Bluebonnet Learning
Secondary Mathematics
EDITION 1

Grade 8

Skills Practice

STUDENT EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

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GRADE 8

MODULE 1

Transforming Geometric Objects

TOPIC 1: Rigid Motion Transformations

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Name _____ Date _____

I. Introduction to Congruent Figures

Topic Practice

A. Consider the figures. Make a conjecture about which figures are congruent to the original figure shown in the center. Then, use patty paper to investigate your conjecture. Finally, justify your conjecture by stating how you can move from the original figure to each congruent figure by sliding, flipping, or spinning the original figure.

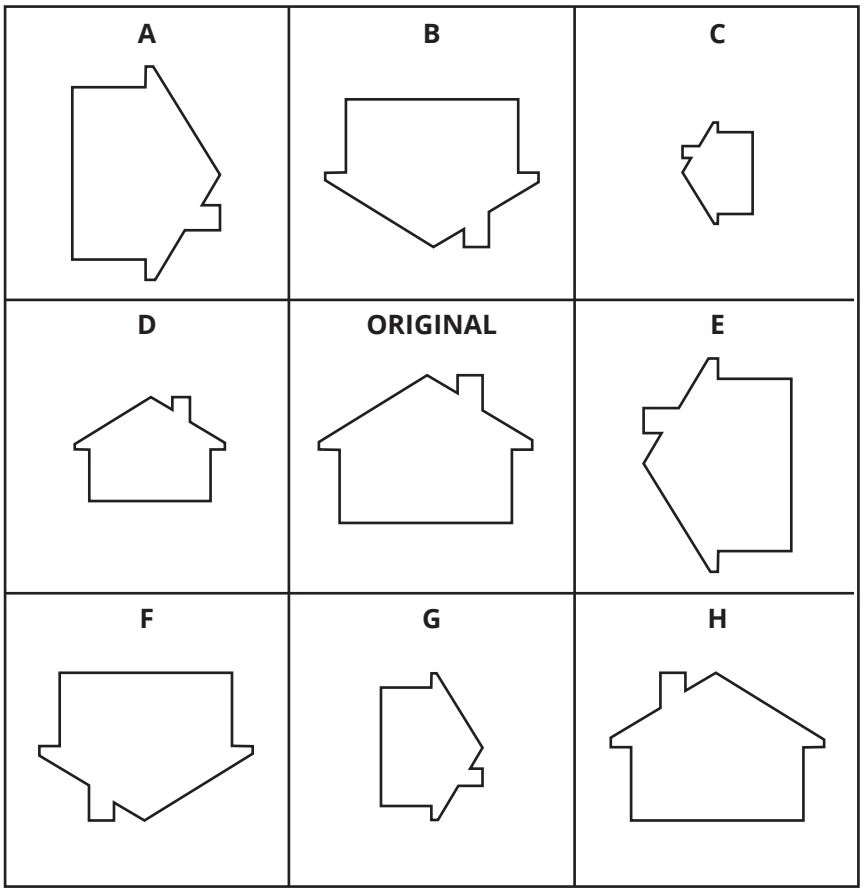

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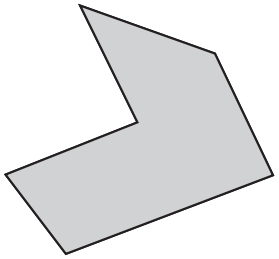
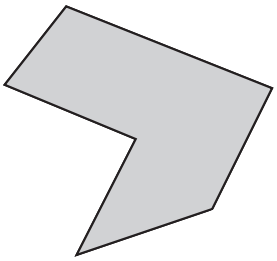
	Figure	Congruent to the original figure?	How do you move the original figure onto the congruent figure?
1.	A		
2.	B		
3.	C		
4.	D		
5.	E		
6.	F		
7.	G		
8.	H		

Extension

1. The figure on the left reflects, or flips, over a *line of reflection* to create the figure on the right. Describe the effect on the reflection and determine the location of the line of reflection.





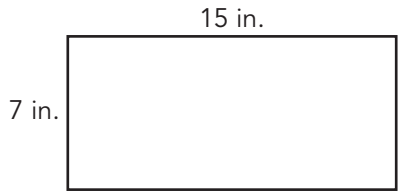
Spaced Practice

1. Determine whether the equation $a^2 + b^2 = c^2$ is true for the given values of a, b, and c.

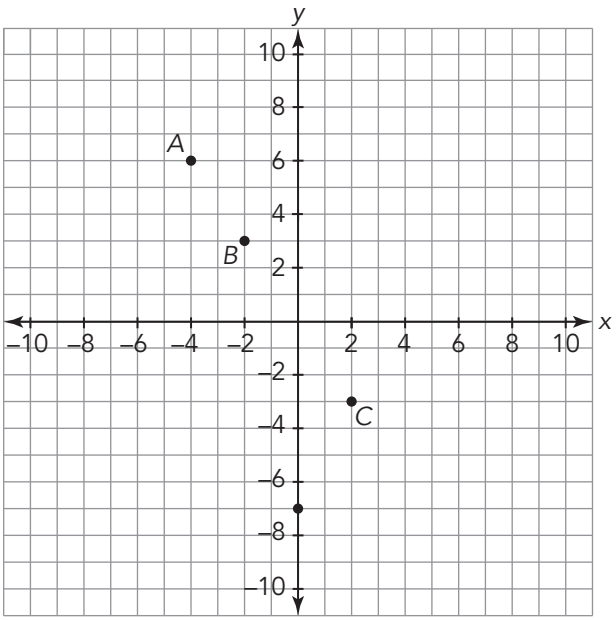
   a. $a = 4$, $b = 3$, and $c = 5$

   b. $a = 24$, $b = 7$, and $c = 25$

2. Calculate the area of the figure. Then, determine what happens to the area of the rectangle when you double the length of each side.



3. Write the ordered pair for each point plotted on the coordinate plane.

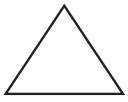


II. Introduction to Rigid Motions

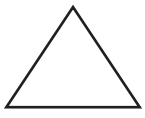
Topic Practice

A. Sketch each rigid motion transformation.

1. Translate the triangle down.

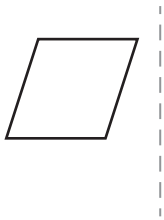


2. Translate the triangle to the right.

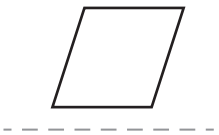


3. What changes about a figure after a translation? What stays the same?

4. Reflect the rhombus over the vertical line.

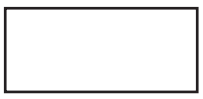


5. Reflect the rhombus over the horizontal line.



6. What changes about a figure after a reflection? What stays the same?

7. Rotate the rectangle 90 degrees clockwise.



8. Rotate the rectangle 90 degrees counterclockwise.



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9. What changes about a figure after a rotation? What stays the same?

Extension

1. Assume that an image is created by rotating a pre-image figure 180°. Explain how you could determine the location of the center of rotation.

Spaced Practice

1. Determine which figures are congruent to Figure *A*. Follow the steps given as you investigate each shape.
   - Make a conjecture about which figures are congruent to Figure *A*.
   - Justify your conjecture by stating how you can move from Figure *A* to each congruent figure by translating, reflecting, or rotating Figure *A*.

Figure *A*

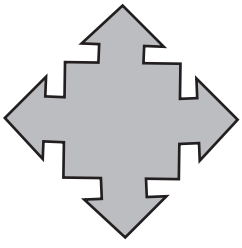


Figure *B*

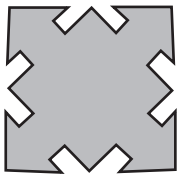
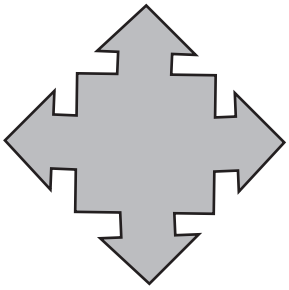


Figure *C*

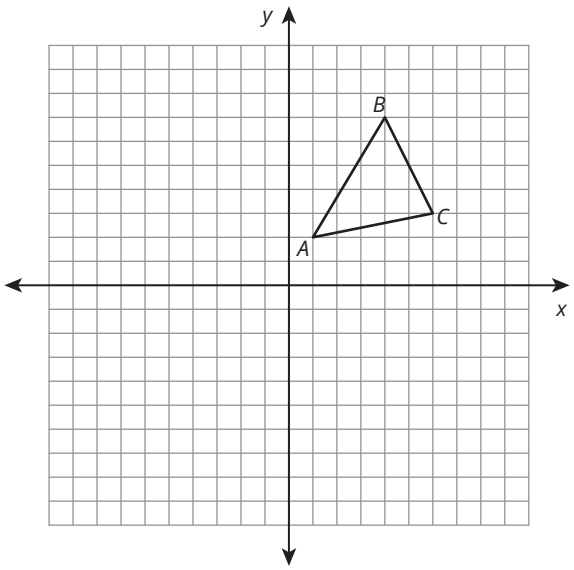


III. Translations of Figures on the Coordinate Plane

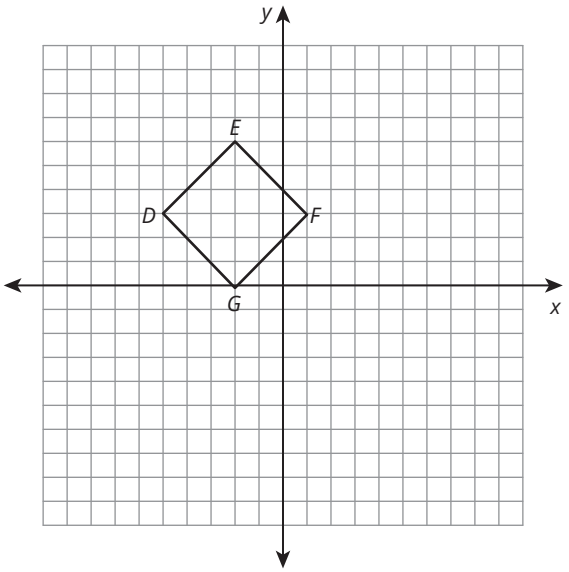
Topic Practice

A. Sketch the translation of each given figure in the coordinate plane.

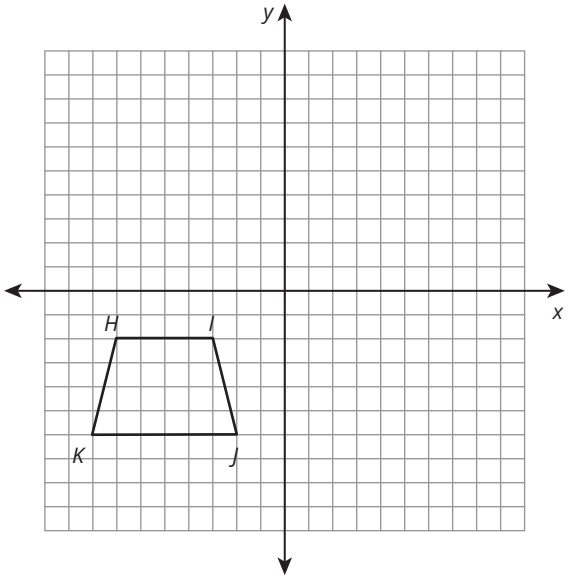
1. Translate the given figure −7 units horizontally.



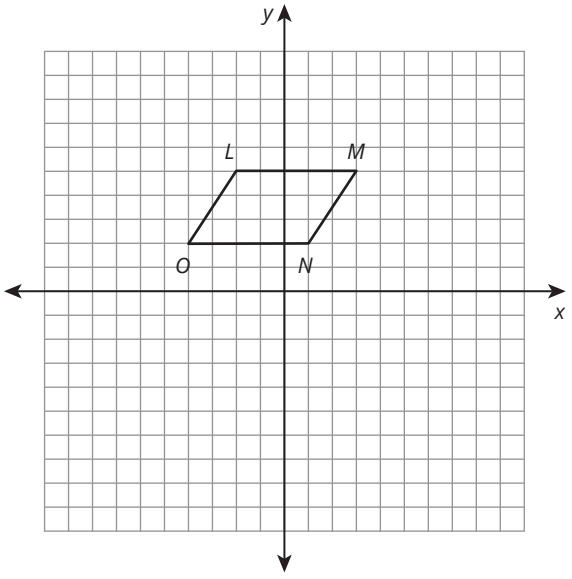
2. Translate the given figure 3 units horizontally.



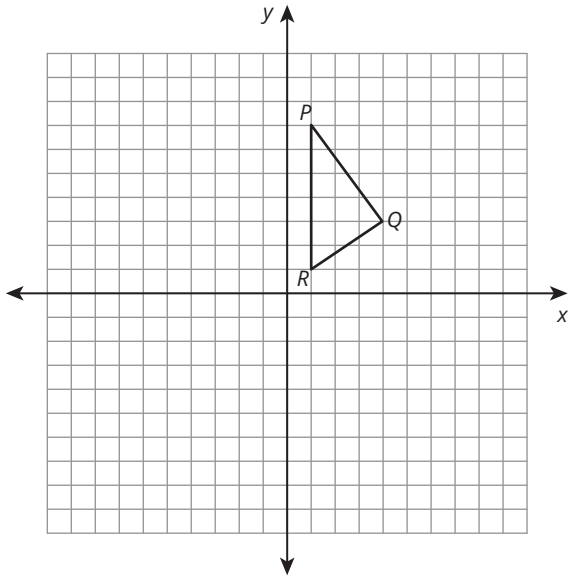
3. Translate the given figure 8 units vertically.



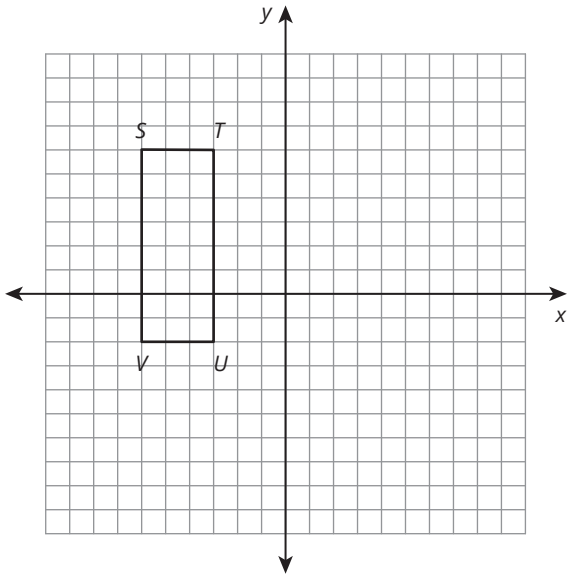
4. Translate the given figure −7 units vertically.



5. Translate the given figure 3 units horizontally and −8 units vertically.

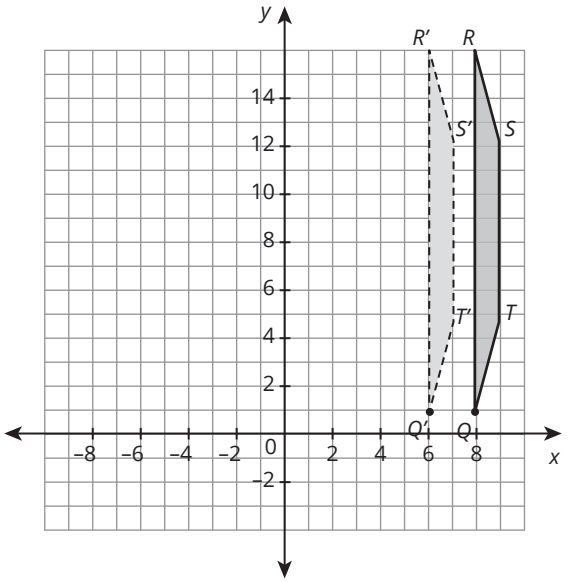


6. Translate the given figure 9 units horizontally and −4 units vertically.

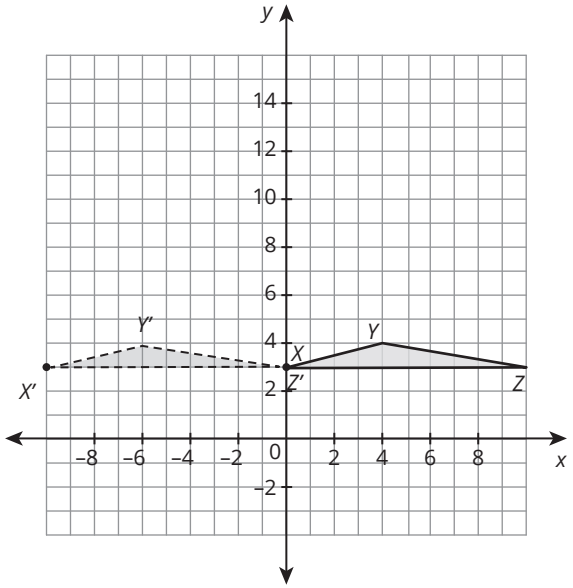


B. Describe the translation needed to map each pre-image onto each congruent image. Write the algebraic rule for the given transformation.

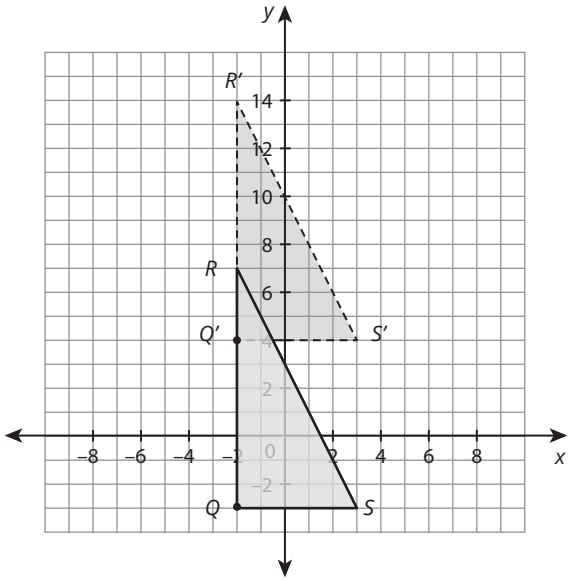
1. Pre-image: Quadrilateral *RSTQ*
Image: Quadrilateral *R′S′T′Q′*



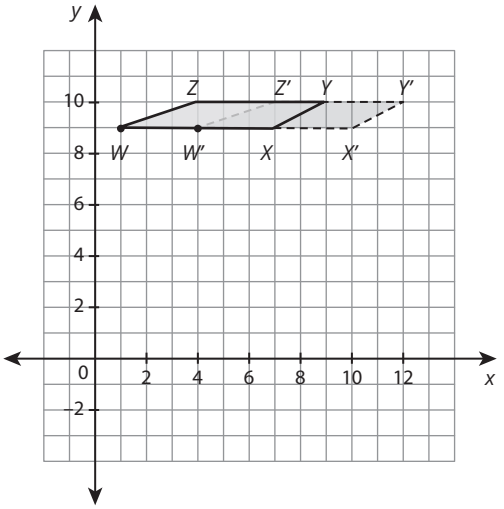
2. Pre-image: Triangle *XYZ*
Image: Triangle *X′Y′Z′*



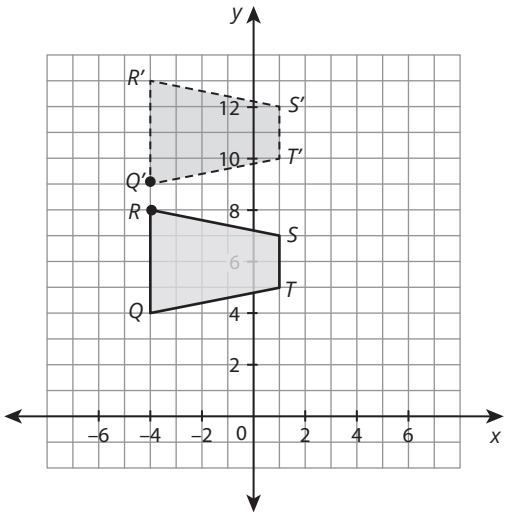
3. Pre-image: Triangle *QRS*
Image: Triangle *Q′R′S′*



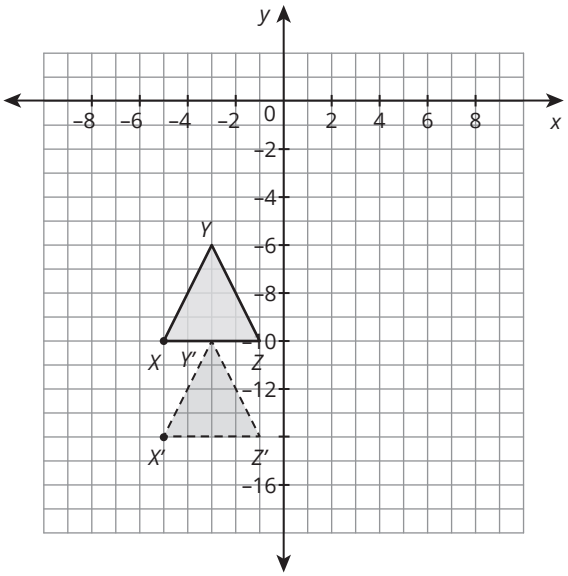
4. Pre-image: Quadrilateral *WXYZ*
Image: Quadrilateral *W′X′Y′Z′*



5. Pre-image: Quadrilateral *QRST*
Image: Quadrilateral *Q′R′S′T′*



6. Pre-image: Triangle *XYZ*
Image: Triangle *X′Y′Z′*



C. Write an algebraic rule that represents the given translation. Determine the coordinates of the new image.

1. Triangle *ABC* with coordinates *A*(2, 4), *B*(3, 6), and *C*(5, 1) is translated 4 units horizontally.

2. Parallelogram *DEFG* with coordinates *D*(0, 2), *E*(1, 5), *F*(6, 5), and *G*(5, 2) is translated −7 units horizontally.

3. Trapezoid *HIJK* with coordinates *H*(−1, 3), *I*(−1, −3), *J*(−4, −1), and *K*(−4, 1) is translated 3 units vertically.

4. Square *LMNO* with coordinates *L*(−1, 7), *M*(3, 7), *N*(3, 3), and *O*(−1, 3) is translated −5 units vertically.

5. Rectangle *WXYZ* with coordinates *W*(−8, −1), *X*(−2, −1), *Y*(−2, −3), and *Z*(−8, −3) is translated 13 units horizontally.

6. Rhombus *ABCD* with coordinates *A*(7, 8), *B*(9, 5), *C*(7, 2), and *D*(5, 5) is translated −9 units vertically.

7. Triangle *PQR* with coordinates *P*(3, −4), *Q*(6, −1), and *R*(6, −6) is translated −3 units horizontally and 6 units vertically.

8. Triangle *STU* with coordinates *S*(0, 0), *T*(4, 4), and *U*(5, 0) is translated 10 units horizontally and −2 units vertically.

9. Triangle *DEF* with coordinates *D*(0, 12), *E*(−3, −7), and *F*(−5, 1) is translated −12 units horizontally and −8 units vertically.

10. Parallelogram *GHIJ* with coordinates *G*(0, 0), *H*(2, 8), *I*(8, 8), and *J*(6, 0) is translated −8 units horizontally and −8 units vertically.

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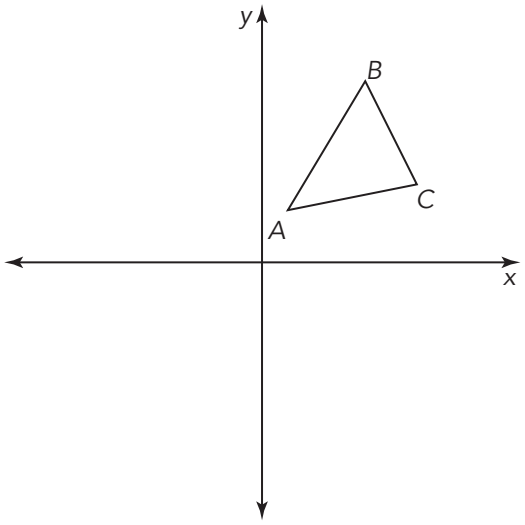
Extension

1. A point at the origin is repeatedly translated c units horizontally and d units vertically. Write the coordinates of the translated point if the translation sequence is repeated n times.

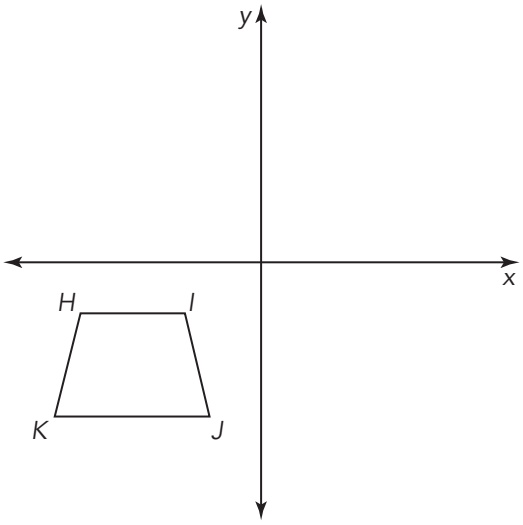
Spaced Practice

1. Sketch the translation of each figure.

   a. Translate the figure to the left



   b. Translate the figure up.



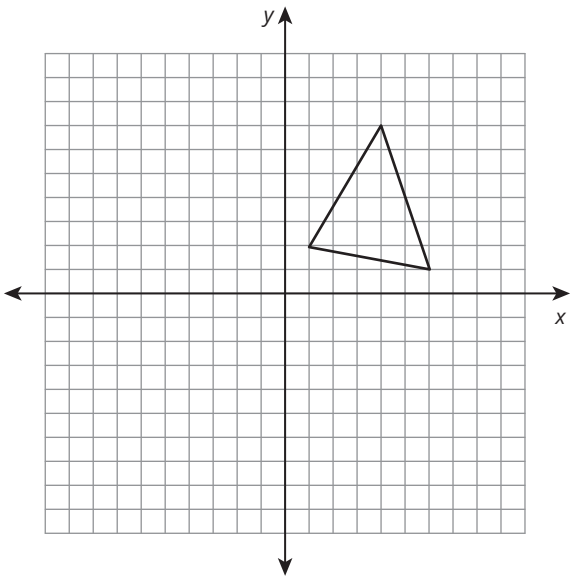
2. What is true about the relationship between the image and pre-image in each translation?

IV. Reflections of Figures on the Coordinate Plane

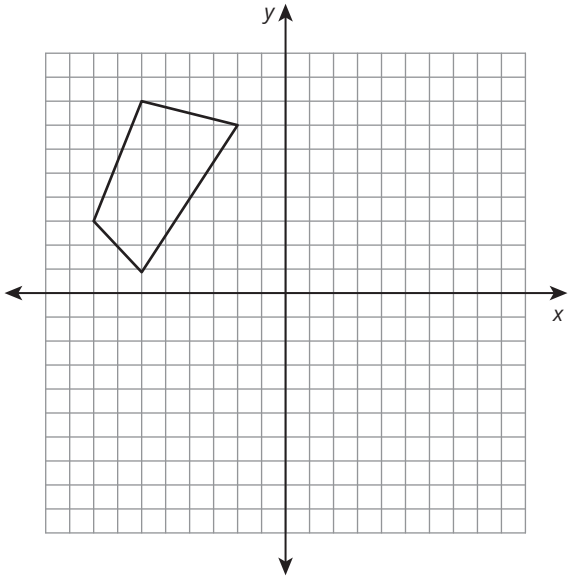
Topic Practice

A. Sketch the reflection of the figure in each given coordinate plane over the specified axis or line.

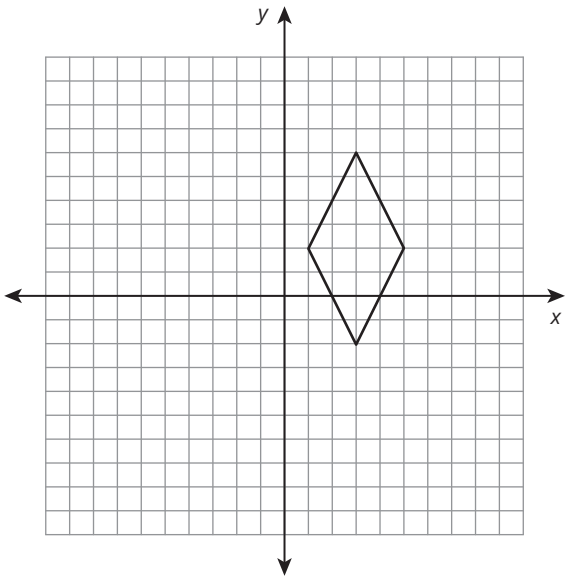
1. Reflect the triangle over the *x*-axis.



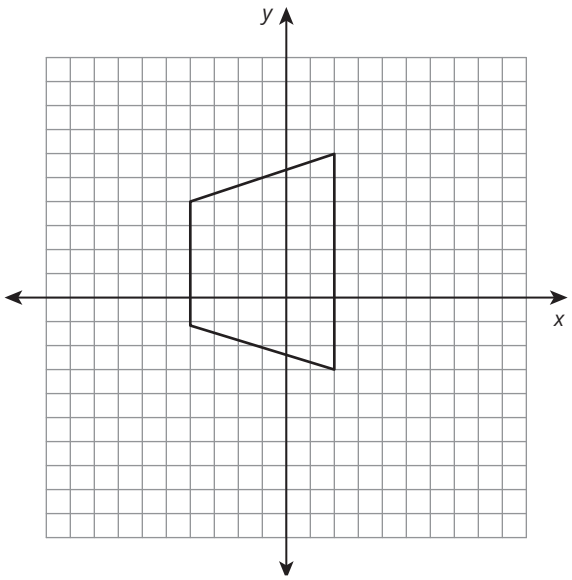
2. Reflect the quadrilateral over the *y*-axis.



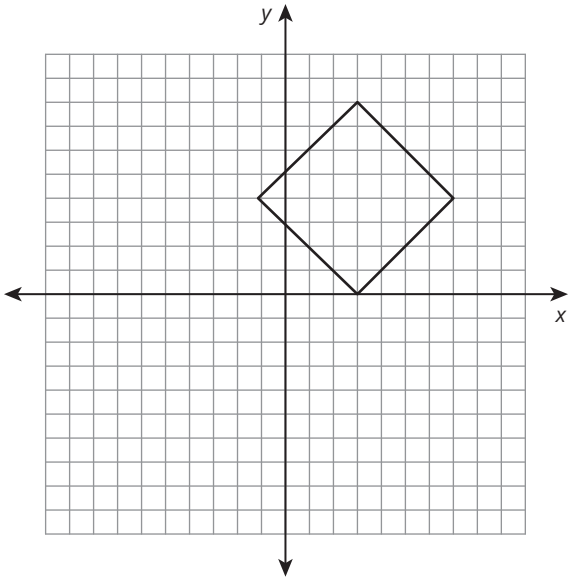
3. Reflect the rhombus over the *x*-axis.



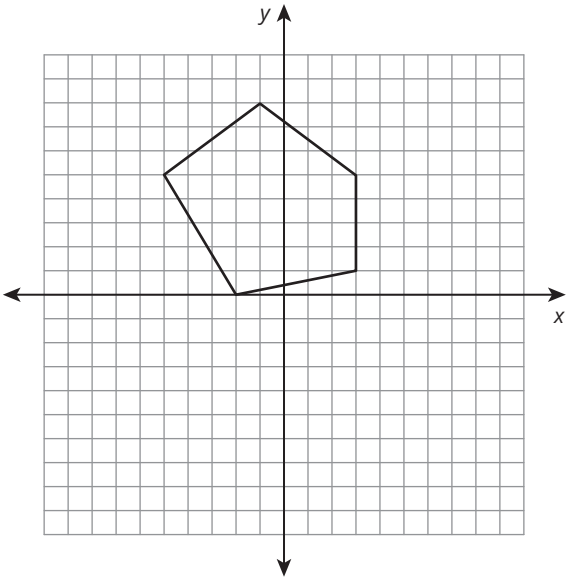
4. Reflect the trapezoid over the *y*-axis.



5. Reflect the square over the *x*-axis.

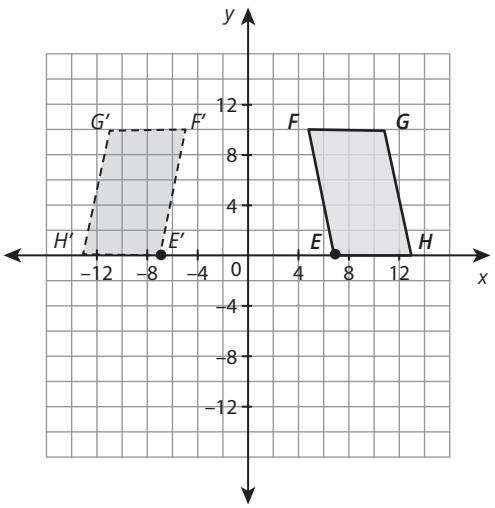


6. Reflect the pentagon over the *y*-axis.

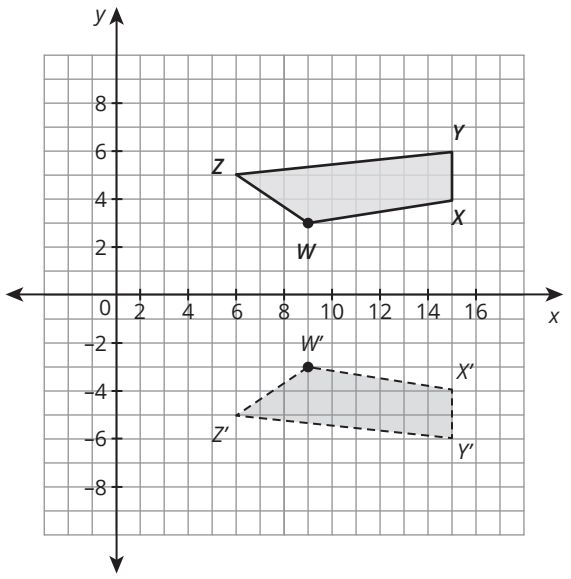


B. Describe the reflection needed to map each pre-image onto each congruent image. Write the algebraic rule for the given transformation.

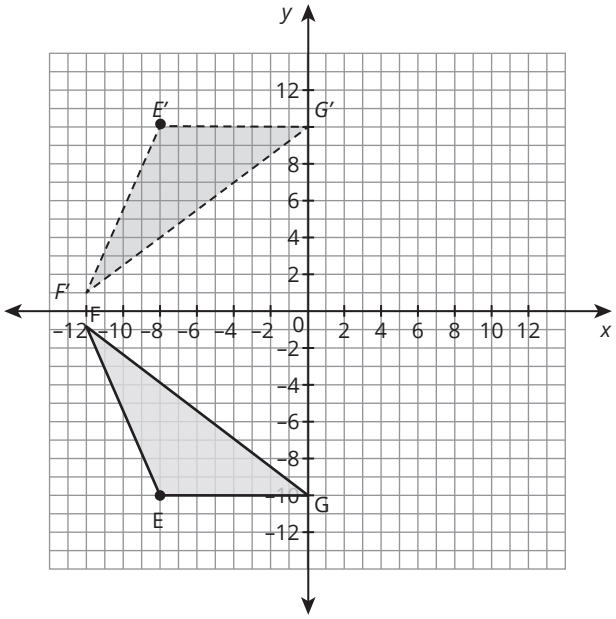
1. Pre-image: Quadrilateral *EFGH*
Image: Quadrilateral *E′F′G′H′*



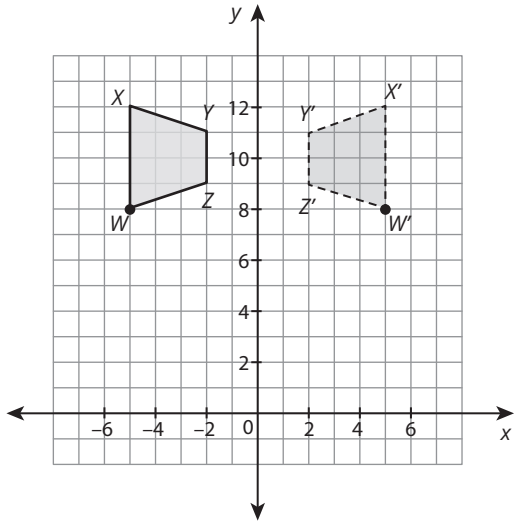
2. Pre-image: Quadrilateral *WXYZ*
Image: Quadrilateral *W′X′Y′Z′*



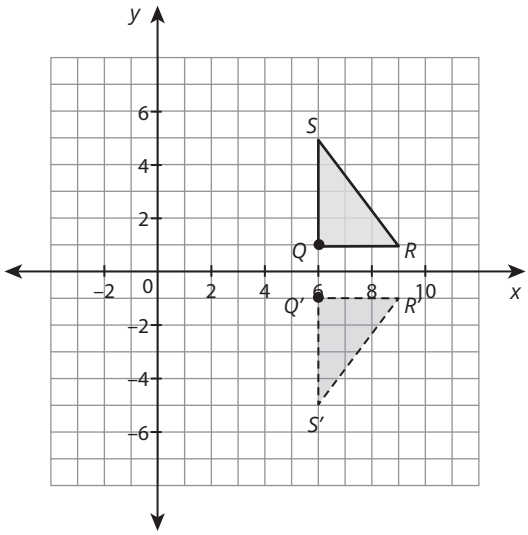
3. Pre-image: Triangle *EFG*
Image: Triangle *E′F′G′*



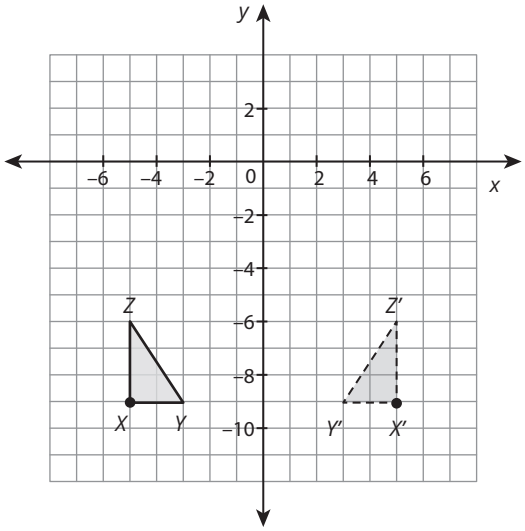
4. Pre-image: Quadrilateral *WXYZ*
Image: Quadrilateral *W′X′Y′Z′*



5. Pre-image: Triangle *QRS*
Image: Triangle *Q′R′S′*



6. Pre-image: Triangle *XYZ*
Image: Triangle *X′Y′Z′*



C. Write an algebraic rule that represents the given reflection. Determine the coordinates of the new image.

1. A triangle with vertices $A(1, 3)$, $B(4, 8)$, and $C(5, 2)$ is reflected over the x-axis.

2. A triangle with vertices $A(1, 3)$, $B(4, 8)$, and $C(5, 2)$ is reflected over the y-axis.

3. A triangle with vertices $D(-2, 5)$, $E(-1, 1)$, and $F(3, 6)$ is reflected over the x-axis.

4. A triangle with vertices $D(-2, 5)$, $E(-1, 1)$, and $F(3, 6)$ is reflected over the y-axis.

5. A square with vertices $G(0, 2)$, $H(-2, 4)$, $J(0, 6)$, and $K(2, 4)$ is reflected over the x-axis.

6. A square with vertices $G(0, 2)$, $H(-2, 4)$, $J(0, 6)$, and $K(2, 4)$ is reflected over the y-axis.

7. A trapezoid with vertices $L(-4, 0)$, $M(-4, -8)$, $N(-6, -5)$, and $O(-6, -3)$ is reflected over the x-axis.

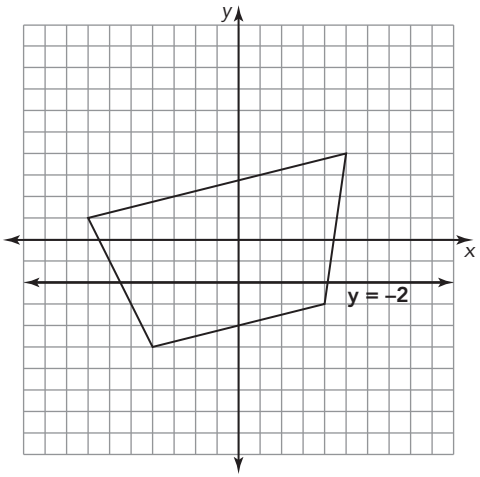
8. A triangle with vertices $P(0, 0)$, $Q(-5, 0)$, and $R(0, 5)$ is reflected over the y-axis.

9. A pentagon with vertices $S(-4, 2)$, $T(0, 5)$, $U(4, 2)$, $V(2, -3)$, and $W(-2, -3)$ is reflected over the x-axis.

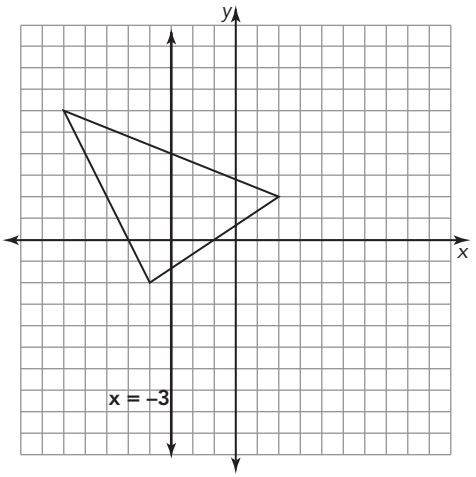
10. A triangle with vertices $X(2, 5)$, $Y(4, 1)$, and $Z(6, 8)$ is reflected over the x-axis and then reflected over the y-axis.

Extension

1. Reflect the quadrilateral across the line $y = -2$.



2. Reflect the triangle across the line $x = -3$.



Spaced Practice

Determine the coordinates of the image following each given translation.

1. Triangle *ABC* with coordinates *A*(2, 4), *B*(3, 6), and *C*(5, 1) is translated 4 units horizontally.

2. Parallelogram *DEFG* with coordinates *D*(0, 2), *E*(1, 5), *F*(6, 5), and *G*(5, 2) is translated (−7) units horizontally.

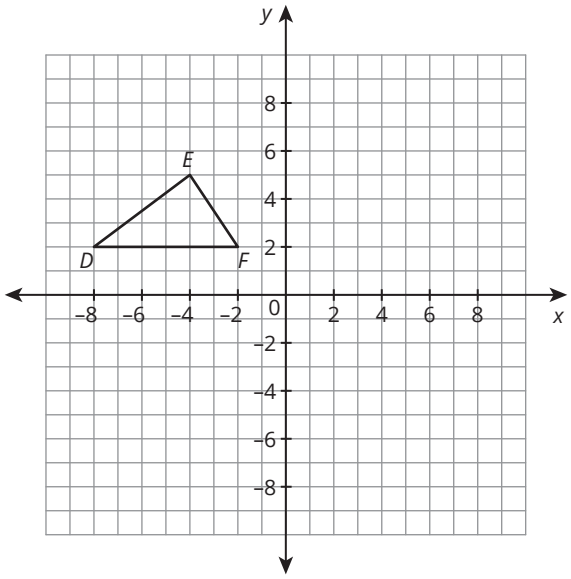
3. For each translation described, what is the relationship between the image and pre-image?

V. Rotations of Figures on the Coordinate Plane

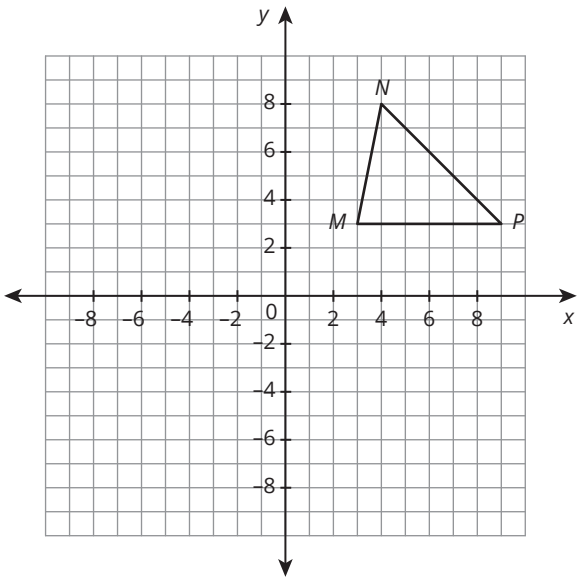
Topic Practice

A. Sketch the rotation of each given figure on the coordinate plane.

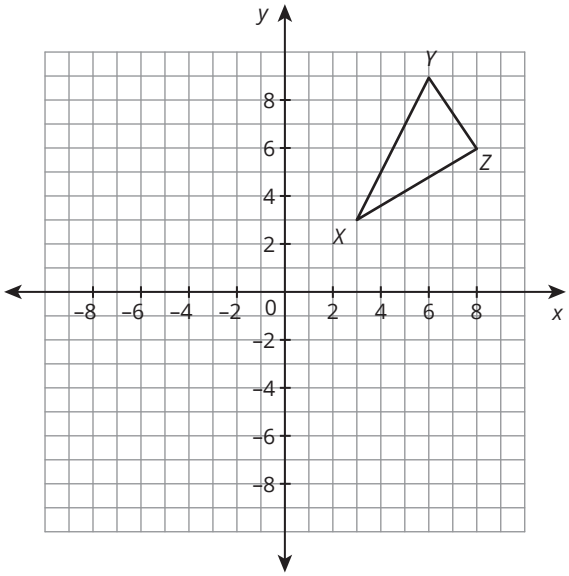
1. Rotate △*DEF* 90° counterclockwise about the origin.



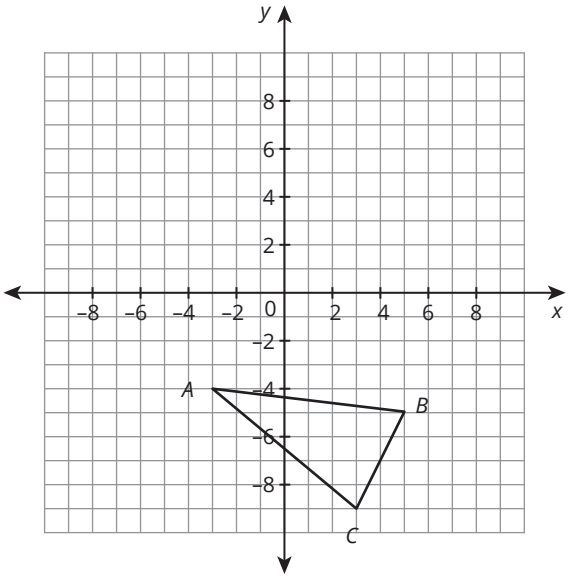
2. Rotate △*MNP* 90° clockwise about the origin.



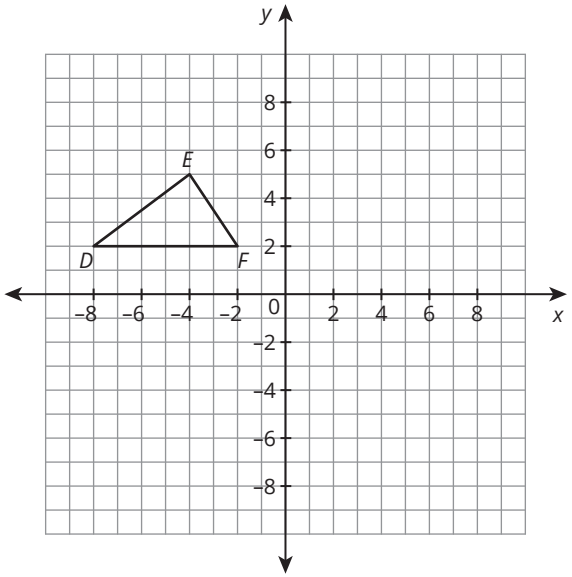
3. Rotate △*XYZ* 180° about the origin.



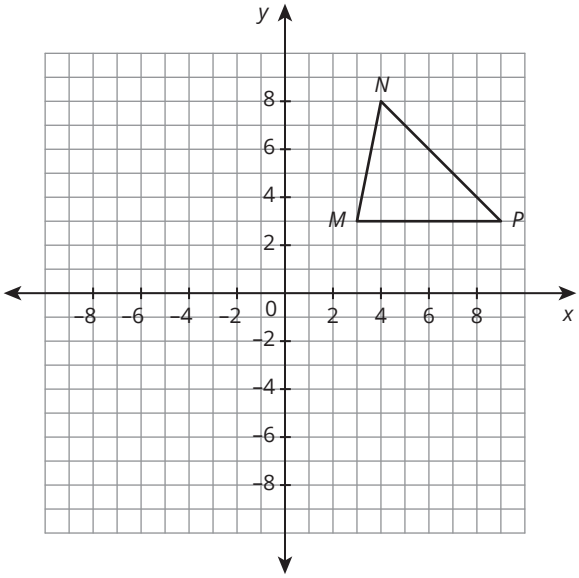
4. Rotate △*ABC* 360° counterclockwise about the origin.



5. Rotate $\triangle DEF$ 270° clockwise about the origin.

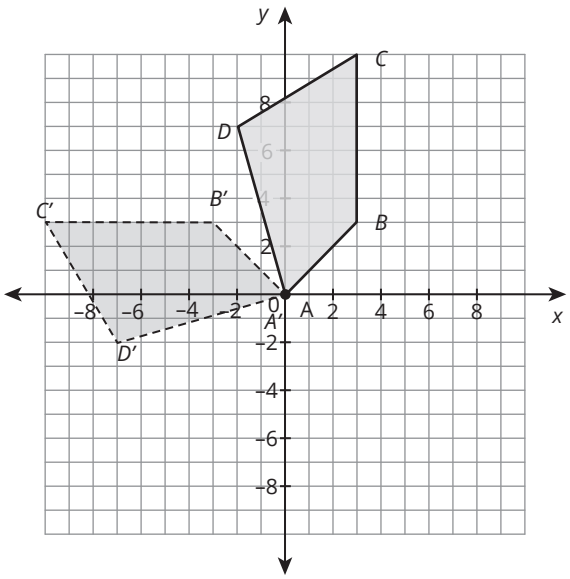


6. Rotate $\triangle MNP$ 270° counterclockwise about the origin.

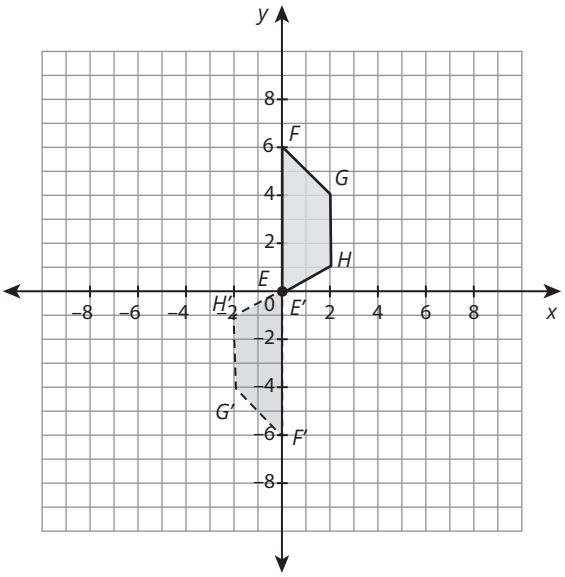


B. Describe the rotation needed to map each pre-image onto each congruent image. Write the algebraic rule for the given transformation.

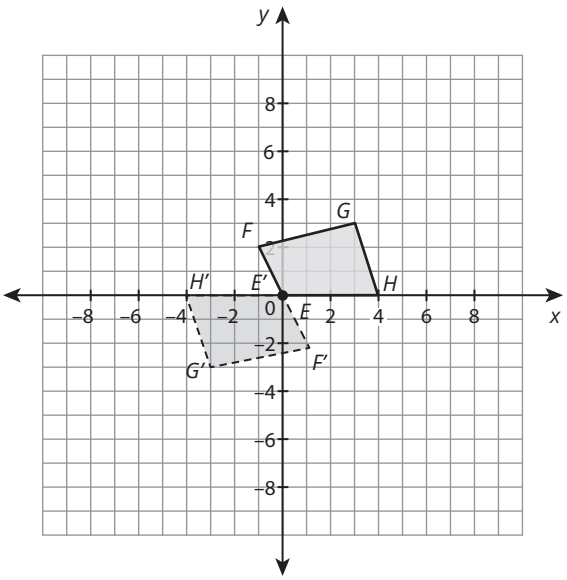
1. Pre-image: Quadrilateral *ABCD*
Image: Quadrilateral $A'B'C'D'$



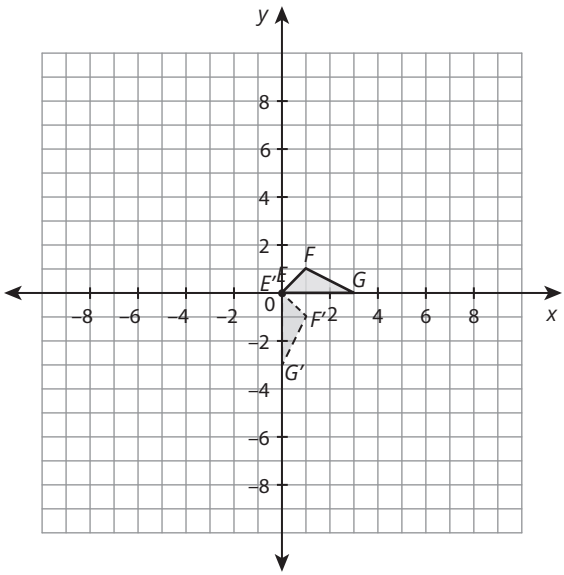
2. Pre-image: Quadrilateral *EFGH*
Image: Quadrilateral $E'F'G'H'$



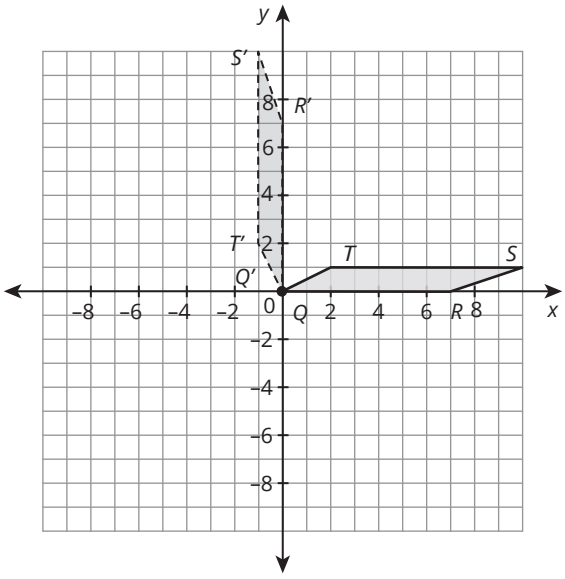
3. Pre-image: Quadrilateral *EFGH*
Image: Quadrilateral *E′F′G′H′*



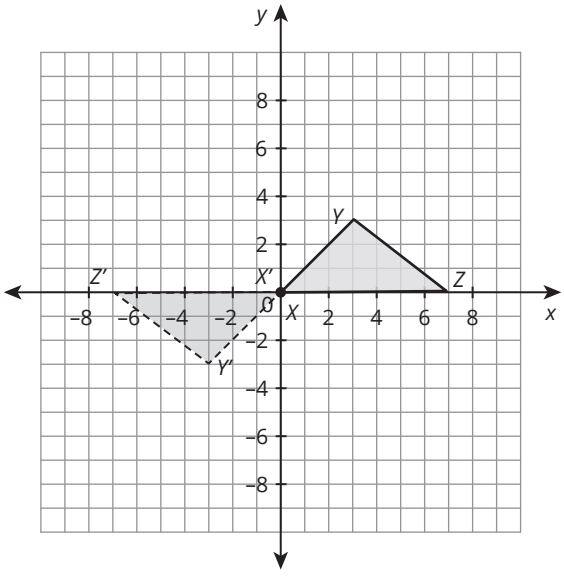
4. Pre-image: Triangle *EFG*
Image: Triangle *E′F′G′*



5. Pre-image: Quadrilateral *QRST*
Image: Quadrilateral *Q′R′S′T′*



6. Pre-image: Triangle *XYZ*
Image: Triangle *X′Y′Z′*



C. Write an algebraic rule that represents the given rotation. Determine the coordinates of the new image.

1. Triangle *DEF* with coordinates *D*(−2, 2), *E*(1, 5), and *F*(4, −1) is rotated 90° counterclockwise about the origin.

2. Triangle *NPQ* with coordinates *N*(12, −3), *P*(1, 2), and *Q*(9, 0) is rotated 180° about the origin.

3. Triangle *ABC* with coordinates *A*(− 2, 2), *B*(1, 5), and *C*(4, −1) is rotated 90° clockwise about the origin.

4. Triangle *XYZ* with coordinates *X*(10, −10), *Y*(8, 4), and *Z*(1, 9) is rotated 270° clockwise about the origin.

5. Triangle *GHJ* with coordinates *G*(2, −9), *H*(3, 8), and *J*(1, 6) is rotated 270° counterclockwise about the origin.

6. Triangle *KLM* with coordinates *K*(−4, 2), *L*(−8, 7), and *M*(3, −3) is rotated 360° about the origin.

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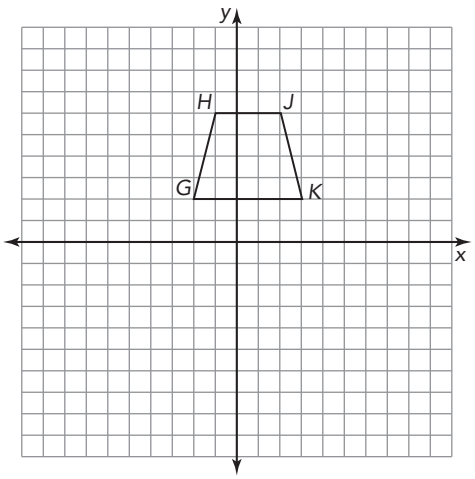
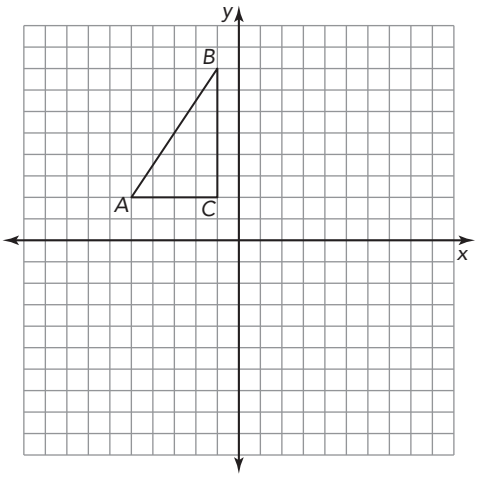
Extension

1. Rotate Trapezoid *GHJK* 90° clockwise around point *G*.



2. Rotate $\triangle ABC$ 135° clockwise around point *C*.



Spaced Practice

Consider a triangle with the vertices *A*(1, 3), *B*(4, 8), and *C*(5, 2).
Determine the vertices of each described transformation.

1. A reflection across the *x*-axis.

2. A reflection across the *y*-axis.

3. A translation 5 units horizontally.

4. A translation −4 units vertically.

Rewrite each expression using properties.

5. $2(x + 4) - 3(x - 5)$

6. $10 - 8(2x - 7)$

VI. Congruence and Rigid Motions

Topic Practice

A. Answer each question related to congruency.

1. Draw and label a pair of congruent triangles.

2. Write a congruence statement for the triangles.

3. Identify each pair of congruent line segments in the drawing you made for Question 1.

4. Identify each pair of congruent angles in the drawing you made for Question 1.

5. Identify each pair of corresponding sides in the drawing you made for Question 1.

6. Identify each pair of corresponding angles in the drawing you made for Question 1.

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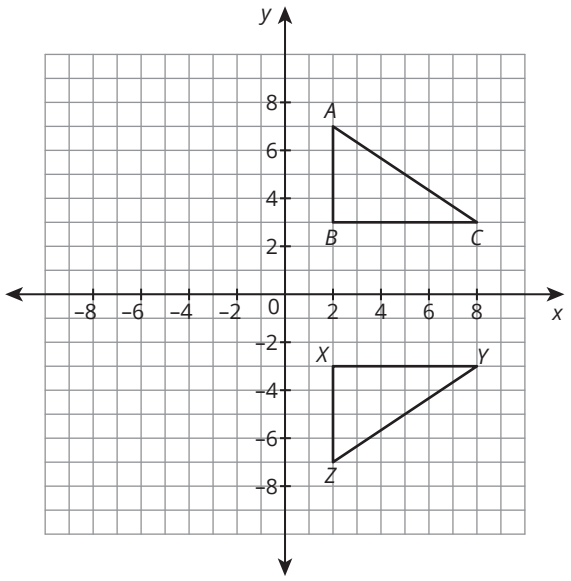
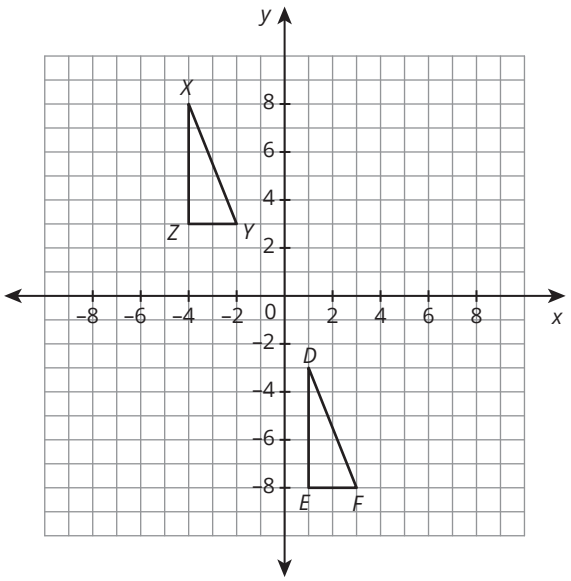
B. Identify the transformation used to create $\triangle XYZ$ in each.

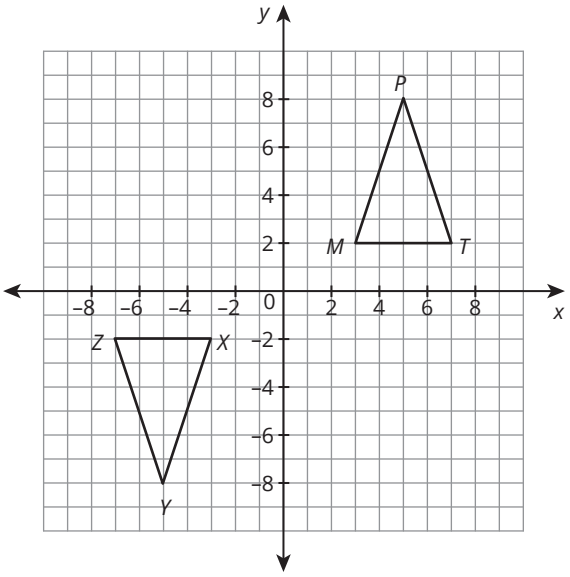
1.



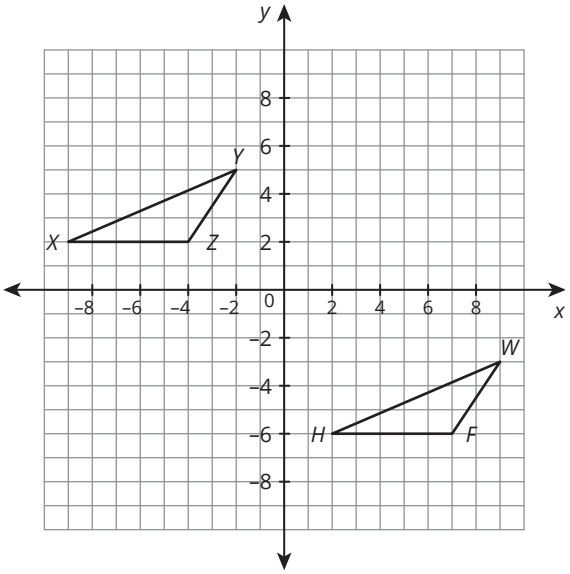
2.



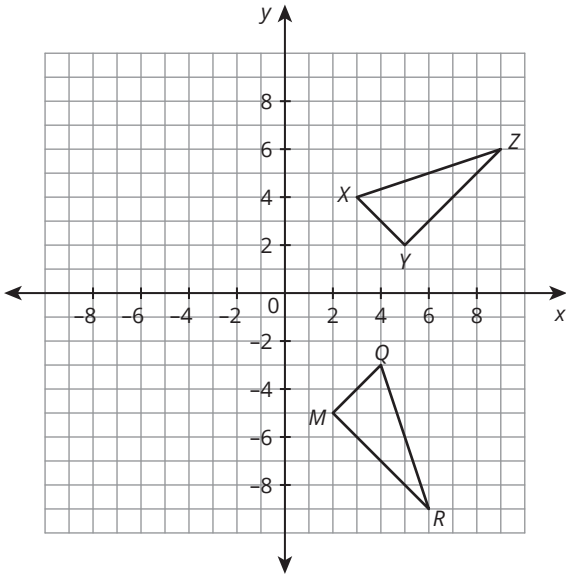
3.



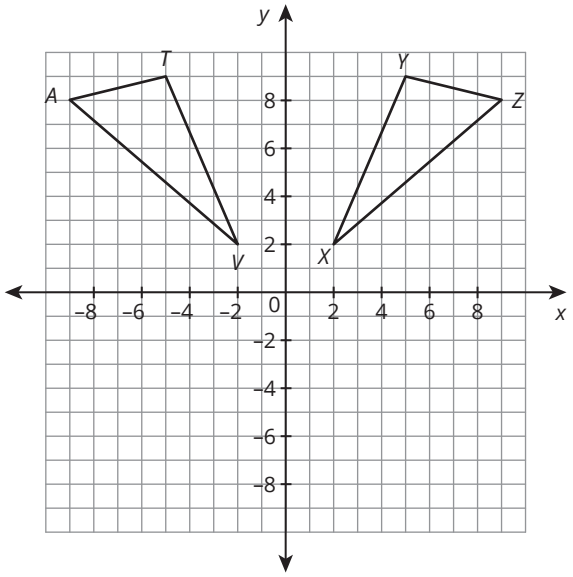
4.



5.



6.



7. Given that two figures are congruent, what else can you determine about the figures?

C. Describe the transformation used to form each triangle in words and with an algebraic rule.

1. The vertices of $\triangle ABC$ are $A(2, 4)$, $B(5, 7)$, and $C(3, 1)$. Describe the translation used to form $\triangle A'B'C'$ for $A'(-5, 4)$, $B'(-2, 7)$, and $C'(-4, 1)$.

2. The vertices of $\triangle DEF$ are $D(0, 3)$, $E(1, 8)$, and $F(-3, 4)$. Describe the rotation used to form $\triangle D'E'F'$ for $D'(3, 0)$, $E'(8, -1)$, and $F'(4, 3)$.

