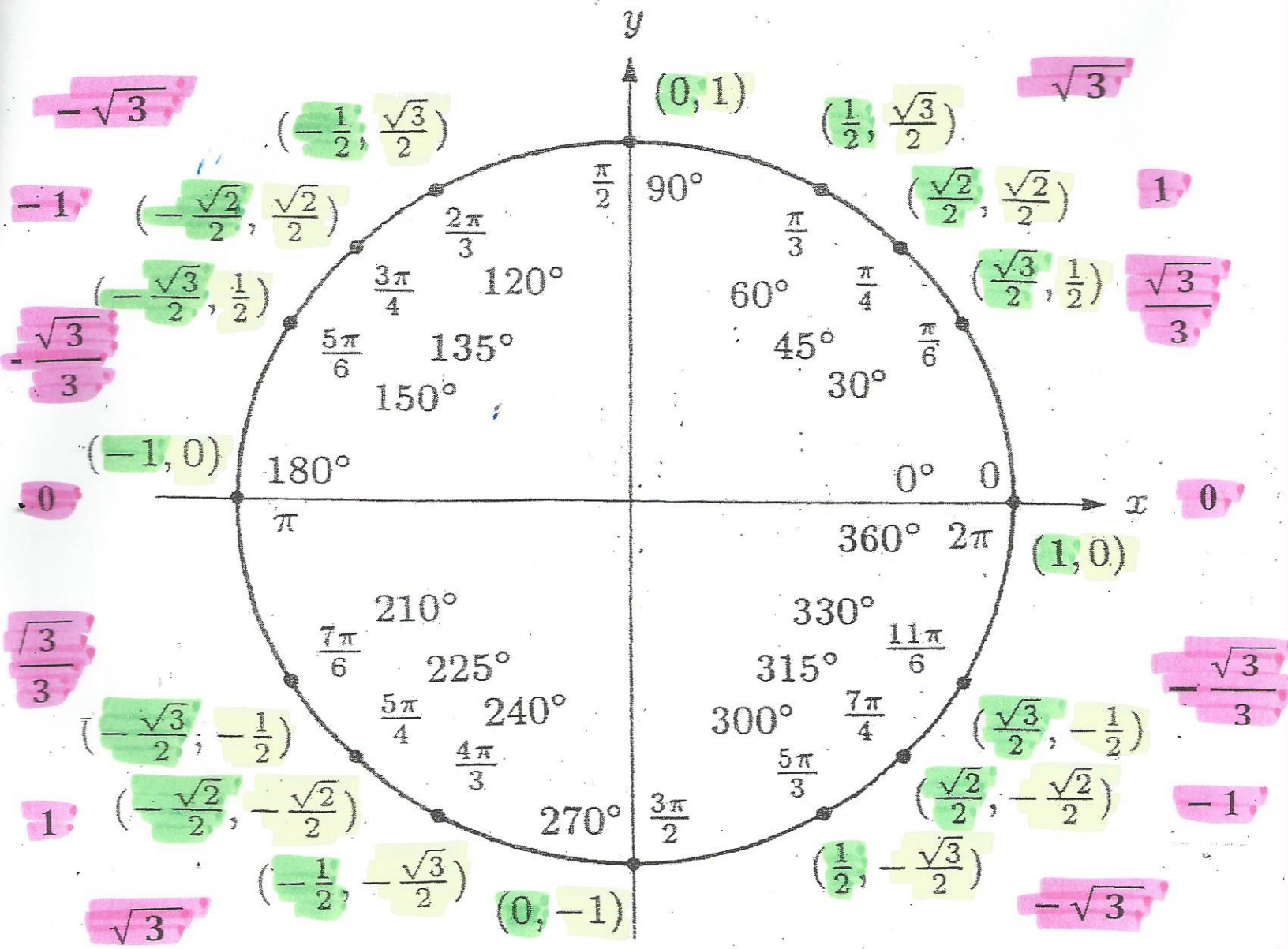
P: (x , y)

