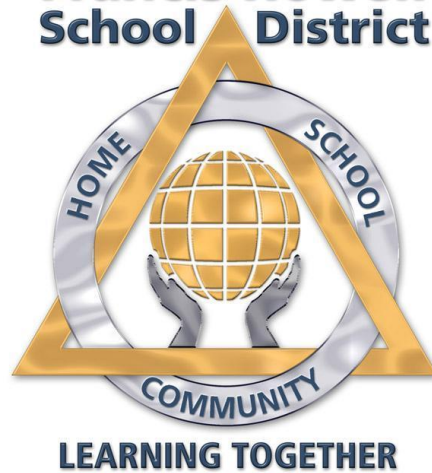


Mathematics

Discrete Mathematics

Curriculum

**Francis Howell
School District**



Board Approved:
July 21, 2022

Francis Howell School District

Mission Statement

The mission of the Francis Howell School District is to empower students to be lifelong learners prepared for the future.

Vision Statement

The Francis Howell High School Community will provide a high quality educational experience that will result in students possessing the necessary knowledge and skills to become life-long learners and be positive contributors within their community.

The Francis Howell High School Community will provide an environment which develops a sense of personal and school pride, cultural awareness and tolerance.

Values

Francis Howell School District is committed to:

High Expectations

Continuous Improvement

Engagement, Collaboration, and Partnerships

Innovation

Safety

Trust, Respect, and Inclusiveness

Customer Service and Satisfaction

Accountability and Transparency

Graduate Goals

Upon completion of their academic study in the Francis Howell School District, students will be able to:

1. Gather, analyze and apply information and ideas.
2. Communicate effectively within and beyond the classroom.
3. Recognize and solve problems.
4. Make decisions and act as responsible members of society.

Mathematics Graduate Goals

Upon completion of their mathematics study in the Francis Howell School District, students will be able to:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Course Rationale

In order to be effective citizens in the 21st century, students need to understand mathematics. Students often encounter problem situations that require reasoning, computation and communication. Students regularly study the most efficient methods for reaching solutions, but also realize that examining different solution methods help develop more flexible problem solving skills. The instruction and assessment is focused on instilling students with enduring understandings of mathematics.

Course Description

Discrete Mathematics explores non-traditional concepts in mathematics beyond algebra and geometry. This course will require students to think critically and apply basic mathematics to draw conclusions and solve problems within real world contexts. Among possible topics to be studied are game theory, cryptography, circuits, and paths.

Content Curriculum Team

Curriculum Committee

Stacey Bevill
Zachary Forsythe
Steven Le

Francis Howell North HS
Francis Howell HS
Francis Howell Central HS

Secondary Math & Science Content Leader
Director of Curriculum & Assessment
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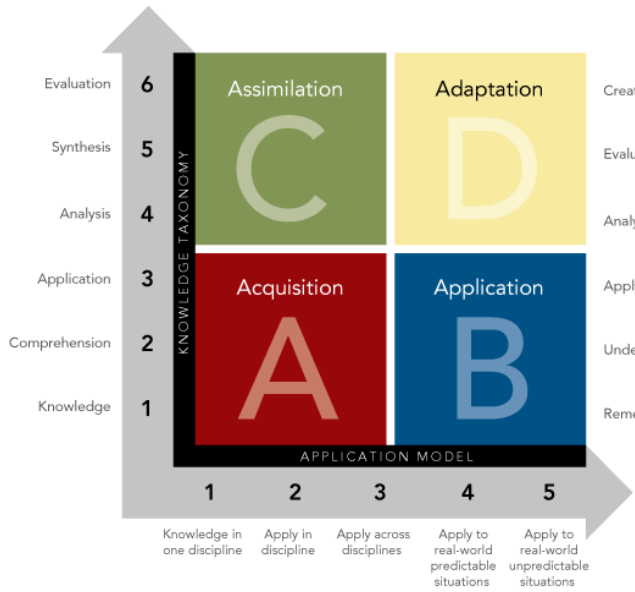
Tim Besse
David Brothers
Dr. Connie Buckman
Dr. Ken Roumpos

Curriculum Notes

All FHSD performance tasks and sample learning activities are aligned not only to understandings and standards, but also the [Rigor and Relevance Framework](#) and [21st Century Skills](#). Information on these two things is provided below or by clicking on the hyperlinks.