3. The vertices of $\triangle GHJ$ are $G(-5, 3)$, $H(2, 6)$, and $J(-6, 2)$. Describe the reflection used to form $\triangle G'H'J'$ for $G'(5, 3)$, $H'(-2, 6)$, and $J'(6, 2)$.

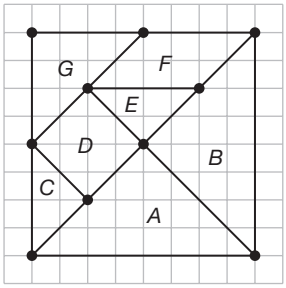
4. The vertices of $\triangle KMN$ are $K(12, 3)$, $M(-5, 2)$, and $N(8, -4)$. Describe the translation used to form $\triangle K'M'N'$ for $K'(18, 0)$, $M'(1, -1)$, and $N'(14, -7)$.

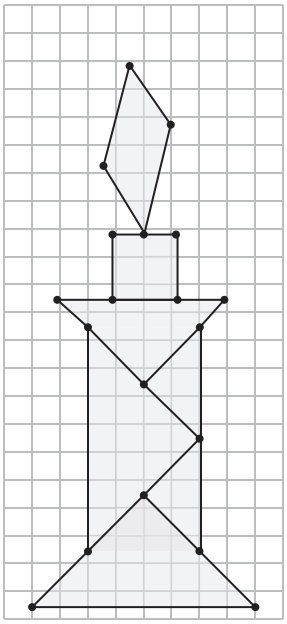
5. The vertices of $\triangle PQR$ are $P(9, -2)$, $Q(1, 0)$, and $R(-7, 3)$. Describe the rotation used to form $\triangle P'Q'R'$ for $P'(-9, 2)$, $Q'(-1, 0)$, and $R'(7, -3)$.

6. The vertices of $\triangle STW$ are $S(15, -6)$, $T(-2, 3)$, and $W(-8, -8)$. Describe the reflection used to form $\triangle S'T'W'$ for $S'(15, 6)$, $T'(-2, -3)$, and $W'(-8, 8)$.

Extension

1. The tangram is a popular Chinese puzzle that consists of seven geometric shapes. The shapes are composed into figures using all seven pieces. The seven pieces fit together to form a square. Determine the transformations of each shape required to create the candle pictured.





Spaced Practice

1. Triangle *HOP* has coordinates *H*(2, 1), *O*(−3, 4), and *P*(5, 7). Determine the coordinates of the image of △*HOP* after each rotation.

   a. Rotation 90° clockwise about the origin

   b. Rotation 90° counterclockwise about the origin

   c. Rotation 180° about the origin

2. Combine like terms to rewrite each expression.

   a. $\left(4\frac{1}{2}x - 3\right) + \left(-2 + 1\frac{3}{4}x\right)$

   b. $4 - (2.3x - 7)$

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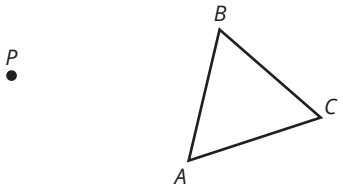
Name _____ Date _____

I. Dilation of Figures

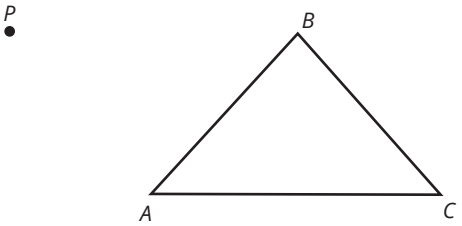
Topic Practice

A. Dilate each triangle using *P* as the center of dilation and the given scale factor.

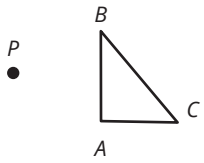
1. Scale factor: 2



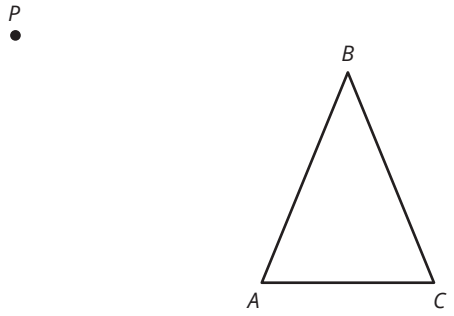
2. Scale factor: $\frac{1}{2}$



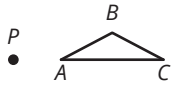
3. Scale factor: 3



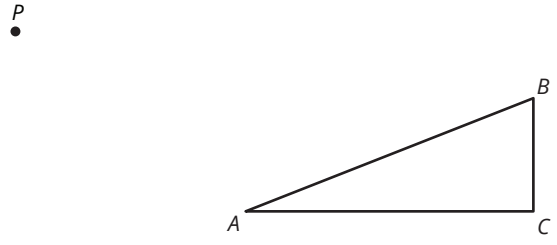
4. Scale factor: $\frac{1}{3}$



5. Scale factor: 4



6. Scale factor: $\frac{1}{4}$



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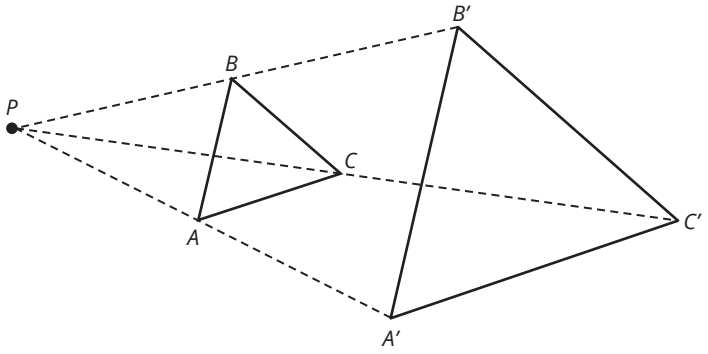
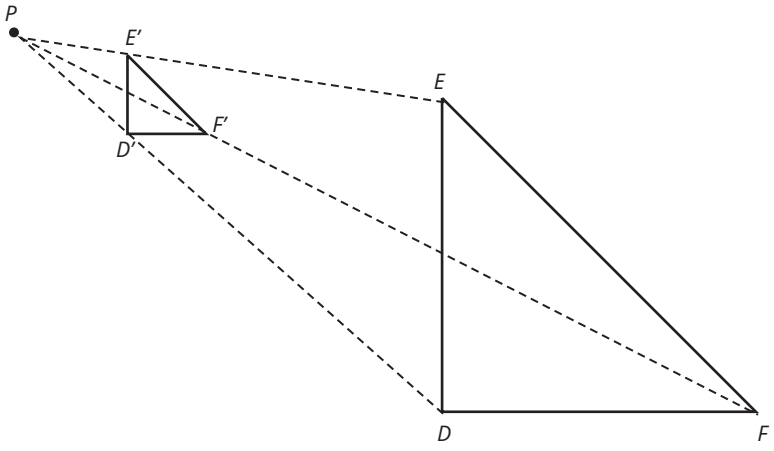
B. The triangles in each pair are similar. Identify the congruent corresponding angles and write ratios to identify the proportional sides of each triangle.

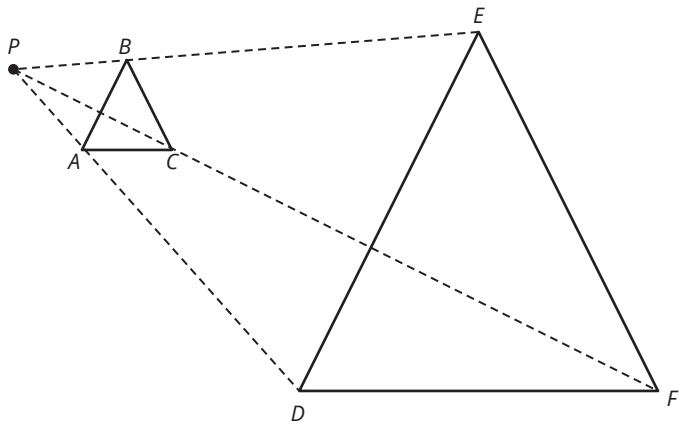
1. $\triangle ABC \sim \triangle A'B'C'$



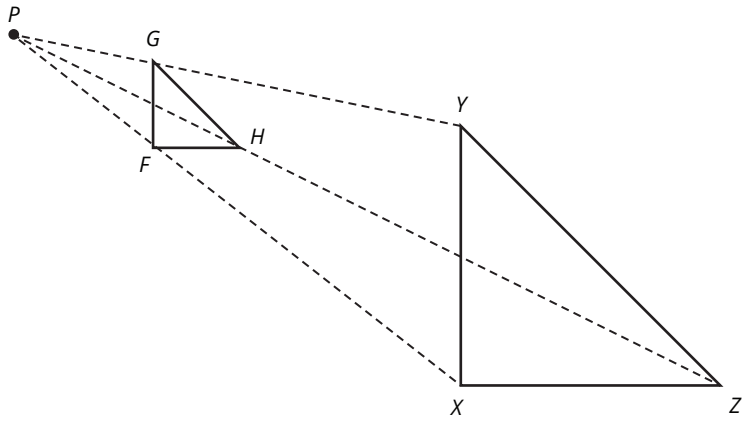
2. $\triangle DEF \sim \triangle D'E'F'$



3. $\triangle ABC \sim \triangle DEF$



4. $\triangle FGH \sim \triangle XYZ$

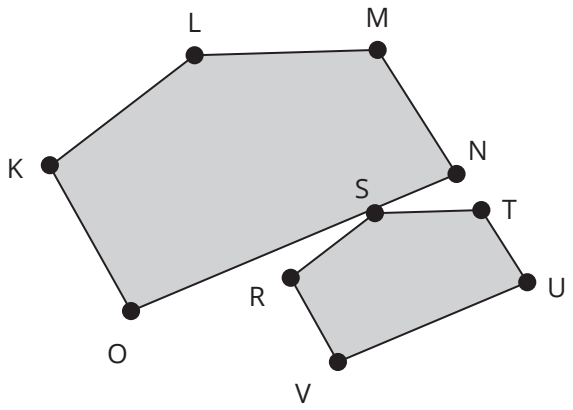


5. $\triangle ABC \sim \triangle MNP$

6. Quadrilateral $RSTU \sim$ Quadrilateral $WXYZ$

7. $\triangle XYZ \sim \triangle GHJ$

8. Pentagon *KLMNO* ~ Pentagon *RSTUV*



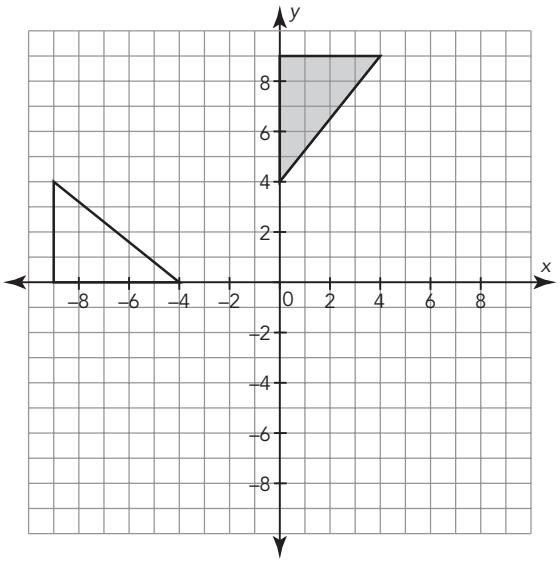
Extension

1. What happens if you dilate a figure by a negative scale factor? Use examples to explain your reasoning and justify your answer.

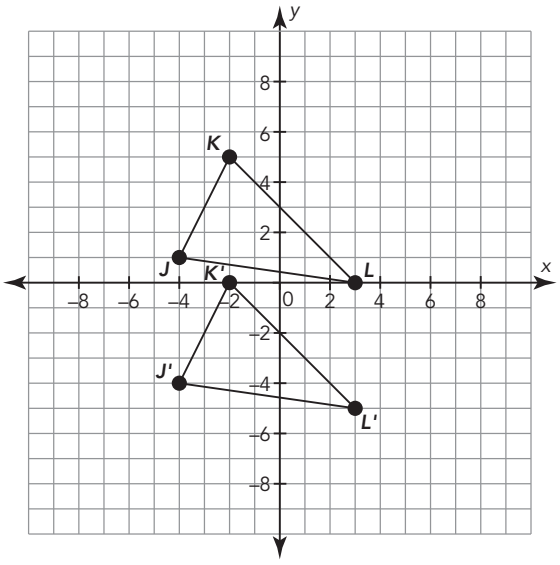
Spaced Practice

1. Describe a sequence of transformations that exhibits the congruence between each pair of figures.

a.

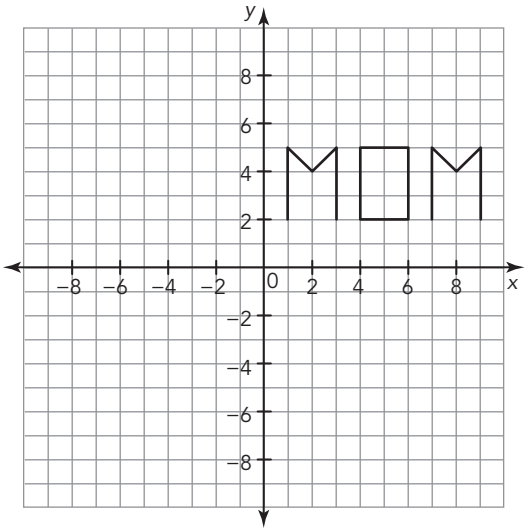


b.



2. Use what you know about reflections to answer each question.

   a. Reflect the word MOM across the *y*-axis. Is it still a word?



   b. Consider the coordinates of the vertices of Hexagon *ABCDEF*. Write the coordinates of the hexagon reflected across the *y*-axis and across the *x*-axis.

Vertex	Reflection Across *y*-Axis	Reflection Across *x*-Axis
A(1, 6)		
B(3, 4)		
C(5, 6)		
D(5, 4)		
E(3, 2)		
F(1, 4)		

3. Calculate the circumference and area of a circle with the given measure. Use 3.14 for π.

   a. Radius = 3 cm

   b. Diameter = 4 ft

II. Dilating Figures on the Coordinate Plane

Topic Practice

A. Use the given information to answer each question.

1. A flag is represented by the coordinates $A(3, 9)$, $B(15, 9)$, $C(15, 3)$, and $D(3, 3)$. Suppose you were to dilate the figure by a scale factor of $\frac{1}{3}$ using the origin as the center of dilation.

   a. What are the coordinates of the dilated figure?

   b. Compare and contrast the corresponding angles and corresponding side lengths of the original figure and the dilated figure.

   c. Is the image congruent to the pre-image? Why or why not?

   d. How does the perimeter of the original figure compare to the perimeter of the dilated figure?

   e. How does the area of the original figure compare to the area of the dilated figure?

   f. How does the orientation of the original figure compare to the orientation of the dilated figure?

2. How does dilating a figure affect its perimeter?

3. How does dilating a figure affect its area?

4. Does dilating an image create an image congruent to the pre-image? Explain your reasoning.

5. Triangle ABC is dilated by a scale factor of 2 to form $\triangle A'B'C'$.

   Perimeter $\triangle ABC = 18$ in. Perimeter $\triangle A'B'C'$

   Area $\triangle ABC = 20$ in.^{2} Area $\triangle A'B'C'$

6. Triangle LMN is dilated by a scale factor of 4 to form $\triangle L'M'N'$.

   Perimeter $\triangle LMN = 8$ cm Perimeter $\triangle L'M'N'$

   Area $\triangle LMN = 12$ cm^2 Area $\triangle L'M'N'$

7. Triangle GHI is dilated by a scale factor of 3 to form $\triangle G'H'I'$.

   Perimeter $\triangle GHI = 13.2$ ft Perimeter $\triangle G'H'I'$

   Area $\triangle GHI = 15.4$ ft^2 Area $\triangle G'H'I'$

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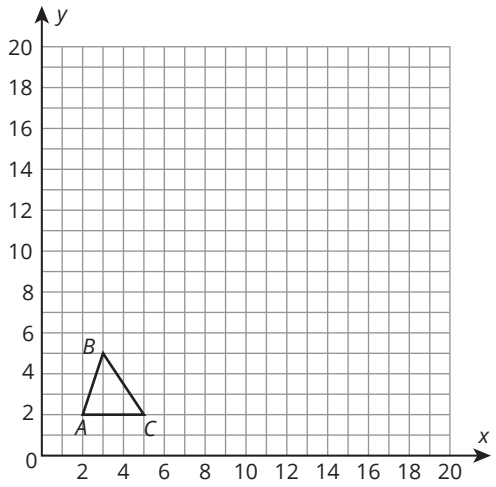
8. Triangle MNO is dilated by a scale factor of 2.5 to form $\triangle M'N'O'$.

Perimeter $\triangle MNO = 22.5$ m Perimeter $\triangle M'N'O'$

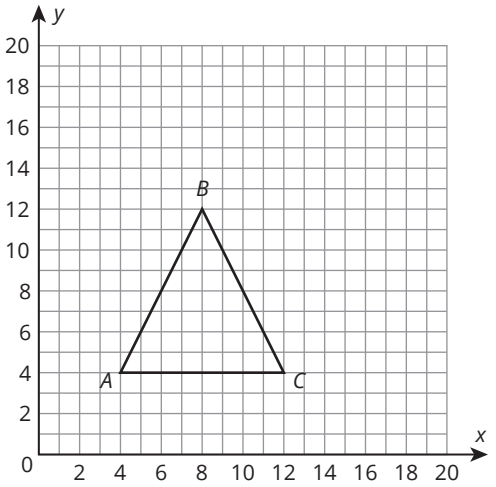
Area $\triangle MNO = 9.8$ m^2 Area $\triangle M'N'O'$

B. Dilate each figure or answer the question using the information provided. Then, compare and contrast the corresponding angles and side lengths of the original figure and dilated figure. In addition, compare the area of the original figure to the area or the dilated figure. Finally, write the algebraic representation for a dilation of the point (x, y) using the given scale factor.

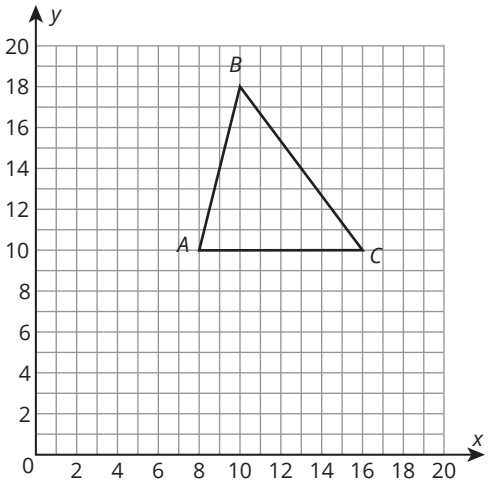
1. Dilate $\triangle ABC$ on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of 2.



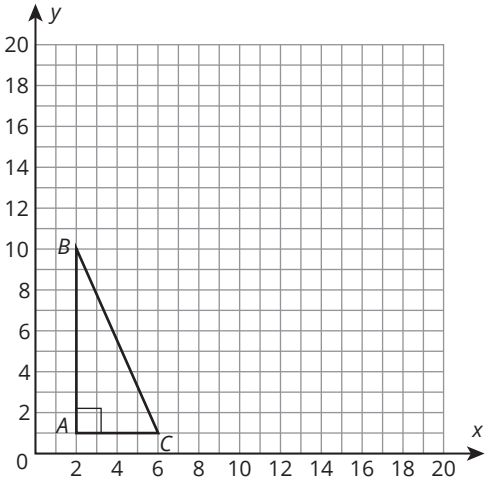
2. Dilate $\triangle ABC$ on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of $\frac{1}{2}$.



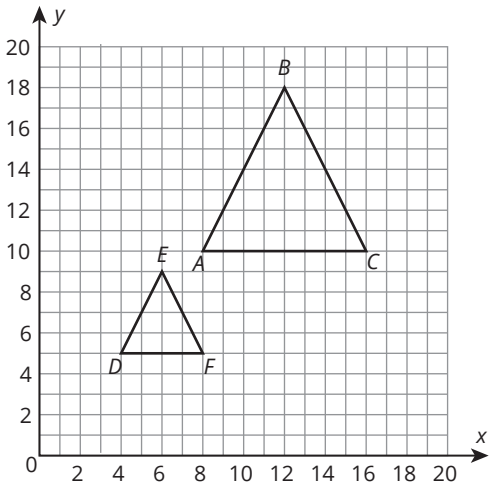
3. Dilate $\triangle ABC$ on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of $\frac{1}{2}$.



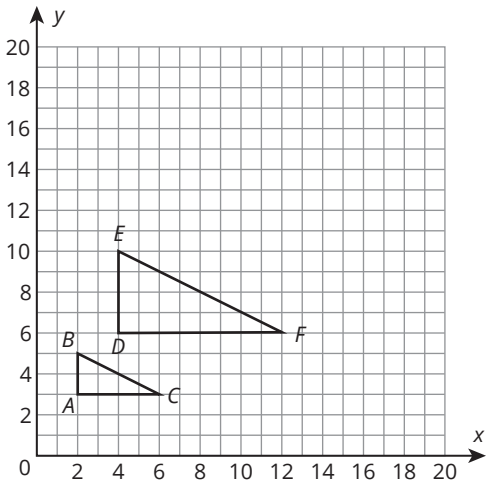
4. Dilate $\triangle ABC$ to form $\triangle DEF$ using the origin (0, 0) as the center of dilation and a scale factor of 2.



5. Triangle *DEF* is the image that resulted from a dilation of $\triangle ABC$ using the origin as the center of dilation. What scale factor was used to dilate $\triangle ABC$?



6. Triangle *DEF* is the image that resulted from a dilation of $\triangle ABC$ using the origin as the center of dilation. What scale factor was used to dilate $\triangle ABC$?

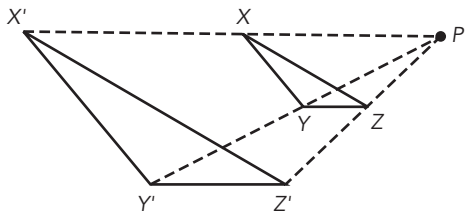


Extension

1. Square *ABCD* has coordinates *A*(4, 4), *B*(8, 4), *C*(8, 0), and *D*(4, 0). A dilation of Square *ABCD* has coordinates *A′*(0, 0), *B′*(2, 0), *C′*(2, −2), and *D′*(0, −2). What is the center of dilation?

Spaced Practice

1. Triangle *XYZ* has been enlarged with *P* as the center of dilation to form △*X′Y′Z′*. Identify the equivalent ratios that are equal to the scale factor.



2. A triangle is dilated with center of dilation at point *U*. Point *E* is a vertex of the triangle, and point *E′* is the corresponding vertex of the image. If $UE = 2$ centimeters and $UE' = 10$ centimeters, what is the scale factor?

3. The coordinates of Quadrilateral *ABCD* are *A*(−6, 2), *B*(−5, 3), *C*(7, 3), and *D*(0, −4). What are the coordinates of the image if the quadrilateral is translated 4 units right and 3 units down?

4. The coordinates of △*JKL* are *J*(0, 1), *K*(6, 0), and *L*(−6, 0). What are the coordinates of the image if the triangle is translated 8 units left?

III. Mapping Similar Figures Using Dilations

Topic Practice

A. Each figure is dilated using the origin as the center of dilation to create a new figure. Determine the scale factor and write the rule applied to create the new figure.

1. $X(10,10)$, $Y(15,6)$, $Z(20,10)$

   $X'(3.3, 3.3)$, $Y'(5, 2)$, $Z'(6.7, 3.3)$

2. $A(14, 2)$, $B(18, 6)$, $C(22,2)$

   $A'(3.5, 0.5)$, $B'(4.5, 1.5)$, $C'(5.5, 0.5)$

3. $E(4,0)$, $F(4, -5)$, $G(12, -5)$, $H(10, -2)$

   $E'(2,0)$, $F'(2, -2.5)$, $G'(6, -2.5)$, $H'(5, -1)$

4. $Q(2, -3)$, $R(5, -3)$, $S(5, -5)$, $T(2, -5)$

   $Q'(8, -12)$, $R'(20, -12)$, $S'(20, -20)$, $T'(8, -20)$

5. $A(3, -4)$, $B(4, -1)$, $C(2, 2)$

   $A'(6, -8)$, $B'(8, -2)$, $C'(4, 4)$

6. $W(6, 2)$, $X(12, 2)$, $Y(12, 4)$, $Z(8, 6)$

   $W'(21, 7)$, $X'(42, 7)$, $Y'(42, 14)$, $Z'(28, 21)$

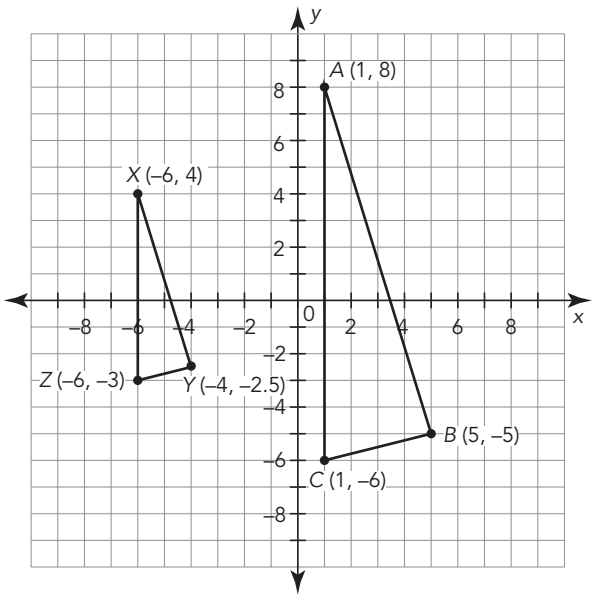
7. Do any of the dilation(s) in Questions 1–6 preserve congruence? Explain your reasoning.

8. Which transformation(s) preserve congruence?

9. Which transformation(s) do not preserve congruence?

Extension

Triangle *XYZ* is the image after a dilation of $\triangle ABC$.



1. Determine the scale factor.

2. Determine the center of dilation.

3. Explain how you could verify that the ratio of corresponding sides is constant.

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Spaced Practice

1. Give the coordinates of $\triangle A'B'C'$ after a transformation of $\triangle ABC$ with the coordinates $A(6, -3)$, $B(9, 5)$, and $C(5, 6)$. Use the origin as the center of dilation or rotation, as needed.

   a. Dilate $\triangle ABC$ by a scale factor of $\frac{1}{3}$.

   b. Dilate $\triangle ABC$ by a scale factor of 4.

   c. Rotate $\triangle ABC$ 180 degrees.

   d. Reflect $\triangle ABC$ across the x-axis.

2. In the equation $y = 4x + 7$, x is the number of items and y is the total cost. What is the unit rate? Include units in your response.

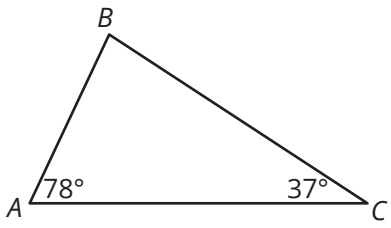
Name _____ Date _____

I. Exploring Angle Theorems

Topic Practice

A. Determine the measure of the unknown angle in each triangle.

1.



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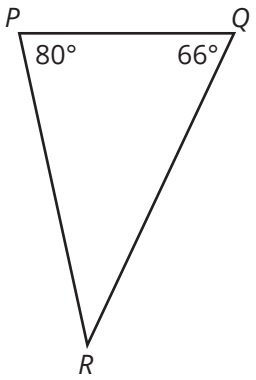
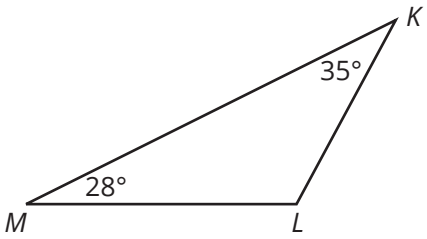


3.



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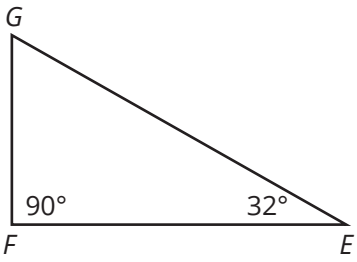

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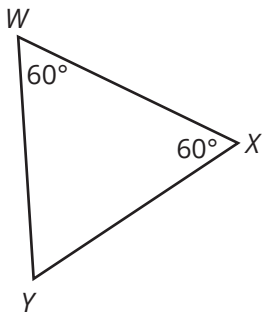

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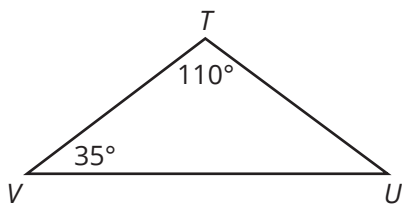
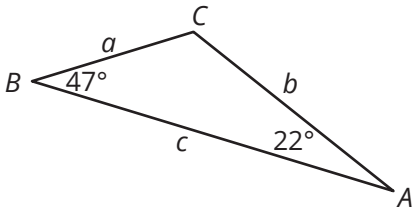
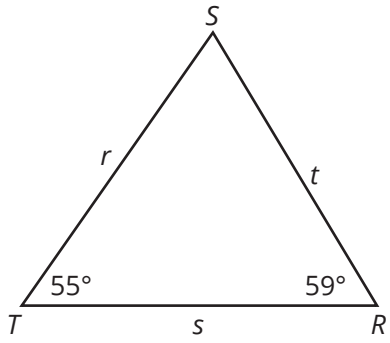


B. Determine the measure of each unknown angle. List the side lengths from shortest to longest for each diagram. Explain your reasoning.

1.



2.



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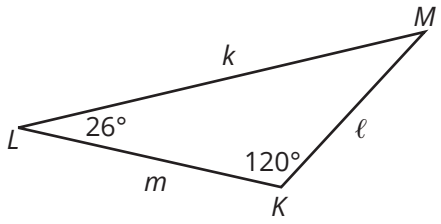

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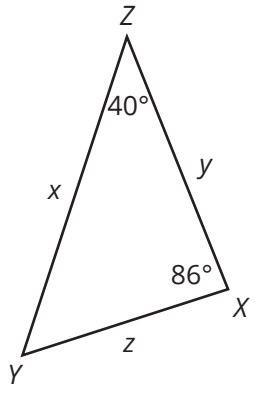

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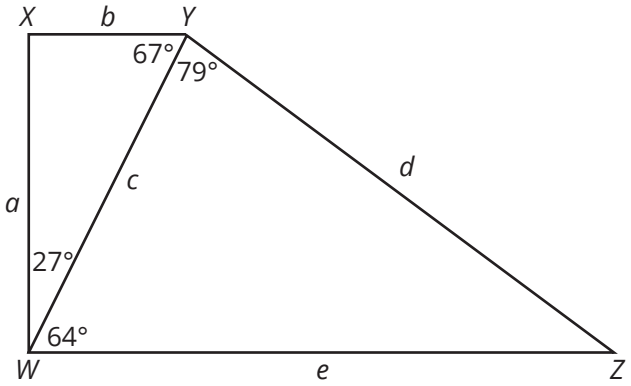

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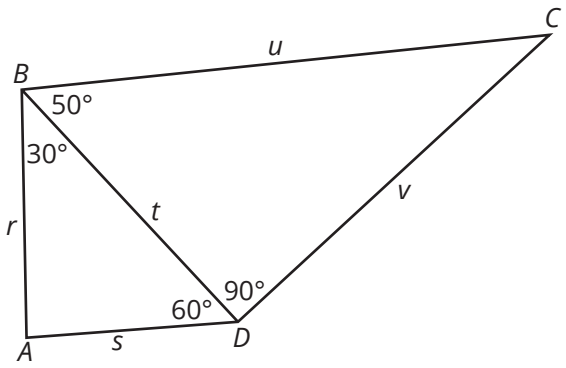
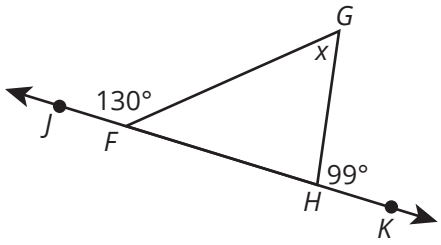
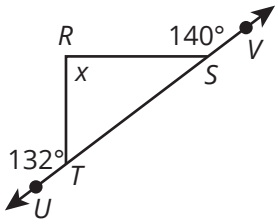


C. Solve for *x* in each diagram.

1.



2.



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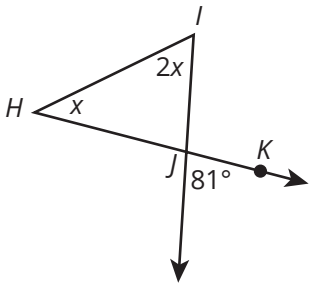

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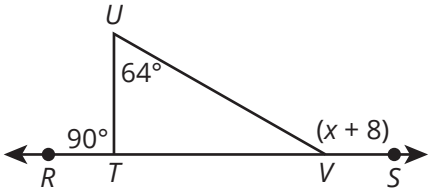

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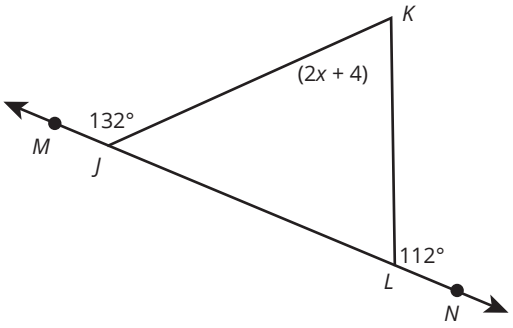
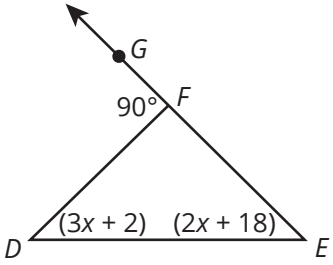


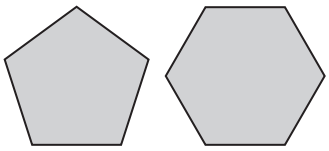
6.



Extension

1. To tessellate a plane means to cover a surface by repeated use of a single shape or design without gaps or overlaps. M.C. Escher was a Dutch graphic artist who was famous for his tessellations, perspective drawings, and impossible spaces.

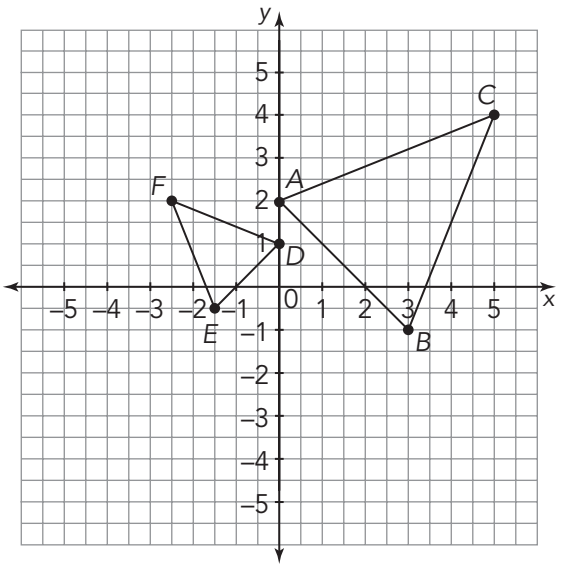
   Not all shapes or patterns can be tessellated. Use what you know about interior and exterior angles to show why it is possible to tessellate with a regular hexagon but not with a regular pentagon.



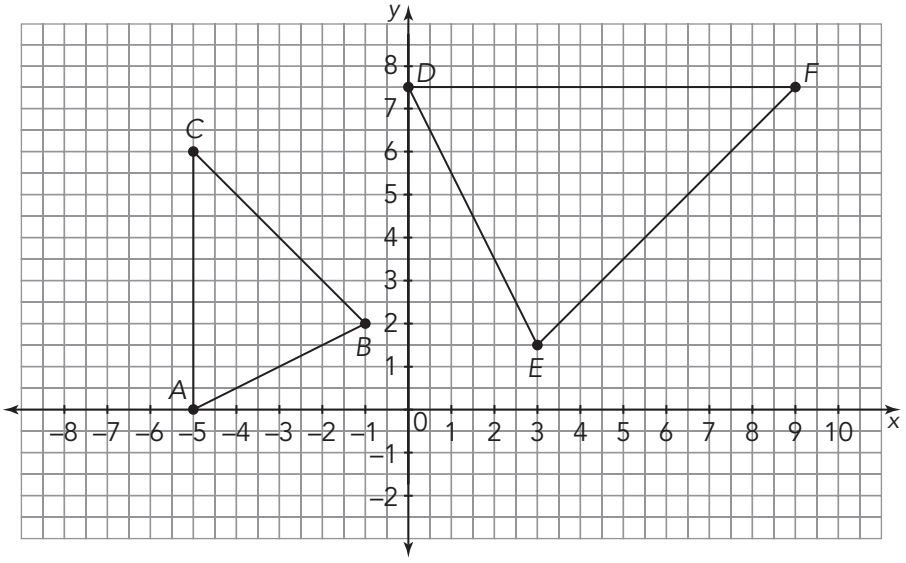
Spaced Practice

1. Triangle *ABC* is similar to Triangle *DEF*. Determine a sequence of transformations that maps $\triangle ABC$ onto $\triangle DEF$.

a.

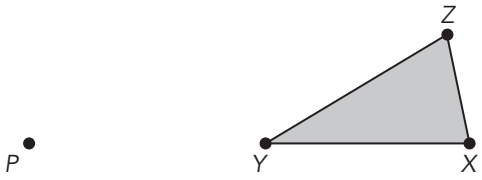


b.

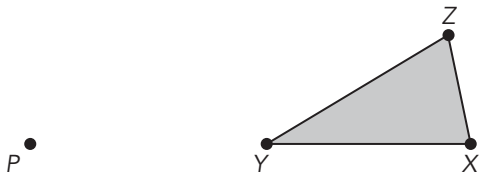


2. Dilate $\triangle XYZ$ by the given scale factor, using point P as the center of dilation.

a. Dilate by a scale factor of $\frac{3}{4}$.

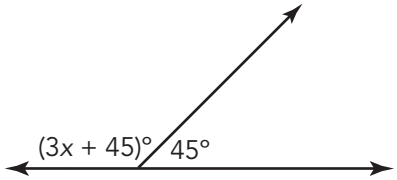


b. Dilate by a scale factor of 1.5.

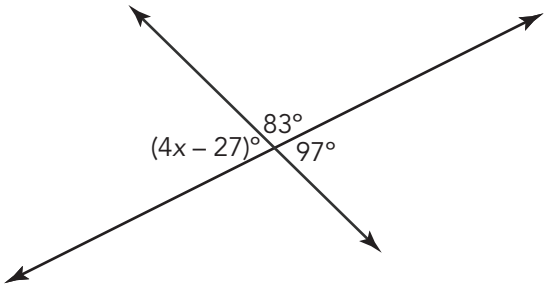


3. Calculate the measure of each angle.

a.



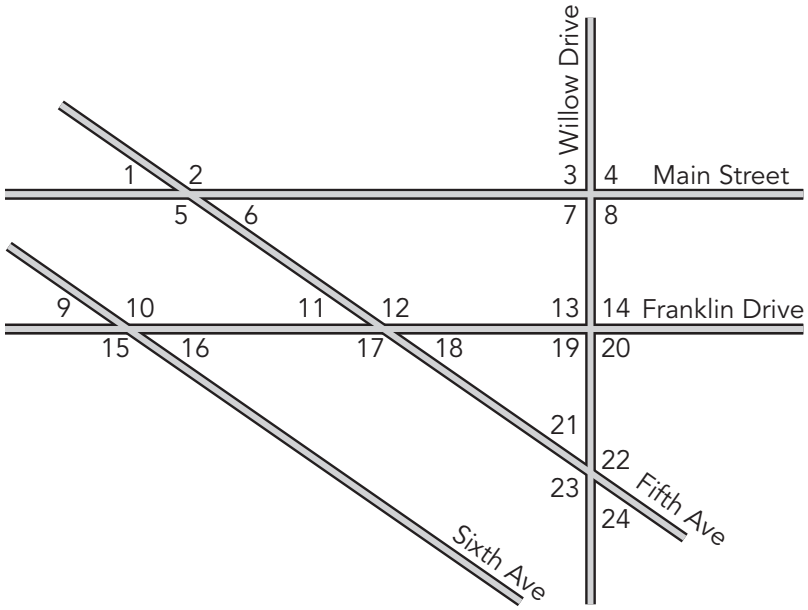
b.



II. Exploring the Angles Formed by Lines Intersected by a Transversal

Topic Practice

A. Use the map to give an example of each relationship.



1. Congruent angles

2. Vertical angles

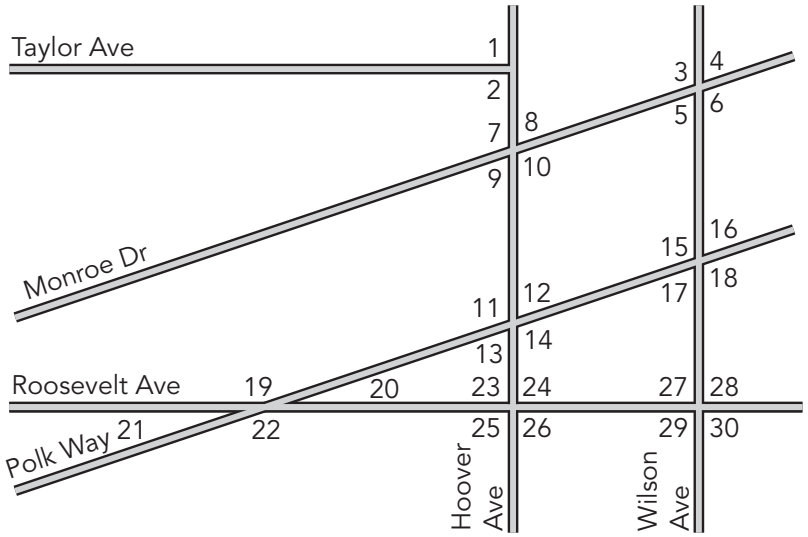
3. Supplementary angles

4. Linear pair

5. Adjacent angles

6. Corresponding angles

B. Use the map to answer each question. Assume the streets extend beyond the edges of the map. Explain your reasoning.



1. Identify each street that is a transversal to Hoover and Wilson.

2. Identify each street that is not a transversal to Hoover and Wilson.

3. Identify each street that is a transversal to Monroe and Polk.

4. Identify each street that is not a transversal to Monroe and Polk.

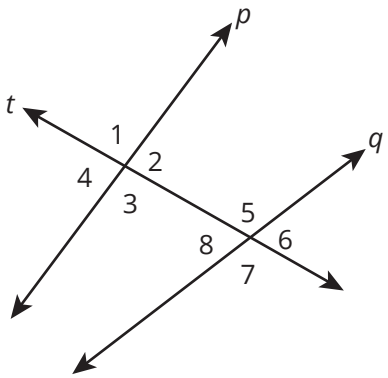
5. Identify all the angles that are same-side exterior to $\angle 11$.

6. Identify all the angles that are alternate interior to $\angle 11$.

7. Identify all the angles that are corresponding to $\angle 11$.

8. Identify all the angles that are vertical to $\angle 11$.

C. In the diagram, transversal *t* intersects lines *p* and *q*. Classify each pair of angles as *vertical*, *linear*, *corresponding*, *same-side exterior*, *same-side interior*, *alternate interior*, or *alternate exterior*.



1. Angle 1 and angle 2

2. Angle 1 and angle 3

3. Angle 1 and angle 6

4. Angle 3 and angle 7

5. Angle 2 and angle 8

6. Angle 1 and angle 7

7. Angle 4 and angle 7

8. Angle 6 and angle 8

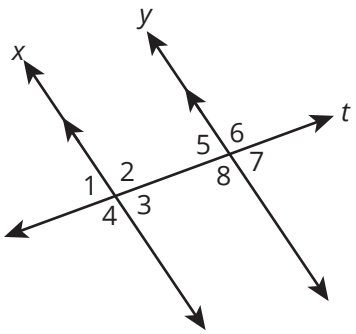
9. Angle 3 and angle 4

10. Angle 2 and angle 6

11. Angle 2 and angle 5

12. Angle 3 and angle 5

D. Use the diagram to answer each question.



1. Identify the angles that are congruent to $\angle 6$.

2. Identify the angles that are supplementary to $\angle 6$.

3. Identify the angles that are neither congruent nor supplementary to $\angle 6$.

4. Identify the angles that are congruent to $\angle 3$.

5. Identify the angles that are supplementary to $\angle 3$.

6. Identify the angles that are neither congruent nor supplementary to $\angle 3$.

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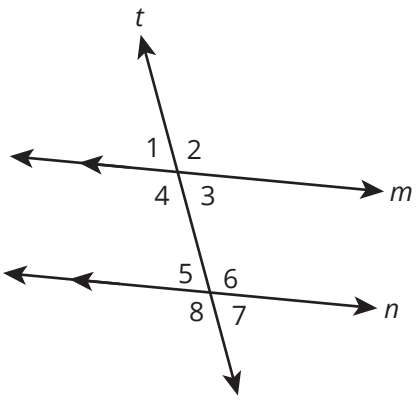
E. In the diagram, transversal *t* intersects parallel lines *m* and *n*. Suppose that the measure of ∠4 is 106°. Classify the given angle pair. Then determine each measure.



1. ∠4 and ∠1, m∠1 = _____

2. ∠4 and ∠2, m∠2 = _____

3. ∠4 and ∠3, m∠3 = _____

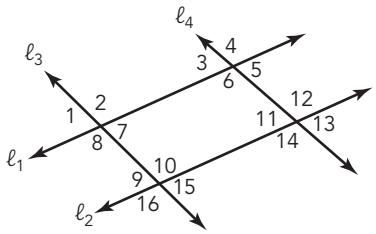
4. ∠4 and ∠8, m∠8 = _____

5. ∠4 and ∠5, m∠5 = _____

6. ∠5 and ∠7, m∠7 = _____

Extension

Given: $\ell_1 \parallel \ell_2$ and $\ell_3 \parallel \ell_4$.



1. Explain why every angle in the diagram is congruent to $\angle 6$ or $\angle 7$.

2. What can you conclude about the sum of the measures of $\angle 6$, $\angle 7$, $\angle 10$, and $\angle 11$? Explain your reasoning.

3. Use what you learned in this lesson to explain what you know about the angles in any parallelogram.

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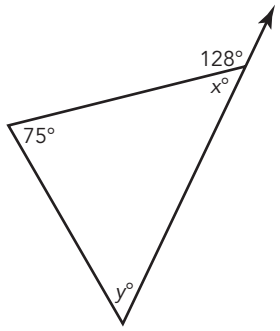
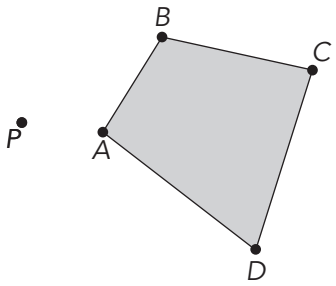
Spaced Practice

1. Determine the unknown angle measures.



2. Triangle *ABC*, with coordinates *A* (−2, 5), *B* (0, 7), and *C* (1, 3), is dilated by a scale factor of $\frac{1}{2}$ with a center of dilation at the origin. Determine the coordinates of Triangle *A′B′C′*.

3. Dilate Quadrilateral *ABCD* by a scale factor of 2, using point *P* as the center of dilation.

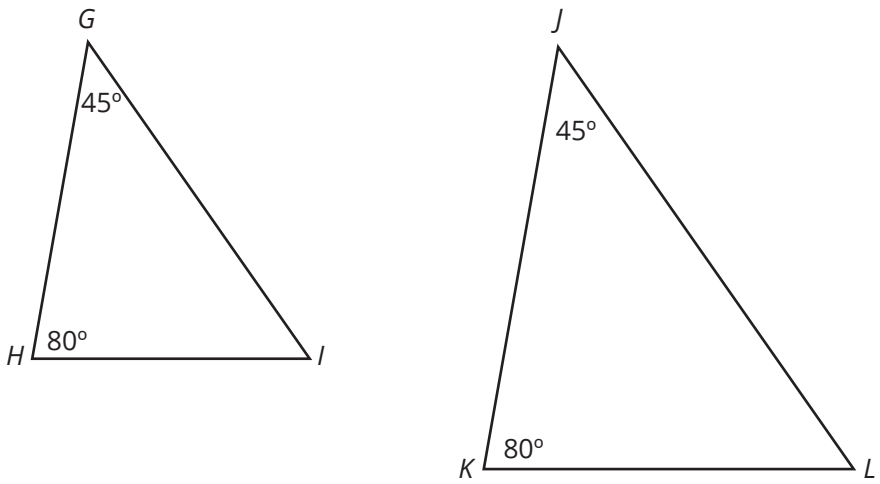


III. Exploring the Angle-Angle Similarity Theorem

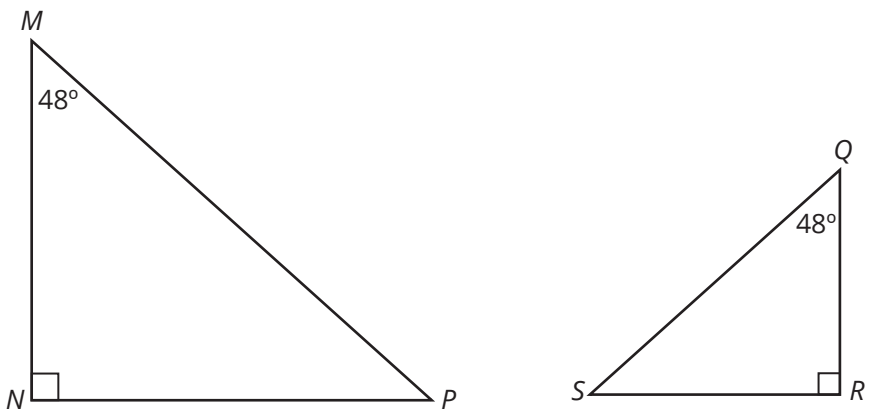
Topic Practice

A. Use the Angle-Angle Similarity Theorem to explain how the triangles are similar.

1.



2.



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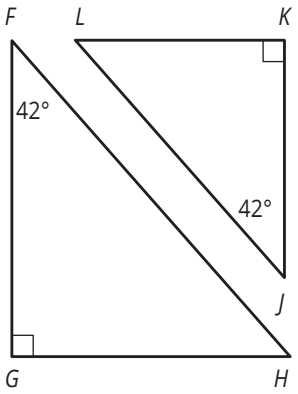
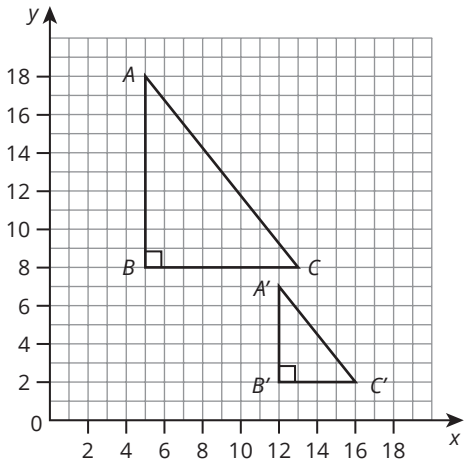
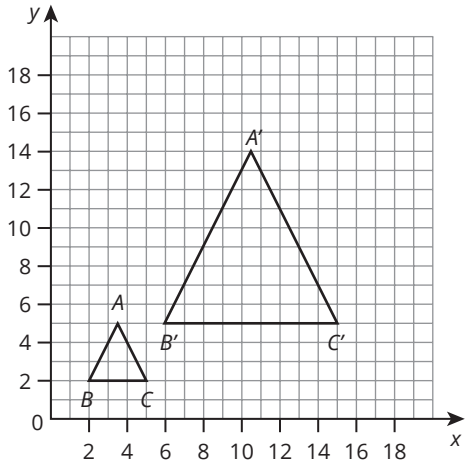
3. 

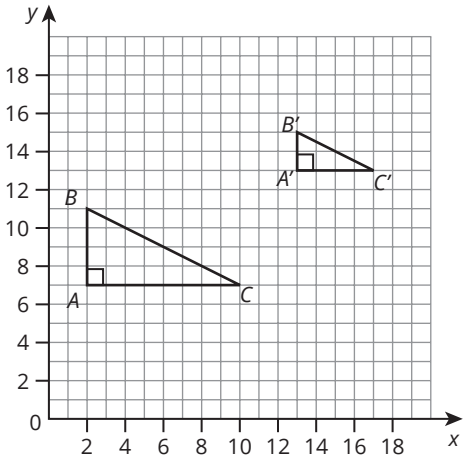
4. Use patty paper or a protractor to show that if $\triangle ABC$ and $\triangle A'B'C'$ are similar triangles.



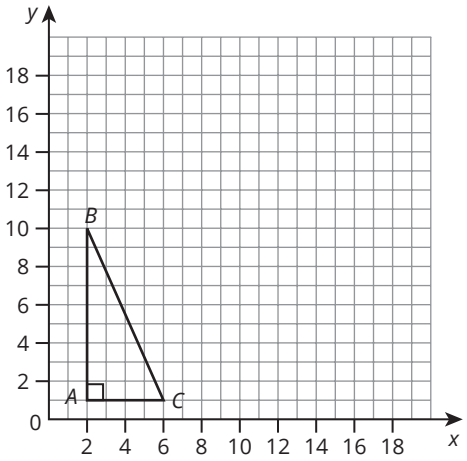
5. Use patty paper or a protractor to show that if $\triangle ABC$ and $\triangle A'B'C'$ are similar triangles.



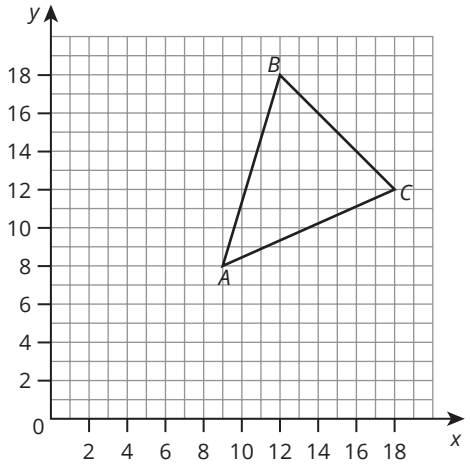
6. Use patty paper or a protractor to show that if △ABC and △A′B′C′ are similar triangles.



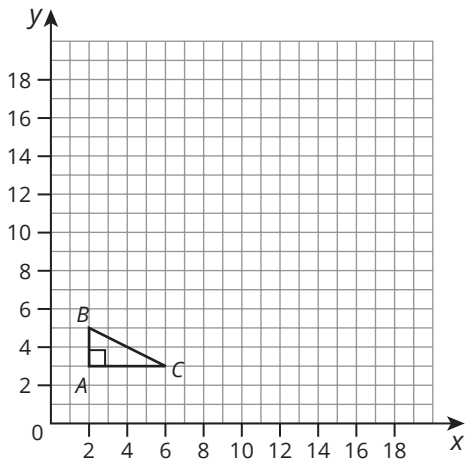
7. Dilate △*ABC* to form △*DEF* using the origin as the center of dilation and a scale factor of 2. Show that △*ABC* is similar to △*DEF* using patty paper or a protractor.



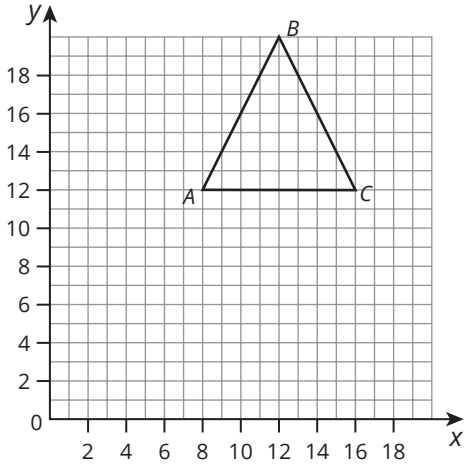
8. Dilate $\triangle ABC$ to form $\triangle DEF$ using the origin as the center of dilation and a scale factor of $\frac{1}{2}$. Show that $\triangle ABC$ is similar to $\triangle DEF$ using patty paper or a protractor.



9. Dilate $\triangle ABC$ to form $\triangle DEF$ using the origin as the center of dilation and a scale factor of 2. Show that $\triangle ABC$ is similar to $\triangle DEF$ using patty paper or a protractor.



10. Dilate $\triangle ABC$ to form $\triangle DEF$ using the origin as the center of dilation and a scale factor of $\frac{1}{4}$. Show that $\triangle ABC$ is similar to $\triangle DEF$ using patty paper or a protractor.

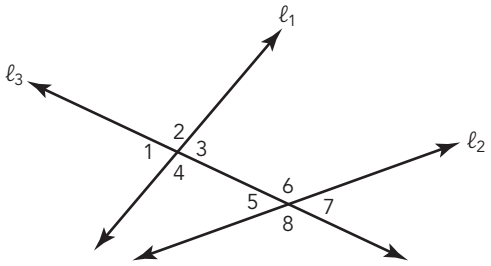


Extension

Madison says that any two right triangles with two congruent angles are similar. William says that the triangles are similar and congruent. Who is correct? Explain how you know.

Spaced Practice

1. In the figure shown, lines ℓ_1 and ℓ_2 are intersected by transversal ℓ_3. Name the corresponding angles.



2. Sketch an example of alternate interior angles.

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3. A photo has a width of 250 pixels and a height of 320 pixels. Determine the new dimensions and tell whether the enlarged or reduced photo is similar.

   a. Width: 150%, height: 200%

   b. Width: 75%, height: 75%

4. Solve each equation.

   a. $3(x + 3) = -6$

   b. $-20 = -2(4 - x)$

GRADE 8

MODULE 2

Developing Function Foundations

TOPIC 1: From Proportions to Linear Relationships

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TOPIC 2: Linear Relationships

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V. Defining Functional Relationships . . . 170

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Name _____ Date _____

I. Representations of Proportional Relationships

Topic Practice

A. At a large company, the total number of employees varies directly with the number of employees that drive to work. The company determines that two out of every three of its total employees drive to work. The other employees walk or use public transportation. Use the information to determine each number of employees.

1. The number of total employees if there are 150 employees that drive to work

2. The number of total employees if there are 85 employees that do not drive to work

3. The number of employees that do not drive to work if there are 240 total employees

4. The number of employees that drive to work if there are 570 total employees

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5. The number of employees that drive to work if there are 330 employees that do not drive to work.

6. The number of employees that do not drive to work if there are 770 employees that drive to work.

B. Use the information given to complete each table. Then, write an equation in the form $y = kx$ to represent the situation.

1. On the school bus, Jamal notices that there are four seventh-grade students for every three eighth-grade students. How many eighth-grade students, y, are there for x number of seventh-grade students?

Seventh-grade Students	Eighth-grade Students
0	
	16
24	
	44

2. While flipping through the television channels, Mariana determines that one out of every five shows is a reality show. How many reality shows, y, are there for x number of total shows?

Total Shows	Reality Shows
10	
35	
	37
	52
	63
350	

3. Mario notices that four out of every seven books on his bookshelf are comic books. How many comic books, *y*, are there for *x* number of total books?

Total Books	Comic Books
	8
	20
56	
	48
105	
154	

4. While lounging on the beach, Matthew notices that eight out of every twelve people on the beach are children. How many children, *y*, are there for *x* number of people?

Total People	Children
	24
	104
288	
456	
636	
	512
	680

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5. Ashley is sorting through donations she received at the thrift store. She determines that for every three pieces of adult clothing she receives nine pieces of children's clothing. How many adult's items, y, are there for x number of children's items?

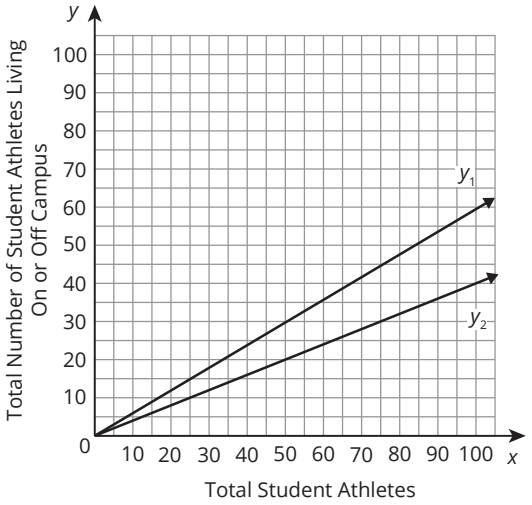
Children's Items	Adult's Items
	1
18	
36	
	18
	24
126	
198	

6. While walking through the art museum, Josh notices that for every six sculptures, there are ten paintings. How many paintings, y, are there for x number of sculptures?

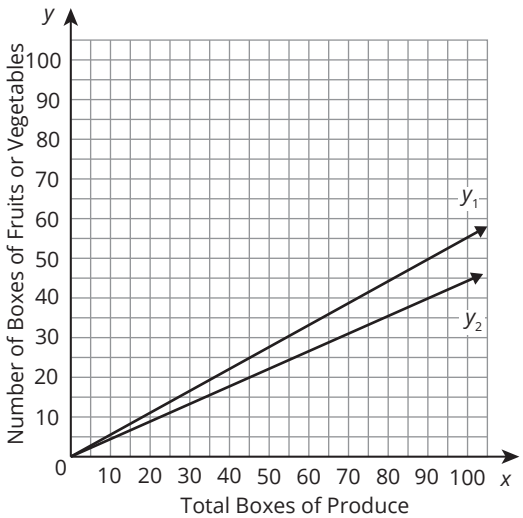
Sculptures	Paintings
	5
	30
48	
72	
108	
	250
186	

C. Each graph represents two proportional relationships. Determine the constant of proportionality for each line. Then, identify which graph represents each proportional relationship.

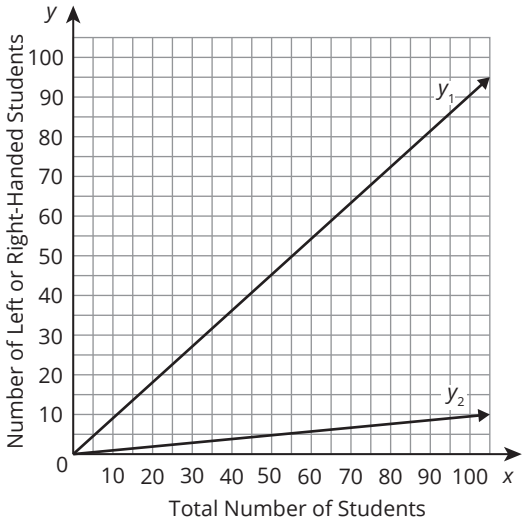
1. A local college determines that two out of every five of their student athletes live off campus.



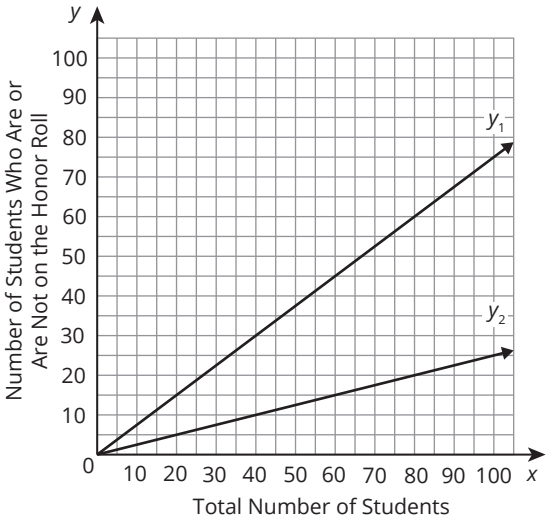
2. While unloading a produce truck of fruits and vegetables, Mario realizes that five out of every nine boxes he unloads are fruit.



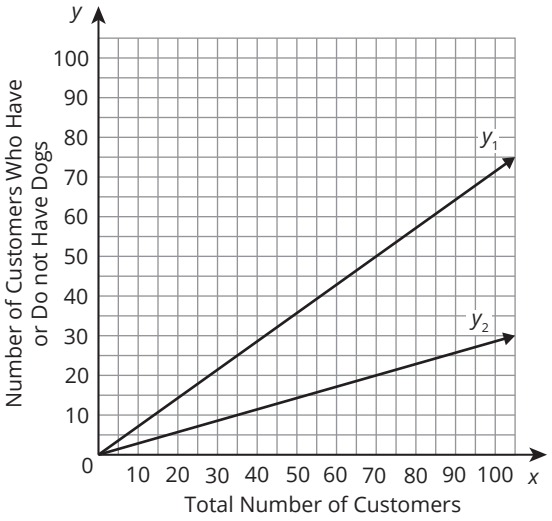
3. A teacher notices that only one out of every ten of her students is left-handed.



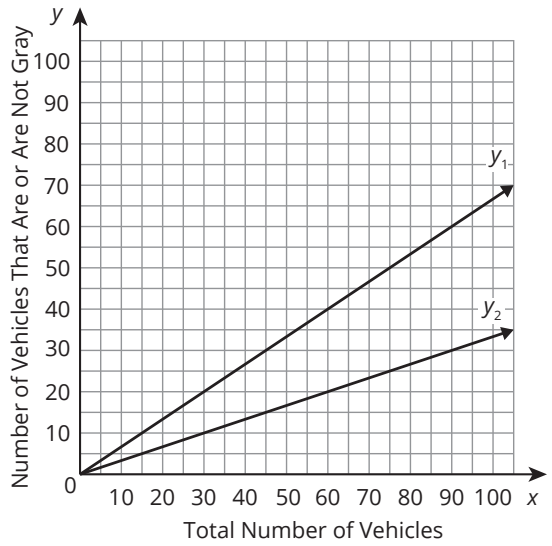
4. Principal Patel determines that three out of every four students at his high school are honor roll students.



5. A recent survey at a pet supply store shows that five out of every seven of their customers' pets are dogs.



6. A car dealership notices that two out of every six vehicles for sale are gray.



D. Solve each problem.

1. Tiara is trying to start an Intramural Club at her school. The principal tells her she must get signatures from students to show support. Each filled sheet contains 25 signatures.

