



## Algebra 2

<b>Grade(s):</b>	<input type="checkbox"/> K <input type="checkbox"/> 1 <input type="checkbox"/> 2 <input type="checkbox"/> 3 <input type="checkbox"/> 4 <input type="checkbox"/> 5	<input type="checkbox"/> 6 <input type="checkbox"/> 7 <input type="checkbox"/> 8	<input checked="" type="checkbox"/> 9 <input checked="" type="checkbox"/> 10 <input checked="" type="checkbox"/> 11 <input checked="" type="checkbox"/> 12 <input type="checkbox"/> Other _____
<b>Discipline/Course:</b>	<b>Discipline:</b> Mathematics <b>Course:</b> Algebra 2		
<b>Course Title:</b>	Algebra 2		
<b>Prerequisite(s):</b>	Successful completion of Geometry. With permission, students may take Algebra 2 concurrently with Geometry.		
<b>Course Description:</b> <i>Program of Studies</i>	Algebra 2 is an advanced study of functions that began in Algebra 1. Building on their work with linear and quadratic functions, students extend their repertoire of functions to include piecewise functions and parent exponential, polynomial, rational, and radical functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Throughout the course, students will explore patterns and determine how to best model functions using graphing technology. By the end of the course, students will be able to compare linear, quadratic, and exponential growth models.		
<b>Course Essential Questions:</b>	<ul style="list-style-type: none"> <li>● How is thinking algebraically different from thinking arithmetically?</li> <li>● How is mathematics used to measure, model, and calculate change?</li> <li>● How can we communicate mathematical ideas clearly and effectively?</li> <li>● How can we generalize relationships using algebraic equations and expressions from specific cases?</li> <li>● How do patterns and functions help us describe data and physical phenomena and solve a</li> </ul>		

	<p>variety of problems?</p> <ul style="list-style-type: none"> <li>• How can we use representations of functions graphically, numerically, symbolically, and verbally to quantify and compare situations, events, and phenomena?</li> <li>• How can mathematics be used to provide models of data and physical phenomena to help us describe, interpret, and make predictions?</li> <li>• How does the choice of method, tool, or representation affect the efficiency and reliability of problem-solving?</li> <li>• How do algebraic transformations inform how we use functions?</li> </ul>	
<b>Course Enduring Understandings:</b>	<ul style="list-style-type: none"> <li>• Algebraic reasoning allows us to represent, analyze, and explain patterns and relationships in real-world contexts, extending beyond basic arithmetic computation.</li> <li>• Change in real-world situations can be measured, represented, and predicted using algebraic expressions, equations, tables, graphs, and functions.</li> <li>• Algebra provides tools to move from particular examples to general rules by representing relationships with variables, expressions, and equations.</li> <li>• Identifying mathematical patterns reveals structure in data and natural phenomena, which can be modeled to make meaningful predictions and solve problems.</li> <li>• Multiple representations of functions (verbal, numerical, graphical, symbolic) how quantities change and allow us to compare and analyze relationships effectively.</li> <li>• Quantities change in predictable ways that can be modeled, analyzed, and interpreted using linear, quadratic, and exponential relationships.</li> <li>• While there are multiple ways to analyze or solve a problem, selecting appropriate representations, strategies, and tools leads to more efficient, accurate, and reliable conclusions.</li> </ul>	
<b>Duration: Credit:</b>	<input type="checkbox"/> Semester <input checked="" type="checkbox"/> Full-Year	<input type="checkbox"/> 0.5 Credit (s) <input checked="" type="checkbox"/> 1.0 Credit(s) <input type="checkbox"/> 1.5 Credit(s) <input type="checkbox"/> N/A
<b>Course Materials/Resources:</b>	enVision Algebra 2, Savvas Learning Company College Board Pre-AP Algebra II	

<b>FPS Course Academic Expectation(s):</b>	<input checked="" type="checkbox"/> Exploring and Understanding (EU) <input type="checkbox"/> Synthesizing and Evaluating (SE) <input type="checkbox"/> Creating and Constructing (CC) <input checked="" type="checkbox"/> Conveying Ideas (CI) <input type="checkbox"/> Collaborating Strategically (CS) <input type="checkbox"/> Using Communication Tools (UCT)
<b>Unit Overview</b>	<b>Unit 1:</b> Functions (~6 weeks) <b>Unit 2:</b> Quadratic Functions (~7 weeks) <b>Unit 3:</b> Polynomials (~6 weeks) <b>Unit 4:</b> Rational Functions (~6 weeks) <b>Unit 5:</b> Inverses and Radical Functions (~6 weeks) <b>Unit 6:</b> Exponential and Logarithmic Functions (~7 weeks)

<b>Unit Number and Title:</b>	<b>Unit 1 - Functions</b>
<b>Duration:</b>	~6 weeks
<b>Resource(s):</b>	enVision Algebra 2, College Board Pre-AP Algebra II
<b>Unit Overview:</b>	In this module, students expand their experience with functions to include more specialized functions—linear, and those that are piecewise-defined, including absolute value and step. They examine how a function can be transformed algebraically and how that is represented graphically. Students model phenomena using the modeling cycle. Students display and interpret graphical representations of data, and if appropriate, choose regression techniques when building a model that approximates a linear relationship between quantities. They analyze their knowledge of the context of a situation to justify their choice of a linear model. With linear models, they plot and analyze residuals to informally assess the goodness of fit.
<b>Learning Goals</b>	
<b>Standard(s):</b>	<b>Build a function that models a relationship between two quantities</b>
	<b>F-BF.1</b> Write a function that models a relationship between two quantities.
	a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
	<b>Build new functions from existing functions</b>
	<b>F-BF.3</b> Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

