Name: [redacted] Period: [redacted]

Ch 2 Differentiation (AP Exam MCQ's)

1998

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

$4 + 2y = 10$

$y = 3$

$4 + 2\frac{dy}{dx} + 3 = 0$

$2\frac{dy}{dx} = -7$

$\frac{dy}{dx} = -\frac{7}{2}$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

$h' = fg' + gf' = fg'$

$\therefore gf' = 0$

$f(x)$ is constant

$f(0) = 1$, so $f(x) = 1$

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

$f'(2) = \frac{(x-1)2x - (x^2-2)}{(x-1)^2} = \frac{4-2}{1} = 2$

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$v(t) = 2t - 6 = 0$

$t = 3$

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point (0, 1) is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

$y' = 1 - \sin x$

$y'(0) = 1 - \sin 0$

$= 1$

$y - 1 = 1(x - 0)$

$= x + 1$

24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

$a(t) = 3t^2 - 6t + 12$

$a'(t) = 6t - 6 = 0$

$t = 1$ on $[0, 3]$

$a(0) = 12$

$a(1) = 9$

$a(3) = 27 - 18 + 12 = 21$

28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

$f'(x) = 2\sec^2(2x)$

$f'\left(\frac{\pi}{6}\right) = 2\sec^2\left(\frac{\pi}{3}\right)$

$= 2\sec\left(\frac{\pi}{3}\right)^2$

$= 2(2)^2 = 8$

$\sec(60) = \frac{1}{\cos(60)}$

$= \frac{1}{1/2} = 2$

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

$\frac{dr}{dt} = -0.1$

$A = \pi r^2$ $\quad C = 2\pi r$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= C(-0.1)$

87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

$f'(x) = 4x^3 + 4x = 1$

$x = 0.2367329$

$f(.236\ldots) = 0.115221996\,9$

$y - 0.115 = 1(x - 0.237)$

$y = x - 0.237 + 0.115$

$y = x - 0.122$

2003

1. If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$ $2(x^3+1)(3x^2) = 6x^2(x^3+1)$

(A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

4. If $y = \frac{2x + 3}{3x + 2}$, then $\frac{dy}{dx} =$ $\frac{(3x+2)\cdot 2 - (2x+3)\cdot 3}{(3x+2)^2} = \frac{6x+4-6x-9}{(3x+2)^2} = \frac{-5}{(3x+2)^2}$

(A) $\frac{12x + 13}{(3x + 2)^2}$ (B) $\frac{12x - 13}{(3x + 2)^2}$ (C) $\frac{5}{(3x + 2)^2}$ (D) $\frac{-5}{(3x + 2)^2}$ (E) $\frac{2}{3}$

14. If $y = (x^2)\sin 2x$, then $\frac{dy}{dx} =$ $x^2(2\cos(2x)) + \sin(2x)\,2x$
$= 2x(x\cos(2x) + \sin(2x))$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

$m = \frac{-2-7}{-2-1} = \frac{-9}{-3} = 3$

$f(-1) = -4+5+3 = 4$

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 7x - 3$

(B) $y = 7x + 7$

(C) $y = 7x + 11$

(D) $y = -5x - 1$

(E) $y = -5x - 5$

$f' = 12x^2 - 5$

$f'(-1) = 12 - 5 = 7$

$y - 4 = 7(x+1)$

$y = 7x + 11$

25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

$v(t) = 6t^2 - 42t + 72 = 0$

$6(t^2 - 7t + 12)$

$6(t-3)(t-4) = 0$

$t = 3, t = 4$

26. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6$ at the point $(3, 2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