$\tan\Theta = \dfrac{\sin\Theta}{\cos\Theta}$

P: ($\cos\Theta$, $\sin\Theta$)

undefined



undefined

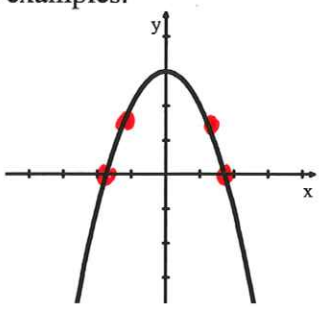
AP Precalc

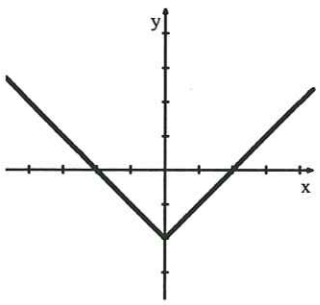
1.5B Even and Odd Polynomials

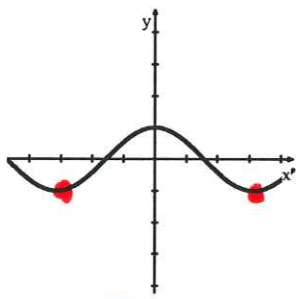
1.5B Notes

Write your questions and thoughts here!

Even functions are symmetrical over the y-axis. The following graphs are examples.

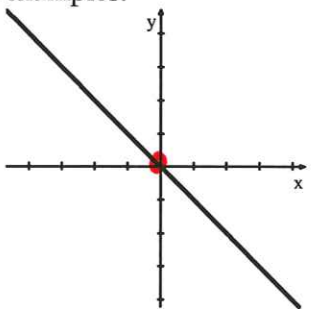


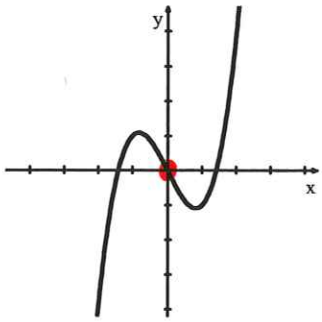


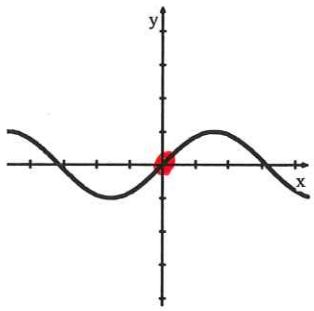


$(x,y) \rightarrow (-x,y)$

Odd functions are symmetrical over the origin, (0,0). The following graphs are examples.





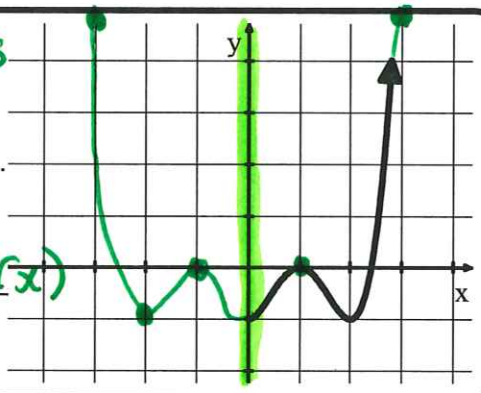


$(x,y) \rightarrow (-x,-y)$

An EVEN Function…

y-axis

- is graphically symmetric over the line $x = 0$.
- analytically has the property $f(-x) = f(x)$



You can prove that a function is even by substituting in $-x$ and see if you get the same original function.

1. Show analytically that $f(x) = x^6 - 4x^2$ is an even function.

$f(-x) = (-x)^6 - 4(-x)^2$

$= x^6 - 4x^2$

Same as original function

$f(-x) = f(x)$

EVEN

Write your questions and thoughts here!

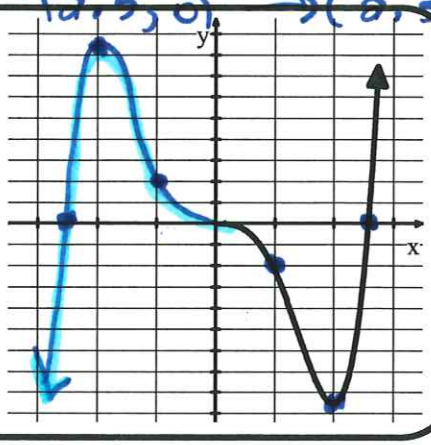
(1, -2) → (-1, 2)
(2, -8.5) → (-2, 8.5)
(-2.5, 0) → (2.5, 0)

An ODD Function…

origin

- is graphically symmetric over the point (0,0).
- analytically has the property _____

$f(-x) = -f(x)$



You can prove that a function is odd by substituting in $-x$ and see if you get the opposite of the original function.

