PREP FOR PRECALCULUS

A) Factor all of the following completely, if possible:

1) $x^2 - x - 6$

2) $81y^4 - 16x^8$

$(9y^2 - 4x^4)(9y^2 + 4x^4)$

$(3y - 2x^2)(3y + 2x^2)(9y^2 + 4x^4)$

3) $9x^2 - 15x + 6$

$3(3x^2 - 5x + 2)$

$3(3x - 2)(x - 1)$

4) $(2x-3)^2 - 6(2x-3) + 8$

Let $N = 2x - 3$

$N^2 - 6N + 8$

$(N - 4)(N - 2)$

$(2x - 3 - 4)(2x - 3 - 2)$

$(2x - 7)(2x - 5)$

5) $6x^2 + 3xy + 10x + 5y$

6) $8A^3 - 27B^3$

$(2A - 3B)(4A^2 + 6AB + 9B^2)$

7) $36N^4 + 44N^3 + 24N^2$

8) $4x^2y - 36y$

$4y(x^2 - 9)$

$4y(x - 3)(x + 3)$

9) $9x^4 + 16$

PRIME

10) $m(n-2) - 3(2-n)$

11) $5x^2 - 3x - 2$

$(5x + 2)(x - 1)$

12) $3x^2 - 17x - 6$

$(3x + 1)(x - 6)$

B) Evaluate WITHOUT A CALCULATOR - making sure to leave NO Radicals in the denominator of your answer and no negative exponents.

1) $(2^{\sqrt{2}})^{\sqrt{8}}$ $= 2^{\sqrt{16}} = 2^4$ $= 16$

2) $\log_2 32$ $= 5$

3) $\log_3 \frac{1}{81}$

4) $\ln(e)$ $= 1$

5) $\log 100^5$

$= 5 \log 100$

$= 5(2) = 10$

6) $27^{\frac{2}{3}}$ $= \left[\sqrt[3]{27}\right]^2 = 3^2 = 9$

7) $\frac{3^{\sqrt{5}-2}}{3^{\sqrt{5}+1}}$

8) $25^{-\frac{1}{2}}$ $= \frac{1}{\sqrt{25}} = \frac{1}{5}$

9) $4^{x+1} \cdot 8^x$

$(2^2)^{x+1} (2^3)^x$

$2^{2x+2} = 2^{3x}$

$2x+2 = 3x$

$2 = x$

10) $\frac{\sqrt[3]{9}}{\sqrt[6]{3}} = \frac{(3^2)^{\frac{1}{3}}}{(3)^{\frac{1}{6}}}$

$= \frac{3^{\frac{2}{3}}}{3^{\frac{1}{6}}} = 3^{\frac{2}{3}-\frac{1}{6}} = 3^{\frac{3}{6}}$

$= 3^{\frac{1}{2}} = \sqrt{3}$

11) $\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

12) $\sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

13) $\frac{3}{2-\sqrt{3}}$ $\frac{2+\sqrt{3}}{2+\sqrt{3}}$

$\frac{6+3\sqrt{3}}{4-3} = \frac{6+3\sqrt{3}}{1}$

14) $\frac{3}{7-i}$

15) $\left(\sqrt{3}+\sqrt{5}\right)^2$

$(\sqrt{3}+\sqrt{5})(\sqrt{3}+\sqrt{5})$

$3+\sqrt{15}+\sqrt{15}+5$

$8+2\sqrt{15}$

16) $\left(5\sqrt{2}-\sqrt{10}\right)\left(3\sqrt{2}+2\sqrt{10}\right)$

17) $\frac{x^2y^3}{y^{-4}} \cdot \frac{y^4}{x^{-2}y^{-3}}$

$(x^2y^7)(x^2y^7)$

x^4y^{14}

18) $\frac{xy}{3}(xy)^{-1}$

19) $\sqrt{420x^2y^3}$

$2xy\sqrt{105y}$

20) $(3x)^3$

$27x^3$

21) $\sqrt[3]{-64xy^7}$

$-4y^2\sqrt[3]{xy}$

C) Quadratics/Polynomials

1) State the QUADRATIC FORMULA: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

