Bluebonnet Learning
Secondary Mathematics
EDITION 1

Grade 6

Volume 2

STUDENT EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

If you have further product questions or to report an error, please email openeducationresources@tea.texas.gov.

Bluebonnet Learning

Secondary Mathematics

EDITION 1

Grade 6

Course Guide

GRADE 6

Welcome to the Course Guide for Secondary Mathematics, Grade 6

Instructional Design FM-6

Lesson Structure. FM-10

Research-Based Strategies FM-15

The Crew FM-17

TEKS Mathematical Process Standards FM-18

Understanding the Problem-Solving Model. FM-21

The Problem-Solving Model Graphic Organizer FM-22

Academic Glossary. FM-23

What is Productive Struggle? FM-25

Resources for Students and Families. FM-26

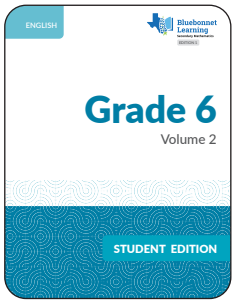
Course Table of Contents FM-31

Instructional Design

The instructional materials help you learn math in different ways. There are two types of resources: Learning Together and Learning Individually. These resources provide various learning experiences to develop your understanding of mathematics.

Learning Together

On Learning Together days, you spend time engaging in active learning to build mathematical understanding and confidence in sharing ideas, listening to one another, and learning together. The Student Edition is a consumable resource that contains the materials for each lesson.



STUDENT EDITION

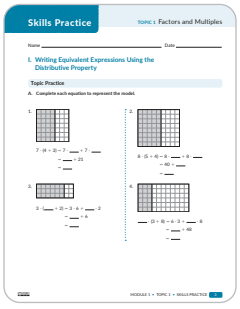
I am a record of your thinking, reasoning, and problem solving.

My lessons allow you to build new knowledge from prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

Learning Individually

On Learning Individually days, Skills Practice offers opportunities to engage with skills, concepts, and applications that you learn in each lesson. It also provides opportunities for interleaved practice, which encourages you to flexibly move between individual skills, enhancing connections between concepts to promote long-term learning. This resource will help you build proficiency in specific skills based on your individual academic needs as indicated by monitoring your progress throughout the course.



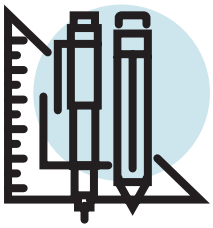
SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student edition.

My purpose is to provide problem sets for additional practice, enrichment, and extension.

Instructional Design

The instructional materials intentionally emphasize active learning and sense-making. Deep questions ensure you thoroughly understand the mathematical concepts. The instructional materials guide you to connect related ideas holistically, supporting the integration of your evolving mathematical understanding and developing proficiency with mathematical processes.



Intentional Mathematics Design

Mathematical Coherence: The path through the mathematics develops logically, building understanding by linking ideas within and across grades so you can learn concepts more deeply and apply what you've learned to more complex problems.

TEKS Process Standards: The instructional materials support your development of the TEKS mathematical process standards. They encourage you to experiment, think creatively, and test various strategies. These mathematical processes empower you to persevere when presented with complex real-world problems.

Multiple Representations: The instructional materials connect multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: The instructional materials focus on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

What principles guide the design and organization of the instructional materials?

Active Learning: Studies show that learning becomes more robust as instruction moves from entirely passive to fully interactive (Chi, 2009). Classroom activities require active learning as students engage by problem solving, explaining and justifying reasoning, classifying, investigate, and analyze peer work.

Discourse Through Collaborative Learning: Collaborative problem solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities intentionally promote active dialogue centered on structured activities.

Personalized Learning: Research has proven that problems that capture your interests are more likely to be taken seriously (Baker et. al, 2009). In each lesson, learning is linked to prior knowledge and experiences, creating valuable and authentic opportunities for you to build new understanding on the firm foundation of what you already know. You move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures.

Focus on Problem Solving: Solving problems is an essential life skill that you need to develop. The problem-solving model provides a structure to support you as you analyze and solve problems. It is a strategy you can continue to use as you solve problems in everyday life.

1 Writing Equivalent Expressions Using the Distributive Property

1 OBJECTIVES

- Write, read, and evaluate equivalent numeric expressions.
- Identify the adjacent side lengths of a rectangle as factors of the area value.
- Identify parts of an expression, such as the product and the factors.
- Write equivalent numeric expressions for the area of a rectangle by decomposing one side length into the sum of two or more numbers.
- Apply the distributive property to rewrite the product of two factors.

3 NEW KEY TERMS

- numeric expression
- equation
- distributive property

2

You know how to add, subtract, multiply, and divide numbers using different strategies. Taking apart numbers before you perform a mathematical operation can highlight important information or make calculations easier.

How can taking apart numbers help you to express number sentences in different ways?

CC BY NC

LESSON STRUCTURE

1 Objectives

Objectives are stated for each lesson to help you take ownership of the objectives.

2 Essential Question

Each lesson begins with a statement connecting what you have learned with a question to ponder. Return to this question at the end of this lesson to gauge your understanding.

3 New Key Terms

The new key terms for each lesson are identified to help you connect your everyday and mathematical language.

CC BY NC

4 Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

4 Getting Started

Ask Yourself . . .
How can you use the area of rectangles in everyday life?

Break It Down to Build It Up

Sofia is building a rectangular walkway up to her house. The width of the walkway is 5 feet and the length is 27 feet. She needs to calculate the area of the walkway to determine the amount of materials needed to build it.

1. Mark and label two different ways you could divide an area model to determine the area of the walkway.

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

2. Determine the areas of each of the subdivided parts of your models.

3. What is the total area of the walkway?

5

ACTIVITY 2.1

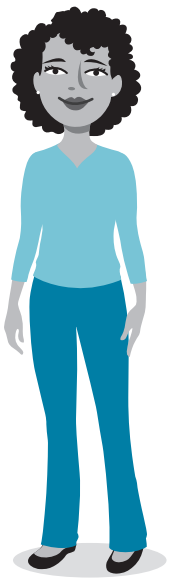
Prime Factors

You just determined the factors for a given number as well as the common factors that two numbers share.

In this activity, you will learn to determine the prime factors of a given number.

A factor tree is a way to organize the prime factors of a number. Choose any factor pair to get started.

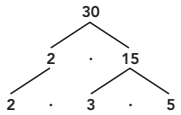
When you express any whole number as a product of primes, only primes, and nothing, you are determining the **prime factorization** of that number.



WORKED EXAMPLE

Use a factor tree to write the prime factorization for 30.

- Begin with the number 30.
- Pick any whole number factor pair of 30, other than 1 and 30.
- Draw a branch from 30 to each factor, 2 and 15.
- Since one of the factors is not prime, you are not finished.
- Use branches to write a factor pair for 15.
- Because 2, 3, and 5 are all prime, this factor tree is complete.



1. Use the factor tree to write the prime factorization of 30.

The factor tree in the Worked Example is not the only factor tree you can create for 30.

2. How many different factor trees are there for 30?

CC BY NC

5 Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about getting the answer. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

CC BY NC

6 Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

Talk the Talk 6

The Floor Is Yours

You can apply the distributive property to solve real-world problems. Consider the situation.

Diego is setting up the gym floor for an after-school program. He wants to include a rectangular area for playing volleyball and another for dodgeball. He also wants to have an area for kids who like to play board games or just sit and read. The gym floor is already 50 feet by 84 feet, or 4200 square feet.

1. Create a diagram to show how you would split up the gym floor. Represent your diagram using the distributive property and write an explanation for the areas assigned to each activity.

Lesson 1 Assignment

7 Write

Explain the distributive property in terms of composing and decomposing numbers.

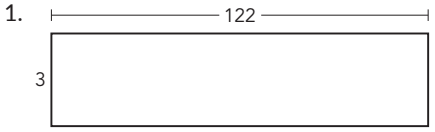
Remember 8

There are many ways to rewrite equivalent expressions using properties. The distributive property of multiplication over addition states that for any numbers a, b, and c, $a(b + c) = ab + ac$.

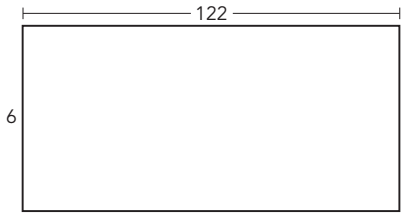
9 Practice

Decompose each rectangle into two or three smaller rectangles to demonstrate the distributive property. Then, write each area in the form $a(b + c) = ab + ac$.

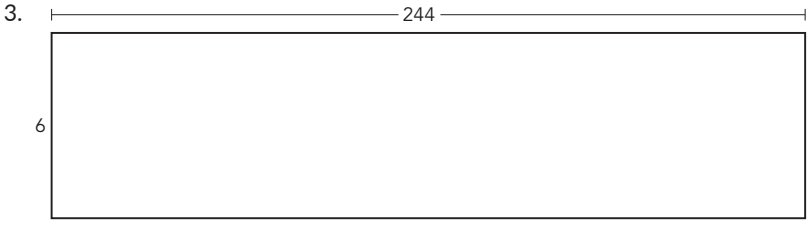
1.



2.



3.



ASSIGNMENT

7 Write

Reflect on your work and clarify your thinking.

8 Remember

Take note of the key concepts from the lesson.

9 Practice

Use the concepts learned in the lesson to solve problems.

Lesson 1 Assignment

4. $6(12 + 4)$

5. $10 + 4(2 + 20)$

6. $7(4 + 19)$

10 Prepare

1. In the array of numbers shown, circle the prime numbers, cross out the composite numbers, and use a box to identify any number that is neither prime nor composite.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

ASSIGNMENT

10 Prepare

Get ready for the next lesson.

CC BY NC

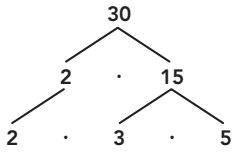
CC BY NC

Research-Based Strategies

WORKED EXAMPLE

Use a factor tree to write the prime factorization for 30.

- Begin with the number 30.
- Pick any whole number factor pair of 30, other than 1 and 30.
- Draw a branch from 30 to each factor, 2 and 15.
- Since one of the factors is not prime, you are not finished.
- Use branches to write a factor pair for 15.
- Because 2, 3, and 5 are all prime, this factor tree is complete.



WORKED EXAMPLE

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

Ask Yourself . . .

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

Jayden and Mateo want to determine the number of miles in 31,680 feet using unit analysis.

Mateo

$31{,}680\ \text{ft} \cdot \frac{1\ \text{mi}}{5280\ \text{ft}} = 6\ \text{mi}$

Jayden

$31{,}680\ \text{ft} \left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right) =$

$167{,}270{,}400\ \text{mi}$

Ask Yourself . . .

- Why is this method correct?
- Have I used this method before?

Ask Yourself . . .

- Where is the error?
- Why is it an error?
- How can I correct it?

THUMBS UP

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

THUMBS DOWN

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

WHO'S CORRECT?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine whether the work is correct or incorrect.



4. Jamal and Tiara are solving the equation $5 = \frac{p}{7}$.
   a. Jamal says that to solve $5 = \frac{p}{7}$, he would divide by 7. The value of p that makes the equation true is $\frac{5}{7}$. Tiara disagrees and says that they should divide by $\frac{1}{7}$ and the solution is 35. Who is correct?

   b. How can Jamal and Tiara check to see whose answer is correct?

Ask Yourself . . .

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

The Crew

The Crew is here to help you on your journey. Sometimes, they will remind you about things you already learned. Sometimes, they will ask you questions to help you think about different strategies. Sometimes, they will share fun facts. They are members of your group—someone you can rely on!



Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.



CC BY NC

TEKS Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I Can" expectations listed below align with the TEKS mathematical process standards and encourage students to develop their mathematical learning and understanding.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I CAN:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I CAN:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology, as appropriate; and techniques including mental math, estimation, and number sense, as appropriate, to solve problems.

I CAN:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language, as appropriate.

I CAN:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Create and use representations to organize, record, and communicate mathematical ideas.

I CAN:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

CC BY NC

Analyze mathematical relationships to connect and communicate mathematical ideas.

I CAN:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I CAN:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.



The Problem-Solving Model

Productive mathematical thinkers are problem solvers. These instructional materials include a problem-solving model to help you develop proficiency with the TEKS mathematical process standards. An opportunity to use the problem-solving model is present every time you see the problem-solving model icon.

The problem-solving model represents a strategy you can use to make sense of problems you must solve. The model provides questions you can use to guide your thinking and the graphic organizer provides a place for you to show and organizer your work.

Understanding the Problem-Solving Model



Notice | Wonder

Understand the situation by asking these questions.

- What do I notice?
- What do I wonder?
- How do I analyze the given information to identify what is important?
- Do I have enough information to formulate a plan and determine a solution?



Organize | Mathematize

Devise a plan for your mathematical approach by asking these questions.

- What mathematical relationships exist between this problem and similar problems I have solved?
- What plan or strategy can I use to solve this problem?
- How can I efficiently solve this problem?
- How can I organize, record, and communicate my mathematics?



Predict | Analyze

Carry out your plan to determine a solution. Then, ask yourself the following questions.

- Did I display my work using multiple representations?
- Did I explain my reasoning in terms of the problem situation?
- Did I communicate the strategy used to determine the solution?
- Did I justify my mathematical argument clearly using precise mathematical language?
- Can I use my mathematical reasoning to make any predictions?



Test | Interpret

Look back at your work and ask these questions.

- Does my solution clearly and completely answer the original question/problem?
- Is my solution reasonable?
- Does my solution make sense in terms of the problem situation?
- Can I solve the problem using a different strategy? Would another strategy be more efficient?
- Can I justify my solution?



Report

As you share your mathematical reasoning with others ask these questions.

- Did you use multiple representations to represent your mathematics?
- Did you justify your mathematical reasoning?
- Can others understand my process and solution?

The Problem-Solving Model Graphic Organizer



Understand the Problem



Devise a Plan



Carry Out the Plan



Look Back



Report

Academic Glossary

Knowing what these terms mean and using them will help you demonstrate proficiency with the TEKS mathematical process standards as you think, reason, and communicate your ideas.

Analyze

Definition

Study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

Explain Your Reasoning

Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself...

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

- Show your work
- Explain your calculation
- Justify
- Why or why not?

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Represent

Definition

Display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

- Predict
- Approximate
- Expect
- About how much?

Estimate

Definition

Make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Describe

Definition

Represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

What is Productive Struggle?

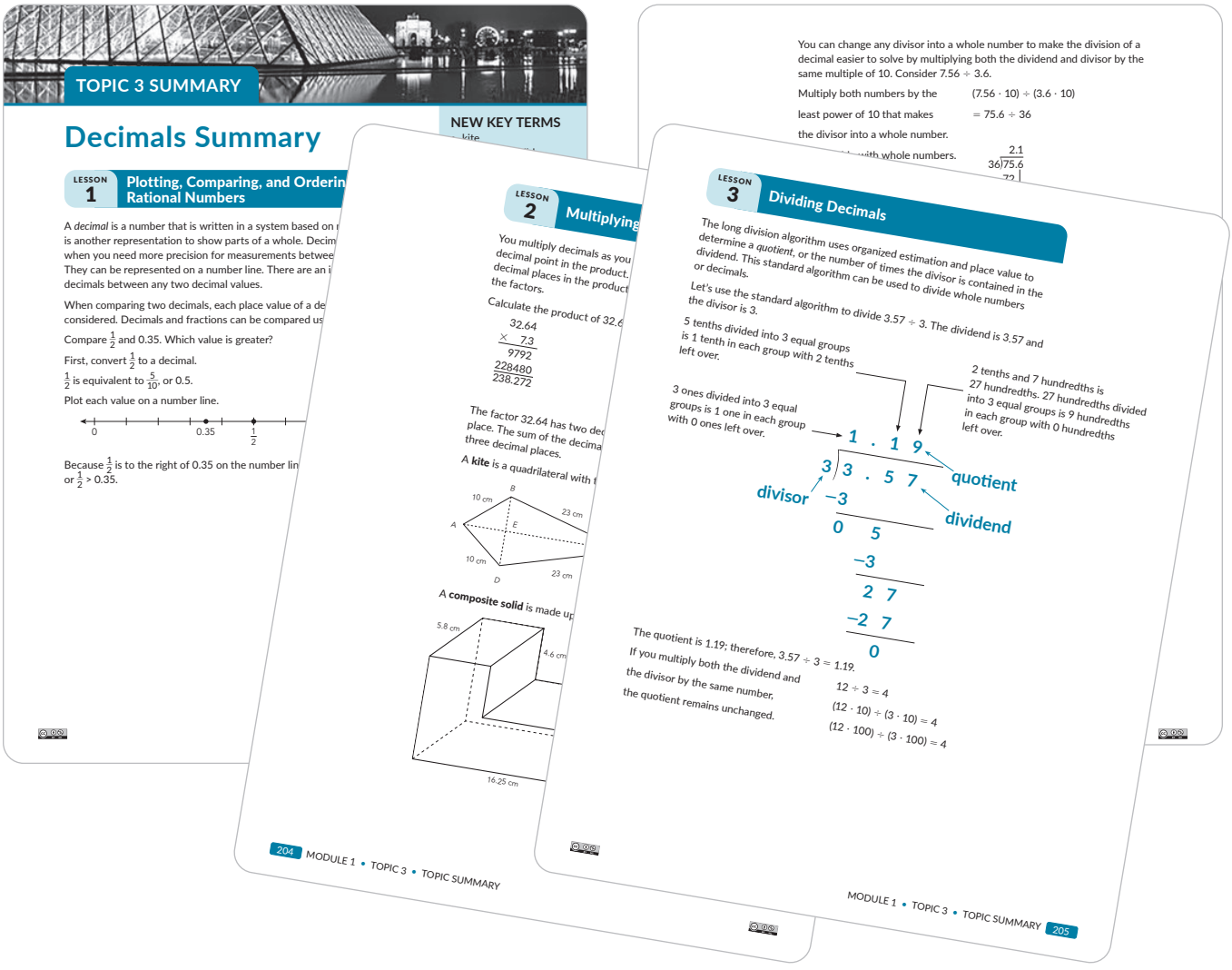
Imagine you are trying to beat a game or solve a puzzle. At some point, you are not sure what to do next. You don't give up; you keep trying repeatedly until you accomplish your goal. You call this process *productive struggle*. Productive struggle is when you choose to persist, think creatively, and try various strategies to accomplish your goal. Productive struggle supports you as you develop characteristics and habits that you will use in everyday life.

Things to do:	Things not to do:
• Persevere. • Think creatively. • Try different strategies. • Look for connections to other questions or ideas. • Ask questions that help you understand the problem. • Help your classmates without telling them the answers.	• Get discouraged. • Stop after trying your first attempt. • Focus on the final answer. • Think you have to make sense of the problem on your own.

This course will challenge you by providing you with opportunities to solve problems that apply the mathematics you know to the real-world. As you work through these problems, your first approach may not work. This is okay, even when your initial strategy does not work, you are learning. In addition, you will discover multiple ways to solve a problem. Looking at various strategies will help you evaluate the problem-solving process and identify an efficient approach. As you persist in problem-solving, you will better understand the math.

Topic Summary

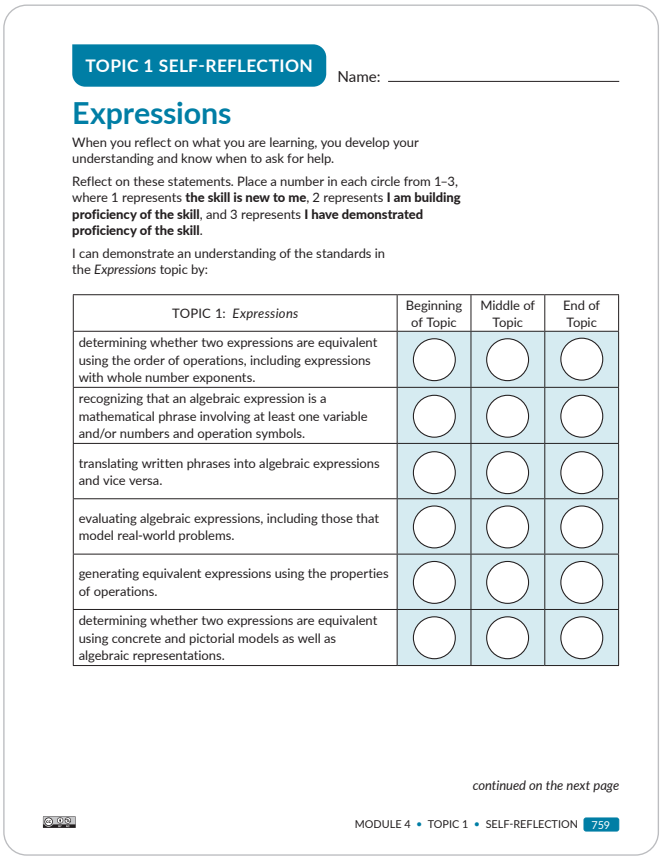
Each topic includes a Topic Summary. The Topic Summary contains a list of all new key terms addressed in the topic and a summary of each lesson, including worked examples and new key term definitions. Use the Topic Summary to review each lesson's major concepts and strategies as you complete assignments and/or share your learning outside of class.



Topic Self-Reflection

The Topic Self-Reflection, provided at the end of each topic, empowers you to develop confidence in your mathematical understanding and monitor your own learning processes. Taking the time for self-reflection helps you identify your strengths and where you want to focus your efforts to improve.

Use the Topic Self-Reflection throughout the topic to monitor your progress toward the mathematical goals for the topic.



TOPIC 1 SELF-REFLECTION

Name: _____

Expressions

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

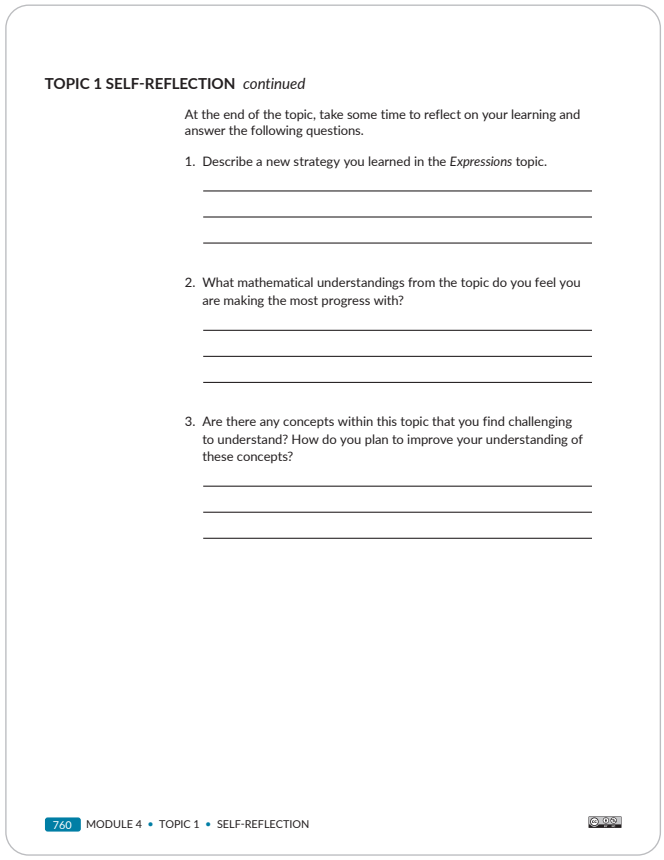
Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Expressions* topic by:

TOPIC 1: *Expressions*	Beginning of Topic	Middle of Topic	End of Topic
determining whether two expressions are equivalent using the order of operations, including expressions with whole number exponents.	○	○	○
recognizing that an algebraic expression is a mathematical phrase involving at least one variable and/or numbers and operation symbols.	○	○	○
translating written phrases into algebraic expressions and vice versa.	○	○	○
evaluating algebraic expressions, including those that model real-world problems.	○	○	○
generating equivalent expressions using the properties of operations.	○	○	○
determining whether two expressions are equivalent using concrete and pictorial models as well as algebraic representations.	○	○	○

continued on the next page

© 2021



TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Expressions* topic.

