

Accelerated Math 7

Middle School



Unit #A

Proportional Reasoning: Percents & Statistical Samples

Essential Question

How can proportional reasoning be used to solve problems about percents and statistical sampling?

Unit Summary

In this unit, students will use proportions to solve various percent problems. Students will use data from a relevant sample to gain information and make inferences about a larger population. Students will apply the appropriate measures of center and variability to make comparisons between sets of data.

Guiding Questions

Content

- How can proportions be used to solve multi-step ratio and percent problems?
- How are statistics used to gain information about a population?
- What determines whether a sample is an accurate representation of a population?
- How can data from a sample be used to generate inferences about a population?

Process

- How can multiple samples reveal the accuracy of a prediction or estimation?
- How are measures of center and variability used to make inferences and comparisons between two sets of data?

Reflective

- What coupon would you rather have, 20% off or \$20 off?
- What is a situation where it would be more appropriate to use the median instead of the mean? Why?
- If the average height of one sports team at your school is 10 cm taller than another team, what can you infer about the two teams?

Power Standards

7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Supporting Standards

7.SP.1. Use statistics to gain information about a population by examining a sample of the population;

- **7.SP.1a.** Know that generalizations about a population from a sample are valid only if the sample is representative of that population and generate a valid representative sample of a population.
- **7.SP.1b.** Identify if a particular random sample would be representative of a population and justify your reasoning.

7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to informally gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability (requires introduction of mean absolute deviation). *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.4. Use measures of center (mean, median and/or mode) and measures of variability (range, interquartile range and/or mean absolute deviation) for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a*

seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (NOTE: Students should not have to calculate mean absolute deviation but use it to interpret data).

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Unit #B

Probability

Essential Question

How can properties of probability be used to make educated decisions in real world situations?

Unit Summary

Students will collect, organize and display data to find the probability of events and determine which outcomes are more likely to occur. Students understand the difference between experimental and theoretical probability and when to use each one. Students can conduct a probability experiment and analyze the results through a probability model to make educated decisions.

Guiding Questions

Content

- How can probability be expressed as a number between 0 and 1?
- What happens to the experimental probability as more trials are conducted?
- How can creating probability models be helpful to compare results with expected outcomes?
- What factors cause differences between experimental and theoretical probabilities?
- How can probability models (uniform and non-uniform) be used to determine the likelihood of an event?

Process

- How do lists, tables, tree diagrams, and simulations help determine the probability of compound events?
- How can a model (list, table, tree diagram) be used to represent the sample space of compound events and express the probability as a fraction?

- How can a simulation be designed and used to generate frequencies for compound events?

Reflective

- When rolling two dice, what is the probability that the sum of the two numbers is 8?
- What are three ways to express the probability of rolling an odd number on a six-sided die?
- How can you explain to a friend why theoretical and experimental probabilities can differ?

Power Standards

None

Supporting Standards

7.SP.5. Express the probability of a chance event as a number between 0 and 1 that represents the likelihood of the event occurring. (Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.)

7.SP.6. Collect data from a chance process (probability experiment). Approximate the probability by observing its long-run relative frequency. Recognize that as the number of trials increases, the experimental probability approaches the theoretical probability. Conversely, predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- **7.SP.7a.** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

- **7.SP.7b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- **7.SP.8a.** Know that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.8b.** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g. “rolling double sixes”), identify the outcomes in the sample space which compose the event.
- **7.SP.8c.** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

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Unit #C

Real Numbers: Rational, Irrational, & Pythagorean Theorem

Essential Question

How can the Pythagorean Theorem be used to solve problems?

Unit Summary

In this unit, students will build a visual understanding of irrational numbers. Students will investigate and apply the Pythagorean Theorem to solve real-world and mathematical problems.

Guiding Questions

Content

- What is the difference between rational and irrational numbers?
- How does a number line help approximate and compare irrational numbers?
- How does understanding inverse operations help solve equations containing square or cube numbers?
- How can the square root of a number be classified as rational or irrational?

Process

- What is the relationship between the area and the side length of a square?
- How is the converse of the Pythagorean Theorem used to show a triangle is right?
- How does applying the Pythagorean Theorem help find unknown lengths in right triangles and problems in two and three dimensions?
- How can the Pythagorean Theorem be used to find the distance between two points in a coordinate system?

Reflective

- How can I find the area of any square by sub-dividing it or surrounding it with a larger square?
- What strategies did you use to find the area and side length of a square with an irrational length?
- How would you estimate the value of a square root, such as $\sqrt{60}$?
- How could you find the length of any line segment on a grid?

Power Standards

8.G.8. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. *For example: Finding the slant height of pyramids and cones.*

Supporting Standards

8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.