Rigor and Relevance Framework

The Rigor/Relevance Framework is a tool developed by the International Center to examine curriculum, instruction, and assessment along the two dimensions of higher standards and student achievement.



The Rigor/Relevance Framework has four quadrants.

Quadrant A represents simple recall and basic understanding of knowledge for its own sake. Examples of Quadrant A knowledge are knowing that the world is round and that Shakespeare wrote Hamlet.

Quadrant C represents more complex thinking but still knowledge for its own sake. Quadrant C embraces higher levels of knowledge, such as knowing how the U.S. political system works and analyzing the benefits and challenges of the cultural diversity of this nation versus other nations.

Quadrants B and D represent action or high degrees of application. Quadrant B would include knowing how to use math skills to make purchases and count change. The ability to access information in wide-area network systems and the ability to gather knowledge from a variety of sources to solve a complex problem in the workplace are types of Quadrant D knowledge.

A	B	C	D
Students gather and store bits of knowledge and information. Students are primarily expected to remember or understand this knowledge.	Students use acquired knowledge to solve problems, design solutions, and complete work. The highest level of application is to apply knowledge to new and unpredictable situations.	Students extend and refine their acquired knowledge to be able to use that knowledge automatically and routinely to analyze and solve problems and create solutions.	Students have the competence to think in complex ways.

21st Century Skills

These skills have been pared down from 18 skills to what are now called the 4Cs. The components include critical thinking, communication, collaboration and creativity. Critical thinking is focused, careful analysis of something to better understand and includes skills such as arguing, classifying, comparing, and problem solving. Communication is the process of transferring a thought from one mind to another and receiving thoughts back and includes skills such as choosing a medium (and/or technology tool), speaking, listening, reading, writing and evaluating messages. Collaboration is working together with others to achieve a common goal and includes skills such as delegating, goal setting, resolving conflicts, team building, decision-making and managing time. Creativity is expansive, open-ended invention and discovery of possibilities and includes skills such as brainstorming, creating, designing, imagining, improvising and problem-solving.

AP Mathematical Practices

https://drive.google.com/file/d/1Ts2i9JdA_A6LKudBb8kSd5Y3TdP1U_te/view?usp=sharing

Statement on Standards: In most curriculum documents, the district aligns their course to the current, approved state standards. However, in some cases, particularly in non-core areas, state standards may be very dated or do not provide material to support an entire course. In some instances, DESE even recommends using the national standards for these courses. For Discrete Mathematics, DESE does not provide standards beyond Algebra 2, therefore the decision was made to write the curriculum aligned to the AP Mathematical Practices. The standards specify that students will implement mathematical processes, connect representations, justify reasoning and solutions, and communicate results or solutions.

Units & Standards Overview

Semester

Unit 1:	Unit 2:	Unit 3:	Unit 4:	Unit 5:
Historic Numeration Systems	Logic	Number Theory	Graph Theory	Matrices & Game Theory
PE Standards: 1, 2, 3, 4	PE Standards: 1, 2, 3	PE Standards: 1, 2, 3, 4	PE Standards: 1, 2, 4	PE Standards: 1, 2, 3, 4
PE Assessment: Unit 1 Assessment	PE Assessment: Unit 2 Assessment	PE Assessment: Unit 3 Assessment	PE Assessment: Unit 4 Assessment	PE Assessment: Unit 5 Assessment