   _____

   _____

   _____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   _____

   _____

   _____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   _____

   _____

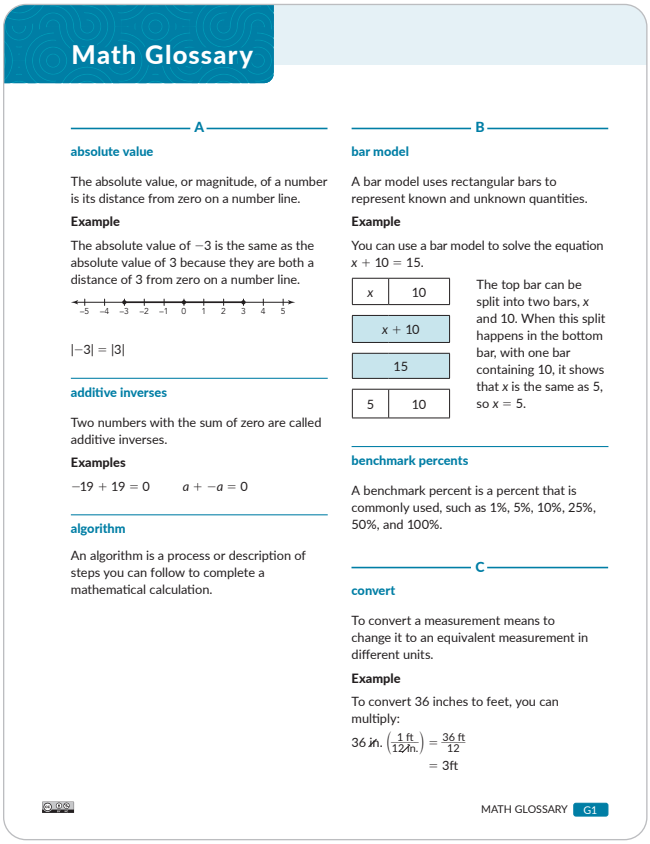
   _____

© 2021

CC BY NC

Math Glossary

A course-specific math glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.



Math Glossary

A

absolute value

The absolute value, or magnitude, of a number is its distance from zero on a number line.

Example

The absolute value of -3 is the same as the absolute value of 3 because they are both a distance of 3 from zero on a number line.

$|-3| = |3|$

additive inverses

Two numbers with the sum of zero are called additive inverses.

Examples

$-19 + 19 = 0$ $a + -a = 0$

algorithm

An algorithm is a process or description of steps you can follow to complete a mathematical calculation.

B

bar model

A bar model uses rectangular bars to represent known and unknown quantities.

Example

You can use a bar model to solve the equation $x + 10 = 15$.

x	10
$x + 10$	
15	
5	10

The top bar can be split into two bars, x and 10. When this split happens in the bottom bar, with one bar containing 10, it shows that x is the same as 5, so $x = 5$.

benchmark percents

A benchmark percent is a percent that is commonly used, such as 1%, 5%, 10%, 25%, 50%, and 100%.

C

convert

To convert a measurement means to change it to an equivalent measurement in different units.

Example

To convert 36 inches to feet, you can multiply:

$36\ \text{in.} \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right) = \frac{36\ \text{ft}}{12}$

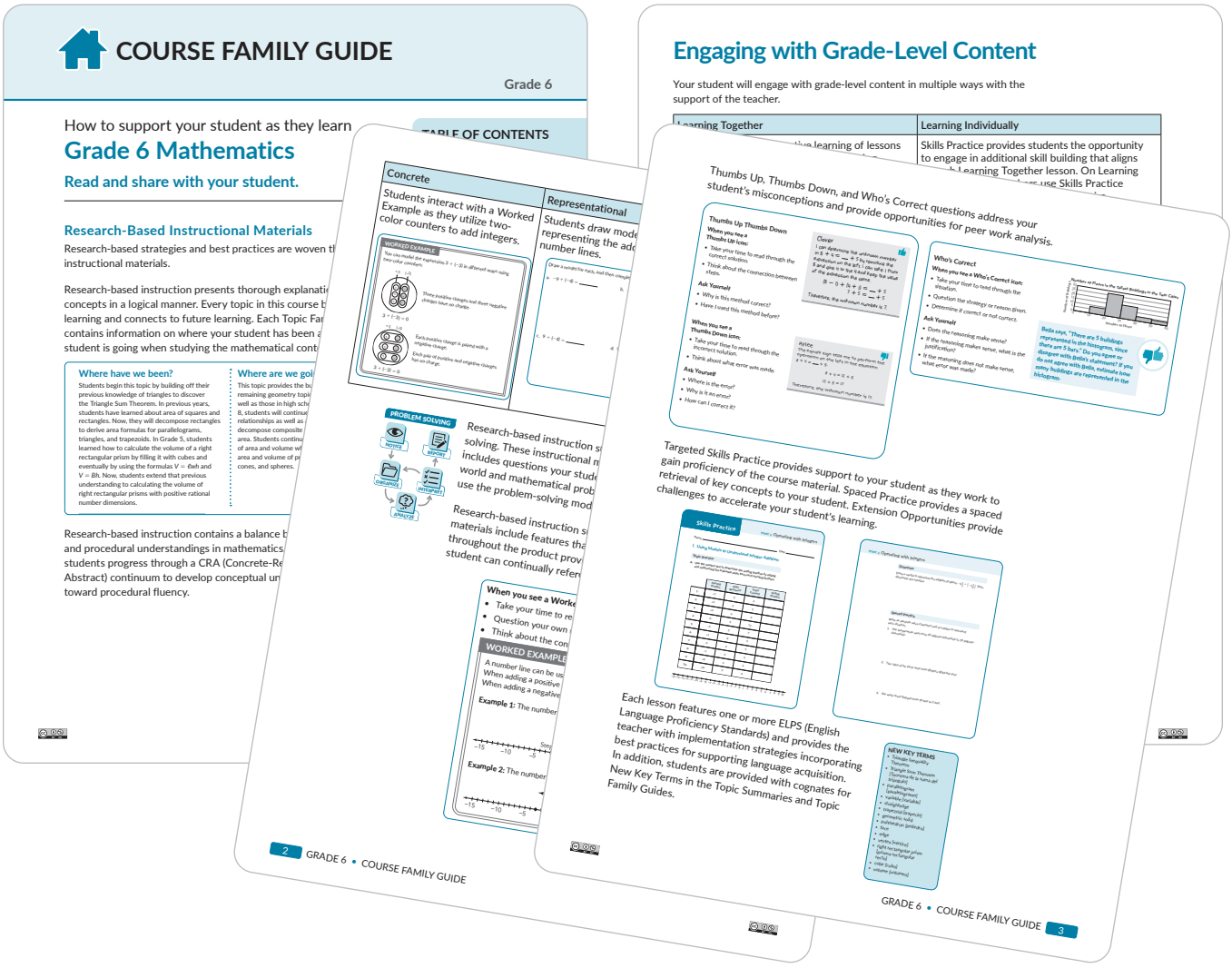
$= 3\text{ft}$

MATH GLOSSARY G1

Course Family Guide

The Course Family Guide provides you and your family with an overview of the course design. The guide details the resources available to support your learning, such as the Math Glossary, the Topic Family Guide, the Topic Self-Reflection, and the Topic Summaries.

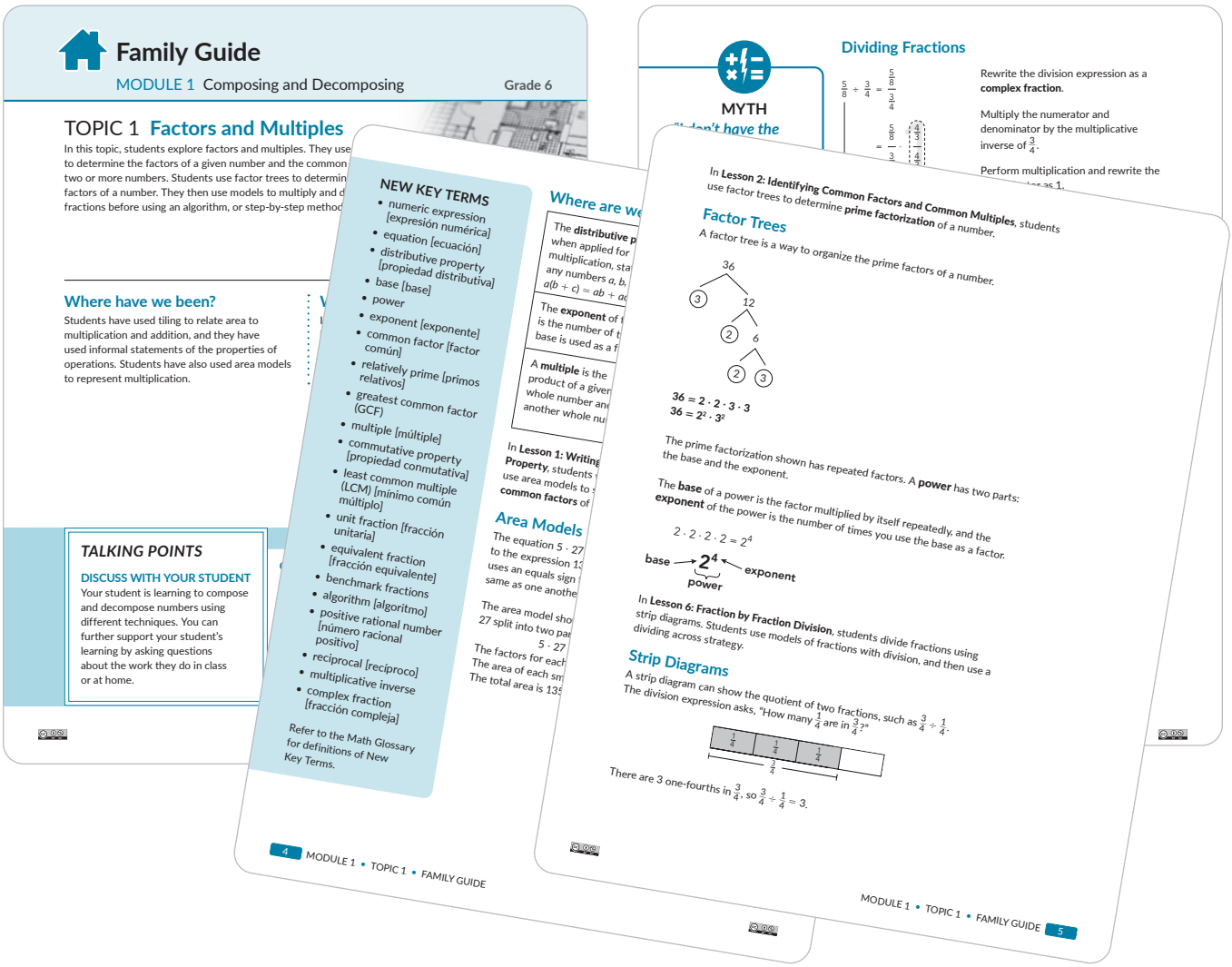
The purpose of the Course Family Guide is to bridge your learning in the classroom to your learning at home. The goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together.



Topic Family Guides

Each topic contains a Topic Family Guide that provides an overview of the math of the topic and answers the questions, "Where have we been?" and "Where are we going?" Additional components of the Topic Family Guide are an example of a math model or strategy taught in the topic, definitions of a few key terms, busting of a math myth, and questions family members can ask you to support your learning.

Learning outside of the classroom is crucial to student success at school. The Topic Family Guide serves to assist families in talking to students about the learning that is happening in the classroom.



Course Table of Contents

MODULE 1 Composing and Decomposing

TOPIC 1 Factors and Multiples 3

Introduction to the Problem-Solving Model and Learning Resources

LESSON 1 Writing Equivalent Expressions Using the Distributive Property

LESSON 2 Identifying Common Factors and Common Multiples

LESSON 3 Dividing a Whole into Fractional Parts

LESSON 4 Benchmark Fractions

LESSON 5 Multiplying Fractions

LESSON 6 Fraction by Fraction Division

TOPIC 2 Shapes and Solids 95

LESSON 1 Constructing Triangles Given Sides

LESSON 2 Triangle Sum Theorem

LESSON 3 Area of Triangles and Quadrilaterals

LESSON 4 Volume of Rectangular Prisms

TOPIC 3 Decimals 159

LESSON 1 Plotting, Comparing, and Ordering Rational Numbers

LESSON 2 Multiplying Decimals

LESSON 3 Dividing Decimals

CC BY NC

MODULE 2 Relating Quantities

TOPIC 1 Ratios and Rates 209

LESSON 1 Introduction to Ratio and Ratio Reasoning

LESSON 2 Comparing Ratios and Rates to Solve Problems

LESSON 3 Determining Equivalent Ratios

LESSON 4 Using Tables to Represent Equivalent Ratios

LESSON 5 Graphs of Ratios

LESSON 6 Using and Comparing Ratio Representations

TOPIC 2 Percents 323

LESSON 1 Percent, Fraction, and Decimal Equivalence

LESSON 2 Using Estimation and Benchmark Percents

LESSON 3 Determining the Part and the Whole in Percent Problems

TOPIC 3 Unit Rates and Conversions 391

LESSON 1 Using Ratio Reasoning to Convert Units

LESSON 2 Introduction to Unit Rates

LESSON 3 Multiple Representations of Unit Rates

MODULE 3 Moving Beyond Positive Quantities

TOPIC 1 Signed Numbers and the Four Quadrants . . . 451

LESSON 1 Introduction to Negative Numbers

LESSON 2 Absolute Value

LESSON 3 Rational Number System

LESSON 4 Extending the Coordinate Plane

TOPIC 2 Operating with Integers **541**
LESSON 1 Using Models to Understand Integer Addition
LESSON 2 Adding Integers, Part I
LESSON 3 Adding Integers, Part II
LESSON 4 Subtracting Integers
LESSON 5 Multiplying and Dividing Integers

MODULE 4 Determining Unknown Quantities

TOPIC 1 Expressions **667**
LESSON 1 Evaluating Numeric Expressions
LESSON 2 Introduction to Algebraic Expressions
LESSON 3 Equivalent Expressions
LESSON 4 Verifying Equivalent Expressions

TOPIC 2 Equations and Inequalities **765**
LESSON 1 Reasoning with Equal Expressions
LESSON 2 Solving One-Step Addition Equations
LESSON 3 Solving One-Step Multiplication Equations
LESSON 4 Solving Equations to Solve Problems
LESSON 5 Solving Inequalities with Inverse Operations

TOPIC 3 Graphing Quantitative Relationships **881**
LESSON 1 Independent and Dependent Variables
LESSON 2 Using Graphs to Solve Problems
LESSON 3 Multiple Representations of Equations
LESSON 4 Relating Distance, Rate, and Time
LESSON 5 Problem Solving on the Coordinate Plane

CC BY NC

TOPIC 4 Financial Literacy: Accounts, Credit, and Careers 987

LESSON 1 Checking Accounts
LESSON 2 Debit Cards vs. Credit Cards
LESSON 3 Understanding Credit Reports
LESSON 4 Career Exploration
LESSON 5 Paying for College

MODULE 5 Describing Variability of Quantities

TOPIC 1 The Statistical Process 1081

LESSON 1 Understanding the Statistical Process
LESSON 2 Analyzing Numerical Data Displays
LESSON 3 Using Histograms to Display Data

TOPIC 2 Numerical Summaries of Data 1155

LESSON 1 Analyzing Data Using Measures of Center
LESSON 2 Displaying the Five-Number Summary
LESSON 3 Collecting, Displaying, and Analyzing Data

Math Glossary G-1

GRADE 6

MODULE
4

Determining Unknown Quantities

TOPIC 1 Expressions 667

TOPIC 2 Equations and Inequalities 765

TOPIC 3 Graphing Quantitative Relationships 881

TOPIC 4 Financial Literacy: Accounts, Credit, and Careers 987

CC BY NC

GRADE 6

TOPIC 1



If you think of the surface of the ocean as 0, then a diver is in the negative numbers until he comes back up.

Expressions

LESSON 1 Evaluating Numeric Expressions 669

LESSON 2 Introduction to Algebraic Expressions 691

LESSON 3 Equivalent Expressions 707

LESSON 4 Verifying Equivalent Expressions 735

1 Evaluating Numeric Expressions

OBJECTIVES

- Interpret a number raised to a positive integer power as a repeated product.
- Identify perfect square numbers and perfect cube numbers.
- Write and evaluate numeric expressions involving whole-number exponents.
- Model numeric expressions with two- and three-dimensional figures.
- Evaluate numeric expressions using the order of operations.

NEW KEY TERMS

- perfect square
- perfect cube
- evaluate a numeric expression
- order of operations

You have written and evaluated expressions equivalent to given numbers.

Other than the four operations—addition, subtraction, multiplication, and division—are there other structures that can be used in numeric expressions?

CC BY NC

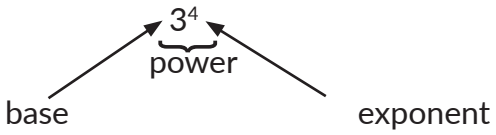
Getting Started

Square and Cube Numbers

You can read a power in different ways:
"3 to the fourth power"
"3 raised to the fourth power"

Remember that repeated multiplication can be represented as a power, which has two elements: the base and the exponent.

$3 \cdot 3 \cdot 3 \cdot 3 = 3^4$



The *base* of a power is the factor that is multiplied repeatedly in the power, and the *exponent* of the power is the number of times the base is used as a factor.

1. Identify the base and exponent in each power. Then, write each power in words.

   a. 5^6

   b. 8^7

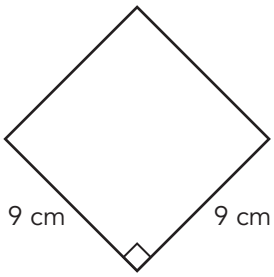
In the power s^2, the base is the side length, s, and the exponent is 2.

You know that the area of a rectangle is calculated by multiplying its length by its width. Because all sides of a square have the same length, the area of a square, A, is calculated by multiplying the length of the side, s, by itself. The formula for the area of a square, $A = s \cdot s$, can be written as $A = s^2$.

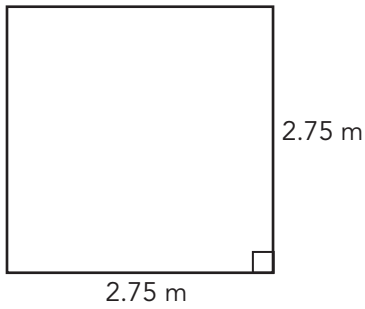
In the same way, to calculate the square of a number, you multiply the number by itself.

2. Write the area of each square as a repeated product, as a square number, and as an area in square units.

a.



b.



One of the areas that you wrote in Question 2 is called a **perfect square** because it is the square of an integer. For example, 9 is a perfect square because $3 \cdot 3 = 9$. Another way you can write this mathematical sentence is $3^2 = 9$.

You can read 3^2 as "3 squared."

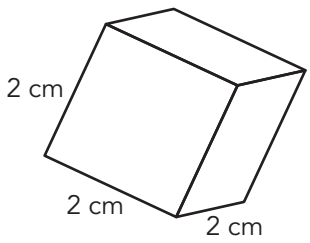
Recall that the volume of a cube is calculated by multiplying its length by its width and its height. Since the length, width, and height of a cube are all the same, the formula for the volume, V, of a cube can be written as $V = s \cdot s \cdot s$, or $V = s^3$.

In the power s^3, the base is the side length, s, and the exponent is 3.

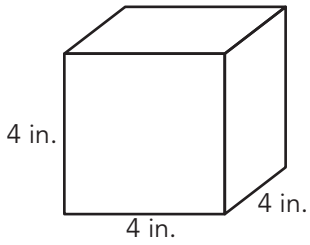
In the same way, to calculate the cube of a number, you use the number as a factor three times.

3. Write the volume of each cube as a repeated product, as the cube of a number, and as a volume in cubic units.

a.



b.



A **perfect cube** is the cube of an integer. For example, 216 is a perfect cube because 6 is a whole number and $6 \cdot 6 \cdot 6 = 216$.

You can read 6^3 as "6 cubed."

ACTIVITY
1.1 Modeling Expressions

Previously, you may have thought about expressions as recipes. For example, the expression $2 + 2$ might have meant "start with 2 and add 2 more." But as a relationship, $2 + 2$ means "2 combined with 2."

Remember. . .
A numeric expression is a mathematical phrase that contains numbers and operations.

The Expression Cards at the end of this lesson contain a variety of numeric expressions and models that represent numeric expressions. Cut out the Expression Cards.

1. Consider the different structures of the expressions and the models.

   a. Sort the models in a mathematically meaningful way.

   b. Sort the expressions in a mathematically meaningful way.

   c. Explain how you sorted the Expression Cards.

2. Match the numeric expressions with the models. Select two pairs of cards and explain why each expression matches the model.

Now, it's your turn!

3. Think of a numeric expression. Draw a model to represent that expression. Trade your model with a classmate and write the numeric expression that represents their model. When you both have written your answers, trade back and check your work!

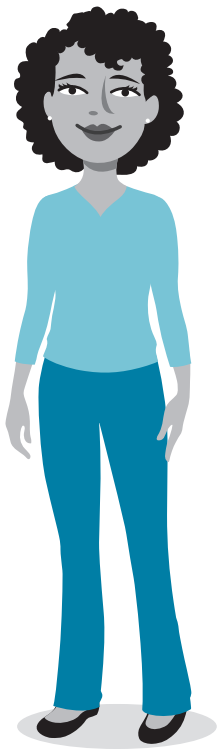
CC BY NC

ACTIVITY
1.2 Writing Equivalent Expressions

The diagram can be used to determine perfect squares. Mario drew on the diagram to show that the expression $(4 + 4)^2$ is equivalent to 8^2.

	$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$	$6^2 = 36$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$	$11^2 = 121$	$12^2 = 144$	$13^2 = 169$	$14^2 = 196$	$15^2 = 225$
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1															
2															
3															
4															
5															
6															
7															
8															
9															
10															
11															
12															
13															
14															
15															

How can you use the grid to determine the square of any number from 1 to 15?



1. Explain why $(4 + 4)^2$ is equivalent to 8^2 and not equivalent to $4^2 + 4^2$. Then, use the diagram to write other expressions that are equivalent to 8^2.

2. Write an equivalent numeric expression for each perfect square.

   a. 6^2

   b. 12^2

To **evaluate a numeric expression** means to simplify the expression to a single numeric value.

3. Use the diagram to rewrite the expression $(7 - 3)^2 + (10 - 7)^2$ with fewer terms. Explain your work.

4. Use the diagram to write four numeric expressions. Then, explain how to evaluate each expression.

The table shows the cubes of the first 10 whole numbers.

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$6^3 = 216$	$7^3 = 343$	$8^3 = 512$	$9^3 = 729$	$10^3 = 1000$

5. Write two more equivalent expressions for each. Show how to evaluate the expressions.

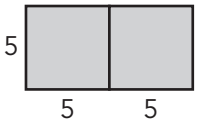
   a. 5^3

   b. 2^3

ACTIVITY
1.3 Evaluating Expressions

Consider the numeric expression $2 \cdot 5^2$.

1. Natalia drew a model to represent the expression. Explain how Natalia's model represents the expression. Then, evaluate the expression.