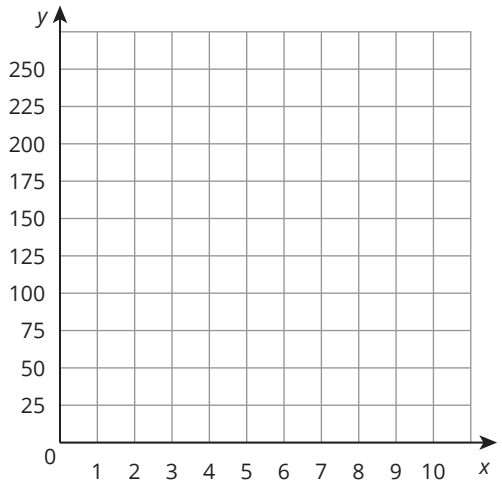
   a. If the number of signatures, *y*, Tiara gets varies directly with the number of filled sheets, *x*, write an equation to represent this relationship.

   b. The principal tells Tiara she must have 7 sheets filled with signatures. If she fills all of these, how many signatures will she have in all?

   c. On Tuesday, Tiara has 225 signatures. How many sheets has she filled?

   d. What is the unit rate in this situation? What does it represent?

   e. Graph your equation from part (a) on a coordinate plane. Label the axes of the graph.



2. Lizzie has started a business making doghouses. Her local hardware store saw her work and wants to buy as many as she can build for $45 per doghouse.

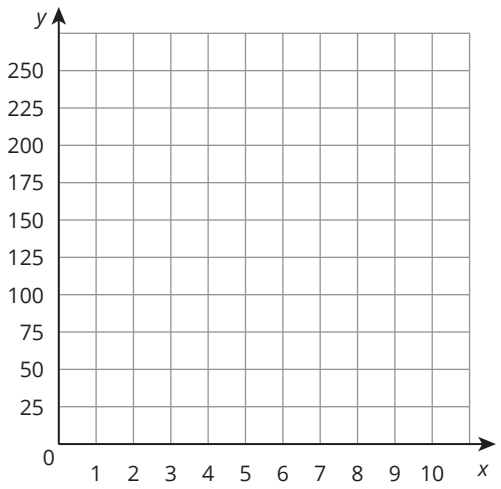
   a. If the amount of money Lizzie receives, *y*, varies directly with the number of doghouses built, *x*, write an equation to represent this relationship.

   b. Lizzie believes she can build 5 doghouses in a week. How much money will she receive from the hardware store if she meets this goal?

   c. Lizzie surpassed her goal and received $360 from the hardware store. How many doghouses did she build?

   d. What is the rate of change in this situation? What does it represent?

   e. Graph your equation from part (a) on a coordinate plane. Label the axes of the graph.



3. Andrew uses a fitness tracker to determine the number of calories he burns while running. He burns about 110 calories per mile of running.

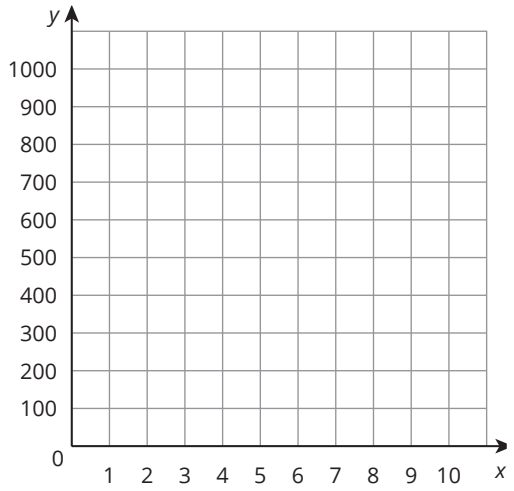
   a. If the number of calories Andrew burns, y, varies directly with the number of miles he runs, x, write an equation to represent this relationship.

   b. Andrew runs 5 miles on Saturday. How many calories did he burn while running?

   c. After a run on Sunday, Andrew's fitness tracker shows he burned 385 calories. How many miles did he run?

   d. What is the rate of change in this situation? What does it represent?

   e. Graph your equation from part (a) on a coordinate plane. Label the axes of the graph.



4. Dr. Davis, a vet, is running a free rabies clinic for her town. She estimates it will take her six minutes for each animal she treats.

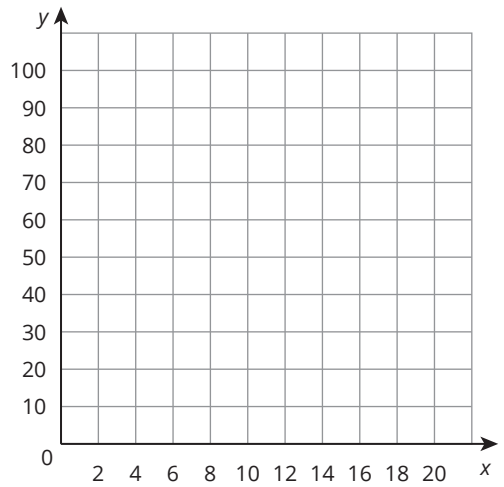
   a. If the time in minutes, y, Dr. Davis works at the clinic varies directly with the number of animals treated, x, write an equation to represent this relationship.

   b. After treating 11 animals, how many minutes has Dr. Davis worked at the clinic?

   c. Dr. Davis has been working at the clinic 1 hour and 42 minutes. How many animals has she treated?

   d. What is the constant of proportionality in this situation? What does it represent?

   e. Graph your equation from part (a) on a coordinate plane. Label the axes of the graph.



5. Alyssa wants to increase the number of kumquat trees in her orchard, which she has divided into equal size units of land. Each kumquat tree will take up four units of land.

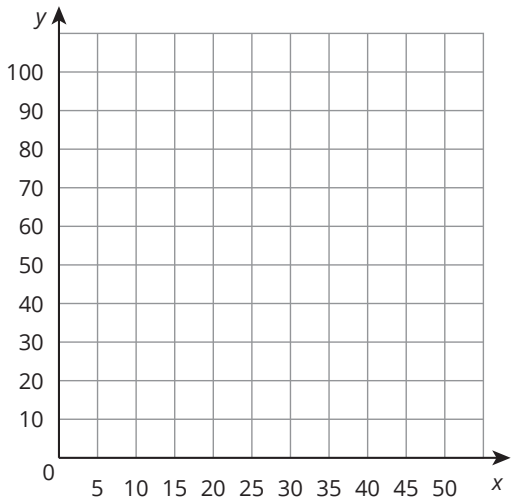
   a. If the number of units of land used, y, varies directly with the number of kumquat trees planted, x, write an equation to represent this relationship.

   b. Alyssa plants 25 kumquat trees. How many units of land will be used?

   c. If 132 units of land in the orchard are used for kumquat trees, how many kumquat trees did Elizabeth plant?

   d. What is the unit rate in this situation? What does it represent?

   e. Graph your equation from part (a) on a coordinate plane. Label the axes of the graph.



6. Alejandro works at the circus making balloon animals, charging $3 for a balloon animal.

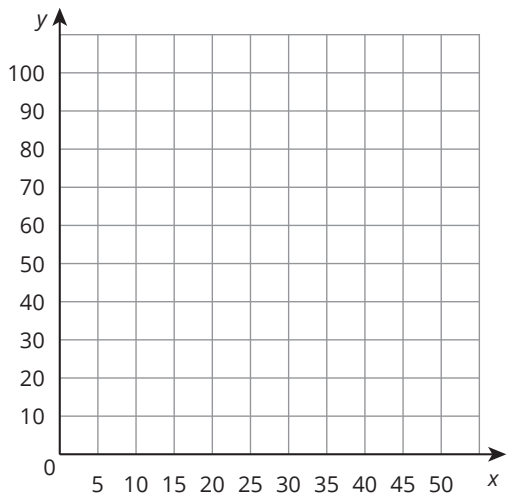
   a. If the total amount of money, *y*, Alejandro receives varies directly with the number of balloon animals made, *x*, write an equation to represent this relationship.

   b. Alejandro sells twenty-one balloon animals by lunchtime. Determine the total amount of money he receives.

   c. How many balloon animals would Alejandro need to sell in order to make $117?

   d. What is the constant of proportionality in this situation? What does it represent?

   e. Graph your equation from part (a) on a coordinate plane. Label the axes of the graph.

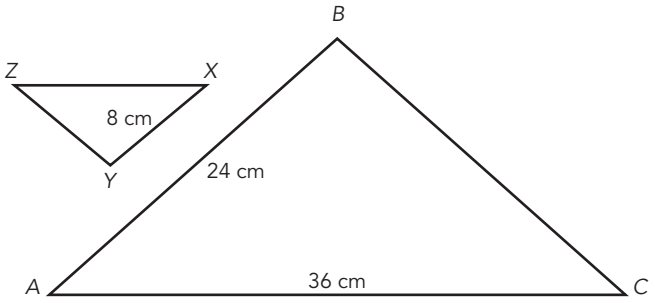


Extension

Consider the relationship between the side length of a square and the area of the square. Does this represent a proportional relationship? Use a table of values, equation, and graph to justify your answer.

Spaced Practice

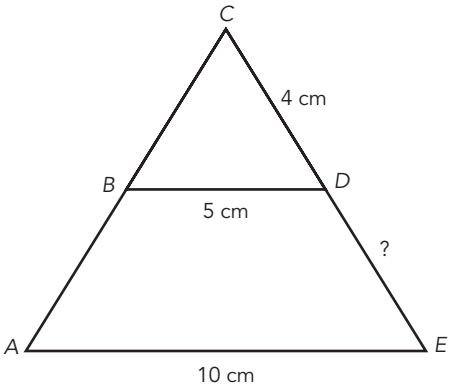
1. In the diagram, $\triangle ABC$, $\triangle XYZ$. State the corresponding sides and angles.



2. In the diagram, $\overline{BD} \parallel \overline{AE}$.

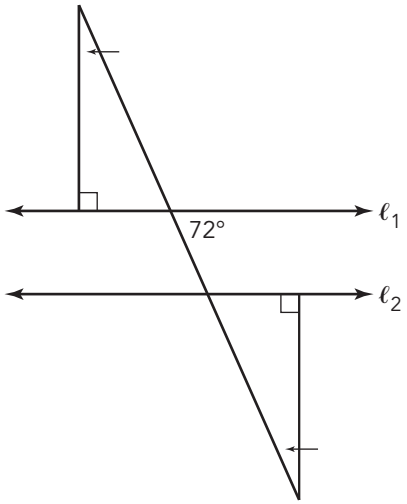
   a. Explain why $\triangle BDC \sim \triangle AEC$.

   b. Determine the length of $\overline{DE}$.

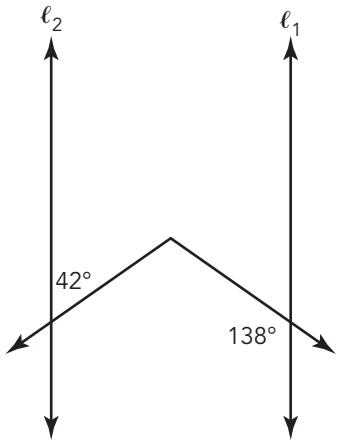


3. Solve for each unknown angle measure given that $\ell 1 \parallel \ell 2$.

a.

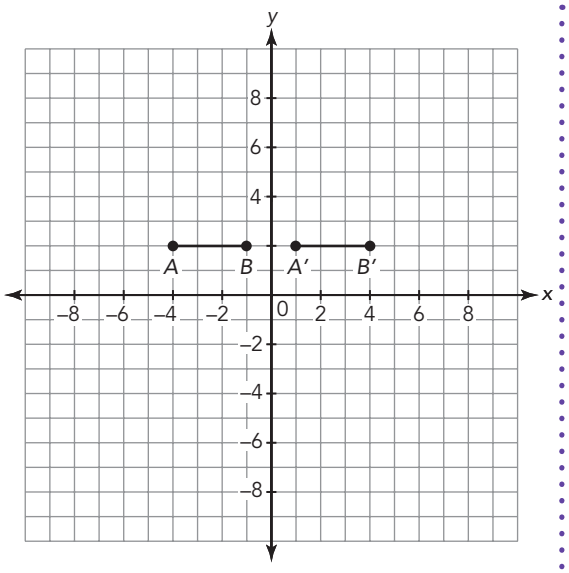


b.

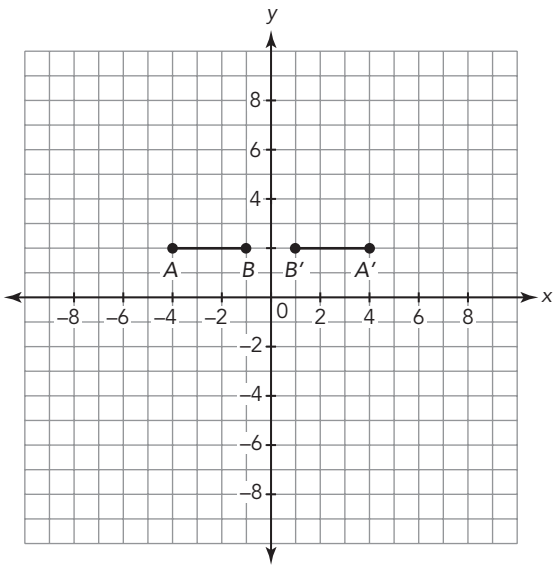


4. Describe a transformation or sequence of transformations to generate line segment $A'B'$ from original line segment AB.

a.



b.

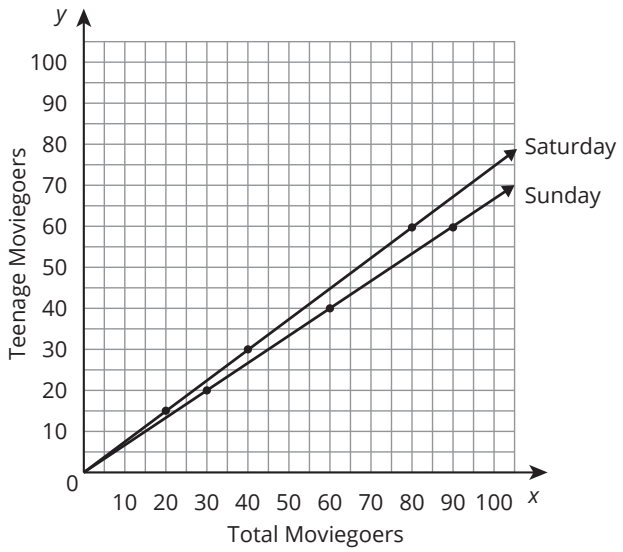


II. Using Similar Triangles to Describe the Steepness of a Line

Topic Practice

A. Read each problem situation and use the information to answer the questions.

1. The employees at a movie theater keep track of the moviegoers over the weekend. On Saturday, they notice that 3 out of every 4 moviegoers are teenagers. On Sunday, they notice that 2 out of every 3 moviegoers are teenagers. Each point on the graph represents a time they gathered data.



   a. Identify the proportional and non-proportional relationships.

   b. What is the unit rate of change for the proportional relationship(s)?

   c. Write an equation to represent the proportional relationship(s).

2. Natalia and Jorge use different cell phone providers, but they both kept track of their bills over the past year. The total amount paid by month is shown.

Natalia	
Time (months)	Amount Paid (dollars)
3	166.50
5	267.50
9	469.50
12	621

Jorge	
Time (months)	Amount Paid (dollars)
2	110
6	330
8	440
10	550

a. Identify the proportional and non-proportional relationships.

b. What is the unit rate of change for the proportional relationship(s)?

c. Write an equation to represent the proportional relationship(s).

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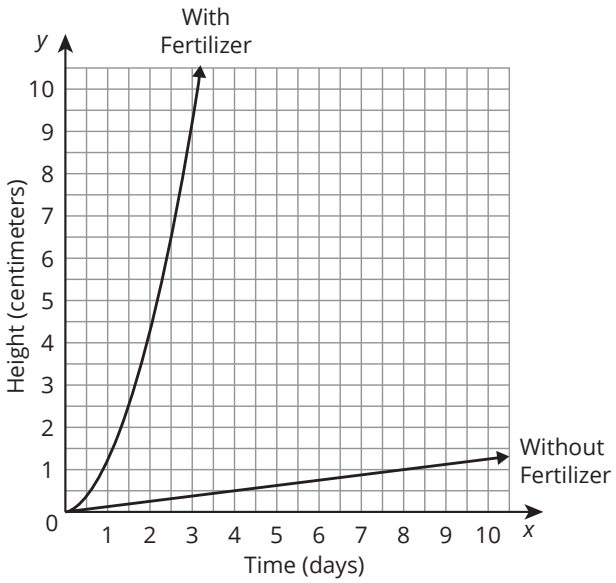
3. For his school's science fair, Jamal measures the rate of growth of a plant with and without the use of fertilizer for 10 days. His results are shown on the graph.



a. Identify the proportional and non-proportional relationships.

b. What is the unit rate of change for the proportional relationship(s)?

c. Write an equation to represent the proportional relationship(s).

4. Elena likes to stick to a budget when buying gifts for family and friends. Each year she sets a budget of $500 to spend on all her gifts. She then keeps track of the number of gifts she buys each month and the amount she spends over the year. Her budget for two previous years is shown.

2021		
Month	Number of Gifts	Total Amount Paid (dollars)
Jan.	4	100
May	2	50
June	6	150
Oct.	8	200

2022		
Month	Number of Gifts	Total Amount Paid (dollars)
Feb.	3	125
April	5	200
Aug.	2	75
Nov.	4	100

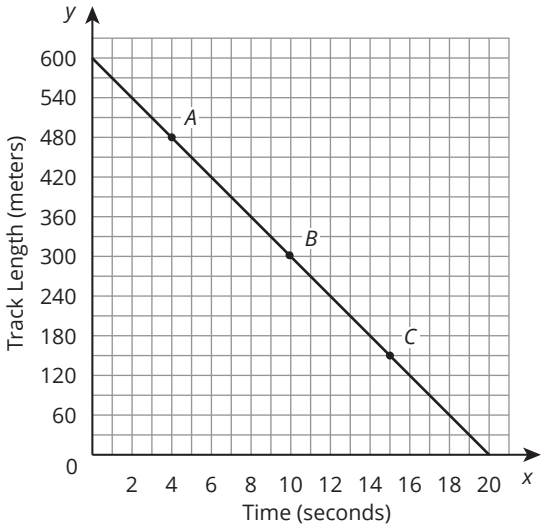
a. Identify the proportional and non-proportional relationships.

b. What is the unit rate of change for the proportional relationship(s)?

c. Write an equation to represent the proportional relationship(s).

B. Answer each question.

The linear graph shown is a model of the length of track remaining in a bobsled run, over time.



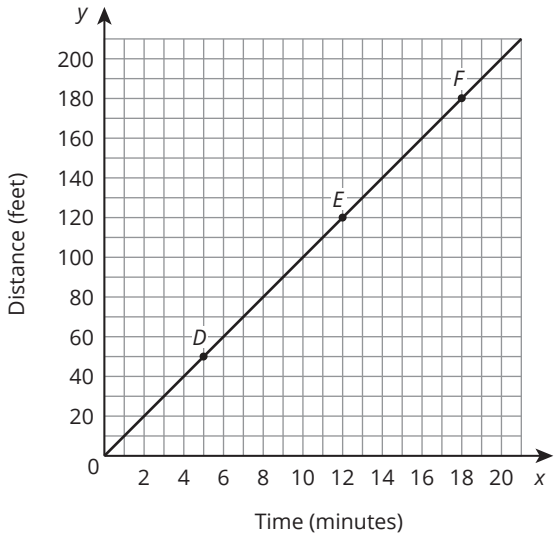
1. Does the situation represent a proportional or non-proportional relationship?

2. Use the graph to write a rate that compares the change in the track length remaining to the change in time from the start to point *A*. State whether the rate is a rate of increase or a rate of decrease.

3. Use the graph to write a rate that compares the change in the track length remaining to the change in time from the start to point *B*. State whether the rate is a rate of increase or a rate of decrease.

4. Use the graph to write a rate that compares the change in the track length remaining to the change in time from the start to point *C*. State whether the rate is a rate of increase or a rate of decrease.

The linear graph shown is a model of the distance ascended by a mountain climber, over time.



5. Does the situation represent a proportional or non-proportional relationship?

6. Use the graph to write a rate that compares the change in elevation to the change in time from the start to point *D*. State whether the rate is a rate of increase or a rate of decrease.

7. Use the graph to write a rate that compares the change in elevation to the change in time from the start to point *E*. State whether the rate is a rate of increase or a rate of decrease.

8. The linear graph shown is a model of the distance ascended by a mountain climber, over time. Use the graph to write a rate that compares the change in elevation to the change in time from the start to point *F*. State whether the rate is a rate of increase or a rate of decrease.

C. Solve each problem.

1. Matthew is making curtains for several windows in his aunt's house. He determines that he needs 8.5 yards of fabric for each window.

   a. What is the rate of change in this situation? What does it represent?

   b. Write an equation to represent the number of yards of fabric needed given a certain number of windows. Does this represent a linear proportional or linear non-proportional relationship?

c. While Matthew was at his aunt's house, he decides he also needs 10 yards of fabric to recover a chair. How will this additional fabric to recover the chair impact the graph? Impact the equation?

d. Write an equation to represent the number of yards of fabric needed given a certain number of windows and the fabric Matthew will need to recover the chair. Does this represent a linear proportional or linear non-proportional relationship?

2. Tiara is given a bunch of nickels as her change after buying a snack. To figure out how much money she has been given, she starts counting the nickels.

   a. What is the rate of change in this situation? What does it represent?

   b. Write an equation to represent Tiara's total change in dollars given a certain number of nickels received. Does this represent a linear proportional or linear non-proportional relationship?

   c. When Tiara reached in her wallet, she found an additional $5 that she had forgotten about. How will this additional money impact the graph? Impact the equation?

   d. Write an equation to represent the total amount of money Tiara has now, given a certain number of nickels received. Does this represent a linear proportional or linear non-proportional relationship?

3. A citizen's group pays Josh to collect signatures to stop a local park from being sold to an industrial developer. He earns $0.15 per signature.

   a. What is the rate of change in this situation? What does it represent?

   b. Write an equation to represent how much money Josh earns given a certain number of signatures collected. Does this represent a linear proportional or linear non-proportional relationship?

   c. When the citizen's group hired Josh to collect signatures he earned a consulting fee of $50. How will this additional money impact the graph? Impact the equation?

   d. Write an equation to represent how much money Josh earns given a certain number of signatures collected with his consulting fee added in. Does this represent a linear proportional or linear non-proportional relationship?

4. Trung is swimming laps during swim team practice. He knows it takes him about 1.75 minutes to swim each lap.

   a. What is the rate of change in this situation? What does it represent?

   b. Write an equation to represent the total time in minutes Walter swims given a certain number of laps. Does this represent a linear proportional or linear non-proportional relationship?

c. Before Trung started swimming laps, he took some time to warm up in the pool. Trung spent 10 minutes warming up before starting his laps. How will his warm up minutes impact the graph? Impact the equation?

d. Write an equation to represent how many minutes Trung swims in the pool given a certain number of laps swam and his warm up time. Does this represent a linear proportional or linear non-proportional relationship?

5. Ashley is a translator who was hired to translate documents for a legal firm. To meet her deadline, she can only spend $\frac{1}{2}$ hour on each document she translates.

a. What is the rate of change in this situation? What does it represent?

b. Write an equation to represent the number of hours spent on the project given a certain number of documents translated. Does this represent a linear proportional or linear non-proportional relationship?

c. Last week Ashley spent 5 hours translating documents for the legal firm. How does this additional time spent on the project impact the graph? Impact the equation?

d. Write an equation to represent the number of hours spent on the project given a certain number of documents translated and the time Ashley already spent last week. Does this represent a linear proportional or linear non-proportional relationship?

6. The Drama Club is selling small bags of popcorn at an outdoor movie to raise money for their club. The club raises $3.75 for each bag it sells.

   a. What is the rate of change in this situation? What does it represent?

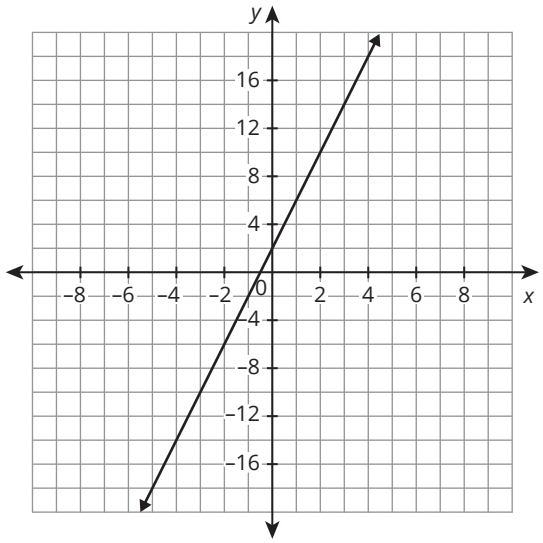
   b. Write an equation that represents the total amount raised by the club given a certain number of bags of popcorn sold. Does this represent a linear proportional or linear non-proportional relationship?

   c. The Drama Club was surprised with an anonymous donation a week before the popcorn fundraiser. The Drama Club was given $100 for their club. How does this donation impact the graph? Impact the equation?

   d. Write an equation to represent the total amount raised by the club given a certain number of bags of popcorn sold and the anonymous donation received. Does this represent a linear proportional or linear non-proportional relationship?

D. Determine the slope of each line. Then, write an equation in the form of $y = mx + b$ to represent the line.

1.



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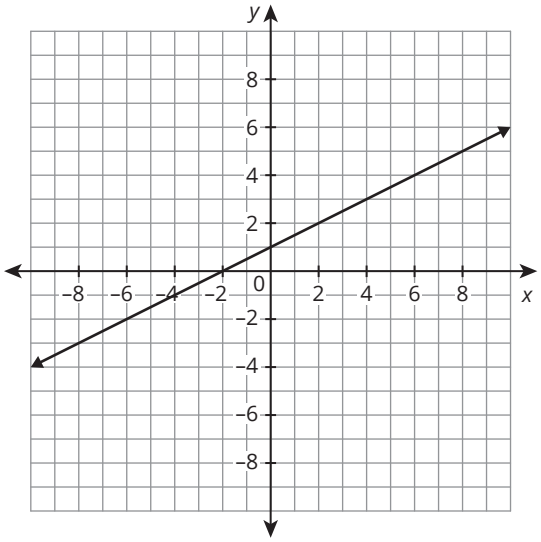

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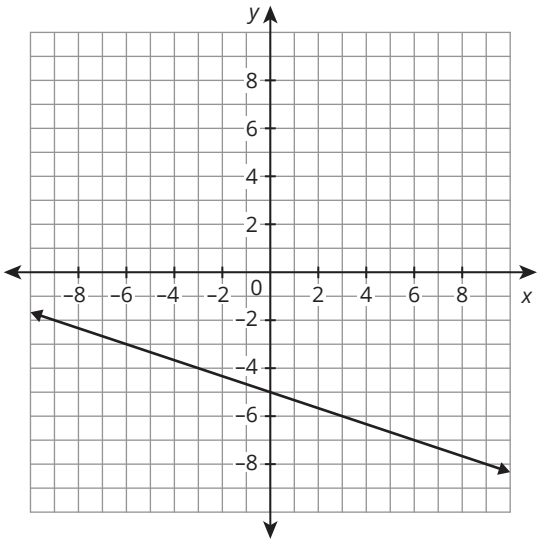

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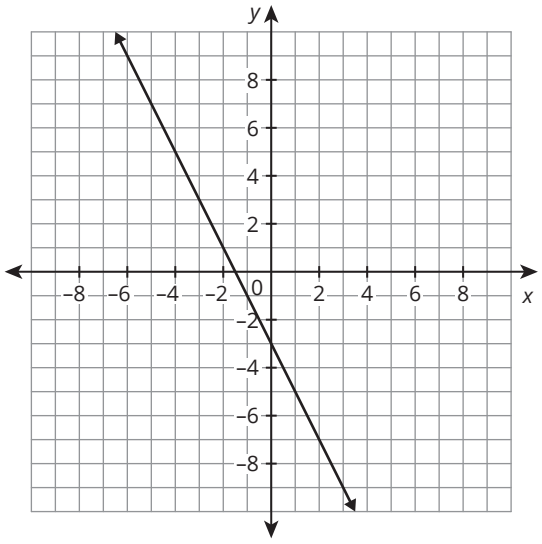

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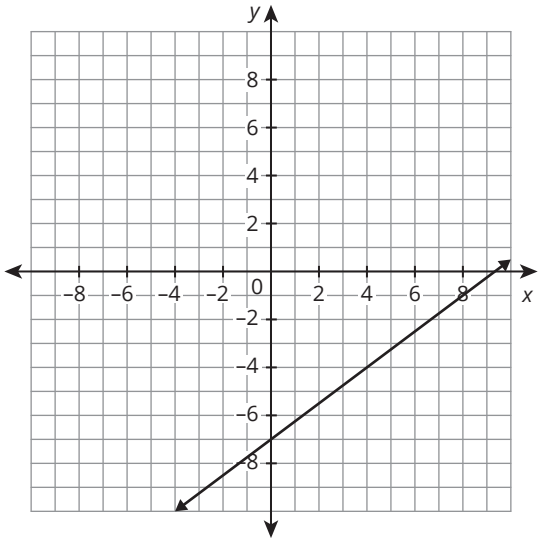

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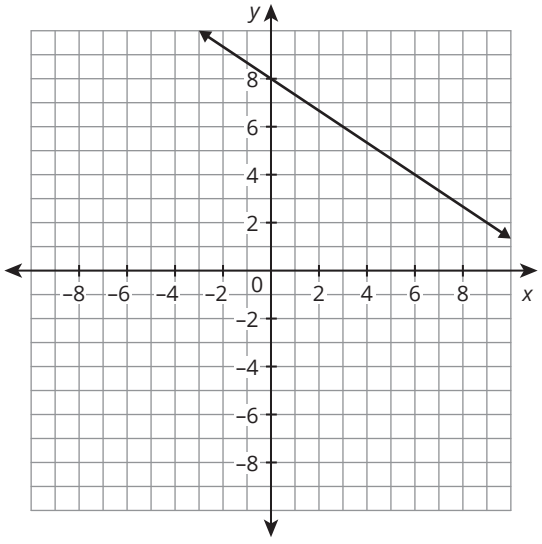

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Extension

1. Write an equation that determines where the graph crosses the *y*-axis, given the slope and the coordinates of one point.

Spaced Practice

1. Determine whether each equation represents a proportional or non-proportional relationship.

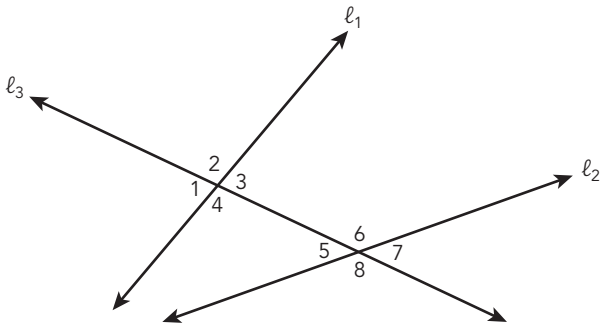
   a. $y = 2.5x$

   b. $y = x - 4$

2. Examine the figure shown.

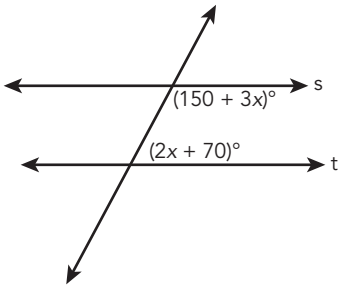
   a. Name 2 pairs of same-side interior angles.

   b. Name 2 pairs of congruent angles.



   c. Name 2 pairs of supplementary angles.

3. In the diagram shown, line *s* and line *t* are parallel. Determine the measures of all the angles.

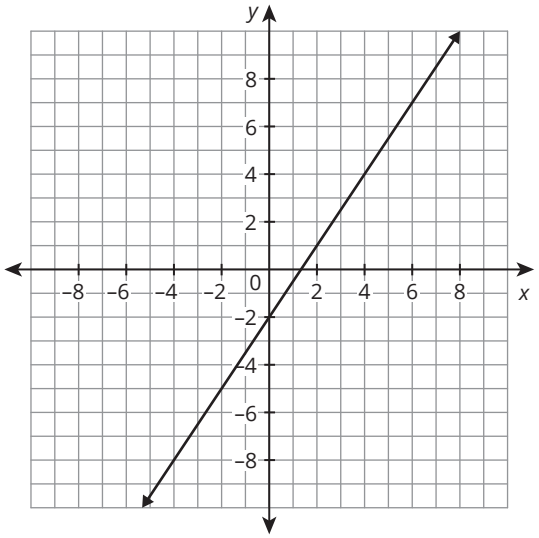


III. Exploring Slopes Using Similar Triangles

Topic Practice

A. Use similar triangles to show that the rate of change is the same for any two points on each line. What is the slope of the line?

1.



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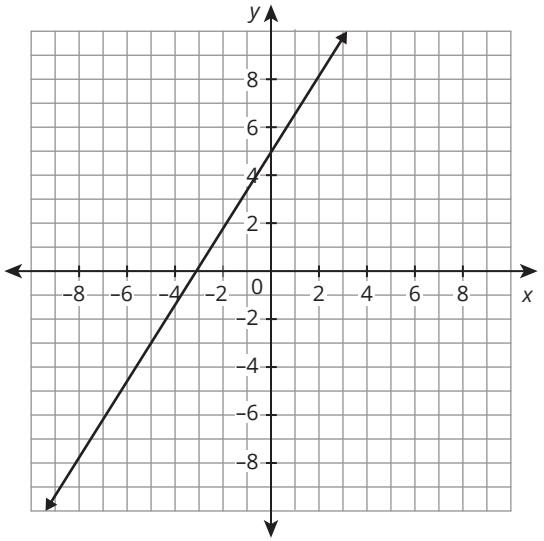

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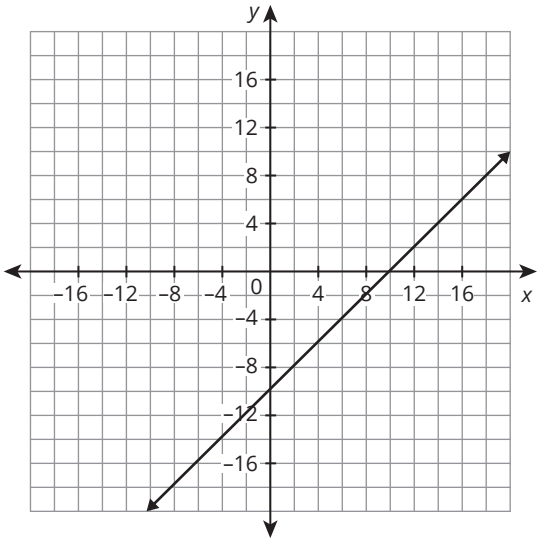
3.


y
x
16
12
8
4
0
−4
−8
−12
−16
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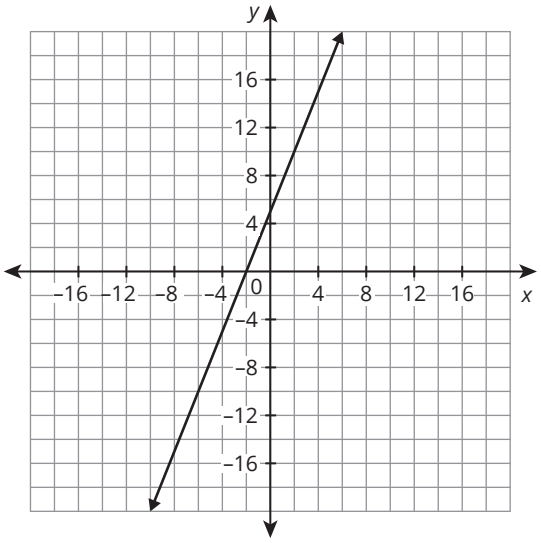
−8
−4
4
8
12
16

4.


y
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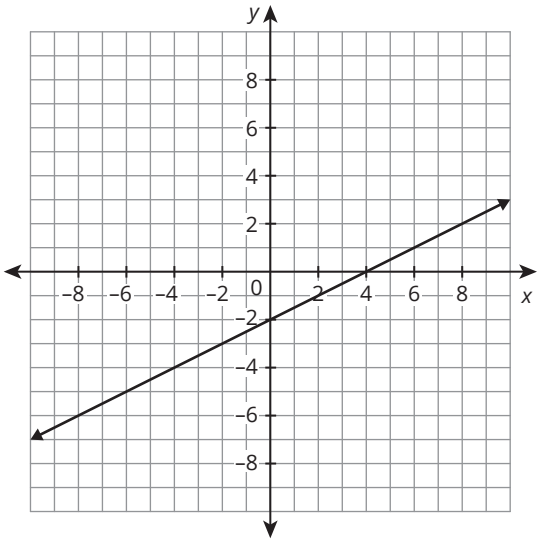
5.



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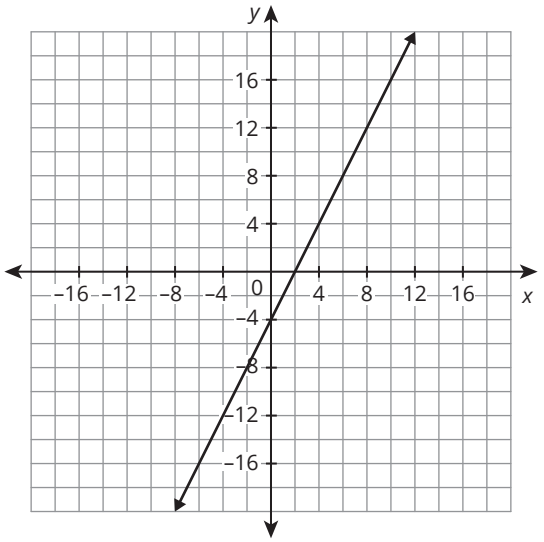

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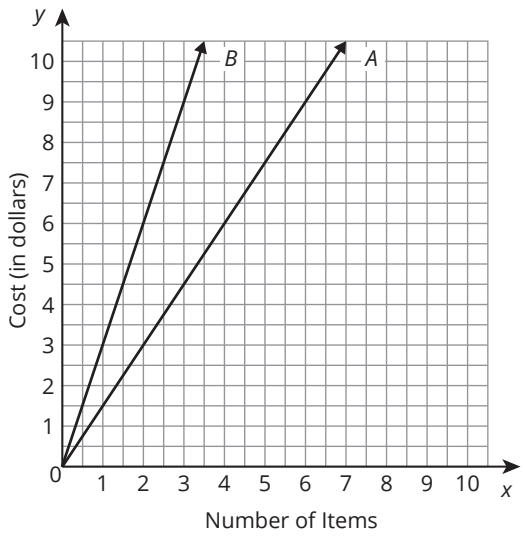
B. Determine the slope of each line and then write the equation to represent each line.

1.



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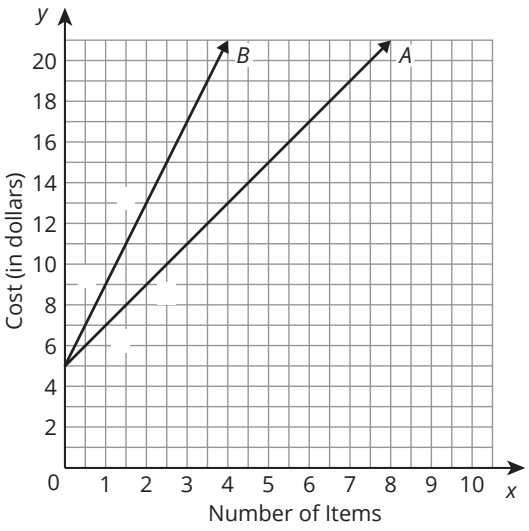
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12
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8
6
4
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2
3
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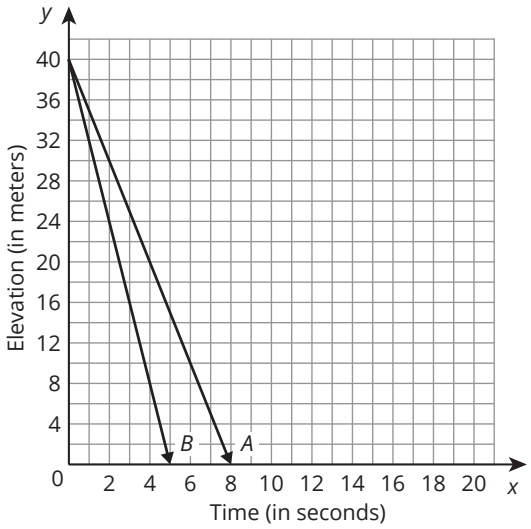
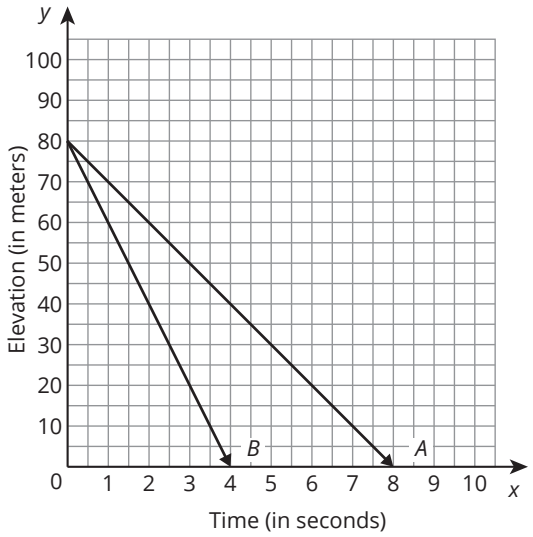
5
6
7
8
9
10
x
B
A
Cost (in dollars)
Number of Items

3.


y
40
36
32
28
24
20
16
12
8
4
0
Elevation (in meters)
B
A
2 4 6 8 10 12 14 16 18 20
x
Time (in seconds)

4.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x
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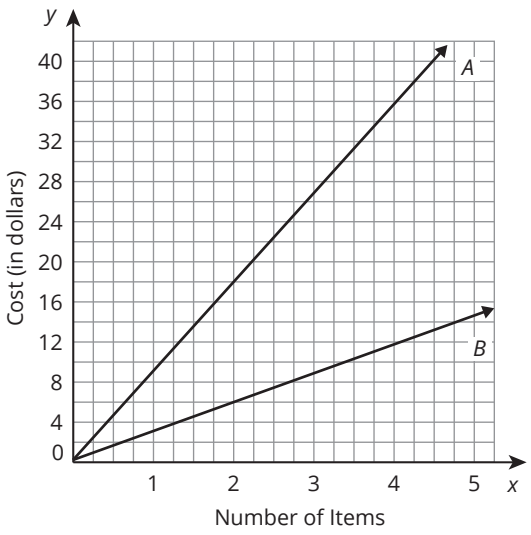
A
Elevation (in meters)
Time (in seconds)

5.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Cost (in dollars)
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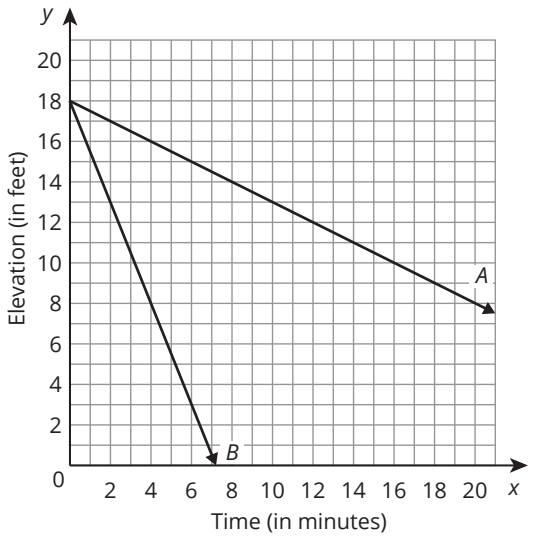
4
5
x
Number of Items

6.


y
20
18
16
14
12
10
8
6
4
2
0
Elevation (in feet)
A
B
2 4 6 8 10 12 14 16 18 20 x
Time (in minutes)

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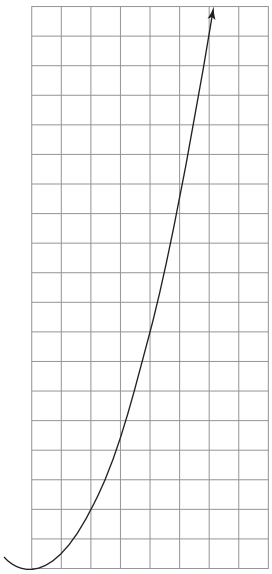
Extension

Create a table of values for the equation $y = x^2$. Use the points with x-values of 0, 1, 2, and 3 to create triangles with the length of each base equal to 1 unit.

- Describe the relationship between the heights of the resulting triangles.
- Are the triangles similar? Explain your reasoning.



x	y

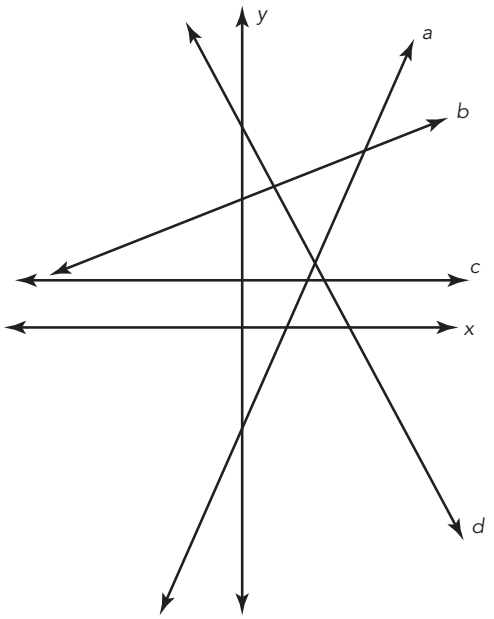
Spaced Practice

1. Determine the unknown angle measure for each triangle.

   a. $m\angle A = 46°$, $m\angle B = 90°$, $m\angle C = ?$

   b. $m\angle P = ?$, $m\angle Q = 10°$, $m\angle R = 110°$

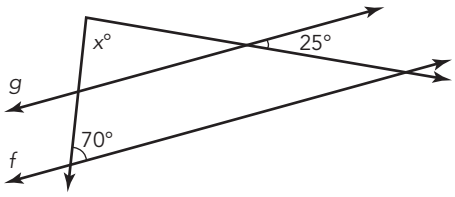
2. Consider the graph of lines a, b, c, and d.



   a. Name the line(s) with a positive slope.

   b. Name the line(s) with a negative slope.

3. Solve for the unknown angle measure given that $f \parallel g$.

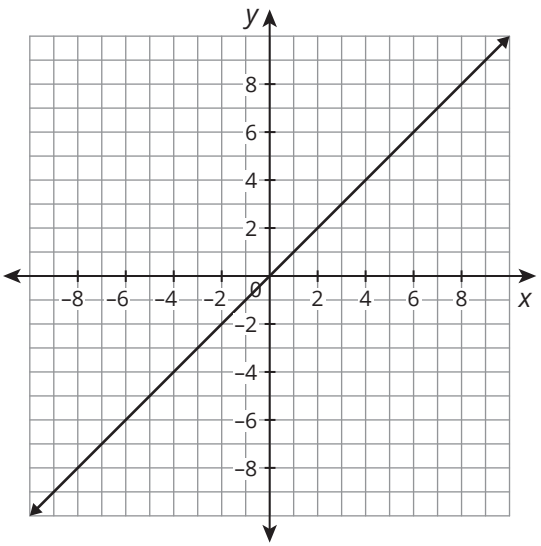


IV. Transformations of Lines

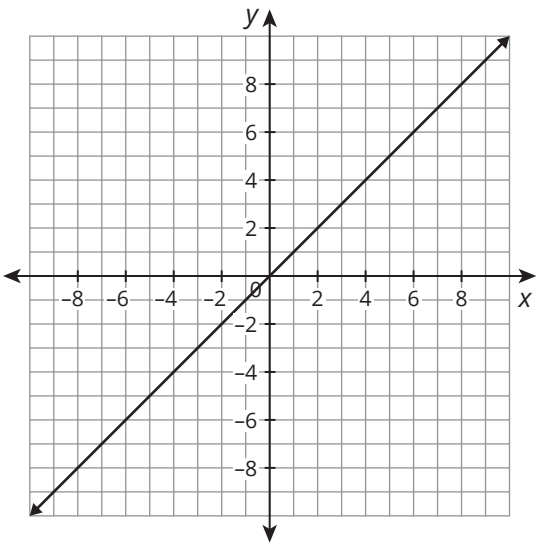
Topic Practice

A. For each graph: write an equation to represent the line, perform the given translation on the coordinate plane, and write an equation to represent the translation.

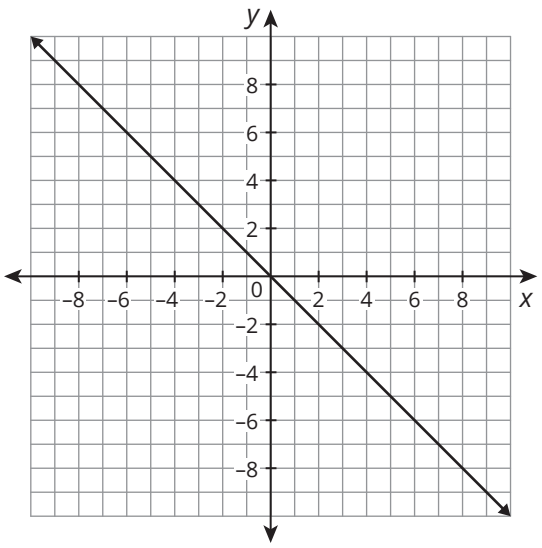
1. Translate the line up 6 units.



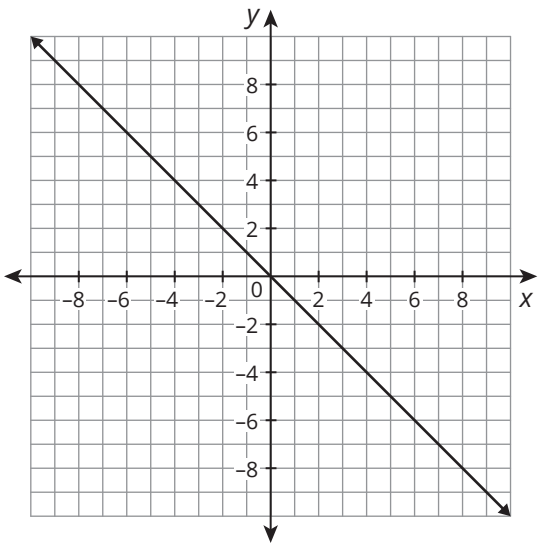
2. Translate the line down 2 units.



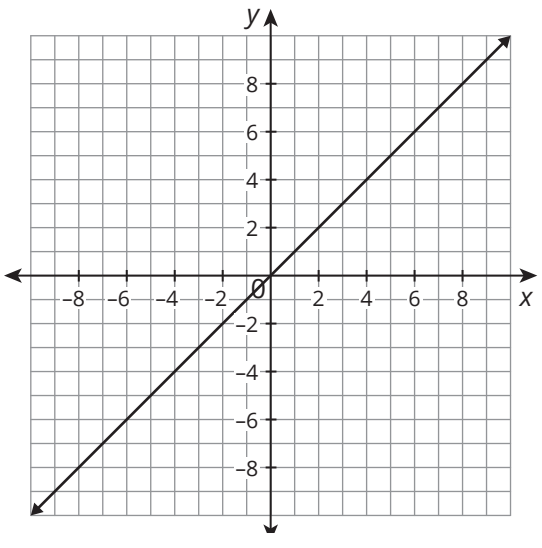
3. Translate the line down 3 units.



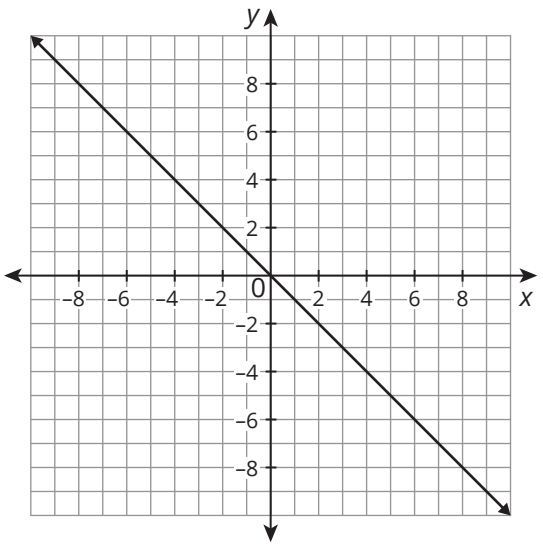
4. Translate the line up 2 units.



5. Translate the line up 5 units.



6. Translate the line down 8 units.

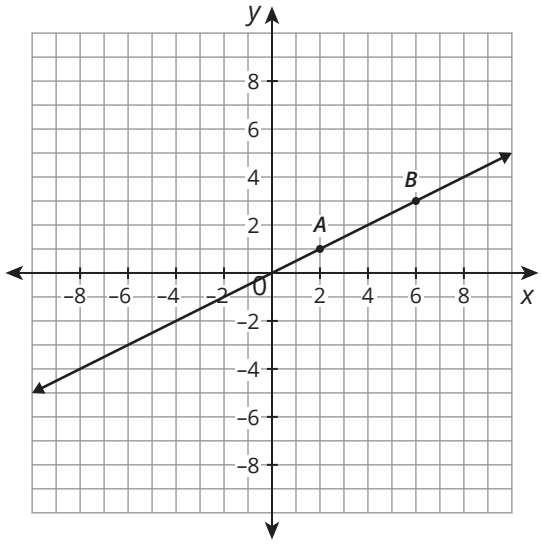


7. Consider your graphs. What do you notice about the pair of lines in each example?

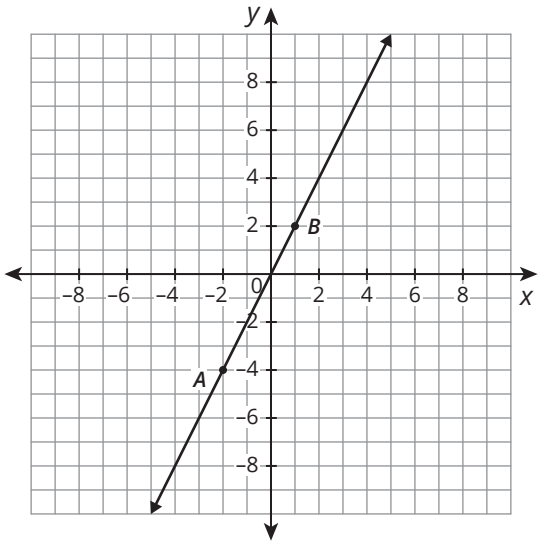
8. Consider your equations. What do you notice about the pair of equations in each example?

B. For each graph, suppose the line $y = x$ was dilated to form the given line. Write the equation of the dilated line. Then, perform the translation and graph the new line on the coordinate plane. Finally, write the equation of the dilated and translated line.

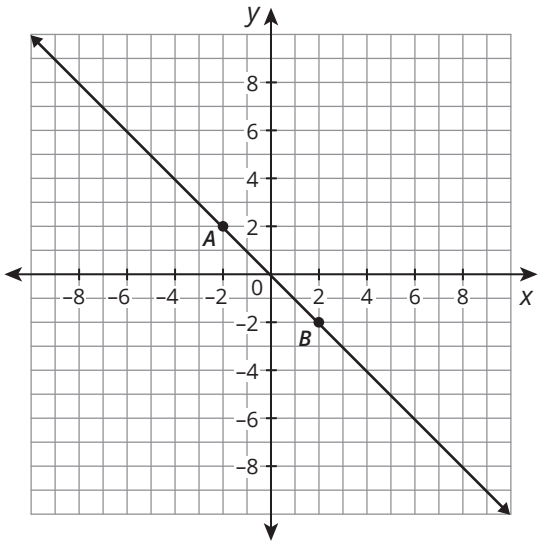
1. Vertically translate line *AB* 4 units to create line *CD*.



2. Vertically translate line *AB* −6 units to create line *CD*.



3. Vertically translate line *AB* 5 units to create line *CD*.

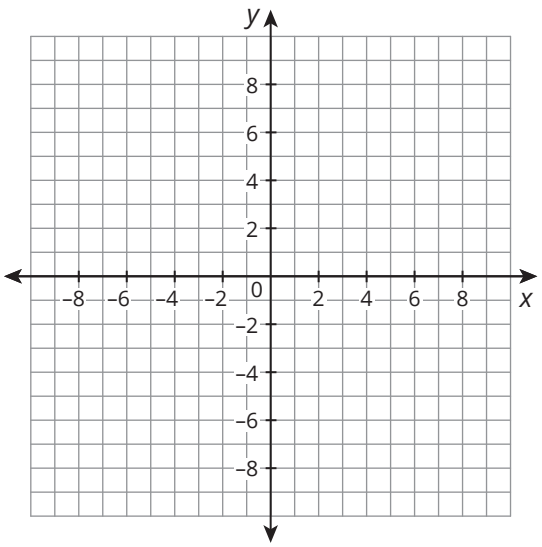


4. Consider your graphs. What do you notice about the relationship among the original line, $y = x$, the dilated line shown and the dilated and translated line that you added in each example?

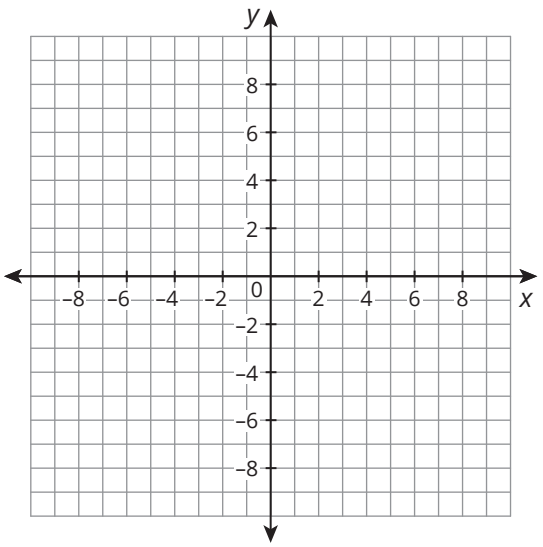
5. Consider your equations. What do you notice about the relationship among the original equation, $y = x$, the equation of the dilated line and the equation of the dilated and translated line in each example?

C. Graph each equation using transformations. Specify which transformations you use.

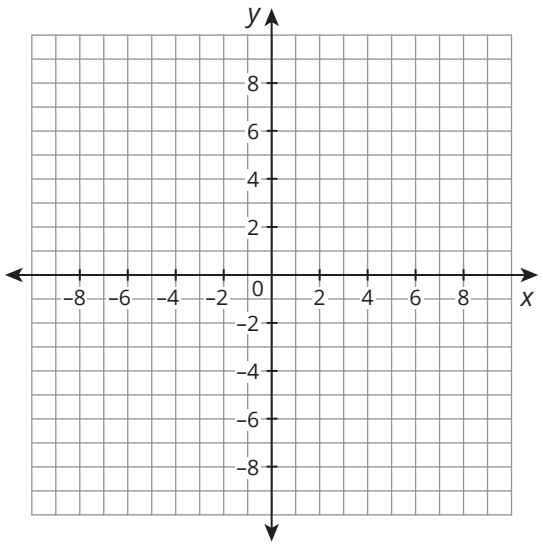
1. $y = 3x + 2$



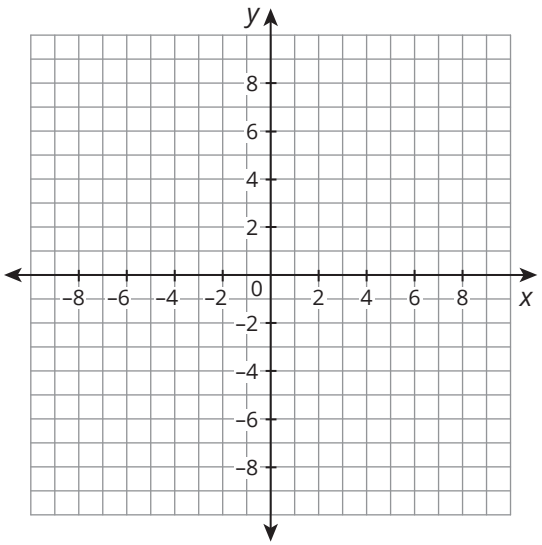
2. $y = -\frac{2}{3}x + 5$



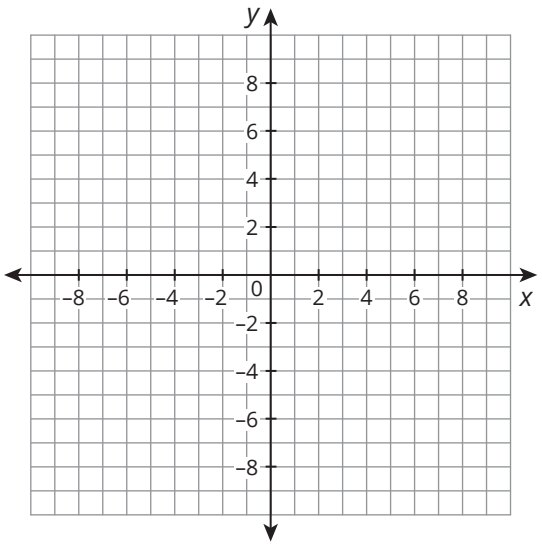
3. $y = \frac{1}{2}x - 6$



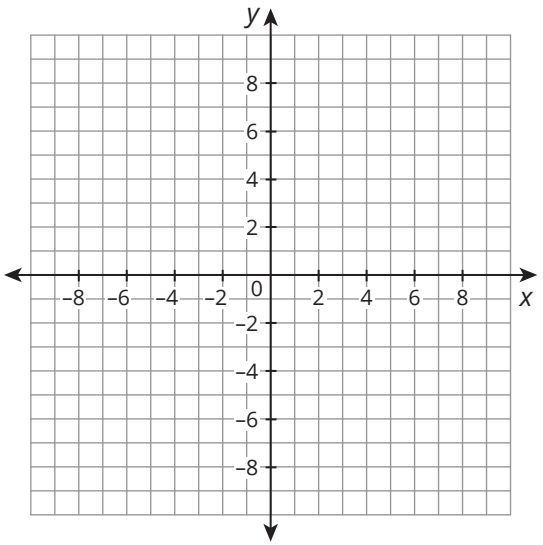
4. $y = -4x - 3$



5. $y = -\frac{3}{4}x - 4$



6. $y = \frac{3}{8}x + 1$



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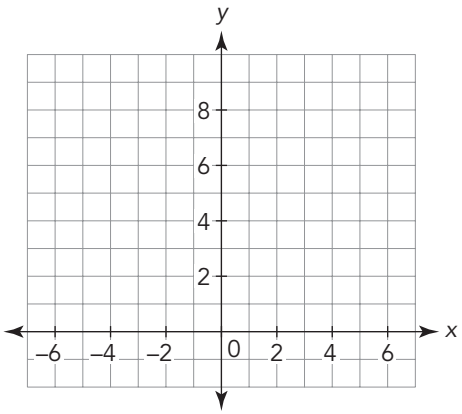
Extension

Graph each given sequence of transformations. Are the equations the same? Explain why the equations must be the same or why they are not the same. Use transformations to support your answer.

1. Translate $y = x$ up 4 units and then dilate by a factor of 2.

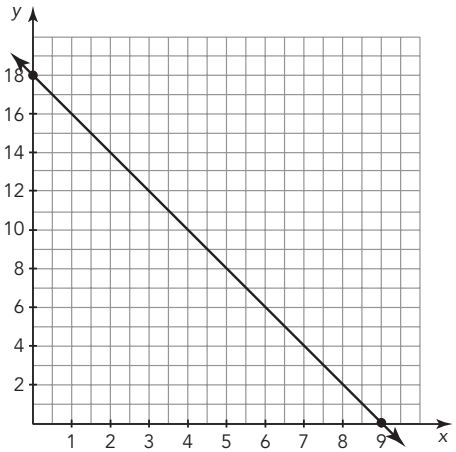
2. Dilate $y = x$ by a factor of 2 and then translate up 4 units.



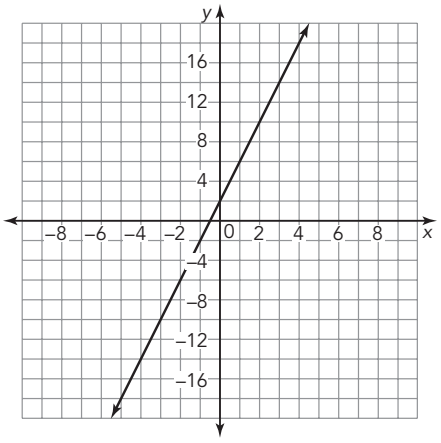
Spaced Practice

Draw similar triangles on the graph to determine each slope.

1.

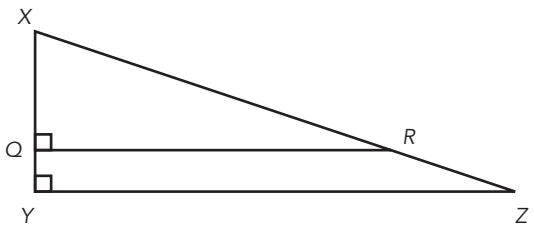


2.

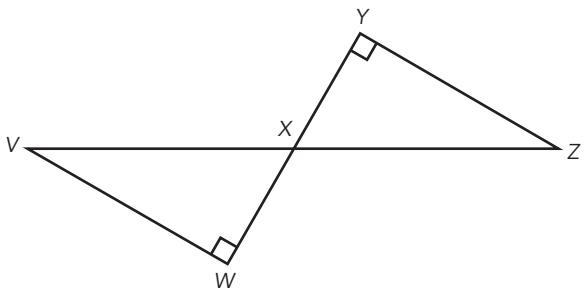


Identify the similar triangles and explain how the triangles are similar by the Angle-Angle Similarity theorem.

3.



4.



Determine whether each equation represents a proportional or non-proportional relationship.

5. $y = 17x - 4$

6. $y = -3x + 9$

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Name _____ Date _____

I. Using Tables, Graphs, and Equations

Topic Practice

A. For each problem situation, complete the table of values. Identify the variable quantities and the constant quantities.

1. You pay $5 to enter a raffle, plus $1 for each raffle ticket.

Number of Raffle Tickets Purchased	Total Cost (dollars)
10	
11	
12	
13	
14	
15	

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2. A company pays $500 for each computer, plus an $80 shipping charge.

Number of Computers in Each Order	Total Cost (dollars)
5	
6	
7	
8	
9	
10	

3. Ashley has already read 55 pages of a novel. Each day, she reads another 30 pages.

Number of Days	Number of Pages Read
2	
4	
6	
8	
10	
12	

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4. Mr. Patel is writing an autobiography. He has already written 32 pages. Each day, he writes another 5 pages.