2) Solve by using the Quadratic Formula

a) $x^2 - x + 7 = 0$

$x = \frac{1 \pm \sqrt{1-4(1)(7)}}{2}$

$x = \frac{1 \pm \sqrt{-27}}{2} = \frac{1 \pm 3i\sqrt{3}}{2}$

b) $x^2 = 6x - 2$ $\quad x^2 - 6x + 2 = 0$

$x = \frac{6 \pm \sqrt{36-4(1)(2)}}{2} = \frac{6 \pm \sqrt{28}}{2}$

$= 3 \pm \sqrt{7}$

3) Solve by Completing the Square.

a) $x^2 - 2x - 8 = 0$

$x^2 - 2x \cdot = 8$

$x^2 - 2x + 1 = 8 + 1$

$\sqrt{(x-1)^2} = \pm\sqrt{9}$

$x - 1 = \pm 3$

$x = 1 \pm 3$

$x = 4, -2$

b) $2x^2 = 9x + 3$

4) Solve by taking the roots.

a) $x^2 - 16 = 0$

$(x-4)(x+4) = 0$

$x = 4, -4$

b) $2(x+3)^2 - 5 = 7$

$+5 \quad +5$

$\frac{2(x+3)^2}{2} = \frac{12}{2}$

$\sqrt{(x+3)^2} = \pm\sqrt{6}$

$x + 3 = \pm\sqrt{6}$

$x = -3 \pm \sqrt{6}$

5) Solve any way you wish.

a) $x = (x-6)^2$

$x = x^2 - 12x + 36$

$0 = x^2 - 13x + 36$

$0 = (x-4)(x-9)$

$x = 4$ $x = 9$

b) $y^2(y-3)(y^2-9) = 0$

$y^2 = 0 \mid y-3=0 \mid (y-3)(y+3) = 0$

$y = 0 \quad y = 3 \quad y = 3 \quad y = -3$

$\{-3, 0, 3\}$

c) $y^4 - 5y^2 + 4 = 0$

d) $(x-4)^3 - 4(x-4) = 0$

e) $(x+1)(x-5) = 7$

$x^2 - 5x + 1x - 5 = 7$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6, -2$

6) The product of three consecutive integers is 21 more than the cube of the smallest integer. Find the three integers

Let $N = 1^{st}$

$N+1 = 2^{nd}$

$N+2 = 3^{rd}$

$N(N+1)(N+2) = N^3 + 21$

$N(N^2 + 3N + 2)$

$N^3 + 3N^2 + 2N = N^3 + 21$

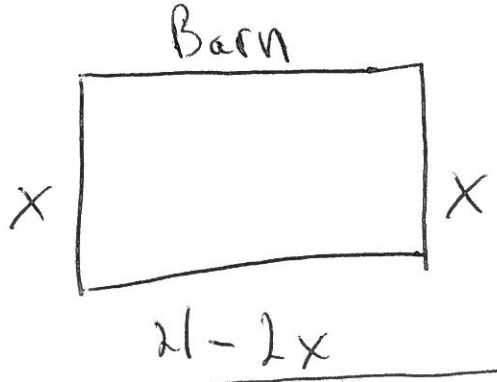
$3N^2 + 2N - 21 = 0$

$(3N-7)(N+3)$ $\rightarrow N = -3$

So... $(-3, -2, -1)$

7) A rectangular garden has a perimeter of 66 ft and an area of 216 ft^2. Find the dimensions of the rectangle.

8) A farmer plans to use 21 ft of fencing to enclose a rectangular corral having an area of 55 ft^2. Since the corral is attached to the back of the barn, the farmer only needs to build 3 sides with fencing using the barn as the fourth side. Find the dimensions of the rectangular corral.



$x(21-2x) = 55$

$21x - 2x^2 = 55$

$0 = 2x^2 - 21x + 55$

$0 = (2x-11)(x-5)$

$x = 5.5$ or $x = 5$

Dimensions are 5ft x 11ft or 5.5ft by 10ft.