2. Kaya and Samantha each evaluated the expression differently.

Samantha

$2 \cdot 5^2$

$5^2 = 25$

$2 \cdot 25 = 50$

Kaya

$2 \cdot 5^2$

$2 \cdot 5 = 10$

$10^2 = 100$

   a. What does Samantha's solution tell you about how to evaluate a numeric expression with both multiplication and exponents?

   b. Draw a model to represent Kaya's solution. Explain how the model is different from Natalia's.

Parentheses are symbols used to group numbers and operations. You can think about expressions inside parentheses as a single value. In Question 3, take a careful look at how the parentheses are used in each expression.

3. This model represents the expression $(6 + 4) \cdot 3$.



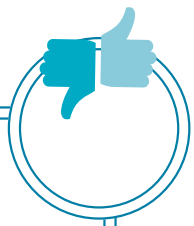
   a. Evaluate the expression that the model represents.

   b. Draw a model that would represent the expression $6 + (4 \cdot 3)$ and evaluate the expression.

   c. Compare the models and the expressions. How does moving the parentheses change how you draw the model and how you evaluate the expression?

4. Consider the numeric expression $(5 + 3)^2$.

   a. Draw a model to represent this expression.



   b. People evaluated the numeric expression two different ways, resulting in two different values. Determine which solution is correct. Explain why one solution is correct and state the error that was made in the other solution.

Solution A	**Solution B**
$(5 + 3)^2$	$(5 + 3)^2$
$= 8^2$	$= 25 + 9$
$= 64$	$= 34$

5. Consider the numeric expression $3 \cdot (7 - 2)$.

   a. Draw a model to represent this expression.



   b. People evaluated the numeric expression two different ways, resulting in two different values. Determine which solution is correct. Explain why one solution is correct and state the error that was made in the other solution. Cross out the incorrect solution.

Solution A	**Solution B**
$3 \cdot (7 - 2)$	$3 \cdot (7 - 2)$
$= 21 - 2$	$= 3(5)$
$= 19$	$= 15$

6. A band is playing at a local restaurant for a total of 8 Fridays and will receive payment after their last performance. The band advertises their 8 appearances in the local newspaper for a total cost of $400. When the band makes $500 for each appearance, which numeric expression correctly shows the amount of money each of the four members will earn? Explain your reasoning.

Expression A	**Expression B**
$(8 \cdot 500 - 400) \div 4$	$8 \cdot 500 - 400 \div 4$

ACTIVITY
1.4 The Order of Operations

There is an *order of operations*, an order in which operations are performed when evaluating any numeric expression. The **order of operations** is a set of rules that ensures the same result every time an expression is evaluated.

Order of Operations Rules

1. Evaluate expressions inside parentheses or grouping symbols.
2. Evaluate exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Keep in mind that multiplication and division are of equal importance and evaluated in order from left to right. The same is true for addition and subtraction.

A mnemonic is a strategy using a pattern of letters or words to help you remember something. What is a mnemonic could you use to remember the order of operations?



A mnemonic may help you remember the order. The important thing is to understand WHY the order of operations works.



Evaluate each expression using the order of operations.

1. $6(16 - 10) \div 2^2 - 36 \div (-3)^2$

2. $-12 - (25 \div 5)^2$

3. $(12^2 - 48) \cdot (-2)$

4. $-168 \div 2^3 + (-3)^3 - 20$

5. $10 \div (5 - 3) + 2^3$

ACTIVITY
1.5

Applying the Order of Operations and Prime Factorization

You have previously learned about prime factorization, and you have learned the order of operations.

You can use what you know about prime factorization and the order of operations to determine the value of expressions or rewrite expressions.

1. Determine the value of the expression $6(-2) + (-5) + (-4)(-8)$.

2. Determine the value of the expression $72 - ((-4)(3) + 6 + (2)(5) - 8)$.

3. A gym teacher purchased 5 basketballs for $27 each and 2 volleyballs for $10 each. To determine the balance change of the school gym checking account the gym teacher calculated $5(-27) + 2(-10)$. What is the balance change in the checking account?

4. Determine the value of the expression $-3 \cdot 2 - (6 + (-2)^3)$.

CC BY NC

You can use prime factorization to write equivalent expressions.
$5^2 + 5 \cdot 45$.

WORKED EXAMPLE

To rewrite the expression $5^2 + 5 \cdot 75$ using prime factorization.

Step 1: Rewrite the prime factorization for 75 using powers.

$$75 = 3 \cdot 5^2$$

Step 2: Since, 75 is multiplied by 5.

Rewrite the expression $5^2 + 5 \cdot (3 \cdot 5^2)$ as $5^2 + 5^3 \cdot 3$

5. Why can't you combine 5^2 and 5^3 when rewriting the expression with powers?

6. Rewrite each expression using prime factorization with powers.

   a. $2^2 + 3 \cdot 28$

   b. $3^2 + 2 \cdot 42$

   c. $225 + 144$

7. Determine the value of each expression.

   a. $3^3 + 3^2 - (3 \cdot 6) + 7$

   b. $(-2)^3 - ((-4)^2 \cdot 2)$

CC BY NC

Talk the Talk

Order of Operations

Determine whether or not each expression was evaluated correctly. Show the correct work for any incorrect answers.

1. $18 \div 2 \cdot 3^2$
   $18 \div 2 \cdot 9$
   $18 \div 18$
   1

2. $(15 + 10 \div 5) - 19$
   $(15 + 2) - 19$
   $17 - 19$
   -2

3. $60 - (10 - 6 + 1)^2 \cdot 2$
   $60 - (10 - 7)^2 \cdot 2$
   $60 - (3)^2 \cdot 2$
   $60 - 9 \cdot 2$
   $60 - 18$
   42

Each numeric expression has been evaluated correctly and incorrectly. For those that have been evaluated correctly, state how the order of operations was used to evaluate the expression. For those expressions that have been evaluated incorrectly, determine the error that was made.

4. Strategy A

$-2(10 - 1) - 3 \cdot 2$
$-2(9) - 3 \cdot 2$
$-18 - 3 \cdot 2$
$-21 \cdot 2$
-42

Strategy B

$-2(10 - 1) - 3 \cdot 2$
$-2(9) - 3 \cdot 2$
$-18 - 6$
-24

5. Strategy A

$4 + 3^2$
$4 + 9$
13

Strategy B

$4 + 3^2$
7^2
49

6. Strategy A

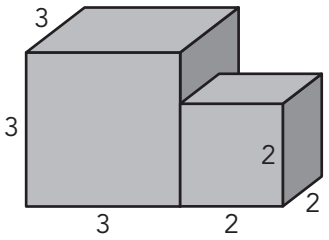
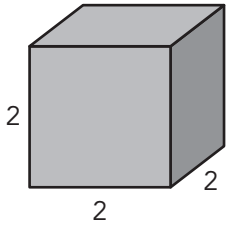
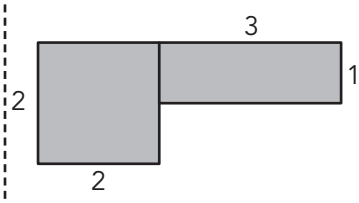
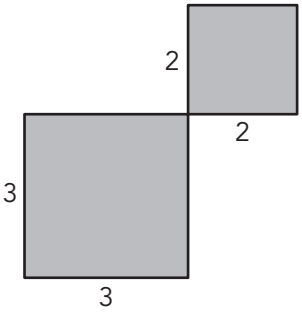
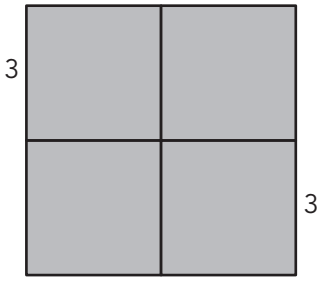
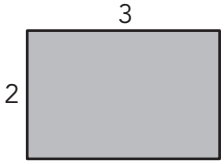
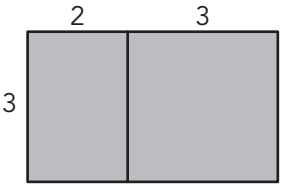
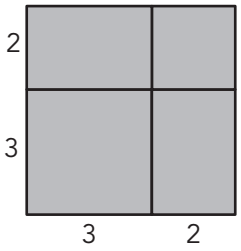
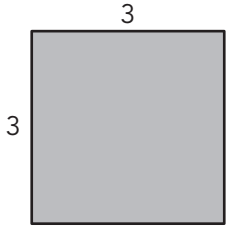
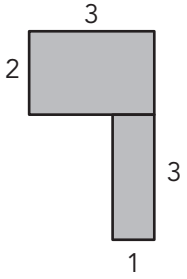
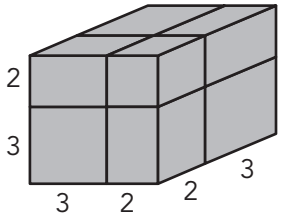
$(2 + 6)^2$
8^2
64

Strategy B

$(2 + 6)^2$
$4 + 36$
40

CC BY NC

Expression Cards

$3 \cdot 2$	$(3 \cdot 2)^2$		$(3 + 2)^3$
	3^2	$3 \cdot (2 + 3)$	
$3 + 2^2$			$(3 + 2)^2$
$3^3 + 2^3$			
	$3 \cdot 2 + 3$	$3^2 + 2^2$	
2^3			

Why is this page blank?

So you can cut out the expression cards on the other side

Lesson 1 Assignment

Write

What is important to remember when simplifying expressions?

Remember

Memorize the first 15 squares and 10 cubes.

Perfect Squares				
$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$6^2 = 36$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$
$11^2 = 121$	$12^2 = 144$	$13^2 = 169$	$14^2 = 196$	$15^2 = 225$
Perfect Cubes				
$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
$6^3 = 216$	$7^3 = 343$	$8^3 = 512$	$9^3 = 729$	$10^3 = 1000$

Practice

Use the order of operations to evaluate each numeric expression.

1. $4^2 \cdot 3$

2. $3^3 - 14 \div 2 + 5$

3. $-17 - 2^3$

4. $144 \div 6^2 \cdot 8 + 2^2$

Lesson 1 Assignment

5. $32 \div 4^2$

6. $2^4 - 3 \cdot 5 - 9$

7. $9 - 5^2 - 2 \cdot 3^2$

8. $11^2 - 7 \cdot 6 - 4^3 \div (-2)$

Rewrite each expression using prime factorization with powers.

9. $2^2 + 6 \cdot 2 \cdot 18$

10. $5^3 + 4 \cdot 7 \cdot 21$

Prepare

In the school cafeteria, soft pretzels sell for $1.25 each.
Determine how much money the cafeteria earns in each situation.

1. On Monday, the cafeteria sold 14 soft pretzels.

2. On Wednesday, the cafeteria sold 35 soft pretzels.

3. On Thursday, the cafeteria sold 50 soft pretzels.

2 Introduction to Algebraic Expressions

OBJECTIVES

- Write algebraic expressions to represent real-world and mathematical situations.
- Match algebraic and verbal expressions.
- Identify parts of an algebraic expression using mathematical terms.
- Evaluate algebraic expressions at specific values of their variables.

NEW KEY TERMS

- variable
- algebraic expression
- coefficient
- term
- evaluate an algebraic expression

You have written and evaluated expressions made up of numbers, but often expressions are made up of numbers and letters.

What situations can be represented by expressions with letters, and how do you evaluate them?

Getting Started

Do You Speak Math?

You are writing a numeric expression to represent each statement.

Rewrite each statement using symbols.

1. Fourteen more than six

2. Six more than fourteen

3. Seven less than thirteen

4. Thirteen less than seven

5. Twenty-three subtracted from thirty

6. Thirty subtracted from twenty-three

7. The quotient of twelve divided by four

8. The quotient of four divided by twelve

9. One-fourth of twenty-eight

10. Two to the seventh power

11. Seven squared



ACTIVITY
2.1 Writing Expressions with Variables

Consider the quantity that changes as you think about the situations in Question 1.

1. A school lunch costs $1.85 for each student. For each situation, write a numeric expression to determine the amount of money that is collected. Then, evaluate the expression.

   a. Fifty-five students purchase a school lunch.

   b. One hundred twenty-six students purchase a school lunch.

   c. Two hundred thirteen students purchase a school lunch.

   d. One thousand five hundred twelve students purchase a school lunch.

2. Write a sentence to describe how you can determine the amount of money collected for any number of students buying school lunches.

In Question 1, there is one quantity that changes or varies—the number of students who bought school lunches. In mathematics, letters are often used to represent quantities that vary. These letters are called **variables**, and they help you write algebraic expressions to represent situations. An **algebraic expression** is an expression that has at least one variable.

CC BY NC

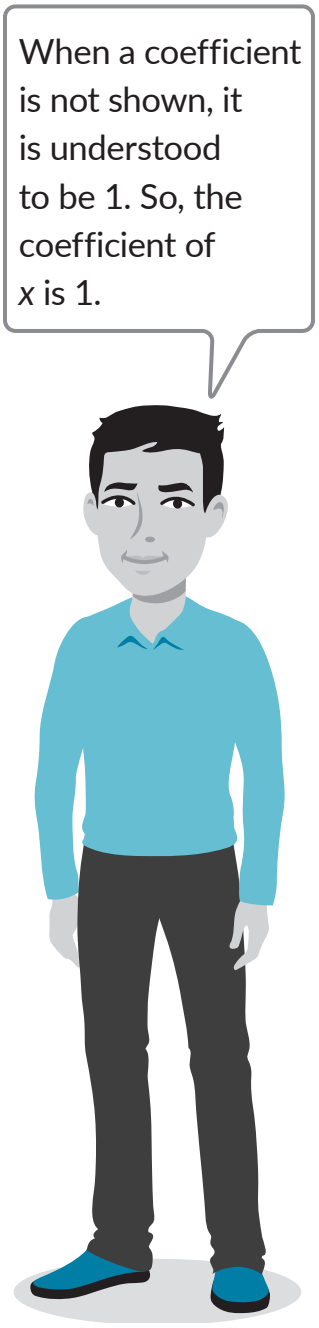
PROBLEM SOLVING



3. Write an algebraic expression to represent the total amount of money collected for any number of students buying school lunches.

A number that is multiplied by a variable in an algebraic expression is called a **coefficient**.

4. Identify a coefficient in the expression you wrote in Question 3.



5. The cost to rent a skating rink is $215 for a two-hour party. All the people who attend the party will share the cost equally. For each number of attendees, write a numeric expression to determine how much each person will pay. Then, evaluate the expression.

   a. 25 attendees

   b. 100 attendees

   c. Write an algebraic expression to represent how much each person will pay to attend the skate party.

6. Andrew has three 300-minute international calling cards.

   a. Complete the table to determine how many minutes are left on each card after each call.

Minutes on Card	Duration of Call	Minutes Left on Card
300	33 min	
300	1 h 20 min	

   b. Write an algebraic expression that represents the number of minutes remaining after each call on each card.

7. Write an algebraic expression to represent each situation. Identify the coefficient(s).

   a. A plane descends to $\frac{5}{6}$ of its cruising altitude, a. What is its new altitude?

   b. A cube has an edge length of s. What is the volume of the cube?

   c. Used paperback books cost $6.25 each, with an additional shipping and handling cost of $8.75. What is the cost of x books?

   d. Chairs cost $35, and sofas cost $75. How much does it cost to purchase x chairs and y sofas?

8. Write an algebraic expression to represent each word expression.

   a. The quotient of a number n divided by 7

   b. Fourteen less than three times a number n

   c. One-fourth of a number n minus 6

   d. Ten times the square of a number w divided by 12

CC BY NC

ACTIVITY
2.2

Matching Algebraic and Verbal Expressions

Let's play Expression Explosion! You teacher will hand out cards. Your goal is to identify the written or algebraic expression that corresponds to your card.

Record your pair of matching algebraic and written expressions.

1. How can you be sure that you have found the correct match?

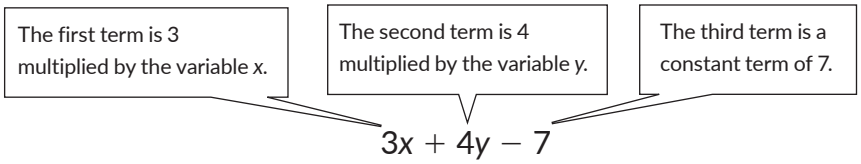
ACTIVITY
2.3 Parts of Algebraic Expressions

As you learned previously, an algebraic expression contains at least one variable and sometimes numbers and operations. A **term** of an algebraic expression is a number, a variable, or a product of numbers and variables.

WORKED EXAMPLE

Consider the expression $3x + 4y - 7$.

The expression has three terms: $3x$, $4y$, and 7. The operation between the first two terms is addition, and the operation between the second and third term is subtraction.



1. Consider two algebraic expressions: $8 + 5x$ and $8 - 5x$.

   a. Identify the terms in each algebraic expression.

   b. Identify the operation between each term in each algebraic expression.

   c. What is the same in both expressions?

   d. What is different in the expressions?

2. Identify the number of terms, and then the terms themselves, for each algebraic expression.

   a. $4 - 3x$

   b. $4a - 9 + 3a$

   c. $7b - 9x + 3a - 12$

ACTIVITY
2.4 Evaluating Algebraic Expressions

To **evaluate an algebraic expression** means to determine the value of the expression for a given value of each variable. When you evaluate an algebraic expression, you substitute the given values for the variables and then determine the value of the expression.

Don't forget to use the order of operations when evaluating an algebraic expression.

1. Write a sentence to describe the meaning of each algebraic expression. Then, evaluate the algebraic expression for the given value.

a. $3x - 4$, for $x = 10$

b. $11 - s$, for $s = 14$

c. $-10 - z$, for $z = 8$

d. $5 - \frac{y}{4}$, for $y = 2$

e. $7 - 5a$, for $a = 20$

f. $\frac{b}{4}$, for $b = 8$

CC BY NC

2. Complete each table.

a.

h	$3h - 2$
-2	
$\frac{7}{3}$	
5.1	
$\frac{5}{6}$	

b.

m	$1 - m$
0	
$\frac{2}{3}$	
4	
1.7	

c.

z	$\frac{2z}{3} + 1$
1	
2	
-5	
11	

d.

p	$0.5p$
0	
-1	
1.5	
2.5	

Talk the Talk

Expression Construction

1. Construct an algebraic expression for each description.

   a. There are 2 terms. The first term is a constant. Add the first term to the second term, which is a product of a number and a variable.

   b. There are 4 terms. The first term is a variable divided by 11. Add the first term to a second term, which is a constant. The third term is a second variable multiplied by three-fourths. Subtract the third term from the first 2 terms. The last term is a different constant. Add the last term to the other 3 terms.

   c. The cube of a variable is subtracted from a constant and then added to the square of the same variable.

   d. A number is multiplied by the square of a variable minus a number multiplied by the same variable minus a constant.

It is your turn to challenge your classmates!

2. Create a description for an algebraic expression and swap descriptions with a classmate. After you receive the algebraic expression back from your classmate, answer Question 3.

3. Did your classmate write an expression that fits your description?

Lesson 2 Assignment

Write

Complete each statement with the correct term: *algebraic expression*, *variable*, *evaluate an algebraic expression*, *constant*, *coefficient*.

1. A(n) _____ is a letter used to represent a quantity that varies.
2. A(n) _____ is a number, or quantity, that you multiply a variable by in an algebraic expression.
3. A number, or quantity, that does not change its value is called a(n) _____.
4. A mathematical phrase involving at least one variable is called a(n) _____.
5. To _____ means to determine the value of the expression.

Remember

Whenever you perform the same mathematical process over and over, you can write a mathematical phrase, called an *algebraic expression*, to represent the situation.

Practice

Write an algebraic expression to represent each situation.

1. A T-shirt costs $5.99.

   a. How much will you spend when you buy x T-shirts?

   b. Evaluate your expression to calculate the amount of money you will spend when you buy 4 shirts or 10 shirts.

Lesson 2 Assignment

2. You have 7 folders and you want to put the same number of pages in each folder.

   a. When you have a total of p pages, how many pages will be in each folder?

   b. Evaluate your expression to calculate the number of pages in each folder when you have 147 pages or 245 pages.

3. You have a coupon for $5 off your total bill at Mama's Meals on Main.

   a. How much will you pay after using the coupon when your bill was b dollars?

   b. Evaluate your expression to calculate the amount you will pay when your bill was $23.45 or $54.83.

4. You have already read two and a half hours for the Read-a-Thon.

   a. How long will you have read when you read an additional h hours?

   b. Evaluate your expression to calculate the amount of time you will have read when you read 3 or $5\frac{1}{2}$ additional hours.

Lesson 2 Assignment

Write an algebraic expression that represents each verbal expression.

5. 6 times a number plus 3

6. 4 times a number subtracted from 2

7. A number squared divided by 2 and then added to 16

8. 5 plus a number and then multiplied by 8

Identify the number of terms, and then the terms themselves, for each algebraic expression.

9. $6y + 14$

10. $7x - 3y + 12z$

11. $104a + 224b$

Evaluate each algebraic expression for the given value.

12. $34 - y^2$ for $y = 5$

CC BY NC

Lesson 2 Assignment

13. $m^3 - 18$ for $m = 2$

14. $\frac{d}{5} + 42$ for $d = -70$

Prepare

Evaluate each expression.

1. $5 \div \frac{3}{4}$

2. $0.24 \div 0.6$

3. $\frac{(14 + 8)}{2}$

4. $\frac{14}{2} + \frac{18}{2}$

5. What do you notice about the answers to Questions 3 and 4?

3 Equivalent Expressions

OBJECTIVES

- Model algebraic expressions with algebra tiles.
- Simplify algebraic expressions using algebra tiles.
- Simplify algebraic expressions using the associative, commutative, and distributive properties.
- Apply properties of operations to create equivalent expressions.
- Rewrite expressions as the product of two factors.

NEW KEY TERM

- like terms

You have evaluated numeric expressions and written and evaluated algebraic expressions.

How do you combine algebraic expressions, like you did with numeric expressions, into as few terms as possible?

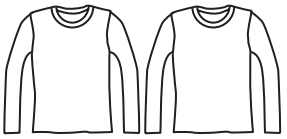
CC BY NC

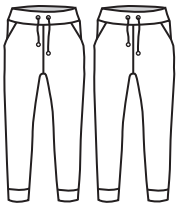
Packing for a Camping Trip

Alejandro and Eduardo, twin brothers, are packing for a weekend camping trip. They lay out the following items to go in the suitcase.

Alejandro:

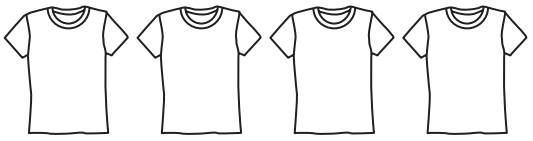








Eduardo:





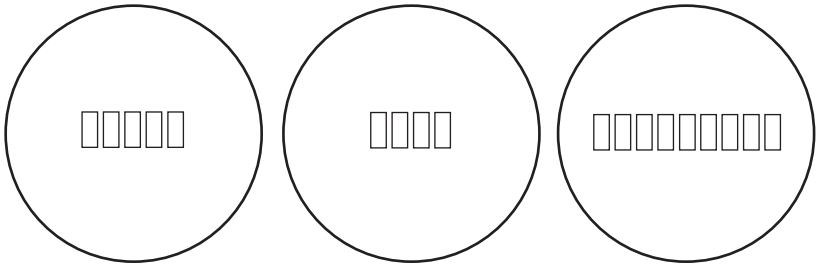


1. How many shirts and bottoms is each brother packing? Together, how many shirts and bottoms are they packing?

	Shirts	Bottoms
Alejandro		
Eduardo		
Together		

Your teacher has provided you with algebra tiles.

2. How can you use algebra tiles to model the number of shirts each brother packed and the number of shirts they packed together?