Number of Days	Number of Pages Written
10	
15	
20	
25	
30	
35	

5. A flower is 9.3 cm tall when it is planted and grows 2.5 cm per week.

Number of Weeks	Height (cm)
3	
6	
9	
12	
15	

6. Mariana pays $18 to join an online library and $6.99 for each electronic book she downloads.

Number of Books	Total Cost (dollars)
2	
4	
6	
8	
10	

B. Create a graph of the data in each given table. Write an equation to represent each given linear relationship.

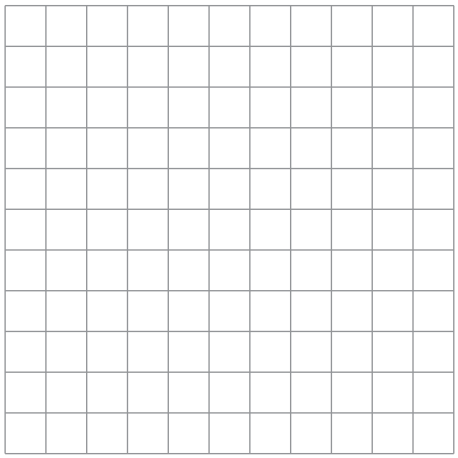
1. A video game store sells each game for $30. The store's total fixed expenses are $500 each month. They record the number of games sold, *g*, and their total profit, *p*, in the table below.

Number of Games Sold	20	30	40	50	60	70
Total Profit (dollars)	100	400	700	1000	1300	1600



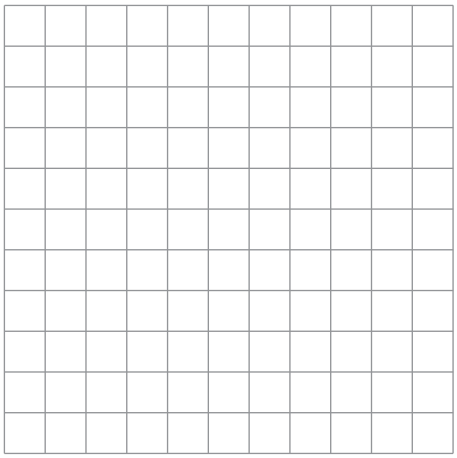
2. A paper store sells each ream of paper for $40. They will deliver it for a one-time fee of $25. An office records their total paper costs, c, in dollars, and the total number of reams delivered, r.

Number of Reams Delivered	Total Paper Costs (dollars)
6	265
8	345
10	425
12	505
14	585
16	665



3. An architect is designing a new hotel. He is planning to have 10 windows in the lobby, and each hotel room will have 2 windows. The hotel will not have any other windows. He records the total number of windows, w, for different numbers of hotel rooms, r.

Number of Hotel Rooms	Total Number of Windows
50	110
75	160
100	210
125	260
150	310
175	360



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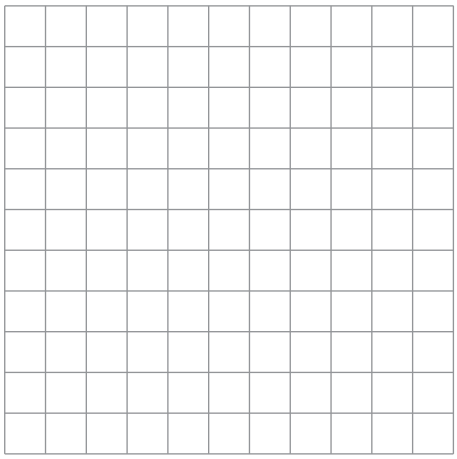
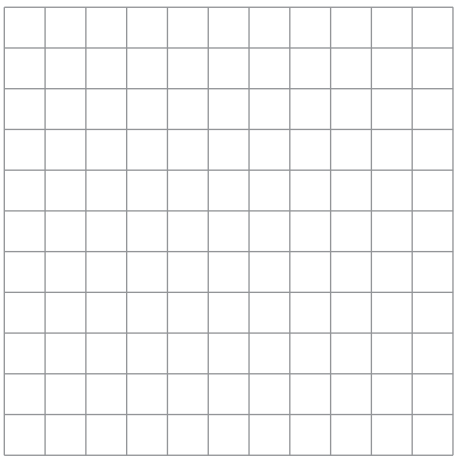
4. A bookstore spends $300 on a new shipment of 200 books. They sell each book for $12. They record their total profit, p, in dollars, after selling b books.

Number of Books Sold	Total Profit (dollars)
40	180
60	420
80	660
100	900
120	1140
140	1380



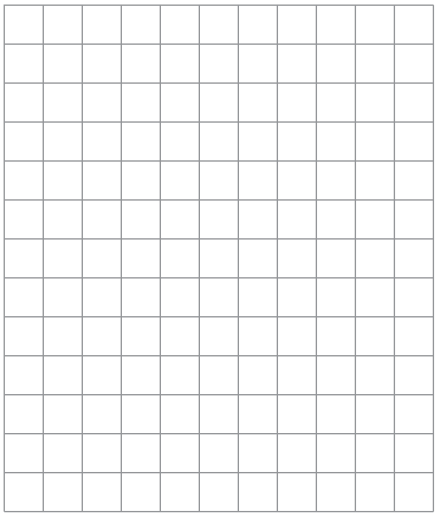
5. Kaya's grass is 3 inches tall. Her grass grows at a rate of 2.5 inches per week. She records the height, h, in inches, after w weeks.

Number of Weeks	Height (in.)
1	5.5
2	8
3	10.5
4	13
5	15.5



6. Jamal purchases a bowling pass for $12. He pays $3.50 per game. He keeps track of the cost in dollars, d, after g games.

Number of Games	Total Cost (dollars)
2	19
4	26
6	33
8	40
10	47



C. For each problem situation, define variables and write an algebraic equation.

1. A person's nails grow at an average rate of 1 millimeter per week. You measure one of your nails to be 10 millimeters long.

2. An employee earns $500 each week, plus $15 for each hour of overtime.

3. A manager is hired at a starting salary of $60,000 per year. Each year, her salary increases by $5000.

4. A hair stylist earns $70 per day, plus $20 for each haircut.

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5. A clothing company charges $5 for each belt that it sells. It also charges a fixed delivery fee of $8 per order.

6. A local gym charges $10 for each class that you attend. It also charges a fixed membership fee of $50 a month.

Extension

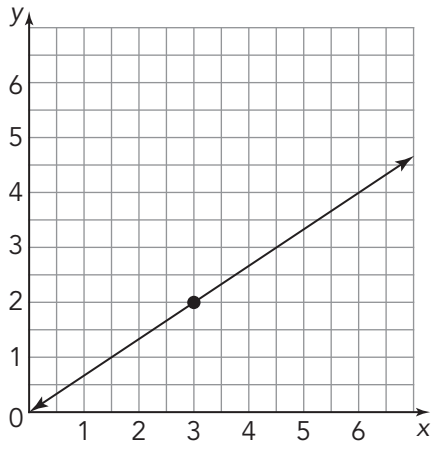
Two catering companies have different one-time fixed fees. Company *A* charges a fixed fee of $75, and Company *B* charges a fixed fee of $100. Each company also has a cost per person.

Suppose the independent quantity is the number of people and the dependent quantity is the cost. The graphs for the two companies never intersect. What does this tell you about how much each company charges?

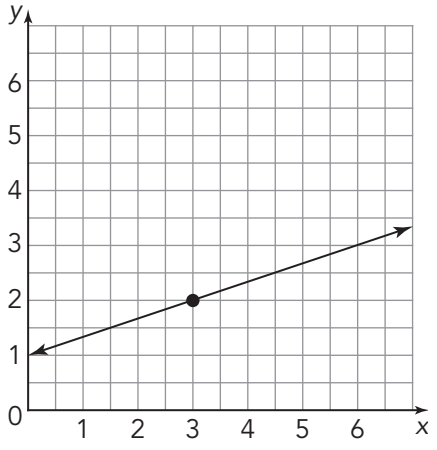
Spaced Practice

1. Draw a line through the point and label the graph to represent each linear relationship. Then, write an equation to represent the relationship.

   a. Linear proportional relationship

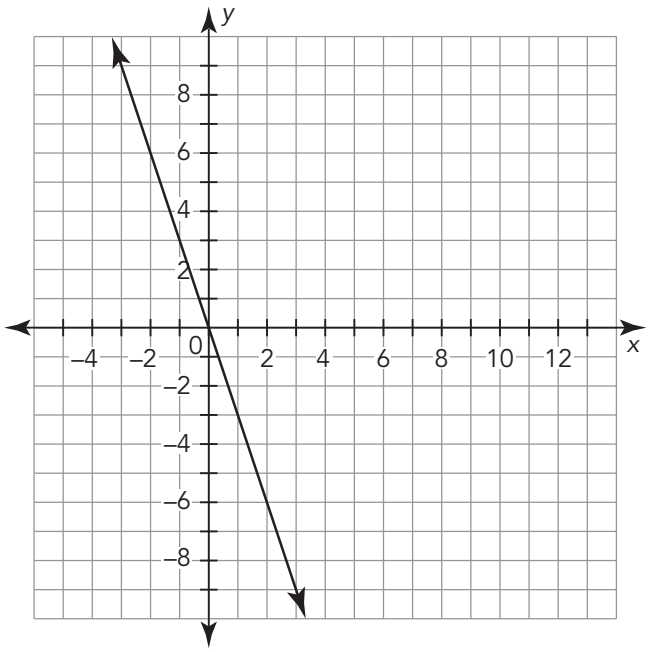


   b. Linear non-proportional relationship



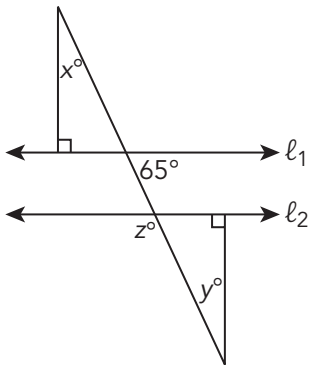
2. Use the equation $y = -3x$ to complete the table of values. Graph the equation. Then, use the points on the graph to sketch similar triangles that may be used to show the rate of change of the line is the same between any two points.

x	y
−2	
−1	
0	
1	
2	

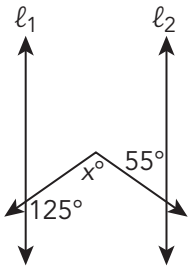


3. Solve for each unknown angle measure, given that $\ell 1 \parallel \ell 2$.

a.



b.

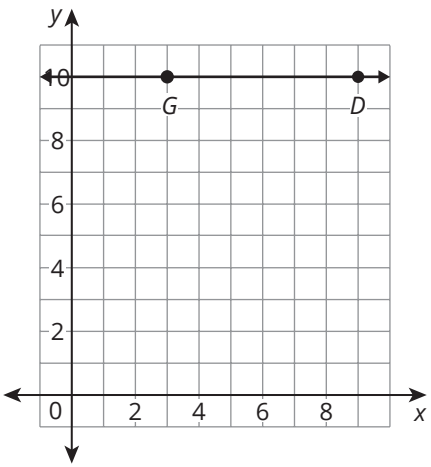


II. Linear Relationships in Tables

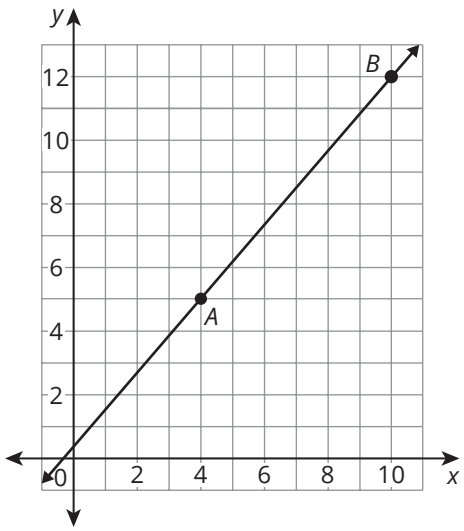
Topic Practice

A. Use the graph and the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to determine the slope of each line. Verify your answer with similar triangles.

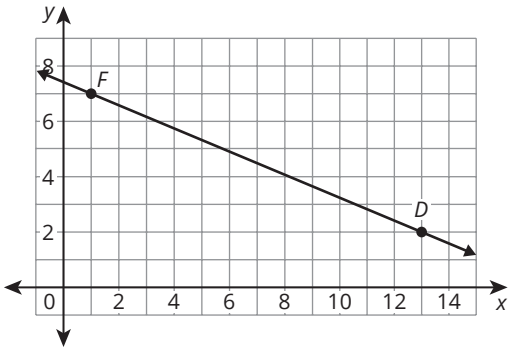
1.



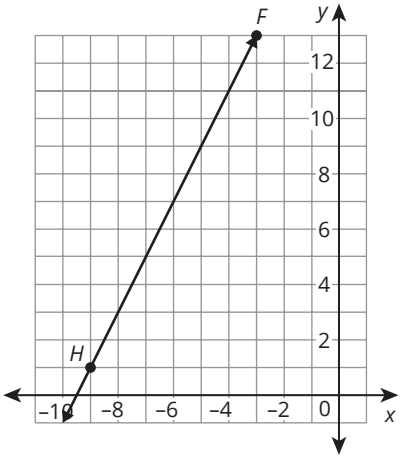
2.



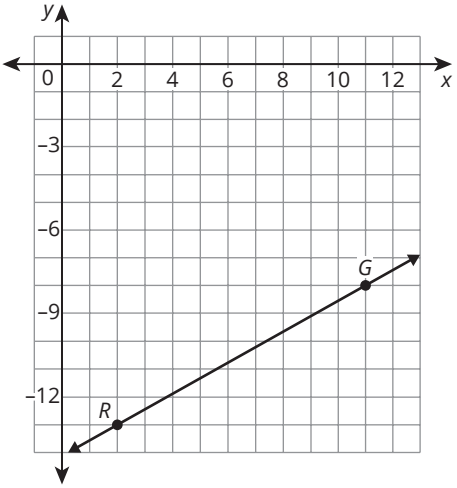
3.


y
x
F
D
0
2
4
6
8
10
12
14

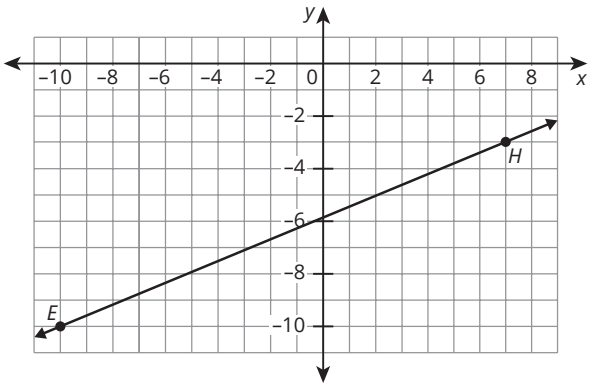
4.


y
x
F
H
12
10
8
6
4
2
−10
−8
−6
−4
−2
0

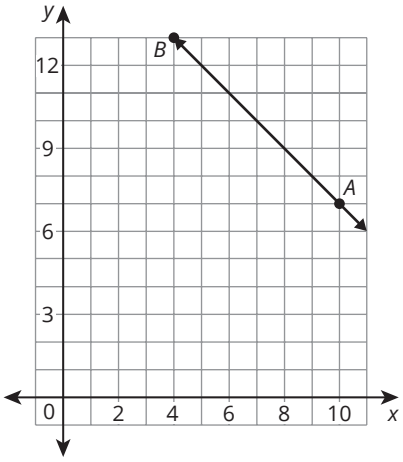
5.


y
x
G
R
0
2
4
6
8
10
12
−3
−6
−9
−12

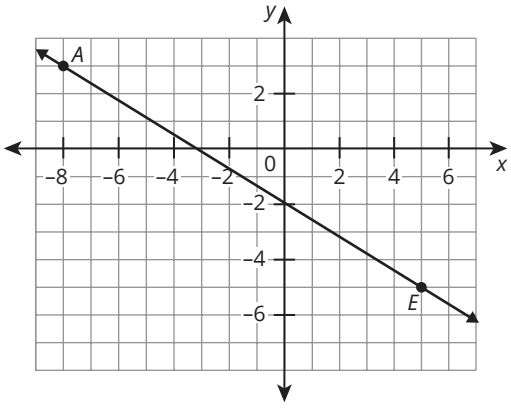
6.


y
x
H
E
−10
−8
−6
−4
−2
0
2
4
6
8
−2
−4
−6
−8
−10

7.


y
x
B
A
12
9
6
3
0
2
4
6
8
10

8.


y
x
A
E
2
−2
−4
−6
−8
0
2
4
6

B. Analyze each table of values. Calculate the rate of change between the points listed in each table. Determine if the table represents a proportional relationship.

1.

x	y
8	34
20	70
32	106
44	142
60	190

2.

x	y
4	60
12	180
23	345
37	555
48	720

3.

x	10	16	22	29	35
y	25	40	66	87	105

4.

x	4	12	28	36	44
y	9	15	27	33	39

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5.

x	y
6	48
12	96
23	345
36	540
42	756

6.

x	y
3	4.5
6	9
9	13.5
12	18
15	22.5

C. Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the unit rate of change for the data in each table. The rate of change is constant for the data in each table.

1.

Number of Raffle Tickets	Total Cost of Raffle Tickets (dollars)
2	1
4	2
8	4
10	5

2.

x	*y*
−2	8
0	0
2	−8
4	−16

3.

Number of Photos Printed	Total Cost of Photos (dollars)
10	2
20	4
30	6
40	8

4.

x	*y*
3	27
5	45
7	63
9	81

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5.

Number of Greeting Cards	2	3	6	8
Total Cost of Greeting Cards (dollars)	6.50	9.75	19.50	26.00

6.

x	−3	1	3	7
y	54	−18	−54	−126

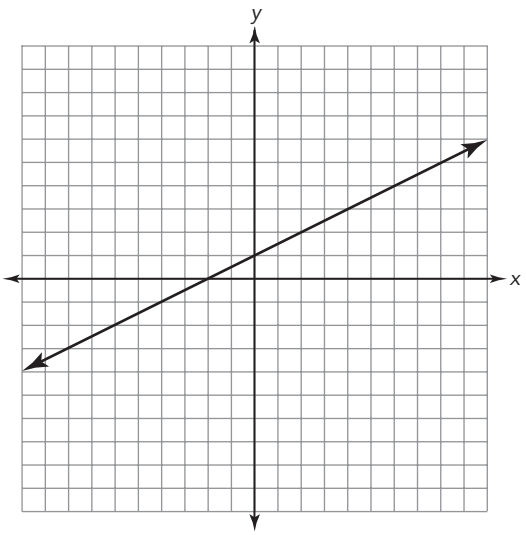
Extension

Is the relationship described by the equation $y = x^2$ linear? Is it proportional? Describe how you determined your answer.

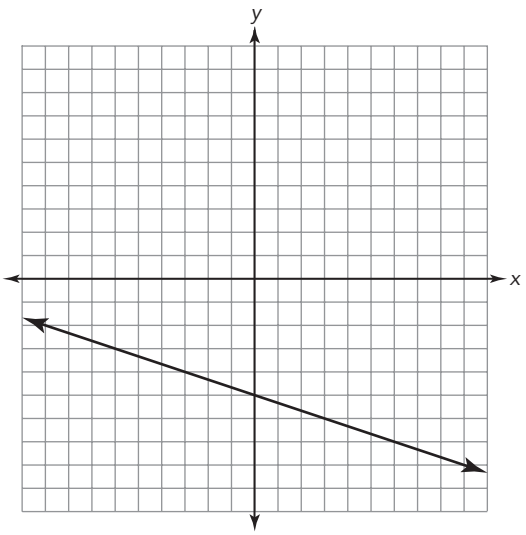
Spaced Practice

1. Determine the slope of each linear relationship.

a.



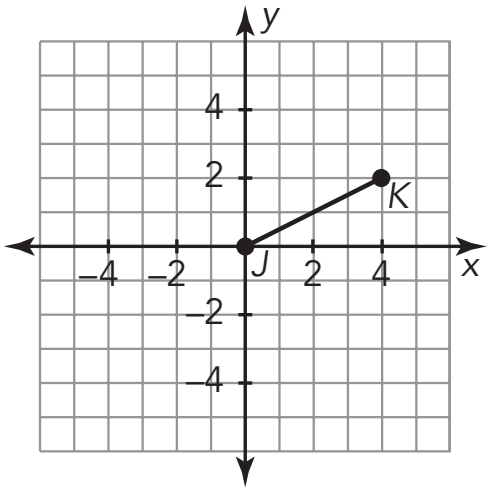
b.



c. $y = 2x$

d. $\frac{5}{6} = \frac{y}{x}$

2. Consider the graph shown.



a. Segment JK is rotated 90° clockwise, resulting in segment $J'K'$. What are the coordinates of K'?

b. Segment JK is reflected across the line $x = -1$, resulting in segment $J'K'$. What are the coordinates of K'?

III. Linear Relationships in Contexts

Topic Practice

A. Read each problem situation and identify the independent and dependent variables. Then, convert the information to coordinate points and use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to answer the question.

1. Eduardo is traveling to a vacation destination 715 miles from home. On the first day of his trip, he travels 390 miles in 6 hours. On the second day of his trip, he leaves at 8:00 a.m. and arrives at his destination at 1:00 p.m. What is the unit rate of the total number of miles per hour traveled?

2. Samantha loves reading and is participating in a read-a-thon to raise money for a charity. She plans to read 15 books during the 90-day read-a-thon. During the first 30 days, she reads 7 books. What is the unit rate of the number of remaining days she has to read each book to meet her goal?

3. Elena paid $19 to park in a downtown parking garage for a 3-hour event. After spending 5 hours downtown she paid $25 for parking. What is the unit rate for garage parking in dollars per hour?

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4. Jamal is traveling on a toll road. If he exits the road 5 miles ahead at First Avenue, he pays $1.75. If he exits the road 9 miles ahead at Butler Street, he pays $2.75. What is the unit rate of the toll cost per mile?

5. Eduardo is saving for a new video game. After adding 2 weeks of his allowance to a savings account, he has $105. After adding 5 more weeks of his allowance to his savings, he has $150. What is the unit rate of Eduardo's allowance in dollars per week?

6. Josh is renewing a magazine subscription. Josh can renew the magazine for 3 years for $26. He can renew the magazine for 5 years for $38. What is the unit rate of the subscription cost per year?

7. Lizzie and her friend Alyssa both saw multiple movies this past month. After going to 5 movies, Lizzie had spent $64. After going to 2 movies, Alyssa had spent $40. What is the unit rate of the cost per movie?

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Extension

Create a situation that can be represented by a linear relationship in which its unit rate value doesn't change when you switch the independent and dependent quantities.

Spaced Practice

Determine whether the relationships represented in the tables are linear. If so, calculate the rate of change.

1.

Number of Bull's-Eyes Made	Points Displayed
0	12,000
3	36,000
5	52,000
9	84,000

2.

x	6	-4	-12	-22
y	12	7	-3	-8

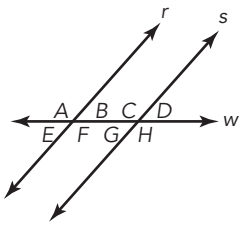
Determine whether the slope of the line represented by each equation is positive, negative, zero, or undefined.

3. $y = -x + 5$

4. $x = 0$

In the figure, parallel lines r and s are cut by transversal w.

5. List all pairs of corresponding angles.



6. List all pairs of alternate interior angles.

IV. Slope-Intercept Form of a Line

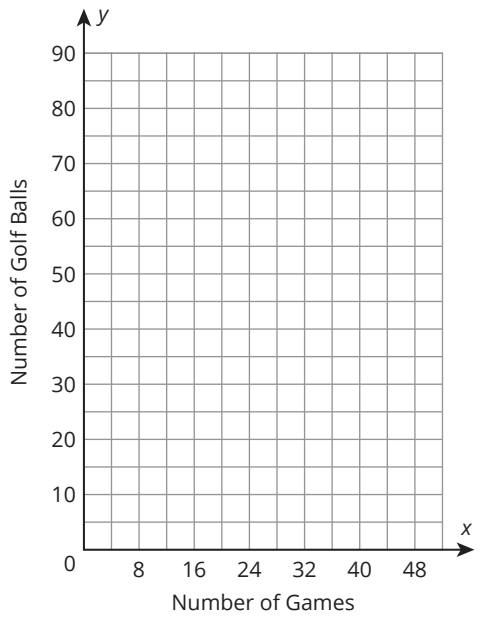
Topic Practice

A. Graph each situation. Identify the slope and *y*-intercept of each graph.

1. At the beginning of the golf season, Chris buys 84 golf balls. He loses 2 balls each time he plays a game.

   a. Write an equation that represents the number of golf balls, *y*, Chris has left given a number of times he plays *x* games.

   b. Graph the equation.



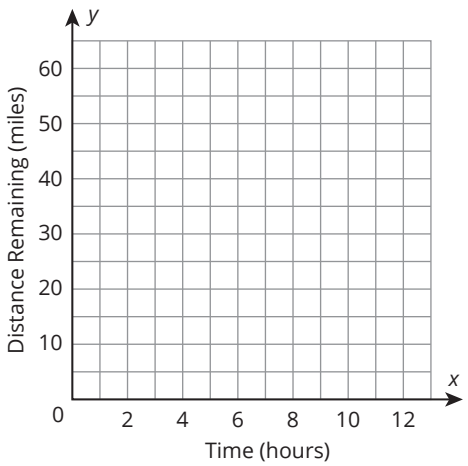
   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

d. How many golf balls will Chris have remaining after he plays 12 games?

2. Matthew averages a speed of 20 miles per hour during his bike race. The race is 60 miles long.

a. Write an equation that represents the number of miles, *y*, Matthew has left in the race given *x* number of hours he has been racing.

b. Graph the equation.



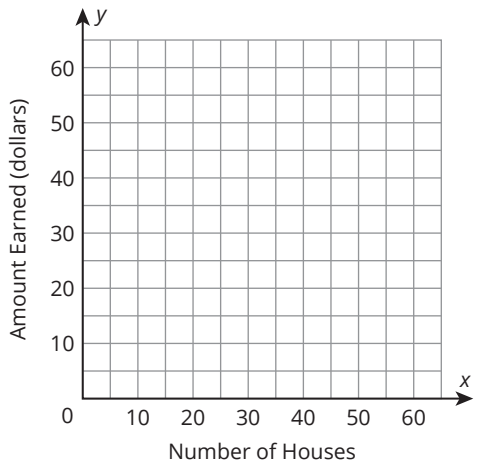
c. Identify the slope and the *y*-intercept of the graph. Explain what each means in terms of the situation.

d. How many miles does Matthew have remaining after 2 hours of racing?

3. Jorge has a job delivering newspapers. Each Sunday, she earns $24.75 plus $0.20 for each house she delivers to.

   a. Write an equation that represents the amount Jorge earns on Sunday, y, given x number of houses she delivers to.

   b. Graph the equation.



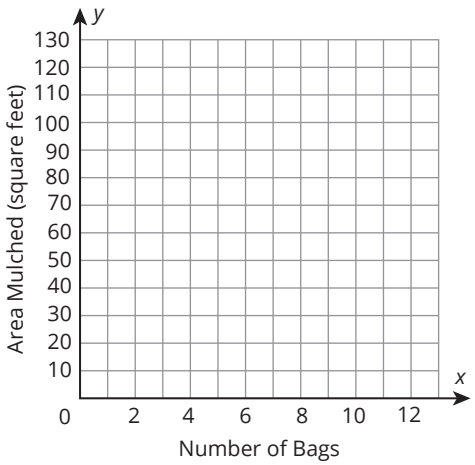
   c. Identify the slope and y-intercept of the graph. Explain what each means in terms of the situation.

   d. Jorge has earned $36.75 delivering newspapers. How many houses has she delivered to?

4. Paola is mulching her flower garden. She was able to mulch 48 square feet of garden before lunch. She plans to finish the job after lunch. Each bag of mulch covers 24 square feet of garden.

   a. Write an equation that represents the number of square feet of garden, *y*, given *x* number of bags of mulch.

   b. Graph the equation.



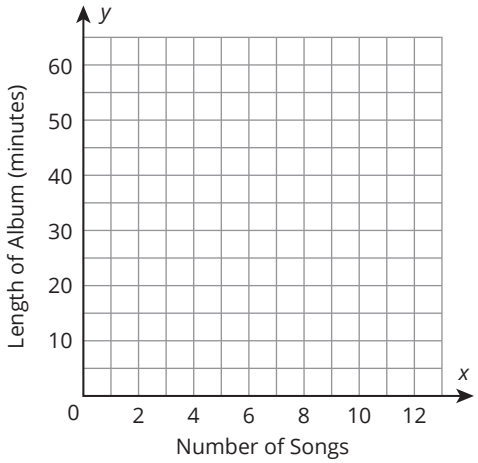
   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

   d. If Paola has 4 bags of mulch still after lunch. How many square feet of her garden will she be able to mulch?

5. Trung and her band are recording a CD. After reading an article about great songs, they determine that each song they record should be about 3.1 minutes. The band then decides to add a special behind the lyrics feature that is 3.7 minutes long.

   a. Write an equation that represents the total length of the CD in minutes, *y*, given *x* number of songs recorded.

   b. Graph the equation.



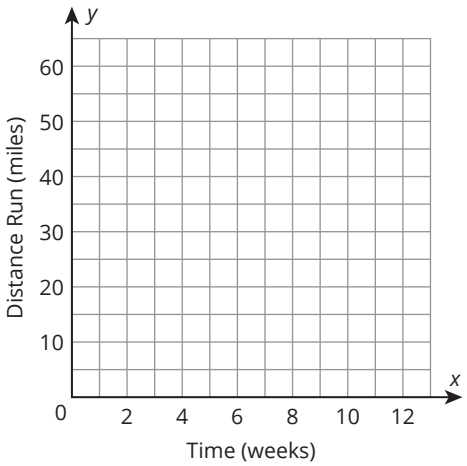
   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

   d. Trung and her band decide to put 12 songs on the CD, plus the behind the lyrics feature. How many minutes long will the CD be?

6. You are planning a training program to run a marathon. You start your program by running 5 miles per week and want to add 3.9 more miles to your program each week.

   a. Write an equation that represents the distance you run in miles, *y*, given *x* number of weeks.

   b. Graph the equation.



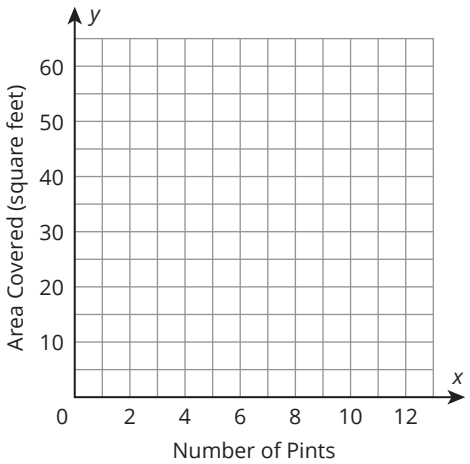
   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

   d. After training for 5 weeks, how many miles will you be able to run?

7. Mario is painting a canvas with one color that he will use as a background. He has already painted 5 square feet. The directions on his paint container state that each pint of paint will cover 16 square feet of canvas.

   a. Write an equation that represents the number of square feet of canvas covered, y, given x number of pints of paint used.

   b. Graph the equation.



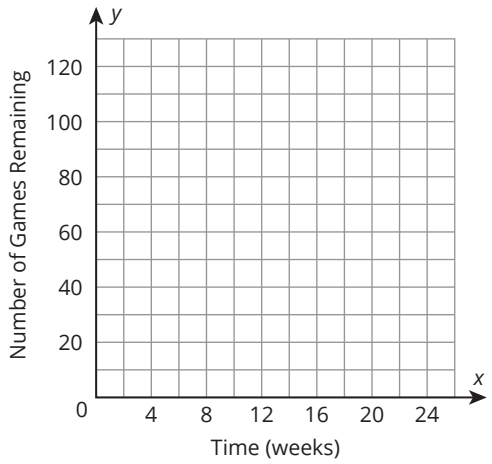
   c. Identify the slope and y-intercept of the graph. Explain what each means in terms of the situation.

   d. Mario was able to cover 37 square feet of his canvas. How many pints of paint did Mario use?

8. Mariana just bought a new baseball bat. She expects the bat to last for 75 games. She plays 3 games a week.

   a. Write an equation that represents the number of games remaining, *y*, given *x* number of weeks.

   b. Graph the equation.



   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

   d. Mariana is down to only have 30 games left on her bat. How many weeks has Mariana been playing?

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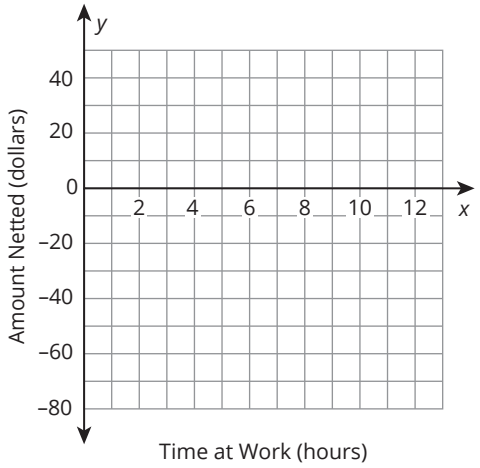
9. Jaylen got a job paying $9.50 an hour at the coffee shop, but the first week there he spent $70.00 on smoothies, muffins, and frozen macchiatos.

   a. Write an equation that represents the amount netted, *y*, given *x* number of hours worked.

   b. Graph the equation.



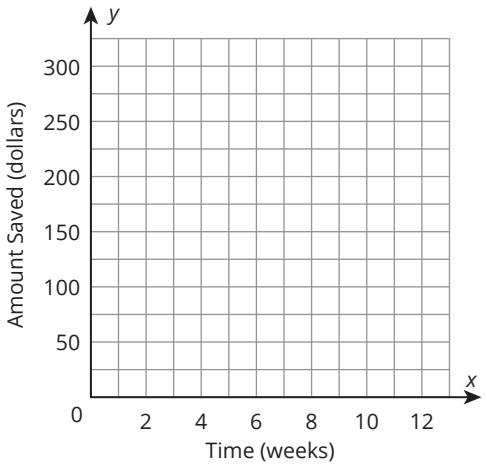
   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

   d. Jaylen worked 35 hours this past week. What is the amount netted on his paycheck?

10. Andrew is saving his money to purchase a smartphone. He already has $57 saved, and he saves $10 a week from his part-time job walking neighborhood dogs.

    a. Write an equation that represents the total amount saved, *y*, given *x* number of weeks.

    b. Graph the equation.



    c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

    d. Andrew has $97 saved towards his new smartphone. How many weeks has Andrew been saving?

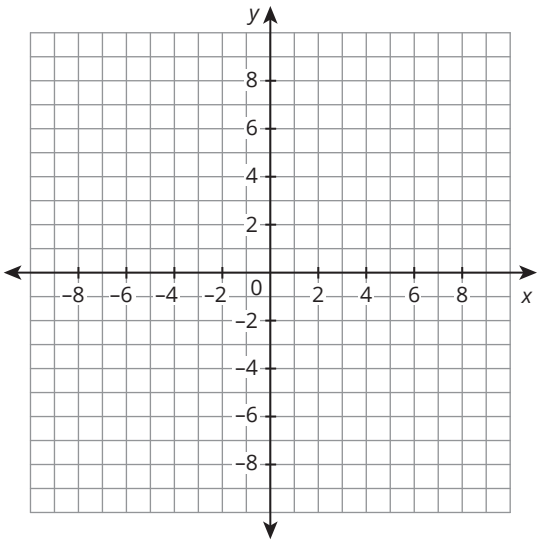
B. Use the slope and *y*-intercept to graph each equation.

1. Graph the equation $y = -3x - 6$.

   a. Determine the *y*-intercept by substituting 0 for *x*.

   b. Use the slope to determine another point on the line. Write the coordinates of the point.

   c. Draw a line to graph the equation.

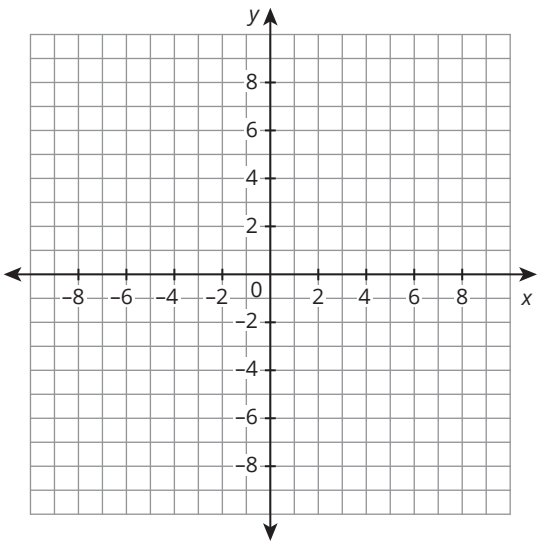


2. Graph the equation $y = -2x - 9$.

   a. Determine the *y*-intercept by substituting 0 for *x*.

   b. Use the slope to determine another point on the line. Write the coordinates of the point.

   c. Draw a line to graph the equation.

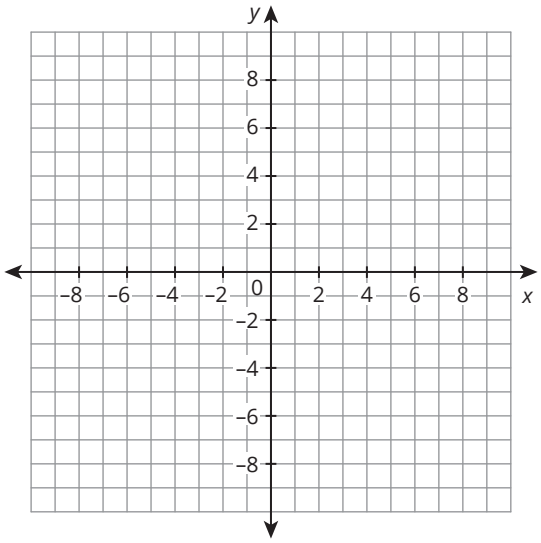


3. Graph the equation $y = x + 4$.

   a. Determine the *y*-intercept by substituting 0 for *x*.

   b. Use the slope to determine another point on the line. Write the coordinates of the point.

   c. Draw a line to graph the equation.

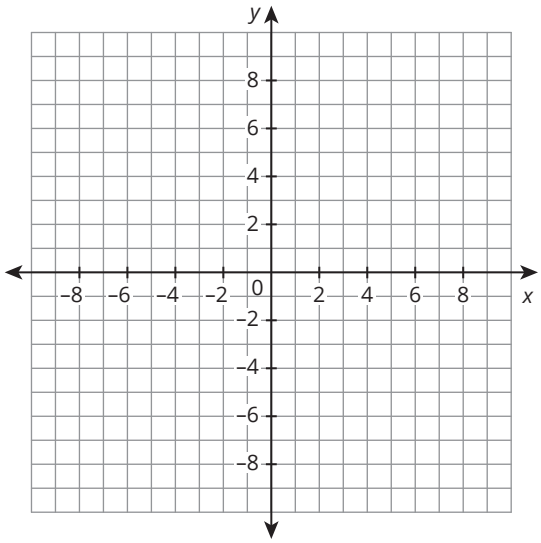


4. Graph the equation $y = -8x + 9$.

   a. Determine the *y*-intercept by substituting 0 for *x*.

   b. Use the slope to determine another point on the line. Write the coordinates of the point.

   c. Draw a line to graph the equation.

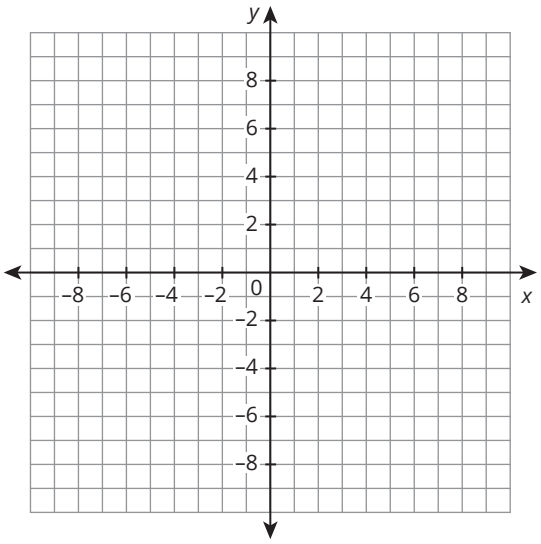


5. Graph the equation $y = 2x + 5$.

   a. Determine the *y*-intercept by substituting 0 for *x*.

   b. Use the slope to determine another point on the line. Write the coordinates of the point.

   c. Draw a line to graph the equation.

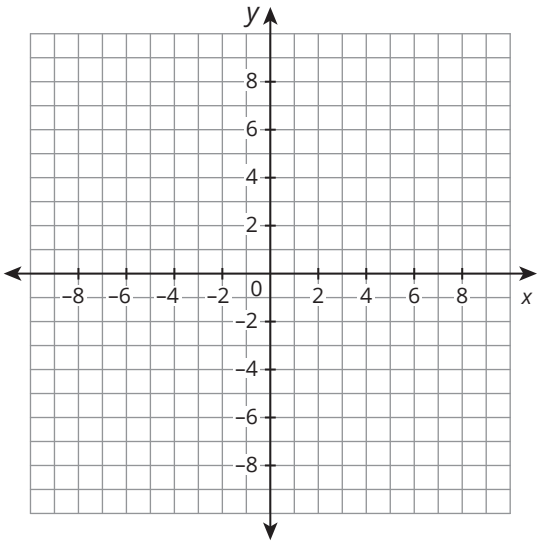


6. Graph the equation $y = -3x + 1$.

   a. Determine the *y*-intercept by substituting 0 for *x*.

   b. Use the slope to determine another point on the line. Write the coordinates of the point.

   c. Draw a line to graph the equation.

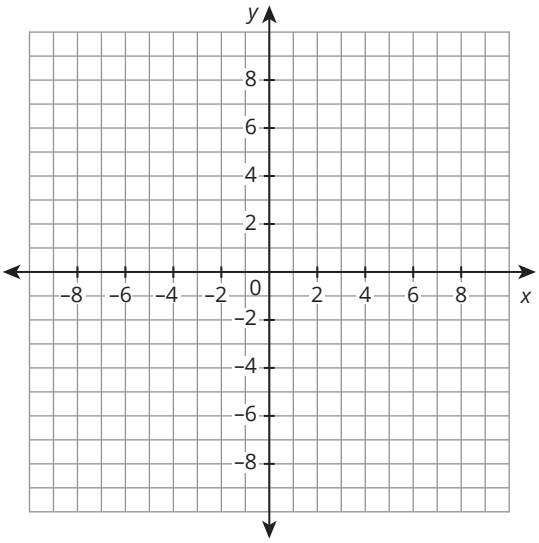


7. Graph the equation $y = 4x - 1$.

   a. Determine the y-intercept by substituting 0 for x.

   b. Use the slope to determine another point on the line. Write the coordinates of the point.

   c. Draw a line to graph the equation.

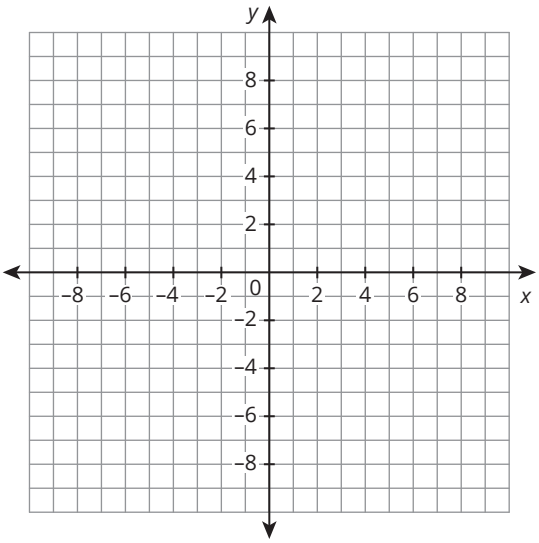


8. Graph the equation $y = 3x - 7$.

   a. Determine the y-intercept by substituting 0 for x.

   b. Use the slope to determine another point on the line. Write the coordinates of the point.

   c. Draw a line to graph the equation.

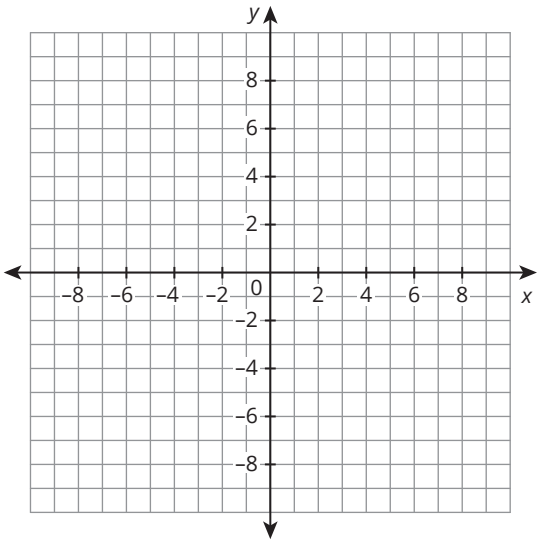


9. Graph the equation $y = -x - 2$.

a. Determine the y-intercept by substituting 0 for x.

b. Use the slope to determine another point on the line. Write the coordinates of the point.

c. Draw a line to graph the equation.

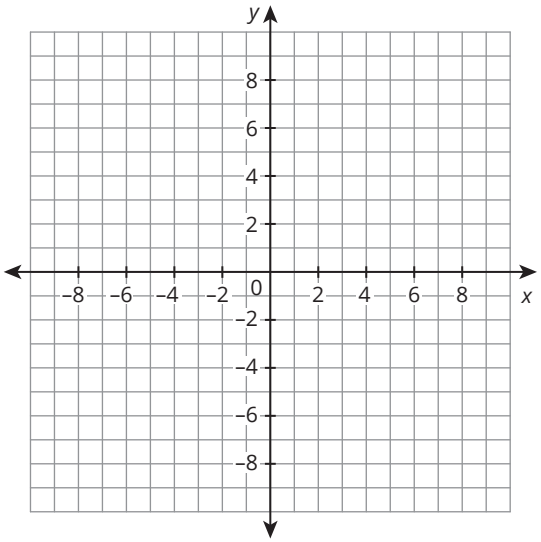


10. Graph the equation $y = 6x + 4$.

a. Determine the y-intercept by substituting 0 for x.

b. Use the slope to determine another point on the line. Write the coordinates of the point.

c. Draw a line to graph the equation.

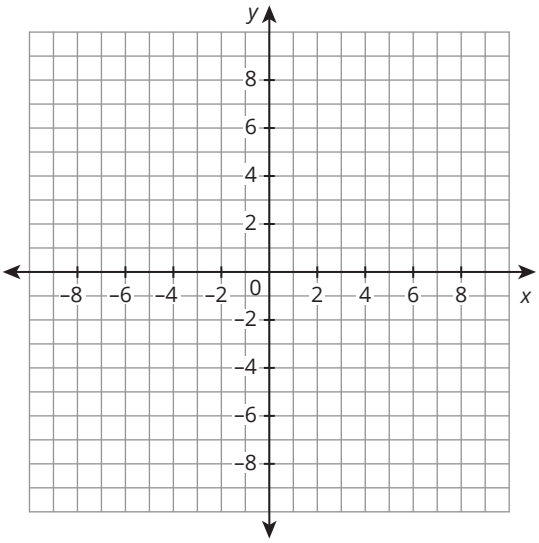


11. Graph the equation $y = -5x + 4$.

a. Determine the *y*-intercept by substituting 0 for *x*.

b. Use the slope to determine another point on the line. Write the coordinates of the point.

c. Draw a line to graph the equation.

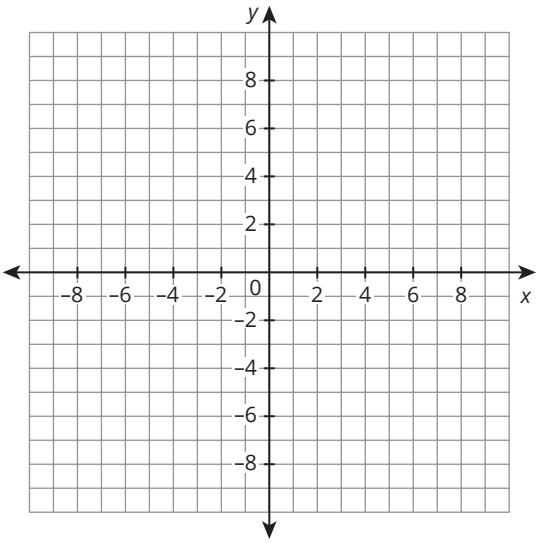


12. Graph the equation $y = 7x - 4$.

a. Determine the *y*-intercept by substituting 0 for *x*.

b. Use the slope to determine another point on the line. Write the coordinates of the point.

c. Draw a line to graph the equation.



C. Use the given information to determine each equation.

1. Point D is located at $(-13, -1)$. Point Q is located at $(0, 12)$. Determine the slope-intercept equation of $\overline{DQ}$.

2. Point S is located at $(-9, 8)$. Point G is located at $(-3, 3)$. Determine the slope-intercept equation of $\overline{SG}$.

3. Point Q is located at $(-8, -10)$. Point B is located at $(-3, -4)$. Determine the slope-intercept equation of $\overline{QB}$.

3. The slope of $\overline{FL}$ is $\frac{3}{2}$. The y-intercept of $\overline{FL}$ is $(0, -6)$. Determine the slope-intercept form equation of $\overline{FL}$.

5. Point V is located at $(-2, 7)$. Point T is located at $(10, -8)$. Determine the slope-intercept equation of $\overline{VT}$.

6. Point E is located at $(-5, -4)$. Point C is located at $(1, 2)$. Determine the slope-intercept equation of $\overline{EC}$.

7. Point R is located at $(-2, 5)$. Point S is located at $(4, -2)$. Determine the slope-intercept equation of $\overline{RS}$.

8. The slope of $\overline{AB}$ is -0.2. The y-intercept of $\overline{AB}$ is $(0, 2)$. Determine the slope-intercept form equation of $\overline{AB}$.

9. Point W is located at $(-13, -13)$. Point T is located at $(-7, -6)$. Determine the slope-intercept equation of $\overline{WT}$.

10. Point Z is located at $(-10, -6)$. Point D is located at $(5, 11)$. Determine the slope-intercept equation of $\overline{ZD}$.

11. Point M is located at $(5, 1)$. Point N is located at $(13, 12)$. Determine the slope-intercept equation of $\overline{MN}$.

12. The slope of $\overline{OP}$ is 9. The y-intercept of $\overline{OP}$ is $(0, -3)$. Determine the slope-intercept form equation of $\overline{OP}$.

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Extension

Determine the equation for a vertical line and the equation for a horizontal line. What are the slope and *y*-intercept for each type of line?

Spaced Practice

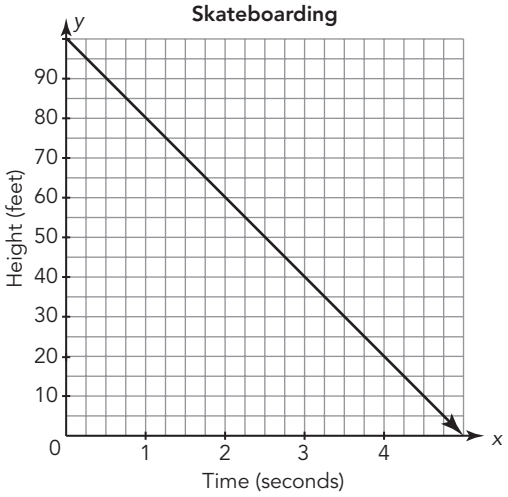
Determine the rate of change for each situation.

1. Alyssa is ordering a submarine sandwich from the corner deli. The deli charges $6.25 for a 7-inch sub. Additional toppings cost extra. Alyssa's sandwich with two extra toppings costs $7.75. What is the cost per additional topping?

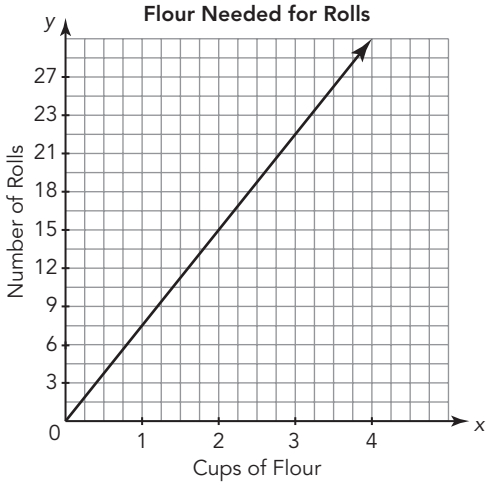
2. Lizzie is selling pies at the cherry festival to raise money for her local volunteer fire department. She sells 85 pies for $12 each. The supplies to make the pies cost Lizzie $340. What is the unit rate of the profit made for each pie?

For each graph, determine the slope and explain what the slope means in terms of the independent and dependent quantities. Then, write an equation in the form $y = mx$ or $y = mx + b$ to represent the relationship between the independent and dependent quantities.

3. Alejandro is riding her skateboard down a hill, as shown in the graph.

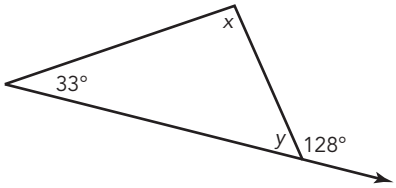


4. Andrew needs a specific amount of flour to bake rolls, as shown in the graph.



Determine the measure of each unknown angle.

5.



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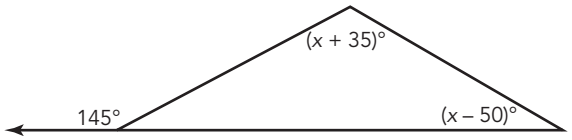
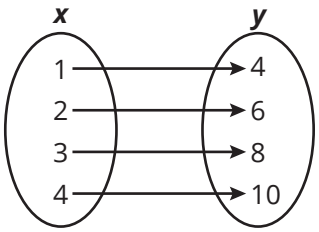


V. Defining Functional Relationships

Topic Practice

A. Write the corresponding ordered pairs and tell whether each relation is a function.

1.



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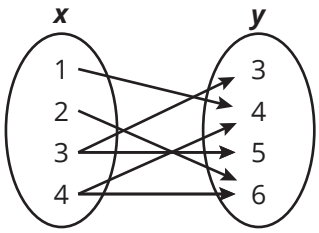

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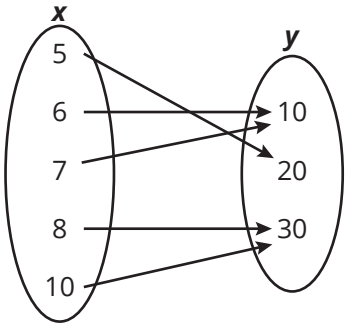

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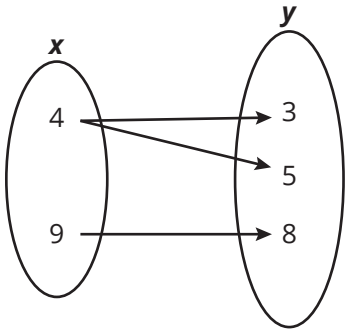
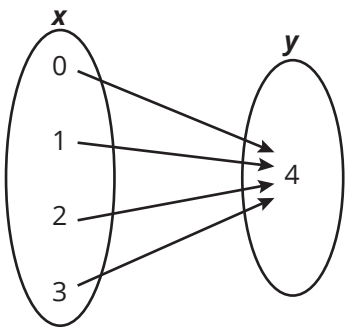


5.



6.

Input	Output
4	5
8	12
12	16
16	20
20	24

7.

Input	15	10	5	10	15
Output	0	5	10	15	20

8.

x	0	5	10	15	20
y	15	10	5	10	15

9.

x	*y*
5	0
5	1
5	2
5	3
5	4

10.

x	*y*
−3	9
−1	1
0	0
1	1
3	9

B. Determine whether each given situation represents a function. Explain your answer.

1. Input: Elena mails 6 different valentines to her friends.
   Output: Each of Elena's 6 friends receives a valentine from her.

2. Input: There is 1 copy of a popular book in the library.
   Output: The book has been checked out by 45 different people.

3. Input: The principal of a school sends the same memo to all of the teachers.
   Output: There are 28 teachers in the school.

4. Input: There are 13 cats for adoption at the animal shelter.
   Output: Each cat is adopted by a different family.

5. Input: There are 3 showings of a play.
   Output: Over 200 people attend each showing.

6. Input: Chris bakes 2 dozen cookies for the bake sale.
   Output: Twenty-four people buy a cookie.

7. Input: The new issue of Sports Today is published.
   Output: Issues are sent to millions of readers.

8. Input: Eduardo has French on Mondays, Wednesdays, and Fridays, and German on Tuesdays and Thursdays.
   Output: Eduardo's language classes per day.

9. Input: There is a flight from Los Angeles to New York.
   Output: There are 350 passengers on the flight.

10. Input: There are five different trails in a park.
    Output: Three friends take a different trail.

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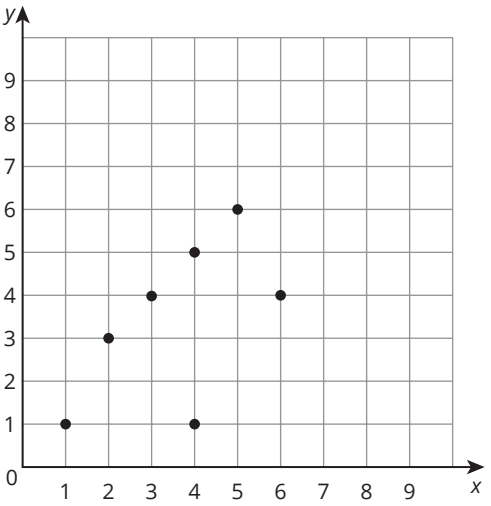
C. Determine whether each graph represents a function.

1.



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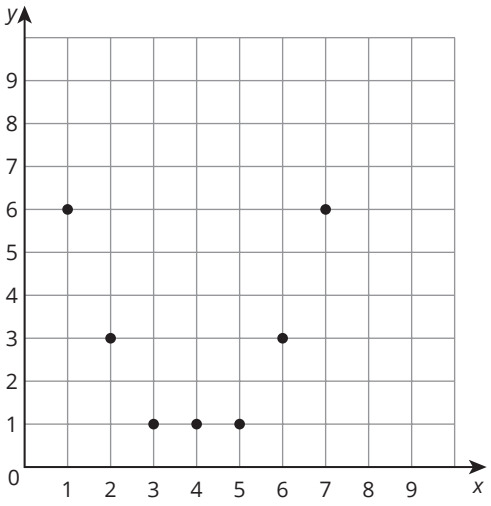

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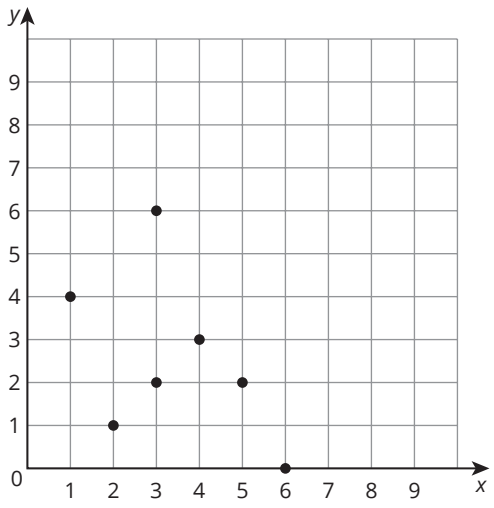

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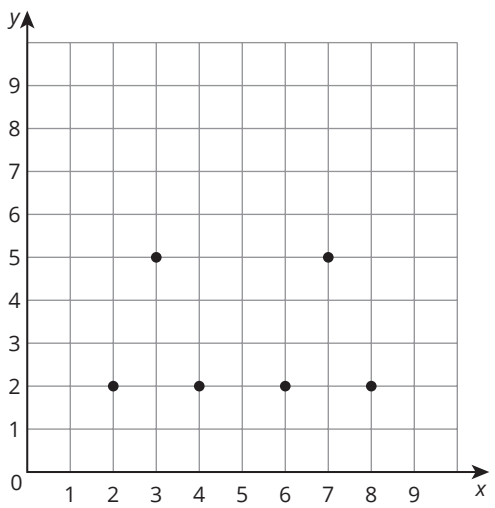

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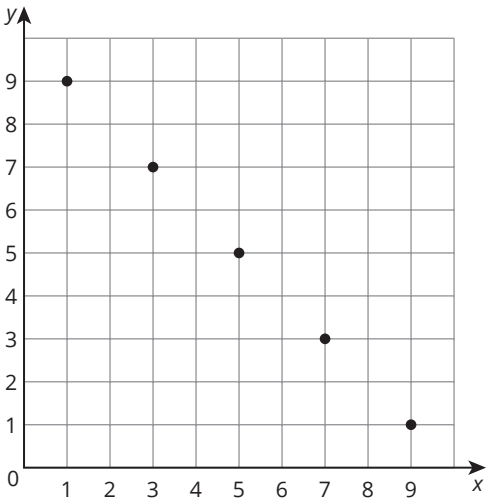

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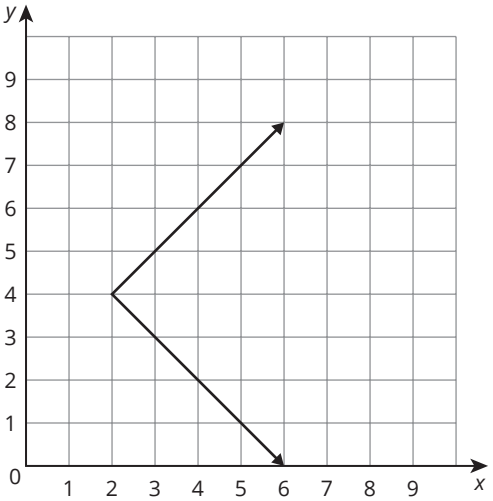

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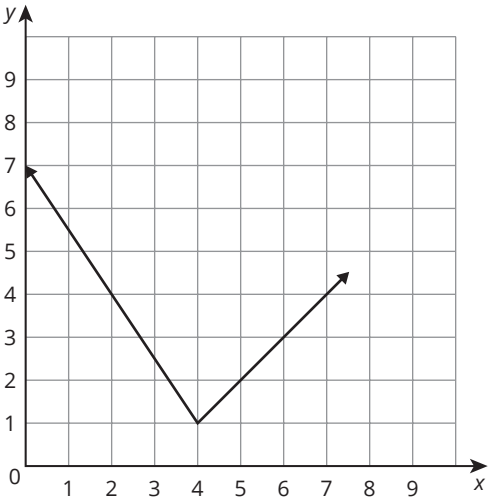

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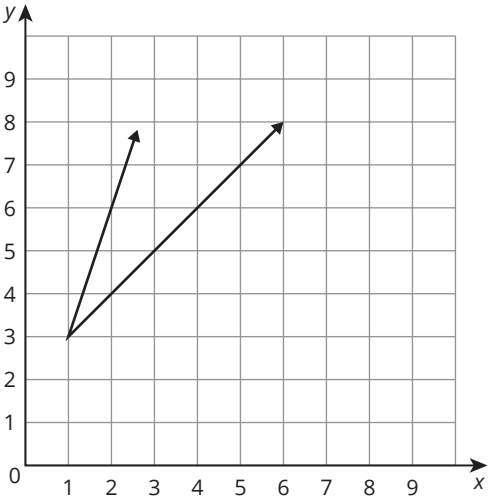

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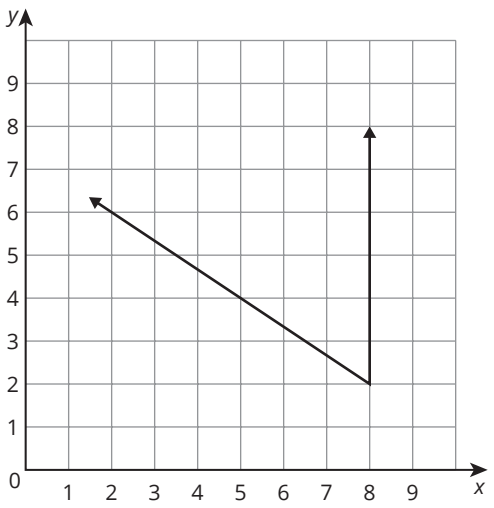

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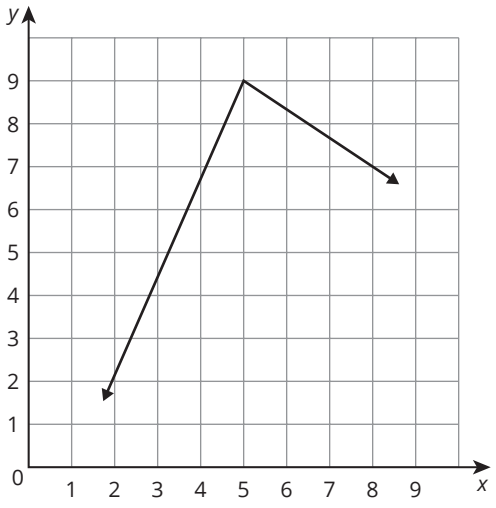


D. Determine whether each equation is a function. If it is not a function, explain why not.

1. $y = 3x + 1$

2. $y = x^2$

3. $y^2 = x$

4. $y = \sqrt{x + 5}$

5. $y = -|x|$

6. $\sqrt{y} = x - 8$

7. $y = \frac{1}{2}x$

8. $|y| = 6 + 4x$

9. $y = x^3$

10. $x = 1$

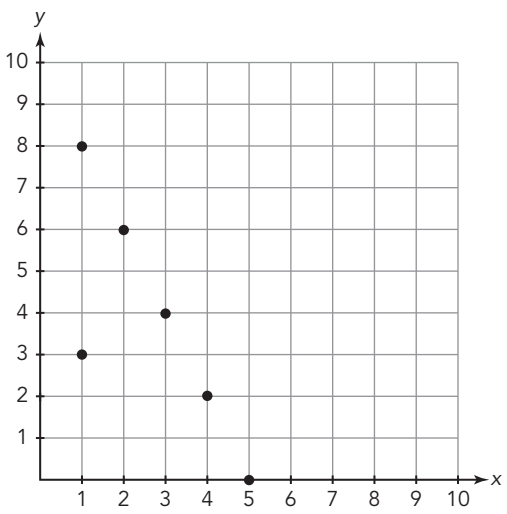
Extension

Describe how you can tell from an equation whether a function is increasing, decreasing, or constant.

Spaced Practice

Tell whether each graph is a function.

1.