<b>Create equations that describe numbers or relationships</b>	
<b>A-CED.3</b>	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as variable and nonviable options in a modeling context.
<b>Understand the concept of a function and use function notation</b>	
<b>F-IF.1</b>	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
<b>F-IF.2</b>	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
<b>Interpret functions that arise in applications in terms of the context.</b>	
<b>F-IF.4</b>	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity..
<b>F-IF.5</b>	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function
<b>F-IF.6</b>	Calculate and interpret the average rate of change of a function (presented symbolically or as a table over a specified interval). Estimate the rate of change from a graph

<b>Analyze functions using different representations</b>	
<b>F-IF.7</b>	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
	a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
	b. Graph square root, cube root, and piecewise defined functions, including step functions and absolute value functions.
<b>Summarize, represent, and interpret data on two categorical and quantitative variables</b>	
<b>S-ID.6</b>	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
	a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use the given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</i>
	c. Fit a linear function for a scatter plot that suggests a linear association
<b>Interpret linear models</b>	
<b>S-ID.7</b>	Interpret the slope (rate of change) and intercept (constant term) of a linear model in the context of data.
<b>S-ID.8</b>	Compute (using technology) and interpret the correlation coefficient of a linear fit.
<b>S-ID.9</b>	Distinguish between correlation and causation.
<b>Create equations that describe numbers or relationships</b>	

	<table border="1"> <tr> <td data-bbox="569 250 730 391"><b>A-CED.1</b></td> <td data-bbox="730 250 1894 391">Create equations and inequalities in one variable and use them to solve equations. . <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></td> </tr> <tr> <td data-bbox="569 391 730 532"><b>A-CED.3</b></td> <td data-bbox="730 391 1894 532">Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as variable and nonviable options in a modeling context.</td> </tr> </table>	<b>A-CED.1</b>	Create equations and inequalities in one variable and use them to solve equations. . <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>	<b>A-CED.3</b>	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as variable and nonviable options in a modeling context.
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<b>A-CED.3</b>	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as variable and nonviable options in a modeling context.				
<b>Essential Question(s):</b>	<ul style="list-style-type: none"> <li>● How do algebraic transformations inform how we use functions?</li> <li>● How does the correlation coefficient help us evaluate the accuracy and usefulness of a regression model?</li> <li>● How can we use piecewise-defined functions to model a situation?</li> <li>● What are the limitations of models when making real-world decisions?</li> </ul>				
<b>Enduring Understanding(s):</b>	<ul style="list-style-type: none"> <li>● Regression, residuals, and the correlation coefficient are used to evaluate how well a model fits the data.</li> <li>● Graphing and writing non-linear functions such as step functions, absolute value, and piecewise functions, can model data that is composed of several functions, and when the conditions, such as the rate of change, vary over each interval of a domain.</li> <li>● Mathematical models are approximations, and their usefulness depends on how well they represent the given situation.</li> <li>● Understand that the transformations on a graph always have the same effect regardless of the type of the underlying function.</li> </ul>				
<b>. Learning Goal(s):</b> <i>Students will know and will be able to use their learning to:</i> (Content/ Skills)	<b>Content:</b> (Students will know:) <ul style="list-style-type: none"> <li>● Linear regression</li> <li>● Quantifying predictability             <ul style="list-style-type: none"> <li>○ correlation coefficient</li> <li>○ causation</li> </ul> </li> </ul>				

- extrapolation
- interpolation
- Graphs of piecewise functions
  - Graphs of absolute value functions
  - Graphs of step functions
  - Writing and graphs of piecewise linear functions
  - Sketching graphs from verbal descriptions
  - Key features of graphs
  - Domain and range of functions
- Transformation of functions
  - stretch, compress, shift
  - vertical
  - horizontal
  - reflection
- Representations of functions: algebraic, graphical, numeric, and verbal description

**Skills:** (Students will be able to...)

- identify key features of functions.
- make connections between a table, an algebraic representation, or a graph of a linear function both in context and not in context.
- calculate the line of best fit using linear regression technologies.
- analyze a line of best fit using the correlation coefficient.
- fit linear models to data represented in a scatterplot.
- transform a parent function.

<b>Unit Number and Title:</b>	<b>Unit 2 - Quadratic Functions</b>						
<b>Duration:</b>	~7 weeks						
<b>Resource(s):</b>	enVision Algebra 2 College Board Pre-AP Algebra II						
<b>Unit Overview:</b>	In Unit 2, students expand on their understanding of quadratic functions from Algebra I by examining the effect of transformations on the parent $y = x^2$ . Students will learn how to transform the standard form of a quadratic equation by completing the square. Students will learn about the complex number system and perform operations with complex numbers. Students will solve quadratics using the quadratic formula, extending their understanding of the discriminant to include imaginary roots. Students identify zeros of polynomials, including complex zeros of quadratic polynomials. Through regularity in repeated reasoning, they make connections between zeros of quadratic polynomials and solutions of quadratic polynomial equations. Students analyze the key features of a graph or table of a quadratic polynomial function and relate those features back to the two quantities in the problem that the function is modeling. They will solve quadratic-linear systems of equations algebraically, building on the graphical representations in Algebra I. Lastly, students will analyze their knowledge of the context of a situation to justify their choice of a quadratic model. With quadratic models, they will use technology to do quadratic regression.						
<b>Learning Goals</b>							
<b>Standard(s):</b>	<table border="1" style="width: 100%;"> <tr> <td colspan="2"><b>Interpret the structure of expressions</b></td> </tr> <tr> <td style="width: 20%;"><b>A-SSE.2</b></td> <td>Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i> (factoring perfect cubes, etc).</td> </tr> <tr> <td colspan="2"><b>Write expressions in equivalent forms to solve problems</b></td> </tr> </table>	<b>Interpret the structure of expressions</b>		<b>A-SSE.2</b>	Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i> (factoring perfect cubes, etc).	<b>Write expressions in equivalent forms to solve problems</b>	
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<b>Write expressions in equivalent forms to solve problems</b>							