ACTIVITY 3.1

Algebra Tiles and Combining Like Terms

Ask Yourself . . .
Which tool or strategy is most efficient?

As you may have seen in the previous activity, when using algebra tiles to model situations and expressions, it is important to have a shared meaning for each differently sized algebra tile.

Your teacher will hold up each differently sized algebra tile and tell you the conventional value of each.

1. Sketch each tile and record its value.

Your addition expressions should each have 3 terms. Why?



2. Represent each numeric or algebraic expression using algebra tiles. Write an addition expression that highlights the different tiles used in the model. Then, sketch the model below the expression.

   a. 3

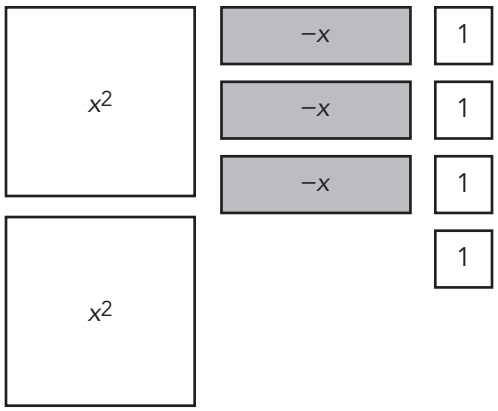
b. $3x$

c. $-3x^2$

In an algebraic expression, **like terms** are two or more terms that have the same variable raised to the same power. The coefficients of like terms can be different. Let's start our exploration of combining like terms with a review of the properties of arithmetic and algebra that you will use to combine terms.

The expression you wrote in each part of Question 2 was made up of like terms. All tiles that are the same size and have the same value represent like terms.

3. Given the algebra tile model, write an addition expression that highlights the different tiles in the model. Then, when necessary, combine like terms and write the expression using as few terms as possible.



4. Analyze the last expression you wrote in Question 3.

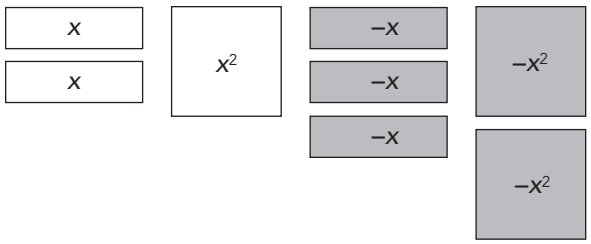
   a. How many terms are in your expression with the fewest terms? What is the relationship between this and the algebra tile model?

   b. What is the greatest exponent in the expression?

   c. What is the coefficient of x in the expression? What is the relationship between this and your algebra tile model?

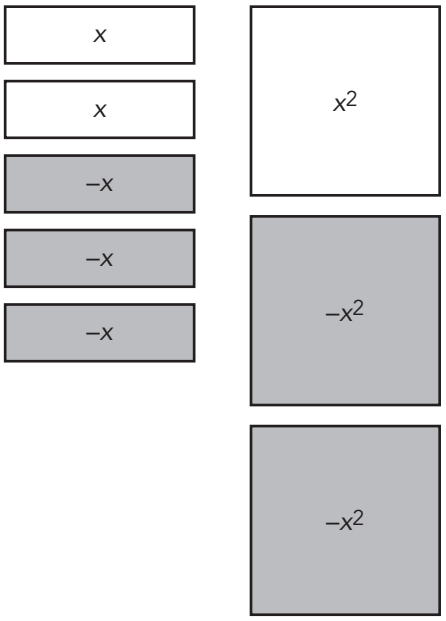


5. Consider the model.



   a. Write an addition expression that highlights the different tiles in the model.

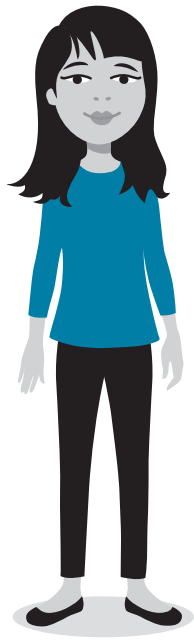
b. Rearrange the tiles to combine all of the like tiles. How many terms does your expression have now?



c. Write the new algebraic expression represented.

6. Represent the algebraic expression $3x^2 + x - 2$ using algebra tiles. How many types of tiles do you need?

So, combining like terms means to add or subtract terms with the same variables. For example, $3x + 5x$ is $8x$.



CC BY NC

Algebra tiles are helpful tools for combining like terms in algebraic expressions. However, because they only represent whole number tiles, they cannot be used to model all algebraic expressions.

7. Use what you have learned about combining like terms to rewrite each algebraic expression with as few terms as possible.

   a. $2x + 3x - 4.5x$

   b. $3\frac{1}{2}x + 2 + 4x + 1\frac{1}{4}$

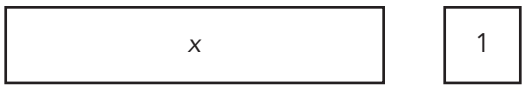
   c. $4.5x + 6x - 3.5x + 7$

   d. $5x + 2x + \frac{1}{3}x^2 - 3x$

ACTIVITY 3.2

Algebra Tiles and the Distributive Property

Let's use algebra tiles to explore rewriting algebraic expressions with the distributive property.

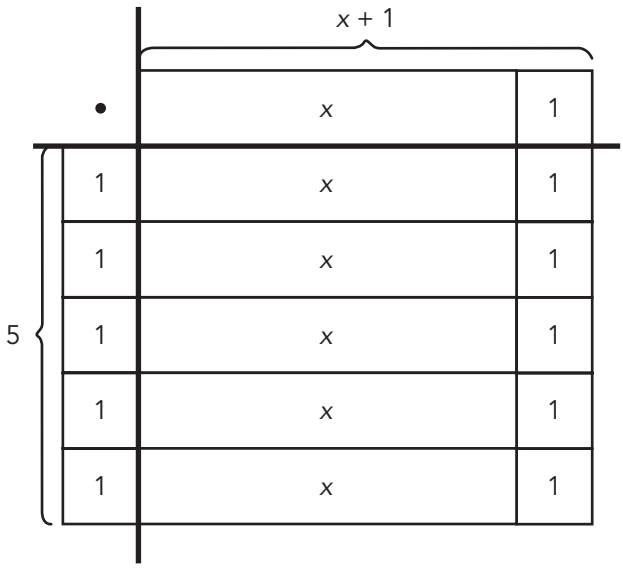


The algebra tiles model the expression $x + 1$.

When you are speaking about an algebraic expression that is grouped together with parentheses, use the words *the quantity*. For example, $2(x + 3)$ in words would be "two times the quantity x plus three."

WORKED EXAMPLE

Consider the expression $5(x + 1)$. This expression has two factors: 5 and the quantity $(x + 1)$. You can use the distributive property to rewrite this expression. In this case, multiply the 5 by each term of the quantity $(x + 1)$. The model using algebra tiles is shown.



$$5(x + 1) = 5x + 5$$



1. Analyze the parts of the mathematical expressions in the Worked Example. Explain each response.

   a. Which expression, $5(x + 1)$ or $5x + 5$, shows a product of two factors?

   b. How many terms are in $5x + 5$?

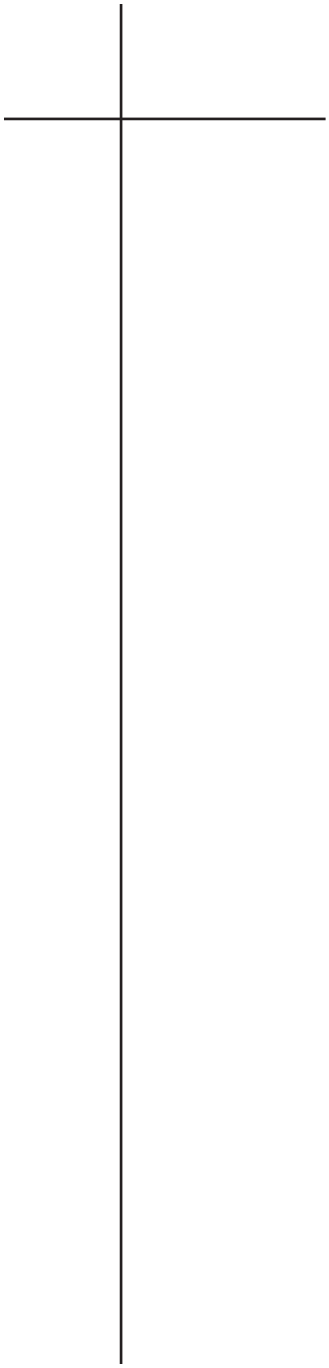
   c. The number 5 is a coefficient in which expression?

2. Create a model of each expression using your algebra tiles. Then, sketch the model and rewrite the expression using the distributive property.

   a. $4(-2x + 1)$



b. $(3x + 1)2$



CC BY NC

3. Rewrite each expression using the distributive property. Then, combine like terms when possible.

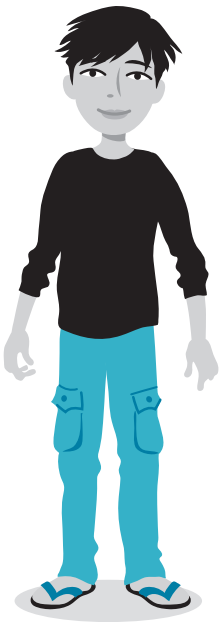
a. $\frac{2}{3}(6x + 12)$

b. $2(-x + 5) + 4(x + 7)$

c. $5x + 2(3x - 7)$

Ah, so I distribute like this: $2(x + 2)$ $2 \cdot x + 2 \cdot 2$ $2x + 4$.

d. $\frac{1}{2}(4x + 2) + 8x$



So far in this activity, you have multiplied expressions together using the distributive property. Now, let's think about how to divide expressions.

How do you think the distributive property will play a part in dividing expressions? Let's find out.

4. Consider the expression $(4x + 8) \div 4$, which you could also rewrite as $\frac{4x + 8}{4}$.

   a. First, represent $4x + 8$ using your algebra tiles. Sketch the model you create.

   b. Next, divide your algebra tile model into four equal groups. Then, sketch the model you created with your algebra tiles.

   c. Write an expression to represent each group from your sketches in part (b).

   d. Verify you created equal groups by multiplying your expression from part (c) by 4. The product you calculate should equal $4x + 8$.



Let's consider the division expression from Question 4.

To rewrite the expression, divide the denominator into both terms in the numerator.

WORKED EXAMPLE

You can rewrite an expression of the form $\frac{4x + 8}{4}$ using the distributive property.

$$\frac{4x + 8}{4} = \frac{4x}{4} + \frac{8}{4}$$

$$= 1x + 2$$

$$= x + 2$$

So, $\frac{4x + 8}{4} = x + 2$

The model you created in Question 4 is an example that shows that the distributive property can be used with division as well as with multiplication.

5. Consider the expression $\frac{2x^2 - 6x + 4}{2}$.

   a. Use algebra tiles to represent the division expression.

b. Rewrite the division expression using the distributive property. Then, simplify the expression.

$$\frac{2x^2 - 6x + 4}{2} = \frac{2x^2}{\square} - \frac{6x}{\square} + \frac{4}{\square} = \_\_\_\_\_$$

c. Verify that your answer is correct.

Trung thinks he can simplify algebraic expressions that use the distributive property with division without using algebra tiles. He wants to rewrite $\frac{6 + 3(x + 1)}{3}$ in as few terms as possible and proposes two different methods.

6. Analyze each correct method.

Method 1

$$\frac{6 + 3(x + 1)}{3} = \frac{6}{3} + \frac{3(x + 1)}{3}$$
$$= 2 + (x + 1)$$
$$= x + 3$$

Method 2

$$\frac{6 + 3(x + 1)}{3} = \frac{6 + 3x + 3}{3}$$
$$= \frac{3x + 9}{3}$$
$$= \frac{3x}{3} + \frac{9}{3}$$
$$= x + 3$$

a. Explain the reasoning used in each method.

b. Which method do you prefer. Why?

CC BY NC

ACTIVITY
3.3 Factoring Algebraic Expressions

You have used the distributive property to multiply and divide algebraic expressions by a given value. The distributive property can also be used to rewrite an algebraic expression as a product of two factors: a constant and a sum of terms.

You can write any expression as a product of two factors. In many types of math problems, you often need the coefficient of a variable to be 1. Let's explore how to use the distributive property—without algebra tiles—to rewrite expressions so that the coefficient of the variable is 1.

1. Consider the expression $3x + 6$.

   a. Identify the coefficient of the variable term.

   b. Use the distributive property to rewrite the expression as the product of two factors: the coefficient and a sum of terms.

   c. How can you check your work?

Using the distributive property to rewrite the sum of two terms as the product of two factors is also referred to as *factoring expressions*. In the expression $3x + 6$, you factored out the common factor of 3 from each term and rewrote the expression as $3(x + 2)$. In other words, you divided 3 from each term and wrote the expression as the product of 3 and the sum of the remaining factors, $(x + 2)$.

You can use the same strategy to rewrite an algebraic expression so that the coefficient of the variable is 1 even when the terms do not have common factors.

WORKED EXAMPLE

Let's rewrite the expression $4x - 7$ so the coefficient of the variable is 1.

To rewrite the expression, factor out the coefficient 4 from each term. The equivalent expression is the product of the coefficient and the difference of the remaining factors.

$$4x - 7 = 4\left(\frac{4x}{4} - \frac{7}{4}\right)$$
$$= 4\left(x - \frac{7}{4}\right)$$

Remember. . .

You can multiply or divide any expression by 1 and not change its value.

2. Use the distributive property to check that the new expression is equivalent to the original expression in the Worked Example.

3. Rewrite each expression as the product of two factors. Check your answers.

a. $4x + 5$

b. $-8x - 3$

c. $\frac{1}{2}x - 4$

d. $1.1x + 1.21$

ACTIVITY
3.4 Simplifying Algebraic Expressions

Rewrite each expression using the distributive property.

1. $\frac{32 - 4x}{4}$

2. $15x - 10$

3. $\frac{3(x + 1) + 12}{3}$

4. $2\frac{1}{2} + \frac{1}{4}x$

Rewrite each algebraic expression in as few terms as possible.

5. $3x + 5x^2 - 3x + 2x^2$

6. $4x^2 - 4x + 3x + 2x^2$

7. $7x + 5 - 6x + 2$

8. $x^2 + 5x + 4x^2 - 3x$

Rewrite each algebraic expression by applying the distributive property and then combining like terms.

9. $4(x + 5x) - 3x$

10. $2(2x - 5x) + 3(x + 3x)$

11. $3x + 5(2x + 7)$

12. $\frac{-4x + 6}{2} + 3x$

13. $3(x + 2x) + \frac{3x - 9x}{3}$

14. $2(x + 3) + 4(x + 5) - 3x$

CC BY NC

Talk the Talk

Write Right

Mr. Patel asked his class to write expressions equivalent to $7(3a + 5)$ and $8 + 3(2x + 5)$ and got five different responses for each. For each response, determine whether the students rewrote the original expression correctly. For those not rewritten correctly, describe the mistake that was made in rewriting the expression.

1. $7(3a + 5)$

   a. $10a + 12$

   b. $7(3a) + 7(5)$

   c. $21a + 5$

   d. $21a + 35$

   e. $7(8a)$

2. $8 + 3(2x + 5)$

   a. $8 + 3 \cdot 2x + 3 \cdot 5$

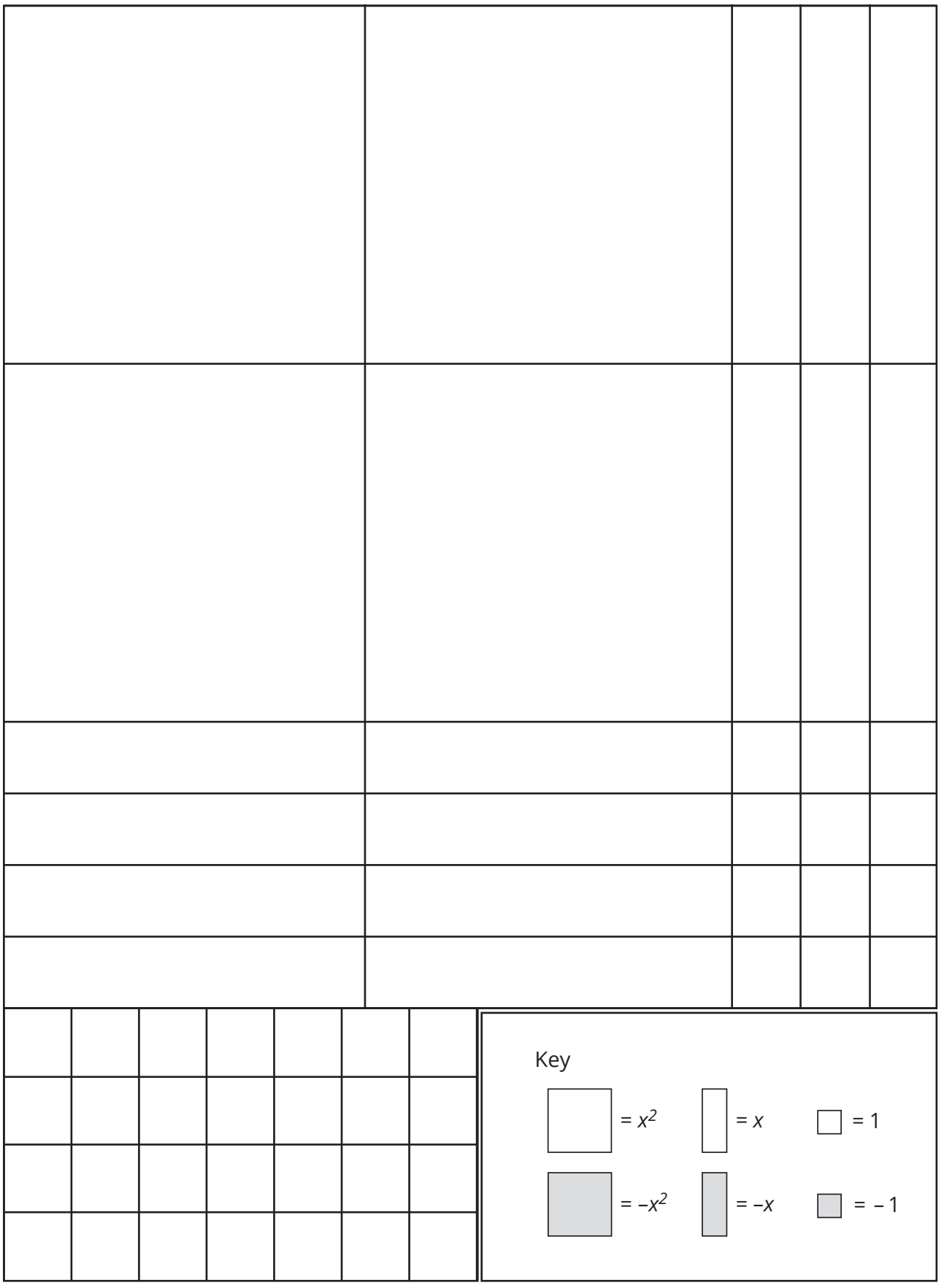
   b. $23 + 6x$

   c. $11(2x + 5)$

   d. $8 + 6x + 15$

   e. $13 + 6x$

Algebra Tiles

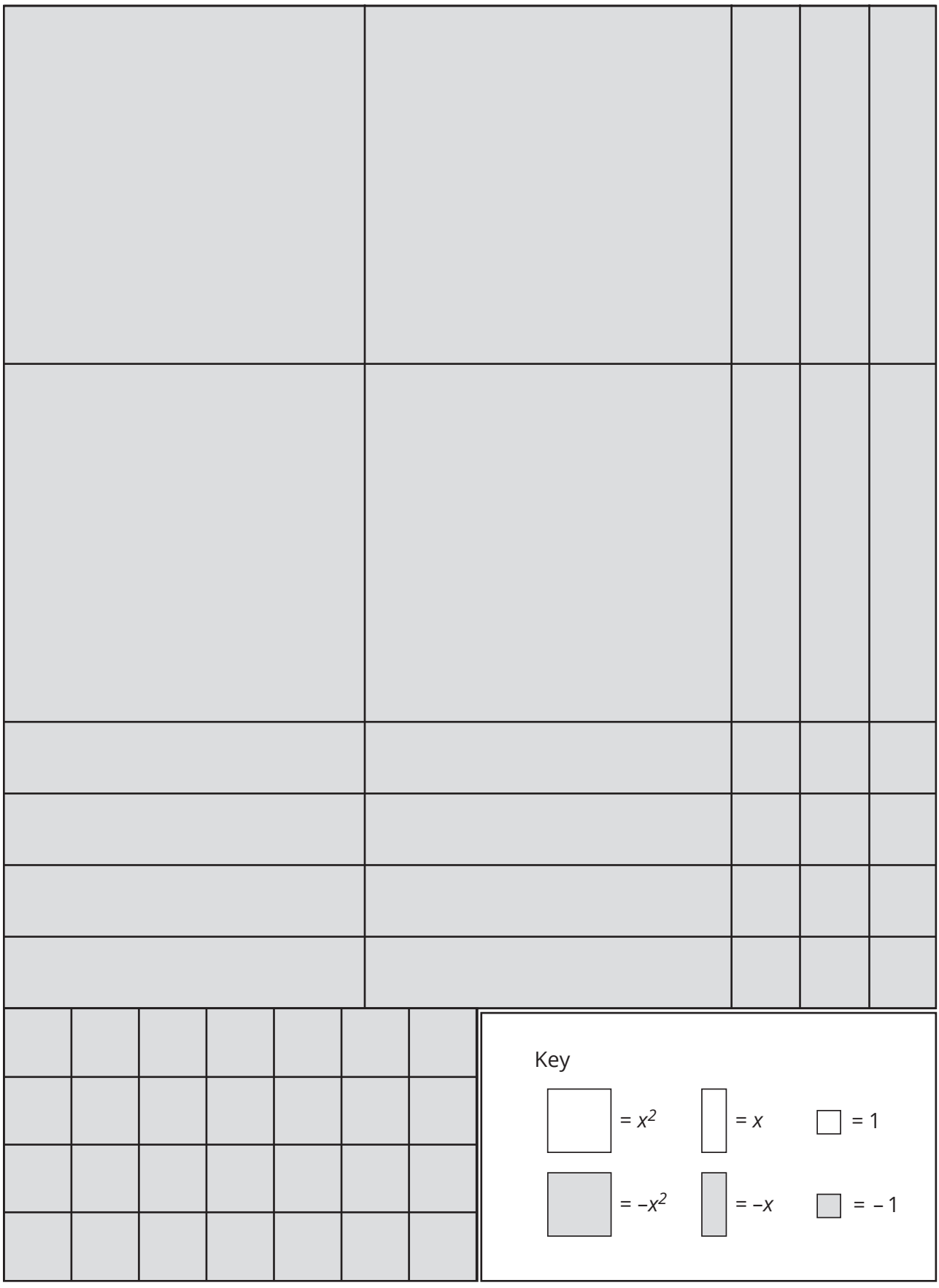


CC BY NC

Why is this page blank?

So you can cut out the algebra tiles on the other side

Algebra Tiles



CC BY NC

Why is this page blank?

So you can cut out the algebra tiles on the other side

Lesson 3 Assignment

Write

Describe three different ways that you can use the distributive property to rewrite expressions. Provide an example for each.

Remember

To rewrite an algebraic expression with as few terms as possible, use the properties of arithmetic and the order of operations.

An algebraic expression containing terms can be written as the product of two factors by applying the distributive property.