2. Show analytically that $f(x) = -2x^3 + 5x$ is an odd function.

$f(-x) = -2(-x)^3 + 5(-x)$
$= 2x^3 - 5x$
$f(-x) = -f(x)$ ODD

3. Show analytically if $f(x) = 6x^4 - 2x$ is even, odd, or neither.

$f(-x) = 6(-x)^4 - 2(-x)$
$= 6x^4 + 2x$ Neither

Shortcut for simple polynomial form

Let p be a polynomial of the form $p(x) = a_n x^n$, where $n \geq 1$ and $a_n \neq 0$.

- If n is even, then p is an even function.
- If n is odd, then p is an odd function.

simple, one term polynomial functions

If you see one of these in the practice, you can just state that it is an odd or even function without having to go through all the analytical work!

State if the following functions are odd or even.

4. $f(x) = -10x^3$

$f(-x) = -10(-x)^3$
$= 10x^3$
ODD

5. $f(x) = 7x^4$

$f(-x) = 7(-x)^4$
$= 7x^4$
EVEN

6. $f(x) = -3x^6$

$f(-x) = -3(-x)^6$
$= -3x^6$
EVEN

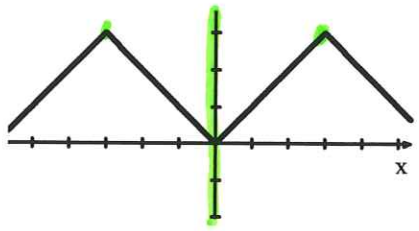
1.5B Even and Odd Polynomials

AP Precalculus

1.5B Practice

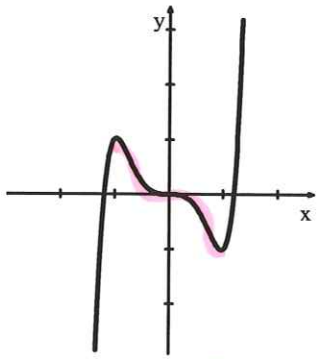
State whether the following graphs represent functions that are even, odd, or neither.

1.



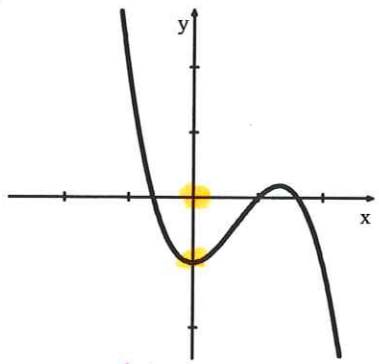
Even

2.



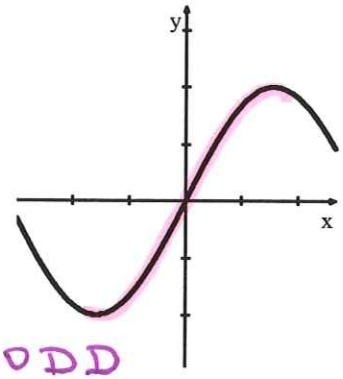
ODD

3.



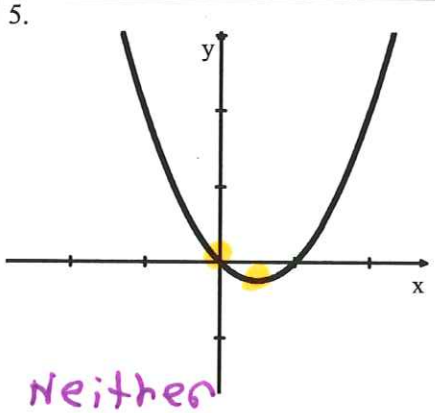
Neither

4.



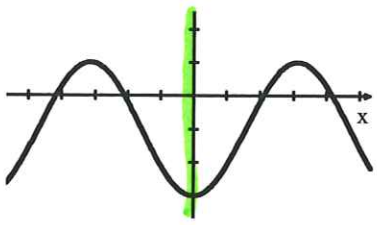
ODD

5.



Neither

6.



Even

State if the following functions are even, odd, or neither. Using algebra.