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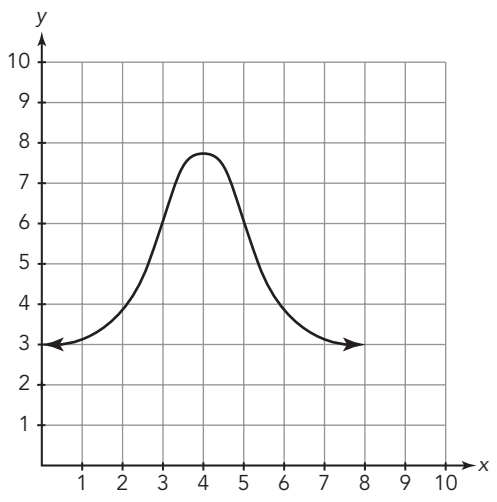

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Determine the slope and *y*-intercept of the linear relationship described by each equation.

3. $y = \frac{x}{2} + 5$

4. $y = \frac{x}{4}$

Calculate the slope of the line represented by each table.

5.

x	*y*
2	−1
3	1.5
4	4
5	6.5

6.

x	*y*
2	8
4	2
6	−4
9	−13

GRADE 8

MODULE 3

Data Data Everywhere

TOPIC 1: Patterns in Bivariate Data

I. Analyzing Patterns in Scatterplots 181
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Name _____ Date _____

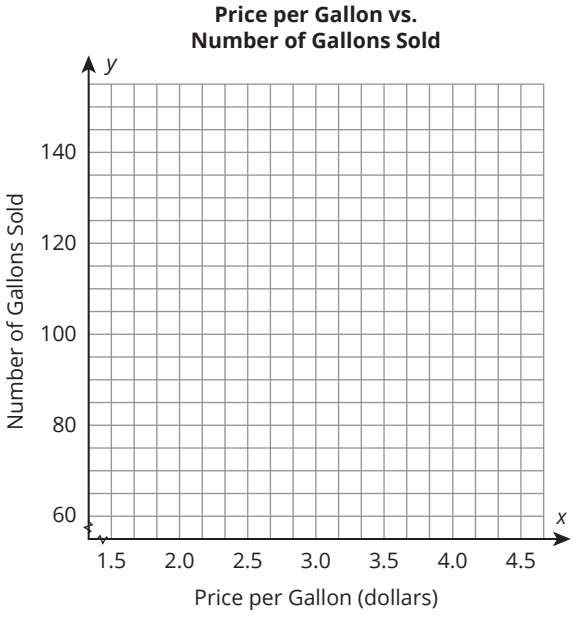
I. Analyzing Patterns in Scatterplots

Topic Practice

A. Construct a scatterplot for each given table of values. Describe any patterns you observe. Circle the third point from the table on the graph and explain what it means.

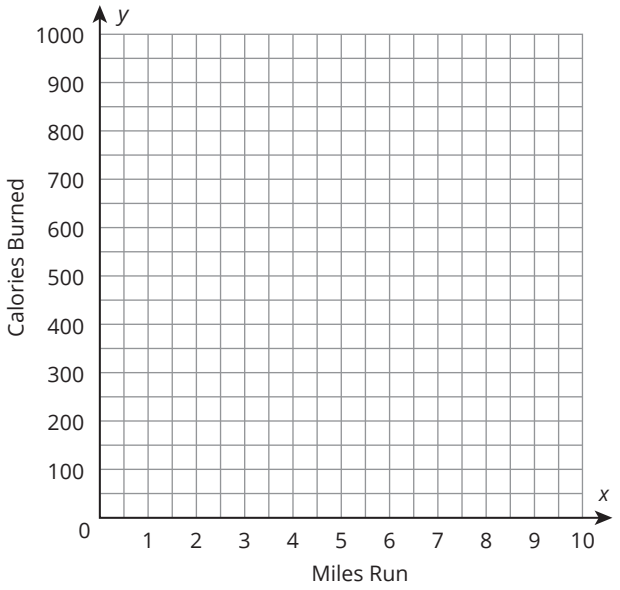
1. A grocery store tracks the number of gallons of milk it sold on 9 different days. The table lists the daily price per gallon of milk and the number of gallons sold each day.

Price per Gallon (dollars)	2.75	3.30	2.25	2.50	3.50	3.25	3.00	2.00	3.55
Number of Gallons Sold	120	90	140	130	80	100	110	150	70



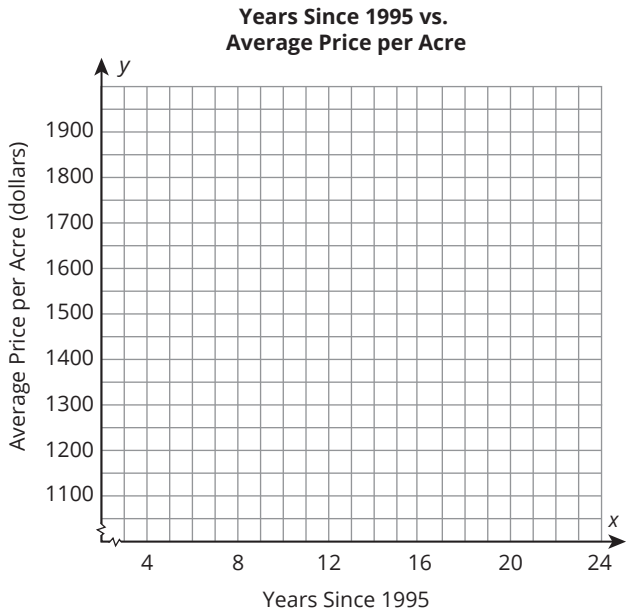
2. A personal trainer tracks the calories burned by 9 clients. The table lists the number of miles run and the number of calories burned for each person.

Number of Miles Run	Number of Calories Burned
3	275
4.5	475
5	600
2.5	300
7	850
8	800
6	650
5.5	525
4	400



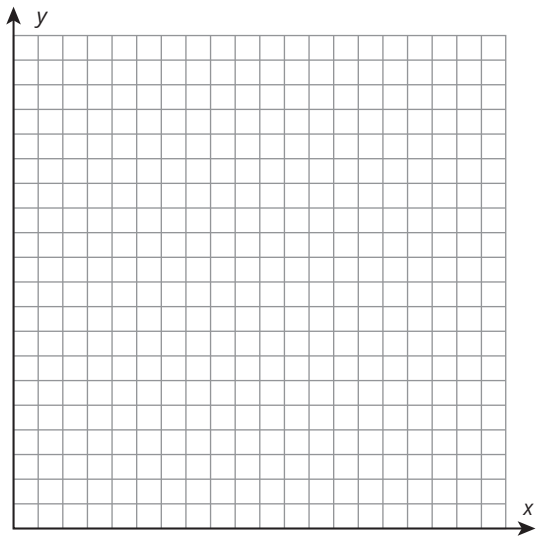
3. Linh a local real estate agent, has collected data about the average cost of an acre of land in his county for various years.

Years Since 1995	5	20	10	24	15	13	17	0	7	22
Price Per Acre (dollars)	1300	1650	1200	1800	1500	1350	1700	1100	1250	1750



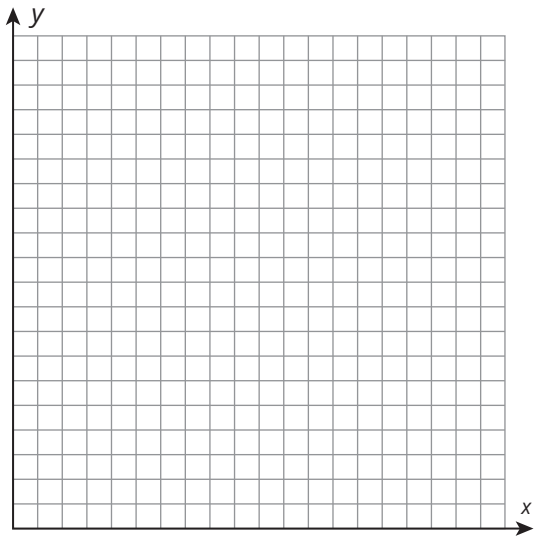
4. Lauren and Jackson the managers a basketball team, have collected information on the average points scored by each player and the average minutes played by each player.

Average Minutes Played	10	20	15	12	5	8	12	14	9	16	6	3
Average Points Scored	7	14	10	8	3	5	9	9	7	12	4	1



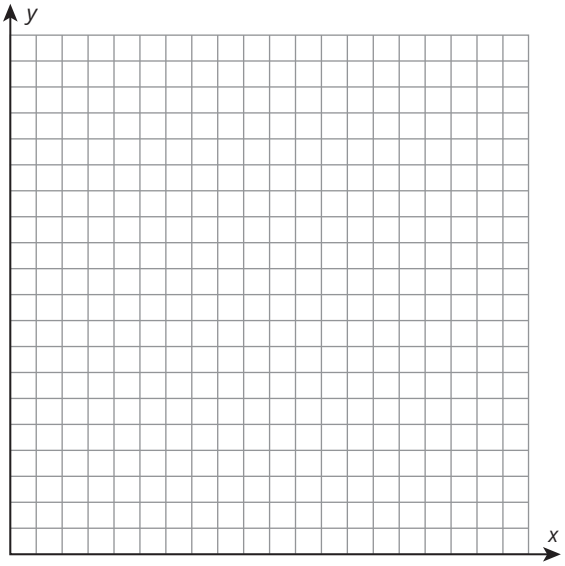
5. Nakota a meteorologist, is studying the average temperatures in the month of June for different latitudes in the northern hemisphere.

Latitude (degrees)	45	20	60	5	15	25	30	10	50
Average June Temp. (°F)	75	86	50	102	90	84	81	96	68



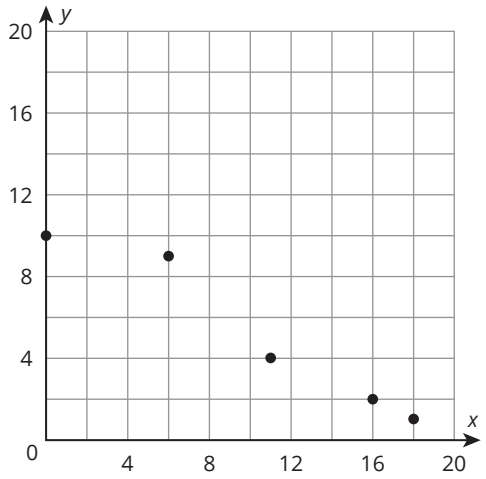
6. A mathematics teacher tracked the number of student absences and the students' math grades for the year.

Number of Absences	3	2	5	1	0	8	3	2	6
Math Grades (%)	85	90	65	90	100	60	80	95	75



B. Determine whether each graph shows a positive linear association, a negative linear association, a non-linear association, or no association.

1.



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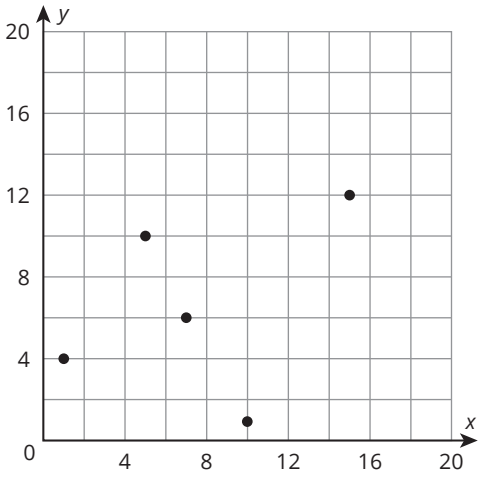

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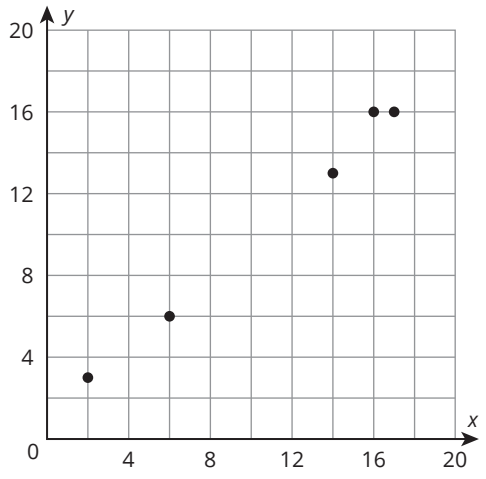

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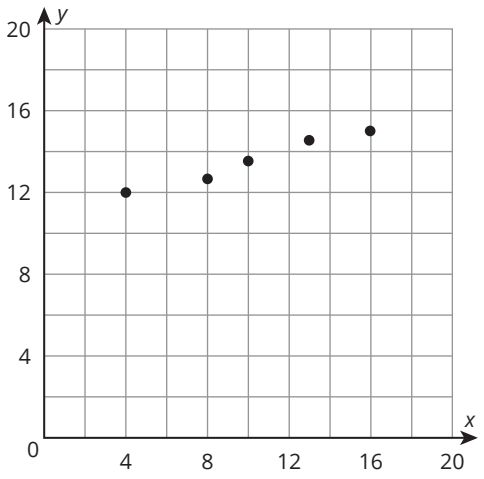

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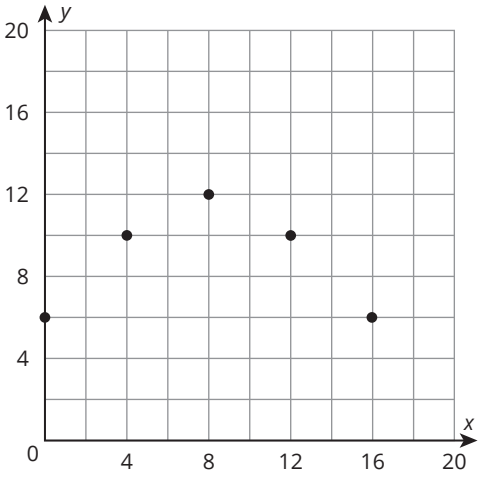
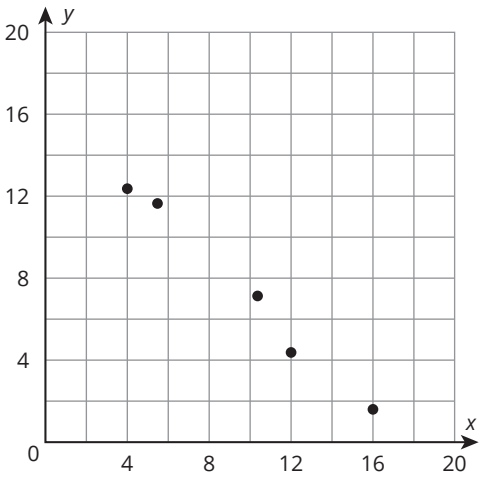

y
20
16
12
8
4
0
4
8
12
16
20
x

6.


y
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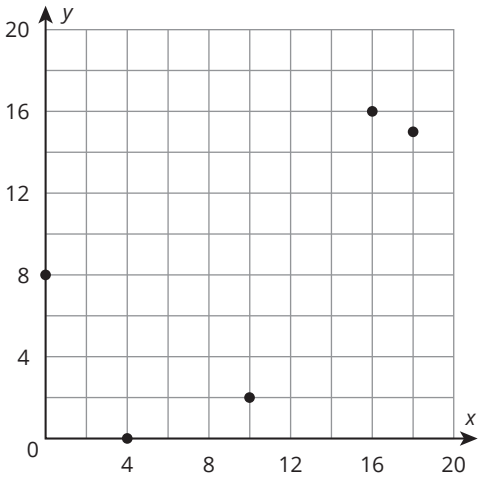
16
12
8
4
0
4
8
12
16
20
x

7.


y
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0
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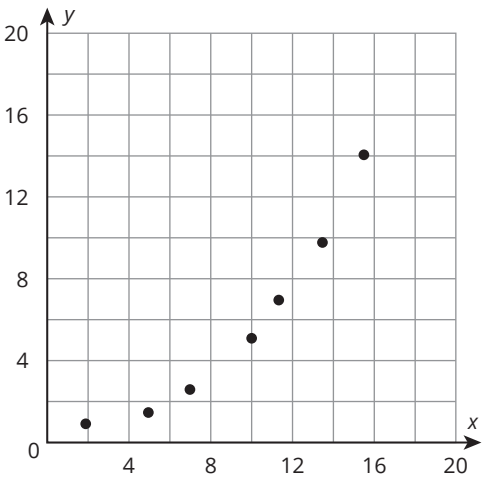
16
20
x

8.


y
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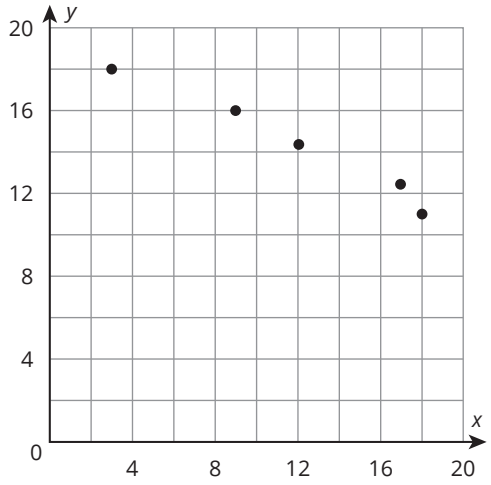
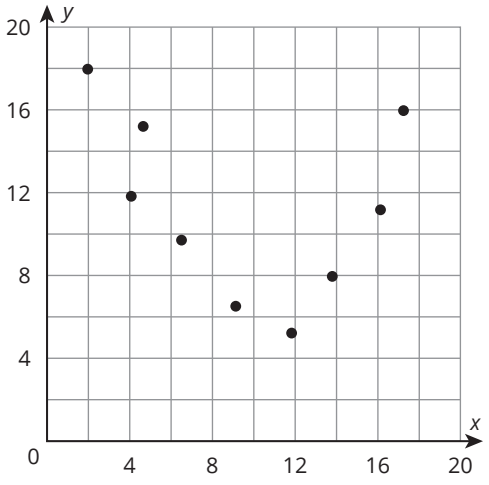
16
12
8
4
0
4
8
12
16
20
x

9.


y
20
16
12
8
4
0
x
4
8
12
16
20

10.


y
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16
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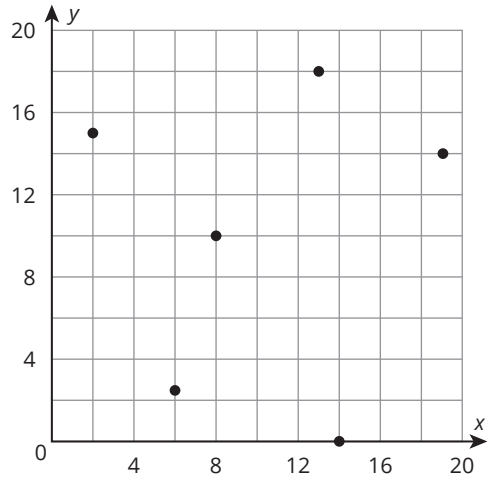
0
x
4
8
12
16
20

11.


y
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16
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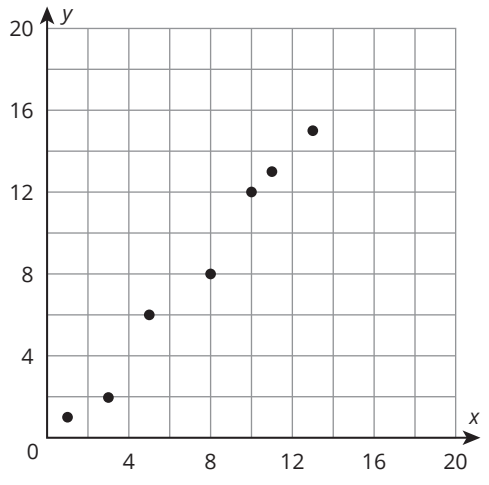
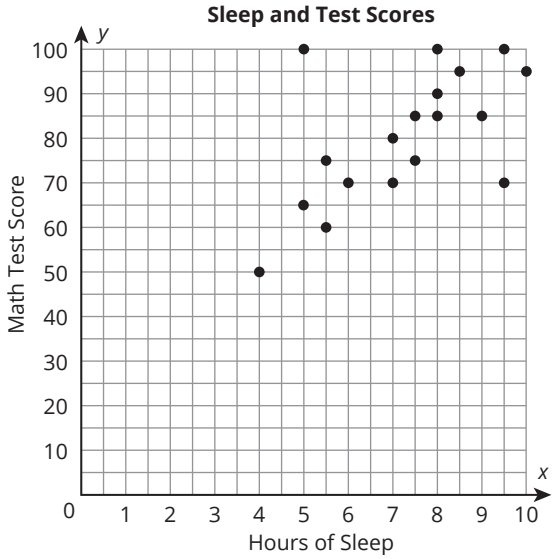
8
4
0
x
4
8
12
16
20

12.

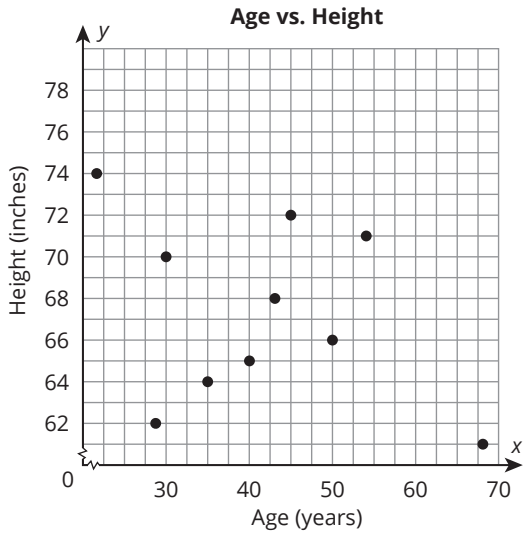

y
20
16
12
8
4
0
x
4
8
12
16
20

C. For each graph, identify the explanatory variable and the response variable. Determine whether the scatterplot shows an association or not, and when it does, tell whether it is positive or negative. State the association in terms of the variables. Identify any outliers.

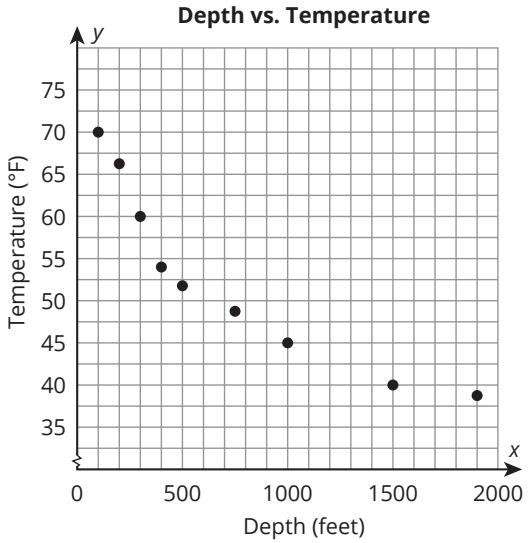
1. A teacher surveyed students about the amount of sleep they got the night before the math test.



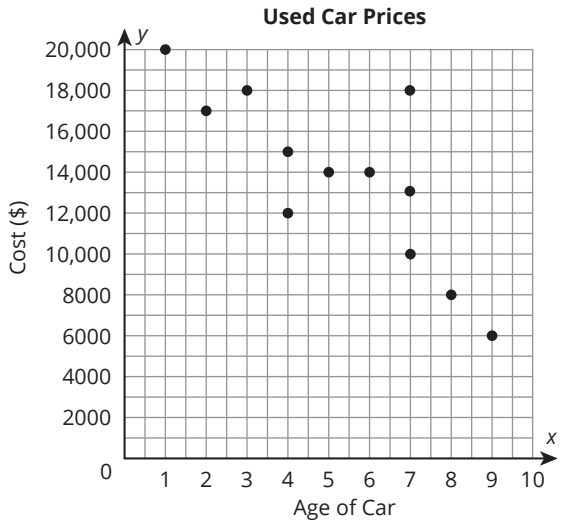
2. Alejandra is collecting data for a research paper. The graph shows the age and height data Suzanne collected for 10 adults.



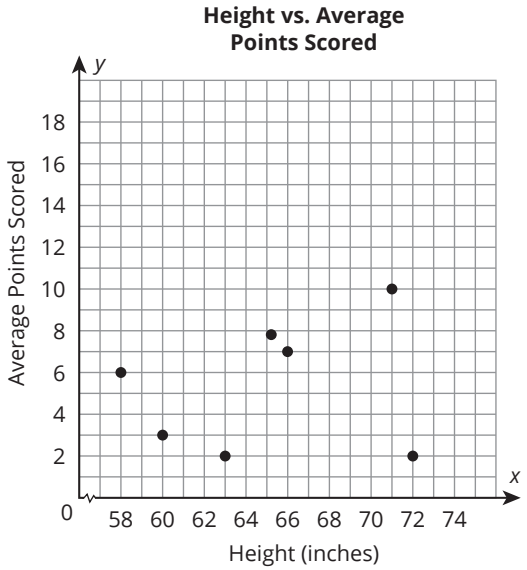
3. Scientists studying the Bermuda Triangle measured the ocean's temperature at various depths.



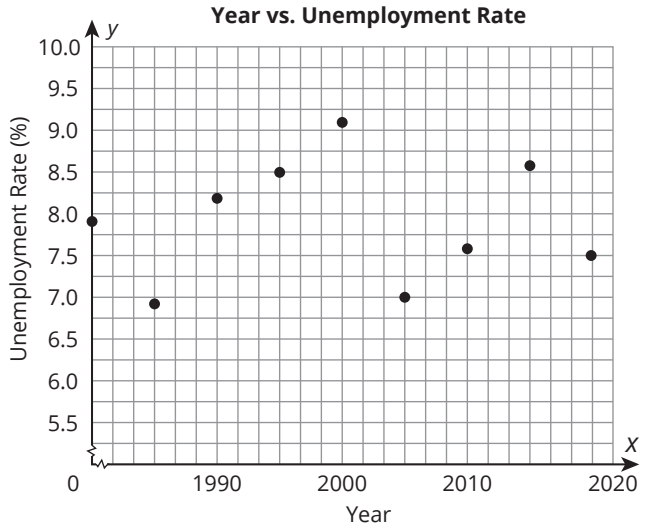
4. The graph shows the relationship between the age of different cars sold at a dealership and the cost for each car.



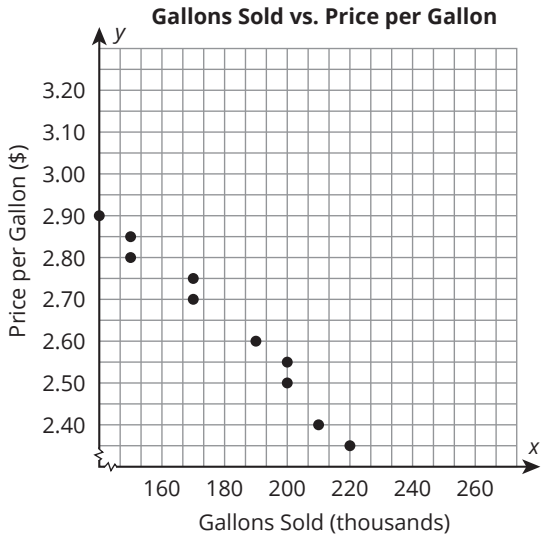
5. Hailey and Joey, the managers of a basketball team, have collected data on the players' heights and the average number of points scored by each player.



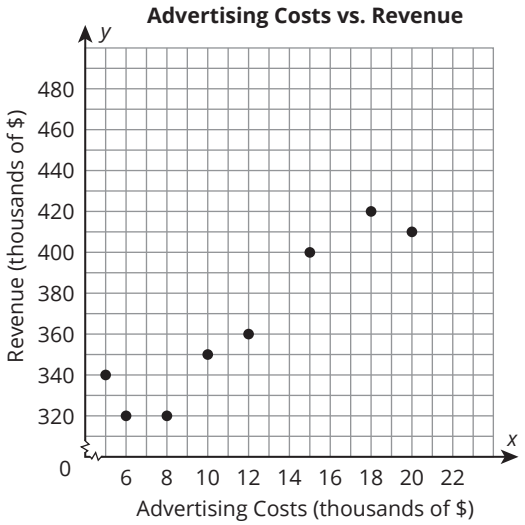
6. Brianna has collected data about the unemployment rate in her county over the last several decades.



7. A gas station has collected data about the number of gallons of regular unleaded gas it sells each week and the average price charged for each gallon during that week.



8. Gracie a department store manager, has collected data on the store's advertising costs for a certain year and the corresponding revenue for that year.



Extension

Pose a statistical question that can be answered by collecting bivariate data. Identify the explanatory and response variables. Collect and record the data. Construct a scatterplot. Describe the relationship between the variables and note any possible outliers.

Spaced Practice

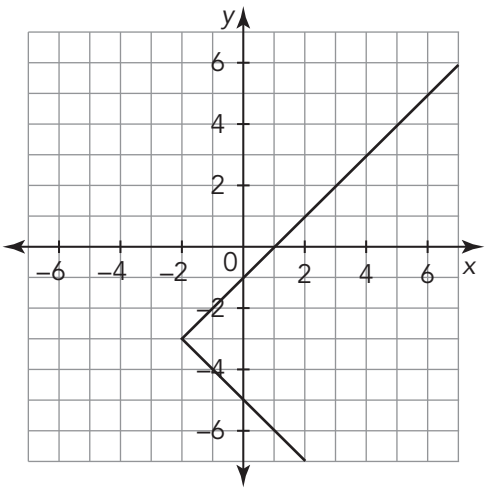
1. Determine whether congruence is preserved or not for each transformation applied to an original shape.

   a. A hexagon is graphed on a coordinate plane. The hexagon is translated 7 units right.

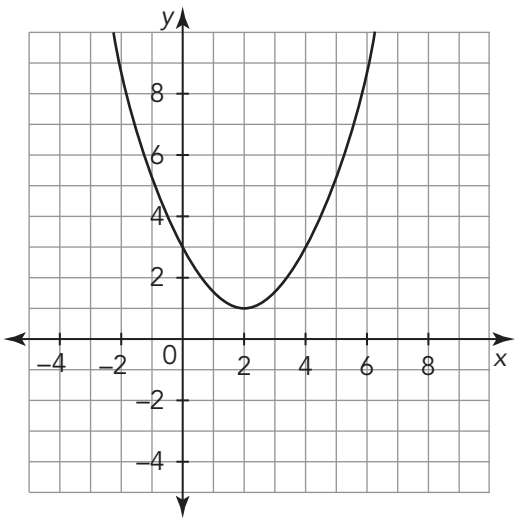
b. A square drawn on a coordinate grid is dilated by a scale factor of 3.

2. Tell whether each graph shows a function or non-function.

a.



b.



3. Calculate the slope of the line represented by each table.

a.

x	y
3	6
6	8
15	14
21	18

b.

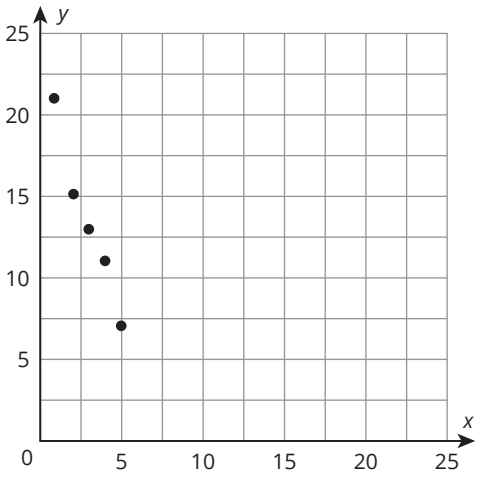
x	y
0	10
4	5
8	0
16	−10

II. Drawing Trend Lines

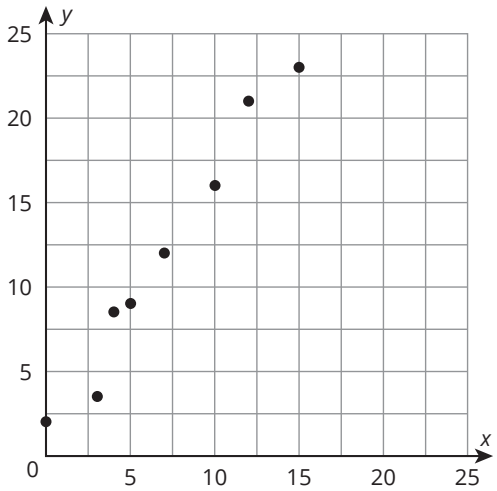
Topic Practice

A. Use a straightedge to draw a line that best fits each scatterplot.

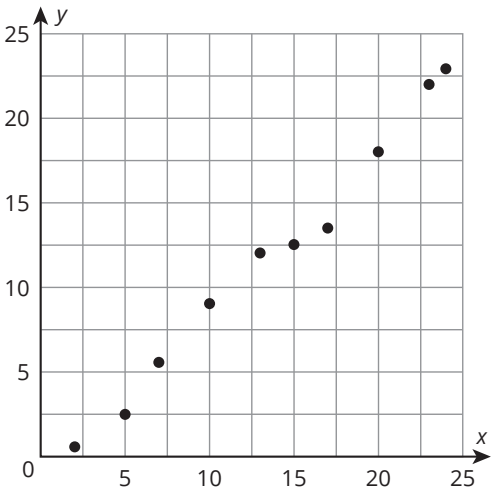
1.



2.



3.



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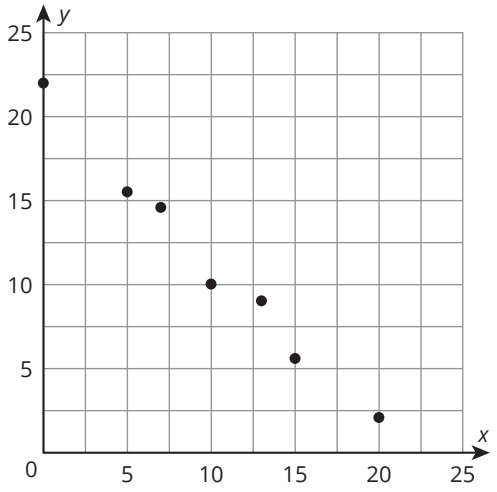
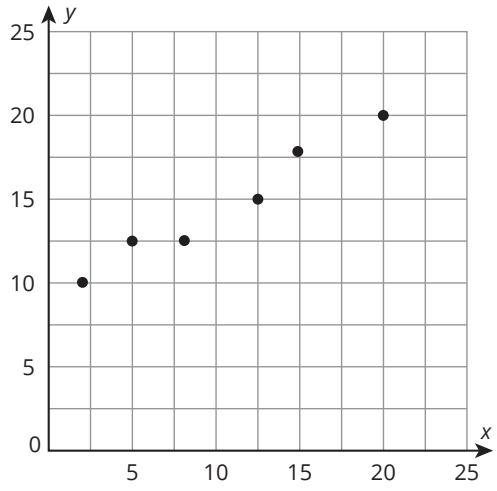
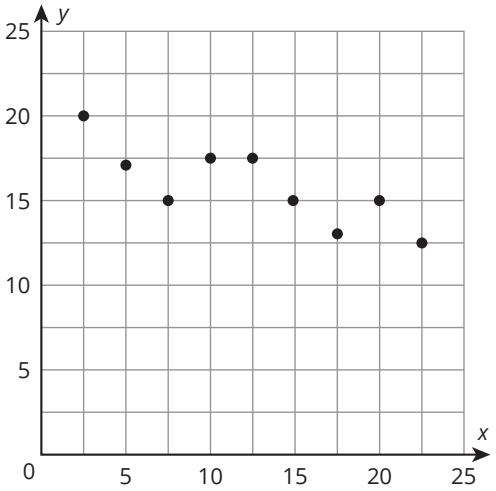


5.

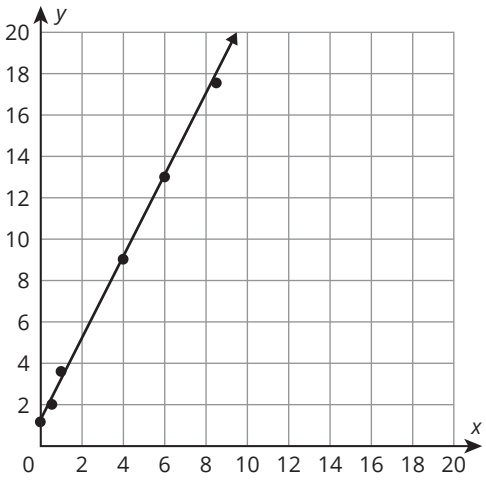


6.

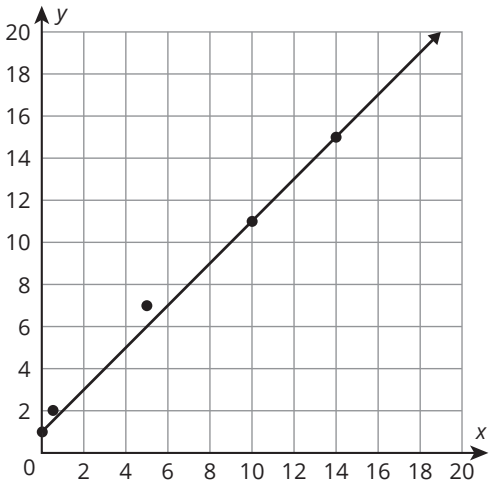


B. Estimate the equation of the trend line shown in each scatterplot.

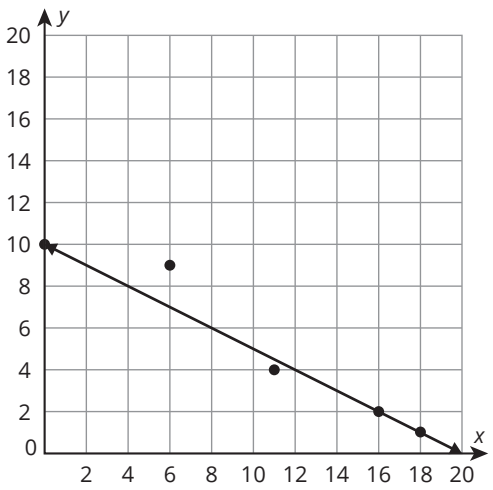
1.



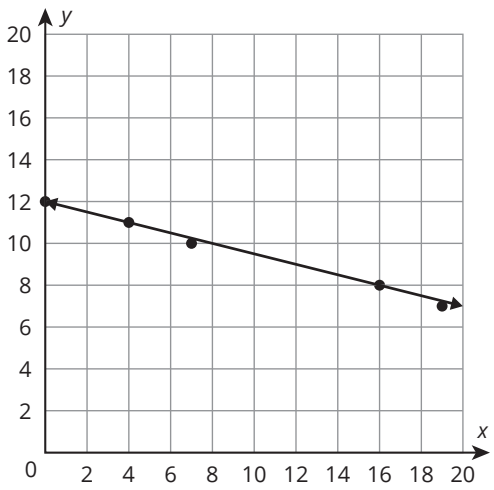
2.



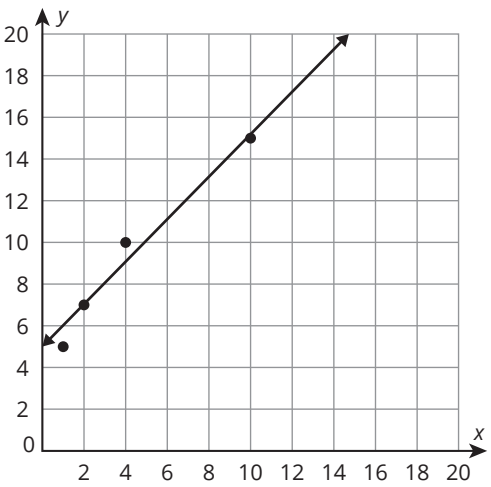
3.


y
20
18
16
14
12
10
8
6
4
2
0
2 4 6 8 10 12 14 16 18 20
x

4.


y
20
18
16
14
12
10
8
6
4
2
0
2 4 6 8 10 12 14 16 18 20
x

5.


y
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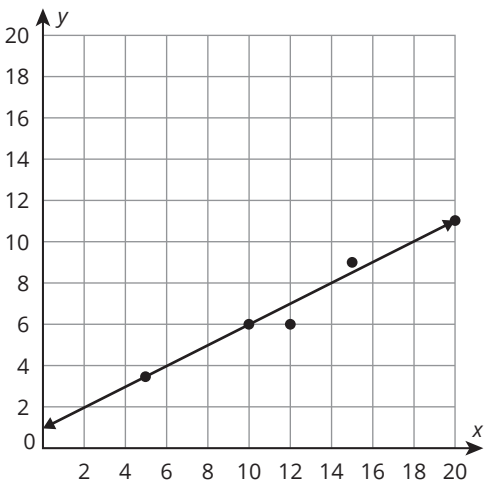
x
2 4 6 8 10 12 14 16 18 20

6.


y
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18
16
14
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6
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2
0
x
2 4 6 8 10 12 14 16 18 20

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C. Use the given equation to answer each question.

1. A grocery store's earnings in dollars can be modeled by the equation $y = 0.6x$, where x represents the number of tomatoes that they sell. If they sell 200 tomatoes in one day, how much money do they earn?

2. The total cost in dollars, y, that a t-shirt company charges can be modeled by the equation $y = 5x + 62$, where x represents the number of t-shirts sold. If they sell 74 shirts, what will the total cost be?

3. The distance a car can travel in miles is modeled by the equation $y = 23x - 6$, where x represents the number of gallons of gas the car uses. If the car used 4 gallons of gas, how many miles did the car travel?

4. The amount a shoe store charges in dollars to deliver x pairs of shoes can be modeled by the equation $y = 40x + 30$. If 8 pairs of shoes were delivered, how much was the total bill?

5. The number of gallons of water, y, in a swimming pool is modeled by the equation $y = 7.5x + 500$, where x represents the time in minutes after the pump is turned on. How many gallons of water are in the pool if the pump runs for 200 minutes?

6. The height of a tree in feet, y, is modeled by the equation $y = 2.5x + 3$, where x represents the age of the tree in years. How tall will the tree be when it is 12 years old?

Extension

An *influential point* is an outlier that greatly affects the slope of the trend line. One way to test if an outlier is an influential point is to determine a trend line with and without the point and then compare the slopes of the lines. For the given data, determine a trend line. Then, identify a potential influential point and determine the trend line without that point. Compare the slopes of the lines and state if you think the point was influential.

x	0	1	2	4	5	6	8	10	11	25
y	95	80	93	70	85	70	84	60	80	70

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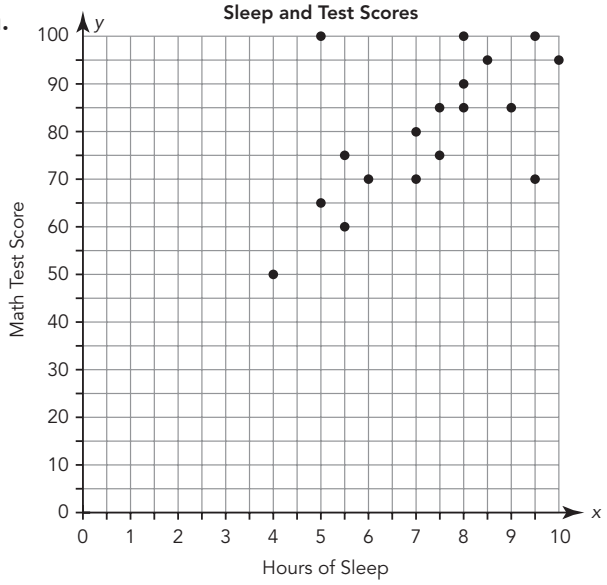
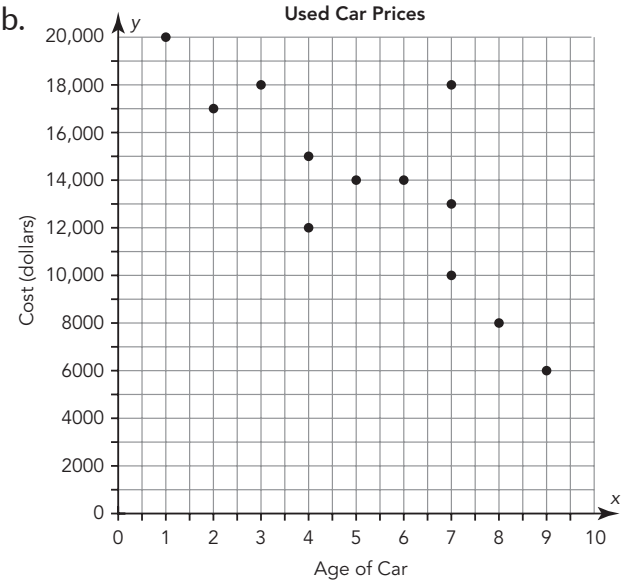
Spaced Practice

1. Identify the explanatory and response variables in each. Determine whether the scatterplot shows an association or not, and if so, tell if it is positive or negative. State the association in terms of the variables. Identify any outliers.

a.



b.

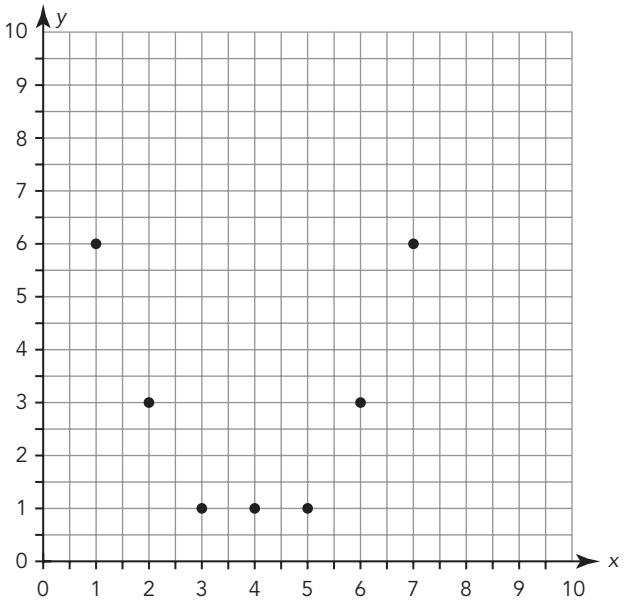


2. Tell whether each relation is a function. Justify your answer.

a.

Input	Output
15	0
10	5
5	10
10	15
15	20

b.



3. Determine the slope and y-intercept of the line represented by each equation.

a. $2x - 5y = 30$

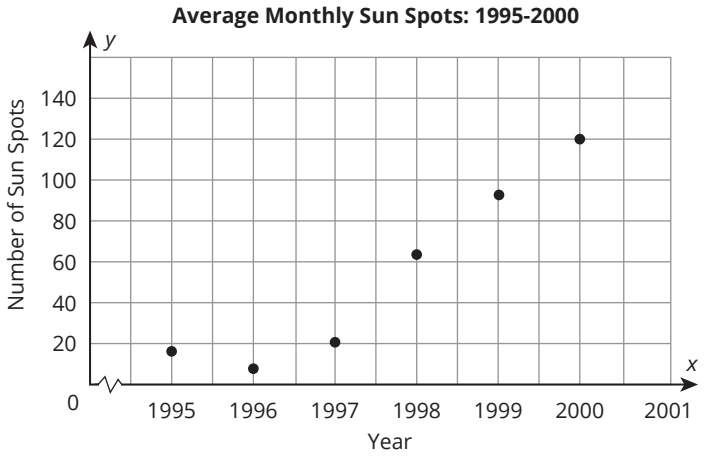
b. $4y = 40 - 8x$

III. Analyzing Trend Lines

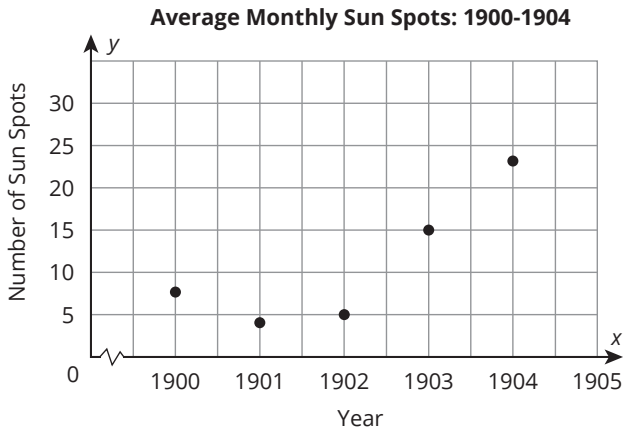
Topic Practice

A. For each graph, draw a trend line and make a prediction.

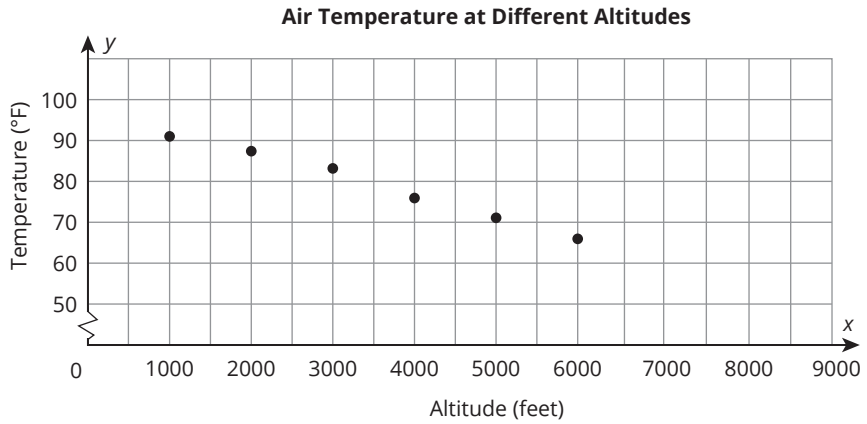
1. Predict the average monthly number of sun spots that occurred in 2001.



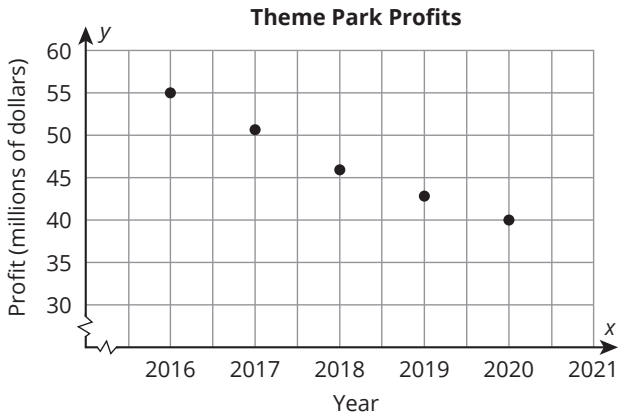
2. Predict the average monthly sun spots that occurred in 1905.



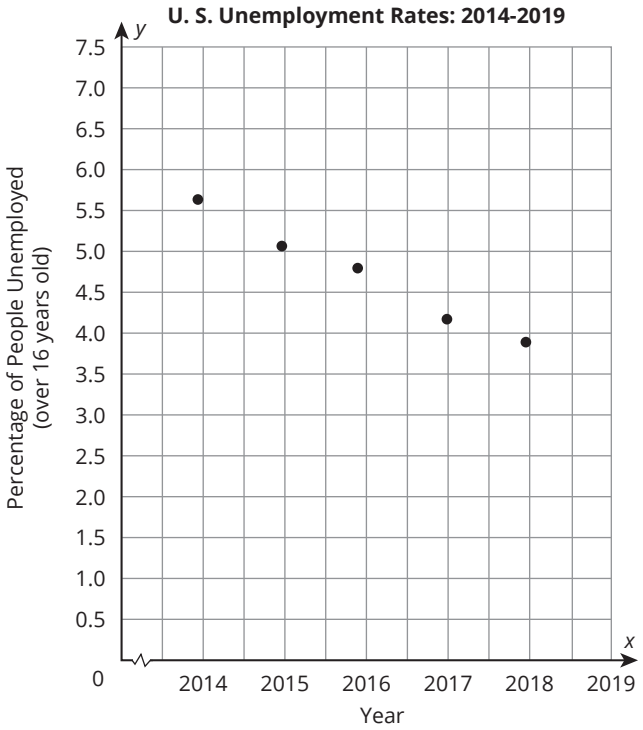
3. Predict the air temperature at an altitude of 9000 feet.



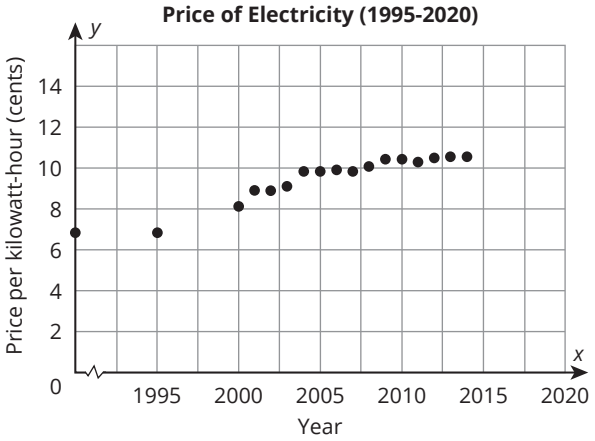
4. Predict the theme park's profit in the year 2021.



5. Predict the U.S. unemployment rates from the year 2019.

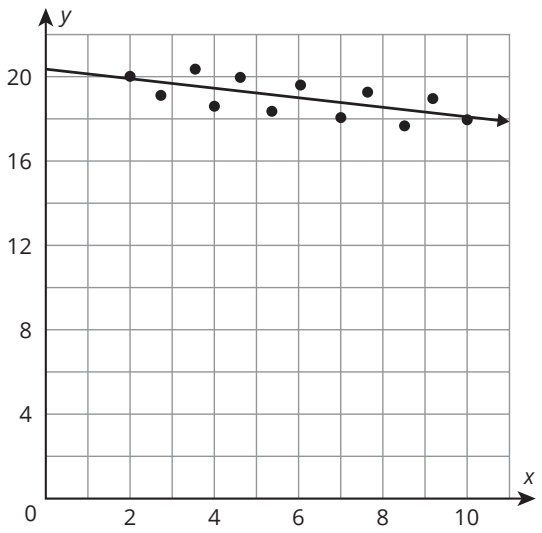


6. Predict the price of electricity from the year 2022.

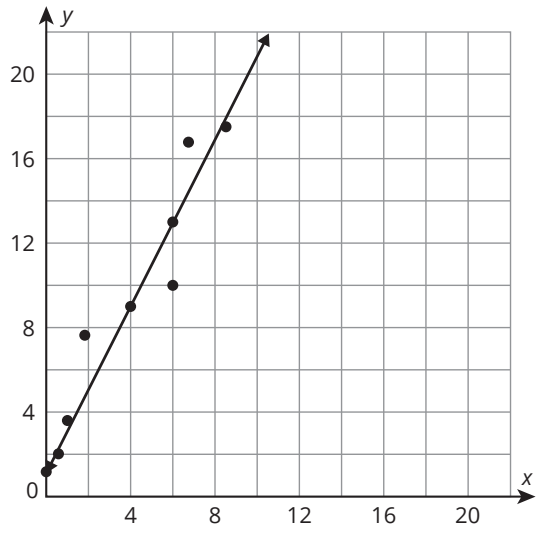


B. For each scatterplot, the trend line is given. Use two points to estimate the slope of the line that best fits the data. Use the graph to estimate the y-intercept. Then, write the equation of the line.

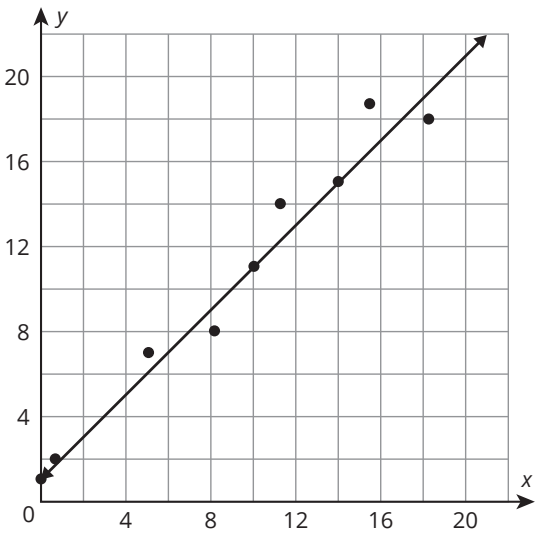
1.



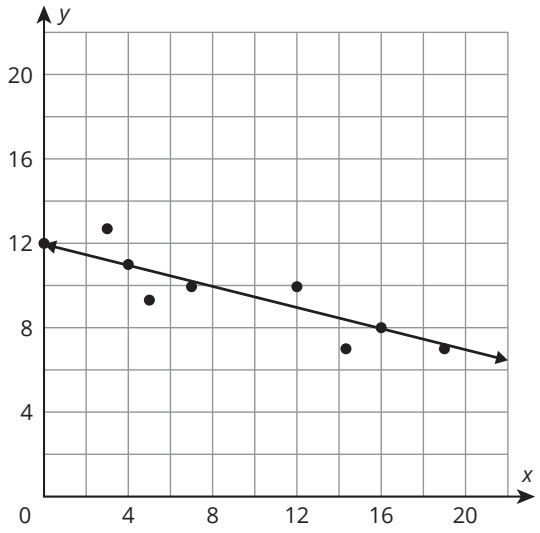
2.



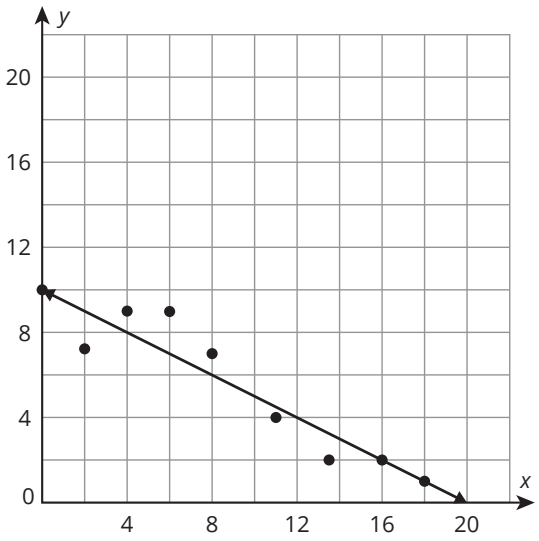
3.



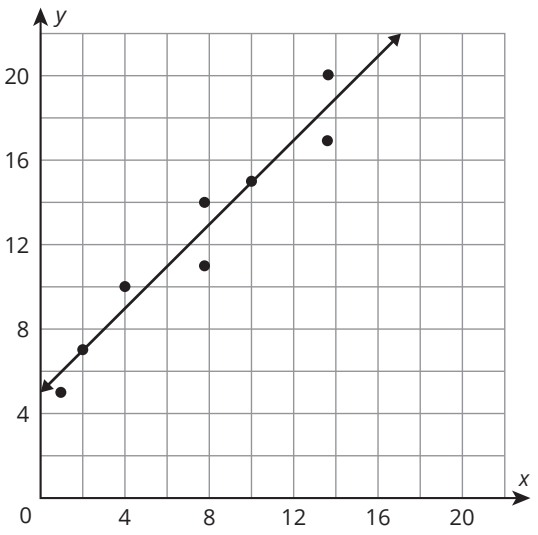
4.



5.



6.



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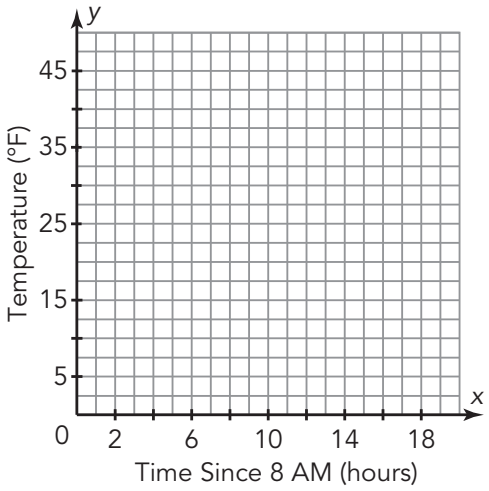
Extension

Because the relationships in bivariate data can change for different ranges of values of the explanatory variable (e.g., age, time), sometimes the best model for a data set includes more than one line.

Consider the data set for the temperature in Washington, DC since 8 am on a day in winter.

- Create a scatterplot.
- Split the data into sections that show increasing, decreasing, or constant associations, and draw lines for each section.
- Determine an equation for the trend line for each section of the scatterplot. Specify for which domain each equation is the trend line.

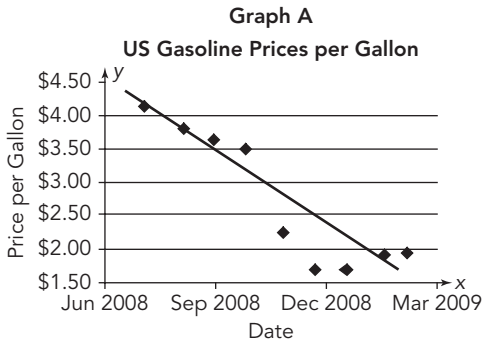
Time Since 8 AM (hours)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Temperature (°F)	20	23	31	35	38	45	45	45	45	45	40	29	27	16	11

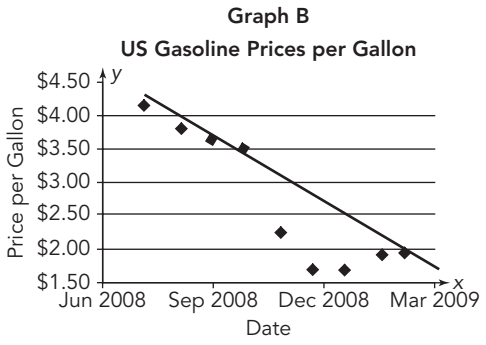


Spaced Practice

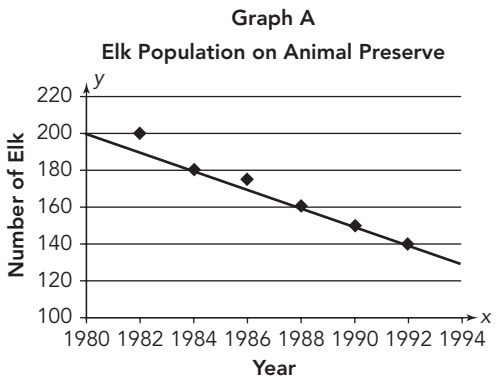
1. Compare each pair of graphs to determine which line is a better fit for the data.

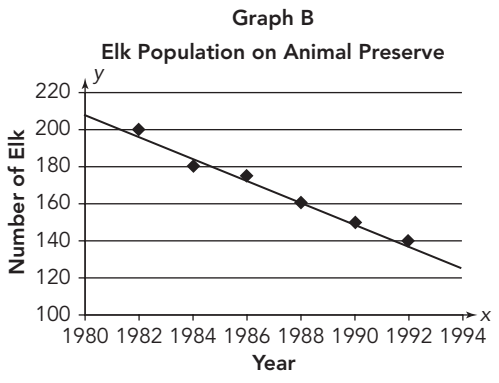
a.





b.

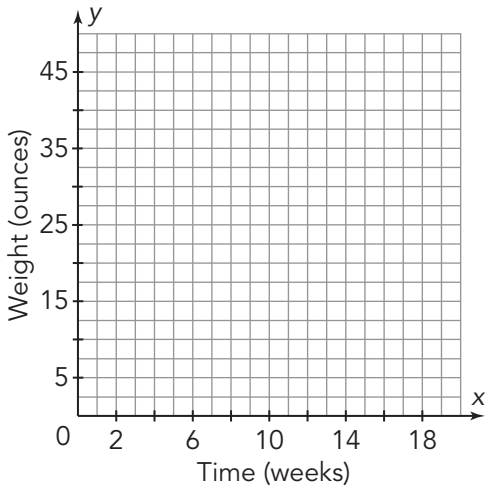




2. Nicky puts a different type of fertilizer on each of his two pumpkin patches. A pumpkin from Patch A measures 13 ounces at the end of the first week of growth and grows at a rate of 3.2 ounces per week. A pumpkin from Patch B measures 9 ounces at the end of the first week of growth and grows at a rate of 3.6 ounces per week.

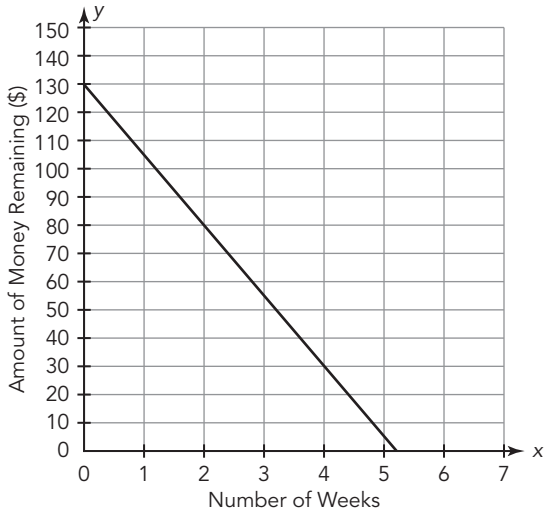
   a. Write equations for the weight of pumpkins from each patch over time.

b. Create a graph that contains both lines.

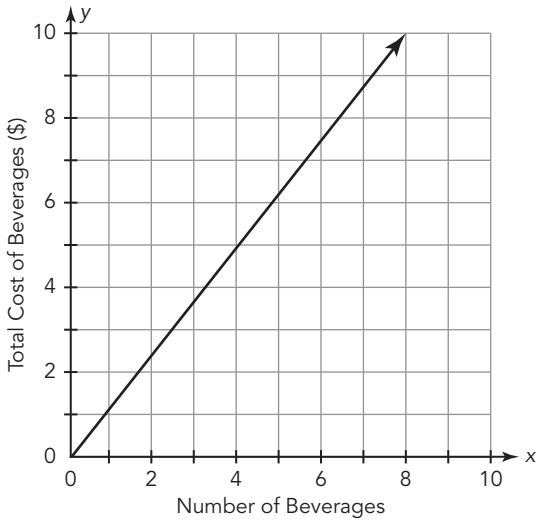


c. Explain the conditions for which Nicky should use each fertilizer.

3. The graph models the amount of money Nia has in her savings account. What is Nia's rate of saving? Write your answer as a unit rate and include units.



