Calculus: Derivatives

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to estimate and calculate limits in order to describe continuity, derivatives and integrals.***
- ***Students will demonstrate the ability to calculate derivatives geometrically, numerically and analytically in order to describe the rate of change of functions.***
- ***Students will demonstrate the ability to analyze and summarize text and integrate knowledge to make meaning of discipline-specific materials.***
- ***Students will demonstrate the ability to produce coherent and supported writing in order to communicate effectively for a range of discipline-specific tasks, purposes, and audiences.***
- ***Students will demonstrate the ability to speak purposefully and effectively by strategically making decisions about content, language use, and discourse style.***

Content Standards:

- A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- HSF.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified

Transfer

Students will be able to independently use their learning to find maximum and minimum values to determine the solutions to a problem.

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- the derivative is one of the two central concepts of calculus.
- the derivative is a key element connecting the concepts of position, velocity, and acceleration in physics.

ESSENTIAL QUESTIONS

- Why is the derivative important?

Acquisition

Students will know…

- *the common notations for the derivative of a functions*
- *that the derivative at a point in graphical terms can be thought of as slope.*
- *that right-handed and left-handed derivatives may be defined at any point of a function's domain.*
- *that the usual relationship between one-sided and two-sided limits holds for derivatives.*
- *that $f^{-1}(a)$ might fail to exist.*
- *that it is useful to think of differentiable functions as locally linear.*
- *that differentiability implies continuity.*
- *the Intermediate Value Theorem for Derivatives.*
- *the derivative of a constant function is 0.*
- *the Power Rule for Positive Integer*

Students will be skilled at…

- *applying the definition of derivative.*
- *applying the definition of derivative at a point.*
- *sketching the graph of f^{-1} by estimating the slopes at various points along the graph of f.*
- *sketching the graph of f by analyzing the graph of f^{-1}.*
- *graphing the derivative from data.*
- *Comparing right- and left-handed derivatives to show that the function is not differentiable at a point.*
- *using a calculator to approximate derivatives.*
- *identifying why functions fail to be differentiable at a point.*
- *finding higher order derivatives.*
- *finding instantaneous rate of change.*

amplitude, frequency, and midline. • MP1 Make sense of problems and persevere in solving them. • MP2 Reason abstractly and quantitatively. • MP4 Model with mathematics. • MP5 Use appropriate tools strategically. • MP7 Look for and make use of structure. • MP8 Look for and express regularity in repeated reasoning.	*Powers of x.* • *the Constant Multiple Rule.* • *the Sum and Difference Rule.* • *the Product Rule.* • *the Quotient Rule.* • *the Power Rule for Negative Integer Powers of x.* • *that derivatives can be used as functions to represent the rates at which things change in the world around us.* • *the Chain Rule.* • *the Power Chain Rule.* • *that implicit differentiation can be used to find derivatives of higher order.* • *the Power Rule for Rational Powers of x.* • *that if f is differentiable at every point of an interval I and $\frac{df}{dx}$ is never zero on I, then f has an inverse and f^{-1} is differentiable at every point of the interval $f(I)$.* • *that the relationship between the graph of a function and its inverse allows us to see the relationship between their derivatives.* • *the Inverse Function-Inverse Cofunction Identities.* • *the Calculator Conversion Identities .* • *the Power Rule for Arbitrary Real Powers .* • *the Extreme Value Theorem.* • *that the Mean Value Theorem for Derivatives is an important theoretical tool to connect the average and instantaneous rates of change.* • *that functions whose first derivative is zero are constant.* • *that functions with the same derivative differ by a constant.* • *the procedure for Newton's Method*	● *modeling with differentiation.* ● *using a graphing calculator to see motion.* ● *finding the derivatives of basic trigonometric functions.* ● *finding equations for lines that are tangent and normal to a curve.* ● *differentiating with a parameter.* ● *differentiating implicitly* ● *applying the differentiation process* ● *differentiating a variety of functions (algebraic, trigonometric, logarithmic) by using the rules of differentiation.* ● *determining extreme values* ● *finding functions who have a certain derivative.* ● *using the First and Second Derivative Tests to find the local extrema of a function.* ● *using the Concavity Test to determine the concavity of a given function.* ● *using the first and second derivatives to graph a function.* ● *using strategies for solving max-min problems.* ● *finding a linearization of a function at a point.* ● *using Newton's Method to solve an equation.* ● *finding differentials of functions.* ● *estimating change with differentials.* ● *finding related rate equations.* ● *using strategies for solving related rate problems.*

	• *that differential dx is an independent variable.* • *that differential dy is always a dependent variable depending on both x and dx.* • *that related rate problems are at the heart of Newtonian mechanics; it was essentially to solve such problems that calculus was invented.* <u>vocabulary:</u> *derivative, differentiable function, differentiable on a closed interval, corner, cusp, vertical tangent, discontinuity, symmetric difference quotient. first derivative, nth derivative, instantaneous rate of change, displacement, average velocity, instantaneous velocity, speed, acceleration, marginal cost, marginal revenue, simple harmonic motion, jerk, implicit differentiation, antiderivative, concavity, point of inflection, linearization, locally linear*	

Content Area Literacy Standards	***21st Century Skills***
• RST.9-10.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. • RST.9-10.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9-10 texts and topics*. • RST.9-10.7 Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words. • WHST.11-12.2.D Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.	• *reason effectively* • *use systems thinking* • *solve problems* • *access and evaluate information* • *use and manage information* • *be self-directed learners*

Evaluative Criteria	***Assessment Evidence***

	PERFORMANCE TASK(S):
	OTHER EVIDENCE:

Summary of Key Learning Events and Instruction

Science Integration	***College, Career, and Civic Life Integration***	***Technology Integration***
District Materials	***Distance Learning/Field Trips***	***Technology Resources***