7. $f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x)$

$= -x^3 - x$

ODD

8. $g(x) = x^4 + 3x^2$

$g(-x) = (-x)^4 + 3(-x)^2$

$= x^4 + 3x^2$

Even

9. $h(x) = -7x^6$

$h(-x) = -7(-x)^6$

$= -7x^6$

Even

10. $w(t) = 8t^5 + 2t$

$w(-t) = 8(-t)^5 + 2(-t)$

$= -8t^5 - 2t$

ODD

11. $a(w) = 1 - w - w^3$

$a(-w) = 1 - (-w) - (-w)^3$

$= 1 + w + w^3$

Neither

12. $f(x) = 2x^{10} + x^6 - 7x^2 - 2$

$f(-x) = 2(-x)^{10} + (-x)^6 - 7(-x)^2 - 2$

$= 2x^{10} + x^6 - 7x^2 - 2$

Even

13. $h(t) = 4t^9$

$h(-t) = 4(-t)^9$

$= -4t^9$

ODD

14. $g(x) = x^2 + x - 3$

$g(-x) = (-x)^2 + (-x) - 3$

$= x^2 - x - 3$

Neither

15. $b(x) = -4x^7 + x^3 - x$

$b(-x) = -4(-x)^7 + (-x)^3 - (-x)$

$= 4x^7 - x^3 + x$

ODD

16. Is $f(x) = 7x^6 - x^4 + 5$ even, odd, or neither

$f(-x) = 7(-x)^6 - (-x)^4 + 5$

$= 7x^6 - x^4 + 5$

Even

1.5B Even and Odd Polynomials

1.5B Test Prep

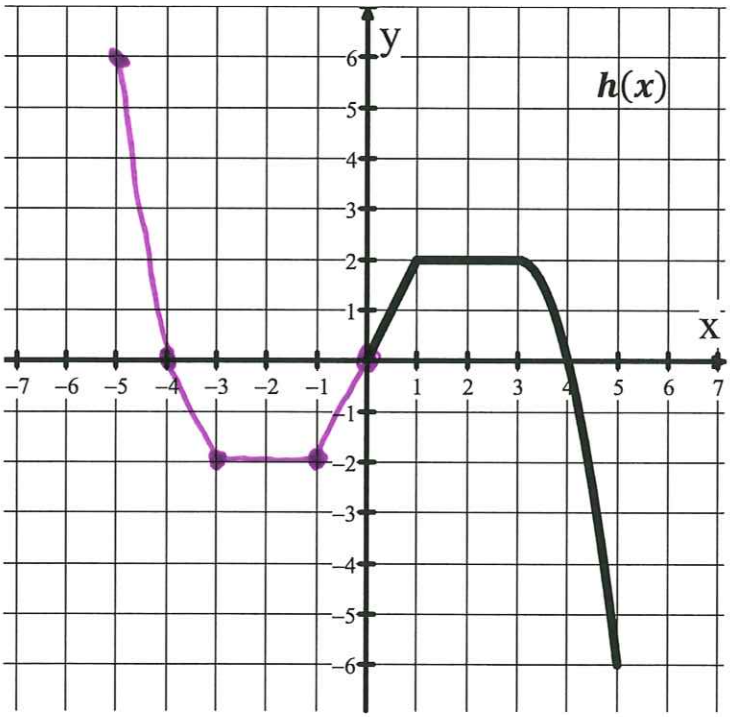
17. Certain values of $g(x)$ are given in the table below. Given that $g(x)$ is odd, fill in the missing values of the table.

x	$g(x)$
-5	10.5
7	-22.5
	-38.5
-7	22.5
5	-10.5
9	38.5

$(x, y) \rightarrow (-x, -y)$

ODD

18. Given that $h(x)$ is continuous on $-5 \leq x \leq 5$ and odd, draw the graph $h(x)$ from $-5 \leq x \leq 0$.



AP Precalc

2.12 Logarithmic Function Manipulation — 2.12 Notes

Write your questions and thoughts here!