4. The graph models the sale of beverages. What is the cost of one beverage?



IV. Comparing Slopes and Intercepts of Data from Experiments

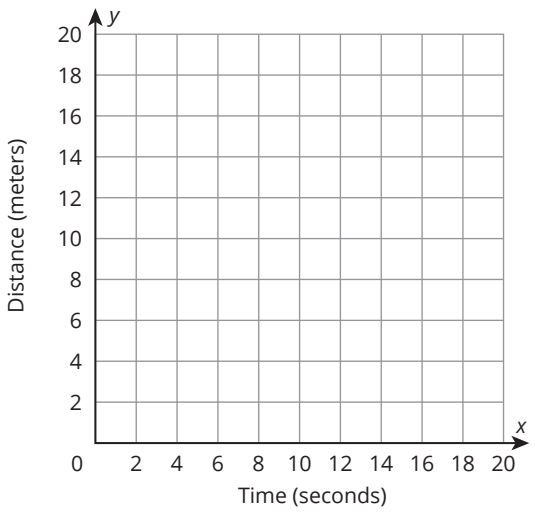
Topic Practice

A. For each problem situation:

- **Create a scatterplot of the ordered pairs on the grid provided**
- **Draw a trend line then write an equation for the trend line.**
- **Use your trend line to answer the given question**

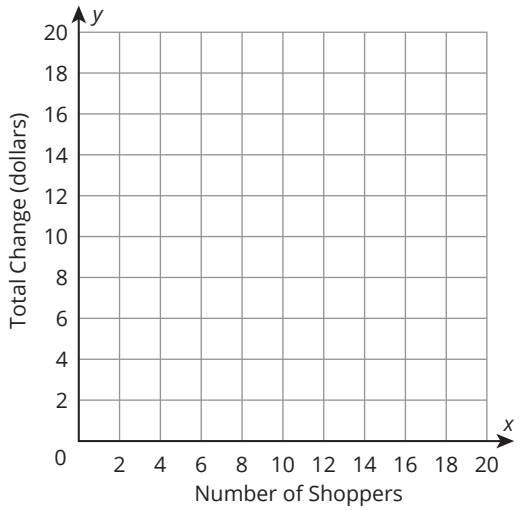
Tracking a Turtle	
Time (seconds)	Distance (meters)
5	3
6	3.3
10	4.1
11	4.5
15	6.3
20	7.9

1. Your little sister's classroom has a pet baby turtle. They let it swim in a tank and record the distance it travels at certain times. Using your trend line to predict how far the baby turtle will swim in 18 seconds.



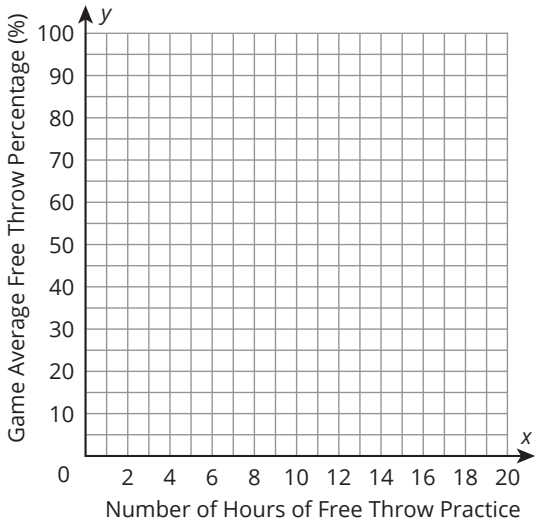
2. Your class conducts an experiment. You ask shoppers at different stores how much change they have, and you record the total number of shoppers and the total dollar amount of change among them. Should you encounter 16 shoppers, how much change, in dollars, would you expect them to have altogether? Use your trend line to approximate a solution.

Counting Change	
Number of Shoppers	Total Change (dollars)
2	1.5
5	3
7	4.2
10	4.5
14	7
18	11



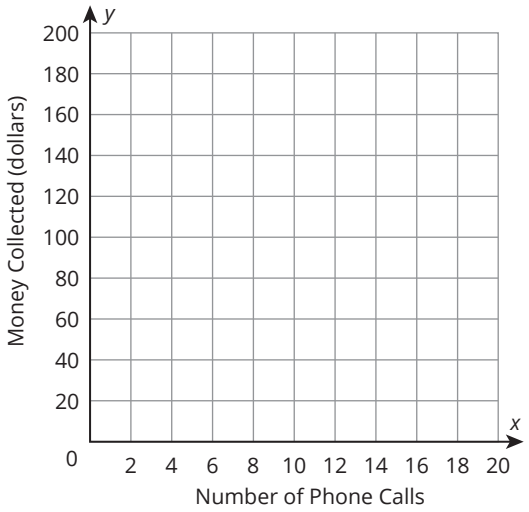
3. The basketball coach wanted to prove to her players that practice does make perfect. She recorded the amount of hours per week each player practiced free throw shots, and then calculated each player's average free throw percentage from each week's games. Using your trend line, predict the average free throw percentage a basketball player would expect in that week's games when they practiced for 20 hours.

Free Throw Practice Hours and Game Free Throw Percentage per Week	
Number of Hours of Free Throw Practice	Game Average Free Throw Percentage (%)
1	40%
2	47%
5	73%
10	73%
15	80%



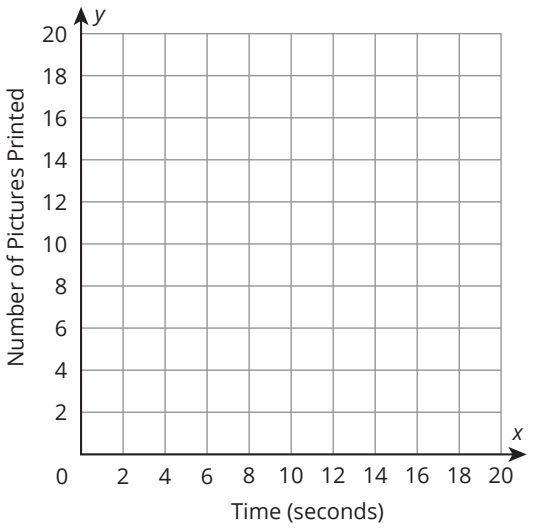
4. A fundraiser uses telemarketers to raise money. Every so often, they stop to determine how many phone calls they've made and how much money they've collected. Using your trend line, approximately how much money would the telemarketers raise when they made 20 phone calls?

Fundraising Totals	
Number of Phone Calls	Money Collected (dollars)
5	60
10	82.30
12	92
13	98.70
15	100.30
20	146



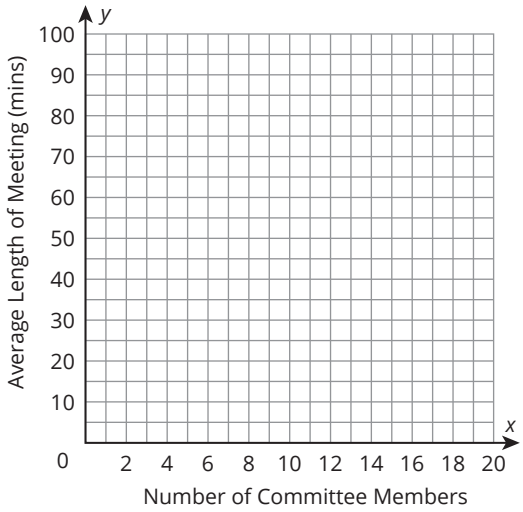
5. Miguel prints his digital pictures. After they start printing, he starts recording the time elapsed and the total number of pictures printed. Using your trend line, how many pictures could Miguel print in 30 seconds?

Printing Pictures	
Time (seconds)	Number of Pictures Printed
7	6.5
12	9.4
15	11.5
17	13.2
20	15



6. The student council tracked data on various committees that were set up throughout the year to work on different projects. They tracked the size of the committee (the number of committee members) and the average length of their meetings. Using your trend line, approximately how long would the meeting be when there were 30 committee members?

Committee Meeting Lengths	
Number of Committee Members	Average Meeting Length (mins)
3	10
12	25
8	20
20	60
5	15
10	30
5	20
15	35
18	45



B. Use the given information to answer each question.

1. A professor is interested in determining the average reading time of his students. He performs two experiments. In the first experiment, students read in a quiet room and are uninterrupted. In the second experiment, students read in a busy cafeteria.

   The trend line for the first experiment is $w = 259.5t - 53.2$, and the trend line for the second experiment is $w = 178.8t + 12$, where w equals the total number of words read and t is the time in minutes. Explain the difference between the number of words read in 20 minutes by a student in each experiment.

2. A car review website wants to compare the total gas used between small cars and pick-up trucks. They conduct two experiments. In the first experiment, they record the number of miles driven and the total gas used by small cars. In the second experiment, they record the number of miles driven and the total gas used by pick-up trucks. The trend line for the first experiment is $d = 29.4g - 3.9$, and the trend line for the second experiment is $d = 17.4g - 2.2$, where d equals the total number of miles driven and g is the amount of gasoline in gallons. Explain the difference between the number of miles driven by a small car and a pick-up truck when both vehicles use 10 gallons of gasoline.

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3. A teacher wants to determine how far her students travel to school. She collects the times and distances traveled for students who walk, and she separately collects the times and distances traveled for students who ride the bus. For the students who walk, the trend line is $d = 2.5t - 0.15$, and for students that ride the bus, the trend line is $d = 17.6t - 0.7$, where d is the distance in miles and t is the time in hours. When two students, one walking and one riding the bus, take 15 minutes (or 0.25 hour) to get to school, how much farther does the student travel who rides the bus?

4. Two experiments are conducted to compare how long it takes copiers to copy in black-and-white and how long it takes them to copy in color. The number of pages copied in black-and-white can be expressed by the trend line $p = 33.8t + 5.3$, and the number of pages copied color can be expressed by the trend line $p = 21t + 2.7$, where p is the total number of pages printed and t is the time in minutes. When you only have 15 minutes to use a copier, how many black-and-white pages could you copy compared to color pages?

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5. The average speed of a horse is 30 miles per hour. The total distance traveled by a horse is given by the equation $d = 30h$, where h is the number of hours. A buffalo runs at an average speed of 35 miles per hour. The total distance traveled by the buffalo is given by the equation $d = 35h$, where h represents the time in hours. Explain the difference between the number of minutes it takes for a horse and a buffalo to run a distance of 2.5 miles.

6. You make an investment of \$50 in your friend's business. According to your friend, you will have a total of $p = 50 + 5.5m$ dollars when you invest in the business for m months. Your uncle put aside some college savings for you when you were born. His initial investment into your college savings was \$5000. The total amount saved is $A = 5000 + 15y$, where y is the time in years the money has been invested. Explain the difference between investing in your friend's business and your college savings for 10 years.

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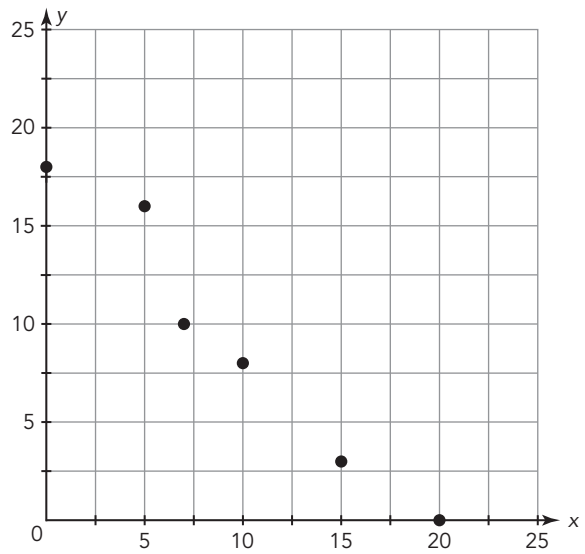
Extension

Two experiments are conducted to compare how long it takes copiers to copy in black-and-white and how long it takes them to print in color. The number of pages copied using black-and-white can be expressed by the trend line $p_b = 33.8t + 5.3$, and the number of pages copied using color can be expressed by the trend line $p_c = 21t + 2.7$, where p is the total number of pages copied, and t is the time in minutes. If you only had 30 minutes to use a copier, how many more black-and-white pages could you copy than color pages?

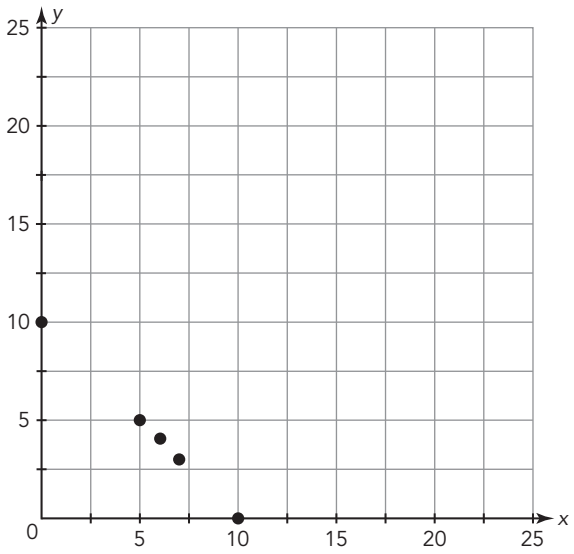
Spaced Practice

Draw a trend line for each scatterplot. Then, write the equation for each line.

1. (0, 18), (5, 16), (7, 10), (10, 8), (15, 3), (20, 0)



2. (0, 10), (5, 5), (6, 4), (7, 3), (10, 0)



Data are collected on the number of minutes a basketball player plays in a game and the number of points they score. The trend line for the data is $y = 0.63x + 1.25$, where x is the number of minutes a basketball player plays in a game and y is the number of points they score. Predict the number of points scored given the minutes a basketball plays in a game.

3. 20 minutes

4. 15 minutes

Write the equation of the translated line.

5. Given the equation of the line $y = -3x - 1$, rewrite the equation to represent this line translated 2 units up.

6. Given the equation of the line $y = 2x + 6$, rewrite the equation to represent this line translated 8 units down.

Name _____ Date _____

I. Mean Absolute Deviation

Topic Practice

A. Calculate the mean of the data in each table. Then describe the deviation from the mean for each data point.

1. Mean:

Data Point	Description of the Deviation from the Mean
15	
26	
9	
19	
21	

2. Mean:

Data Point	Description of the Deviation from the Mean
123	
107	
99	
131	
115	

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3. Mean:

Data Point	Description of the Deviation from the Mean
28	
16	
18	
10	
13	

4. Mean:

Data Point	Description of the Deviation from the Mean
9	
3	
5	
0	
8	

5. Mean:

Data Point	Description of the Deviation from the Mean
87	
98	
93	
88	
94	

6. Mean:

Data Point	Description of the Deviation from the Mean
5	
20	
13	
6	
1	

B. Calculate the deviation from the mean and the absolute deviation for each data value.

1. Mean:

Data Point	Deviation from the Mean	Absolute Deviation
29		
59		
36		
44		
62		
34		

2. Mean:

Data Point	Deviation from the Mean	Absolute Deviation
12		
6		
8		
19		
10		
5		

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3. Mean:

Data Point	Deviation from the Mean	Absolute Deviation
145		
205		
243		
160		
57		
174		

4. Mean:

Data Point	Deviation from the Mean	Absolute Deviation
6		
5		
6		
3		
4		
30		

5. Mean:

Data Point	Deviation from the Mean	Absolute Deviation
78		
87		
69		
74		
66		
70		

6. Mean:

Data Point	Deviation from the Mean	Absolute Deviation
100		
100		
96		
85		
99		
99		

C. Calculate the mean absolute deviation for each data set.

1. Data set: 4, 5, 9, 4, 8

2. Data set: 7, 11, 8, 35, 14

3. Data set: 60, 65, 66, 67, 67, 65

4. Data set: 22, 26, 29, 23, 26, 21, 28, 24, 25, 26

5. Data set: 180, 150, 110, 230, 90, 190, 100, 90, 130, 200

6. Data set: 55, 74, 90, 20, 47, 59, 26, 83, 77, 62

Extension

1. Create a data set of 5 numbers that has a mean absolute deviation of 1. Explain how you arrived at your solution.

2. Create a data set of 6 numbers that has a mean absolute deviation of 10. Explain how you arrived at your solution.

Spaced Practice

1. The rate at which crickets chirp is affected by the temperature. In fact, you can estimate the outside temperature by counting cricket chirps. As a homework assignment, Mr. Ortega asks each of his students to count the number of chirps they hear in 15 minutes at 8:00 PM. The results are shown.

   36, 37, 41, 39, 35, 39, 35, 39, 42, 37

   Determine the median and mean number of cricket chirps heard in 15 seconds.

2. Patrick recorded the number of emails he sent over two weeks: 11, 5, 6, 9, 10, 5, 4, 2, 9, 10. What is the median of his data?

3. Order the integers in each group from least to greatest.

   a. 0, 115, −35, 32, −116, 92

   b. 2, 31, −5, 27, 0, 90

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II. Collecting Random Samples

Topic Practice

A. Answer the question using the information provided.

1. Ms. Tanaka concludes that 85% of the students in the school have at least one pet after she conducts a survey of the students in her class. Does this represent a census or a sample?

2. After examining eight cartons of eggs at a local supermarket, Juliana concludes that one out of every 12 eggs packaged in the U.S. is cracked. Does this represent a census or a sample?

3. After surveying each of the members of the Horse Club, Lucia reports that four out of every five members own two or more horses. Does this represent a census or a sample?

4. According to an online poll, 35% of all U.S. citizens live in an apartment. Is this a parameter or a statistic?

5. A local newspaper conducted a survey last week. Of the 2300 subscribers, 1276 responded. Based on the survey, the editor concludes that 95% of the newspaper's subscribers will renew their subscription next year. Is this a parameter or a statistic?

6. Mr. Brown gives a survey to each of the students in the Art Club. He concludes that 90% of the Art Club members would like to visit the Dallas Museum of Art. Is this a parameter or a statistic?

7. James chooses each player on the soccer team whose last name starts with a vowel to participate in a survey about team sports. Is the sample random or not random?

8. Ms. Flores, the chess team captain, writes the names of each of the twelve team members on separate slips of paper. She places the slips in a box, shakes it, and selects five slips with her eyes closed. The five members whose names are chosen will complete a survey about the chess team. Is the sample random or not random?

9. A local restaurant is conducting a survey to determine the eating habits of the county's residents. They hand a survey to the first 50 customers of the day. Is the sample random or not random?

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B. Use Ms. Tanaka's Class List and the Random Number Table on the next page to answer each question.

Ms. Tanaka's Class List of 7th and 8th Grade Students			
Student Name	Student Number	Student Name	Student Name
Jasmine (7)	11	Gabriel (7)	21
Luna (7)	12	Diego (8)	22
Lucas (8)	13	Valentina (7)	23
Nahimana (8)	14	Logan (8)	24
Mason (8)	15	Emily (8)	25
Sebastian (7)	16	Sarah (8)	26
Kayla (7)	17	Camilla (7)	27
Adriana (7)	18	Avery (8)	28
Angelina (8)	19	Parker (8)	29
Mei (7)	20	Carlos (7)	30

1. The students in Ms. Tanaka's class are each given a student number (11 through 30) as shown in the class list. Use line 14 of the Random Number Table to determine the first four random student numbers.

2. Ms. Tanaka randomly selects 3 students in her class according to their student number. She uses line 18 of the Random Number Table. What are the students' names?

3. Ms. Tanaka randomly selects three 7th graders in her class using line 8 of the Random Number Table. What are the students' names?

4. Ms. Tanaka randomly selects three 8th graders in her class using line 10 of the Random Number Table. What are the students' names?

5. Ms. Tanaka randomly selects two 8th graders in her class using line 20 of the Random Number Table. What are the students' names?

6. Ms. Tanaka randomly selects two 7th graders in her class using line 15 of the Random Number Table. What are the students' names?

Random Number Table										
Line 1	65285	97198	12138	53010	94601	15838	16805	61404	43516	17020
Line 2	17264	57327	38224	29301	18164	38109	34976	65692	98566	29550
Line 3	95639	99754	31199	92558	68368	04985	51092	37780	40261	14479
Line 4	61555	76404	86214	11808	12840	55147	97438	60222	12645	62090
Line 5	78137	98768	04689	87130	79225	08153	84968	64539	79493	74917
Line 6	62490	97198	84987	28759	19107	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	11415	36457
Line 8	18975	95428	33226	55901	31605	43816	22259	00317	46999	98571
Line 9	59138	39542	71168	57609	91510	27904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825

Random Number Table										
Line 11	96155	95009	27429	72918	08457	78134	48407	26061	58754	05326
Line 12	29621	66583	62966	12468	20245	14015	04014	35713	03980	03024
Line 13	12639	75291	71020	17265	41598	64074	64629	63293	53307	48766
Line 14	14544	37134	54714	02401	63228	26831	19386	15457	17999	18306
Line 15	83403	88827	09834	11333	68431	31706	26652	04711	34593	22561
Line 16	67642	05204	30697	44806	96989	68403	85621	45556	35434	09532
Line 17	64041	99011	14610	40273	09482	62864	01573	82274	81446	32477
Line 18	17048	94523	97444	59904	16936	39384	97551	09620	63932	03091
Line 19	93039	89416	52795	10631	09728	68202	20963	02477	55494	39563
Line 20	82244	34392	96607	17220	51984	10753	76272	50985	97593	34320

Extension

1. Variations of random samples are often preferred to truly random samples. Two variations are called **stratified random samples** and **systematic random samples**. Research each type of random sampling. Define each type of sample and give an example, perhaps from the lesson, of when each might provide a more representative sample than a truly random sample.

Spaced Practice

1. A square is represented by the coordinates *A* (2, 2), *B* (2, 8), *C* (8, 2) and *D* (8, 8). Suppose you were to dilate the figure by a scale factor of $\frac{1}{8}$ using the origin as the center of dilation. What are the coordinates of the dilated figure?

2. A triangle is represented by the coordinates *X* (1, 2), *Y* (8, 3), and *Z* (4, 5). Suppose you were to dilate the figure by a scale factor of 3 using the origin as the center of dilation. What are the coordinates of the dilated figure?

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3. A hexagon has a perimeter of 48 inches. The hexagon is dilated by a scale factor of $\frac{1}{4}$. What is the perimeter of the dilated figure?

4. A rectangle has an area of 8 square centimeters. The rectangle is dilated by a scale factor of 3. What is the area of the dilated figure?

III. Sample Populations

Topic Practice

A. Suppose that Mr. Lopez gives a survey to the 20 students in his U.S. History class. Use the results of the survey and a random number generator tool to answer each question.

Student Number	Student Name	Hours Spent Online per Week	Hours Spent Watching TV per Week
20	Michael	14	10
21	Olivia	10	12
22	Javier	7	15
23	Fernando	0	14
24	Alexander	8	5
25	Jacob	14	3
26	Amir	0	9
27	Daniela	20	10
28	Koda	5	20
29	Destiny	10	0
30	Gabriela	10	10
31	Hannah	25	4
32	Madison	0	5

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Student Number	Student Name	Hours Spent Online per Week	Hours Spent Watching TV per Week
33	William	14	14
34	Ana	11	10
35	Maria	5	5
36	Abby	15	5
37	Angel	7	14
38	Luis	5	20
39	Malik	20	10

1. Select 2 students from Mr. Lopez's class using a random number generator tool. Determine the mean number of hours spent online weekly for the 2 students in your random sample.

2. Select 5 students from Mr. Lopez's class using a random number generator tool. Determine the mean number of hours spent online weekly for the 5 students in your random sample.

3. Select 8 students from Mr. Lopez's class using a random number generator tool. Determine the mean number of hours spent online weekly for the 8 students in your random sample.

4. Determine the actual mean number of hours spent online weekly for the 20 students in the class. Discuss how the means changed as the random sample size increased.

5. Select 2 students from Mr. Lopez's class using a random number generator tool. Determine the mean number of hours spent watching television weekly for the 2 students in your random sample.

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6. Select 5 students from Mr. Loepz's class using a random number generator tool. Determine the mean number of hours spent watching television weekly for the 5 students in your random sample.

7. Select 8 students from Mr. Lopez's class using a random number generator tool. Determine the mean number of hours spent watching television weekly for the 8 students in your random sample.

8. Determine the actual mean number of hours spent watching television weekly for the 20 students in the class. Discuss how the means changed as the random sample size increased.

B. Use the random sample to answer the questions.

California is the most populous state in our country. There are more people living in California than people living in the 21 least populous states combined! As of July 2012, California has a total population of 38,041,430 people living in 58 counties.

The following is a random sample of 10 of California's counties and each county's population.

County	Population	County	Population
Amador	35,100	Kings	19,461
Colusa	18,804	Napa	124,279
Glenn	26,453	Orange	2,846,289
Humboldt	126,518	Shasta	163,256
Inyo	17,945	Trinity	13,022

Use the random sample to answer each question.

1. Calculate the mean for the sample. Show your work.

2. Calculate the median for the sample. Show your work.

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3. Calculate the mode for the sample. Show your work.

4. Calculate each quartile for the sample. Show your work.

5. Create a box plot to represent the sample.

6. Calculate the mean absolute deviation of the sample.

Population	Calculation of Deviation from the Mean	Deviations from the Mean	Absolute Deviations
35,100			
18,804			
26,453			
126,518			
17,945			
19,461			
124,279			
2,846,289			
163,256			
13,022			

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Extension

1. Formulate a question and collect data for a population. Then, choose a sample of the data. Compare the sample statistics to the population parameters. What are some predictions or generalizations about the population parameter based on your sample?

Spaced Practice

1. Mr. Brown, the student council sponsor, wants to survey the 8th grade class about upcoming events. Which sampling method would result in representative samples? Explain your reasoning.

   a. Surveying the first 5 members who arrive at the basketball game.

   b. Drawing 5 names out of a box containing all the students' names and surveying those 5 members.

   c. Surveying the 5 members who volunteer first.

2. The basketball coach is asked to choose 3 students to represent the team at a county-wide banquet. The coach decides to randomly select 3 players out of the 15 students on the team.

   a. What is the population for this situation?

   b. What is the sample for this situation?

   c. Suggest a method for selecting the random sample of 3 players.

3. Lauren is saving money to buy a new video game console. She put an initial amount of money into an envelope Each month she puts an additional amount of money into the envelope. The table shows the linear relationship between the number of months Lauren has saved, x, and the total amount of money in the envelope, y.

   a. How much money does Lauren add to the envelope each month?

   b. What is the initial amount of money Lauren put into the envelope?

Number of months, x	Total Amount in the Envelope, y (dollars)
4	205
7	265
10	325
12	365

4. The volleyball team is ordering T-shirts. They pay a one-time fee of $18 to design the shirts. They also pay $12 for each T-shirt ordered. Write an equation to determine the total cost, c, when the team orders s T-shirts.

5. The value of y varies directly with x. When $y = 50$, $x = 5$. What is the value of y when $x = 2\frac{1}{2}$?

6. The cost of gasoline varies directly with the number of gallons of gasoline purchased. When the cost of 12 gallons of gasoline is $45, how much does it cost for 5 gallons of gasoline?

GRADE 8

MODULE 4

Modeling Linear Equations

TOPIC 1: Solving Linear Equations and Inequalities

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Name _____ Date _____

I. Equations with Variables on Both Sides

Topic Practice

A. Solve each equation using algebra tiles.

1. $3x + 7 = 4x + 2$

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2. $6x + 9 = 3x + 3$

3. $3x + 4 = x - 8$

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4. $5x - 5 = 2x + 4$

5. $2x + 3 = 6x - 5$

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6. $4x - 6 = 6x + 2$

B. Solve for *x*.

1. $3.5x - 7 = 5.6x$

2. $-4x + 5 = -10 + 10x$

3. $\frac{1}{6}x + 2 = \frac{5}{6}x$

4. $9x + 7 = 1 - 8x$

5. $9 + \frac{3}{10}x = \frac{7}{10}x$

6. $1 + \frac{3}{8}x = \frac{5}{8}x$

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7. $\frac{2}{3}x = 5 + \frac{1}{3}x$

8. $-8x = -9x - 9$

9. $-5x - 7 = -x$

10. $6.7 - x = -10x$

11. Solve $7x - 28 = -35 - 21x$ by factoring a number from both sides of the equation.

12. Solve $-3.4x + 17.2 = -4.6x - 9.8$ by multiplying both sides of the equation by a power of 10.

13. Solve $-13x - 39 = -26x + 78$ by factoring a number from both sides of the equation.

14. Solve $-34.5x - 219 = 14.5x - 72$ by multiplying both sides of the equation by a power of 10.

Extension

You can solve an equation with two variables by trying different values. What is the solution to the equation $2x + 3y = 13$?

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Spaced Practice

1. Noah had $450 in his checking account when Ava opened a checking account with zero dollars.

   - Noah deposited $40 into his account each week for x weeks.
   - Ava deposited $60 into her account each week for x weeks.
   - The accounts did not earn interest.

   Write an inequality that represents this situation when the amount of money in Noah's account was greater than the amount of money in Ava's account.

2. Calculate the slope of the line represented by each table.

a.

x	y
4	8
10	11
16	14
20	16

b.

x	y
2	5
4	3
5	2
8	−1

II. Analyzing and Solving Linear Equations

Topic Practice

A. Write an equation for each situation, define your variable, and solve the equation. Then, verify your solution.

1. A pet store charges $35 to groom each pet. A mobile groomer charges a one-time fee of $20 and $25 for each pet. How many pets would make their fees equal?

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2. Sofia earns $3.50 per hour mowing lawns. She also gets $10 per week as an allowance. Chloe earns $5.25 per hour babysitting and spends $11 per week on books. Sofia and Chloe work the same number of hours per week and have the same amount of money at the end of each week. How many hours per week do they each work?

3. Minh wants to purchase a custom bookcase. The hardware store charges a $50 consultation fee and $75 per shelf needed in the bookcase. A local carpenter charges $100 per shelf needed in the bookcase but does not charge a consultation fee. At how many shelves would the cost of the bookcase be the same?

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4. There are two candy machines in a restaurant. One is filled with 200 gumballs, and the other is filled with 900 chocolate candies. The gumballs are dispensed 1 at a time, and the chocolate candies are dispensed 8 candies at a time. How many dispenses from each machine will make the number of candies in the machines equal?

5. Minh and Victoria are the same height. Minh is $\frac{5}{6}$ the height of their friend Aaliyah, and Victoria is $\frac{7}{8}$ the height of Aaliyah minus $2\frac{1}{4}$ inches. How tall are Aaliyah, Minh, and Victoria?

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6. Store *A* rents boats for $60 the first hour and $30 each hour after that. Store *B* rents boats for $40 per hour. At what number of hours are the rental fees for a boat from each shop equal?

B. Define a variable and write algebraic expressions for each of the given quantities. Then, answer the question by writing and solving an equation.

1. The middle school dance team had 3 separate car washes on Saturday to raise money for new uniforms. The Main Street location raised $50 more than the Back Street location. The Washington Street location raised $50 more than half the amount raised at the Back Street location. How much did the dance team raise at each location when they raised a total of 3 times the amount of money raised at the Back Street location?

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2. Three brothers spent different amounts of money on their recent vacation. Ethan spent $25 less than Samuel. Diego spent 3 times as much as Ethan. How much did each brother spend when they spent a total of 3 times the amount Samuel spent?

3. Three waiters at a local restaurant earned different amounts of money the past week. Emma earned $40 less than twice the amount Daniel earned. Juan earned $60 more than Daniel. How much did each waiter earn when they earned a total of 250 more than 3 times what Daniel made?

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4. Four friends have different numbers of cows on their farms. Jayden has 3 times as many cows as Harper. Mateo has half as many cows as Jayden. Sofia has 2 less than 4 times as many cows as Harper. Together, the four friends have 90 less than 5 times the number of cows Jayden has. Determine the number of cows each friend has.

5. Minh gives baseball cards to four of his friends. He gives Chloe 3 cards less than he gives Liam. He gives Elijah 7 cards more than he gives Chloe. He gives Mia twice as many as he gives Elijah. How many cards does Minh give to each friend should he give out a total of 6 times what Chloe has?

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6. Victoria types twice as many words per minute as Aaliyah. Kai types 5 words per minute less than Victoria. Ava types 10 words per minute more than Victoria. How many words per minute does each person type if the total number of words per minute that Ava and Aaliyah type equals the total number of words per minute that Victoria and Kai type?

C. Solve each equation. Tell whether the equation has one solution, no solution, or infinitely many solutions. For an equation with one solution, write a real-world problem that corresponds to the equation.

1. $10(x - 2) + 15 = 8x + 7$

2. $7(x - 1) = 7(3 + x)$

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3. $8x - 12 = 8(x - 1.5)$

4. $5 + 5x - 3 = 5x + 9$

5. $5(x + 3) = 8x - 33$

6. $4x - 9 = 12\left(\frac{1}{3}x - \frac{3}{4}\right)$

7. $2.25x + 15 = 3.5x$

8. $-4(3 + 2.5x) = \frac{1}{2}(-20x - 24)$

Extension

When an equation is not a linear equation, it can have more than one solution. The equation $x^2 = 9$ has two solutions, -3 and 3. What are the solutions to the equation $2x^2 + 5 = 77$?

Spaced Practice

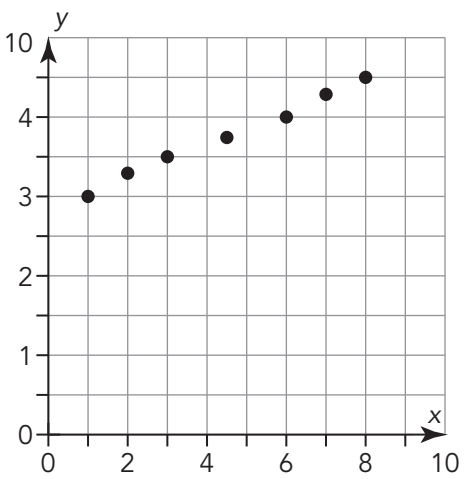
Solve each equation.

1. $\frac{2}{3}x + \frac{4}{3} = \frac{1}{6}x + \frac{1}{3}$

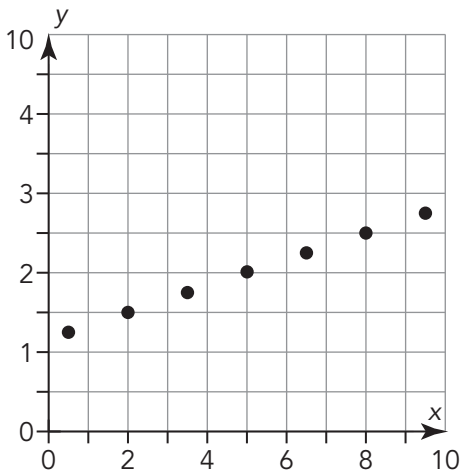
2. $2.5x - 1 = 10 - 7.5x$

Determine whether the data suggest a linear relationship or a nonlinear relationship.

3.



4.



Determine the slope and *y*-intercept of the line represented by each equation.

5. $36 = 24y + 48x$

6. $y - 14 = 7x + 9$

III. Solving Linear Inequalities

Topic Practice

A. Define a variable and write an inequality to represent each problem situation.

1. Emma read 33 pages of her book on the weekend and then read 19 pages per day. Ava read 45 pages of her book on the weekend and then read 23 pages per day. Write an inequality to represent the number of days that Emma must read in order to read more pages than Ava.

2. Noah bought a large bottle of apple juice for $2.99 and some individual juice boxes that cost $0.29 per box. Kai bought a large bottle of grape juice for $2.45 and some individual juice boxes that cost $0.30 per box. Write an inequality to represent the number of juice boxes that Kai must buy to pay less than Noah.

3. Elijah went running for 14 minutes and then did a series of exercises for 2 minutes each. Aaliyah went running for 22 minutes and then did a series of exercises for 3 minutes each. Write an inequality to represent the number of exercises that Aaliyah must do to exercise longer than Elijah.

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4. Liam bought a watermelon at the grocery store for $3.25 and bananas for $0.19 each. Victoria bought a pineapple for $2.59 and apples for $0.65 each. Write an inequality to represent the number of pieces of fruit that Liam must buy to pay less at the grocery store than Victoria.

5. Minh watched the news for 9 minutes and then watched a series of videos that were 12 minutes each. Mia watched the news for 11 minutes and then watched a series of videos that were 10 minutes each. Write an inequality to represent the number of videos that Mia must watch in order to watch for more minutes than Minh.

6. Chloe joined her local yoga studio for $29.99 per month plus a one-time membership fee of $35. Mateo joined his local kickboxing gym for $19.99 per month plus a one-time membership fee of $50. Write an inequality to represent the number of months that Mateo must be a member to pay less than Chloe.

B. Write a real-world problem that corresponds to each equation or inequality.

1. $0.25p + 3.75 > 0.55p + 2.49$

2. $\frac{2}{3}x = \frac{1}{3}x + 3$

3. $17d < 9d + 34$

4. $4.5b + 7.99 = 3.29b + 12.84$

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5. $15v + 7 > 21v + 4$

6. $1.99f + 3.85 = 1.24f + 4.6$

Extension

1. Consider the equation $2x - 5(x - 1) = 50 + 3x$. Solve the equation for x.

2. Kai solves the inequality $2x - 5(x - 1) < 50$. Her answer is $x < -7.5$. Substitute in any value for x less than -7.5 to determine whether Kai is correct. If not, determine the correct solution.

Spaced Practice

1. Write a real-world situation that models each equation.

   a. $10x = 12x - 2$

   b. $10x - 5 = 9x + 3$

2. Diego is training for a charity bike ride. He takes a ride on 49-Mile Scenic Drive in San Francisco. The table shows his time at the beginning and end of the ride. What is Diego's average speed for the ride?

Time (hours)	Distance (miles)
0	0
3.5	49

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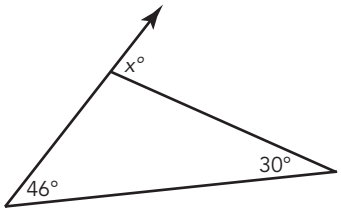
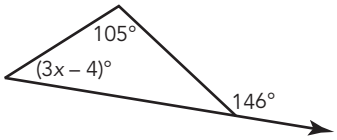
3. In each figure, solve for x.

a.



b.



4. Determine the slope and y-intercept of the line represented by each equation.

a. $36 = 24y + 48x$

b. $y - 14 = 7x + 9$

Name _____ Date _____

I. Point of Intersection of Linear Graphs

Topic Practice

A. Complete each table of values. Use the table to identify when the production cost and the income are equivalent.

1.

Number of Mugs	Income (dollars)	Production Cost (dollars)	Profit (dollars)
x	$5x$	$2x + 45$	$3x - 45$
0			
5			
15			
25			
50			
100			

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2.

Number of Box Lunches	Income (dollars)	Production Cost (dollars)	Profit (dollars)
x	$4x$	$3x + 10$	$x - 10$
0			
2			
5			
10			
20			
50			

3.

Number of Lawns Mowed	Income (dollars)	Production Cost (dollars)	Profit (dollars)
x	$2x$	$4x + 400$	$20x - 400$
0			
5			
10			
15			
20			
30			

4.

Number of Scarves	Income (dollars)	Production Cost (dollars)	Profit (dollars)
x	$6x$	$3.5x + 30$	$2.5x - 30$
0			
2			
4			
6			
8			
12			

5.

Number of Flower Wreaths	Income (dollars)	Production Cost (dollars)	Profit (dollars)
x	$37x$	$12x + 500$	$25x - 500$
0			
10			
12			
14			
20			
100			

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6.

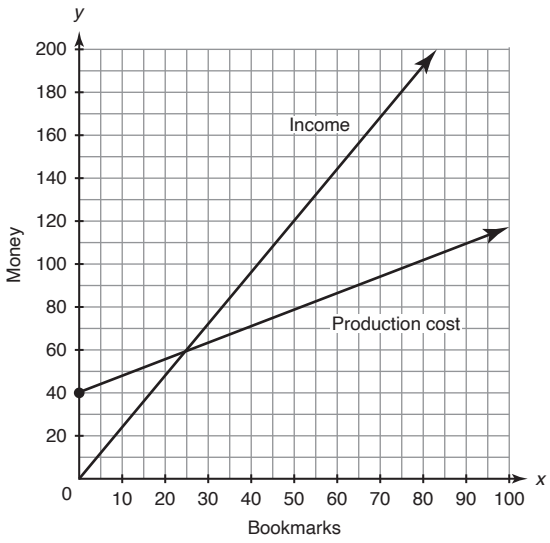
Number of Video Games	Income (dollars)	Production Cost (dollars)	Profit (dollars)
x	$35x$	$10x + 1500$	$25x - 1500$
0			
25			
50			
60			
200			
300			

B. Determine the point of intersection and the break-even point for each graph. Then, state the least number of items that must be sold to make a profit.

1. Point of intersection:

   Break-even point:

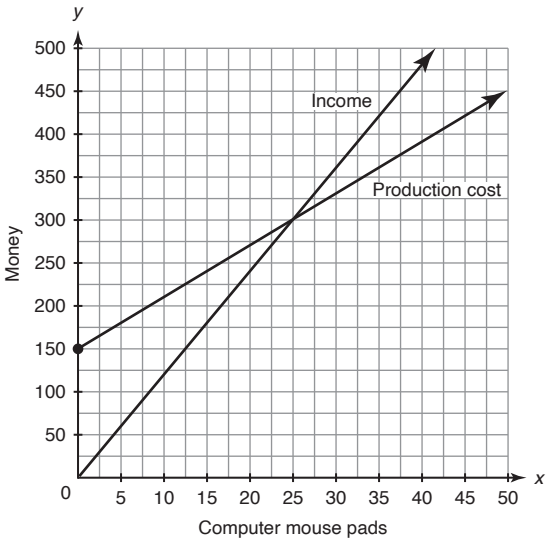
   At least _____ bookmarks must be sold to make a profit.



2. Point of intersection:

   Break-even point:

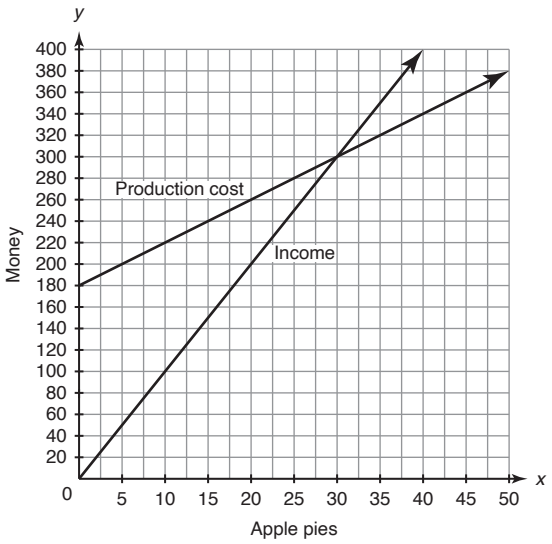
   At least _____ computer mouse pads must be sold to make a profit.



3. Point of intersection:

   Break-even point:

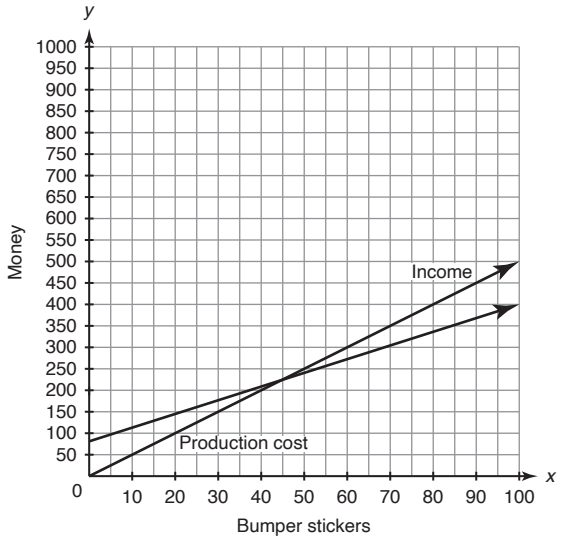
   At least _____ apple pies must be sold to make a profit.



4. Point of intersection:

   Break-even point:

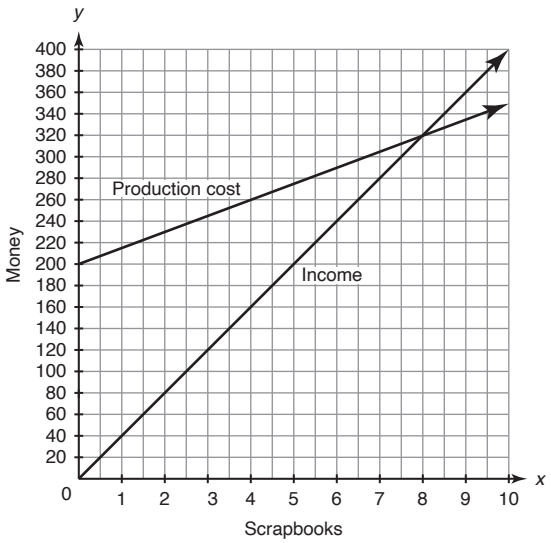
   At least _____ bumper stickers must be sold to make a profit.



5. Point of intersection:

   Break-even point:

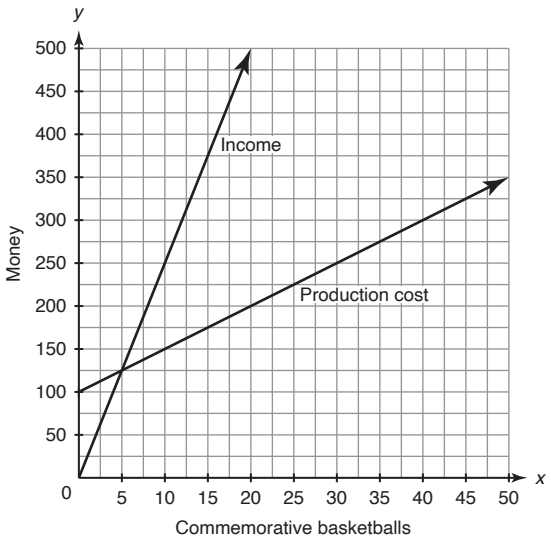
   At least _____ scrapbooks must be sold to make a profit.



6. Point of intersection:

   Break-even point:

   At least _____ commemorative basketballs must be sold to make a profit.



Extension

Suppose a jet plane is traveling at 500 miles per hour at a height of 30,000 feet. If you took off from the ground in a flying car, traveling straight up at 100 miles per hour, how far away would the plane need to be when you take off for the car to meet the plane at the same height and time?

Spaced Practice

Determine whether each equation has one solution, no solution, or infinitely many solutions.

1. $3x - 4 = 6x - 8$

2. $2x + 1 = 2x - 1$

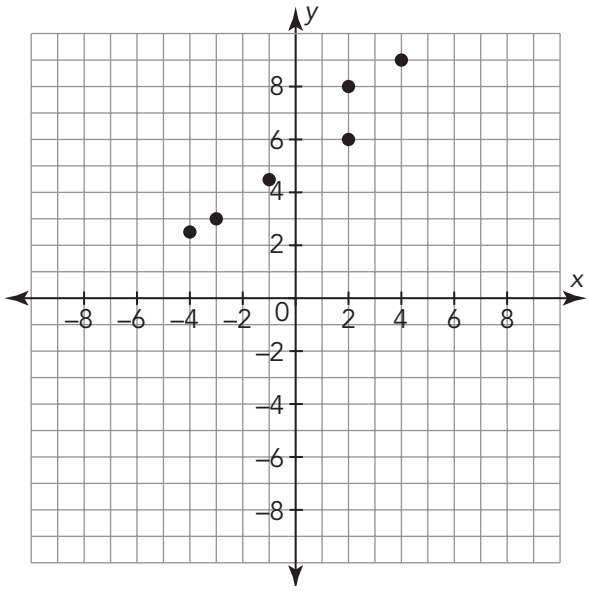
Solve each equation.

3. $-4x - 2 = 6x + 2$

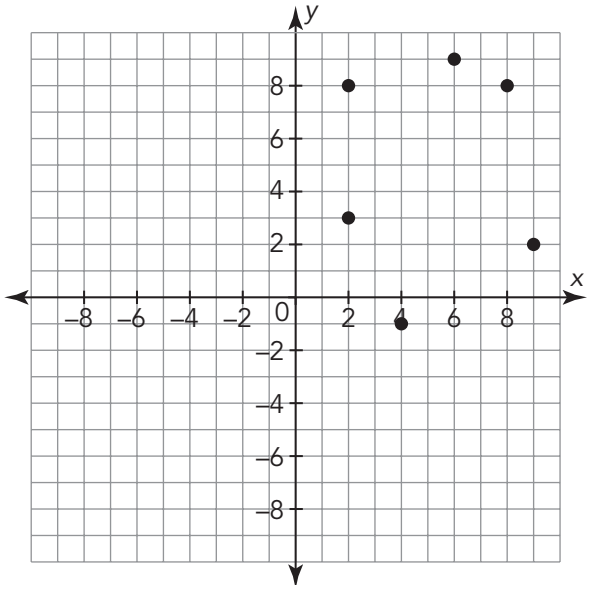
4. $\frac{1}{2}x - 5 = 8 + 2x$

Describe the pattern of association between the two quantities in each scatterplot.

5.



6.



II. Systems of Linear Equations

Topic Practice

A. Write a system of equations that represents each situation. Let *y* represent the total amount of money, in dollars, in each person's savings account in terms of the number of weeks, *x*, that he or she places money in the account. Then, solve for the indicated amounts.

1. Kai has $65 in her savings account. Her friend Emma has $119 in her savings account. Kai plans to add $23 per week to her account. Emma plans to add $14 per week. How much money will each friend have after 6 weeks? 10 weeks?

2. Elijah and his brother each received $100 to open their own savings accounts. Elijah tutors two students after school and is able to save $30 per week. Elijah's brother can only save $12 per week. How much will each brother have after 4 weeks? 12 weeks?

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3. Jayden and Isabella make $50 per week helping a neighbor, and they split the money evenly and add it to their savings accounts. Jayden already has $150 in his account and Isabella has $40 in her account from babysitting. How much will each person have saved after 3 weeks? 13 weeks?

4. Ethan has saved all of his allowance for a year to pay for guitar lessons and has a total of $550 in savings. His friend Harper is just starting to save and has $50 so far. Each week Ethan will pay Harper $25 for a guitar lesson and Ethan will save the money in her savings account. How much will each person have in their accounts after 10 weeks? 20 weeks?

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5. Samuel has $160 in his savings account and Diego has $142 in his savings account. Diego saves $2 per week and Samuel spends a quarter every week for a gumball. How much will each person have after 4 weeks? 8 weeks?

6. At her job, Chloe earns $120 per week plus a one-time $300 bonus. Mia teaches art lessons and earns $24 per week plus a $60 art-supply fee for each student she teaches. Should Mia have 5 students, how much money will each person earn after 5 weeks? 15 weeks?

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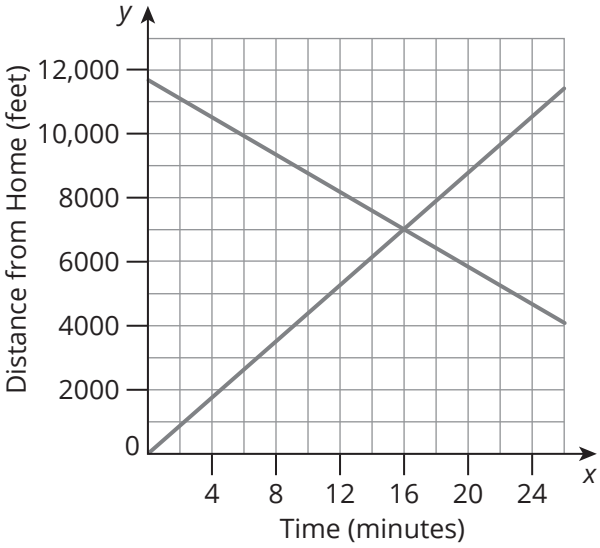
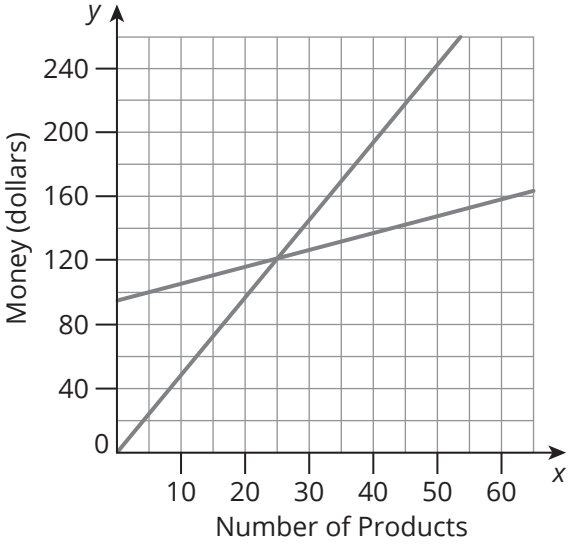
B. Use each graph to determine the solution to the system of equations. Interpret the solution in terms of the problem situation.

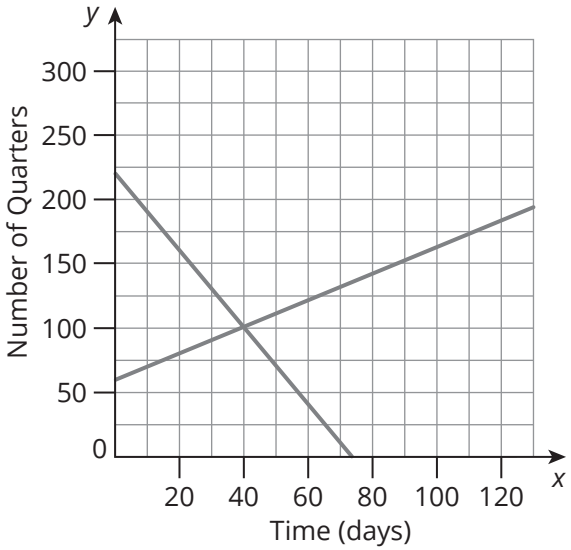
1. Juan started walking from home to school just as his brother Daniel was leaving school to walk home. The distance from their home to school is 11,680 feet. Daniel was walking at a slow pace averaging about 292.5 feet per minute. Juan, who was excited to get to class, was walking more quickly, averaging about 437.5 feet per minute. Assume they both left at the same time and maintained their rate of speed for the entire trip.



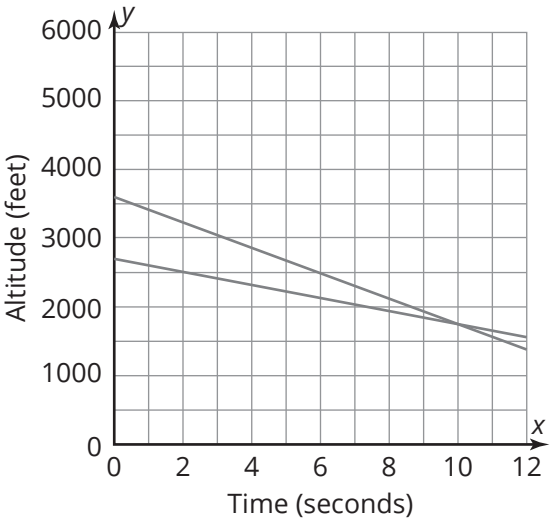
2. Sofia runs a company that produces clothing accessories—hats, patches, scarves, etc.—for events such as a Presidential inauguration. To produce each item costs her $1. She also pays an initial startup cost for production of $95, regardless of the number of products that she produces. She plans to sell each product for $4.80.



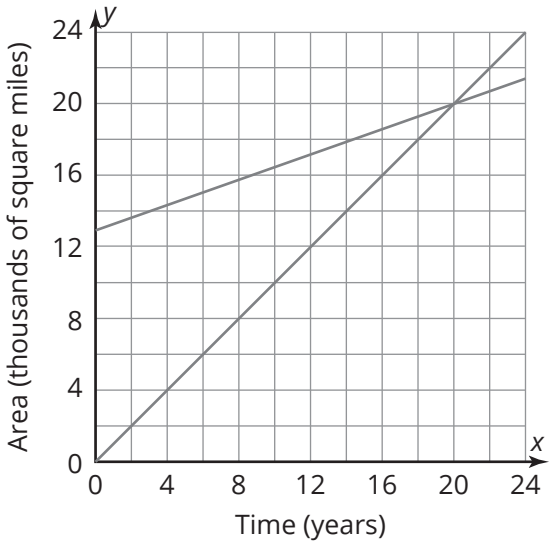
3. Victoria and Jayden both have coffee cans in which they keep quarters. Victoria has 60 quarters in her can while Jayden has 220 in his. This summer, Jayden is working at the library five days per week and spends 3 of his quarters every day in the parking meter. Victoria, however, receives 1 quarter every day in change from her lunch and is putting them into her can. (Victoria also works five days per week.)



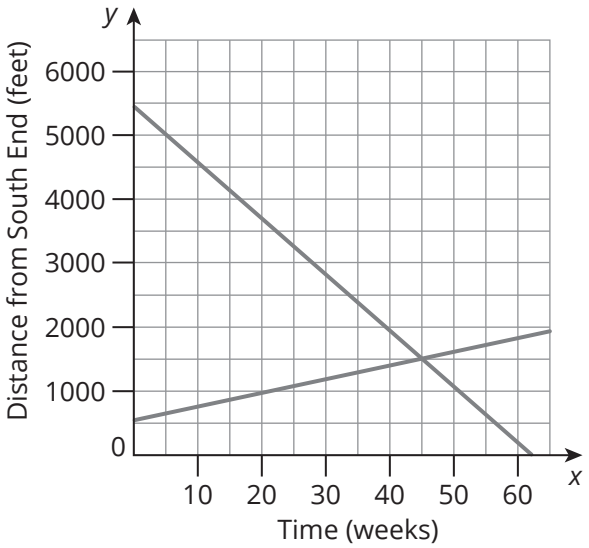
4. Two skydivers are jumping out of two separate planes to do a stunt for a movie. The first skydiver is at an altitude of 2700 feet and will fall at the rate of 95 feet per second by stretching out his arms and legs to slow his descent. The second skydiver will jump from an altitude of 3600 feet but will fall at the rate of 185 feet per second.



5. Old growth forests are forests that have not experienced unnatural or man-made disturbances. Currently, the United States has about thirteen thousand square miles designated as old growth forest, and this is increasing at the rate of 340 square miles a year. The logging industry proposes that it be permitted to harvest 990 square miles of old growth timber per year.



6. A company was hired to build a tunnel through a mountain. The company started at the south end of the mountain and completed only 555 feet of the required 5460 feet before going bankrupt. A different company was hired to complete the job, but they decided to use two crews. Crew A would start where the other company left off at the southern end, while Crew B would start at the northern end and dig towards the other crew. Crew A was able to dig 21 feet of the tunnel per week. Crew B, which was larger, was able to dig 88 feet of the tunnel per week.

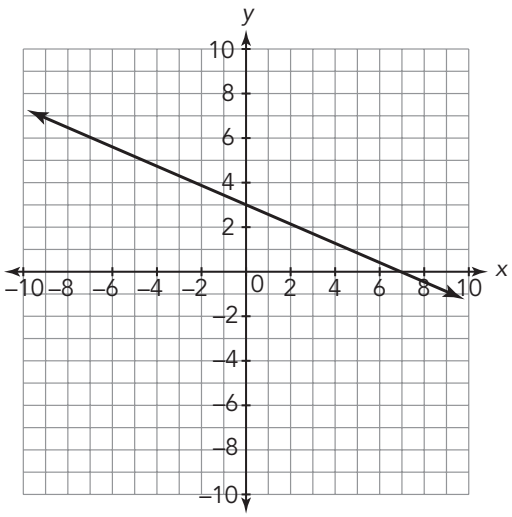


Extension

Minh has a booth at the flea market where she sells purses and wallets. All of her purses are the same price and all of her wallets are the same price. The first hour of the day, she sells 10 purses and 6 wallets for a total of $193. The second hour, she sells 2 purses and 1 wallet for a total of $37.50. Write a system of equations to represent the problem situation.

Spaced Practice

1. Write a linear equation that represents the graph.



2. Write a linear equation that represents the table.

x	*y*
0	1
1	3
5	11
6	13
9	19

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3. Determine whether the equations have one solution, no solution, or infinitely many solutions.

   a. $1.5x + 6.5 = \frac{3}{2}x + \frac{13}{2}$

   b. $-\frac{1}{5}x - 12 = -0.2x - \frac{24}{2}$

4. Solve each equation.

   a. $4(x + 5) = 6(x + 4)$

   b. $-3(p - 4) = -2p + 1$

III. Multiple Representations of Systems of Linear Equations

A. Complete each table for the given system of linear equations. Then, identify the ordered pair that satisfies the system of equations.

1. $\begin{cases} y = 4x - 1 \\ y = -2x + 5 \end{cases}$

x	$y = 4x - 1$	$y = -2x + 5$
-2		
-1		
0		
1		
2		

2. $\begin{cases} y = 3x + 2 \\ y = -5x - 6 \end{cases}$

x	$y = 3x + 2$	$y = -5x - 6$
-2		
-1		
0		
1		
2		

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3. $\begin{cases} y = -2x + 6 \\ y = -4x + 12 \end{cases}$

x	$y = -2x + 6$	$y = -4x + 12$
-1		
0		
1		
2		
3		

4. $\begin{cases} y = -\frac{2}{3}x - 1 \\ y = 4x - 15 \end{cases}$

x	$y = \frac{2}{3}x - 1$	$y = 4x - 15$
-1		
0		
1		
3		
6		

5. $\begin{cases} y = 5x + 11 \\ y = -3x + 3 \end{cases}$

x	$y = 5x + 11$	$y = -3x + 3$
-2		
-1		
0		
1		
2		

6. $\begin{cases} y = \frac{3}{4}x + \frac{3}{2} \\ y = -\frac{1}{5}x - \frac{2}{5} \end{cases}$

x	$y = \frac{3}{4}x + \frac{3}{2}$	$y = -\frac{1}{5}x - \frac{2}{5}$
-3		
-2		
-1		
0		
1		

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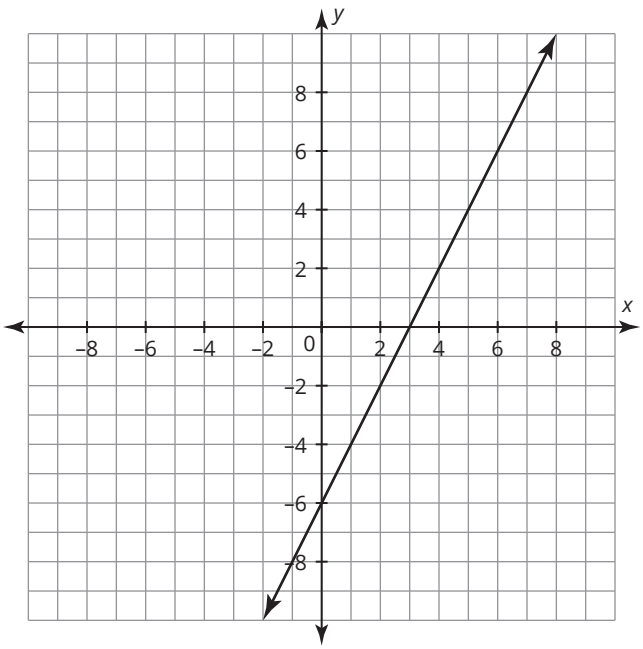
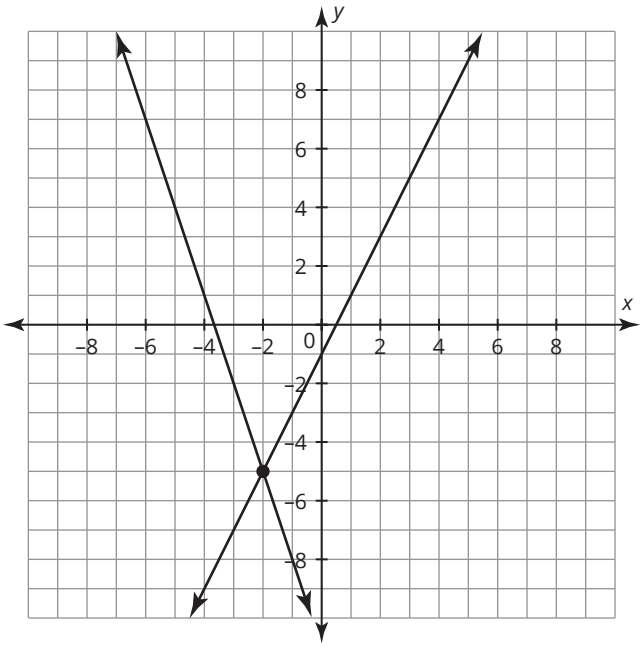
B. Use each graph to identify whether the system has one solution, no solution, or infinitely many solutions. When the system has one solution, write the values of the variables that make the equations true. Use algebraic verification to show that the values satisfy both equations in the system.

