordinates defined using the parameter t:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt} \cdot \frac{dt}{dx}, \text{ provided } \frac{dx}{dt} \neq 0.$$

Since dy/dx is in terms of t, students must be particularly careful when determining $\frac{d^2y}{dx^2}$. Similarly, using definite integrals to represent lengths and areas defined by polar curves is based on the same principles as calculating lengths and areas defined by the graphs of more familiar functions (i.e., the limit of a Riemann sum). Students will need to practice with trigonometric identities, radian measures and formulas for arc length and area of a sector to reinforce practice with associated calculus topics

Essential Questions:
- How can we model motion not constrained to a linear path?
- How does the chain rule help us to analyze graphs defined using parametric equations or polar functions?

Building Mathematical Practices: As students transition to parametric and vector-valued functions, they'll need to practice previously learned concepts and skills to reinforce the new procedures and representations they're learning in Unit 9. As with particle motion on a line, students learning to handle motion in the plane will need to practice interpreting which procedure is needed for different scenarios (differentiation or integration) and solving for speed, velocity, distance traveled, or initial position. Reinforce the importance of precise notation, particularly regarding the variable of differentiation, as well as correct application of the chain rule. Leibniz notation helps students to remember how to find the derivative of y with respect to x for coordinates defined using the parameter t: dy/dx= dy/dt/dx/dt=dy/dt x dt/dx, provided dx/dt ≠ 0. Since dy/dx is in terms of t, students must be particularly careful when determining d^2y/dx^2 . Similarly, using definite integrals to represent lengths and areas defined by polar curves is based on the same principles as calculating lengths and areas defined by the graphs of more familiar functions (i.e., the limit of a Riemann sum). Students will need to practice with trigonometric identities, radian measures and formulas for arc length and area of a sector to reinforce practice with associated calculus topics.

Unit 9 Outcomes: BC ONLY

Topic	Suggested Skills
9.1 Defining and Differentiating Parametric Equations	2.D Identify how mathematical characteristics or properties of functions are related in different representations.
9.2 Second Derivatives of Parametric Equations	1.E Apply appropriate mathematical rules or procedures, with and without technology.
9.3 Finding Arc Lengths of Curves Given by Parametric Equations	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.
9.4 Defining and Differentiating Vector-Valued Functions	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.
9.5 Integrating Vector-Valued Functions	1.E Apply appropriate mathematical rules or procedures, with and without technology.
9.6 Solving Motion Problems Using Parametric and Vector-Valued Functions	1.E Apply appropriate mathematical rules or procedures, with and without technology.
9.7 Defining Polar Coordinates and Differentiating in Polar Form	2.D Identify how mathematical characteristics or properties of functions are related in different representations.
9.8 Find the Area of a Polar Region or the Area Bounded by a Single Polar Curve	3.D Apply an appropriate mathematical definition, theorem, or test.
9.9 Finding the Area of the Region Bounded by Two Polar Curves	3.D Apply an appropriate mathematical definition, theorem, or test.

Sample Unit 9 Assignments: **BC ONLY**

Students will work with parametric equations to analyze motion in the plane, calculating derivatives, speed, and total distance traveled, and sketching graphs of particle paths. Another task involves representing and analyzing vectors—finding components, magnitudes, directions, and using vector notation to describe motion. Students also complete assignments focused on polar curves, where they graph a variety of polar equations, find areas bounded by polar curves, and apply calculus techniques such as finding slopes and arc lengths in polar form. Students practice these topics in both Free Response and Multiple Choice form so as to prepare them for the AP Test.
Sample Assignment 1 Unit 9
Sample Assignment 2 Unit 9

AP Classroom: Personal Progress Check 9

- Multiple-choice: 25 questions
- Free-response: 3 questions

Sample unit 9 Assessment: **BC ONLY**

Students will demonstrate an understanding of parametric equations, vector-valued functions, and polar curves through a mix of multiple-choice and "free-response" questions. Assessments emphasize conceptual understanding, accurate use of calculus methods, and clear mathematical communication across multiple representations. These tasks prepare students for the rigor of the AP exam by requiring them to apply both graphical and analytical approaches to non-rectangular coordinate systems.

Sample Assessment 1 Unit 9 **BC ONLY**

Unit Title: ***Unit 10: Infinite Sequences and Series*** **BC ONLY**

Unit 10 Summary:
Students will need to develop proficiency with complex series notation and the ability to communicate their reasoning. Emphasize appropriate use of notation, precision of language, and establishing conditions for using a particular test. Remind students that a sound justification relies upon both mathematical evidence and reasons why that evidence supports the conclusion.

Additionally, students will need to practice determining which application is appropriate for different scenarios (for example, using the definitions of harmonic or p-series to classify certain infinite series) and then applying associated procedures accurately. Students will also need to practice using Taylor polynomials to approximate the value of a function, choosing and implementing an appropriate method to bound the error involved in the approximation, and effectively communicating supporting work.

Connecting representations is an important skill to develop in this unit. For example, students will need to identify infinite power series to represent functions presented symbolically or move between graphic and symbolic representations of an interval of convergence.

Essential Questions:

- How can the sum of infinitely many discrete terms be a finite value or represent a continuous function?