Properties of Logarithmic Functions

	Property	….but also
Product Property	$\log_b(xy) = \log_b x + \log_b y$ $\log 4x = \log 4 + \log x$	$\ln 6 + \ln y = \ln 6y$
Quotient Property.	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$ $\log_a\left(\frac{x}{10}\right) = \log_a x - \log_a 10$	$\ln(3y) - \ln z = \ln\left(\frac{3y}{z}\right)$
Power Property.	$\log_b x^k = k \cdot \log_b x$ $\log_3(x)^2 = 2 \cdot \log_3 x$	$10 \cdot \ln(yz) = \ln(yz)^{10}$

Expanding and Condensing

a) $\log(4x^2y)$

$\log 4 + \log x^2 + \log y$

$\log 4 + 2 \cdot \log x + \log y$

b) $\log\left(\frac{\sqrt[3]{x+5}}{y^2}\right)$

$\log(x+5)^{\frac{1}{3}} - \log y^2$

$\frac{1}{3} \cdot \log(x+5) - 2 \cdot \log y$

$\frac{\log(x+5)}{3} - 2 \cdot \log y$

LOG ↓ common log, base 10

c) $2\log_2 x + \frac{1}{2}\log_2(y+1)$

$\log_2 x^2 + \log_2 (y+1)^{\frac{1}{2}}$

$\log_2\left(x^2 \cdot \sqrt{(y+1)}\right)$

d) $2\log_3(ab^2) - 3\log_3 c + \frac{1}{3}\log_3 d$

$\log_3(ab^2)^2 - \log_3 c^3 + \log_3 \sqrt[3]{d}$

$\log_3 \frac{(ab^2)^2 \cdot \sqrt[3]{d}}{c^3}$

Change of Base Formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$\log_4 12$

$\frac{\log 12}{\log 4} \approx 1.792$

LN ↓ Natural log, $\log_e$

*Dirty little secret: MATH ↑↑↑ A: log Base(enter

$\log_{\square}(\square)$

Write your questions and thoughts here!

Logarithmic Function Manipulation Graphically

Expand the function: $f(x) = log_2(4x)$ $= \log_2 4 + \log_2 x$

$= \boxed{2} + \log_2 x$

What happens to the graph?

vertical shift up 2

$\log_2 4 = x$

$2^x = 4$

$2^x = 2^2$

$\boxed{x = 2}$

Expand the function: $f(x) = \ln(x-4)^2$ $= 2 \cdot \ln(x-4)$

What happens to the graph?

horiz. shift right 4 and vertical dilation of 2

Try on your own!

Find the domain/range and describe any transformations on each function.

$log_3(18 - 9x)$ $= \log_3(9(2-x))$

$= \log_3 9 + \log_3(2-x)$

$= 2 + \log_3(-(x-2))$

vert. shift up 2, horiz. reflection, horiz. shift right 2

$\log_3 9 = x$

$3^x = 9$

$3^x = 3^2$

$\boxed{x = 2}$

2.12 Logarithmic Function Manipulation

AP Precalculus

2.12 Practice

Instructions: Let x and y be positive constants. Write each expression as a sum or difference of logarithms.

1. $log_3(xy^3)$

$\log_3 x + \log_3 y^3$

$\log_3 x + 3 \cdot \log_3 y$

2. $\ln\frac{x^2}{y^3}$

$\ln x^2 - \ln y^3$

$2 \cdot \ln x - 3 \cdot \ln y$

3. $\log 1000x^3$

$\log 1000 + \log x^3$

$\log 1000 + 3 \cdot \log x$

4. $log_2(8\sqrt{x})$

$\log_2 8 + \log_2 x^{\frac{1}{2}}$

$3 + \frac{1}{2} \cdot \log_2 x$

5. $log_2\sqrt{\frac{16x^3}{y^2}}$ $= \frac{1}{2}(\log_2 16x^3 - \log_2 y^2)$

$= \frac{1}{2}(\log_2 16 + \log_2 x^3 - 2 \cdot \log_2 y)$

$= \frac{1}{2}(4 + 3 \cdot \log_2 x - 2 \cdot \log_2 y)$

6. $log_3(9x - 27)$

$\log_3[9(x-3)]$

$\log_3 9 + \log_3(x-3)$

$2 + \log_3(x-3)$

Instructions: Let x and y be positive constants. Write each as a single logarithm.