- π^2). *For example, for the approximation of $\sqrt{68}$, show that $\sqrt{68}$ is between 8 and 9 and closer to 8.*

8.EE.1. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of whole number perfect squares with solutions between 0 and 15 and cube roots of whole number perfect cubes with solutions between 0 and 5. Know that $\sqrt{2}$ is irrational.

8.G.7. Explain a proof of the Pythagorean Theorem and its converse.

8.G.9. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

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Unit #D

3D Geometry

Essential Question

How can understanding the relationships between two-dimensional and three-dimensional shapes help us solve real-world problems involving area, surface area, and volume?

Unit Summary

In this unit, students explore how two-dimensional and three-dimensional shapes are connected through slicing, measurement, and real-world applications. They learn to calculate area, surface area, volume, arc length, and sector area while comparing and generalizing formulas for prisms, cylinders, pyramids, cones, and spheres. By the end of the unit, students use these relationships to solve problems and explain their mathematical thinking.

Guiding Questions

Content

- If you slice a cylinder straight across (parallel to the base), what 2D shape is formed? How would the shape change if the slice is at an angle?
- A cone and a cylinder have the same base and height. How does the volume of the cone compare to the volume of the cylinder?

Process

- How can area, surface area, and volume be applied to solve real-world problems?

Reflective

- When solving real-world problems involving surface area or volume, how do you decide which formula to use? What clues help you?
- How would you explain how to find the volume of a prism or cylinder to a friend?

Power Standards

None

Supporting Standards

7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right cylinder.

7.G.6. Solve real-world and mathematical problems involving area of two-dimensional objects and volume and surface area of three-dimensional objects including cylinders and right prisms. (Solutions should **not** require students to take square roots or cube roots. *For example, given the volume of a cylinder and the area of the base, students would identify the height.*)

8.G.10 Use the formulas or informal reasoning to find the arc length, areas of sectors, surface areas and volumes of pyramids, cones, and spheres. For example, given a circle with a 60° central angle, students identify the arc length as $\frac{1}{6}$ of the $\left(\frac{1}{6} = \frac{60}{360}\right)$ total circumference

8.G.11 Investigate the relationship between the formulas of three dimensional geometric shapes;

- 8.G.11a. Generalize the volume formula for pyramids and cones ($V = 1/3Bh$).
- 8.G.11b Generalize surface area formula of pyramids and cones ($SA = B + 1/2Pl$).

8.G.12 Solve real-world and mathematical problems involving arc length, area of two-dimensional shapes including sectors, volume and surface area of three-dimensional objects including pyramids, cones and spheres.

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Unit #E

Angle Relationships

Essential Question

How can angle relationships be used to solve mathematical problems?

Unit Summary

Students explore and understand concepts of angles and angle measurement. Students investigate and informally prove the relationship between angles that are created when parallel lines are cut by a transversal. Students examine the relationships between exterior and interior angles of a triangle and construct triangles given a variety of conditions.

Guiding Questions

Content

- What is the relationship between the number of degrees in an angle and a circle measuring 360 degrees?
- How can we classify angles that are created when two rays share a common endpoint based on the degree measurement?
- How can an angle be measured and constructed using a protractor?
- How can algebraic reasoning and the angle properties be used to solve for unknown angle measures in a variety of problems?

Process

- How can the properties of complementary, supplementary, adjacent and vertical angles be used to solve problems?
- What is the relationship between angle sums and exterior angle sums of triangles?
- How can angles be used to prove triangles are similar?
- How can you use angle measures and side lengths to create multiple and unique triangles, or determine that no triangle is possible?

Reflective

- How can I use the relationships between parallel lines cut by a transversal to find multiple missing angle measures in a diagram?
- How can I construct triangles using given angle measures and/or side lengths?
- How do I use algebra to create and solve equations to find unknown angle measures in a problem?

Power Standards

8.G.1. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

- 8.G.1a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles.
- 8.G.1b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

8.G.4. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure.

8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Supporting Standards

8.G.2. Measure angles in whole-number degrees using a protractor. Draw angles of specified measure using a protractor and straight edge.

8.G.3. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, *e.g. by using an equation with a symbol for the unknown angle measure.*

8.G.6. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on drawing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

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Unit #F

Linear Relationship: Slope, Linear Equations, & Systems

Essential Question

How can algebraic equations and inequalities be used to model, analyze, and solve real-world situations?

Unit Summary

In this unit, students will solve equations and inequalities in one variable. Students will investigate and identify equations with a variety of solutions. Students will write equations and inequalities to model real world situations. Students can distinguish between proportional and non-proportional relationships.