Course Map

	Unit Description	Performance Event Summary	PE Standards
Unit 1: Historic Numeration Systems 2-3 weeks	This unit will cover early computational methods and notations of the past before modern age mathematics and connect these systems to more modern numerical systems.	Students will convert from Hindu-Arabic to Roman, Egyptian, and Mayan systems. Students will convert from decimal to binary and hexadecimal, and vice-versa. Students will complete multiplication using historic methods of calculation. Students will demonstrate an understanding of place-value and different number bases through a collection of justified responses.	1, 2, 3, 4
Unit 2: Logic 3 weeks	This unit will cover how logic can be used to analyze statements in everyday lives. This will include truth tables, statements, arguments, and logic circuits.	Students will identify statements in logic. They will be able to identify if statements are true, false, or inconclusive based on given information. Students will be able to complete truth tables with accuracy. Students will be able to complete proofs based on their understanding of logic.	1, 2, 3
Unit 3: Number Theory 3 weeks	This unit will cover an introduction to number theory and their properties. This will include Fibonacci sequence, cryptography, and a deeper understanding of basic mathematical rules (i.e., prime numbers, modular arithmetic, factorization) and how they are applied.	Students will determine the accuracy of statements covering the properties of different numbers. They will correct inaccurate statements. Students will utilize divisibility rules on a number to determine potential factors. Students will define the fundamental theorem of arithmetic. Students will define and apply modular arithmetic. Students will demonstrate an understanding of Fibonacci's sequence. Students will decrypt a Caesar cipher.	1, 2, 3, 4

<p>Unit 4: Graph Theory</p> <p>3 weeks</p>	<p>This unit will cover making connections between various locations or objects. This unit can be used to help determine an efficient solution to problems such as determining the shortest path to make deliveries. This includes Euler paths, Euler circuits, Hamilton paths, trees, etc.</p>	<p>Students will understand and be able to identify an Euler circuit and an Euler path. Students will understand and be able to identify a Hamilton circuit and a Hamilton path. Students will be able to use the Hamilton circuits to solve the traveling salesperson problem. Students will be able to effectively use PERT (Program Evaluation and Review Technique) to schedule projects.</p>	<p>1, 2, 4</p>
<p>Unit 5: Matrices and Game Theory</p> <p>3 weeks</p>	<p>This unit will review/introduce concepts of matrix arithmetic (adding, multiplying, etc.) and relate these concepts to Markov chains and game theory. Students will explore how to look at a game matrix and determine best strategies.</p>	<p>Students will be able to understand what strategies work best for certain games using matrices. They will be able to identify which player has a dominant strategy. Students will perform the iterated elimination of strictly dominated strategies (IESDS) to determine which strategy to choose.</p>	<p>1, 2, 3, 4</p>
<p>Final Exams:</p>	<p>Sem 1 Final Exam Blueprint: (to be completed by the Curriculum Team Fall 2022)</p>	<p>Students will be able to convert numbers from Hindu-Arabic to other ancient systems, and convert to different number bases. Students will display an understanding of number sets and modular arithmetic</p>	<p>1, 2, 3, 4</p>

Unit 1: Historic Numeration Systems

Content Area: Mathematics	Course: Discrete Mathematics	UNIT: Historic Numeration Systems
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Unit Description: This unit will cover early computational methods and notations of the past before modern age mathematics and connect these systems to more modern numerical systems.	Unit Timeline: 2 - 3 weeks
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DESIRED Results

Transfer Goal - *Students will be able to independently use their learning to...*

Understand that mathematics has changed and evolved over time and continues to change as mankind has discovered better and more efficient ways of solving problems.

Understandings – *Students will understand that... (Big Ideas)*

1. Numbers can be counted in ways other than the base-10 system.
2. Computations can be done in other numerical systems.

Essential Questions: *Students will keep considering...*

1. Why would it be appropriate to use a non-base-10 number system?
2. How are the early numerical systems different from our current system?
3. What are the merits and drawbacks between a placement and a grouping system?

Students will know/understand ...	Students will be able to ...
<ul style="list-style-type: none">● Students will know what types of numerical systems were used in the past and understand how they work.● Students will know how ancient civilizations did calculations and understand the difference between place value and grouping systems.● Students will know the most effective way to convert between multiple numeration systems. <p>Essential Vocabulary:</p> <ul style="list-style-type: none">● Numeration systems - Any notation for the representation of numbers● Napier rod - a set of graduated rods (as of wood or bone) invented by John Napier and used for multiplication and division based on the principles of logarithms● Expanded form - A way to write a number that expresses the value of each digit.● Hexadecimal system - Base-16 number system; Uses 0-9 as well as A-F to show 11-15.● Place-value system - Numeration system in which the position of a digit determines its value.● Binary - a number system based only on the numerals 0 and 1	<ul style="list-style-type: none">● Students will be able to represent Arabic numerals in ancient numeral systems (Roman, Mayan, Egyptian).● Students will be able to make computations in ancient numeral systems and other base systems.● Students will be able to convert between multiple numeration systems.