$(2x + 6y)\frac{dy}{dx} = 4x - 2y$

$\frac{dy}{dx} = \frac{4x - 2y}{2x + 6y} = \frac{8}{18} = \frac{4}{9}$

76. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.978

$\frac{dr}{dt} = 0.2$

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) 0.04π m^2/sec

(B) 0.4π m^2/sec

(C) 4π m^2/sec

(D) 20π m^2/sec

(E) 100π m^2/sec

$A = \pi r^2$

$C = 2\pi r = 20\pi$

$r = 10$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(10)(0.2)$

$= 4\pi$

$g(2) = 2f(2) = 2(3) = 6$

89. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

(C) $y - 6 = -5(x - 2)$

(D) $y - 6 = -7(x - 2)$

(E) $y - 6 = -10(x - 2)$

$g' = xf' + f$

$g'(2) = 2f'(2) + f(2)$

$= -10 + 3 = -7 = m$

$y - 6 = -7(x - 2)$

2008

3. If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) =$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

$(x-1)3(x^2+2)^2(2x) + (x^2+2)^3$

$= (x^2+2)^2(6x(x-1)+(x^2+2))$

$= (x^2+2)^2(7x^2-6x+2)$

8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$

$f' = -3\sin(3x)$

$f'\left(\frac{\pi}{9}\right) = -3\sin\left(\frac{\pi}{3}\right) = -3\cdot\frac{\sqrt{3}}{2} = \frac{-3\sqrt{3}}{2}$

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$

$\cos(xy)\left(x\frac{dy}{dx}+y\right) = 1$

$x\cos(xy)\frac{dy}{dx} + y\cos(xy) = 1$

$\frac{dy}{dx} = \frac{1-y\cos(xy)}{x\cos(xy)}$

$y = -x + k$
$m = -1$

$y = 4 - 6 + 1 = -1$

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

$y' = 2x + 3 = -1$
$x = -2$

82. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

(A) -0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

$\frac{dr}{dt} = -2$, $r = 3$

88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$.)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$
$= 8\pi(3)(-2)$
$= -48\pi$

2012

1. If $y = (x)(\sin x)$, then $\frac{dy}{dx} =$ $x\cos x + \sin x$

$f \cdot g$

(A) $\sin x + \cos x$

(B) $\sin x + x\cos x$

(C) $\sin x - x\cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

7. If $y = (x^3 - \cos x)^5$, then $y' =$ $5(x^3-\cos x)^4(3x^2+\sin x)$

(A) $5(x^3 - \cos x)^4$

(B) $5(3x^2 + \sin x)^4$

(C) $5(3x^2 + \sin x)$

(D) $5(3x^2 + \sin x)^4 \cdot (6x + \cos x)$

(E) $5(x^3 - \cos x)^4 \cdot (3x^2 + \sin x)$

14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$

$f = (x^2-4)^{1/2}$

$f' = \frac{1}{2}(x^2-4)^{-1/2}(2x)$

$= \frac{x}{\sqrt{x^2-4}}$

$f'(g(3))g'(3)$

$= f'(7) \cdot 3$

$\frac{7}{3\sqrt{5}} \cdot 3 = \frac{7}{\sqrt{5}}$

$f'(7) = \frac{7}{\sqrt{49-4}} = \frac{7}{\sqrt{45}} = \frac{7}{3\sqrt{5}}$

16. A particle moves along the x-axis with its position at time t given by $x(t) = (t - a)(t - b)$, where a and b are constants and $a \neq b$. For which of the following values of t is the particle at rest?

(A) $t = ab$

(B) $t = \frac{a+b}{2}$

(C) $t = a + b$

(D) $t = 2(a + b)$

(E) $t = a$ and $t = b$

$v(t) = (t-a) + (t-b) = 0$

$2t = a+b$

$t = \frac{a+b}{2}$

18. $\lim_{h \to 0} \frac{\ln(4+h) - \ln(4)}{h}$ is $\ln x \quad \frac{1}{x}$

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

def of der

$f(0)=0$, $f(-4)=2$

19. The function f is defined by $f(x)=\frac{x}{x+2}$. What points (x,y) on the graph of f have the property that the line tangent to f at (x,y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2},\frac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4,\frac{2}{3}\right)$

(E) There are no such points.