	<b>A-SSE.3</b>	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
		a. Factor a quadratic expression to reveal the zeros of the function it defines.
		b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
	<b>Understand the relationship between zeros and factors of a polynomial</b>	
	<b>A-APR.3</b>	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
	<b>Create equations that describe numbers or relationships</b>	
	<b>A-CED.2</b>	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
	<b>Solve equations and inequalities in one variable</b>	
	<b>A-REI.4</b>	Solve quadratic equations in one variable.
		a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
		b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ .
	<b>Solve systems of equations</b>	

<b>A-REI.7</b>	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
<b>Represent and solve equations and inequalities graphically</b>	
<b>A-REI.11</b>	Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make a table of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
<b>Interpret functions that arise in applications in terms of context</b>	
<b>F-IF.4</b>	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>
<b>F-IF.7</b>	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
	a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
<b>F-IF.8</b>	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
	a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

<b>F-IF.9</b>	Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
<b>Building new functions from existing functions</b>	
<b>F-BF.3</b>	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
<b>Construct and compare linear and exponential models and solve problems</b>	
<b>F-LE.3</b>	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function
<b>Summarize, represent, and interpret data on two categorical and quantitative variables</b>	
<b>S-ID.6</b>	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
<b>Perform arithmetic operations with complex numbers</b>	
<b>N.CN.1</b>	Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ with $a$ and $b$ real.
<b>N.CN.2</b>	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
<b>Use complex numbers in polynomial identities and equations.</b>	

	<table border="1"> <tr> <td data-bbox="569 318 747 383"><b>N.CN.7</b></td> <td data-bbox="747 318 1898 383">Solve quadratic equations with real coefficients that have complex solutions.</td> </tr> </table>	<b>N.CN.7</b>	Solve quadratic equations with real coefficients that have complex solutions.
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<b>Essential Question(s):</b>	<ul style="list-style-type: none"> <li>• How are quadratic functions and equations used to model and optimize real-world situations (such as area and vertical motion)?</li> <li>• How does the structure of a quadratic equation determine the most efficient method for finding its solution(s)?</li> <li>• What do the solutions and intersections of quadratic functions represent in real-world and mathematical contexts?</li> <li>• How can we extend from real numbers to complex numbers?</li> <li>• How can we solve equations with complex and real roots?</li> </ul>		
<b>Enduring Understanding(s):</b>	<ul style="list-style-type: none"> <li>• Quadratic functions are transformations of the parent function <math>y = x^2</math>, and their graphs (parabolas) are defined by a vertex, axis of symmetry, direction, and width of opening.</li> <li>• The structure of a quadratic expression reveals important features of the function, including its zeros, maximum or minimum value, and symmetry.</li> <li>• Quadratic relationships are useful for modeling situations involving area, optimization, and parabolic motion.</li> <li>• Different forms of a quadratic function (standard, factored, and vertex) highlight different information and support different problem-solving strategies.</li> <li>• Quadratic equations can be solved using multiple strategies, and the most efficient method depends on the form of the equation and the context of the problem.</li> <li>• The discriminant provides information about the number and type of solutions a quadratic equation has.</li> </ul>		
<b>Learning Goal(s):</b> <i>Students will know and will be able to use their</i>	<b>Content:</b> (Students will know:) <ul style="list-style-type: none"> <li>• Vertex form, standard form, factored form</li> </ul>		

*learning to:*  
 (Content/ Skills)

- Transformations of quadratic functions
- Completing the square
- Quadratic formula
  - discriminant
  - number of real or imaginary roots
  - complex solutions
- Complex numbers & operations
- Quadratic-linear systems of equations
  - algebraically
  - graphically
- Modeling with quadratic functions
- Quadratic regression

**Skills:** (Students will be able to...)

- determine the conditions under which a quadratic equation has no real solutions, one real solution, or two real solutions.
- perform operations with complex numbers.
- use properties of transformations to write a quadratic function in vertex form and to identify key features of the graph.
- recognize complex solutions when using the quadratic formula in the form  $a + bi$ .
- fluently solve quadratic equations in one variable, written as a quadratic expression in standard form, where using the quadratic formula or completing the square is the most efficient method for solving the equation.
- create and use quadratic functions to solve problems in a variety of contexts.
- identify or create an appropriate quadratic function to model a relationship between quantities.
- for a quadratic function that represents a context:
  - interpret the meaning of an input/output pair, including an intercept or initial value, including situations where seeing structure provides an advantage.
  - interpret the meaning of a constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage

- for a quadratic function in a context:
  - interpret a point on the graph.
  - interpret parts of the graph (other than a point or intercept)
- make connections between a table, an algebraic representation, or a graph of a quadratic function, both in context and not in context.
- understand that for the graph of  $y = f(x)$ , the solutions to  $f(x) = 0$  correspond to x-intercepts of the graph and  $f(0)$  corresponds to the y-intercept of the graph; make connections between the input/output pairs and points on a graph; interpret this information in a context.
- use function notation to represent and interpret input/output pairs for quadratic functions, and find the input value for a corresponding output.
- distinguish between linear and exponential relationships in tables, graphs, and real-world scenarios, correctly identifying which model is appropriate (additive vs. multiplicative change).
- construct and interpret exponential growth and decay functions to model real-world situations (e.g., compound interest, population growth).
- given an equation or formula in two or more variables, view it as an equation in a single variable of interest where the other variables are parameters, and solve for the variable of interest.
- relate the solutions of a system of linear and exponential equations in two variables to the graphs of the equations in the system.
- make strategic use of algebraic structure, the properties of operations, and reasoning about equality to quadratic equations in one variable, presented in a wide variety of forms
- make strategic use of algebraic structure, the properties of operations, and reasoning about equality to solve systems of linear and quadratic equations in two variables.
- relate the solutions of a system of linear and quadratic equations in two variables to the graphs of the equations in the system.
- determine which model – linear or quadratic – best fits a set of data.

<b>Unit Number and Title:</b>	<b>Unit 3 - Polynomial Expressions</b>	
<b>Duration:</b>	~ 6 weeks	
<b>Resource(s):</b>	enVision Algebra 2 College Board Pre-AP Algebra II	
<b>Unit Overview:</b>	Unit 3 focuses on extending students' previous knowledge of polynomials. Students identify the key features of polynomial functions and interpret graphs of polynomial functions. They learn methods to add, subtract, and multiply polynomial expressions, use multiple theorems as tools to understand the roots of polynomial functions, and transform graphs from cubic or quartic parent functions.	
<b>Learning Goals</b>		
<b>Standard(s):</b>	<b>Interpret the structure of expressions</b>	
	<b>A-SSE.1</b>	Interpret expressions that represent a quantity in terms of its context.
		a. Interpret parts of an expression, such as terms, factors, and coefficients.
		b. Interpret complicated expressions by viewing one or more of their parts as a single entity
	<b>A-SSE.2</b>	Use the structure of an expression to identify ways to rewrite it. For example, see $(x^4 - y^4)$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .
	<b>Perform arithmetic operations on polynomials.</b>	

<b>A-APR.1</b>	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
<b>Understand the relationship between zeros and factors of polynomials.</b>	
<b>A-APR.2</b>	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .
<b>A-APR.3</b>	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
<b>Use polynomial identities to solve problems</b>	
<b>A-APR.4</b>	Prove polynomial identities and use them to describe numerical relationships.
<b>Rewrite rational expressions</b>	
<b>A-APR.7</b>	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
<b>Understand solving equations as a process of reasoning and explain the reasoning</b>	
<b>A-REI.1</b>	Explain each step in solving a simple equation as following from the equality of numbers assured at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
<b>A-REI.2</b>	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

<b>Interpret functions that arise in applications in terms of context</b>	
<b>F-IF.4</b>	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>
<b>Analyze the function using different representations</b>	
<b>F-IF.7</b>	Graph functions expressed symbolically and show key features of the graph [of a function], by hand in simple cases and using technology for more complicated cases.
	c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
<b>F-IF.9</b>	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
<b>Build a function that models a relationship between two quantities</b>	
<b>F-BF.1</b>	Write a function that models a relationship between two quantities.
	b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model</i>
<b>Building new functions from existing functions</b>	
<b>F-BF.3</b>	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the

	<p>graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p>
<b>Essential Question(s):</b>	<ul style="list-style-type: none"> <li>● In what ways are polynomials like integers, and how does this analogy help us understand the operations of addition, subtraction, and multiplication of polynomial expressions?</li> <li>● Why is identifying the structure of a polynomial (terms, coefficients, degrees, and number of terms) the essential first step in choosing an efficient strategy to simplify, multiply, or factor?</li> <li>● How does multiplying polynomials using the Distributive Property and area models help explain the structure of the resulting expression?</li> <li>● How does the process of factoring an expression directly reverse the multiplication process, and how does this enable us to solve related problems?</li> <li>● How can we use the Greatest Common Factor (GCF) to rewrite polynomials and reveal important features of the expression?</li> <li>● How does recognizing special patterns, such as a difference of squares and perfect-square trinomials, improve efficiency and accuracy when factoring?</li> <li>● How can polynomial expressions and factoring be used to model and solve real-world situations involving area, volume, and constraints?</li> <li>● What can an equation for a polynomial function tell about its graph, and what can a graph of a polynomial function tell about the solutions to the equation?</li> </ul>
<b>Enduring Understanding(s):</b>	<ul style="list-style-type: none"> <li>● Polynomials are closed under the operations of addition, subtraction, and multiplication, meaning the result of these operations is always another polynomial.</li> <li>● Polynomials are added and subtracted by combining like terms.</li> <li>● Polynomials are multiplied using the Distributive Property or visual tools like area models, and the degree of the product is the sum of the degrees of the factors.</li> </ul>