Practice

1. Represent each algebraic expression by sketching algebra tiles. Rewrite the expression with a lower number of terms, when possible.

   a. $x^2 + 2x + 5$

   b. $-x^2 + 3x + 1 - x$

Lesson 3 Assignment

2. Rewrite each expression by combining like terms.

   a. $-4.5x + (6 - 3.5x) + 7$

   b. $\left(\frac{2}{3}x + \frac{5}{8}x + \frac{1}{4}\right) + \left(\frac{1}{4}x + \frac{1}{2}\right)$

3. Paola is going on an overnight family reunion camping trip. She is in charge of bringing the wood for the campfire. She will start the fire with 6 logs and then plans to add 3 logs for each hour the fire burns.

   a. Represent the number of logs she will use as an algebraic expression.

   b. Suppose the family decides to stay for 2 nights next year. Write the expression for the number of logs they would need for 2 nights.

Lesson 3 Assignment

c. Create a model of the situation in part (b) using your algebra tiles and then sketch the model.



d. Rewrite the expression in part (c) using as few terms as possible.

e. Paola's cousin believes they will only need one-third of the firewood Paola brings for one night. Represent this as an expression and then use the distributive property to rewrite the expression.

4. Rewrite each expression by applying the distributive property and combining like terms.

a. $-7(-2x + 1) + 5(x + 4)$

b. $9x + 6 + \frac{12 + 16x}{4}$

c. $\frac{6(-x + 1) + 30}{6}$

CC BY NC

Lesson 3 Assignment

5. Rewrite each expression as a product of two factors so that the coefficient of the variable is 1.

   a. $-6x + 7$

   b. $\frac{2}{3}x + 8$

Prepare

Determine which pairs of ratios are equivalent. Explain how you know.

1. 5 : 7 and 100 : 140

2. 42 : 48 and 14 : 15

3. 105 : 100 and 20 : 21

4. 9 : 12 and 60 : 80

4 Verifying Equivalent Expressions

OBJECTIVES

- Compare expressions using properties of numbers and operations, tables, and graphs.
- Identify when two expressions are equivalent.
- Determine whether two expressions are equivalent.

NEW KEY TERM

- equivalent expressions

You know how to use the distributive property and combine like terms to write equivalent expressions.

How can you determine whether two given expressions are equivalent?

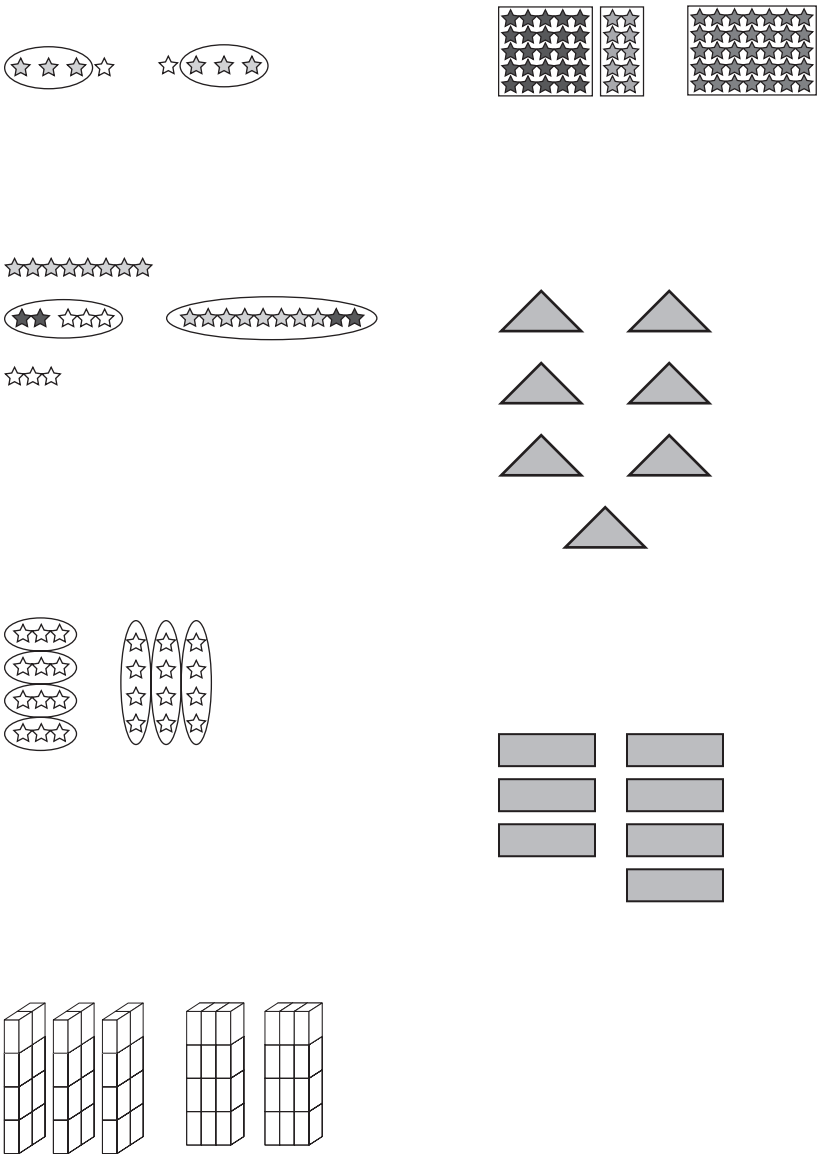
Getting Started

Property Sort

Cut out the Property Cards located at the end of the lesson.

On each card is one representation of a property of numbers or operations that you have used in the past to rewrite and evaluate numeric expressions.

1. Sort the cards according to the property named or illustrated on the cards. Create a table that shows your final sorting.



ACTIVITY
4.1 Multiple Methods to Demonstrate the Equivalence of Expressions

Two algebraic expressions are **equivalent expressions** when you are able to substitute any values for the variables and the results are equal.

While it's not realistic to test each expression for every possible value for the unknown, you can examine the characteristics of each expression in the different representations:

- rewritten expressions using the properties
- a table of values
- a graph of both expressions
- modeling with algebra tiles

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

Let's explore each representation.

Consider the two expressions $2(x + 2) + 3x$ and $5x + 4$.

1. Use a table to evaluate each expression for different values of the variable.

   a. Complete the table of values for each value of x.

x	$2(x + 2) + 3x$	$5x + 4$
0		
1		
2		
3		

   b. What can you determine based on the values in the table?

c. What would you need to know to be able to verify that the two expressions are equivalent?

2. Rewrite the given expression and identify the property applied at each step.

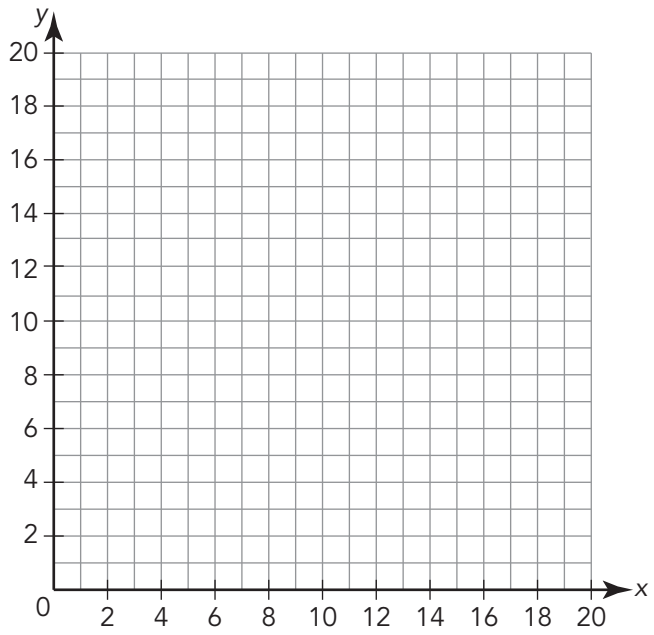
$2(x + 2) + 3x$	given
$= 2x +$ _____ $+ 3x$	
$=$ _____ $x + 4$	combine like terms/addition

3. Are the two expressions from the table equivalent? Explain.

You can also use a graph to determine or verify whether two expressions are equivalent.

4. Use the table of values to sketch the graph of both expressions on the coordinate plane.

   a. Plot the values for each expression on the coordinate plane. Use a ☐ to represent the values from the first expression and a △ to represent the values from the second expression. Then, connect the results for each expression with a line.

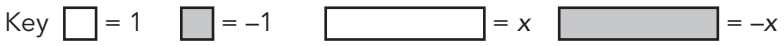


   b. How does the graph demonstrate that the two expressions are equivalent?



CC BY NC

Lastly, you can use algebra tiles to model each expression to determine whether the two expressions are equivalent.



5. Model each algebraic expression using algebra tiles. Draw your model.

6. How do the algebra tile models demonstrate that the two expressions are equivalent?

ACTIVITY
4.2 Determining Whether Expressions Are Equivalent

Let's consider the expressions $2x + 5$ and $2(x + 5)$.

1. Use a table to evaluate each expression for different values of the variable.

   a. Complete the table of values for each value of x.

x	$2x + 5$	$2(x + 5)$
0		
2		
4		
5		

   b. What can you determine, based on the values in the table?

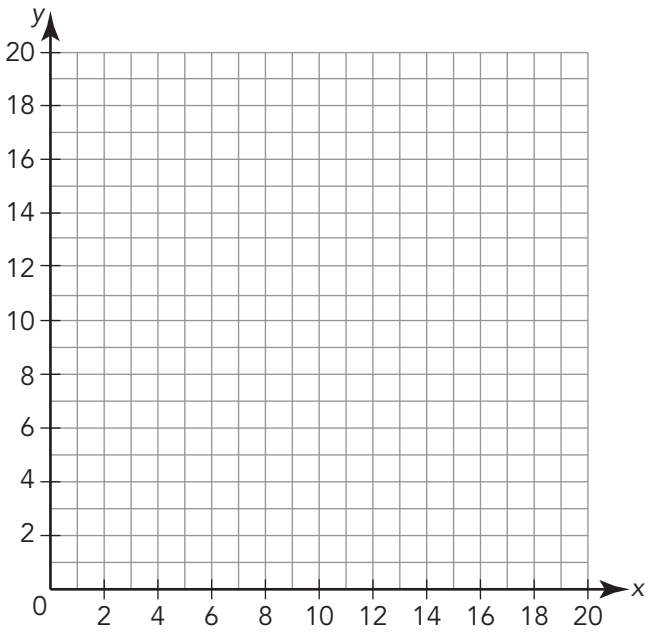
2. Use the distributive property to rewrite the second expression.

CC BY NC

3. Are the two expressions equivalent? Explain your reasoning.

4. Use the table of values to sketch the graph of both expressions on the coordinate plane.

   a. Plot the values for each expression on the coordinate plane. Use a ☐ to represent the values from the first expression and a △ to represent the values from the second expression. Then, connect the results for each expression with a line.



   b. What does the graph tell you about the equivalence of the two expressions?

5. Model each algebraic expression using algebra tiles. Draw your model.

6. How do the algebra tile models demonstrate whether the two expressions are equivalent?

For each pair of expressions, use a table, the properties of numbers and operations, a graph, and algebra tiles to determine whether the expressions are equivalent.

7. $(3x + 8) + (6 - x)$ and $4x + 14$

a.

x	$(3x + 8) + (6 - x)$	$4x + 14$
0		
1		
2		

CC BY NC

b. $(3x + 8) + (6 - x)$ | given

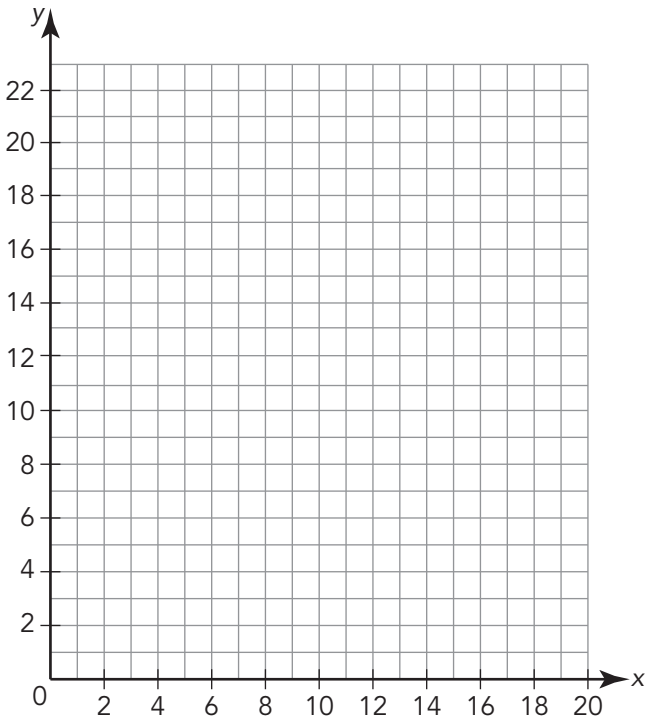
$= 3x + (8 + 6) - x$ | associative property of addition

$= 3x + \_\_\_\_\_ - x$ |

$= \_\_\_\_\_ + 3x - x$ | commutative property of addition

$= \_\_\_\_\_$ |

c.



d. Model each algebraic expression using algebra tiles. Draw your model.

Remember . . .

Just as with two-color counters, tiles of the same shape and opposite color create a zero pair.

e. Are the two expressions from the table equivalent? Explain.

8. $x + 3(2x + 1)$ and $7x + 3$

a.

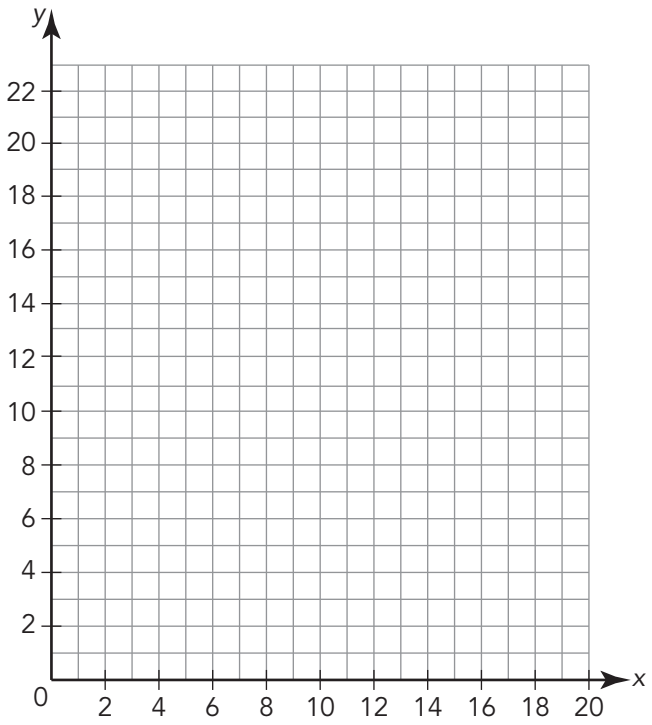
x	$x + 3(2x + 1)$	$7x + 3$
0		
1		
2		

b. $x + 3(2x + 1)$ _____ given _____

$= x +$ _____ _____

$=$ _____ _____

c.



d. Model each algebraic expression using algebra tiles. Draw your model.

e. Are the two expressions equivalent? Explain using all four representations.

Talk the Talk

Property Management

For each step of the simplification of the expression, identify the property or operation applied.

	Number Property or Operation
1. $10 \cdot 4x + 3(2x + 1)$	given
$= (10 \cdot 4)x + 3(2x + 1)$	
$= 40x + 3(2x + 1)$	multiplication
$= 40x + 6x + 3$	
$= 46x + 3$	
2. $20 + (6 + x) + 7$	given
$= 20 + (x + 6) + 7$	
$= 20 + x + (6 + 7)$	associative property of addition
$= 20 + x + 13$	
$= x + 20 + 13$	
$= x + 33$	
3. $7x + \frac{12x - 8}{4} + 5x$	given
$= 7x + 3x - 2 + 5x$	
$= 10x - 2 + 5x$	
$= 10x + 5x - 2$	
$= 15x - 2$	

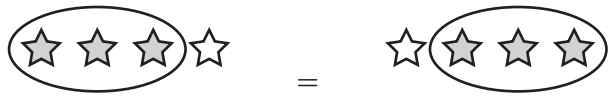
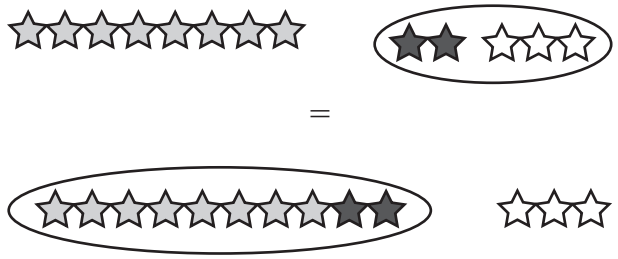
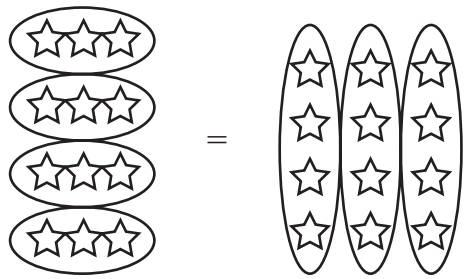
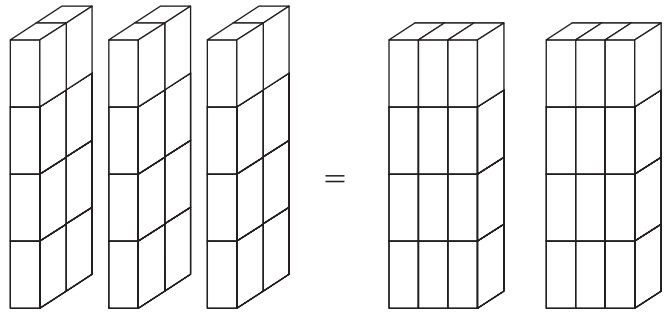
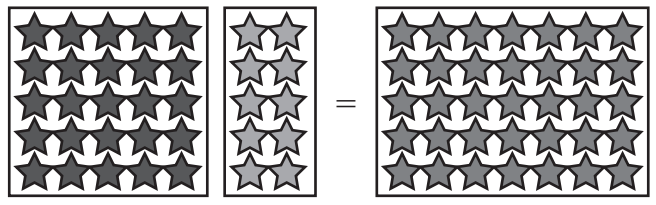
PROBLEM SOLVING



4. Rewrite $\frac{2(x + 5) - 4}{2} - x$ using the fewest terms possible. Justify each step with a property or operation.

5. How can you use another representation to check that your answer in Question 4 is equivalent to $\frac{2(x + 5) - 4}{2} - x$?

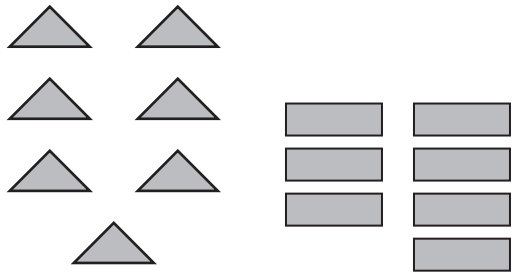
Property Cards

commutative property of addition	associative property of multiplication	$a\,(b + c) = ab + ac$	
distributive property	commutative property of multiplication	$(13 \cdot 2) \cdot 5 = 13 \cdot (2 \cdot 5)$	
$6 \cdot 5 = 5 \cdot 6$	associative property of addition	$5\,(10 + 2) = 5 \cdot 10 + 5 \cdot 2$	
$a + b = b + a$	$x \cdot y = y \cdot x$	$(3 + 4) + 6 = 3 + (4 + 6)$	
$2 + 3 = 3 + 2$	$(j \cdot k) \cdot l = j \cdot (k \cdot l)$	$(l + m) + n = l + (m + n)$	

CC BY NC

Why is this page blank?

So you can cut out the property cards on the other side

identity property of addition	additive inverse property		
$7 + -7 = 0$	$7 + 0 = 7$	$a + 0 = a$	$a + -a = 0$

CC BY NC

Why is this page blank?

So you can cut out the property cards on the other side

Lesson 4 Assignment

Write

For 1–7, match each term to the best definition.

1. commutative property of addition
2. commutative property of multiplication
3. associative property of addition
4. associative property of multiplication
5. like terms
6. additive inverse property
7. identity property of addition

a. For any numbers a and b, $a + b = b + a$

b. For any numbers a, b, and c, $(ab)c = a(bc)$

c. Two or more terms that have the same variable raised to the same power

d. For any numbers a and b, $ab = ba$

e. For any numbers a, b, and c, $(a + b) + c = a + (b + c)$

f. For any number a, $a + 0 = a$

g. For any number a, $a + -a = 0$

8. Explain what it means for two expressions to be equivalent.

Remember

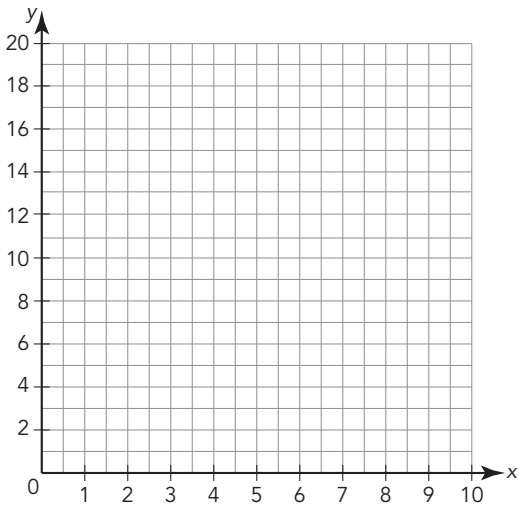
To determine whether two expressions are equivalent, you can create a table of values, graph the expressions rewrite the expressions using properties of numbers and operations, or model the expressions using algebra tiles.

Lesson 4 Assignment

Practice

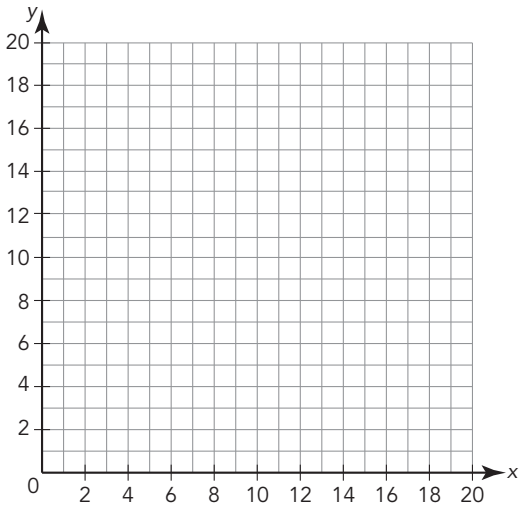
Determine whether the expressions are equivalent. Select a tool (properties, a table, a graph, or algebra tiles) in each problem to verify your answer.