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Unit 1: Assessment

EVIDENCE of LEARNING

<u>Understanding</u> 1, 2	<u>Standards</u> 1, 2, 3, 4	<p>Unit Performance Assessment:</p> <p>Description of Assessment Performance Task(s): Students will convert between multiple systems and bases. Questions will be similar to the following: Why is a symbol for zero necessary in a Place-Value system? Explain. Describe the difference between a Place-Value System and an Additive System. Name at least one advantage to using a number system with a base higher than 10.</p> <p>Teacher will assess:</p> <p><i>The assessment will be graded using a 4 point scale on each individual question unless otherwise stated.</i></p> <table border="1" style="width: 100%; margin-top: 10px;"> <tr> <td style="width: 15%; text-align: center;">Rubric Values</td> <td style="width: 20%;">4 - The target is demonstrated and organized justification leads to a precise proven conclusion.</td> <td style="width: 20%;">3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.</td> <td style="width: 20%;">2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.</td> <td style="width: 25%;">1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.</td> </tr> </table>	Rubric Values	4 - The target is demonstrated and organized justification leads to a precise proven conclusion.	3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.	2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.	1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.	<u>R/R Quadrant</u> A, C <u>21 Century</u> Critical thinking Creativity Communication
Rubric Values	4 - The target is demonstrated and organized justification leads to a precise proven conclusion.	3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.	2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.	1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.				

		<p>Performance:</p> <p>Mastery: <i>Students will show that they really understand when they... score a 75% or higher on the assessment.</i></p> <p>Scoring Guide: See Appendix - Rubric & Unit 1 Assessment</p>	
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Unit 1: Resources

UNIT RESOURCES

Teacher Resources:

This may include:

- *Textbook*
- [Applied Discrete Structures](#)
- [The Discrete Math Hub](#)

Student Resources:

This may include:

- *Textbook*
- *Canvas*
- *Teacher created material*

Unit 2: Logic

Content Area: Mathematics	Course: Discrete Mathematics	UNIT: Logic
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Unit Description: This unit will cover how logic can be used to analyze statements in everyday life. This will include truth tables, statements, arguments, and logic circuits.	Unit Timeline: 3 weeks
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DESIRED Results

Transfer Goal - *Students will be able to independently use their learning to...*

- Create a logical argument or express themselves logically.
- Detect false arguments given by others.

Understandings – *Students will understand that... (Big Ideas)*

1. Logical connectives (and, or, negation, conditional, and bi-conditional) can be used to make compound statements.
2. Truth tables map out all possible outcomes of a statement.
3. Logical statements can be used to prove validity of arguments.

Essential Questions: *Students will keep considering...*

1. What is the difference between truth and validity and when should they be applied?
2. How can students effectively prove the truth of statements and validity of arguments?

Students will know/understand ...	Students will be able to ...
<ul style="list-style-type: none"> ● Students will understand logical connectives (and, or, not) and the symbols that represent them. ● Students will understand the logic behind conditional statements (conditional & bi-conditional). ● Students will know how to read a compound statement and understand the meaning. ● Students will understand how to apply logical reasoning to circuits. ● Students will know the difference between validity and truth. ● Students will know how to read an Euler diagram. <p>Essential Vocabulary:</p> <ul style="list-style-type: none"> ● Negation - Operation of negating or making negative ● Conjunction - A complex statement, true if each component is true. ● Disjunction - A compound statement of two simple statements, joined by <i>or</i>. ● Conditional - A statement, true for certain values that meet a condition. ● Bi-conditional - A relation between two statements, true only when both statements are true or false. ● Truth table - A table that shows the truth-value of a statement, depending on the truth-values of its components. ● Converse - a proposition obtained by interchange of the subject and predicate of a given proposition ● Contrapositive - a proposition formed by contradicting both the subject and predicate ● Syllogism - a deductive scheme of a formal argument consisting of a major and a minor premise and a conclusion ● Euler diagram - a graphic method employing circles to 	<ul style="list-style-type: none"> ● Students will be able to create a truth table given a statement that includes and, or, and not. ● Students will be able to create a truth table using conditional and bi-conditional statements. ● Students will be able to create a truth table involving compound statements. ● Students will be able to write a logical statement to represent a circuit and draw a circuit to represent logical statements. ● Students will be able to determine if an argument is true, valid, both or neither. ● Students will be able to interpret and create Euler diagrams to determine validity.

