Calculus: Integrals

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to connect derivatives and integrals by interpreting an integral both as a limit of a Riemann sum and as an accumulator of change over an interval.***
- ***Students will demonstrate the ability to use a variety of anti-differentiation techniques in order to evaluate integrals and solve problems involving integrals.***
- ***Students will demonstrate the ability to estimate and calculate limits in order to describe continuity, derivatives and integrals.***
- ***Students will demonstrate the ability to calculate derivatives geometrically, numerically and analytically in order to describe the rate of change of functions.***
- ***Students will demonstrate the ability to analyze and summarize text and integrate knowledge to make meaning of discipline-specific materials.***
- ***Students will demonstrate the ability to produce coherent and supported writing in order to communicate effectively for a range of discipline-specific tasks, purposes, and audiences.***
- ***Students will demonstrate the ability to speak purposefully and effectively by strategically making decisions about content, language use, and discourse style.***

Content Standards:

- A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- HSF.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- MP1 Make sense of problems and persevere in solving them.

Transfer

Students will be able to independently use their learning to find solutions by breaking down an irregular whole into regular parts.

Meaning

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
Students will understand that… • Estimating with finite sums sets the foundation for understanding integral calculus. • Derivatives and anti-derivatives have an inverse relationship to each other.	• How do the rules of differentiation lead to an understanding of the rules to integration?

Acquisition

Students will know…	*Students will be skilled at…*
• *that many physical quantities can be estimated by defining a function on a closed interval and finding the sum of function values multiplied by interval lengths.* • *that as the partitions of [a,b] become finer, the rectangles defined by the partitions better approximate the region between the x-axis and the graph of f.* • *that the norm of the partition is required to tend to zero in order to have all Riemann sums of a given function on a given interval converge to a common value.* • *that all continuous functions are integrable.* • *that the value of the definite integral of a function over any particular interval*	• *applying the Rectangular Approximation Method.* • *applying integrals, for example finding the distance traveled when velocity varies.* • *interpreting and writing integral notation.* • *expressing the limit of a sum as definite integral.* • *graphing an integrand and area to evaluate an integral.* • *using NINT to evaluate an expression.* • *finding the average value of a function on an integral without integrating.* • *interpretting the integrand as the rate of change of a quantity.* • *evaluating an integral using the antiderivative.* • *finding total net area analytically and numerically.*

• MP2 Reason abstractly and quantitatively. • MP4 Model with mathematics. • MP5 Use appropriate tools strategically. • MP7 Look for and make use of structure. • MP8 Look for and express regularity in repeated reasoning.	*depends on the function and not on the letter we choose to represent its independent variable.* • *that the definite integral can be interpreted as the net area between the graph of a function and the x-axis.* • *that the integral of a constant function f(x)=c, where c is a constant, on the interval [a,b], is c(b-a).* • *that some functions with discontinuities are also integrable.* • *the rules for definite integrals: order of integration, zero, constant multiple, sum and difference, additivity, max-min inequality, domination.* • *the Mean Value Theorem for Definite Integrals.* • *the Fundamental Theorem of Calculus.* • *that every continuous function is the derivative of some other function.* • *that every continuous function has an antiderivative.* • *that the process of integration and differentiation are inverses of one another.* • *the Integral Evaluation Theorem.* • *that some definite integrals are best found by numerical approximations, and rectangles are not always the most efficient figures to use.* • *the Trapezoidal Rule.* • *Simpson's Rule.* • *Euler's Method.* • *the properties of indefinite integrals* • *the power, trigonometric, exponential and logarithmic formulas for indefinite integrals.* • *the integration by parts formula.* • *not every choice of u and dv leads to*	• *using LRAM, MRAM, RRAM, the Trapezoidal Rule, and Simpson's Rule to find approximations to the integral of a continuous function over a closed interval.* • *performing error analysis.* • *finding a general solution to a differential equation.* • *graphing a general solution.* • *finding the particular solution to a differential equation given an initial condition.* • *constructing slope fields.* • *using Euler's Method for graphing a solution to an initial value problem.* • *evaluating an indefinite integral.* • *using substitution to evaluate integrals.* • *using integration by parts to evaluate integrals.* • *using tabular integration* • *solving by separation of variables* • *using the technique of partial fractions.* • *applying definite integrals to solve real world problems.* • *determining the volume of a solid by the method of slicing.*

	success in antidifferentiation by parts, as there is always a trade-off when replacing $\int u\, dv$ with $\int v\, du$. • *the Law of Exponential Change* • *the principle of partial fraction decomposition with distinct linear denominators.* • *the logistic differential equation.* • *the general logistic formula.* *vocabulary: integral, Reimann Sums, partitian, subinterval,norm of a partition, integrable, definite integral, regular partitions, the definite integral of a continuous function on [a,b], area under a curve (as a definite integral), bounded functions, antiderivative, differential equation, order of a differential equation, slope field, indefinite integral, constant of integration, tabular integration, separable, area between curves*	

Content Area Literacy Standards	***21st Century Skills***
• RST.9-10.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. • RST.9-10.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics. • RST.9-10.7 Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words. • WHST.11-12.2.D Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.	• *reason effectively* • *use systems thinking* • *solve problems* • *access and evaluate information* • *use and manage information* • *be self-directed learners*

Evaluative Criteria	*Assessment Evidence*
	PERFORMANCE TASK(S):
	OTHER EVIDENCE:

Summary of Key Learning Events and Instruction		
Science Integration	***College, Career, and Civic Life Integration***	***Technology Integration***
District Materials	***Distance Learning/Field Trips***	***Technology Resources***