Guiding Questions

Content

- How are properties used to solve one-step, two-step, and multi-step equations and inequalities, including those with rational coefficients?
- What is the difference between the solution(s) of an equation and an inequality?
- What method can be used to prove a solution(s) to an equation or inequality is true?
- What does it mean if an equation has one, no, or infinite solutions?
- How is the distributive property and collecting like terms used to solve linear equations and inequalities?

Process

- How can similar triangles be used to prove the slope is the same between any two points on a line?
- How do I find the slope between any two points?
- What are the differences between proportional and nonproportional linear relationships?

Reflective

- Using the equation $3(2x + 5) = ax + b$, how can you find a value for a and a value for b so that there are infinitely many, one, or no solution for the value of x that make the equation true?
- How would I tell a friend how to solve $\frac{1}{3}x - 5 + 171 = x$?
- How would I explain the differences between various forms of proportional and non-proportional relationships?

Power Standards

8.EE.4. Graph proportional relationships, interpreting its unit rate as the slope (m) of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

8.EE.5. Use similar triangles to explain why the slope (m) is the same between any two distinct points on a non-vertical line in the coordinate plane and extend to include the use of the slope formula ($m = \frac{y_2 - y_1}{x_2 - x_1}$ when given two coordinate points (x_1, y_1) and (x_2, y_2)). Generate the equation $y = mx$ for a line through the origin (proportional) and the equation $y = mx + b$ for a line with slope m intercepting the vertical axis at y -intercept b (not proportional when $b \neq 0$).

8.EE.7. Fluently (efficiently, accurately, and flexibly) solve one-step, two-step, and multi-step linear equations and inequalities in one variable, including situations with the same variable appearing on both sides of the equal sign.

- **8.EE.7a.** Give examples of linear equations in one variable with one solution ($x = a$), infinitely many solutions ($a = a$), or no solutions ($a = b$). Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- **8.EE.7b.** Solve linear equations and inequalities with rational number coefficients, including equations/inequalities whose solutions require expanding and/or factoring expressions using the distributive property and

collecting like terms.

Supporting Standards

None



Unit #G

Functions: Linear & Nonlinear Relationships

Essential Question

How can functions be used to describe real-world situations, model predictions, and solve problems?

Unit Summary

Students will discover the meaning of a function and investigate how it can be presented in various forms, including graphs, tables, equations and visual patterns. Students will construct and analyze functions that model linear relationships and distinguish how they are different from non-linear functions.

Guiding Questions

Content

- When is the relationship between two quantities considered a function?
- How can linear functions be represented and compared in various forms?
- What are examples of linear and nonlinear functions?
- How do you determine and interpret the rate of change and initial value in order to construct various forms of linear functions?

Process

- What can you infer about a situation given a sketch of a graph?
- How can proportional relationships in different forms (graphs, equations, tables) be compared?

Reflective

- How can I translate among representations of functions and describe how aspects of functions are reflected in different representations?
- How would I explain to a friend how to derive an equation for a line?

Power Standards

8.F.2 Compare properties of two linear functions represented in a variety of ways

(algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change, the greater y -intercept, or the point of intersection.*

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Supporting Standards

8.F.1 Explain that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *(Function notation is not required in Grade 8.)*

8.F.3. Interpret the equation

$y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1)$, $(2,4)$ and $(3,9)$, which are not on a straight line.*

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (*e.g. where the function is increasing or decreasing, linear or nonlinear*). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

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Unit #H

Linear Models: What's in the Data?

Essential Question

How does investigating patterns in data help us solve problems?

Unit Summary

Students will investigate patterns of association in bivariate data. Students will use linear models to solve problems.

Guiding Questions

Content

- What kind of patterns of associations can be found in bivariate data represented as a scatter plot?
- How is a straight line used to model relationships between two quantities?

Process

- How is a linear equation used to make predictions about bivariate data and solve problems?
- How can the slope and y-intercept be interpreted in the context of a real world problem?

Reflective

- Why is a scatter plot a good representation of bivariate data?
- How do outliers and/or clustering influence data analysis?
- How do scatter plots and lines of best fit enable you to make predictions about data?
- What types of patterns did you notice when investigating a variety of scatter plots of data?

Power Standards

None

Supporting Standards

8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

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Unit #1

Integer Exponents: Scientific Notation

Essential Question

How can scientific notation help us solve problems about our world?

Unit Summary

Students use scientific notation to represent very large and small quantities and solve problems.

Guiding Questions

Content

- How can very large and very small quantities be represented with standard notation?
- How can numbers written in scientific notation be compared?

Process

- How is standard form converted to scientific notation and scientific notation converted to standard form?

Reflective

- How do you choose an appropriate unit for measurements using scientific notation?
- What quantities in the real world are best represented with scientific notation?

Power Standards

None

Supporting Standards

8.EE.2. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the

world population is more than 20 times larger.

8.EE.3. Read and write numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.