1. $2(3x + 2) - 2x$ and $4x + 2$



Lesson 4 Assignment

2. $1 + 3(3 + x)$ and $4(3 + x)$

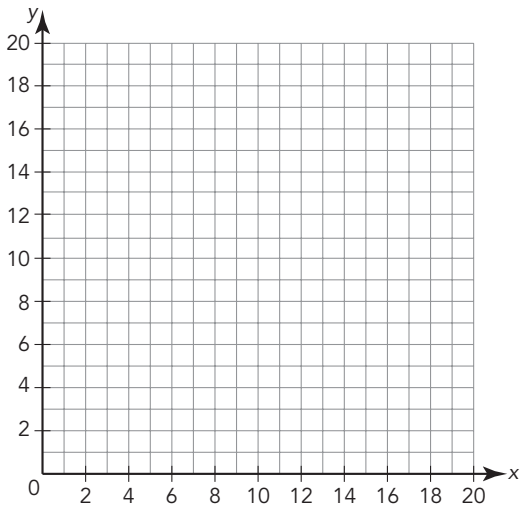


CC BY NC

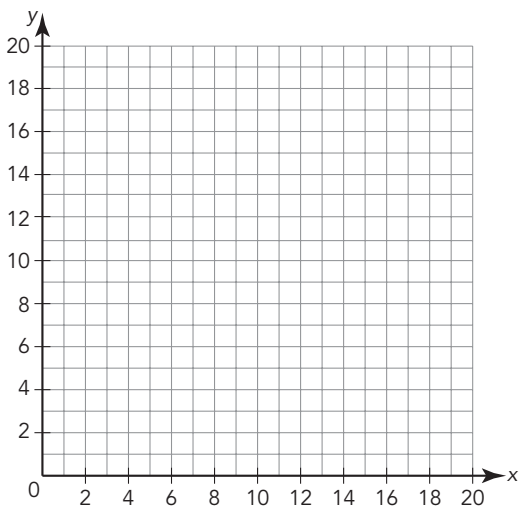
Lesson 4 Assignment

Determine whether the expressions are equivalent. Use the properties, a table, and a graph in each problem to verify your answer.

3. $2x + 1$ and $2\left(x + \frac{1}{2}\right)$



4. $\frac{6x + 9}{3} + 4$ and $2(x + 3.5)$



Lesson 4 Assignment

Prepare

Rewrite each number as an addition, subtraction, multiplication, or division expression. use each operation once.

1. 24

2. $\frac{1}{2}$

3. 0

4. 100

CC BY NC

TOPIC 1 SELF-REFLECTION

Name: _____

Expressions

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Expressions* topic by:

TOPIC 1: *Expressions*	Beginning of Topic	Middle of Topic	End of Topic
determining whether two expressions are equivalent using the order of operations, including expressions with whole number exponents.	○	○	○
recognizing that an algebraic expression is a mathematical phrase involving at least one variable and/or numbers and operation symbols.	○	○	○
translating written phrases into algebraic expressions and vice versa.	○	○	○
evaluating algebraic expressions, including those that model real-world problems.	○	○	○
generating equivalent expressions using the properties of operations.	○	○	○
determining whether two expressions are equivalent using concrete and pictorial models as well as algebraic representations.	○	○	○

continued on the next page

CC BY NC

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Expressions* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?



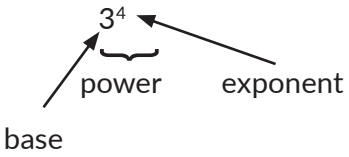
TOPIC 1 SUMMARY

Expressions Summary

LESSON 1 Evaluating Numeric Expressions

Repeated multiplication can be represented as a power. A power has two elements: the base and the exponent. The base of a power is the factor that is multiplied repeatedly in the power, and the exponent of the power is the number of times the base is used as a factor.

$3 \times 3 \times 3 \times 3 = 3^4$



You can read this power in different ways: *3 to the fourth power*, *3 raised to the fourth power*, or *3 to the fourth*.

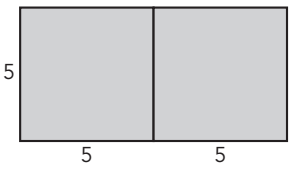
To calculate the square of a number, multiply the number by itself. The squares of integers are called **perfect squares**. For example, 9 is a perfect square because $3 \times 3 = 9$. Another way to write this equation is $3^2 = 9$. You can read 3^2 as "3 squared."

To calculate the cube of a number, use the number as a factor three times. A **perfect cube** is the cube of an integer. For example, 216 is a perfect cube because $6 \times 6 \times 6$, or 6^3, is equal to 216. You can read 6^3 as "6 cubed."

To **evaluate a numeric expression** means to simplify the expression to a single numeric value.

For example, consider the numeric expression $2 \cdot 5^2$, as represented in the model shown.

$5^2 = 25$, and $2 \cdot 5^2 = 2 \cdot 25$, or 50.



Therefore, $2 \cdot 5^2$ has a value of 50.

The **order of operations** is a set of rules that ensures the same result every time an expression is evaluated.

1. Evaluate expressions inside parentheses or grouping symbols.
2. Evaluate exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

NEW KEY TERMS

- perfect square
- perfect cube [cubo perfecto]
- evaluate a numeric expression [evaluar una expresión numérica]
- order of operations [orden de las operaciones]
- variable [variable]
- algebraic expression [expresión algebraica]
- coefficient [coeficiente]
- term [término]
- evaluate an algebraic expression [evaluar una expresión algebraica]
- like terms
- equivalent expressions [expresiones equivalentes]

CC BY NC

For example, evaluate the expression $12 \div (4 + 2) + 4^2$ using the order of operations.

$12 \div (6) + 4^2$	Evaluate the expression in parentheses.
$12 \div 6 + 16$	Evaluate the exponent.
$2 + 16$	Divide from left to right.
18	Add from left to right.

You can use prime factorization to write equivalent expressions.

For example, you can use prime factorization with powers to rewrite $5^2 + 5 \cdot 75$.

Step 1: Rewrite 75 using prime factorization with powers.	$75 = 3 \cdot 5 \cdot 5$ $= 3 \cdot 5^2$
Step 2: Substitute $3 \cdot 5^2$ for 75 in the original expression.	$5^2 + 5 \cdot 3 \cdot 5^2$
Step 3: Use the commutative property to rewrite the multiplication.	$5^2 + 3 \cdot 5 \cdot 5^2$
Step 4: Rewrite $5 \cdot 5^2$ as 5^3.	$5^2 + 3 \cdot 5^3$

LESSON 2 Introduction to Algebraic Expressions

In mathematics, letters are often used to represent quantities that vary. These letters are called **variables**, and they help you write algebraic expressions to represent situations. An **algebraic expression** is an expression that has at least one variable.

For example, if a school lunch costs $2.25 for each student, you can write an algebraic expression to represent the total amount of money collected for any number of students buying school lunches.

The variable s can represent the unknown number of students buying school lunches. The algebraic expression is $2.25s$.

A number that is multiplied by a variable in an algebraic expression is called the numerical **coefficient**. The coefficient in the expression written above is 2.25.

A **term** of an algebraic expression is a number, variable, or product of numbers and variables.

For example, consider the expression $3x + 4y - 7$. The expression has three terms: $3x$, $4y$, and 7. The operation between the first and second is addition, and the operation between the second and third term is subtraction. There are two terms with variables, and the third term is a constant term of 7.

To **evaluate an algebraic expression** means to determine the value of the expression for a given value of each variable. When you evaluate an algebraic

expression, you substitute the given values for the variables and then determine the value of the expression.

For example, evaluate $10 - \frac{x}{3}$ for $x = 9$.

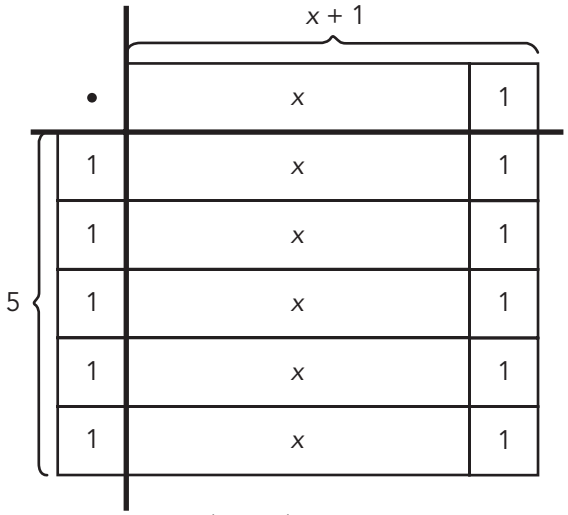
$10 - \frac{9}{3}$ Substitute the given value for x.

$10 - 3 = 7$ Use the order of operations to evaluate the expression.

LESSON 3 Equivalent Expressions

In an algebraic expression, **like terms** are two or more terms that have the same variable raised to the same power. The numerical coefficients of like terms may be different. You can combine like terms in algebraic expressions by adding or subtracting terms with the same variables. For example, $3x + 5x$ combines to make $8x$.

Algebraic expressions can be rewritten using the distributive property.



$5(x + 1) = 5x + 5$

For example, consider the expression $5(x + 1)$, which has two factors: 5 and the quantity $(x + 1)$. In this case, multiply the 5 by each term of the quantity $(x + 1)$. The model using algebra tiles is shown.

You can also rewrite an expression of the form $\frac{4x + 8}{4}$ using the distributive property.

$$\frac{4x + 8}{4} = \frac{4x}{4} + \frac{8}{4}$$
$$= 1x + 2$$
$$= x + 2$$

The distributive property can also be used to rewrite an algebraic expression as a product of two factors: a constant and a sum of terms. This is also referred to as *factoring expressions*.

To rewrite the expression $4x - 7$ so the coefficient of the variable is 1, factor out the coefficient 4 from each term. The equivalent expression is the product of the coefficient and the sum of the remaining factors.

$$4x - 7 = 4\left(\frac{4x}{4} - \frac{7}{4}\right)$$
$$= 4\left(x - \frac{7}{4}\right)$$

CC BY NC

LESSON 4

Verifying Equivalent Expressions

Two algebraic expressions are **equivalent expressions** if, when the same value is substituted for the variable in each expression, the results are equal.

Creating a table of values, graphing the expressions, modeling with algebra tiles, or rewriting the expressions using number properties can help you to determine if two expressions are equivalent.

For example, consider the two expressions $2(x + 2) + 3x - 2x$ and $5x + 4$.

x	$2(x + 2) + 3x$	$5x + 4$
0	4	4
1	9	9
2	14	14
3	19	19

The table of values for each value of x shows that the two expressions are equivalent.

Using the table of values to graph the two expressions results in two lines that lie on top of one another, showing that the expressions are equivalent.

You can also rewrite the given expression to show that the two expressions are equivalent.

$2(x + 2) + 3x$ given

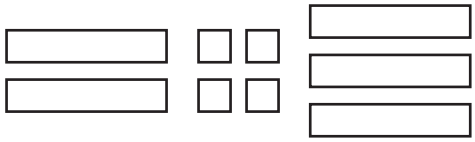
$2x + 4 + 3x$ distributive property

$5x + 4$ combine like terms

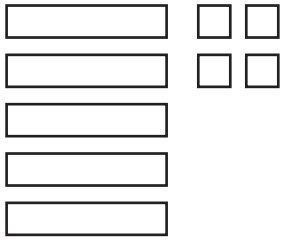
You can model each expression with algebra tiles to determine whether the two expressions are equivalent.

Key $\square = 1$ $\blacksquare = -1$ $\rule{2em}{0.8em} = x$ $\blacksquare\!\!\rule{2em}{0.8em} = -x$

$2(x + 2) + 3x$



$5x + 4$



Both models have 5 of the x tiles and 4 of the unit tiles. So, the expressions are equivalent.

GRADE 6

TOPIC 2



Air traffic controllers use radar to track tens of thousands of commercial airline flights every day. Controllers use quadrants to identify the locations, altitudes, and speeds of the many different flights.

Equations and Inequalities

LESSON 1 Reasoning with Equal Expressions 767

LESSON 2 Solving One-Step Addition Equations 795

LESSON 3 Solving One-Step Multiplication Equations 811

LESSON 4 Solving Equations to Solve Problems 835

LESSON 5 Solving Inequalities with Inverse Operations 847

CC BY NC

1 Reasoning with Equal Expressions

OBJECTIVES

- Compose and decompose numeric and algebraic equations.
- Substitute values into equations to determine whether they make the equation true.
- Construct and analyze equations using properties of equality.
- Analyze, write, and graph inequalities.
- Determine the number of solutions of an equation or inequality.

NEW KEY TERMS

- equation
- reflexive property of equality
- solution
- addition property of equality
- subtraction property of equality
- multiplication property of equality
- division property of equality
- symmetric property of equality
- zero property of multiplication
- identity property of multiplication
- identity property of addition
- graph of an inequality
- solution set of an inequality

You have learned about both numeric and algebraic expressions and how they describe situations and relationships among quantities.

What properties do equal expressions have, and how can you use these properties to reason about solutions?

CC BY NC

Getting Started

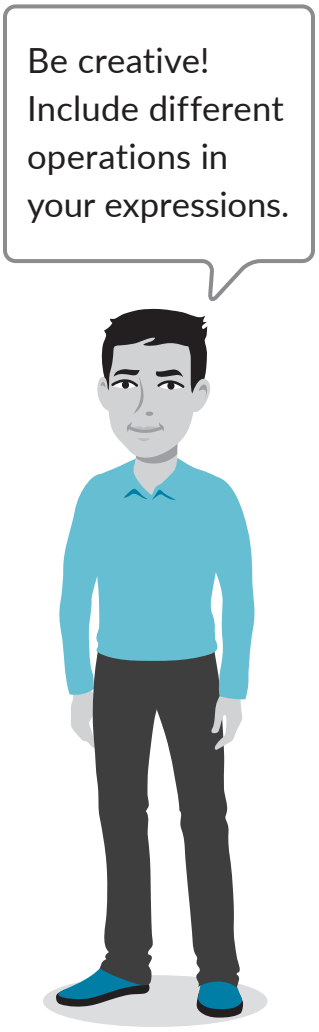
The Same But Different

1. Write different expressions equal to 4.

_____ = 4

4 = _____

4 = _____



2. Now write different expressions equal to 4 + 5.

4 + 5 = _____

_____ = 4 + 5

4 + 5 = _____

3. What can you do to one of the expressions you wrote in Question 1 to make it equal to one of the expressions you wrote in Question 2?

ACTIVITY 1.1

Using Substitution to Understand Equality

An **equation** is a statement of equality between two expressions. An equation can contain numbers, variables, or both in the same mathematical sentence.

Consider the equation $8 + 4 = \_\_\_\_\_ + 5$. It has an unknown number.

One way to determine the unknown number is to rewrite the expressions on both sides of the equals sign until they match.

Consider each reasoning strategy that is used to determine the unknown number in $8 + 4 = \_\_\_\_\_ + 5$.

The **reflexive property of equality** says that when both sides of an equation look exactly the same, their values are equal.

Kaya



The equals sign tells me to perform the operation on the left in the equation $8 + 4 = \_\_\_ + 5$.

$$8 + 4 = 12 + 5$$

$$12 + 5 = 17$$

Therefore, the unknown number is 17.

Natalia



I can determine the unknown number in $8 + 4 = \_\_\_ + 5$ by rewriting the expression on the left. I can take 1 from 8 and give it to the 4 and keep the value of the expression the same.

$$(8 - 1) + (4 + 1) = \_\_\_ + 5$$

$$7 + 5 = \_\_\_ + 5$$

Therefore, the unknown number is 7.

CC BY NC

Samantha

I can determine the unknown number in $8 + 4 = \_\_\_ + 5$ by rewriting both expressions.

$$8 + 4 = \_\_\_ + 1 + 4$$

$$7 + 1 + 4 = \_\_\_ + 1 + 4$$

Therefore, the unknown number is 7.

1. What is the unknown number in the equation $8 + 4 = \_\_\_ + 5$? Explain why this makes sense.

2. Explain the error in Kaya's reasoning.

3. How are Natalia's reasoning and Samantha's reasoning similar? How are they different?

4. Consider the equation $31 + 67 = \_\_\_\_\_ + 12$.

   a. Determine the unknown number by rewriting the expressions on either side of the equals sign until they match.

   b. How can you check your answer to make sure it is correct?

   c. What number property or properties did you use when determining the unknown number?

5. Use your number sense reasoning to determine each unknown number. Show your work.

   a. $85 + 45 = \_\_\_\_\_ + 60$

   b. $9 + 23 = \_\_\_\_\_ + 14$

CC BY NC

ACTIVITY
1.2 Solutions from a Set

Recall that an expression in mathematics is a number or a combination of numbers and operations. An algebraic expression is an expression that has at least one variable.

Equations come in many forms. Because expressions are either numeric or algebraic, equations can be made of just numbers or both numbers and variables. Both expressions and equations can also be represented verbally, using words instead of symbols.

Consider the difference between an equation and an expression.

The expressions 8 and $(-2)^2 + 4$ are both *numeric expressions*. When you write an equal sign between two numeric expressions, you create a statement of equality. For example, $8 = (-2)^2 + 4$ is an equation.

Consider $7x + 3 - 2x$ and -10. They are both expressions. However, $7x + 3 - 2x$ is an *algebraic expression*, and -10 is a numeric expression. When you put an equal sign between an algebraic and a numeric expression, you create a statement of equality. So, $7x + 2 - 2x = -10$ is also an equation.

You can write an equal sign between two algebraic expressions. The expressions $10x - 5$ and $2x + 35$ are both algebraic expressions. When you put an equal sign between two algebraic expressions, you create a statement of equality. So, $10x - 5 = 2x + 35$ is also an equation.

1. Identify whether the given represents a numeric expression, an algebraic expression, or an equation.

   a. $\frac{1}{4}(x - 12) + 6$

   b. Three more than a number squared is nineteen.

   c. $-3x + 2$

   d. $-30 \div 6 + 2 = -3$

e. Eight less than negative seven.

f. $-2 \cdot 7 + 3$

g. $\frac{1}{3}x - 6 = 9$

Equations are statements—they may be always true, never true, or true for certain values of the variable.

Always True	Never True	Sometimes True
$6 = 10 - 4$ $x = x$	$10 = 20$ $x = x + 2$	$x = 5$ $x + 2 = 12$

For equations to be sometimes true, the equation is true for only certain values of the variable.

When you determine that an equation is never true, you can make it a true statement by using the symbol $\neq$. For example, $10 = 20$ should be written as $10 \neq 20$.

2. Create at least five different kinds of equations using the list of expressions. State whether each equation is numeric or algebraic.

Expressions
$6 - 2$
x
4
$2(x + 1)$
$0 + 8$
$2x$
$2x + 2$
8
$3x$
0

3. Identify your equations that are always true, never true, and those equations where you don't yet know whether they are true or false. Explain your reasoning.

A **solution** to an equation is any value for a variable that makes the equation true.

4. Sets of values are given. For each set, decide which value(s), if any, makes each of your equations from Question 2 true. Show your work.

   a. {1, 2, 3, 4}

   b. {1, 3, 5, 7, 9}

   c. {0}

5. Use the list of given expressions to write the type of equation described.

   a. Write an equation with variables that has no possible solution. Explain why the equation has no solution.

   b. Write an equation with variables that is true no matter what number is substituted for the variable. Explain why there are an infinite number of solutions.

CC BY NC

ACTIVITY
1.3 Using Properties of Equality

The **addition property of equality** states that when two values a and b are equal, and when you add the same value c to each, the sums are equal.

The **subtraction property of equality** states that when you subtract the same value c from equal values a and b, the differences are equal.

Properties of equality	For all numbers a, b, and c
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.

1. Suppose you have the equation $x = 15$.

   a. Use the addition property of equality to write at least 3 equations that have the same solution.

   b. Use the subtraction property of equality to write at least 3 equations that have the same solution.

2. Suppose you have the equation $x + 5 = 1 + 9$.

   a. Use the addition property of equality to write at least 3 equations that have the same solution.

   b. Use the subtraction property of equality to write at least 3 equations that have the same solution.

The **multiplication property of equality** states that when two values a and b are equal, and when you multiply each by the same value c, the products are equal.

The **division property of equality** states that when you divide equal values a and b by the same value c, the quotients are equal. The division property of equality is true only when c is not equal to 0.

Properties of equality	For all numbers a, b, and c
Multiplication property of equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

3. Suppose you have the equation $x = 5$.

   a. Use the multiplication property of equality to write at least 3 equations that have the same solution.



b. Use the division property of equality to write at least 3 equations that have the same solution.

Properties of equality are logical rules that allow you to maintain balance and rewrite equations.

4. Suppose you have the equation $\frac{1}{2}x = 10$.

   a. Use the multiplication property of equality to write at least 3 equations that have the same solution.

   b. Use the division property of equality to write at least 3 equations that have the same solution.

5. Describe how you can check the solutions of the equations you wrote in Questions 1 and 3.

ACTIVITY
1.4 Identifying Solutions

Cut out the cards at the end of the lesson. There are Equation Cards and Solution Cards. The Solution Cards are shaded green.

1. Match the Equation Cards with the Solution Cards. Explain how you identified each solution.

The **symmetric property of equality** states that when $a = b$, then $b = a$. So, $x = 3$ is the same as $3 = x$.

2. Which equation(s) have no solution? Explain how you know.

3. Which equation(s) have an infinite number of solutions? Explain how you know.

Equations that have an infinite number of solutions are equations that are true no matter what value you assign to the variable. These kinds of equations often describe important properties of numbers.

Consider each property.

- The **zero property of multiplication** states that the product of any number and 0 is 0.
- The **identity property of multiplication** states that the product of any number and 1 is the number.
- The **identity property of addition** states that the sum of any number and 0 is the number.

4. Study the Equation Cards.

   a. Which equation states the zero property of multiplication?

   b. Which equation states the identity property of multiplication?

c. Which equation states the identity property of addition?

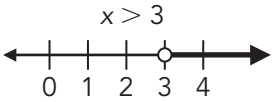
5. Three of the Solution Cards did not match any of the Equation Cards. Write equations that have those values as solutions.

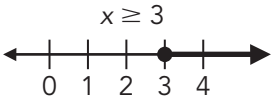
CC BY NC

ACTIVITY
1.5 Inequalities

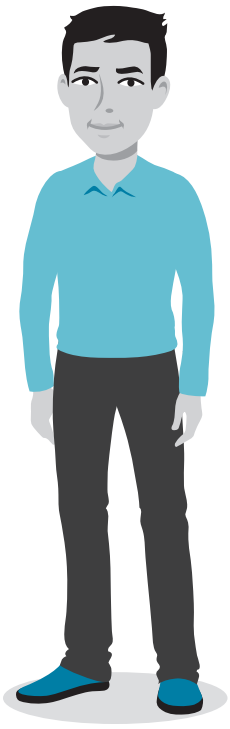
You can use a number line to represent inequalities. The **graph of an inequality** in one variable is the set of all points on a number line that make the inequality true. The set of all points that make an inequality true is the **solution set of the inequality**.

1. Consider the graphs of the inequalities $x > 3$ and $x \geq 3$.





Why does one graph show an open point and the other one a closed point?

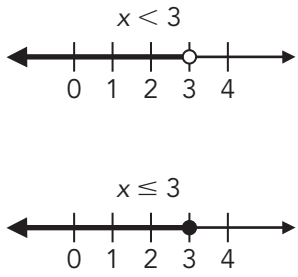


a. Describe each number line representation.

b. Describe the solution set for each inequality.

c. How does the solution set of the inequality $x \geq 3$ differ from the solution set of $x > 3$?

2. Consider the graphs of the inequalities $x < 3$ and $x \leq 3$.



a. Describe each number line representation.

b. Describe the solution set for each inequality.

c. How does the solution set of the inequality $x \leq 3$ differ from the solution set of $x < 3$?

The solution to any inequality can be represented on a number line by a ray. A ray begins at a starting point and goes on forever in one direction.

A closed circle means that the starting point is part of the solution set of the inequality. An open circle means that the starting point is not part of the solution set of the inequality.

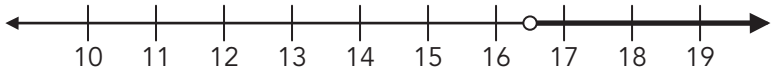
CC BY NC

3. Write the inequality represented by each graph.

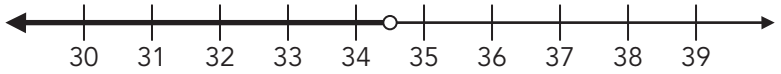
a.



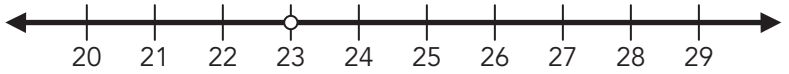
b.