E) Functions

Given $f(x)=2x-5$, $g(x)=x^2+3$, **and** $h(x)=\frac{1}{2}x+5$, **evaluate each of the following, if possible**

1) $f(-2)$

-9

2) $f(g(2))$

3) $h(g(0))$

$h(3) = \frac{1}{2}(3) + 5$

$= 6.5$

4) $f(x+2)$

5) $f(h(x))$

$2\left[\frac{1}{2}x+5\right]-5$

$x+10-5$

$x+5$

6) $f^{-1}(x)$

{Find the inverse of $f(x)$}

7) $f(x)+g(x)$

x^2+2x-2

8) $\frac{g(x)}{f(x)}$

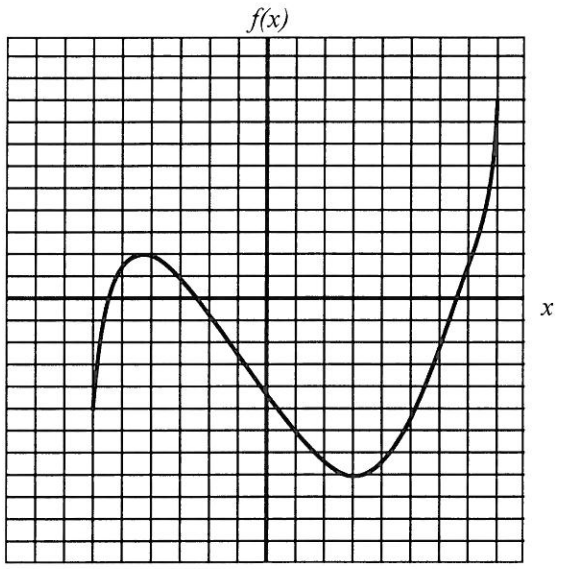
9) What is the domain of $\frac{g(x)}{f(x)}$? $\frac{x^2+3}{2x-5}$

Domain is all $\mathbb{R}$, $x \neq 5/2$

↑ Can't be 0

F) Polynomial and Rational Expressions

1) The graph of $f(x)$ is given to the right.



Use the Graph to estimate the following:

a) domain of f $[-6, 8]$

b) range of f $[-8, 9]$

c) $f(6) = -2$

d) estimate the value(s) of x if $f(x) = -4$

$\{-5.8, 0, 5.5\}$

Perform the indicated operation and simplify. Leave you answer in factored form if necessary.

2) $\dfrac{x^2-2x+1}{x^3+x} \cdot \dfrac{4x^2+4}{x^2+x-2}$

$\dfrac{(x-1)^2}{x(x^2+1)} \cdot \dfrac{4(x^2+1)}{(x+2)(x-1)}$

$\dfrac{4(x-1)}{x(x+2)}$

3) $\dfrac{x+3}{x^2-4} \div \dfrac{x^2-x-12}{x^3-8}$

4) $\dfrac{x^2}{x^2-4} - \dfrac{1}{x}$

5) $\dfrac{x}{x^2+3x+2} + \dfrac{2x-3}{x^2-1}$

$\dfrac{x(x-1)}{(x+1)(x+3)(x-1)} + \dfrac{(2x-3)(x+3)}{(x+1)(x+3)(x-1)}$

$\dfrac{x^2-x+2x^2+3x-9}{(x+1)(x+3)(x-1)} = \boxed{\dfrac{3x^2+2x-9}{(x^2-1)(x+3)}}$

6) $\dfrac{\frac{1}{2}+\frac{3}{x}}{\frac{x+3}{4}}$

7) $\dfrac{\frac{x^2}{x-4}+2}{\frac{2x-2}{x}-1}$

$$\frac{x^3+2x(x-4)}{(2x-2)(x-4)-x(x-4} = \frac{x^3+2x^2-8x}{(x-4)(2x-1-x)}$$

$$= \frac{x^3+2x^2-8x}{(x-4)(x-2)}$$

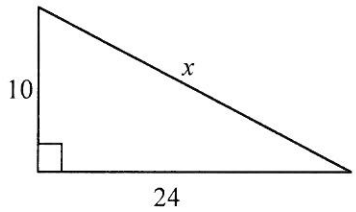
$$= \frac{x(x-2)(x+4)}{(x-4)(x-2)}$$

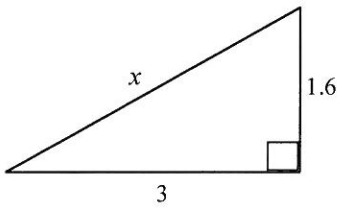
$$= \frac{x(x+4)}{(x-4)}$$

8) Solve for x. $\dfrac{5}{x+4} = 4 + \dfrac{3}{x-2}$

G) Right Triangles and Trigonometry.