	<ul style="list-style-type: none"> <li>• The Greatest Common Factor (GCF) of a polynomial's terms is used to reverse the Distributive Property and write the polynomial in factored form.</li> <li>• You can determine the equation of a graph by first identifying its parent function and then by applying transformations to the parent function.</li> </ul>
<p><b>Learning Goal(s):</b>  <i>Students will know and will be able to use their learning to:</i>        (Content/ Skills)</p>	<p><b>Content:</b> (Students will know...)</p> <ul style="list-style-type: none"> <li>• Key features of polynomials</li> <li>• Graphs of polynomials of <math>y = ax^n</math> with <math>n \geq 3</math> <ul style="list-style-type: none"> <li>○ odd, even, neither</li> <li>○ symmetrical about the y-axis</li> <li>○ symmetrical about the origin</li> <li>○ transformations of parent functions of <math>y = ax^3</math> and <math>y = ax^4</math></li> </ul> </li> <li>• Factoring polynomials           <ul style="list-style-type: none"> <li>○ difference of squares</li> <li>○ GCF</li> <li>○ multivariable polynomials</li> </ul> </li> <li>• Operations with polynomials           <ul style="list-style-type: none"> <li>○ addition</li> <li>○ subtraction</li> <li>○ multiplication</li> <li>○ dividing - synthetic division</li> </ul> </li> <li>• Zeros of a polynomial</li> </ul> <p><b>Skills:</b> (Students will be able to...)</p> <ul style="list-style-type: none"> <li>• find the greatest common factor (GCF) of a polynomial's terms and factor the polynomial completely using the GCF.</li> <li>• factor binomials and trinomials using the special patterns for the difference of two squares and the perfect-square trinomial</li> <li>• interpret the terms, factors, and coefficients of a polynomial expression in a given context.</li> </ul>

- use polynomial multiplication and factoring to model and solve real-world problems involving area and volume.
- express polynomial functions in an equivalent form to reveal properties of the graph and/or the contextual scenario.

<b>Unit Number and Title:</b>	<b>Unit 4 - Rational Functions</b>										
<b>Duration:</b>	~6 weeks										
<b>Resource(s):</b>	enVision Algebra 2 College Board Pre-AP Algebra II										
<b>Unit Overview:</b>	Unit 4 focuses on extending students' previous knowledge of polynomial functions to rational functions. Students identify the equations of the asymptotes of transformations of the reciprocal function. Students use this information to determine how to shift the parent function horizontally and vertically. A theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students learn to rewrite rational expressions in equivalent forms. Students recognize common factors that help them when multiplying rational expressions. Students solve simple rational equations by multiplying by a common denominator. They determine if there are extraneous solutions. Students apply their learning to solving related rate problems.										
<b>Learning Goals</b>											
<b>Standard(s):</b>	<table border="1"> <tr> <td colspan="2"><b>Interpret the structure of expressions</b></td> </tr> <tr> <td><b>A-SSE.1</b></td> <td>Interpret expressions that represent a quantity in terms of its context.</td> </tr> <tr> <td></td> <td>c. Interpret parts of an expression, such as terms, factors, and coefficients.</td> </tr> <tr> <td></td> <td>d. Interpret complicated expressions by viewing one or more of their parts as a single entity</td> </tr> <tr> <td><b>A-SSE.2</b></td> <td>Use the structure of an expression to identify ways to rewrite it. For example, see</td> </tr> </table>	<b>Interpret the structure of expressions</b>		<b>A-SSE.1</b>	Interpret expressions that represent a quantity in terms of its context.		c. Interpret parts of an expression, such as terms, factors, and coefficients.		d. Interpret complicated expressions by viewing one or more of their parts as a single entity	<b>A-SSE.2</b>	Use the structure of an expression to identify ways to rewrite it. For example, see
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<b>A-SSE.2</b>	Use the structure of an expression to identify ways to rewrite it. For example, see										

	$(x^4 - y^4)$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .
<b>Perform arithmetic operations on polynomials</b>	
<b>A-APR.1</b>	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (e.g, multiply two binomials, or a binomial with a trinomial)
<b>A-APR.2</b>	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$
<b>A-APR.3</b>	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
<b>Use polynomial identities to solve problems</b>	
<b>A-APR.4</b>	Prove polynomial identities and use them to describe numerical relationships.
<b>Create equations that describe numbers or relationships</b>	
<b>A-CED.1</b>	Create equations and inequalities in one variable and use them to solve equations. . <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>
<b>A-CED.2</b>	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

	<b>Understand solving equations as a process of reasoning and explain the reasoning</b>	
	<b>A-REI.1</b>	Explain each step in solving a simple equation as following from the equality of numbers assured at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
	<b>A-REI.2</b>	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
	<b>Analyze the function using different representations</b>	
	<b>F-IF.7</b>	Graph functions expressed symbolically and show key features of the graph [of a function], by hand in simple cases and using technology for more complicated cases.
	<b>Building new functions from existing functions.</b>	
	<b>F-BF.3</b>	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
	<b>Reason quantitatively and use units to solve problems</b>	
<b>N.Q.1</b>	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	
<b>Essential Question(s):</b>	<ul style="list-style-type: none"> <li>• What are rational functions, and what are the key features of their graphs?</li> <li>• What is the relationship between operations with rational numbers and operations with rational expressions?</li> </ul>	