$$f' = \frac{(x+2)-x}{(x+2)^2}$$

$$= \frac{2}{(x+2)^2} \neq \frac{1}{2}$$

$$\sqrt{(x+2)^2} = \pm\sqrt{4}$$

$$x+2 = \pm 2$$

$$x=0,\ x=-4$$

27. If $(x+2y)\cdot\frac{dy}{dx}=2x-y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3,0)$?

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

$$\frac{dy}{dx} = \frac{2x-y}{x+2y} = \frac{6}{3} = 2$$

$$\frac{d^2y}{dx^2} = \frac{(x+2y)\left(2-\frac{dy}{dx}\right)-(2x-y)\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$$

$$= \frac{3(0)-(6)(5)}{3^2} = \frac{-30}{9} = -\frac{10}{3}$$

28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

$v(t) = \cos t + \sin t = \dot{x}$

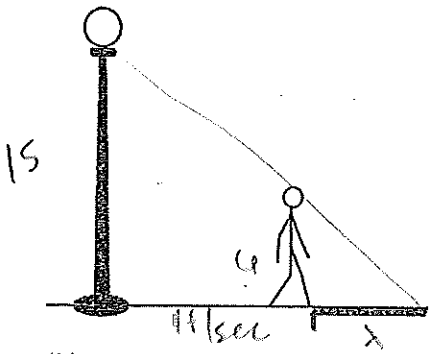
$\sin t = \cos t$

$t = 135° \left(= \frac{3\pi}{4}\right)$

$a(t) = \sin t + \cos t$

$a(135°) = \sin(135°) + \cos(135°)$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$



$\frac{x}{x+4} = \frac{6}{15}$

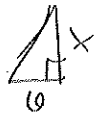
$15x = 6(x+4)$

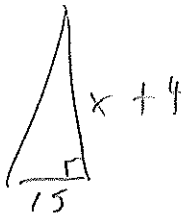
$9x = 24$

$t = \frac{24}{9} \approx 2.667$

88. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

(A) 1.5 ft/sec (B) 2.667 ft/sec (C) 3.75 ft/sec (D) 6 ft/sec (E) 10 ft/sec





Ch 2 FRQ's (Unsecured)

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2015 FRQ

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

$y - 1 = \frac{1}{4}(x+1)$

$\left.\frac{dy}{dx}\right|_{(-1,1)} = \frac{1}{3(1)+1} = \frac{1}{4}$

2014 FRQ

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure. not derivative

$A(0) = 6.687(1) = 6.687$

$A(30) = 6.687(0.931)^{30} = 0.7829\ldots$

$\frac{A(30) - A(0)}{30 - 0} = \frac{0.7829 - 6.687}{30} = -5.904$ lbs/day

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

$\left.\frac{d}{dx}\right|_{t=15} A(t) = -0.164 = A'(15)$

At 15 days, the # of grass clippings in bin is decreasing at a rate of 0.164 lbs/day

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	−120	−150

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(a) Find the average acceleration of train A over the interval $2 \le t \le 8$.

$\frac{v(8) - v(2)}{8 - 2} = \frac{-120 - 100}{6} = \frac{-110}{3}$ m/min^2

2005 FRQ (Form B)

3. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$. $v'(t) = \frac{2t-3}{t^2-3t+3}$

$a(4) = v'(4) = \frac{5}{7}$

$v'(4) = \frac{8-3}{16-12+3} = \frac{5}{7}$

2003 FRQ (Form B)

4. A particle moves along the x-axis with velocity at time $t \ge 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

$v'(t) = e^{1-t}(-1) = -e^{1-t}$

$v'(3) = -e^{1-3} = -e^{-2}$

$a(3) = v'(3) = -e^{-2}$

2008 FRQ (Form B)

6. Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2\left(y^3+1\right)}$. $2x + 2 + 4y^3\frac{dy}{dx} + 4\frac{dy}{dx} = 0$

$(4y^3+4)\frac{dy}{dx} = -2x-2$

$\frac{dy}{dx} = \frac{-2x-2}{4y^3+4}$

$= \frac{-2(x+1)}{4(y^3+1)} = \frac{-(x+1)}{2(y^3+1)}$

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

$\left.\frac{dy}{dx}\right|_{(-2,1)} = \frac{-(-2+1)}{2(1+1)} = \frac{1}{4}$

$y - 1 = \frac{1}{4}(x+2)$