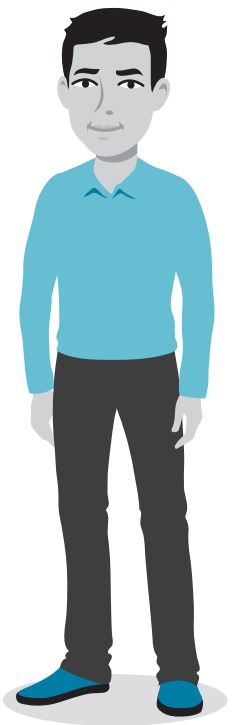
c.



d.

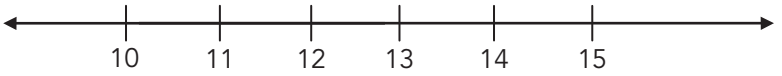


For solution sets that don't start on an exact number, make your best approximation.



4. Graph the solution set for each inequality.

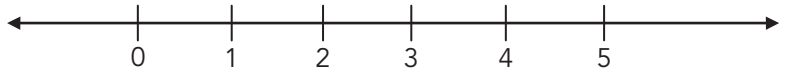
a. $x \leq 14$



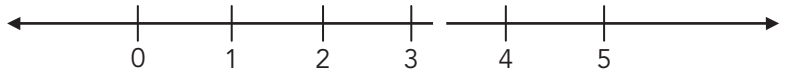
b. $x < 55$



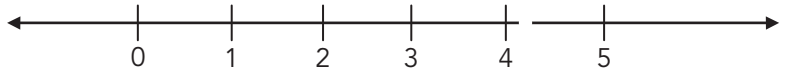
c. $2\frac{1}{2} \leq x$



d. $x > 3.3$



e. $x \neq 4.2$



5. Consider the inequalities in Questions 1 through 4.

   a. How many solutions does each inequality have?

   b. Can you write an inequality that has no solution? Explain.

   c. Can you write an inequality that has just one solution? Explain your reasoning.

6. Explain the meaning of each sentence in words. Then, define a variable and write a mathematical statement to represent each statement. Finally, sketch a graph of each inequality.

   a. The maximum load for an elevator is 2900 lbs.



   b. A car can seat up to 8 passengers.





   c. No persons under the age of 18 are permitted.



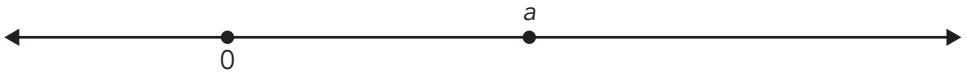
   d. You must be at least 13 years old to join.



Talk the Talk

Not All Variables Are Created Equal

A point at a is plotted on the number line shown.



1. Plot a point to the right of a and label it b.

   a. Write three different inequalities that are true about a and b.

   b. What can you say about all points to the right of point a on the number line?

2. Plot a point to the left of a and label it c.

   a. Write three different inequalities that are true about a and c.

   b. What can you say about all points to the left of point a on the number line?

3. Describe the position of all points on the number line that are:

   a. Greater than a.

   b. Less than a.

Equation Cards

$11 + x = 11$	$10x = 30$	$\frac{x}{3} + 10 = 20$	$x + 3.5 = 14.25$	$\frac{1}{8}x = 8$
$1x + 6 = 9$	$x + 1 = x$	$\frac{5}{8} = x + \frac{1}{2}$	$x - 5 = x - 4$	$\frac{x}{9} = 2$
$5 = x + 5$	$29 = x \cdot 29$	$\frac{x}{10} = 0.1$	$45 = 5x$	$\frac{10}{x} = 0.1$
$x \cdot 1 = x$	$x + 0 = x$	$x + 4 = 4 + x$	$\frac{x}{4} = \frac{1}{4}x$	$0x = 0$

Solution Cards

$x = 10.75$	$x = 0$	$3 = x$	no solution	$x = \frac{1}{8}$
$x = 18$	$30 = x$	$x = 100$	$6 = x$ (six)	$x = 1$
$9 = x$ *(nine)*	$x = \frac{1}{4}$	$64 = x$	infinite solutions	$x = 0.1$

Why is this page blank?

So you can cut out the cards on the other side

Lesson 1 Assignment

Write

Complete each statement with the correct term.

1. The _____ states that when two values a and b are equal, and when you multiply each by the same value c, the products are equal.
2. A(n) _____ to an equation is any value for a variable that makes the equation true.
3. The _____ says that when both sides of an equation look exactly the same, their values must be equal.
4. A(n) _____ is a mathematical sentence created by writing two expressions with an equals sign between them.
5. The _____ is the set of all points on a number line that make the inequality true.
6. The _____ is the set of all points that make an inequality true.

Remember

Properties of equality are logical rules that allow you to maintain balance and rewrite equations.

Properties of equality	For all numbers a, b, and c
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $ac = bc$.
Division property of equality	If $a = b$, and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

CC BY NC

Lesson 1 Assignment

Practice

Indicate whether each equation has one solution, no solution, or an infinite number of solutions and explain your reasoning. When the equation has one solution, determine the solution from the set of values given.

Set of values: {0, 1, 2, 3, 4, 5, 9, 10, 35, 36, 37, 38, 39, 40, 50, 60, 61, 62, 63, 64, 65, 99, 100}

1. $x - 3 = x + 3$

2. $81 = 9x$

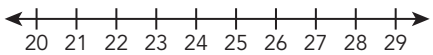
3. $x + 17 = 55$

4. $\frac{x}{3} = 21$

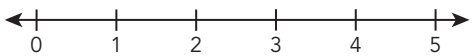
5. $8 + x = x + 8$

Graph the solution set for each inequality

6. $x \geq 22$



7. $3\frac{1}{3} < x$



Lesson 1 Assignment

Prepare

Determine each sum or difference.

1. $5.67 + 8.73$

2. $8.73 - 5.67$

3. $\frac{3}{7} + \frac{4}{5}$

4. $\frac{20}{3} - \frac{15}{4}$

CC BY NC

2 Solving One-Step Addition Equations

OBJECTIVES

- Reason about addition equations.
- Use bar models to represent one-step addition equations.
- Use inverse operations to solve one-step addition equations.
- Solve one-step addition equations.

NEW KEY TERMS

- bar model
- one-step equation
- inverse operations

Throughout this course, you have used a variety of tools to solve mathematical problems, including area models, pictures, tables, strip diagrams, double number lines, graphs, and expressions.

What tools might help you in solving equations?

CC BY NC

Getting Started

Form of 0

Consider the number 0. What comes to mind?

1. Write five different numerical expressions for the number 0.

Think About. . .

As you evaluate your expressions, you will likely use an equal sign. The number sentence is then an equation.



Share your numerical expressions with your classmates.

2. Did you and your classmates use common strategies to write your expressions? How many possible numerical expressions could you write for this number?

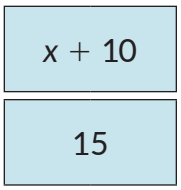
ACTIVITY
2.1 Reasoning About Addition Equations

Reasoning about equations and determining solutions with *bar models* provides a visual representation of the structure of the equations. A **bar model** uses rectangular bars to represent known and unknown quantities.

Remember . . .
A solution to an equation is a value for the variable that makes the equation a true statement.

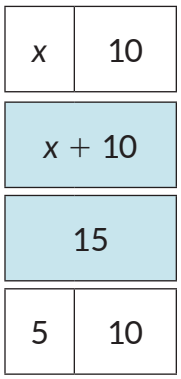
WORKED EXAMPLE

Consider the addition equation $x + 10 = 15$.



This equation states that for some value of x, the expression $x + 10$ is equal to 15. This can be represented using a bar model.

Just like with area models, bar models can be decomposed. The expression $x + 10$ can be decomposed into a part representing x and a part representing 10. The number 15 can be decomposed in a similar way: $15 = 5 + 10$.



The bar model demonstrates that these two equations are equivalent.

$$x + 10 = 15$$

$$x + 10 = 5 + 10$$

By examining the structure of the second equation, you can see that 5 is the value for x that makes this equation true.

Because the expressions $x + 10$ and 15 are equal, their bars are of equal length. Even after the bars are decomposed, the lengths maintain their equality.

You are dealing with unknowns. Do not worry when the lengths of the parts are not accurate.

1. Why is the number 15 decomposed into the numerical expression $5 + 10$?

2. Describe how the model in the Worked Example would be different for each equation. Complete the bar model for each.

   a. $x + 10 = 17$

x	10
$x + 10$	
17	

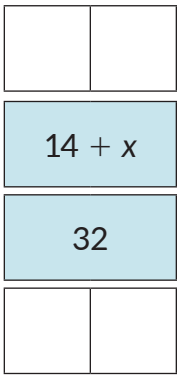


   b. $x + 6 = 15$

x	
$x + 6$	
15	

3. Consider the equation $14 + x = 32$.

   a. Complete the bar model.

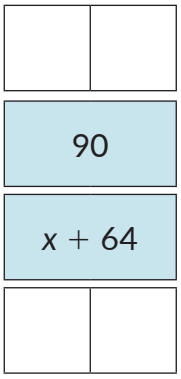


   b. Write the equation represented by the decomposed expressions in the bar model.

   c. Which value for x makes the equation a true statement?

4. Consider the equation $90 = x + 64$.

   a. Complete the bar model.



CC BY NC

b. Write the equation represented by the decomposed expressions in the bar model.

c. Which value for x makes the equation a true statement?

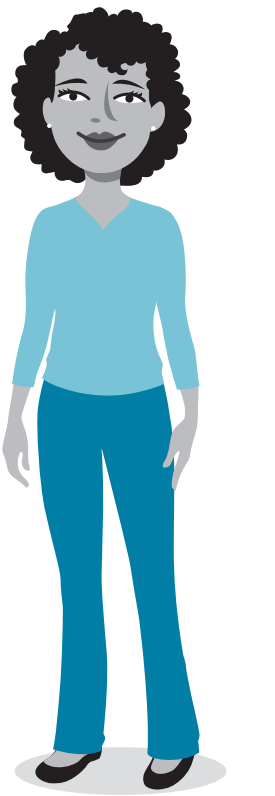
5. In each bar model, how did you determine how to decompose the given expressions?

ACTIVITY
2.2 Solving Addition Equations

In Activity 2.1, *Reasoning About Addition Equations*, you used bar models to solve *one-step equations*. A **one-step equation** is an equation that can be solved using only one operation. How can you use what you learned from creating bar models to solve any equation?

Now that you understand bar models, you can write equivalent equations with the same structure. While you can use reasoning to determine the value for the variable that makes an equation true, you can also use the properties and *inverse operations* to isolate the variable. **Inverse operations** are pairs of operations that reverse the effects of each other. For example, subtraction and addition are inverse operations.

The *additive identity property* states that for any number m, $m + 0 = m$. In other words, when you add 0 to any number, it stays the same. It keeps its identity!



WORKED EXAMPLE

Solve the equation $h + 6 = 19$.

$h + 6 = 13 + 6$	Write equivalent expressions that mirror structure.
$h + 6 - 6 = 13 + 6 - 6$	Use the inverse operations to reverse the addition of 6 to h by creating a zero pair.
$h + 0 = 13 + 0$	Combine like terms and apply the additive identity property.
$h = 13$	

1. Examine the Worked Example.

   a. What is the solution to $h + 6 = 19$?

   b. Are there other solutions to the equation? How do you know?

CC BY NC

2. Use the same strategy to solve each equation.

   a. $35 = 12 + m$

   b. $t + 24 = 85$

3. Analyze Trung's strategy to solve the equation $-11 = m + 7$.

Trung



When solving the addition equation $-11 = m + 7$, I can simply subtract 7 from both sides without first writing an equivalent equation.

$$-11 = m + 7$$

$$-11 - 7 = m + 7 - 7$$

$$-18 = m$$

The value for m that makes this equation true is -18.

   a. What property of equality is Trung using in her strategy?

   b. How could Trung check that her solution is correct?

4. Use Trung's strategy to solve each equation. Check to see that your solution makes the original equation a true statement.

a. $120 + y = 315$

b. $5\frac{3}{4} = x + 4\frac{1}{2}$

c. $b + 5.67 = 12.89$

d. $2356 = -1699 + a$

e. $\frac{7}{12} = g + \frac{1}{4}$

f. $w + 3.14 = -27$

g. $13\frac{7}{8} = c + 9\frac{3}{4}$

h. $p + (-19) = -105$

CC BY NC

Talk the Talk

It All Adds Up

1. What does it mean to solve an equation?

2. Describe how to solve any one-step addition equation. How do you check to see whether a value is the solution to an equation?

3. Write two different one-step equations for each solution provided.

   a. $m = 12$

b. $5 = x$

c. $5.6 = h$

d. $j = 6\frac{4}{7}$

CC BY NC

Lesson 2 Assignment

Write

Write a definition for each term in your own words.

- one-step equation
- solution
- inverse operations

Remember

A solution to an equation is any value(s) for the variable that makes the equation true. To solve a one-step addition equation, perform inverse operations to both sides of the equation to isolate the variable.

Practice

Use a bar model to solve each equation.

1. $x + 7 = 15$

2. $19 = x + 13$

3. $14.5 = 6 + y$

4. $a + \frac{1}{2} = 4\frac{3}{4}$

CC BY NC

Lesson 2 Assignment

Solve each equation. Check each solution.

5. $-34 = x + 17$

6. $a + (-25) = 92$

7. $7\frac{3}{5} + b = 10\frac{3}{4}$

8. $24\frac{1}{2} = t + 5\frac{1}{4}$

9. $r + 3.4 = 13.1$

10. $4.21 = 2.98 + s$

Lesson 2 Assignment

Prepare

Rewrite each fraction as a whole number times a unit fraction.

1. $\frac{4}{5}$

2. $\frac{9}{2}$

3. $3\frac{2}{7}$

4. $\frac{1}{12}$

CC BY NC

3 Solving One-Step Multiplication Equations

OBJECTIVES

- Use bar models to represent one-step multiplication equations, including real-world problems and geometric concepts.
- Use inverse operations to solve one-step multiplication equations.
- Reason about multiplication equations.
- Connect bar models to the algorithm for solving multiplication equations.
- Solve one-step multiplication equations.

You have solved one-step addition equations using bar models and inverse operations.

How can you use similar strategies to solve one-step multiplication problems?

CC BY NC

Getting Started

Form of 1

Consider the number 1. What comes to mind?

1. Write five different numeric expressions for the number 1.



Share your numeric expressions with your classmates.

2. Did you and your classmates use common strategies to write your expressions? How many possible numeric expressions could you write for this number?

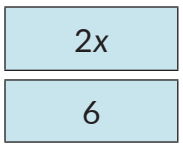
ACTIVITY 3.1

Reasoning About Multiplication Equations

Just as with addition equations, solving multiplication equations involves determining the value for the variable that makes the statement true. You can use bar models to understand the structure of the equation and reason about the solution.

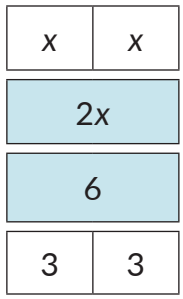
WORKED EXAMPLE

Consider the multiplication equation $2x = 6$.



This equation states that for some value of x, the expression $2x$ is equal to 6.

You can decompose $2x$ by rewriting it as the equivalent expression $1x + 1x$, or $x + x$.



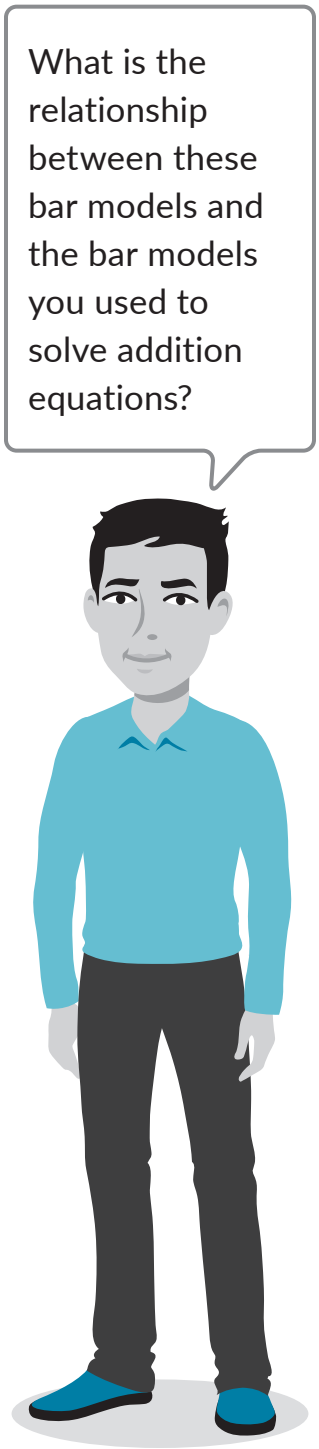
To maintain equivalence, decompose 6 in a similar way.

The bar model demonstrates that these two equations are equivalent.

$$2x = 6$$

$$x + x = 3 + 3$$

By examining the structure of the second equation, you can see that $x = 3$.



1. Why is the number 6 decomposed into the numeric expression $3 + 3$?

CC BY NC

PROBLEM SOLVING

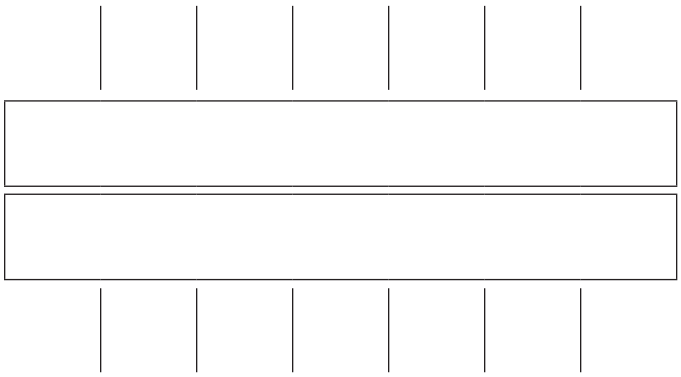


Solve each equation using a bar model.

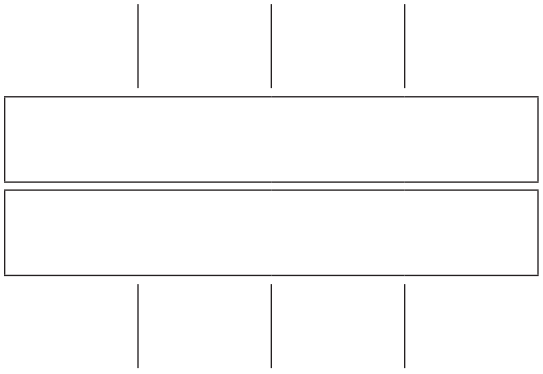
2. $3x = 12$



3. $7x = 63$



4. $4x = 6$

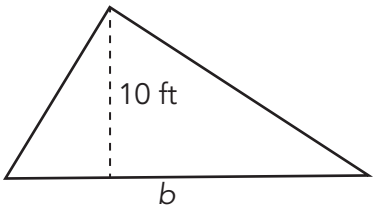


Models help with determining solutions to real-world problems.

WORKED EXAMPLE

The area of a triangle is 65 square feet.

The height of the triangle is 10 feet. What is the length of the base of the triangle?



Write an equation to represent the area of the triangle. Then, use a bar model to solve for the length of the base.

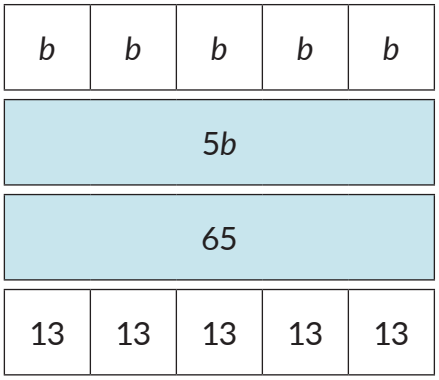
$A = \frac{1}{2}bh$

$65 = \frac{1}{2}b\,(10)$

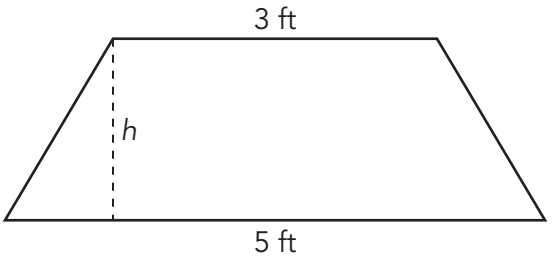
$65 = 5b$

$13 = b$

The length of the base is 13 feet.



5. A trapezoid has bases that are 3 feet and 5 feet in length. The area of the trapezoid is 24 square feet.



   a. Write an equation to represent the area of the trapezoid in terms of its height.

b. Use a bar model to solve for the length of the height.

6. Natalia and Mario are 3 and 4 years old. Their combined age, multiplied by g, results in the age of their grandmother who is 56 years old.

   a. Write an equation to represent the ages of Natalia, Mario and their grandmother.

   b. Use a bar model to determine the value of g.

ACTIVITY 3.2

Solving Equations with Fractional Coefficients

Multiplication equations often include numbers other than whole numbers.

Consider the equation $\frac{1}{3}x = 2$.

1. Explain how this equation compares to the equations in the previous activity.

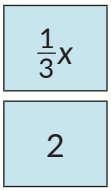
Remember . . .

There are different ways to write equivalent expressions.

$\frac{1}{3}x = \frac{x}{3}$

WORKED EXAMPLE

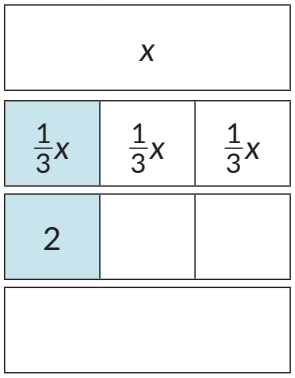
Represent $\frac{1}{3}x = 2$ as a bar model.



To solve this equation for x, compose 3 equally-sized parts to create the whole, x.

To maintain equivalence, compose 3 equally-sized parts for the other expression, too.

This structure allows you to see the value of x.

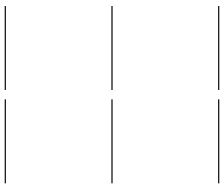


Because you start with a fractional amount of x, you have to compose to get a whole x.

2. Complete the Worked Example by filling in the unknown values. Then, write the solution to the equation $\frac{1}{3}x = 2$.

Solve each equation using a bar model.

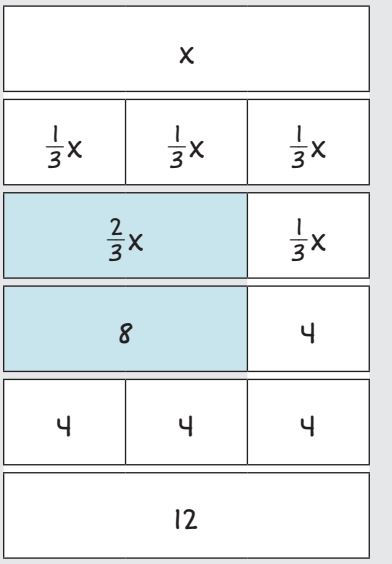
3. $\frac{1}{4}x = 7$



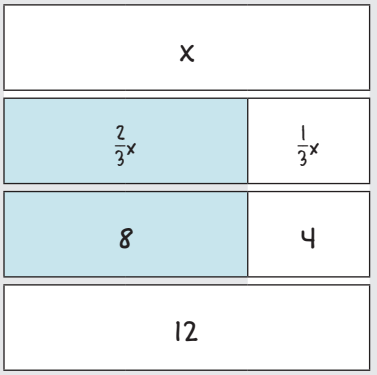
4. $\frac{x}{2} = 5$

5. Consider how to use bar models to solve $\frac{2}{3}x = 8$.
Analyze each strategy.

Tiara's Solution



Alejandro's Solution





a. How is Alejandro's solution strategy different from Tiara's solution strategy?

b. What reasoning might Tiara have used in her solution strategy?