<b>Enduring Understanding(s):</b>	<ul style="list-style-type: none"> <li>● The reciprocal function is used to model inverse variation, which is a proportional relationship between two variables such that when one variable increases, the other decreases.</li> <li>● The properties of operations used to add and subtract rational numbers can be applied to adding and subtracting rational expressions.</li> <li>● Rational equations can be solved by multiplying by a common denominator.</li> </ul>
<b>Learning Goal(s):</b> <i>Students will know and will be able to use their learning to:</i> (Content/ Skills)	<p><b>Content:</b> (Students will know...)</p> <ul style="list-style-type: none"> <li>● Inverse variation</li> <li>● Graph and transformation of the reciprocal function</li> <li>● Operations with rational expressions               <ul style="list-style-type: none"> <li>○ addition</li> <li>○ subtraction</li> <li>○ multiplication</li> </ul> </li> <li>● Simple rational equations               <ul style="list-style-type: none"> <li>○ common denominator of rational Equations</li> <li>○ extraneous solutions</li> </ul> </li> <li>● Rate problems               <ul style="list-style-type: none"> <li>○ units and dimensional analysis</li> </ul> </li> </ul> <p><b>Skills:</b> (Students will be able to...)</p> <ul style="list-style-type: none"> <li>● use their understanding of operations with rational numbers to add and subtract rational expressions.</li> <li>● solve rational equations by multiplying each side of the equation by a common denominator to eliminate the fractions.</li> <li>● any solution that is excluded from the domain in the original equation is extraneous.</li> <li>● make strategic use of algebraic structure and the properties of operations to identify and create equivalent expressions, including rewriting simple rational expressions.</li> <li>● make strategic use of algebraic structure, the properties of operations, and/or reasoning about equality to solve simple rational equations in one variable.</li> </ul>

- make connections between a table, an algebraic representation, or a graph of a simple rational function, both in context and not in context.
- use function notation to represent and interpret input/output pairs for rational functions, and find the input value for a corresponding output.
- solve problems involving one-step unit conversion and multistep or multidimensional unit conversion.
- solve problems involving derived units, including those that arise from products (e.g., kilowatt-hours) and quotients (e.g., population per square kilometer).

<b>Unit Number and Title:</b>	<b>Unit 5 - Inverses and Radical Functions</b>
<b>Duration:</b>	~ 6 weeks
<b>Resource(s):</b>	enVision Algebra 2 College Board Pre-AP Algebra II
<b>Unit Overview:</b>	Unit 5 builds on students' prior knowledge of radicals by introducing radical functions, the square root function, and the cube root function. Students understand properties of rational exponents and radicals using the nth root to solve equations with a variable raised to the nth power. Students learn to graph radical functions, solve radical equations, and combine functions. Students identify inverses of functions and learn to write the equations of inverse functions.
<b>Learning Goals</b>	
<b>Standard(s):</b>	<b>Extend the properties of exponents to rational exponents</b>
	<b>N-RN.1</b> Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
	<b>N-RN.2</b> Rewrite expressions involving radicals and rational exponents using the properties of exponents
	<b>Interpret the structure of expressions.</b>
	<b>A-SSE.2</b> Use the structure of an expression to identify ways to rewrite it. For example, see

	$(x^4 - y^4)$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .
<b>Write expressions in equivalent forms to solve problems.</b>	
<b>A-SSE.3</b>	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
<b>Create equations that describe numbers or relationships</b>	
<b>A-CED.1</b>	Create equations and inequalities in one variable and use them to solve equations. . <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>
<b>A-CED.3</b>	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as variable and nonviable options in a modeling context.
<b>A-CED.4</b>	Arrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance $R$
<b>Understand solving equations as a process of reasoning and explain the reasoning</b>	
<b>A-REI.1</b>	Explain each step in solving a simple equation as following from the equality of numbers assured at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
<b>Interpret functions that arise in applications in terms of context</b>	

<b>F-IF.4</b>	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity..
<b>Analyze the function using different representations</b>	
<b>F-IF.7</b>	Graph functions expressed symbolically and show key features of the graph [of a function], by hand in simple cases and using technology for more complicated cases.
	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
<b>Build a function that models a relationship between two quantities</b>	
<b>F-BF.1</b>	Write a function that models a relationship between two quantities.
	b. Combine standard function types using arithmetic operations
	c. (+) Compose functions
<b>Building new functions from existing functions</b>	
<b>F-BF.3</b>	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
<b>F-BF.4</b>	Find inverse functions.

	<table border="1"> <tr> <td data-bbox="569 248 787 354"></td> <td data-bbox="787 248 1896 354">           a. Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write the expression for the inverse.         </td> </tr> </table>		a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write the expression for the inverse.
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<b>Essential Question(s):</b>	<ul style="list-style-type: none"> <li>• How can the properties of integer exponents be extended and applied to rational exponents and radicals?</li> <li>• How are rational exponents and radical equations used to solve real-world problems?</li> </ul>		
<b>Enduring Understanding(s):</b>	<ul style="list-style-type: none"> <li>• The properties of exponents, including rational exponents, provide a consistent framework for simplifying expressions and solving equations. Radicals can be equivalently expressed using rational exponents.</li> <li>• A rational exponent has an equivalent radical expression and any radical expression can be written with a rational exponent.</li> <li>• Functions can be combined by operations and by composition. The result can be described as a single function. The domain may be different than the domains of the original functions.</li> <li>• The inverse of a function is found by exchanging the roles of the independent and dependent variables.</li> </ul>		
<b>Learning Goal(s):</b> <i>Students will know and will be able to use their learning to:</i> (Content/ Skills)	<b>Content:</b> (Students will know:) <ul style="list-style-type: none"> <li>• Key features of square root and cube root functions</li> <li>• Graphing and transformation of square root and cube root functions</li> <li>• <math>N</math>th roots, radicals, and rational exponents</li> <li>• Properties of exponents and radicals</li> <li>• Equations with rational exponents &amp; radicals</li> <li>• Operations with radicals</li> <li>• Combining functions by operations</li> <li>• Composition of functions</li> <li>• Inverse relations and functions</li> </ul>		