represent relations between and operations on classes and the terms of propositions by inclusion, exclusion, and intersection

Unit 2: Assessment

EVIDENCE of LEARNING

<u>Understanding</u>	<u>Standards</u>	<u>Unit Performance Assessment:</u>	<u>R/R Quadrant</u>					
1, 2, 3	1, 2, 3	<p>Description of Assessment Performance Task(s): Students will use logic to perform multiple tasks. Questions will be similar to the following: Match the sentence with its symbolic equivalent. If A and B are true statements, and X and Y are false statements, determine if the following is true, false, or inconclusive. Given true statements, prove the following.</p> <p>Teacher will assess:</p> <p><i>The assessment will be graded using a 4 point scale on each individual question unless otherwise stated.</i></p> <table border="1" data-bbox="640 971 1549 1260"> <thead> <tr> <th data-bbox="640 971 764 1260">Rubric Values</th> <th data-bbox="764 971 951 1260">4 - The target is demonstrated and organized justification leads to a precise proven conclusion.</th> <th data-bbox="951 971 1140 1260">3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.</th> <th data-bbox="1140 971 1341 1260">2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.</th> <th data-bbox="1341 971 1549 1260">1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.</th> </tr> </thead> </table> <p>Performance: Mastery: <i>Students will show that they really understand when they... score a 75% or higher on the assessment.</i></p>	Rubric Values	4 - The target is demonstrated and organized justification leads to a precise proven conclusion.	3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.	2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.	1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.	<p>A, C</p> <p>21 Century</p> <p>Critical thinking</p>
Rubric Values	4 - The target is demonstrated and organized justification leads to a precise proven conclusion.	3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.	2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.	1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.				

		Scoring Guide: See Appendix - Rubric & Unit 2 Assessment	
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Unit 2: Resources

UNIT RESOURCES

Teacher Resources:

This may include:

- *Textbook*
- [Applied Discrete Structures](#)
- [The Discrete Math Hub](#)

Student Resources:

This may include:

- *Textbook*
- *Canvas*
- *Teacher created material*

Unit 3: Number Theory

Content Area: Mathematics	Course: Discrete Mathematics	UNIT: Number Theory
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Unit Description: This unit will cover an introduction to number theory and their properties. This will include the Fibonacci sequence, cryptography, and a deeper understanding of basic mathematical rules (i.e., prime numbers, modular arithmetic, factorization) and how they are applied.	Unit Timeline: 3 weeks
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DESIRED Results

Transfer Goal - *Students will be able to independently use their learning to...*

Work more efficiently with a better understanding of real number algebra to solve quantitative problems.

Understandings – *Students will understand that... (Big Ideas)*

1. The fundamental theorem of arithmetic is used to express a composite number into a unique product of prime numbers.
2. Each number system has a defined set of properties.
3. Different types of sequences can be used to explain various real life phenomena.
4. There is a relationship between the Fibonacci numbers and the golden ratio.
5. Encryption requires a key and cipher.

Essential Questions: *Students will keep considering...*

1. How well can sequences model real life problems?
2. What are the other connections to the Fibonacci sequence other than the golden ratio?
3. Why are prime numbers important in number theory and cryptography?
4. How can a code keep information private, but also allow the information to be readable by intended recipients of the information?