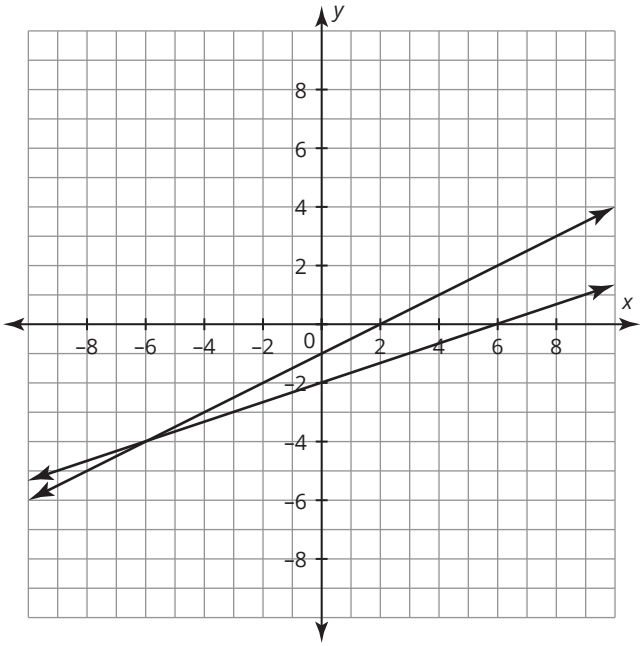
1. $\begin{cases} y = 2x - 6 \\ y = 2x - 6 \end{cases}$



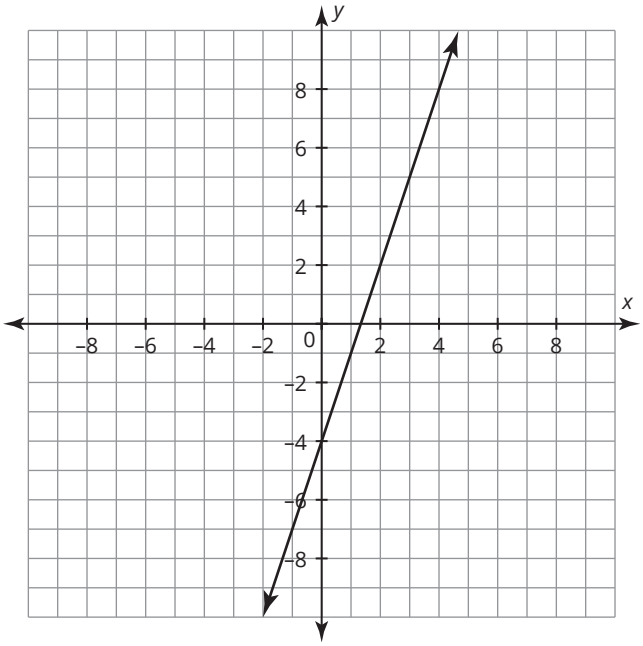
2. $\begin{cases} y = 2x - 1 \\ y = -3x - 11 \end{cases}$



3. $\begin{cases} y = \frac{1}{2}x - 1 \\ y = \frac{1}{3}x - 2 \end{cases}$



4. $\begin{cases} y = 3x - 4 \\ y = 3x - 4 \end{cases}$



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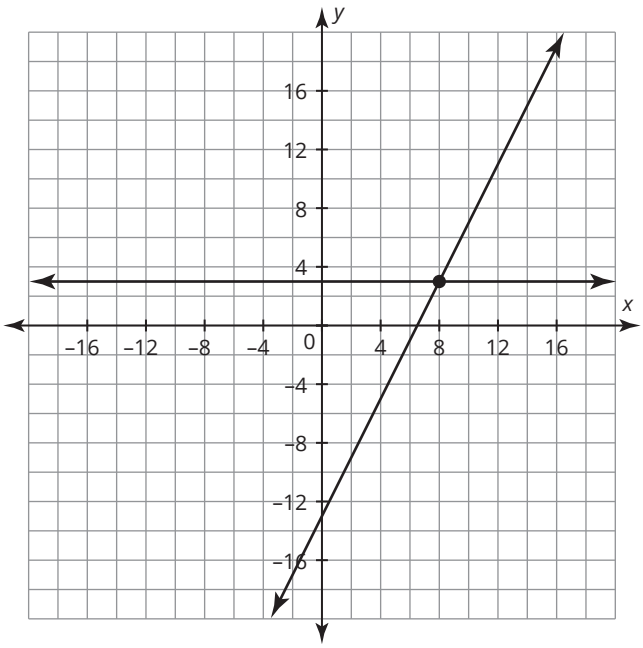
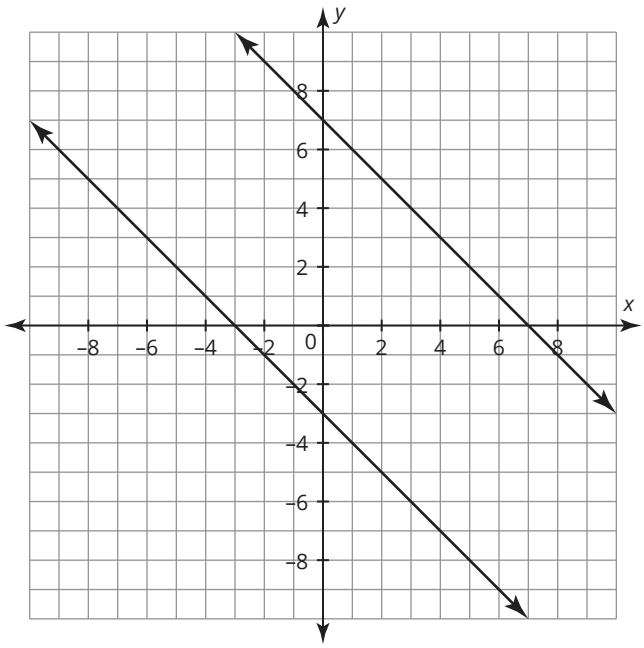
5. $\begin{cases} y = 3 \\ y = 2x - 13 \end{cases}$



6. $\begin{cases} y = -x + 7 \\ y = -x - 3 \end{cases}$



Extension

Ms. Jackson is planning an end-of-the-year party for the entire 8th-grade class. The school gives her a budget of $1380. She is considering an arcade that charges a $500 rental fee plus $7 per student, a mini golf center that only charges $12 per student, and a trampoline park that charges a $1500 flat fee to rent the venue.

Write and solve an inequality to determine the number of students that can attend a party at each of the three locations. Then, interpret each solution in terms of the problem situation.

Spaced Practice

Solve each equation.

1. $8x + 4 - 3x = 10 + 5x - 6$

2. $10(x - 2) + 15 = 8x + 7$

3. $2(x + 3) + 2 = 2(x + 4)$

4. $3(2x + 2) = 6(x + 6)$

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GRADE 8

MODULE 5

Applying Powers

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Name _____ Date _____

I. Sorting Numbers

Topic Practice

A. Classify each number as rational or irrational.

1. 0

2. -5

3. $-\sqrt{2}$

4. 1.3

5. $\sqrt{5}$

6. π

7. 0.33

8. $\sqrt{16}$

9. 6

10. $\frac{3}{4}$

11. 0.67236982158...

12. $\frac{31}{13}$

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B. Provide a rationale for each grouping of numbers.

1. $\frac{2}{3}, \pi, \sqrt{64}, \frac{53}{4}, 0.3\overline{7}, 98\%$

2. $\pi, -\sqrt{7}, \sqrt{3}, \sqrt{2}, -0.121221222...$

3. $\frac{2}{3}, 1.4545..., \frac{74}{3}, -5\frac{6}{7}, 12.\overline{9}$

4. $3, \frac{75}{5}, -18, -\frac{30}{3}$

5. $8, \left(\frac{4}{5}\right)^2, 144, -16^5, 225$

6. $\sqrt{81},\ \sqrt{9}, 2\sqrt{3},\ \sqrt{2^3}$

C. List three numbers that satisfy each grouping.

1. Mixed numbers

2. Natural numbers

3. Numbers between -1 and 0

4. Terminating decimals

5. Improper fractions

6. Irrational numbers

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Extension

1. What is so interesting about the fraction $\frac{61}{64}$? This fraction is called a *digit-canceling fraction*, since you can cross out the common digit in the numerator and denominator and the value of the fraction remains the same.

$$\frac{16}{64} = \frac{1}{4}$$

There are exactly four digit-canceling fractions with two-digit numerators and two-digit denominators that are less than one, not counting examples such as $\frac{30}{50}$, where you cross out the zeros. Research these special fractions and identify the other three.

Spaced Practice

1. A company makes and sells American flags. It costs \$12 to manufacture each flag, and there is a set-up cost of \$200 for the design. The company sells the flags to home improvement stores for \$20 per flag.

   a. Write a system of equations to represent this situation.

   b. What is the break-even point for making and selling flags? Show your work.

2. The school dance team is raising money by charging admission to the spring ballet. They charge $2 for each student ticket and $5 for each adult ticket, with a goal to raise $510 from ticket sales.

   a. If they sell 50 student tickets, how many adult tickets do they need to sell to reach their goal?

   b. If they sell 30 adult tickets, how many student tickets to they need to sell to reach their goal?

3. Solve each equation.

   a. $3(n - 5) = 7 - 2(n + 1)$

   b. $2(5k + 8) = 4(k + 4)$

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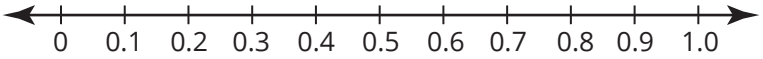
II. Rational and Irrational Numbers

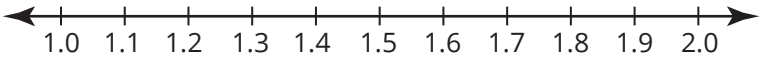
Topic Practice

A. Graph each pair of rational numbers on the number line. Use the graph and write >, <, or = to compare the numbers.

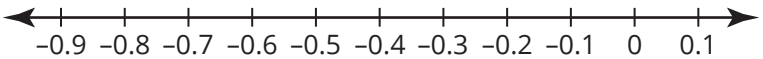
1. $\frac{3}{5} \square 0.25$



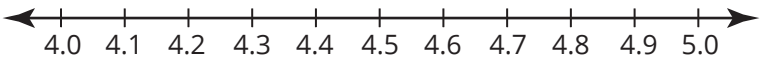
2. $1.7 \square 1\frac{5}{8}$



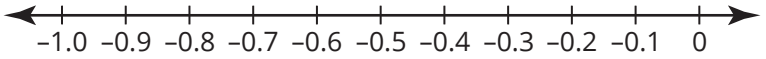
3. $-\frac{4}{5} \square 0.08$



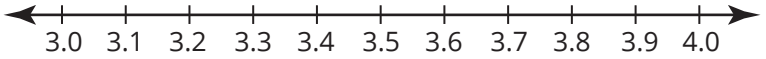
4. $4\frac{3}{4} \square 4.34$



5. $-\frac{11}{15} \square -0.65$



6. $3.8 \square 3\frac{4}{5}$



B. Rewrite each rational number as a decimal to identify whether it is a terminating decimal or a repeating decimal.

1. $\frac{7}{12}$

2. $\frac{8}{11}$

3. $\frac{47}{8}$

4. $\frac{-13}{6}$

5. $\frac{-18}{5}$

6. $4\frac{7}{22}$

Extension

Numbers can be operated on using operations other than addition, subtraction, multiplication, and division. Let's define a new operation called ☆, where $2 ☆ 4 = 2^2 \div 4$ and $6 ☆ 3 = 6^6 \div 3$. Is the set of whole numbers closed under the operation ☆? That is, does $a ☆ b$, where a and b are whole numbers, always result in a whole number? Justify your claim.

Spaced Practice

1. Provide a rationale for each grouping of numbers.

   a. $3, \frac{75}{5}, -18, -\frac{30}{3}$

   b. 25%, $\frac{7}{11}$, 0.912912..., 0.5%

2. Write and solve a system of equations for each problem situation. Interpret the solution in terms of the context.

   a. Mason has 97 athlete cards. In his collection, he has 39 more baseball player cards than football player cards. How many of each type of card does Mason have?

   b. The Perez family is researching venues for their family reunion at the local park. The park charges $150 to reserve a picnic shelter and $20 per hour to use the shelter. The family can also rent a tent. The tent company charges $300 for the rental and setup of a tent and $10 per hour to use their land. When would the cost be the same at for both the pavilion and the tent? What is that cost?

3. Solve each equation.

   a. $6(x + 3) = 3(2x + 5) - 3$

   b. $-5(x - 4) = 2(x + 10)$

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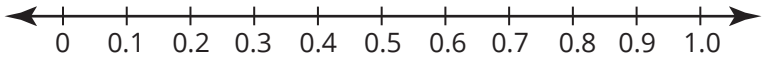
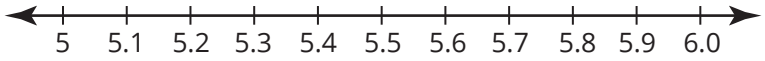
4. Graph each pair of rational numbers on the number line. Use the graph and write $>$, $<$, or $=$ to compare the numbers.

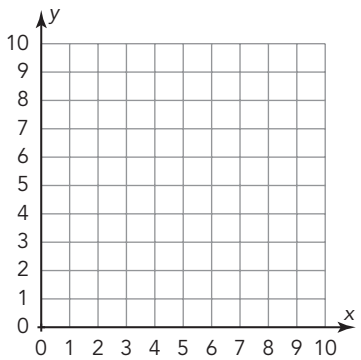
   a. $\frac{7}{8} \square 0.925$



   b. $5.75 \square 5\frac{3}{5}$



5. Michael spent $24.50 to purchase 7 sandwiches. Each sandwich costs the same amount. Graph the proportional relationship on the coordinate grid.



III. The Real Numbers

Topic Practice

A. Estimate each square root to the nearest tenth.

1. $\sqrt{14}$

2. $\sqrt{38}$

3. $\sqrt{7}$

4. $\sqrt{44}$

5. $\sqrt{93}$

6. $\sqrt{147}$

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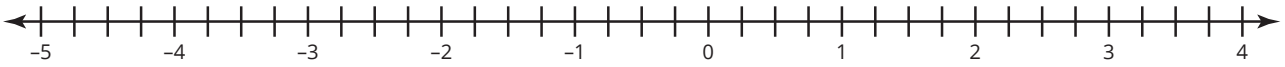
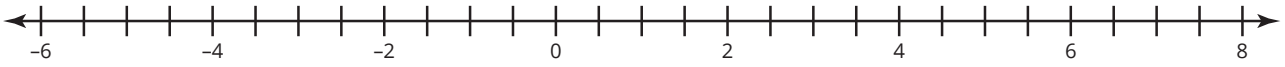
B. Plot points to represent the given numbers on each number line. Then, order the numbers from least to greatest.

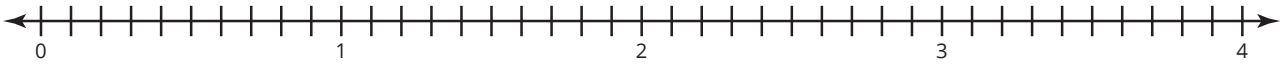
1. $92\%, \pi, -\sqrt{2}, -3\frac{8}{9}$



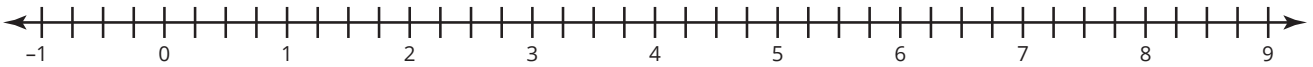
2. $-4\frac{1}{2}, \sqrt{16}, 235\%, \pi$



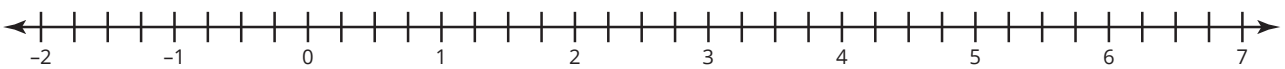
3. $143\%\ \sqrt{4}, \pi, 2.7$



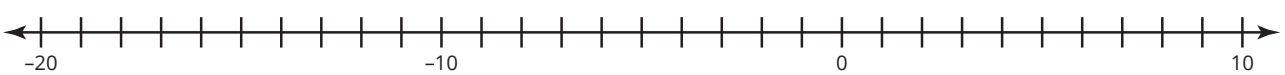
4. $\pi, 7\frac{3}{4}, 10\%, 107\%$



5. $\pi, \sqrt{2}, 5.67, \frac{5}{8}$



6. $375\%, -\sqrt{25}, -\sqrt{74}, 7.589$



C. Use the following number set to answer each question.

π, 5.65, $-\frac{1}{8}$, $\sqrt{7}$, 135%, -12, 4^2, $|-9|$, $4.\overline{56}$, -2.24×10^3, $2\frac{3}{4}$, $-\frac{18}{6}$, $\sqrt{\frac{3}{12}}$, -8^3, 95%, -0.1, 0, $\frac{2}{3}$, $|6|$, 22 , $\frac{6}{2}$, $-\sqrt{11}$, $\frac{\sqrt{3}}{4}$, 5.86348921576...

1. List the positive numbers.

2. Order the numbers from least to greatest.

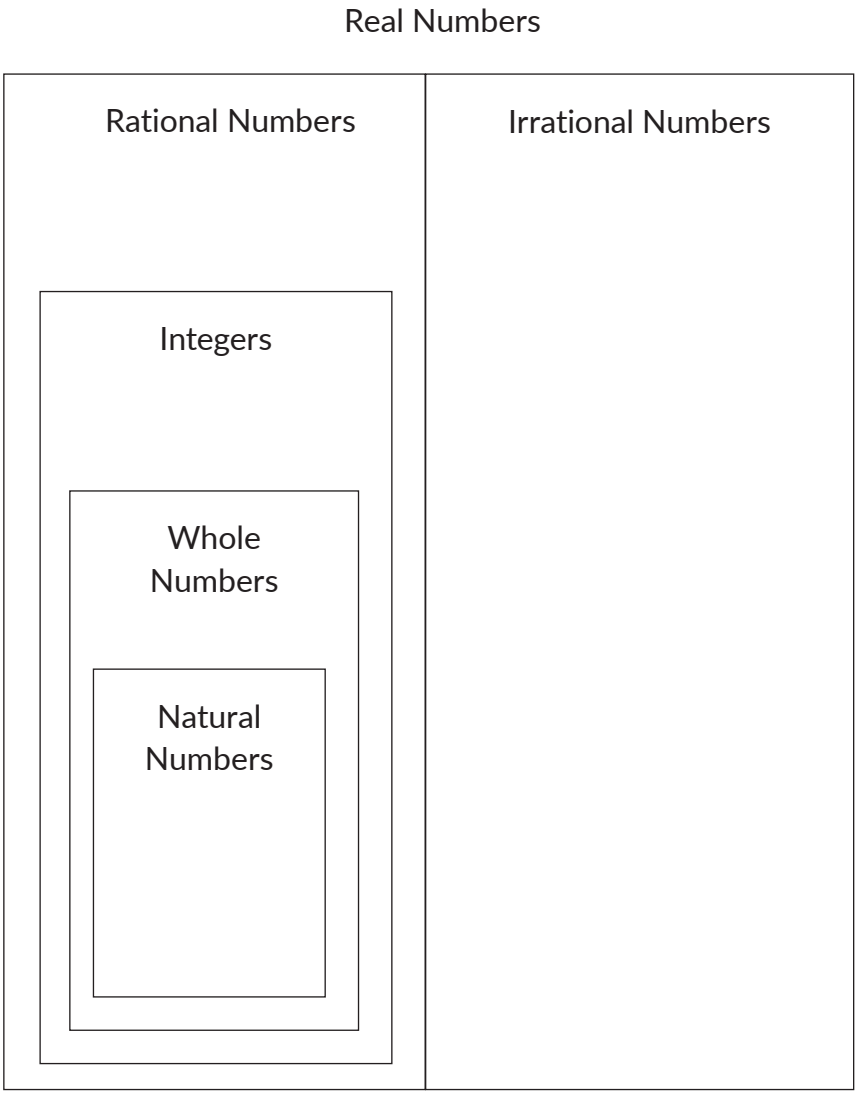
3. List the integers.

4. List the irrational numbers.

5. List the real numbers.

6. List the whole numbers.

7. The Venn diagram shows the relationship between the six sets of numbers shown. Write each number from the list in the most appropriate section of the Venn diagram.



Extension

A number called the *Champernowne constant* is an irrational number formed by placing the digits of successive integers together, like this:

0.12345678910111213141516...

What is the 100th digit of the Champernowne constant, not including the beginning zero?

Spaced Practice

1. Identify whether each statement is true or false.

   a. Some irrational numbers are rational.

   b. All whole numbers are integers.

   c. Some rational numbers are whole numbers.

   d. All real numbers are whole numbers.

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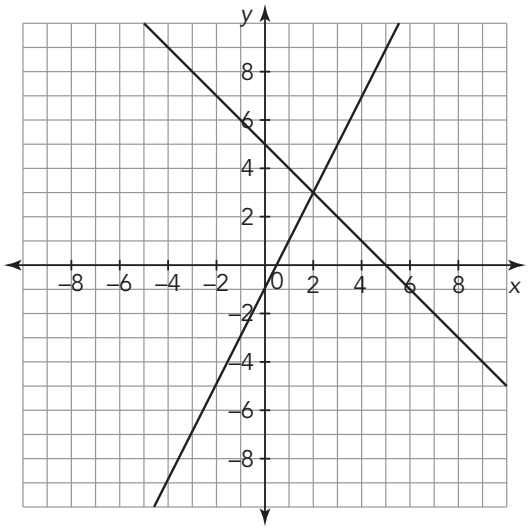
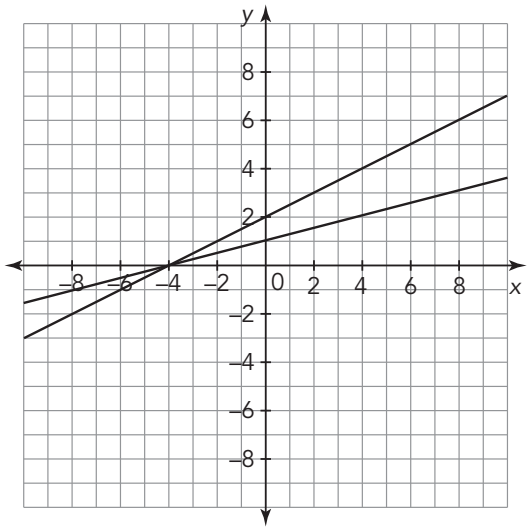
2. Identify and verify the solution for each system of equations.

a. $\begin{cases} y = 2x - 1 \\ y = -x + 5 \end{cases}$



b. $\begin{cases} y = \frac{1}{2}x + 2 \\ y = \frac{1}{4}x + 1 \end{cases}$



IV. Scientific Notation

Topic Practice

A. Write each number in scientific notation or standard notation. Show your work.

1. 45,000,000

2. 3700

3. 562,000

4. There are over 29,000 grains of long-grain rice in a one-pound bag.

5. There are about 100,000,000,000 stars in our galaxy.

6. 5.2×10^7

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7. 3.7×10^8

8. 2.871×10^{11}

9. There are about 1×10^5 strands of hair on the human head.

10. The population of Texas is about 2.4×10^7 people.

B. Write each number in scientific notation or standard notation. Show your work.

1. 0.000067

2. 0.000831

3. 0.00000000253

4. A grain of rice has a mass of about 0.025 gram.

5. The diameter of a red blood cell is about 0.00004 inch.

6. 8.3×10^{-5}

7. 6.22×10^{-9}

8. 4.7×10^{-3}

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9. An oxygen atom has a radius of about 4.8×10^{-11} meters.

10. The diameter of a white blood cell is about 1×10^{-5} meters.

C. Complete the table.

Quantity	Measurement in Standard Form	Measurement in Scientific Notation
The population of California in 2022	39,000,000	
The circumference of the Earth at the equator		4.008×10^{7}
The distance from Earth to the Moon	385,000,000	
The weight of a grain of salt	0.0000585	
The radius of a nitrogen atom		5.6×10^{-11}
The average distance between Earth and the Sun		9.3×10^{7}
The width of a human hair	0.0018	
The thickness of a credit card		2.99×10^{-2}

D. Write $>$ or $<$ to compare each pair of numbers.

1. -4.5×10^{7} ___ 4.5×10^{6}

2. 1.4×10^{-4} ___ 8.6×10^{-8}

3. 7.3×10^{-6} ___ 1.5×10^{-6}

4. 2.2×10^{12} ___ 9.5×10^{4}

5. 1.9×10^{-5} ___ 6.5×10^{-5}

6. 3.7×10^{-2} ___ 9.9×10^{-9}

7. 9.1×10^{5} ___ $3.5 \cdot 10^{3}$

8. 8.4×10^{-2} ___ 8.04×10^{-2}

9. 1.1×10^{-9} ___ 9.9×10^{1}

10. 1.1×10^{-9} ___ 9.9×10^{-1}

11. 6.4×10^{4} ___ 6.09×10^{4}

12. 1.3×10^{-7} ___ 9.1×10^{-7}

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Extension

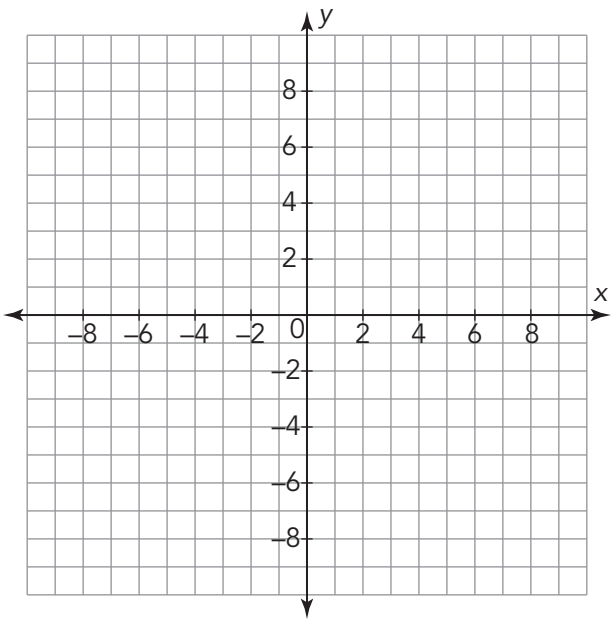
Compare each set of numbers using the appropriate symbol: $>$, $<$, or $=$.

1. 2.478×10^4 ___ 2500
2. 2.478×10^{-4} ___ 0.00025
3. 10.5^3 ___ 5×10^3
4. 0.00012378 ___ 1.3×10^{-4}

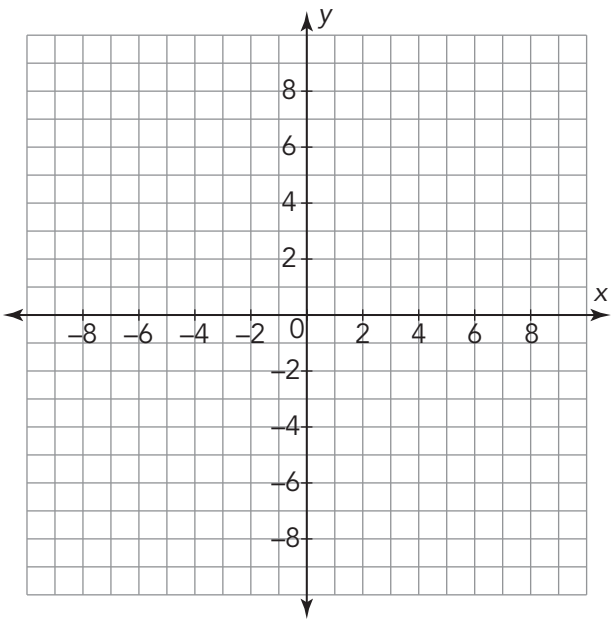
Spaced Practice

1. Graph the equations in each system. Then, use the graph to identify whether the system has one solution, no solutions, or infinite solutions. When the system has one solution, identify the solution.

a. $\begin{cases} y = 2(x - 3) \\ y = 2x - 6 \end{cases}$



b. $\begin{cases} y = 2x - 1 \\ y = -3x - 11 \end{cases}$

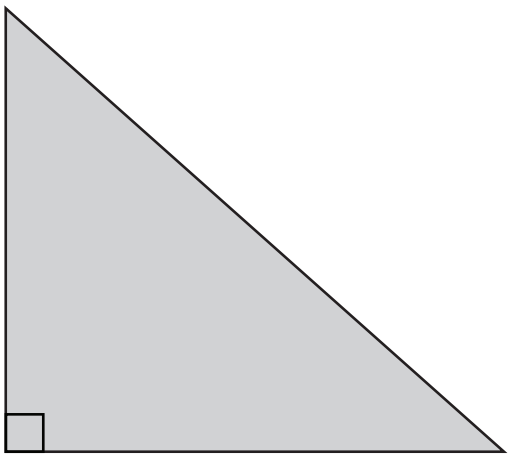


Name _____ Date _____

I. The Pythagorean Theorem

Topic Practice

A. Use the image to answer the questions.



1. What type of triangle is shown?

2. What do you call either of the two shorter sides of a right triangle?

3. What is the longest side of the triangle called?

4. Label the triangle shown with the name for each side.

5. What is the name of the theorem that could be used to determine an unknown side length on a right triangle?

6. Write the formula that can be used to determine an unknown length of a triangle like the one shown.

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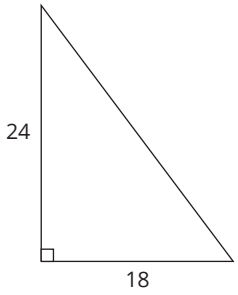
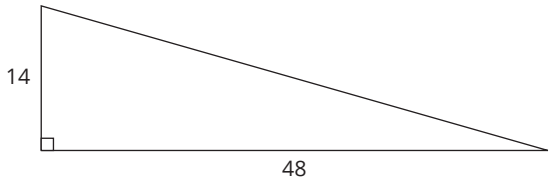
B. Calculate the length of the hypotenuse of each given triangle. Round to the nearest hundredth, when necessary. Then, determine whether the length of the hypotenuse is an irrational real number or a rational real number.

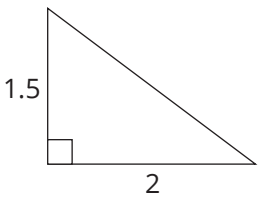
1.



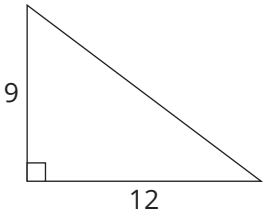
2.



3.



4.



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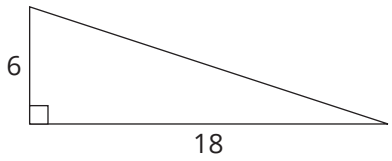

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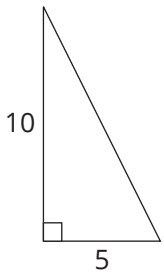

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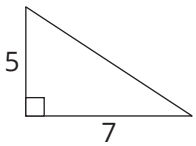

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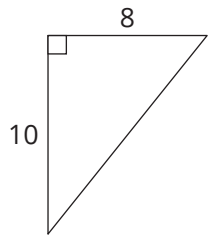
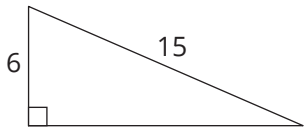
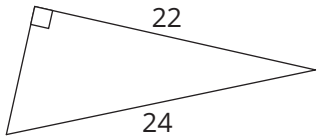
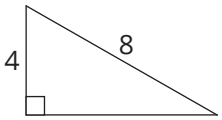
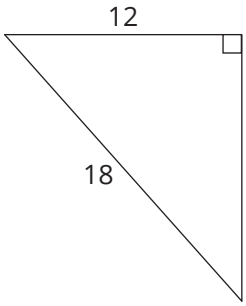


C. Calculate the length of the missing side of each given triangle. Round to the nearest hundredth, when necessary.

1. 

2. 

3. 

4. 

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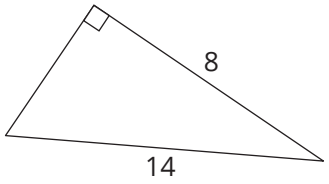
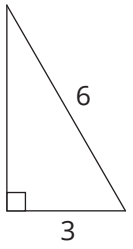
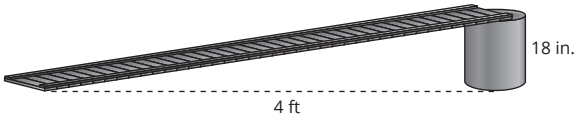


6.

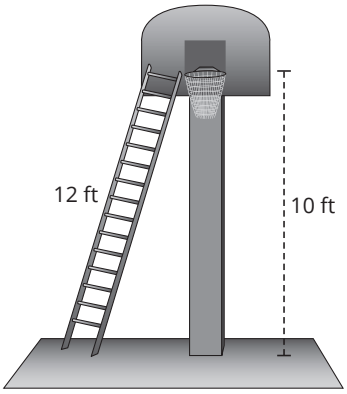


D. Use the scenario to answer each question. Round to the nearest hundredth, when necessary.

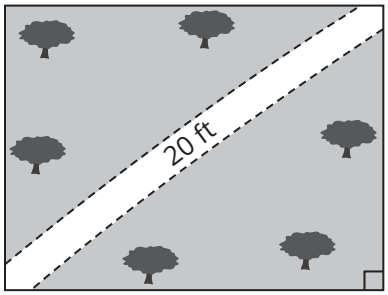
1. Gabriel is building a ramp for his remote control car. He wants the end of the ramp to extend 4 feet from the base of the ramp. The base of the ramp is 18 inches high. How long should the piece of wood for the ramp be?



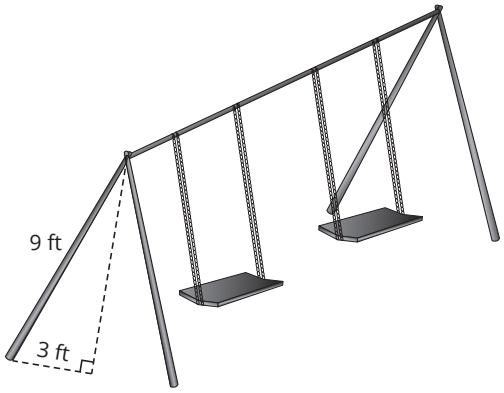
2. Diego wants to replace the net on his basketball hoop. The hoop is 10 feet high and Diego's ladder is 12 feet long. How far from the base of the basketball hoop does Diego need to place his ladder to reach the hoop?



3. Camilla wants to place a fence on the right side of her garden. How much fencing does Camilla need to buy to complete the right side of the fence?



4. Avery is helping to build a swing set at the community park. Each support beam is 9 feet long. The distance from the base of each support beam to the middle point between support beams is 3 feet. What is the height from the middle of the support beams on the ground to the top of the swing set?



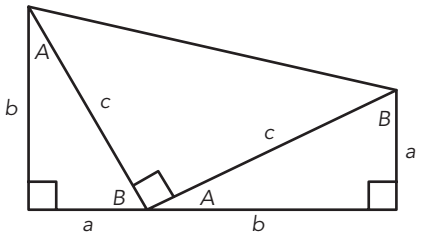
5. Logan wants to use a 12-foot ladder to reach a shelf that is 11 feet above the ground. How far from the wall should Logan place the base of the ladder so that the top of the ladder reaches the shelf?

6. Parker walks to soccer practice on Saturday. She leaves her home and walks 6 blocks north. Parker then turns east and walks 4 blocks to the soccer field. How far is the soccer field from Lea's home?

Extension

Examine President Garfield's proof of the Pythagorean Theorem.

In the figure, an arbitrary right triangle with sides of length a and b and hypotenuse of length c was drawn and copied so that $a \parallel b$. Then, an additional segment was drawn to form a trapezoid.



1. Determine the area of the trapezoid using the formula, $A = \frac{1}{2}(b_1 + b_2)h$, where b_1 and b_2 are the lengths of the parallel bases and h is the perpendicular distance between the bases.

2. Determine the area of each triangle inside the trapezoid. Determine the sum of the areas.

3. How do these area calculations prove the Pythagorean Theorem? (Hint: $(a + b)^2 = a^2 + 2ab + b^2$)

Spaced Practice

1. Estimate each radical to the nearest tenth.

   a. $\sqrt{38}$

   b. $\sqrt{14}$

2. Name all number sets to which each number belongs.

   a. $\frac{2}{3}$

   b. 5

3. Solve each equation.

   a. $4x + 3x + 12 = 2(15 + 2x)$

   b. $3(2c + 5) = 12 + 3(c + 4)$

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II. The Converse of the Pythagorean Theorem

Topic Practice

A. Determine whether each triangle with the given side lengths is a right triangle.

1. 8, 15, 17

2. 6, 9, 14

3. 12, 15, 18

4. 5, 12, 13

5. 6, 8, 10

6. 9, 12, 16

B. The side lengths of a right triangle are given. Use the given side lengths to determine another Pythagorean triple. Verify that the Pythagorean triple satisfies the equation $a^2 + b^2 = c^2$. Be sure to determine which side length is the hypotenuse.

1. 9, 12, 15

2. 5, 13, 12

3. 5, 4, 3

4. 6, 8, 10

5. 25, 15, 20

6. 8, 15, 17

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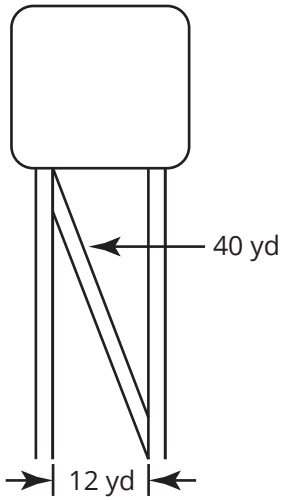
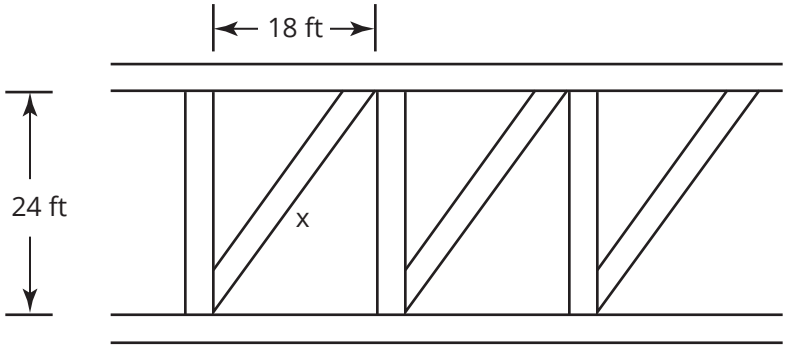
C. Answer each question using the scenario. Draw and label a diagram to represent each scenario as needed. Round your answer to the nearest hundredth, when necessary.

1. The water company installed a 40-yard diagonal brace on a water tower between two vertical beams that are 12 yards apart as shown. Determine the height of each vertical beam.



2. The design for a bridge truss is shown. The distance between the horizontal beams is 24 feet. The distance between the vertical beams is 18 feet. Determine the length, x, of each diagonal brace.



3. A computer monitor is sold by the diagonal length of the screen. A computer monitor has a 15-inch screen. The screen has a width of 13 inches. What is the height of the screen?

4. Jasmine is building a sand box in her backyard. She places four pieces of wood in a rectangle to form the frame. The rectangle is 4 feet long and 3 feet wide. How can she use a measuring tape to make sure that the corners of the frame will be right angles?

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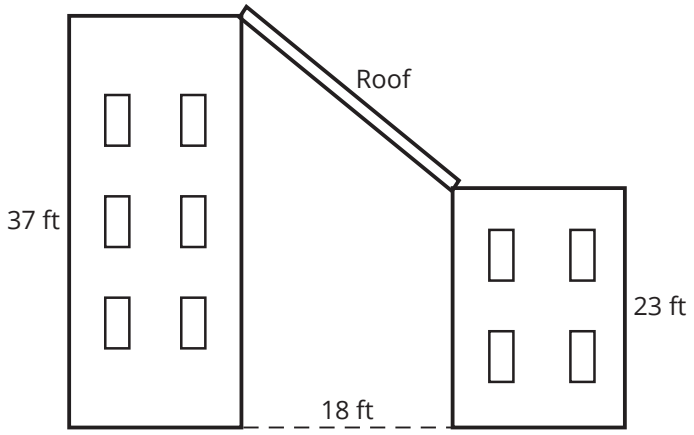
5. A lobster boat leaves a dock and travels 18 miles north to drop a trap into the water. The boat then travels 82 miles southeast to a different dock. What is the direct distance between the two docks?

6. The archery team practices on a rectangular section of the school fields. The length of the practice area is 90 feet. The archers stand in one corner of the field and shoot at a target that is diagonally across the field. The distance from where the archers are standing to the target is 103 feet. Determine the width of the school field.

7. Mason wants to add a roof to his covered patio area that sits between a 37-foot-tall building and a 23-foot-tall building. The buildings are 18 feet apart. What minimum length does the roof need to be to span the two buildings?



Extension

Euclid developed a formula for generating Pythagorean triples given any integers m and n with $m > n > 0$: $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$.

It can be proven that there are exactly eight Pythagorean triples for a right triangle with a perimeter of 840 units.

Use Euclid's Formula, your knowledge of perimeter and algebra, and number sense to determine as many of the eight Pythagorean triples with a perimeter of 840 units as you can.

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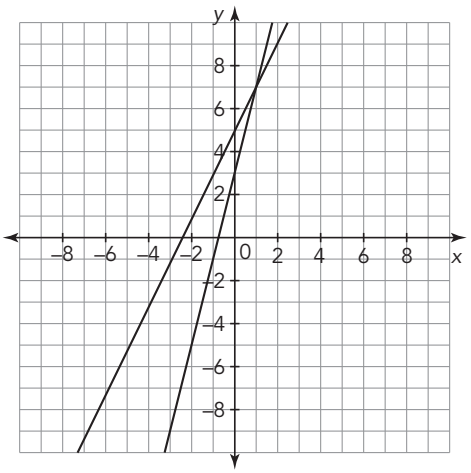
Spaced Practice

1. A carpenter props a ladder against the wall of a building. The base of the ladder is 10 feet from the wall. The top of the ladder is 24 feet from the ground. How long is the ladder?

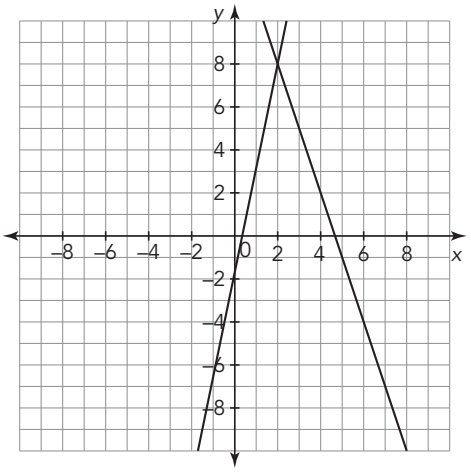
2. The length of the hypotenuse of a right triangle is 50 inches. Determine the length of the legs when each leg is the same length.

3. Use the graph to identify the solution to the system of equations. The, verify the solution algebraically.

   a. $\begin{cases} y = 4x + 3 \\ y = 2x + 5 \end{cases}$



   b. $\begin{cases} y = -3x + 14 \\ y = 5x - 2 \end{cases}$

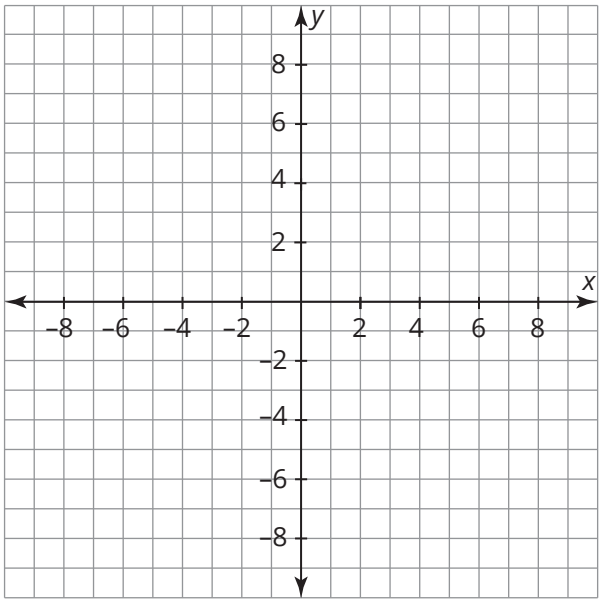


III. Distances in a Coordinate System

Topic Practice

A. Determine the distance between each given pair of points by graphing and connecting the points, creating a right triangle, and applying the Pythagorean Theorem. Round to the nearest hundredth, when necessary.

1. (2, 2) and (8, 5)



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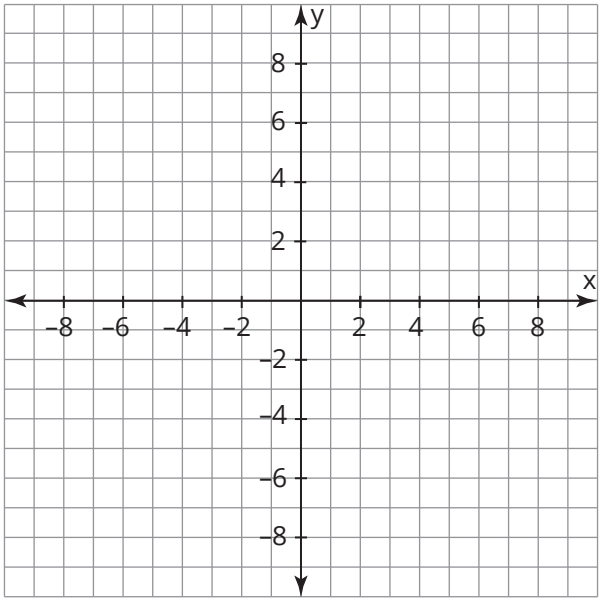
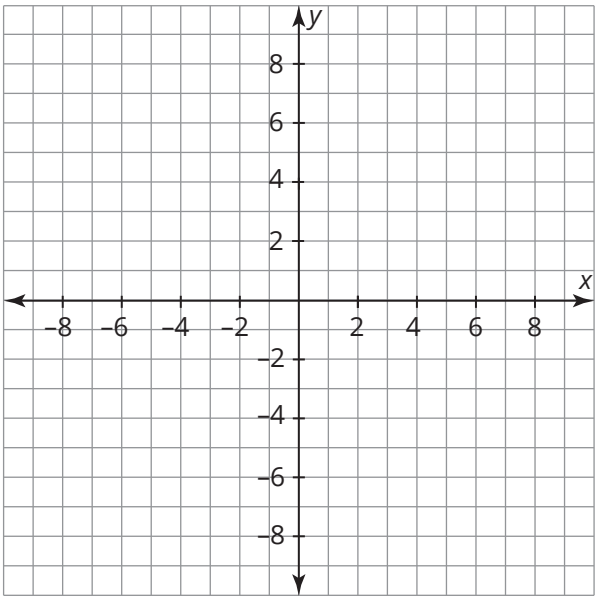
2. (3, 7) and (7, 3)



3. $(-6, 8)$ and $(6, 3)$



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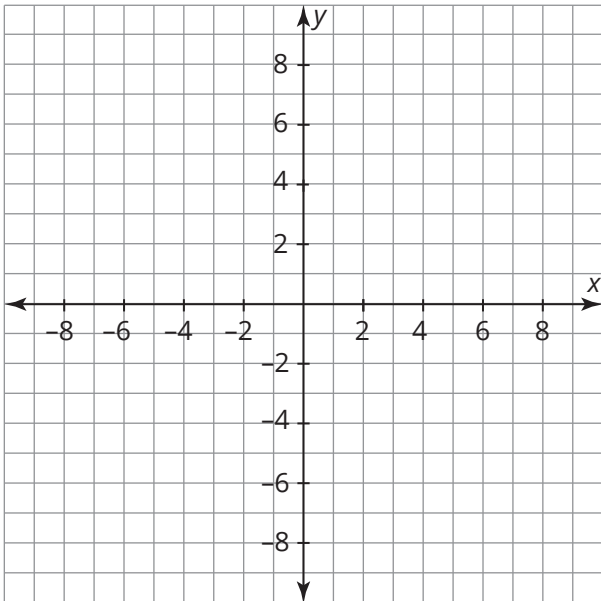
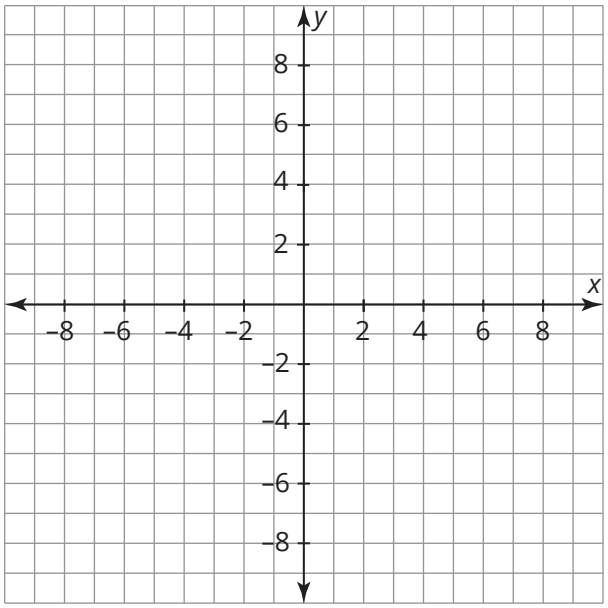
4. (7, 5) and (3, −3)



5. (−4, −4) and (5, 8)



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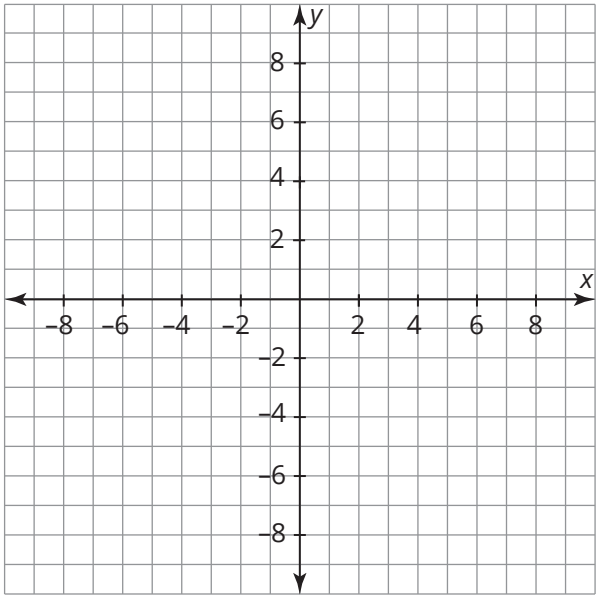
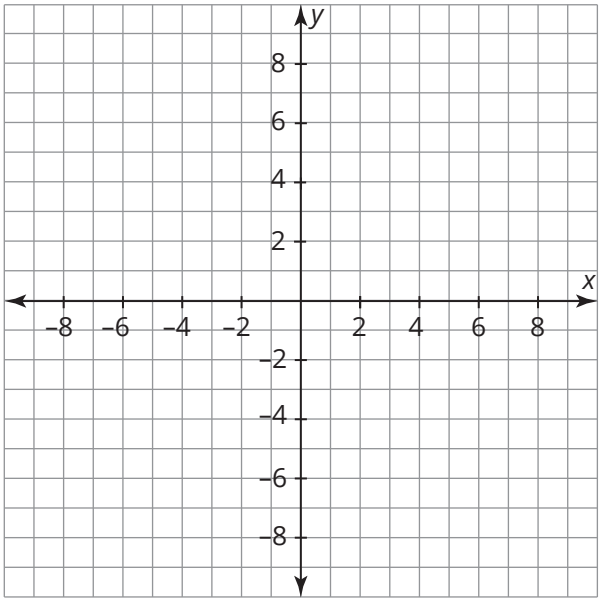
6. (−9, 3) and (7, 5)



7. (−7, 3) and (8, −5)



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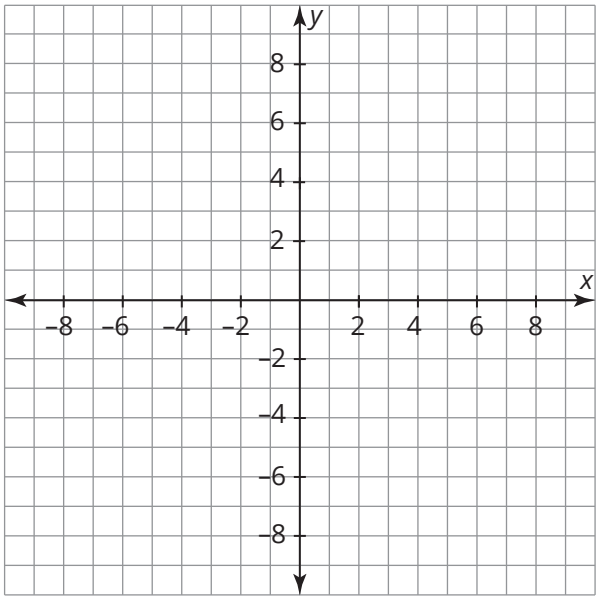
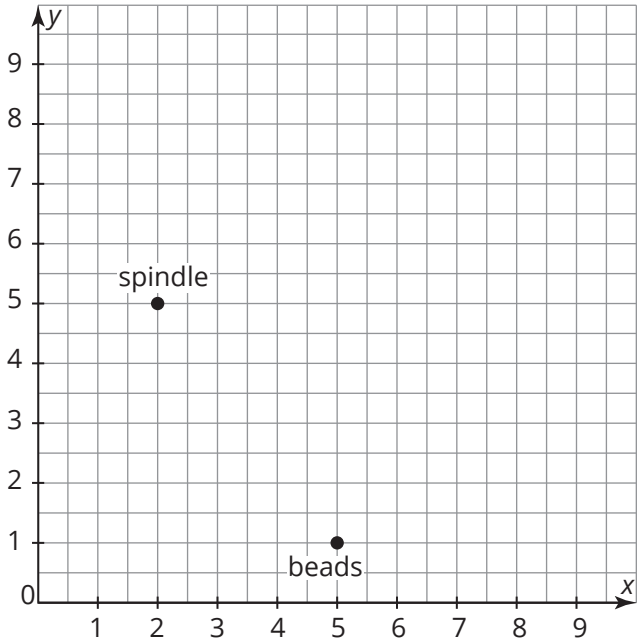
8. (−9, 6) and (8, 1)

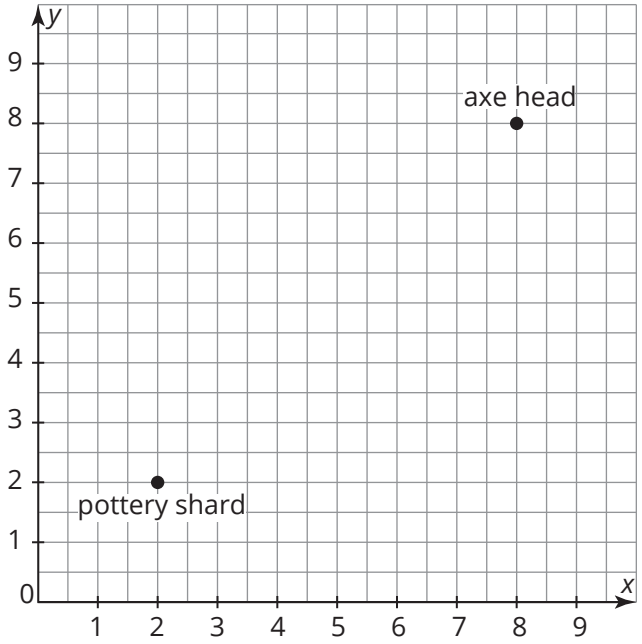


B. Archaeologists map each item they find at a dig on a 1-foot by 1-foot coordinate grid. Calculate the distance between the given pair of objects on the coordinate grid. Round to the nearest hundredth, when necessary.

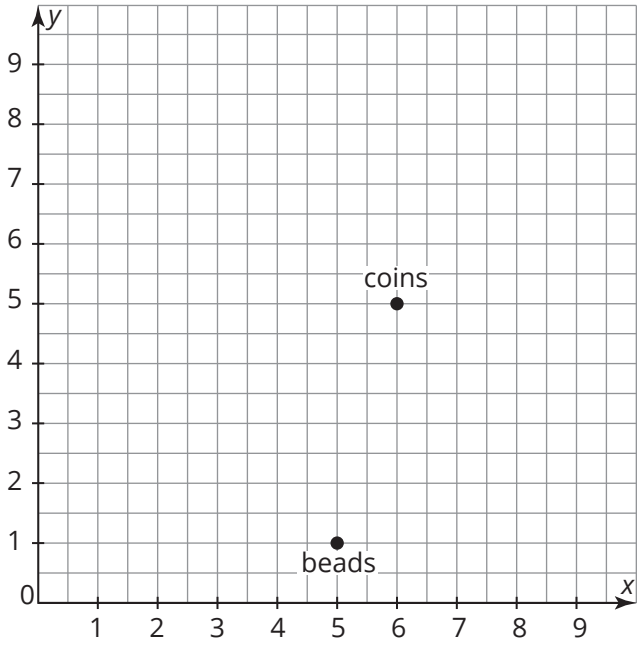
1. Determine the distance between the spindle and the beads.



2. Determine the distance between the pottery shard and the axe head.



3. Determine the distance between the coins and the beads.



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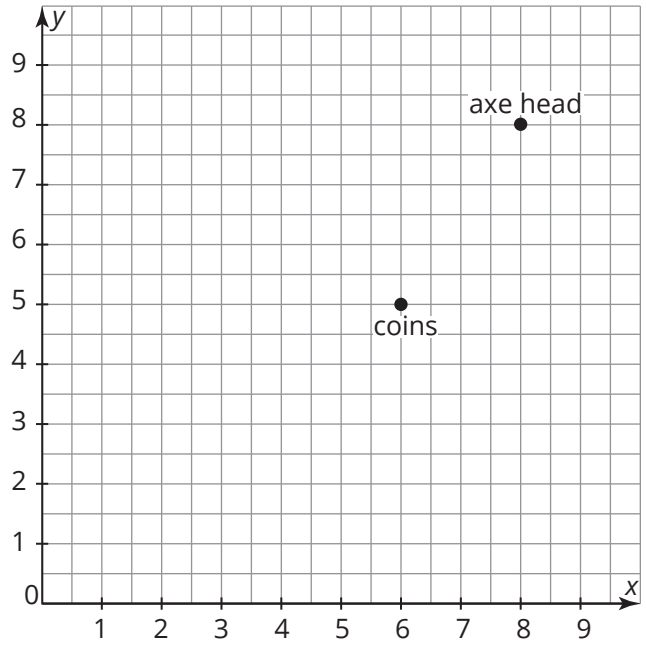
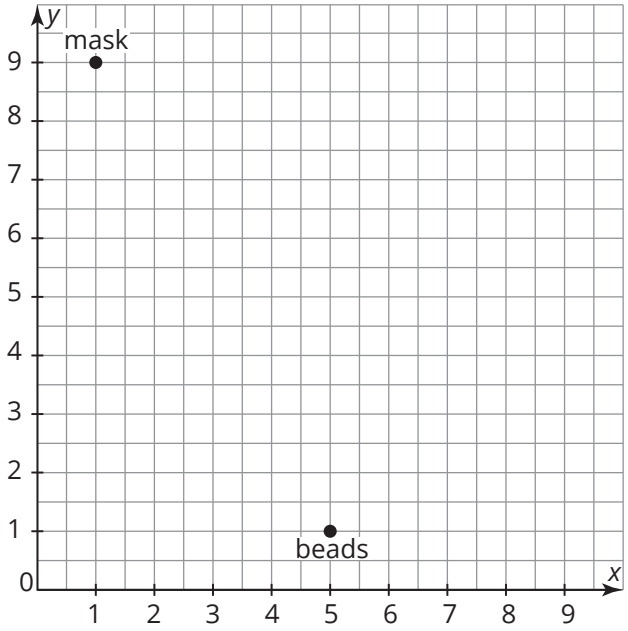
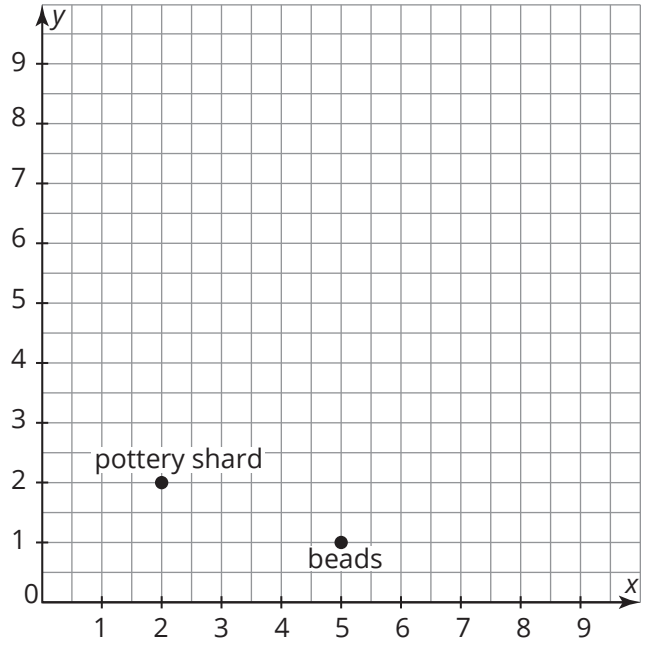
4. Determine the distance between the coins and the axe head.



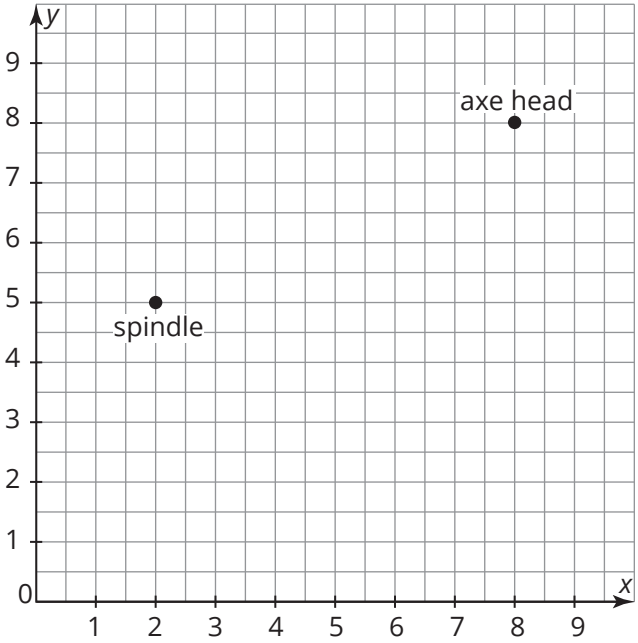
5. Determine the distance between the mask and the beads.



6. Determine the distance between the pottery shard and the beads.



7. Determine the distance between the spindle and the axe head.



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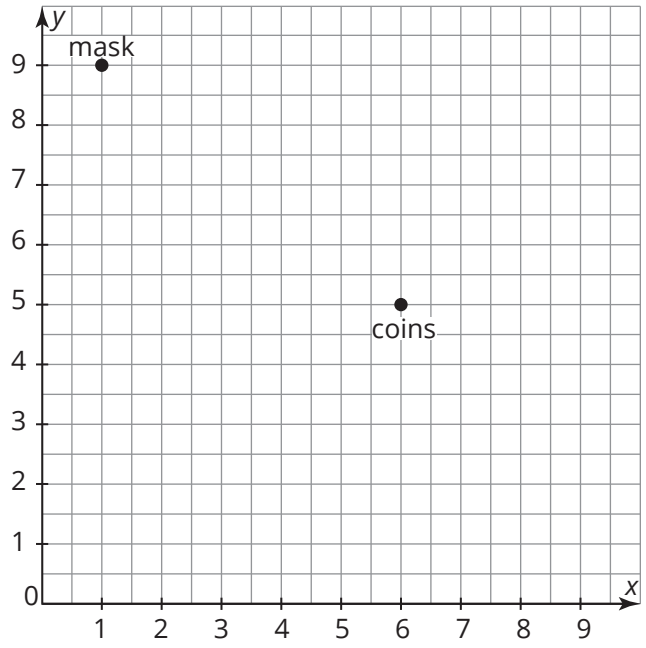
8. Determine the distance between the mask and the coins.



Extension

A right triangle with an area of 6 units is plotted on the coordinate plane. One of the endpoints of the hypotenuse is located at $(-1, 1)$. Where could the other endpoint be located?

Spaced Practice

1. Determine whether the triangle with the given side lengths is a right triangle.

   a. 105, 175, 140

   b. 36, 49, 64

2. Determine whether each number is rational or irrational.

   a. $-\frac{1}{6}$

   b. $\sqrt{81}$

   c. $\sqrt{19}$

   d. 0.3030030003 . . .

3. The area of a square garden is approximately 26 feet. What is the approximate length of the side of the garden to the nearest tenth? Explain your reasoning.

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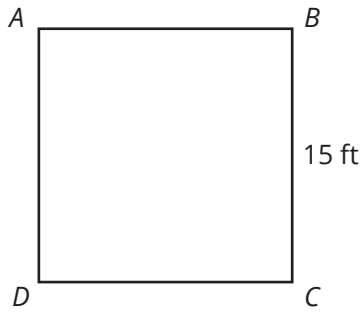
IV. Side Lengths in Two and Three Dimensions

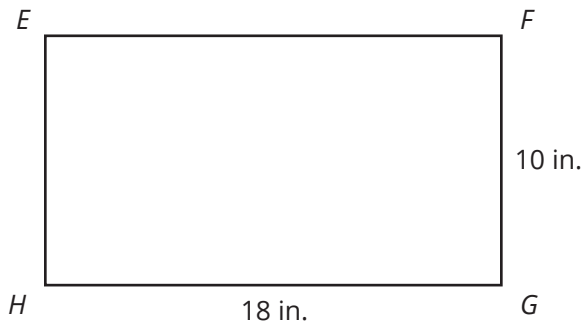
Topic Practice

A. Determine the length of the diagonals in each given quadrilateral. Round to the nearest hundredth, when necessary.

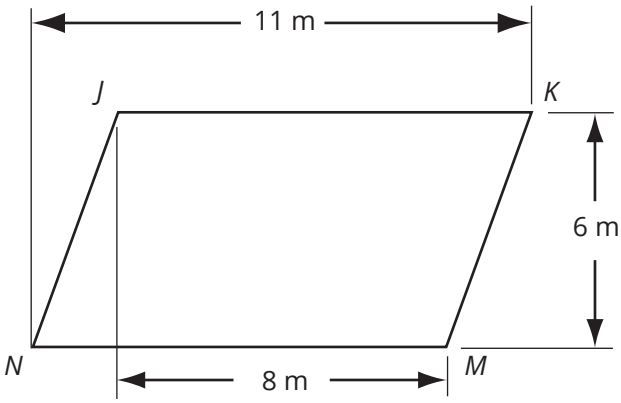
1. The quadrilateral is a square.



2. The quadrilateral is a rectangle.



3. The quadrilateral is a parallelogram.



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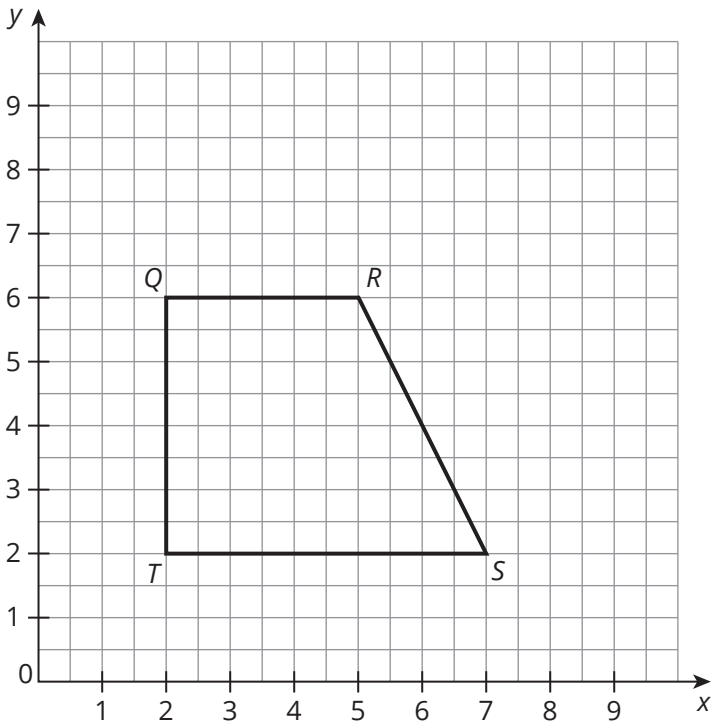
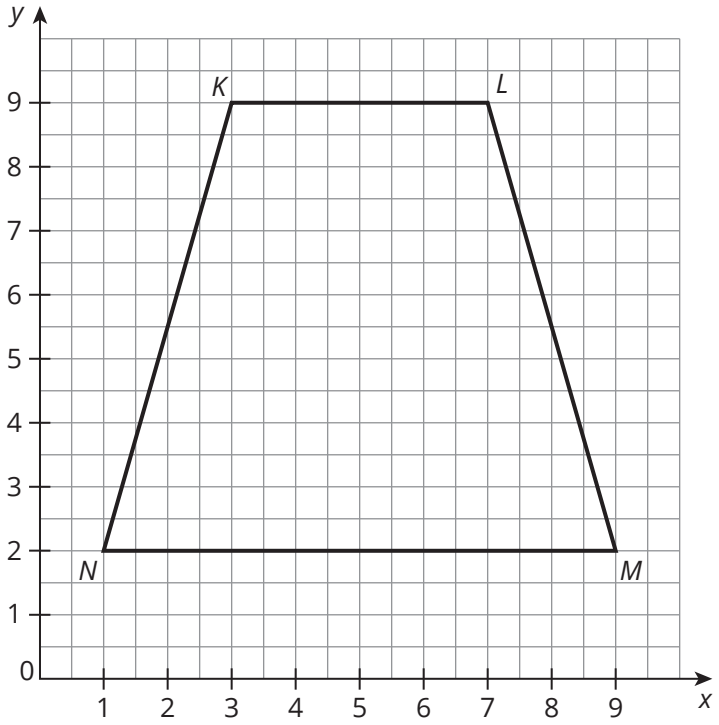
4. The quadrilateral is a trapezoid.