1) Find the unknown side length



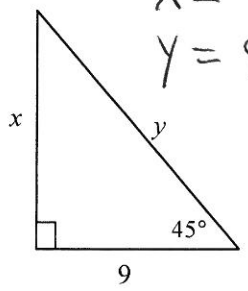


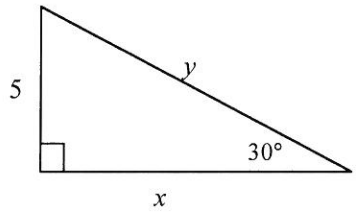
$x = 26$

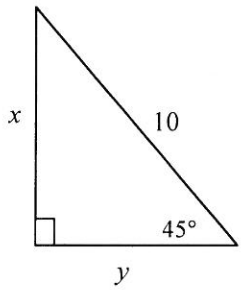
2) Find the value of each variable. Give answer in simplest radical form. (You may have to review special right triangles)

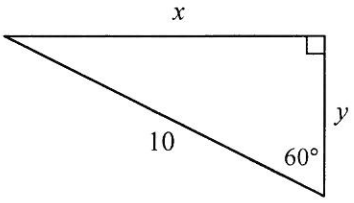
$x = 9$

$y = 9\sqrt{2}$





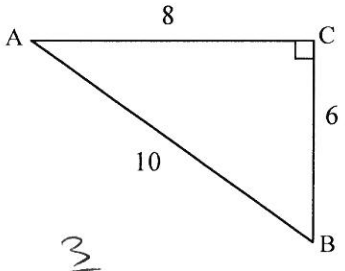




$x = 5\sqrt{2}$

$y = 5\sqrt{2}$

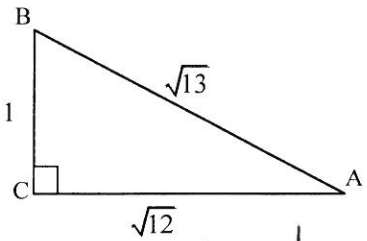
3) Find the sine, the cosine and the tangent of angle A. Express answers in simplest radical form.



$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$

$\tan A = \frac{3}{4}$



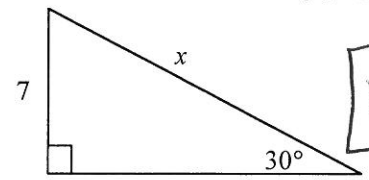
$\sin A = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

$\cos A = \frac{\sqrt{12}}{\sqrt{13}} = \frac{2\sqrt{39}}{13}$

$\tan A = \frac{\sqrt{12}}{12}$

4) Use trig to find the value of each variable. Round answers to the nearest hundredth.

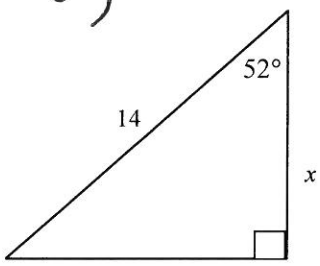
a)



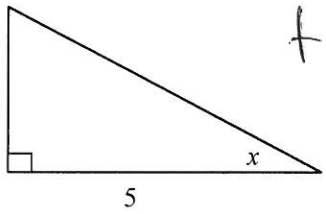
$\sin 30^\circ = \frac{7}{x}$

$x = 14$

b)



c)

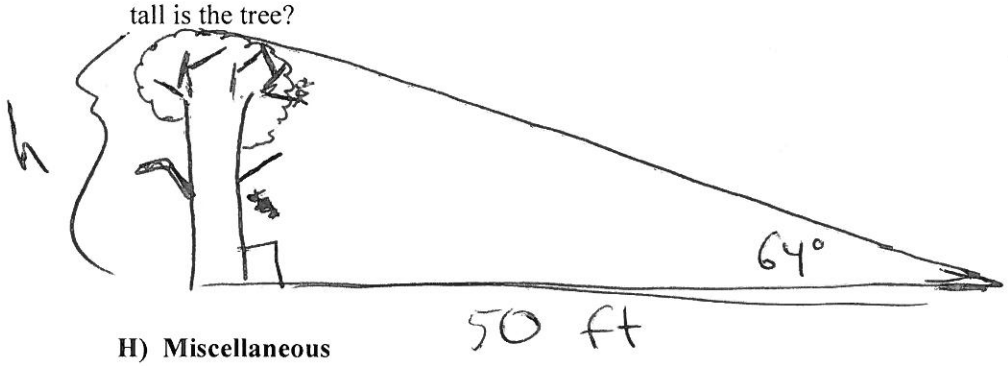


$\tan x = \frac{3}{5}$

$x = \tan^{-1}\left(\frac{3}{5}\right)$

$x \approx 30.96^\circ$

5) A boy is 50 ft from a tall tree. The angle from his feet to the top of the tree is 64°. To the nearest tenth, how tall is the tree?