Students will know/understand ...	Students will be able to ...
<ul style="list-style-type: none"> ● Students will know the different properties of each set of numbers. ● Students will know the fundamental theorem of arithmetic. ● Students will know the divisibility rules. ● Students will know what operations or patterns define each sequence. ● Students will know how to apply the Modular Function to a given number. ● Students will know how to develop a key and cipher. <p>Essential Vocabulary:</p> <ul style="list-style-type: none"> ● Mersenne primes - A prime number that is 1 less than a power of 2. ● Fermat number - A number in the form of $2^{2^n} + 1$ ● Fibonacci sequence - A sequence in which each number is the sum of the previous 2 numbers ● Golden ratio - A ratio of two numbers in which the ratio of the sum to the larger number is the same as the ratio of the larger number to the smaller ● Cryptography - the enciphering and deciphering of messages in secret code or cipher ● Decrypting - To convert coded messages into intelligible form. ● Recursion formula - an equation relating the value of a function for a given value of its argument (or arguments) to its values for other values of the argument(s). ● Primorial primes - In mathematics, a primorial prime is a prime number of the form $p_n \# \pm 1$, where $p_n \#$ is the primorial of p_n (i.e. the product of the first primes) 	<ul style="list-style-type: none"> ● Students will determine which set(s) a number belongs to. ● Students will be able to determine all divisibility rules that apply to a given number. ● Students will be able to use modular arithmetic. ● Students will be able to encrypt and decrypt a message given a key and cipher.

Unit 3: Assessment

EVIDENCE of LEARNING

<u>Understanding</u>	<u>Standards</u>	<u>Unit Performance Assessment:</u>	<u>R/R Quadrant</u>					
1, 3, 4, 5	1, 2, 3, 4	<p>Description of Assessment Performance Task(s): Students will determine if statements about natural and prime numbers are true or false. Any statements claimed false must be supported with evidence. Students will apply the 2, 3, 4, 5, 6, 8, 9, 10, and 12 divisibility rules on a large number. Students must show or explain how they applied each rule. Students will determine if a number is prime or not and explain. They will then apply that thinking to determine if the number 1 is prime. They will restate the fundamental theorem of arithmetic. Students will determine which phrase is false, all pertaining to perfect numbers. Students will apply and explain the Fibonacci sequence, as well as its connection to the golden ratio. Students will describe and decrypt a Caesar cipher.</p> <p>Teacher will assess:</p> <p><i>The assessment will be graded using a 4 point scale on each individual question unless otherwise stated.</i></p> <table border="1" data-bbox="640 1008 1551 1300"> <thead> <tr> <th data-bbox="640 1008 764 1300">Rubric Values</th> <th data-bbox="770 1008 951 1300">4 - The target is demonstrated and organized justification leads to a precise proven conclusion.</th> <th data-bbox="957 1008 1138 1300">3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.</th> <th data-bbox="1144 1008 1325 1300">2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.</th> <th data-bbox="1331 1008 1551 1300">1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.</th> </tr> </thead> </table> <p>Performance: Mastery: <i>Students will show that they really understand when they... score a 75% or higher on the assessment.</i></p>	Rubric Values	4 - The target is demonstrated and organized justification leads to a precise proven conclusion.	3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.	2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.	1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.	<p><u>21 Century</u></p> <p>A, B, C</p> <p>Critical Thinking Communication</p>
Rubric Values	4 - The target is demonstrated and organized justification leads to a precise proven conclusion.	3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.	2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.	1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.				

		Scoring Guide: See Appendix - Rubric & Unit 3 Assessment	
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Unit 3: Resources

UNIT RESOURCES

Teacher Resources:

This may include:

- *Textbook*
- [Discrete Mathematics: An Open Introduction](#)
- [Applied Discrete Structures](#)
- [The Discrete Math Hub](#)

Student Resources:

This may include:

- *Textbook*
- *Canvas*
- *Teacher created material*

Unit 4: Graph Theory

Content Area: Mathematics	Course: Discrete Mathematics	UNIT: Graph Theory
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Unit Description: This unit will cover making connections between various locations or objects. This unit can be used to help determine an efficient solution to problems. This includes Euler paths, Euler circuits, Hamilton paths, trees, PERT (Program Evaluation and Review Technique) diagrams, and conflict graphs.	Unit Timeline: 3 weeks
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DESIRED Results

Transfer Goal - *Students will be able to independently use their learning to...*

Create schedules that involve multiple parameters; create routes that are the most efficient.

Understandings – *Students will understand that... (Big Ideas)*

1. Multiple situations can be represented as graphs.
2. Paths, circuits, and bridges show connections between objects.
3. Trees are used to create, sort, and search to find all possible outcomes in order to optimize and organize.