( linear, quadratic/square root, cubic/cube root functions)

**Skills:** (Students will be able to...)

- find  $n$ th roots of a number and use them to rewrite expressions and solve equations involving rational exponents.
- use the properties of radicals and exponents to rewrite radical expressions.
- graph and transform radical functions, including square root and cube root functions.
- solve radical equations, identifying extraneous solutions.
- add, subtract, multiply, and divide functions
- compose functions.
- write the inverse of a function.
- make strategic use of algebraic structure and the properties of operations to identify and create equivalent expressions, rewriting expressions with rational exponents in radical form.
- make strategic use of algebraic structure, the properties of operations, and/or reasoning about equality to solve radical equations in one variable.
- make connections between a table, an algebraic representation, or a graph of a simple radical function, both in context and not in context.
- use function notation to represent and interpret input/output pairs for radical functions, and find the input value for a corresponding output.
- understand that for the graph of  $y = f(x)$ , the solutions to  $f(x) = 0$  correspond to  $x$ -intercepts of the graph and  $f(0)$  corresponds to the  $y$ -intercept of the graph; make connections between the input/output pairs and points on a graph; interpret this information in a context.

<b>Unit Number and Title:</b>	<b>Unit 6 - Exponential and Logarithmic Functions</b>								
<b>Duration:</b>	~ 7 weeks								
<b>Resource(s):</b>	enVision Algebra 2 College Board Pre-AP Algebra II								
<b>Unit Overview:</b>	This unit extends students' understanding of functions beyond the linear model to introduce the exponential function. By recognizing patterns in data, students will explore exponential growth. They will then explore, graph, and analyze the key features (domain, range, asymptote, intercepts) of exponential functions, including transformations (shifts and stretches). A significant focus is placed on modeling real-world situations, such as population growth, depreciation, and compound interest, by constructing and interpreting exponential growth and decay functions. Students learn that the inverse of a exponential function is a logarithmic function and vice versa. Students will understand logarithms and their properties. They extend their work with exponential functions to include solving exponential equations with logarithms. Students will determine the type of function to best fit a set of data by looking for patterns in the values, their differences, and their ratios, and by looking at the shape of the data. Utilizing technology, students will find a function that best models the data using regression, comparing linear, quadratic, and exponential models.								
<b>Learning Goals</b>									
<b>Standard(s):</b>	<table border="1"> <tr> <td colspan="2"><b>Interpret the structure of expressions.</b></td> </tr> <tr> <td><b>A-SSE.1</b></td> <td>Interpret expressions that represent a quantity in terms of its context.</td> </tr> <tr> <td></td> <td>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i></td> </tr> <tr> <td colspan="2"><b>Interpret the structure of expressions.</b></td> </tr> </table>	<b>Interpret the structure of expressions.</b>		<b>A-SSE.1</b>	Interpret expressions that represent a quantity in terms of its context.		b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i>	<b>Interpret the structure of expressions.</b>	
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<b>Interpret the structure of expressions.</b>									

	<b>A-SSE.2</b>	Use the structure of an expression to identify ways to rewrite it. For example, see $(x^4 - y^4)$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .
	<b>Write expressions in equivalent forms to solve problems.</b>	
	<b>A-SSE.3</b>	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
		c. Use properties of exponents to transform expressions for exponential functions
	<b>Create equations that describe numbers or relationships</b>	
	<b>A-CED.1</b>	Create equations and inequalities in one variable and use them to solve equations. . <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>
	<b>A-CED.4</b>	Arrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law $V = IR$ to highlight resistance $R$
	<b>Understand solving equations as a process of reasoning and explain the reasoning</b>	
	<b>A-REI.1</b>	Explain each step in solving a simple equation as following from the equality of numbers assured at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
	<b>Interpret functions that arise in applications in terms of context</b>	

	<b>F-IF.4</b>	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>
	<b>F-IF.5</b>	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
	<b>F-IF.6</b>	Calculate and interpret the average rate of change of a function (presented symbolically or as a table over a specified interval). Estimate the rate of change from a graph
	<b>Analyze the function using different representations</b>	
	<b>F-IF.7</b>	Graph functions expressed symbolically and show key features of the graph [of a function], by hand in simple cases and using technology for more complicated cases.
		b. Graph exponential and logarithmic functions, showing intercepts and end behavior.
	<b>F-IF.8</b>	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
		b. Use the properties of exponents to interpret expressions for exponential functions.
	<b>F-IF.9</b>	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
	<b>Build a function that models a relationship between two quantities</b>	