5. The quadrilateral is an isosceles trapezoid.



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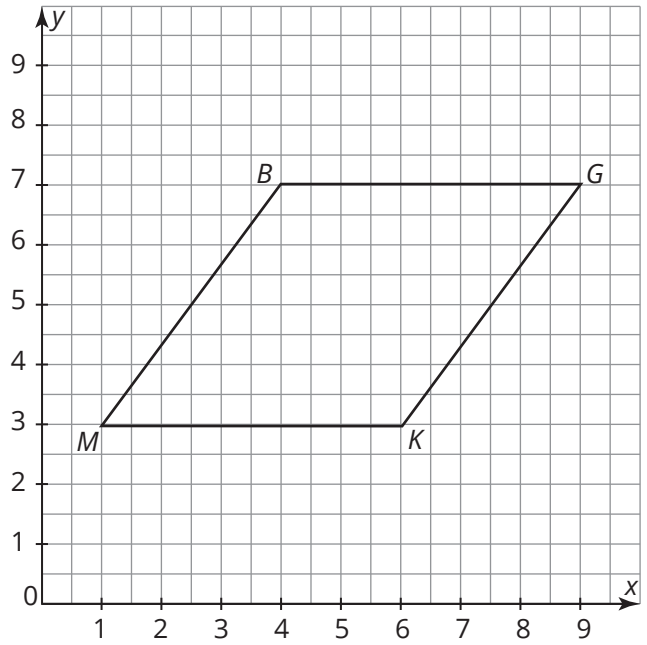
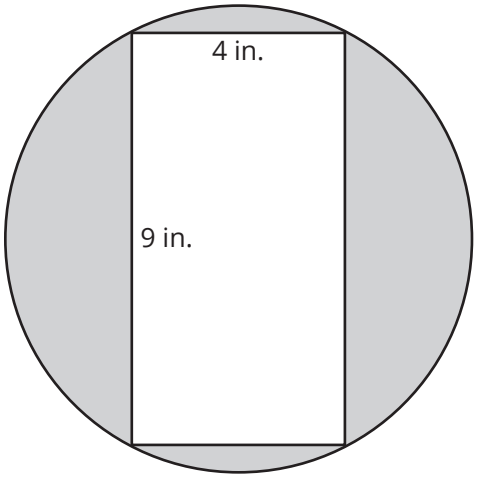
6. The quadrilateral is a rhombus.



B. Calculate the area of each shaded region. Use 3.14 for π. Round to the nearest hundredth, when necessary.

1. The figure is composed of a circle and a rectangle. The diagonal of the rectangle is the same length as the diameter of the circle.



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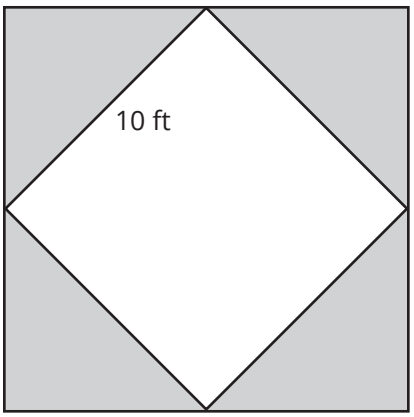
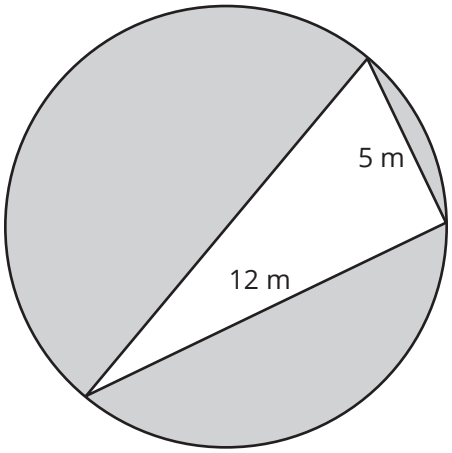
2. The figure is composed of two squares. The length of the diagonal of the smaller square is equal to the width of the larger square.



3. The figure is composed of a right triangle and a circle. The hypotenuse of the right triangle is the same length as the diameter of the circle.



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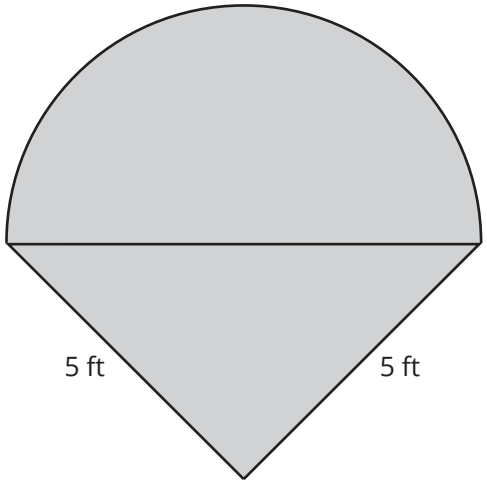
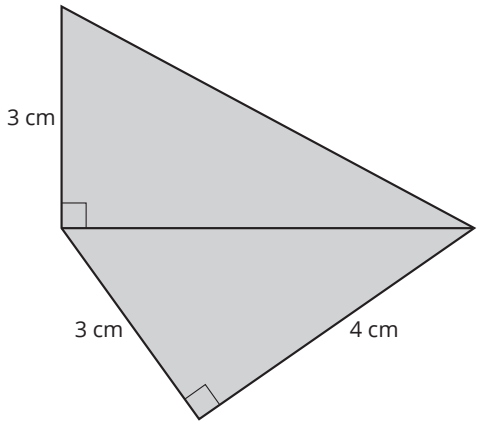
4. The figure is composed of a right triangle and a semi-circle. The hypotenuse of the right triangle is the same length as the diameter of the semi-circle.



5. The figure is composed of two right triangles. The hypotenuse of one right triangle is the leg of the other right triangle.



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Extension

Norton thought he knew a shortcut to determine the length of a three-dimensional diagonal. He said, "All you have to do is calculate the sum of the squares of the rectangular solid's three perpendicular edges (the length, the width, and the height), and that sum would be equivalent to the square of the three-dimensional diagonal." Does this work? Explain your reasoning.

Spaced Practice

Determine the distance between each pair of points.

1. $(-9, -5), (3, 12)$
2. $(5, 5), (1, -10)$

Use the terms *rational*, *irrational*, *integer*, and *counting number* to describe each number.

3. $-\sqrt{100}$
4. $\frac{75}{4}$

Estimate each square root. Round to the nearest tenth.

5. $\sqrt{27}$
6. $\sqrt{75}$

Identify whether each situation represents a proportional or non-proportional relationship.

7. The amount a babysitter, who makes \$12.00 per hour, earns in h hours
8. The weight of a kitten in w weeks, when the kitten gains 1.5 pounds per week and has a starting weight of 7 pounds

Name _____ Date _____

I. Simple and Compound Interest

Topic Practice

A. Complete each table. Then, identify the account as simple interest or compound interest. Explain your reasoning.

1.

Time (years)	Total Amount (dollars)
0	5000
1	5750
2	6500
3	
4	

2.

Time (years)	Total Amount (dollars)
0	2750
1	3506.25
2	4470.47
3	
4	

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3.

Time (years)	Total Amount (dollars)
0	1250
1	1312.50
2	1375
3	
4	

4.

Time (years)	Total Amount (dollars)
0	9500
1	9927.50
3	10,782.50
5	
8	

5.

Time (years)	Total Amount (dollars)
0	10,000
1	10,600
3	11,910.16
5	
7	

6.

Time (years)	Total Amount (dollars)
0	4000
1	4100
5	4525.63
10	
15	

B. Determine the total value for each account given the principal, rate, and time.

1. Simple interest account:

   Principal: $4750

   Rate: 2.5% annual

   Time: 9 years

2. Compound interest account:

   Principal: $2000

   Rate: 3% annual

   Time: 4 years

3. Compound interest account:

   Principal: $8000

   Rate: 5% annual

   Time: 12 years

4. Simple interest account:

   Principal: $4550

   Rate: 4.5% annual

   Time: 15 years

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5. Simple interest account:

   Principal: $9750

   Rate: 2.25% annual

   Time: 10 years

6. Compound interest account:

   Principal: $105,000

   Rate: 4.75% annual

   Time: 8 years

C. Determine how much more money is made in a compound interest account than a simple interest account for each given principal, rate, and time.

1. You wish to invest $1250. How much more money is made from $1250 invested in a 3.25% annual compound interest account over 5 years, compared to investing the money in a simple interest account?

2. You wish to invest $9500 to save for college. How much more money is made from $9500 invested in a 5% annual compound interest account over 12 years, compared to investing the money in a simple interest account?

3. You wish to invest $12,250 to save for retirement. How much more money is made from $12,250 invested in a 2.25% annual compound interest account over 30 years, compared to investing the money in a simple interest account?

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4. You wish to invest $7500 to save for college. How much more money is made from $7500 invested in a 1.5% annual compound interest account over 9 years, compared to investing the money in a simple interest account?

5. You wish to invest $15,000. How much more money is made from $15,000 invested in a 4.25% annual compound interest account over 18 years, compared to investing the money in a simple interest account?

6. You wish to invest $10,250 to save for retirement. How much more money is made from $10,250 invested in a 6% annual compound interest account over 25 years, compared to investing the money in a simple interest account?

Extension

A recent study found that teens between the ages of 16 and 18 years old have an average of $966 in savings. Suppose that you currently have $500 in savings. Create two different fictitious investment options, one that includes simple interest and one that include compound interest. Both options should result in a total balance between $950 and $1050 by the time you are 20 years old. For each investment option include the interest rate, time in years, and a graph that shows the balance over time.

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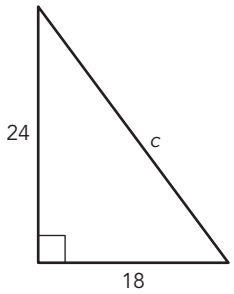
Spaced Practice

Use the Pythagorean Theorem to determine each unknown measure.
Round your answer to the nearest hundredth, when necessary.

1.



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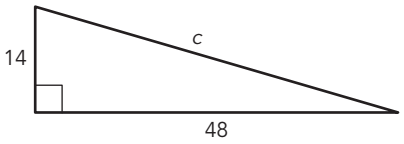

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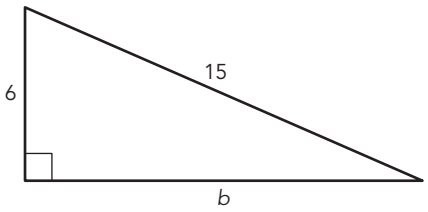

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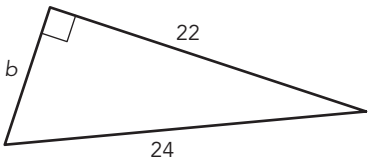

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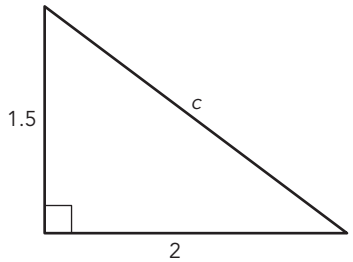

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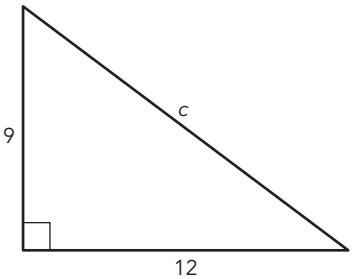

II. Terms of a Loan

Topic Practice

A. Compare each set of loans. Determine which is the better loan. Then, determine the difference in payment amounts between the two loans.

1.

Loan 1	Loan 2
• Loan Amount: $220,000 • Interest Rate: 3% interest • Time: 15 years • Payment: $1519.28/month	• Loan Amount: $220,000 • Interest Rate: 5% interest • Time: 15 years • Payment: $1739.75/month

2.

Loan 1	Loan 2
• Loan Amount: $12,500 • Interest Rate: 2% interest • Time: 8 years • Payment: $141.01/month	• Loan Amount: $12,500 • Interest Rate: 2.5% interest • Time: 8 years • Payment: $143.80/month

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3.

Loan 1

- Loan Amount: $750,000
- Interest Rate: 5.5% interest
- Time: 20 years
- Payment: $5159.15/month

Loan 2

- Loan Amount: $750,000
- Interest Rate: 4.5% interest
- Time: 20 years
- Payment: $4744.87/month

4.

Loan 1

- Loan Amount: $525,000
- Interest Rate: 1.25% interest
- Time: 10 years
- Payment: $4656.41/month

Loan 2

- Loan Amount: $525,000
- Interest Rate: 1.5% interest
- Time: 10 years
- Payment: $4714.05/month

5.

Loan 1

- Loan Amount: $20,000
- Interest Rate: 3% interest
- Time: 12 years
- Payment: $165.56/month

Loan 2

- Loan Amount: $20,000
- Interest Rate: 2.25% interest
- Time: 12 years
- Payment: $158.61/month

6.

Loan 1

- Loan Amount: $65,000
- Interest Rate: 4% interest
- Time: 18 years
- Payment: $422.63/month

Loan 2

- Loan Amount: $65,000
- Interest Rate: 4.5% interest
- Time: 18 years
- Payment: $439.61/month

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B. Use the given loan information to solve each problem situation.

1. When Emily graduated, her student loan balance was $45,000 with a 5% compound interest rate. After graduation she deferred payment for a year until she gets a full-time job. Determine the new balance after the deferment.

2. Avery just graduated from college. He has a student loan balance of $62,000 at a 6.5% compound interest rate to pay back, but decides to defer his loans for two years until he gets settled into his adult life. Determine the new balance after the deferment.

3. At the time of college graduation, Parker had a student loan balance of $26,000 with a 4.5% compound interest rate. After graduation, he decided to defer his loans for one year so he can spend his first year's salary on rent. Determine the additional interest added because of the deferment.

4. Camilla attended college out of state. Her student loan balance at graduation was $75,500 at a compound interest rate of 7%. After graduation she started working a part-time job and decided to defer her loans until she gets a better job. After a year and a half she began repaying her loans. Determine the additional interest added because of the deferment.

5. Gabriel graduated from a four-year college with a student loan balance of $32,250 at a 4% compound interest rate. After college, he joined the military and deferred the loans for four years. Determine the new balance of Gabriel's loans after the deferment.

6. Sarah accumulated $38,250 in student loans with a compound interest rate of 5%. She deferred her payments for two years because of unemployment. Determine the additional interest added because of the deferment.

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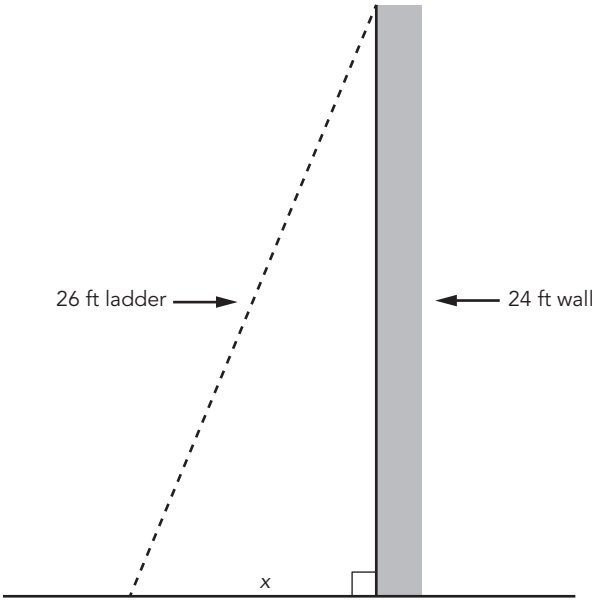
Extension

Kayla and Mei are each utilizing a seven-year loan to purchase a new car for $25,650. Kayla shares that her loan has an interest rate of 3.2%. Mei does not remember the interest rate on her loan, but she knows that her monthly payments are $390.86. For each person, determine their interest rate, monthly payment, amount paid in interest, and total amount paid over the life of the loan. Who has the better terms on their loan? Explain your answer.

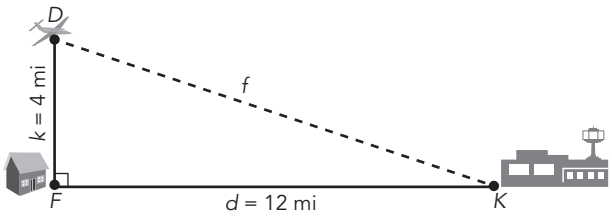
Spaced Practice

Solve each problem.

1. A carpenter needs to do some repairs to the top of a 24-foot wall. He wants the tip of the ladder to rest on the top of the wall. How far must he place his 26-foot ladder from the base of the wall?



2. An airplane is flying directly over your house. The distance between your house and the airport is 12 miles. The altitude of the airplane is 4 miles. Determine the distance between the airplane and the airport to the nearest hundredth of a mile.



3. Write each number in scientific notation.

   a. 680,000,000,000

   b. 0.000000432

4. Write each number in standard notation.

   a. 1.03×10^{-9}

   b. 4.5×10^{6}

III. Online Calculators

Topic Practice

A. Use an online calculator to answer each question about credit card charges. Round each answer to the nearest whole number.

1. Total amount charged: $6000

   Interest Rate: 14%

   Time: 9 months

   Determine the cost of the monthly payments for this charge.

2. Total amount charged: $12,000

   Interest Rate: 10%

   Monthly payment: $450

   Determine how long it will take to pay off the bill.

3. Total amount charged: $6000

   Interest Rate: 14%

   Monthly payment: $300

   Determine the amount of interest paid on the charges.

4. Total amount charged: $6525

   Interest Rate: 18%

   Time: 2 years

   Determine the cost of the monthly payments for this charge.

5. Total amount charged: $2250

   Interest Rate: 16%

   Monthly payment: $135

   Determine the total cost of the credit card purchase.

6. Total amount charged: $8350

   Interest Rate: 20%

   Time: 18 months

   Determine the total cost of the credit card purchase.

B. Use an online calculator to answer each question regarding a credit charge and a cash advance.

1. Total amount charged: $400

   Cash advance charge: 5%

   Interest Rate: 20%

   Time: 6 months

   Determine the cost of the monthly payments for this charge.

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2. Total amount charged: $1200
   Cash advance charge: 4.5%
   Interest Rate: 18%
   Monthly payments: $100
   Determine how long it will take to pay off this charge.

3. Total amount charged: $1430
   Cash advance charge: 4%
   Interest Rate: 24%
   Time: 10 months
   Determine the cost of the monthly payments for this charge.

4. Total amount charged: $1520
   Cash advance charge: 5%
   Interest Rate: 22%
   Monthly payments: $250
   Determine the amount of interest paid on this charge.

5. Total amount charged: $5450

   Cash advance charge: 5%

   Interest Rate: 20%

   Time: 2 years

   Determine the total cost of this credit card purchase.

6. Total amount charged: $6500

   Cash advance charge: 5.5%

   Interest Rate: 25%

   Monthly payments: $400

   Determine the total cost of this credit card purchase.

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C. Use an online calculator to answer each college loan question.

1. Loan Amount: $15,000

   Interest Rate: 5%

   Time: 15 years

   Determine the monthly payments for this loan.

2. Loan Amount: $22,000

   Interest Rate: 5.5%

   Time: 20 years

   Determine the monthly payments for this loan.

3. Loan Amount: $65,500

   Interest Rate: 6%

   Time: 15 years

   Determine the total cost of the loan.

4. Loan Amount: $84,500

   Interest Rate: 7%

   Time: 20 years

   Determine the total cost of the loan.

5. Loan Amount: $112,000

   Interest Rate: 6.25%

   Time: 25 years

   Determine the annual salary necessary to repay this loan.

6. Loan Amount: $96,750

   Interest Rate: 4.5%

   Time: 20 years

   Determine the annual salary necessary to repay this loan.

D. Stephanie had an unexpected home repair and is trying to determine the best option to cover the expense.

1. What are the advantages and disadvantages of Valentina paying with cash?

2. What are the advantages and disadvantages of Valentina paying with a check?

3. What are the advantages and disadvantages of Valentina paying with a credit card?

4. What are the advantages and disadvantages of Valentina paying with a debit card?

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5. What are the advantages and disadvantages of paying with a credit card cash advance?

6. What are the advantages and disadvantages of paying with an easy access loan cash advance?

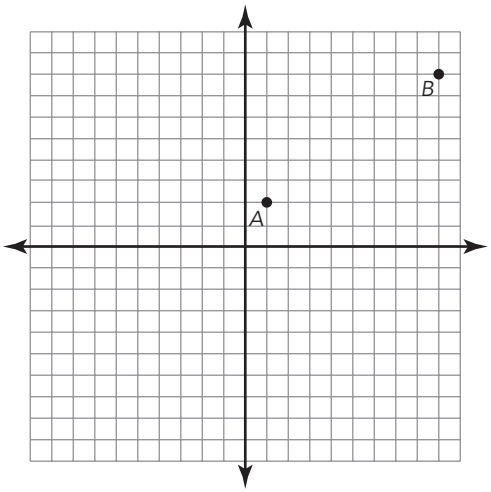
Extension

In 2018, the average American spent $335 shopping online, typically using a credit card to make their purchases. Research the interest rates on three different credit cards. Use an online payoff calculator to determine the monthly payment and amount paid in interest for a balance of $335 paid off in 12 months. Create a table to display the information for all three cards.

Spaced Practice

Calculate the distance from A to B in each diagram. Write your answer as a radical, when necessary.

1.



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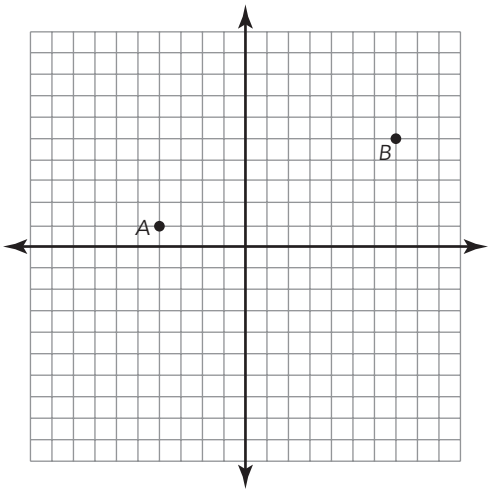

IV. Financing Your Education

Topic Practice

A. Identify each statement as true or false. Explain your reasoning.

1. Malik wants to attend college after finishing high school. He takes courses according to the Recommended High School Program that he received from his guidance counselor.

2. Amir took one year of Italian during his sophomore year of high school. He doesn't plan to take a foreign language again. Jorge doesn't think it will affect his chances of getting into the college of his choice.

3. Madison is taking an Advanced Placement Biology course to earn college credits so that she can get a head start on her degree.

4. Angel is getting ready to graduate from high school. She has no idea what career path she wants to pursue. Angel tells her friends that since she doesn't know what she wants to do, she can't start college next year.

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5. Koda is making plans for his post-secondary education. He wants to attend a 2-year vocational school and become an electrician. Koda's guidance counselor tells him this is an excellent option to pursue after high school.

6. Ana is having difficulty passing her high school math courses. She knows she may have to take an additional math course before starting college.

B. Calculate the estimated cost of each education.

1. Trung plans to attend a local community college for her first year of post-secondary education. The cost of tuition for the year is $3929. She plans to transfer to a state university for the last three years of college. The tuition at the state university is $8760 each semester. What will Trung pay in tuition to attend college for four years?

2. Eduardo is planning to attend a private university for four years. The cost of tuition is $22,630 per year. He plans to live in the campus dorms, which will cost $4190 per semester. Eduardo is eligible for an academic scholarship of $5600 per semester should he maintain a GPA of 3.5 or higher. What will it cost for Eduardo to attend college and live on campus for four years should he be able to meet the GPA requirement?

3. Mariana plans to attend a vocational school for two years after high school. The cost of tuition is $885 per semester. It will cost Mariana $325 per semester for books/supplies. She is able to participate in a work study program to receive $430 per semester toward her tuition. What will Mariana pay to attend school and pay for her books and supplies should she do the work study program?

4. Ashley is going to attend a two-year public community college next year. The tuition is $2370 per semester plus $385 per semester for books. Ashley decides to live at home so that she won't have to pay for housing. What will it cost for Ashley to attend community college and pay for her books for two years?

5. Josh plans to attend an independent university for four years. The cost of tuition is $23,044 per year. Additional costs include $6850 per year for housing and $290 per semester for books. Josh receives a swimming scholarship for half of his yearly tuition each year. How much will it cost for Josh to attend the university for four years?

6. Alyssa plans to attend vocational school for two years and then transfer to a state college for her last two years. The vocational school costs $1879 per year for tuition and the state college costs $12,350 per semester. Alyssa plans to live on campus at the state college. It will cost $5210 per year. How much should Alyssa plan to pay for college and housing?

C. Identify whether each statement is true or false. Explain your reasoning.

1. You cannot work while you are in college because attending post-secondary school is a full-time job.

2. As a Texas resident, you can withdraw money from your Education IRA to pay for post-secondary school.

3. By filling out the FAFSA, you may be eligible for loans, grants, and work-study funds each year.

4. Texas families can use the Texas Tuition Promise Fund to save money for college.

5. You should shop around for the best loan terms when you are planning for college.

6. Texas residents pay less to attend a public university than students from out-of-state.

D. Calculate each amount.

1. Jorge is going to attend a four-year public community college next year. The tuition is $5200 per year plus $425 per semester for books. Jorge decides to live at home so he won't have to pay for housing. What will it cost for Jorge to attend community college and pay for his books for four years?

2. You wish to invest $4000. How much more money is made from $4000 invested in a 2.85% annual compound interest account over 20 years, compared to investing the money in a simple interest account?

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3. Catalina plans to attend a private university for four years. The cost of tuition is $31,450 per year. Additional costs include $7500 per year for housing and $375 per semester for books. How much will it cost for Catalina to attend the university for four years?

4. You wish to invest $7500. How much more money is made from $7500 invested in a 3.5% annual compound interest account over 15 years, compared to investing the money in a simple interest account?

5. Mario plans to attend a private college for four years after high school. The cost of tution is $13,890 per semester. He plans to live in the campus dorms, which will cost $3560 per semester. After his two semesters, Mario plans to become a resident advisor so he can live in the campus dorms for free. What will it cost for Mario to attend college and live on campus for four-years should he become a resident advisor after his first two semesters?

6. You wish to invest $12,000. How much more money is made from $12,000 invested in a 7.5% annual compound interest account over 10 years, compared to investing the money in a simple interest account?

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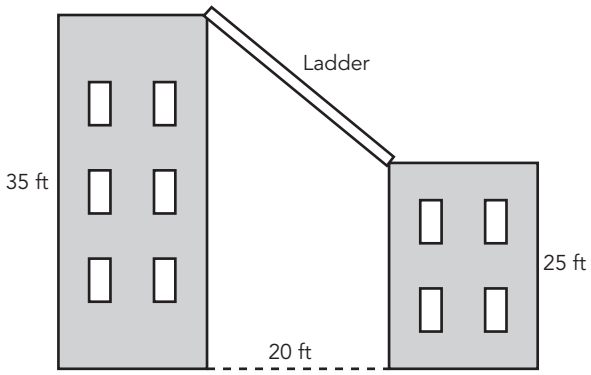
Extension

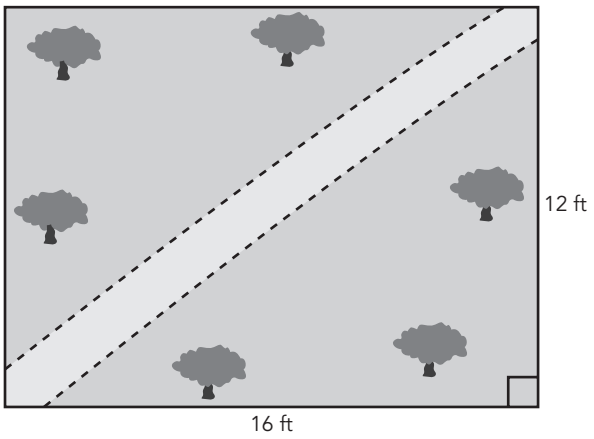
Use an online scholarship director to find at least five scholarships for which you plan to qualify by your senior year of high school. List the name of the scholarship, the sponsor, number of available awards, and the month and day that applications are due.

Spaced Practice

1. Firefighters need to cross from the roof of a 25-foot-tall building to the roof of a 35-foot-tall building using a ladder. The buildings are 20 feet apart. To the nearest whole foot, what minimum length does the ladder need to be to span the two buildings?



2. Nakota wants to create a diagonal path through her flower garden using stepping stones. She would like to place one stone every 2 feet. How many stepping stones does she need?



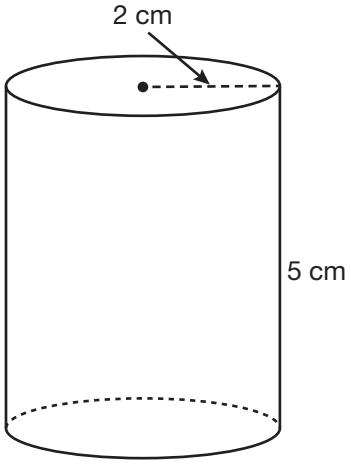
Name _____ Date _____

I. Volume, Lateral, and Total Surface Area of a Cylinder

Topic Practice

A. Calculate and use the area of the base to determine the volume of each given cylinder. Round to the nearest hundredth.

1.



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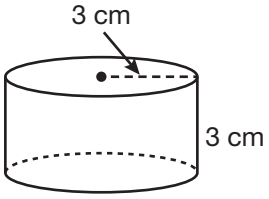

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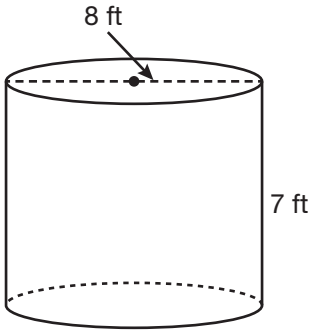

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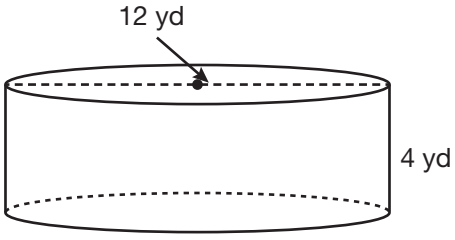

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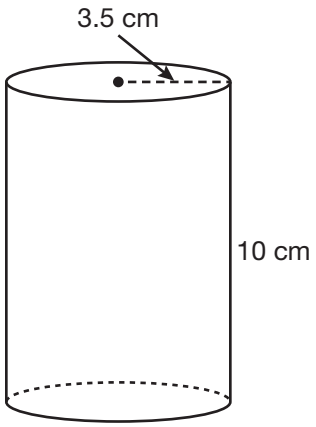

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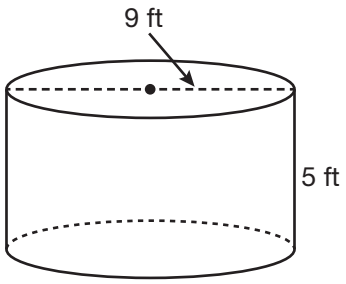
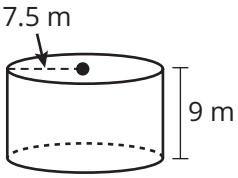


B. Calculate the volume of each cylinder. Write your answer in terms of π and as a decimal rounded to the nearest hundredth.

1.



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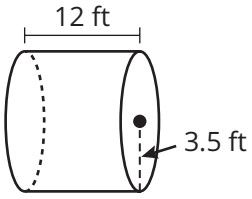

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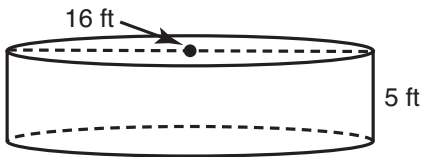

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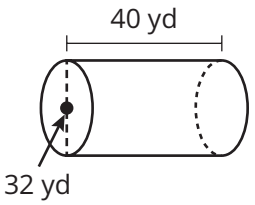

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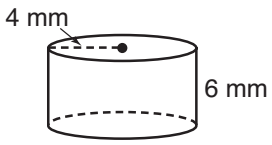

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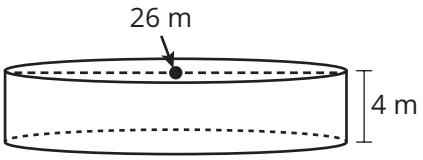
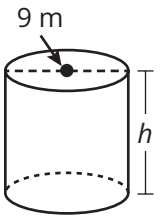


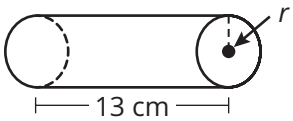
C. Calculate the unknown dimension of each cylinder. Round to the nearest hundredth.

1.



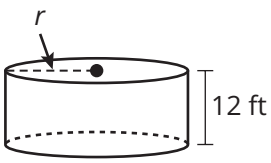
$V = 763 \text{ m}^3$

2.



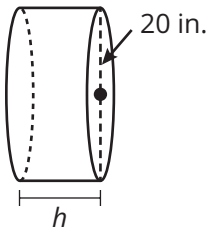
$V = 500 \text{ cm}^3$

3.



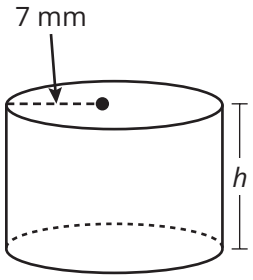
$V = 5425 \text{ ft}^3$

4.



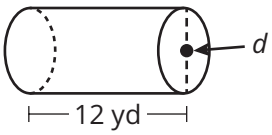
$V = 2826 \text{ in.}^3$

5.



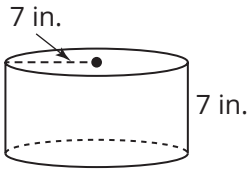
$V = 1231 \text{ mm}^3$

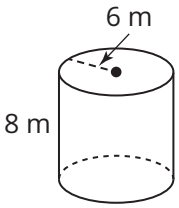
6.

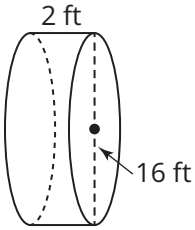


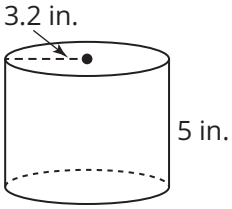
$V = 150 \text{ yd}^3$

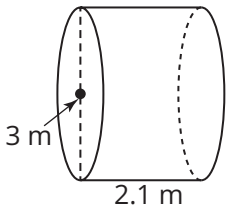
D. Calculate the lateral surface area of each cylinder. Round to the nearest hundredth.

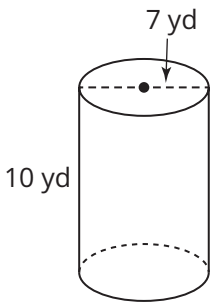
1. 

2. 

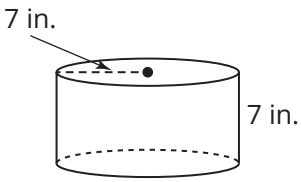
3. 

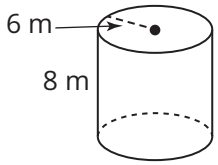
4. 

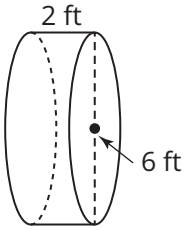
5. 

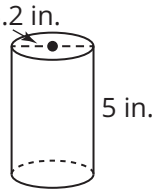
6. 

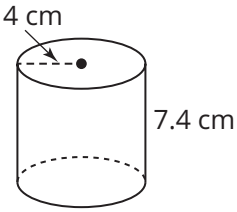
E. Calculate the total surface area of each cylinder. Write your answer in terms of π and as a decimal rounded to the nearest hundredth.

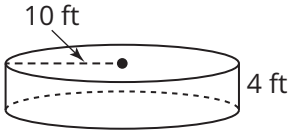
1. 

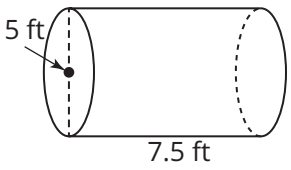
2. 

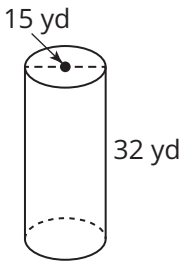
3. 

4. 

5. 

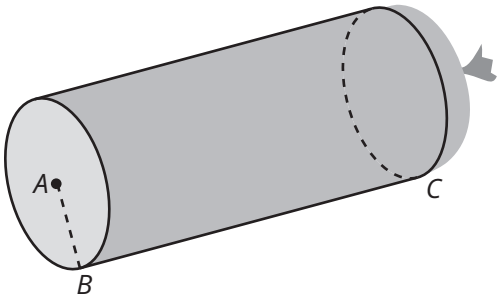
6. 

7. 

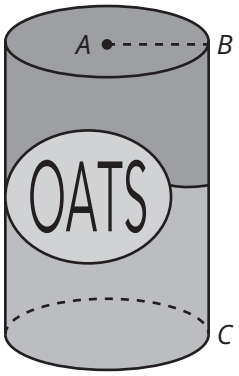
8. 

F. Solve each problem. Round to the nearest hundredth.

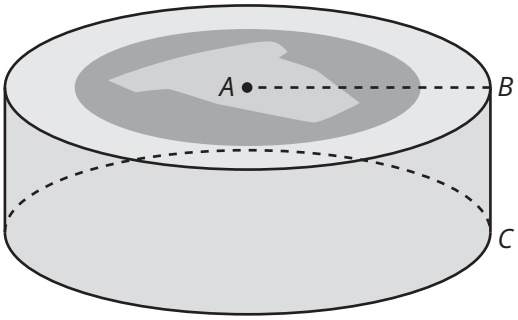
1. The local deli sells sandwich pepperoni. The pepperoni comes in the form of a cylinder. You need to figure out how much paper you will need to wrap the entire pepperoni. The radius of the pepperoni is 22 millimeters and the height of the pepperoni is 110 millimeters. What is the total surface area of the pepperoni?



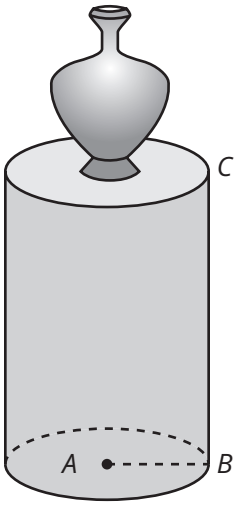
2. You like to eat oatmeal for breakfast. The oatmeal container is a cylinder. You remove the wrapper around the body of the oatmeal and want to use it for an art project. The radius of the container is 5.1 centimeters and the height of the container is 22.3 centimeters. What is the lateral surface area of the container?



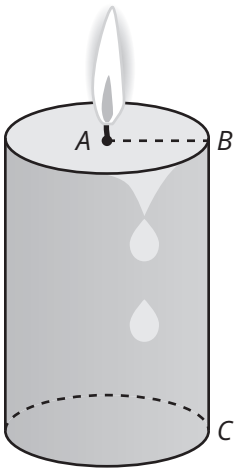
3. The local deli sells cheese wheels. Each cheese wheel is a cylinder. The height of the cheese wheel is 2.8 inches and the volume of the cheese wheel is 155.1 cubic inches. What is the radius of the cheese wheel?



4. Your uncle has a sculpture displayed on a display stand. The display stand is a cylinder and your uncle wants to wrap some fabric around the body of the cylinder as part of the design. The radius of the display stand is 8.7 centimeters and the height of the display stand is 27.8 centimeters. What is the lateral surface area of the display stand?



5. Your aunt keeps a candle on her nightstand. The candle is a cylinder. The radius of the candle is 3.2 centimeters and the height of the candle is 9.4 centimeters. What is the area of the base and the volume of the candle?



6. The local library has a number of pillars at the front entrance. Each pillar is a cylinder. The radius of each pillar is 17 inches and the volume of each pillar is 231,402.3 cubic inches. What is the height of each pillar?



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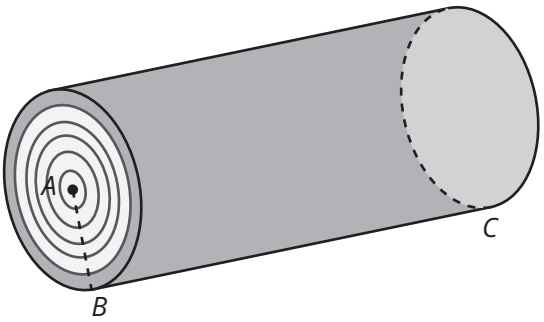
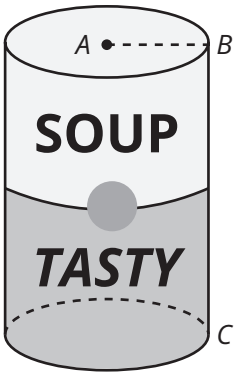
7. You are helping your uncle chop firewood into logs. Each log is a cylinder. The height of each log is 16.5 inches and the diameter is 12 inches. What is the area of the base and the volume of the log?



8. You buy a can of soup at the supermarket. The soup can is a cylinder. The height of the can is 15.7 centimeters and the radius is 3.9 centimeters. What is the area of the base and the volume of the can?



Extension

Provide the dimensions of a cylinder and a pentagonal prism that have the same volume and height.

Spaced Practice

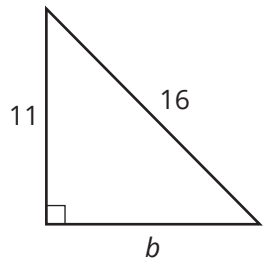
Write each number in standard form.

1. 8.1×10^{-4}

2. 3.2×10^{8}

Determine the unknown side length in the right triangle. Round to the nearest hundredth, when necessary.

3.



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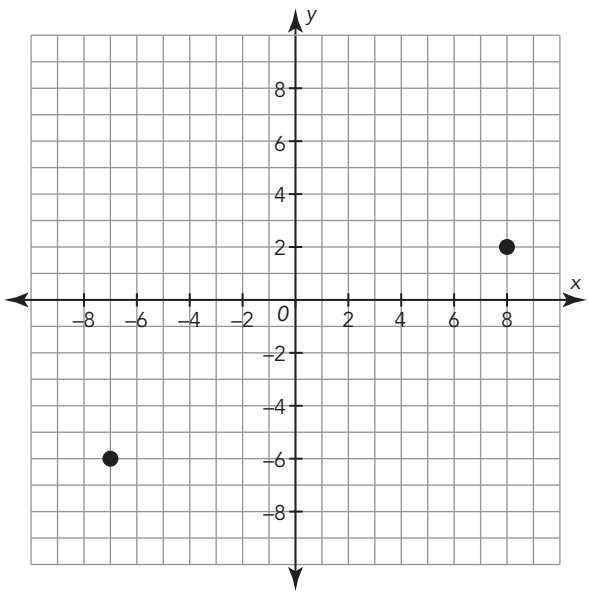
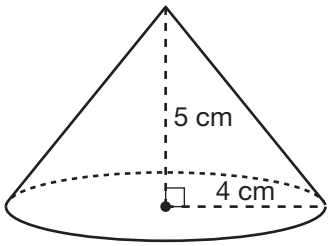


II. Volume of a Cone

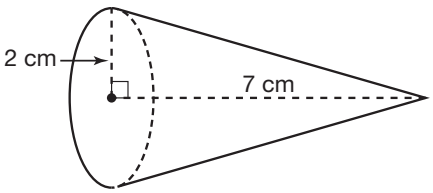
Topic Practice

A. Calculate the volume of each cone. Round to the nearest hundredth.

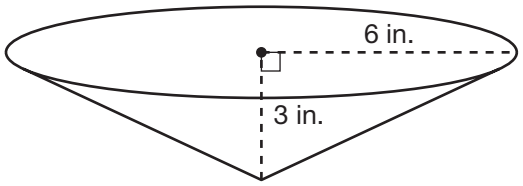
1.



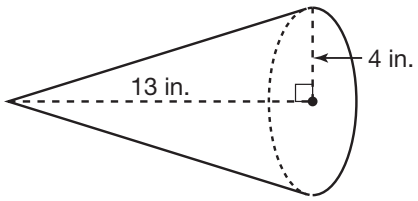
2.



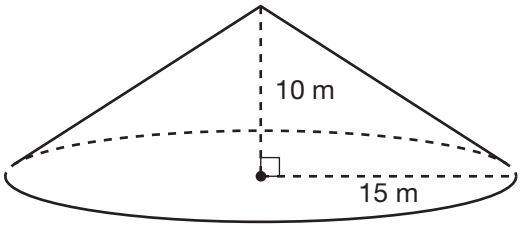
3.



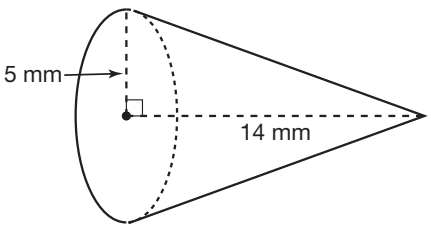
4.



5.



6.



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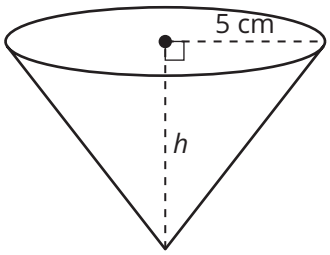
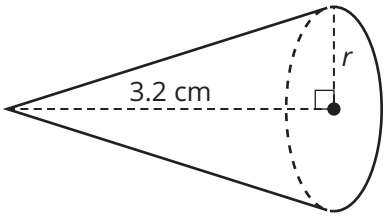
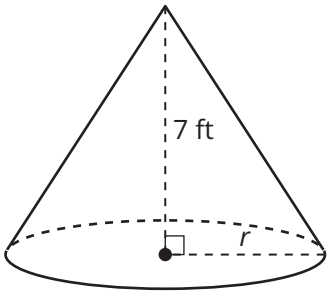
B. Calculate the unknown dimension for each cone. Round to the nearest hundredth.

1. 

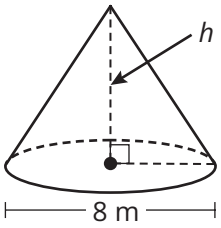
$V = 170 \text{ cm}^3$

2. 

$V = 15 \text{ cm}^3$

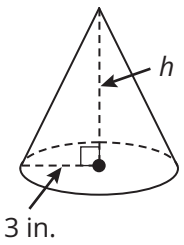
3. 

$V = 148 \text{ ft}^3$

4. 

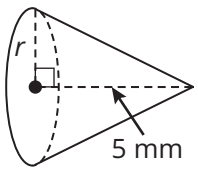
$V = 100 \text{ m}^3$

5.



$V = 75$ in.^{3}

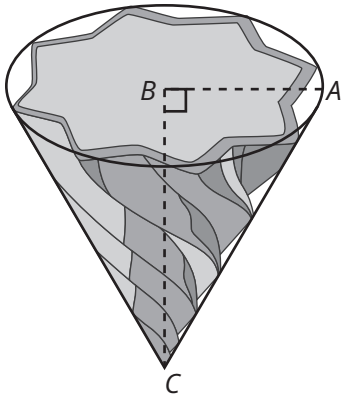
6.



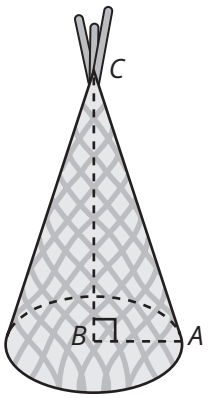
$V = 21$ mm^3

C. Solve each problem. Round to the nearest hundredth.

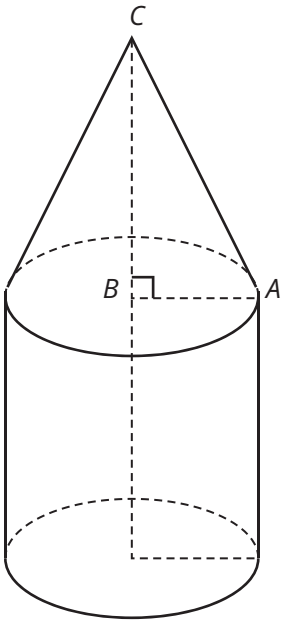
1. You buy your aunt a new drill bit set. Each drill bit is a cone. The radius of the drill bit is 3.9 millimeters and the height of the drill bit is 7.7 millimeters. What is the volume of the drill bit?



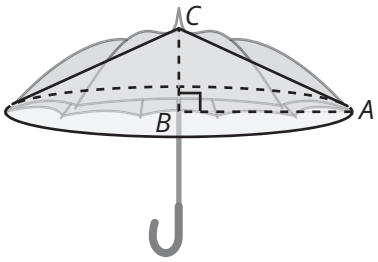
2. The local museum has a teepee on display. The teepee is a cone. The radius of the teepee is 4.7 feet and the height of the teepee is 17.8 feet. What is the volume of the teepee?



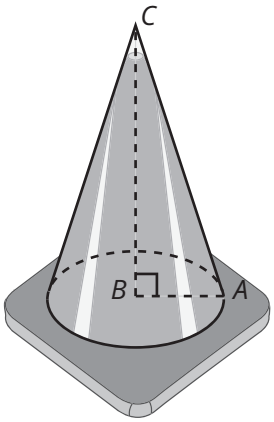
3. A farmer has a silo that is used to store harvested crops. The bottom portion of the silo is a cylinder with a radius of 6 feet. The top portion of the silo is a cone with the same radius. The height of each portion of the silo is the same. If the top portion of the silo can store 452.16 cubic feet of crops, how much can the bottom portion of the silo store? What is the height of each portion of the silo?



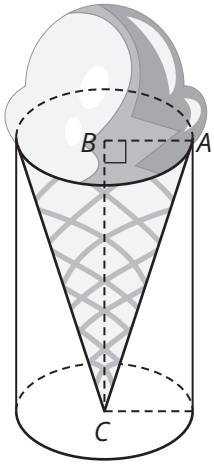
4. Your aunt bought a new umbrella. The umbrella is a cone. The height of the umbrella is 8 inches and volume of the umbrella is 4429.4933 cubic inches. What is the radius of the umbrella?



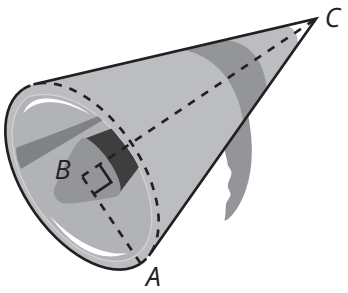
5. Your gym teacher uses traffic cones to create part of an obstacle course. The radius of the traffic cone is 6.4 inches and the volume of the traffic cone is 1045.5335 cubic inches. What is the height of the traffic cone?



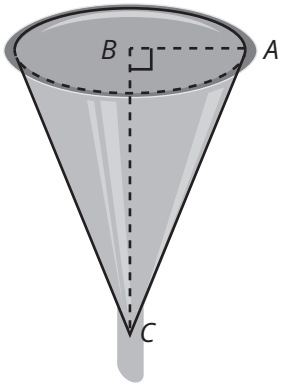
6. You and your friends buy ice cream cones after school. The radius of the ice cream cone is 4 cm. To prevent leaks, the ice cream shop places the cone into a cylindrical cup with the same radius and height. If the volume of the cylindrical cup is 502.4 cubic cm, what is the volume of the ice cream in the cone? What is the height of the ice cream cone and the cylindrical cup?



7. The school cheerleading team uses megaphones in their routines. Each megaphone is a cone. The radius of the megaphone is 5.1 inches and the height of the megaphone is 19.3 inches. What is the volume of the megaphone?



8. Your uncle uses a funnel to change the oil in his car. The funnel is a cone. The height of the funnel is 10.6 centimeters and the volume of the funnel is 335.6137 cubic centimeters. What is the radius of the funnel?



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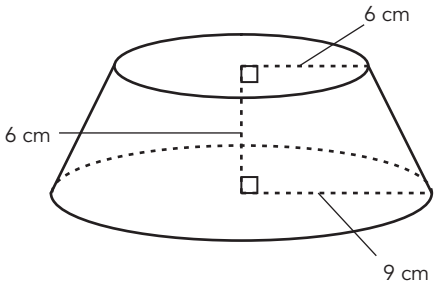
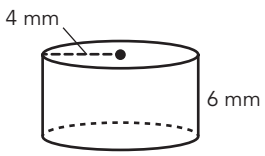
Extension

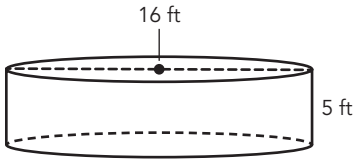
The frustum of a cone is the portion of a cone that remains after its upper part has been cut off by a plane parallel to its base. Calculate the volume of the frustum shown.



Spaced Practice

Calculate the volume, lateral surface area, and total surface area of each cylinder. Round your answer to the nearest hundredth.

1. 

2. 

Estimate each radical to the nearest tenth.

3. $\sqrt{52}$

4. $\sqrt{74}$

5. Emily is considering two plans for college. She has saved $5,000 for college. Find the remaining cost for each plan.

   a. Type of institution: four-year public university
   Tuition per semester: $14,190
   Cost of books per semester: $385
   Housing per semester: $5,400

   b. Type of institution: two-year public community college
   Tuition per semester: $3150
   Cost of books per semester: $360
   Housing per semester: $0 because Emily will live at home

6. Compare the set of loans. Determine which is the better loan, then determine the difference in payment amounts between the two loans.

Loan 1

- Loan Amount: $175,000
- Interest Rate: 4%
- Time: 20 years
- Payment: $1060.47/month

Loan 2

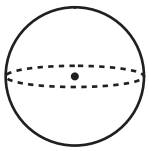
- Loan Amount: $175,000
- Interest Rate: 5.5%
- Time: 20 years
- Payment: $1203.80/month

III. Volume of a Sphere

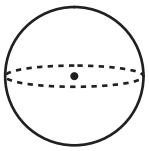
Topic Practice

A. Calculate the volume of each sphere. Round to the nearest hundredth.

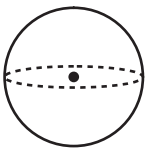
1. $r = 7$ m



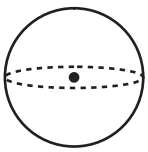
2. $r = 6$ in.



3. $d = 20$ in.



4. $d = 16$ m



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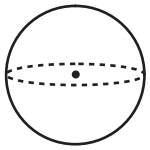
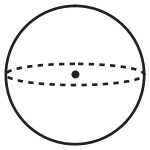
5. $r = 2.5$ cm



6. $r = 11.25$ mm



7. The radius of a sphere is 8 meters.

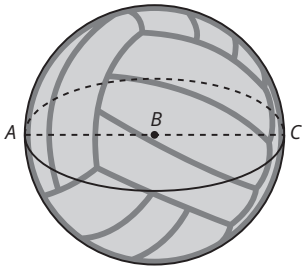
8. The radius of a basketball is 12.13 cm.

9. The diameter of a spherical balloon is 21.6 centimeters.

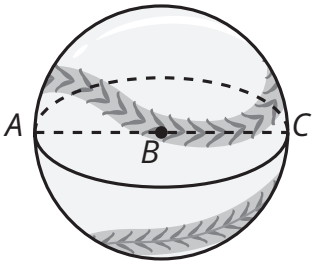
10. The diameter of a sphere is 15 yards.

B. Solve each problem. Round to the nearest hundredth.

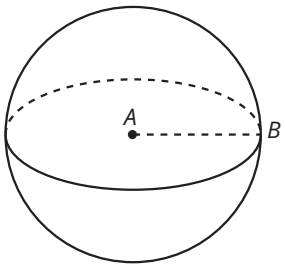
1. You play volleyball in gym class. The volleyball is a sphere. The diameter of the volleyball is 21.4 centimeters. What is the volume of the volleyball?



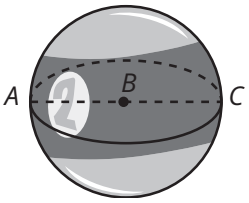
2. Your family decides to attend a baseball game. During the fifth inning, you catch a foul ball. The baseball is a sphere. The radius of the baseball is 36.6 millimeters. What is the volume of the baseball?



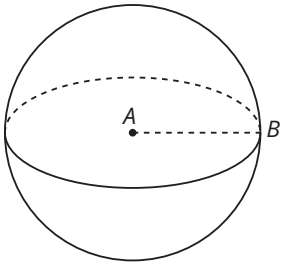
3. The given figure is a sphere. The radius of the sphere is 24 centimeters. What is the volume of the sphere?



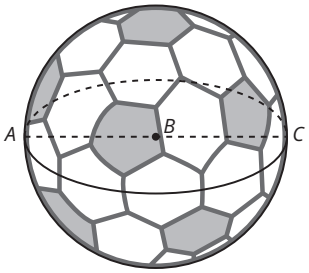
4. Your aunt buys a new pool table. Each billiards ball is a sphere. The diameter of each billiards ball is 56 millimeters. What is the volume of each billiards ball?



5. The given figure is a sphere. The radius of the sphere is 22 meters. What is the volume of the sphere?



6. You enjoy playing soccer with your friends. A soccer ball is a sphere. The diameter of the soccer ball is 10.4 inches. What is the volume of the soccer ball?



Extension

A typical orange has 10 segments and is composed of about 87% water. Suppose an orange has a diameter of 3 inches. What is the volume of water in each segment?

Spaced Practice

Determine the volume of each cone described. Round to the nearest hundredth.

1. Cone with a radius of 4.1 cm and a height of 10 cm

2. Cone with a diameter of 8 in. and a height of 5.03 in.

Indicate whether each real number shown is a rational number, an integer, a whole number, a natural number, or some combination.

3. $\frac{3}{4}$

4. 8

Calculate each amount.

5. You wish to invest $2500. How much more money is made from $2500 invested in a 3.15% annual compound interest account over 15 years, compared to investing the money in a simple interest account?

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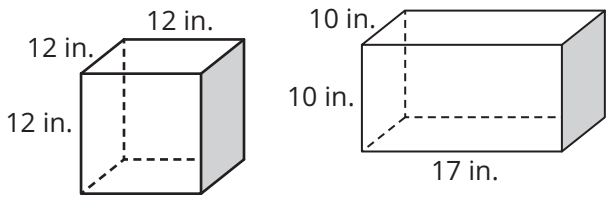
6. You wish to invest $6100. How much more money is made from $6100 invested in a 2.7% annual compound interest account over 20 years, compared to investing the money in a simple interest account?

IV. Volume and Surface Area Problems with Prisms, Cylinders, Cones and Spheres

Topic Practice

A. Use the given information to answer each set of questions.

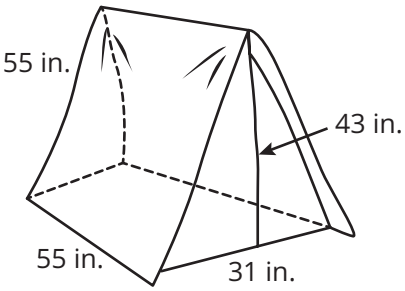
Valentina is making an art project with two different blocks.

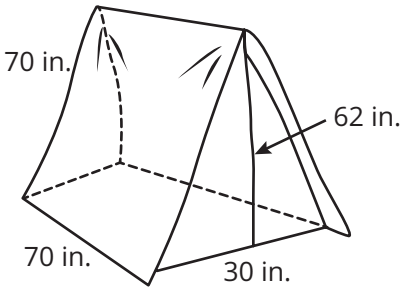


1. Valentina plans to paint the total surface area of each block with blue paint. How much blue paint will she need to paint both both of her blocks?

2. Valentina decides to wrap the lateral surface area of each block with metallic paper. How much metallic paper will she need for both blocks?

A local sporting goods store is running a sale on tents. Two of the most popular tent models are shown.

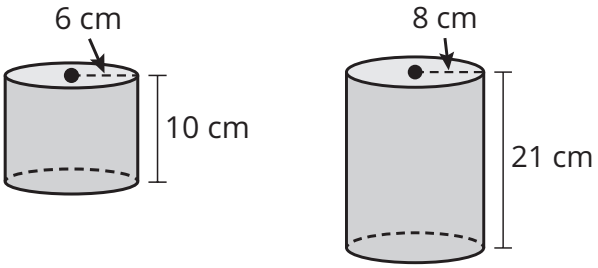




3. The store is offering a deal to spray the lateral surface area of the tent with insect repellent. Which tent will need more insect repellent?

4. The store is offering another deal to spray the entire outside of the tent with a water repellent. Which tent will need more water repellent?

A food company produces canned vegetables in different sized cans. The company produces two different-sized cans of corn.



5. How much more corn can the larger can hold? Explain your reasoning.

6. The company uses paper labels to cover the lateral surface area of their cans. How much less paper is needed to cover the smaller sized can?

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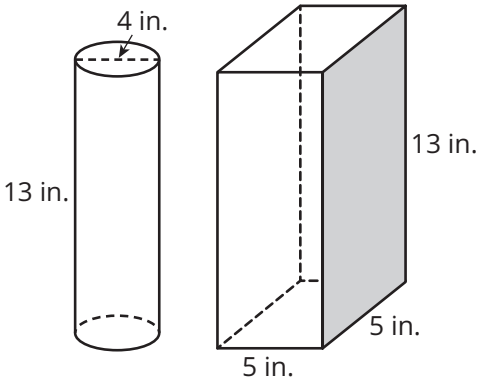
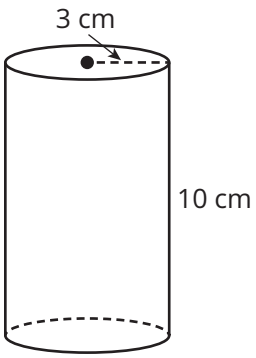
A company is experimenting with different packaging for oatmeal. Their two top options are shown.

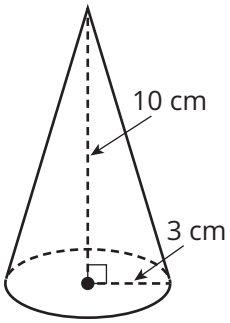


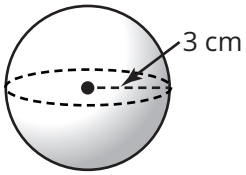
7. Which container can hold the most oatmeal? Explain your reasoning.

8. The company is planning to use cardboard to create each package. Which container uses the least cardboard? What is the difference between the amount of cardboard used for each package?

A company is trying to decide between three different packaging options for slime.





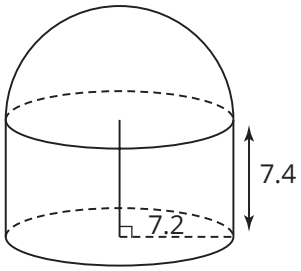


9. Which container would hold the most slime? Explain your reasoning.

10. The company plans to ship each slime container in a rectangular prism shaped box with the dimensions of 7 cm × 7 cm × 11 cm. Which package wastes the least amount of space?

B. Find the volume of the composite solids. Round to the nearest hundredth.

1.



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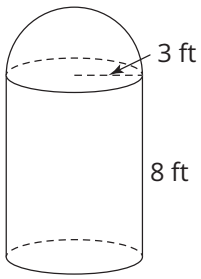

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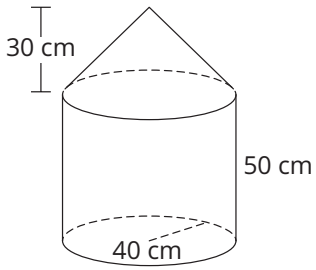

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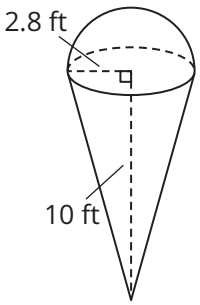

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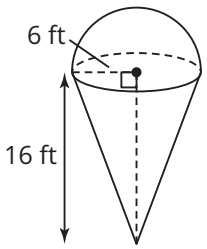

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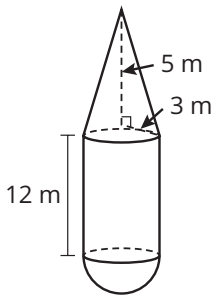


Extension

A container is composed of a half-sphere on top of a cylinder. The container has a volume of 360p cubic inches. The total height of the container is the same as the diameter of the half-sphere. What is the diameter of the half-sphere?

Spaced Practice

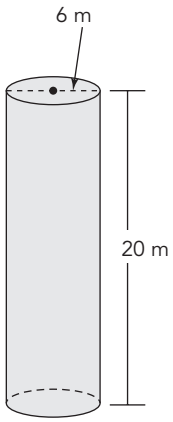
Calculate each volume. Round your answer to the nearest hundredth.

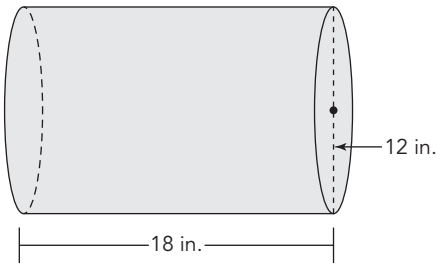
1. What is the volume of a sphere with a diameter of 8.5 inches?

2. What is the volume of a sphere with a radius of 9 millimeters?

Calculate the area of the base and the volume of each cylinder. Round your answer to the nearest hundredth.

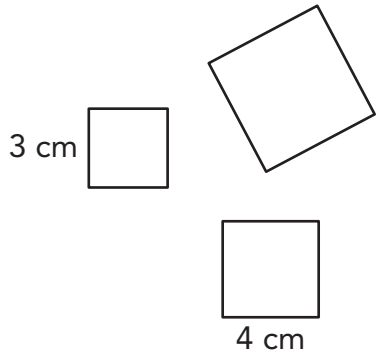
3.



4. 

5. The length of the hypotenuse of a right triangle is 40 inches. Determine the length of the legs if each leg is the same length.

6. The three squares shown will be joined at their vertices to form a right triangle. Given what you know about the Pythagorean Theorem, what is the area of the largest square?



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