Essential Questions: *Students will keep considering...*

1. What is the most efficient path/solution?
2. How can a given situation be represented as a graph or tree?
3. What is the best method to create an optimal path/outcome in a given situation?

Students will know/understand ...	Students will be able to ...
<ul style="list-style-type: none"> ● Students will understand paths, circuits and bridges. ● Students will understand Euler paths and Euler circuits. ● Students will understand that a given graph/tree has a most efficient path. ● Students will understand the traveling salesman problem can be represented by complete graphs. ● Students will know trees are used to represent a wide variety of real-life problems. <p>Essential Vocabulary:</p> <ul style="list-style-type: none"> ● Euler paths - a trail in a finite graph that visits every edge exactly once. ● Euler circuits - Eulerian trail that starts and ends on the same vertex ● Hamilton paths - a graph path between two vertices of a graph that visits each vertex exactly once. ● Hamilton circuits - A Hamilton path that starts and ends at the same vertex, after passing through each vertex exactly once. ● Trees - an undirected graph in which any two vertices are connected by exactly one path ● Spanning trees - a sub-graph of an undirected connected graph, which includes all the vertices of the graph with a minimum possible number of edges. ● Minimum-cost spanning trees - the spanning tree where the cost is minimum among all the spanning trees. ● Brute force method - Calculating the weight of every circuit and choosing the smallest weight ● Nearest-neighbor method - Choose the smallest weight at each vertex, without repeating vertices. ● PERT diagrams - a visual project management tool used to map out and track the tasks and timelines. The name PERT is 	<ul style="list-style-type: none"> ● Students will be able to represent problems using graphs. ● Students will solve problems involving Euler paths and Euler circuits. ● Students will use trees to represent real-life problems to organize conflict and manage projects. ● Students will solve traveling salesman problems using brute force method and nearest-neighbor method. ● Students will be able to solve problems involving spanning trees. ● Students will be able to find the most efficient path on a graph/tree.

<p>an acronym for Project (or Program) Evaluation and Review Technique.</p> <ul style="list-style-type: none"> Conflict graphs - A representation of conflicts using graph coloring. 	
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Unit 4: Assessment

EVIDENCE of LEARNING

<p><u>Understanding</u></p> <p>2, 3</p>	<p><u>Standards</u></p> <p>1, 2, 4</p>	<p><u>Unit Performance Assessment:</u></p> <p>Description of Assessment Performance Task(s): Students will identify graphs as Euler or Hamilton and create graphs based on certain situations. Questions will be similar to the following: Determine if the following has an Euler circuit, Euler path, both, or neither. State the circuit or path and explain your reasoning. Determine if the graph is complete, connected, both, or neither. Explain. Behavior problems restrict the following students from sitting at the same table. Create a conflict graph and color it to determine the number of tables needed to seat the following students with no conflicts.</p> <p>Teacher will assess:</p> <p><i>The assessment will be graded using a 4 point scale on each individual question unless otherwise stated.</i></p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td>Rubric Values</td> <td>4 - The target is demonstrated and organized justification leads to a precise proven conclusion.</td> <td>3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.</td> <td>2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.</td> <td>1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.</td> </tr> </table>	Rubric Values	4 - The target is demonstrated and organized justification leads to a precise proven conclusion.	3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.	2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.	1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.	<p><u>R/R Quadrant</u></p> <p>A, B</p> <p><u>21 Century</u></p> <p>Creativity Collaboration Critical Thinking</p>
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		<p>Performance:</p> <p>Mastery: <i>Students will show that they really understand when they... score a 75% or higher on the assessment.</i></p> <p>Scoring Guide: See Appendix - Rubric & Unit 4 Assessment</p>	
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Unit 4: Resources

UNIT RESOURCES

Teacher Resources:

This may include:

- *Textbook*
- [Discrete Mathematics: An Open Introduction](#)
- [Applied Discrete Structures](#)
- [The Discrete Math Hub](#)

Student Resources:

This may include:

- *Textbook*
- *Canvas*
- *Teacher created material*

Unit 5: Matrices and Game Theory

Content Area: Mathematics	Course: Discrete Mathematics	UNIT: Matrices and Game Theory
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Unit Description: This unit will review/introduce concepts of matrix arithmetic (adding, multiplying, etc.) and relate these concepts to Markov chains and game theory.	Unit Timeline: 3 weeks
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DESIRED Results

Transfer Goal - *Students will be able to independently use their learning to...*

Look at competing opponents' strategies and outcomes and determine their own optimum strategy.