$\tan 64^\circ = \frac{h}{50}$

$50 \tan 64^\circ = h$

$102.5 = h$

The tree is 102.5 ft high

H) Miscellaneous

1) *Systems:* Solve each system of equations, if possible

a)
$$\begin{aligned} 3x - y &= 9 \\ x + 2y &= -4 \end{aligned}$$

b)
$$\begin{aligned} [-x + 2y &= -4] \cdot 3 \\ 3x - 6y &= 12 \end{aligned}$$

$-3x + 6y = -12$

$3x - 6y = 12$

$0 = 0 \therefore$

infinite solutions to the line $-x + 2y = -4$

2) Inequalities: Graph each solution set on a number line

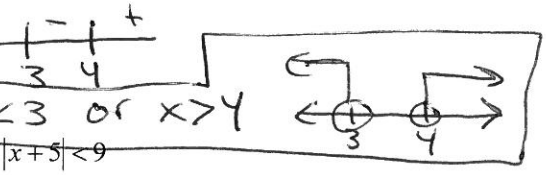
a) $x^2 - 7x + 12 > 0$

$(x-3)(x-4) > 0$

+ | − | +

3 4

$x < 3$ or $x > 4$

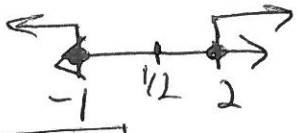


b) $(x^2 - 4x)(x^2 - 4) < 0$

c) $|x+5| < 9$

d) $|1-2x| \geq 3$

$\left|x - \frac{1}{2}\right| \geq \frac{3}{2}$



$x \leq -1$ or $x \geq 2$

Solving: Check for any extraneous solutions. Round your answer to the nearest hundredth if necessary.

3) $\sqrt{2x-5} = 4$

$2x - 5 = 16$

$2x = 21$

$x = \frac{21}{2}$

4) $(10x+15)^{1/2} - 4 = x$

5) $\sqrt[3]{2t-1} = 5$

6) $\frac{3}{x+1} = \frac{2}{x-4}$

$3(x-4) = 2(x+1)$

$3x - 12 = 2x + 2$

$x = 14$

7) $\log_2(4x) = 5$

8) $2\log_2 x + 3\log_2 2 = 7$

$\log_2 x^2 + \log_2 8 = 7$

$\log_2 8x^2 = 7$

$2^7 = 8x^2$

$8x^2 = 128$

$x = \pm 4$

$x = 4$

* X can only equal 4 because of domain

9) $7e^x + 2 = 19$

$7e^x = 17$

$e^x = \frac{17}{7}$

$\ln e^x = \ln \frac{17}{7}$

$\boxed{x = \ln\left(\frac{17}{7}\right)}$

10) $3^{5x} = 20$

11) Divide $(2x^2 - 19x + 24) \div (x+8)$ Use Long Division.

$2x - 35 + \frac{304}{x+8}$

$x+8 \overline{)\,2x^2 - 19x + 24}$

$-2x^2 \mp 16x$

$-35x + 24$

$+35x + 280$

12) Divide $(x^3 - 28x - 48) \div (x+4)$ Use Synthetic Division.

-4	1	0	-28	-48
	↓	-4	16	48
	1	-4	-12	0

$\therefore\ x^2 - 4x - 12$ is the quotient

13) Find an equation of a line having the given characteristics.

a) containing the points (3, -4) and (2, 1)

$m = \frac{1+4}{2-3} = -5$ $\quad Y - 1 = -5(x-2)$

b) perpendicular to the line $3x - y = -4$; containing the point (-2, 4)

$y - 4 = -\frac{1}{3}(x+2)$ $\quad m = 3 \therefore m_\perp = -\frac{1}{3}$

c) vertical containing the point (-3, 4)

$x = -3$