Name: [redacted] 10/10 Period: [redacted]

Sec 6.1 (Euler's Method), 6.3 (Logistic Equation), 7.4 (Arc Length), & Ch 8 (Integration by Parts, Partial Fractions, L'Hopital's Rule, Improper Integrals) AP Calculus BC Exam (MCQ's)

2003

2) $\lim_{x\to 0}\frac{e^x-\cos x-2x}{x^2-2x}$ is $=0$ $=0$

$\frac{e^x+\sin x-2}{2x-2} = \frac{1+0-2}{-2} = \frac{-1}{-2} = \frac{1}{2}$

L'Hopital's

$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

(A) $-\frac{1}{2}$

(B) 0

(C) $\frac{1}{2}$

(D) 1

(E) *nonexistent*

5) Let $y=f(x)$ be the solution to the differential equation $\frac{dy}{dx}=x+y$ with the initial condition $f(1)=2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x=1$ with a step size of 0.5?

$y=y_1+m(\Delta x)$

Euler's

(A) 3

(B) 5

(C) 6

(D) 10

(E) 12

$(1,2)$

$(1.5,3.5)$

$(2,6)$

$y=2+3(0.5)=2+1.5=3.5$

$y=3.5+5(0.5)=3.5+2.5=6$

$y=y_1+m(\Delta x)$

6) What are all values of p for which $\int_1^\infty \frac{1}{x^{2p}}dx$ converges?

improper

$\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$

$2p>1$

$p>\frac{1}{2}$

(A) $p<-1$

(B) $p>0$

(C) $p>\frac{1}{2}$

(D) $p>1$

(E) There are no values of p for which this integral converges.

7. Given that $y(1) = -3$ and $\frac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

(A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

$y = y_1 + m(\Delta x)$

Euler's

$(1, -3)$

$(1.5, -3.5)$

$(2,)$

$y = -3 + -1(0.5) = -3.5$

$y = -3.5 + -0.5(0.5)$

$-3.5 + -0.25 = -3.75$

$2(1) + -3 = -1$

$2(1.5) + -3.5 = -0.5$

$y = y_1 + m(\Delta x)$

improper integrals

11. Let R be the region between the graph of $y = e^{-2x}$ and the x-axis for $x \geq 3$. The area of R is

(A) $\frac{1}{2e^6}$ (B) $\frac{1}{e^6}$ (C) $\frac{2}{e^6}$ (D) $\frac{\pi}{2e^6}$ (E) infinite

$\int_3^\infty$

Area $= \int_3^\infty e^{-2x}\,dx$

improper

$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$

$\lim_{b\to\infty} -\frac{1}{2}\int_3^b 2e^{-2x}\,dx$

$\lim_{b\to\infty} -\frac{1}{2} e^{-2x}\Big|_3^b$

$= \lim_{b\to\infty} -\frac{1}{2}\left(e^{-2b} - e^{-6}\right) = \lim_{b\to\infty} -\frac{1}{2}\left(\frac{1}{e^{2b}} - \frac{1}{e^6}\right)$

$= -\frac{1}{2}\left(\frac{1}{e^{2\infty}} - \frac{1}{e^6}\right) = \frac{1}{2e^6}$

13. $\int_1^e \frac{x^2+1}{x}\,dx = \int_1^e \left(x + \frac{1}{x}\right)dx$

Partial 2

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+1}{2}$ (C) $\frac{e^2+2}{2}$ (D) $\frac{e^2-1}{e^2}$ (E) $\frac{2e^2-8e+6}{3e}$

$= \frac{x^2}{2} + \ln|x|\Big|_1^e = \left(\frac{e^2}{2} + 1\right) - \left(\frac{1}{2} + 0\right) = \frac{e^2}{2} + \frac{1}{2} = \frac{e^2+1}{2}$

2012

4. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

(A) $\int_1^2 \sqrt{1+\frac{1}{x^2}}\,dx$

(B) $\int_1^2 \left(1+\frac{1}{x^2}\right)dx$

(C) $\int_1^2 \sqrt{1+e^{2x}}\,dx$

(D) $\int_1^2 \sqrt{1+(\ln x)^2}\,dx$

(E) $\int_1^2 \left(1+(\ln x)^2\right)dx$

arc length

$\int_1^2 \sqrt{1+\frac{1}{x^2}}\,dx$

14. Let k be a positive constant. Which of the following is a logistic differential equation?

(A) $\frac{dy}{dt} = kt$

(B) $\frac{dy}{dt} = ky$

(C) $\frac{dy}{dt} = kt(1-t)$

(D) $\frac{dy}{dt} = ky(1-t)$

(E) $\frac{dy}{dt} = ky(1-y)$

logistic

16. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

(A) $-\frac{5}{4}$ (B) 1 (C) $\frac{7}{4}$ (D) 2 (E) $\frac{21}{4}$

$y = y_1 + m(\Delta x)$

Euler

(1, 3)
(1.5, 2)
(2,

$y = 3 + -2(0.5) = 3-1 = 2$ $\quad 1-3 = -2$

$y = 2 + -0.5(0.5) = 2 - 0.25$ $\quad 1.5-2 = -0.5$

$= 1.75 = \frac{7}{4}$