Building Mathematical Practices: In Unit 10, students will need to develop proficiency with complex series notation and the ability to communicate their reasoning. Emphasize appropriate use of notation, precision of language, and establishing conditions for using a particular test. Remind students that a sound justification relies upon both mathematical evidence and reasons why that evidence supports the conclusion. Additionally, students will need to practice determining which application is appropriate for different scenarios (for example, using the definitions of harmonic or p-series to classify certain infinite series) and then applying associated procedures accurately. Students will also need to practice using Taylor polynomials to approximate the value of a function, choosing and implementing an appropriate method to bound the error involved in the approximation, and effectively communicating supporting work. Connecting representations is an important skill to develop in this unit. For example, students will need to identify infinite power series to represent functions presented symbolically or move between graphic and symbolic representations of an interval of convergence.

Unit 10 Outcomes: BC ONLY

Topic	Suggested Skills
10.1 Defining Convergent and Divergent Infinite Series	3.D Apply an appropriate mathematical definition, theorem, or test.
10.2 Working with Geometric Series	3.D Apply an appropriate mathematical definition, theorem, or test.
10.3 The *n*th Term Test for Divergence	3.D Apply an appropriate mathematical definition, theorem, or test.
10.4 Integral Test for Convergence	3.D Apply an appropriate mathematical definition, theorem, or test.
10.5 Harmonic Series and *p*-Series	3.B Identify an appropriate mathematical definition, theorem, or test to apply.
10.6 Comparison Tests for Convergence	3.D Apply an appropriate mathematical definition, theorem, or test.
10.7 Alternating Series Test for Convergence	3.D Apply an appropriate mathematical definition, theorem, or test.
10.8 Ratio Test for Convergence	3.D Apply an appropriate mathematical definition, theorem, or test.
10.9 Determining Absolute or Conditional Convergence	3.D Apply an appropriate mathematical definition, theorem, or test.
10.10 Alternating Series Error Bound	1.E Apply appropriate mathematical rules or procedures, with and without technology.

Sample Unit 10 Assignments: BC ONLY

Students will deepen their understanding of infinite processes by exploring sequences, infinite series, and Taylor series. Assignments are designed to develop both conceptual understanding and procedural fluency. Students are expected to explain their reasoning clearly and justify convergence tests and approximation techniques. Through numerical experiments and analytical work, students distinguish between convergent and divergent series. They derive the sum of a geometric series and compare it with the behavior of the harmonic series. Students apply the Alternating Series Test and estimate error using the Alternating Series Error and LaGrange Error bounds. Students derive Taylor series for elementary functions, analyze error using Taylor's Inequality, and compare polynomial approximations to actual function graphs. Finally, from AP Classroom, students practice timed problems with structured feedback. Emphasis is placed on clear justification of convergence and the use of appropriate series techniques.

Sample Assignment 1 Unit 10
Sample Assignment 2 Unit 10
Sample Assignment 3 Unit 10
Sample Assignment 4 Unit 10
Sample Assignment 5 Unit 10

AP Classroom: Personal Progress Check 10
- Multiple-choice: ~45 questions
- Free-response: 3 questions

Sample Unit 10 Assessment: BC ONLY

Students are assessed on their understanding of the behavior of sequences and infinite series. Students will demonstrate an ability to choose and apply appropriate convergence tests, work with Taylor and Maclaurin series as function approximations, and use skill in estimating and interpreting error in series approximations. This assessment encourages a deep understanding of infinite processes, a foundational concept in advanced calculus, and prepares students for success on the AP exam and in future college-level coursework.

Sample Assessment 1 Unit 10

Mathematical Practices (page 21 College Board AP Calculus AB/BC Course and Exam Description)

AP CALCULUS AB AND BC

Mathematical Practices

Practice 1

Implementing Mathematical Processes 1

Determine expressions and values using mathematical procedures and rules.

Practice 2

Connecting Representations 2

Translate mathematical information from a single representation or across multiple representations.

Practice 3

Justification 3

Justify reasoning and solutions.

Practice 4

Communication and Notation 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

SKILLS

1.A Identify the question to be answered or problem to be solved *(not assessed)*.

1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., *Use the chain rule to find the derivative of a composite function*).

1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., *rate of change and accumulation*) or processes (e.g., *differentiation and its inverse process, anti-differentiation*) to solve problems.

1.E Apply appropriate mathematical rules or procedures, with and without technology.

1.F Explain how an approximated value relates to the actual value.

2.A Identify common underlying structures in problems involving different contextual situations.

2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

2.C Identify a re-expression of mathematical information presented in a given representation.

2.D Identify how mathematical characteristics or properties of functions are related in different representations.

2.E Describe the relationships among different representations of functions and their derivatives.

3.A Apply technology to develop claims and conjectures *(not assessed)*.

3.B Identify an appropriate mathematical definition, theorem, or test to apply.

3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

3.D Apply an appropriate mathematical definition, theorem, or test.

3.E Provide reasons or rationales for solutions and conclusions.

3.F Explain the meaning of mathematical solutions in context.

3.G Confirm that solutions are accurate and appropriate.

4.A Use precise mathematical language.

4.B Use appropriate units of measure.

4.C Use appropriate mathematical symbols and notation (e.g., *Represent a derivative using* $f'(x)$, y', *and* $\frac{dy}{dx}$).

4.D Use appropriate graphing techniques.

4.E Apply appropriate rounding procedures.

Recommended Texts and Resources: Suggested textbook pending pilot process and board approval: *Calculus: Early Transcendentals,* 12th Edition, 2021 by Howard Anton, Irl C. Bivens, and Stephen Davis

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UC Area "C"