Understandings – *Students will understand that... (Big Ideas)*

1. Matrices can be used to compare different strategies of two opponents.
2. Elimination of dominant strategies can be used to determine an optimal strategy.
3. When a game has Nash equilibrium there are certain strategies being used to help identify it.
4. A Markov chain is a process that arises naturally in problems.

Essential Questions: *Students will keep considering...*

1. What is the dominating strategy in a given situation? (Is there one?)
2. How to pick the best strategy based on the opponent's likely choices?
3. How can a student change the rules of a "game" to benefit themselves?
4. How does compromise result in the best outcome for both opponents?

Students will know/understand ...	Students will be able to ...
<ul style="list-style-type: none"> ● Students will know how to look at a game matrix to determine best strategies. ● Students will know to look at opponent outcomes to predict their strategies. ● Students will know the Leontief model is a model for the economics of a whole country or region. ● Students will understand the order of a Markov chain refers to how many previous states are used to estimate a new state. ● Students will understand what it means for a game to be fair. ● Students will understand some strategies are more effective than others. <p>Essential Vocabulary</p> <ul style="list-style-type: none"> ● Strategy - A complete plan of action a player will take given the set of circumstances that might arise within the game. ● Game - Any set of circumstances that has a result dependent on the actions of two or more decision-makers (players). ● Game matrix - A matrix of number pairs, indicating the payoffs for the 2 players given the chosen strategies. ● Dominant game strategy - a situation where one player has superior tactics regardless of how their opponent may play. ● Nash Equilibrium - a concept where the optimal outcome of a game is where there is no incentive to deviate from the initial strategy. 	<ul style="list-style-type: none"> ● Students will be able to add, subtract, and multiply matrices. ● Students will be able to use matrices to solve a system of equations. ● Students will be able to apply the Leontief model to predict the output, given a predicted input. ● Students will be able to predict the outcome of a situation using a Markov chain. ● Students will be able to determine if a “game” has a dominating strategy. ● Students will be able to determine if a “game” is fair.

Unit 5: Assessment

EVIDENCE of LEARNING

<u>Understanding</u>	<u>Standards</u>	<u>Unit Performance Assessment:</u>	<u>R/R Quadrant</u> <u>21 Century</u>										
1, 2, 3, 4	1, 2, 3, 4	<p>Description of Assessment Performance Task(s): Students will be able to understand what strategies work best for certain games using matrices. Questions will be similar to the following: Taking into account Player 2's strategy choices, which strategy should Player 1 choose? Create a scenario where a dominant strategy equilibrium exists, and state the scenario. Describe the order in which strategies will be eliminated using IESDS (iterated elimination of strictly dominated strategies) and what the outcome will be for each player.</p> <p>Teacher will assess:</p> <p><i>The assessment will be graded using a 4 point scale on each individual question unless otherwise stated.</i></p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center;">Rubric Values</th> <th style="text-align: center;">4 - The target is demonstrated and organized justification leads to a precise proven conclusion.</th> <th style="text-align: center;">3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.</th> <th style="text-align: center;">2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.</th> <th style="text-align: center;">1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.</th> </tr> </thead> <tbody> <tr> <td style="height: 150px;"></td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>Performance: Mastery: <i>Students will show that they really understand when they... score a 75% or higher on the assessment.</i></p> <p>Scoring Guide: See Appendix - Rubric & Unit 5 Assessment</p>	Rubric Values	4 - The target is demonstrated and organized justification leads to a precise proven conclusion.	3 - The solution is correct or contains minor mathematical errors with detailed and clear mathematical justification.	2 - Some understanding of the target is demonstrated, but justification may not be detailed, clear, complete, or precise.	1 - A very basic understanding of the process to arrive at a solution, but the work is incomplete or has significant errors.						<p>A, C</p> <p>Critical Thinking Creativity</p>
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