7. $\ln 3 + \ln x$

$\ln 3x$

8. $2\log_3 x - 4\log_3 y$

$\log_3 x^2 - \log_3 y^4$

$\log_3 \frac{x^2}{y^4}$

9. $\frac{1}{2}(\log x + 3\log y)$

$\frac{1}{2}(\log x + \log y^3)$

$\frac{1}{2}(\log xy^3) = (\log(xy^3)^{\frac{1}{2}}$

$= \log\sqrt{xy^3}$

10. $3\log_5(x-4) - 2\log_5 y$

$\log_5 (x-4)^3 - \log_5 y^2$

$\log_5 \frac{(x-4)^3}{y^2}$

CALCULATOR ACTIVE: Instructions: Use the change of base to change each to a logarithm with base 10 or base *e*. Then use a calculator and find the value of the logarithm to the nearest thousandth.

11. $\log_4 123$

$\frac{\log 123}{\log 4} \approx 3.471$

12. $\log_9 578$

$\frac{\log 578}{\log 9} \approx 2.894$

Instructions: Describe any transformations on the function, then find the domain and range of the transformed function.

13. $f(x) = \log_4 16x$

$f(x) = \log_4 16 + \log_4 x$

$f(x) = 2 + \log_4 x$

vertical shift up 2

D: $(0, +\infty)$

R: $(-\infty, +\infty)$

14. $g(x) = \log_2\left(\frac{x}{8}\right)$

$g(x) = \log_2 x - \log_2 8$

$g(x) = \log_2 x - 3$

vertical shift down 3

D: $(0, +\infty)$

R: $(-\infty, +\infty)$

15. $f(x) = \log_5 x^2$

$f(x) = 2 \cdot \log_5 x$

vertical dilation of 2

D: $(0, +\infty)$

R: $(-\infty, +\infty)$

16. $h(x) = \log_2(32 - 16x)$

$h(x) = \log_2(16(2-x))$

$h(x) = \log_2 16 + \log_2(-(x-2))$

$h(x) = 4 + \log_2(-(x-2))$

vertical shift up 4

horiz. shift right 2

horiz. reflection

D: $(-\infty, 2)$

R: $(-\infty, +\infty)$

2.12 Logarithmic Function Manipulation

2.12 Test Prep

17. Let x and y be positive constants. Which of the following is equivalent to $5\log x + 6\log y$.

(A) $\log(x^5 + y^6)$
(B) $\log(x^5 y^6)$
(C) $\log(x^5 - y^6)$
(D) $\log(5x + 6y)$

$\log x^5 + \log y^6$
$\log x^5 y^6$

18. Let x and y be positive constants. Which of the following is NOT equivalent to:

$log_3(x+3) + 2log_3(x-1) - log_3(x+2)$.

X (A) $log_3\left(\frac{(x+3)(x-1)}{(x+2)}\right)$ $\log_3(x+3) + \log_3(x-1)^2 - \log_3(x+2)$

✓ (B) $log_3\left(\frac{(x+3)(x-1)(x-1)}{(x+2)}\right)$ $\log_3 \frac{(x+3)(x-1)^2}{(x+2)}$

✓ (C) $log_3\left(\frac{(x+3)(x-1)^2}{(x+2)}\right)$ $\log_3 \frac{(x+3)(x-1)(x-1)}{(x+2)}$

✓ (D) $log_3\left(\frac{(x+3)(x-2x+1)}{(x+2)}\right)$

$(x-1)^2 = (x-1)(x-1) = x^2 - 2x + 1$

19. If $\log 3 = x$ and $\log 4 = y$, which of the following expresses $\log\sqrt{12}$?

(A) $\frac{1}{2}(x-y)$
(B) $\frac{1}{2}(xy)$
(C) $\frac{1}{2}x + 2$
(D) $\frac{1}{2}(x+y)$

$\log\sqrt{3\cdot 4}$
$\log\sqrt{3} + \log\sqrt{4} = \log(3)^{\frac{1}{2}} + \log(4)^{\frac{1}{2}}$
$= \frac{1}{2}\log 3 + \frac{1}{2}\log 4$
$= \frac{1}{2}(\log 3 + \log 4) = \frac{1}{2}(x+y)$

20. Approximate $log_b 18$, given that $log_b 2 \approx .4307$ and $log_b 3 \approx .6827$.

(A) 0.8968
(B) 1.1134
(C) 1.5441
(D) 1.7961

$\log_b 18 = \log_b(2\cdot 9) = \log_b 2\cdot 3^2$
$= \log_b 2 + 2\cdot\log_b 3$
$= 0.4307 + 0.6827$
$= 1.7961$