	<b>F-BF.1</b>	Write a function that models a relationship between two quantities.
	<b>Building new functions from existing functions</b>	
	<b>F-BF.3</b>	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
	<b>F-BF.4</b>	Find inverse functions.
		a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write the expression for the inverse.
		c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse
	<b>F-BF.5</b>	(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents
	<b>Construct and compare linear and exponential models and solve problems</b>	
	<b>F-LE.1</b>	Distinguish between situations that can be modeled with linear functions and with exponential functions.
		a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
		b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

	c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
<b>F-LE.2</b>	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
<b>F-LE.3</b>	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
<b>Interpret expressions for functions in terms of the situation they model</b>	
<b>F-LE.5</b>	Interpret the parameters in a linear or exponential function in terms of a context.
<b>Summarize, represent, and interpret data on two categorical and quantitative variables</b>	
<b>S-ID.6</b>	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
	a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use the given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</i>

<b>Essential Question(s):</b>	<ul style="list-style-type: none"> <li>● How are exponential functions different from linear functions, and how do their key features (intercept, growth rate, asymptote) define their behavior?</li> <li>● How does recognizing whether a real-world situation changes multiplicatively or additively help determine the most appropriate mathematical model?</li> <li>● How do the graph and the algebraic equation of an exponential function reveal key characteristics like domain, range, and growth behavior?</li> <li>● How can exponential functions be constructed, interpreted, and used to model and predict real-world phenomena such as growth, decay, or compound interest?</li> <li>● What is the relationship between exponential functions and logarithmic functions?</li> </ul>
<b>Enduring Understanding(s):</b>	<ul style="list-style-type: none"> <li>● Exponential functions model multiplicative change, whereas linear functions model additive change; identifying the type of change determines the appropriate model.</li> <li>● Exponential functions are defined by a constant growth factor rather than a constant rate of change, and key features like the y-intercept and horizontal asymptote reveal their unique behavior.</li> <li>● Exponential functions can be used to model and predict real-world phenomena, including population growth, decay, and financial applications such as compound interest.</li> <li>● Both the graph and the equation of an exponential function reveal essential characteristics such as asymptotes, domain, range, and growth behavior.</li> <li>● A logarithmic function is the inverse of an exponential function and can be used to solve exponential functions through inverse operations.</li> <li>● Rewriting an exponential function using logarithms and applying properties of logarithms is a more efficient method than writing both sides with a common base.</li> </ul>
<b>Learning Goal(s):</b> <i>Students will know and will be able to use their learning to:</i> (Content/ Skills)	<b>Content:</b> (Students will know...) <ul style="list-style-type: none"> <li>● Rate of change               <ul style="list-style-type: none"> <li>○ constant ratio</li> <li>○ growth factor</li> <li>○ additive change</li> <li>○ multiplicative change</li> </ul> </li> </ul>

- Key features of exponential functions
  - horizontal asymptote
  - domain and range of exponential functions
  - Y-intercept
- Exponential models
  - exponential growth  $f(t) = a(1 + r)^t$
  - exponential decay  $f(t) = a(1 - r)^t$
  - compound interest:  $A = P(1 + \frac{r}{n})^{nt}$
- Exponential - linear systems
- Logarithms
  - logarithmic functions
  - properties of logarithms
- Exponential and logarithmic equations
- Exponential regression
- Comparing linear, quadratic, and exponential growth

**Skills:** (Students will be able to...)

- identify whether growth is linear or exponential.
- identify and describe the domain, range, y-intercept, and horizontal asymptote of a given exponential function from its equation, graph, or table.
- create and use exponential functions to solve problems in a variety of contexts.
- identify or create an appropriate exponential function to model a relationship between quantities.
- for an exponential function that represents a context:
  - interpret the meaning of an input/output pair, including an intercept or initial value, including situations where seeing structure provides an advantage.
  - interpret the meaning of a constant, variable, factor, or term based on the context, including situations where seeing structure provides an advantage
- for an exponential function in a context:
  - interpret a point on the graph.

- interpret parts of the graph (other than a point or intercept)
- sketch the graph of an exponential function, correctly plotting the y-intercept and showing the behavior near the asymptote.
- solve the exponential equation using a common base.
- solve exponential equations using logarithms.
- solve logarithmic equations using the property of equality for logarithmic equations.
- solve logarithmic and exponential equations by graphing.
- make connections between a table, an algebraic representation, or a graph of an exponential function that does not involve a transformation in context and not in context.
- determine the most suitable form of the expression representing the output of the function to display key features for an exponential function.
- understand that for the graph of  $y = f(x)$ , the solutions to  $f(x) = 0$  correspond to x-intercepts of the graph and  $f(0)$  corresponds to the y-intercept of the graph; make connections between the input/output pairs and points on a graph; interpret this information in a context.
- use function notation to represent and interpret input/output pairs for exponential functions, and find the input value for a corresponding output.
- distinguish between linear and exponential relationships in tables, graphs, and real-world scenarios, correctly identifying which model is appropriate (additive vs. multiplicative change).
- construct and interpret exponential growth and decay functions to model real-world situations (e.g., compound interest, population growth).
- given an equation or formula in two or more variables, view it as an equation in a single variable of interest where the other variables are parameters, and solve for the variable of interest.
- relate the solutions of a system of linear and exponential equations in two variables to the graphs of the equations in the system.
- make strategic use of algebraic structure, the properties of operations, and reasoning about equality to solve systems of linear and exponential equations in two variables.
- determine which model – linear, exponential, or quadratic – best fits a set of data.