Solve each equation using a bar model.

6. $\frac{4}{5}x = 12$

7. $\frac{3}{4}x = 8$

8. Consider the equation $\frac{8}{5}x = 64$.

   a. How does this fractional coefficient compare to the fractional coefficients that you have seen in this lesson so far?

   b. Create a bar model and solve for x.



Reflect on the equations you solved in this activity.

9. How were they similar? What was common in how you used the bar models?

ACTIVITY
3.3 Solving More Complex Multiplication Equations

Consider the equation $\frac{4}{5}x = \frac{1}{10}$.

1. How is this equation different from the equation you solved in the previous activity?

Compare the two solution strategies proposed by Mario and Paola.

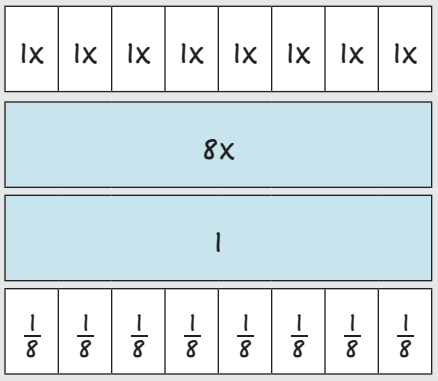
Mario's Solution

$\frac{4}{5}x = \frac{1}{10}$

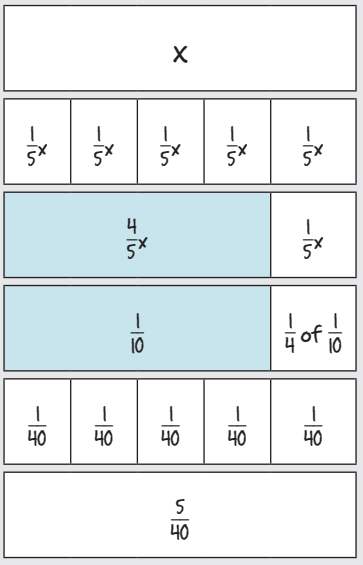
Scale $\frac{4}{5}$ up to $\frac{8}{10}$.

$\frac{8}{10}x = \frac{1}{10}$

$8x = 1$



Paola's Solution



2. Explain Mario's solution strategy.

   a. What type of reasoning did Mario use at the beginning of his solution?

   b. How did he know to write $8x = 1$?

   c. Will scaling up always work?

3. Explain how Paola's solution is similar to the other equations you have solved with bar models.

CC BY NC

Use a bar model to solve each equation.

4. $\frac{3}{4}x = \frac{2}{5}$



5. $\frac{2}{7}x = \frac{4}{9}$

6. You learned to solve addition equations by first reasoning with bar models and then with inverse operations. Now that you have solved multiplication equations by reasoning with bar models, how do you think that you can solve these equations without using the bar models?

CC BY NC

ACTIVITY 3.4

Solving Multiplication Equations Without Models

Like addition equations, all of the multiplication equations you have modeled in this lesson can be solved with one step. You can use the properties of equality and inverse operations to isolate the variable. What operation is the inverse of multiplication?

WORKED EXAMPLE

Solve the equation $4r = 32$.

$4r = 32$	
$4(1r) = 4(8)$	Write equivalent expressions with similar structure.
$\frac{4(1r)}{4} = \frac{4(8)}{4}$	Use inverse operations to reverse the multiplication of 4 and $1r$.
$1r = 1(8)$	Perform division.
$r = 8$	Identity property of multiplication

The *identity property of multiplication* states that for any number m, $m \cdot 1 = m$. In other words, when you multiply by 1, the number stays the same.

1. Examine the Worked Example.

   a. Check the solution to $4r = 32$.

   b. Are there other solutions to the equation? How do you know?

2. Use the same strategy to solve each equation.

   a. $8a = 72$

   b. $11t = -110$

CC BY NC

When you worked with one-step addition equations, you used the subtraction property of equality to more efficiently solve the problem. Similarly, you can use the division property of equality to solve multiplication problems.

3. Write the properties that justify each step.

$6w = 90$

$\frac{6w}{6} = \frac{90}{6}$ _____

$1w = 15$ _____

$w = 15$ _____



4. Andrew and Elena are solving the equation $5 = \frac{p}{7}$.

   a. Andrew says that to solve $5 = \frac{p}{7}$, he would divide by 7. The value of p that makes the equation true is $\frac{5}{7}$. Elena disagrees and says that they should divide by $\frac{1}{7}$, and the solution is 35. Who is correct?

   b. How can Andrew and Elena check to see whose answer is correct?

5. Compare the solution strategies used by Ashley and Mariana. What do you notice?

Ashley



$\frac{2}{5}x = 20$

$\frac{\frac{2}{5}x}{\frac{2}{5}} = \frac{20}{\frac{2}{5}}$

$1x = 20\left(\frac{5}{2}\right)$

$x = 50$

Mariana



$\frac{2}{5}x = 20$

$\left(\frac{5}{2}\right)\frac{2}{5}x = \left(\frac{5}{2}\right)20$

$1x = 50$

$x = 50$

6. Solve each equation. Check to ensure that your solution makes the original equation a true statement.

a. $\frac{n}{4} = -7$

b. $18 = 3y$

c. $\frac{3}{2}h = \frac{5}{2}$

d. $-4.2k = 8.4$

Talk the Talk

What's Your Strategy?

Each equation in this lesson is written as $px = q$, where p and q are positive rational numbers and x is the unknown. You have investigated different strategies to solve these equations.

Analyze each given equation.

- Do you recognize a fact family relationship between the numerical coefficient and the constant?
- Is the numerical coefficient a whole number? A fraction? A decimal?
- Do you recognize a way to form a numerical coefficient of 1 using a property of equality?

$2n = 12$ $\quad \frac{2}{5}x = 14$ $\quad -3x = 55$ $\quad 1.1m = 5.5$

$1.45r = 5.9$ $\quad 7h = -35$ $\quad \frac{x}{4} = \frac{3}{8}$ $\quad 8r = \frac{3}{4}$

1. Sort each equation according to the solution strategy you think is most efficient.

Use Number Sense to Write Equivalent Expressions	Division Property of Equality	Multiplication Property of Equality

2. Provide a rationale for your choice of solution strategy or strategies.

Lesson 3 Assignment

Write

Explain how to solve the equation $px = q$ for x. Be sure to include the properties you use in the process.

Remember

A solution to an equation is the value or values for the variable that makes the equation true. To solve a one-step multiplication equation, isolate the variable using number sense, the division property of equality, or the multiplication property of equality.

Practice

1. Solve each equation using a bar model.

   a. $3x = 10$

   b. $\frac{x}{5} = 6$

CC BY NC

Lesson 3 Assignment

c. $\frac{3}{5}x = 12$

d. $\frac{5}{4}x = \frac{2}{3}$

2. Solve each equation. Check your solutions.

a. $2.5 = 0.5y$

b. $-4r = 32$

c. $\frac{2}{9}h = -8$

d. $\frac{4}{3} = \frac{8}{3}b$

e. $-14 = \frac{s}{3}$

f. $-3.8x = -15.2$

Lesson 3 Assignment

3. Eduardo invites 21 people to his party and wants to give each guest 3 party favors. Let n equal to the total number of party favors Eduardo will need to order. The equation that represents this situation is $\frac{n}{21} = 3$.

   a. Suppose Eduardo orders 58 party favors. Will he be able to give each guest 3 party favors? That is, is 58 a solution to the equation?

   b. Suppose Eduardo orders 62 party favors. Will he be able to give each guest 3 party favors?

   c. How many party favors does Eduardo need to order? Use the equation to determine the solution. State the inverse operation needed to isolate the variable. Then, solve the equation. Check your solution.

CC BY NC

Lesson 3 Assignment

Prepare

Determine each quotient using long division.

1. $435 \div 25$

2. $511 \div 30$

3. $860 \div 23$

4 Solving Equations to Solve Problems

OBJECTIVES

- Use variables to represent quantities in expressions describing real-world values.
- Solve problems by writing and solving equations.

NEW KEY TERM

- literal equation

You know about expressions and equations and how they often represent the structure of real-world situations.

How can you apply your knowledge to write equations and solve real-world and mathematical problems?

CC BY NC

Getting Started

Equations, Literally

You have already learned a lot of important equations in mathematics. Some of these equations are *literal equations*. **Literal equations** are equations in which the variables represent specific measures. You most often see literal equations when you study formulas.

For example, the formula for the area of a triangle, $A = \frac{1}{2}bh$, is a literal equation. The variables in this equation represent the measures of the area, base, and height of a triangle.

1. Consider the formula for area of a parallelogram.

   a. Write the formula for area, A; use b to represent the base and h to represent the height.

   b. Solve the equation for b.

   c. Solve the equation for h.

2. The total cost, t, of an online order is the cost of the items, c, plus the cost of shipping, s.

   a. Write an equation to represent the total cost.

   b. Solve the equation for the cost of the items.

   c. Solve the equation for the cost of shipping.

ACTIVITY
4.1 Using Equations to Solve Problems

Write and solve an equation for each problem. Show your work and label your answers. Describe the strategy you used to determine each solution.

1. Chris's sister is 6 years older than he is. What is Chris's age when Chris's sister is 19 years old?

2. Approximately $\frac{1}{10}$ of the mass of a medium-sized apple is sugar. What is the approximate mass of a medium-sized apple that contains 19 grams of sugar?

3. Jorge made brownies for his class. He tripled the recipe he normally uses. Jorge made 36 brownies for his class. How many brownies does his original recipe make?

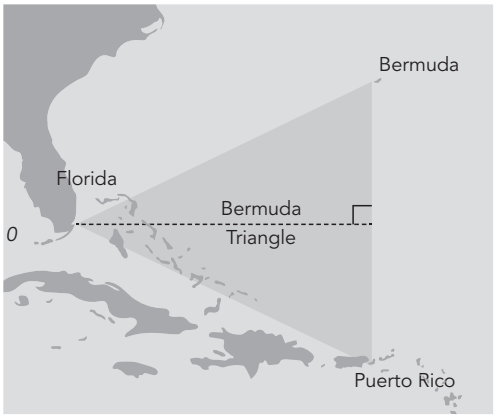
CC BY NC

4. In Jaylen's town, the middle school has 443 more students than the high school. When the middle school has 817 students, how many students are at the high school?

PROBLEM SOLVING



5. The boundaries of the Bermuda Triangle vary according to how sources define the triangle vertices. When you use Miami, Florida; San Juan, Puerto Rico; and Hamilton, Bermuda to define the Bermuda Triangle, it covers an area of about 454,000 square miles. The dashed line on the map shows a distance of about 926 miles.



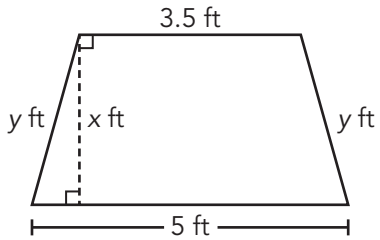
What is the approximate distance from Bermuda to Puerto Rico?

ACTIVITY
4.2 Solving Equations with Geometric Applications

You can write equations that represent problems related to geometric figures. Just like in Getting Started, sometimes you will need to solve the equation for a specific variable.

Jorge dropped his car off at window tinting service.

1. The back window of the car is shaped like a trapezoid. The dimensions of the back window are shown in the diagram.



   a. Write an equation for A, the area of the back window using the given dimensions.

   b. When the area of the window is 10.625 square feet, what is the height of the back window?

2. The driver's window, passenger's window, and the two rear windows are shaped like parallelograms. The table below shows the relationship between A, the area of each window in square inches, b, the length of the base of each window in inches, and h, the height of each window in inches.

	Driver and Passenger Windows	Rear Windows
Area	360	x
Base	y	14
Height	18	18

CC BY NC

a. Write an equation that can be used to represent the relationship between A, b, and h for these parallelograms.

b. Use the information from the table along with your equation from part (a) to solve for x.

c. Use the information from the table along with your equation from part (a) to solve for y.

3. The front windshield is in the shape of a rectangle. The windshield has an area of A square feet. The width of the windshield is 6 feet.

   a. Write an equation that represents h, the height of the windshield in feet.

   b. If the area of the windshield is $19\frac{1}{2}$ square feet, what is the height of the windshield?

You have written equations for two-dimensional geometric figures. You can also write equations for three-dimensional figures.

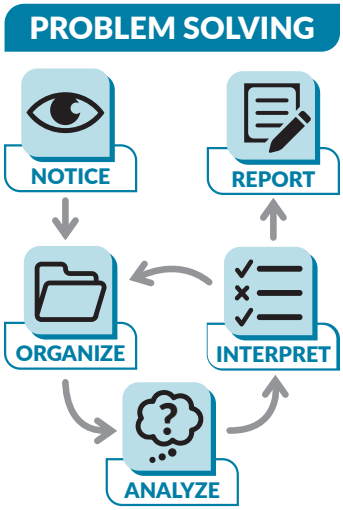
4. A rectangular prism has a volume of V cubic feet. The rectangular prism has a width of 3 feet long and a height of 4 feet.

   a. Write an equation that represents, w, the width of the rectangular prism.

b. When the volume of the prism is 30 cubic feet, what is the width of the rectangular prism?

You can also write equations to represent the sum of angle measures.

5. Jamal is determining the sum of angle *A* and angle *B*.



   a. Write an equation that represents the sum, *s*, of the angle measure when angle *A* is 52°.

   b. Determine the measure of angle *B* when the sum of the angles is 180°.

   c. Determine the measure of angle *B* when the sum of the angle measures is 90°.

Talk the Talk

Write Your Own

1. Write your own word problem that can be solved by writing and solving the equation $2.4 + x = 5$.

2. Write your own word problem that can be solved by writing and solving the equation $4x = 8$.

Lesson 4 Assignment

Write

Write a definition of *literal equation* in your own words.

Remember

You can represent real-world problems and situations by defining variables to represent the measures involved and writing an equation. You can then substitute in the values that you know and solve for the remaining variable to determine the unknown quantity.

Practice

For each problem, write an equation to represent the situation. Then, solve the equation to answer the question.

1. Lizzie's dog is 7 years older than her cat. What is Lizzie's cat's age when her dog is 12 years old?

2. Maple syrup is created by boiling maple tree sap. Approximately $\frac{1}{40}$ of the sap volume remains as syrup. A tree farm makes $18\frac{1}{2}$ gallons of maple syrup. How much maple tree sap did they collect before boiling?

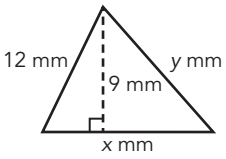
Lesson 4 Assignment

3. A rectangular pool has a width of 24 feet. A second rectangular pool has a perimeter of 48 feet, which is $\frac{1}{3}$ the perimeter of the first pool. What is the perimeter of the first pool? What is the length of the first pool?

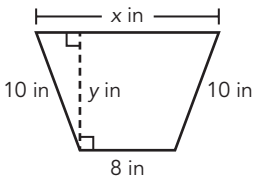
4. The local firefighters collect toys to distribute at various give-away events. They have 4569 toys and will sponsor 129 give-away events. How many toys can they give away at each event? How many toys, if any, will be left over?

Write an equation that could be used to calculate the area, A, of each of the figures.

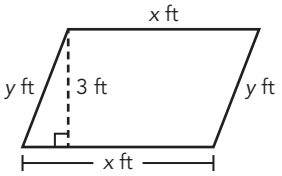
5.



6.



7.

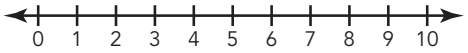


Lesson 4 Assignment

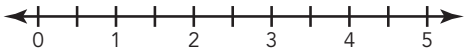
Prepare

Graph each inequality on a number line.

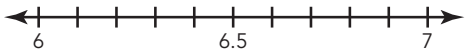
1. $x > 5$



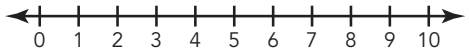
2. $x \geq 2\frac{1}{2}$



3. $x < 6.2$



4. $x \leq 9$



5 Solving Inequalities with Inverse Operations

OBJECTIVES

- Solve and graph one-step inequalities.
- Graph the solution sets of inequalities and interpret the solutions in context.

NEW KEY TERMS

- inequality
- solve an inequality
- properties of inequalities

You have solved a variety of equations.

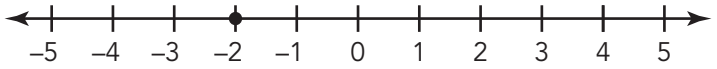
How is solving inequalities similar to or different from solving equations?

CC BY NC

Getting Started

Equations Versus Inequalities

Consider the equation $4x = -8$. The solution is shown on the number line.



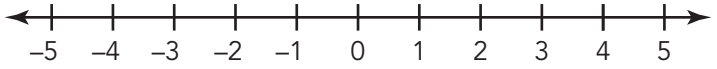
An **inequality** is any mathematical sentence that has an inequality symbol. The solution set of an inequality is all values that make the inequality statement true.

1. Verify the solution is correct. Are there any other solutions to this equation? Explain your reasoning.

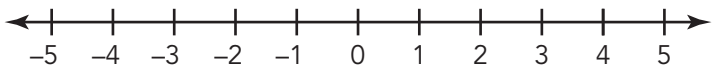
Consider the set of numbers $\{-5, -4, -3, -2, -1, 0, 1\}$.

2. Use substitution to determine which values are solutions to each inequality. Plot the solutions for each inequality on the given number line.

   a. $4x > -8$

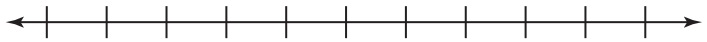


   b. $4x \leq -8$



3. Use your number lines to make predictions about other solutions to each inequality. Create a number line to illustrate each of your conjectures.

(a)



(b)



CC BY NC

ACTIVITY 5.1

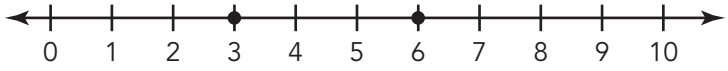
Adding and Subtracting with Inequalities

The values of the variable that make an inequality true are together called the *solution set of the inequality.*

In this lesson, you will learn to *solve an inequality.* To **solve an inequality** means to determine the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation. You want to isolate the variable on one side of the inequality symbol by using the operations of addition, subtraction, multiplication, and division.

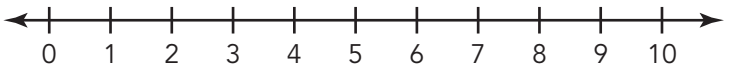
Let's investigate what happens when the same number is added to, or subtracted from, each side of an inequality.

Consider the relationship between the two numbers 3 and 6. Since 3 is to the left of 6, you know that $3 < 6$.



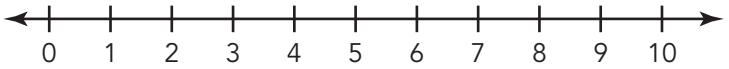
1. Perform each operation to the numbers 3 and 6. Then, plot the new values on the number line. Finally, write a corresponding inequality statement.

   a. Add $\frac{1}{2}$ to each number.

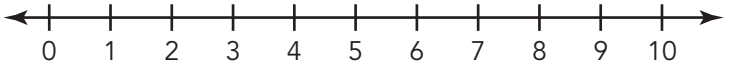
   $3 + \frac{1}{2}$ ____ $6 + \frac{1}{2}$

   b. Add 2 to each number.

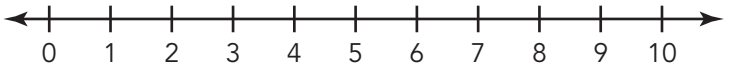
   $3 + 2$ ____ $6 + 2$

   c. Add 3 to each number.

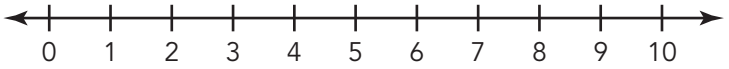
   $3 + 3$ ____ $6 + 3$

d. Subtract $\frac{1}{2}$ from each number.



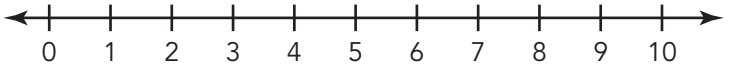
$3 - \frac{1}{2}$ ____ $6 - \frac{1}{2}$

e. Subtract 2 from each number.



$3 - 2$ ____ $6 - 2$

f. Subtract 3 from each number.



$3 - 3$ ____ $6 - 3$

2. When you add the same number to each side of the inequality or subtract the same number from each side of the inequality, what do you notice about the resulting inequality symbol?

3. Explain why Samantha is correct.

Samantha



No matter what number I add to or subtract from both sides of the inequality, the relationship between the two sides of the inequality stays the same:

$$3 < 6$$
$$3 + a < 6 + a$$
$$3 - a < 6 - a$$

CC BY NC

4. Consider the inequality $x - 2 > 6 - 2$.

   a. Write an inequality to describe the possible values of x.

   b. What could you do to both sides of the original inequality to determine your answer to part (a)?

5. Suppose you have the inequality $x - 2 > 6$. Determine the possible values of x and sketch the solution set on a number line. Explain your reasoning.

You can model problem situations with equations and inequalities.

6. Paola is buying clothes from an online store that charges a flat $8.00 shipping fee.

   Write an expression that represents the total cost of the clothing including shipping.

7. Use the expression you wrote in Question 6 to write an equation or inequality that models each situation.

   a. Paola wants to spend exactly $50.

   b. Paola wants to spend at least $50.

   c. Paola wants to spend at most $50.

   d. Paola wants to spend less than $50.

   e. Paola wants to spend more than $50.

You have written inequalities given a problem situation. You can also write a problem situation to model a given inequality.

8. Josh is writing a real-world situation to model the inequality $x - 3.5 \leq 10$.



Josh's Solution

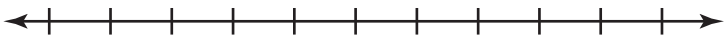
Tiara has at least ten minutes to complete a timed quiz. She has already worked for 3.5 minutes. How much time does she have to finish the quiz?

Is Josh correct? Explain your reasoning.

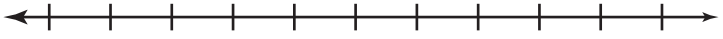
9. Mario is 5 years older than his brother Andrew.
For each question, write and solve an equation or inequality to describe Andrew's possible ages. Then, graph the solution set on the number line.



a. How old will Andrew be when Mario is 29 years old?



b. How old will Andrew be when Mario is at least 25 years old?



Recall that the solution to any inequality is shown on a number line by a ray in which its starting point is an open or closed circle. A closed circle means that the starting point is part of the solution set of the inequality. An open circle means that the starting point is not a part of the solution set of the inequality.

c. How old will Andrew be when Mario is younger than 30 years old?



10. Solve each inequality or equation and graph the solution set on the number line. Then, choose one value from your solution set and one value outside your solution set to check your work.

a. $13 < x + 11$



Ask Yourself . . .
How could you use algebra tiles to model and solve an inequality?

b. $10 + x \geq 45$



c. $\frac{2}{3}x = 5$



d. $x - 3 \leq 21$



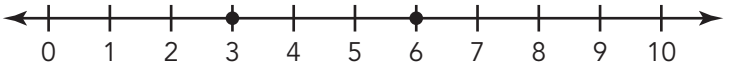
11. Choose one of the inequalities from Question 10 and write a real-world situation that can be modeled by the algebraic statement.

CC BY NC

ACTIVITY
5.2 Multiplying and Dividing by Positive Numbers

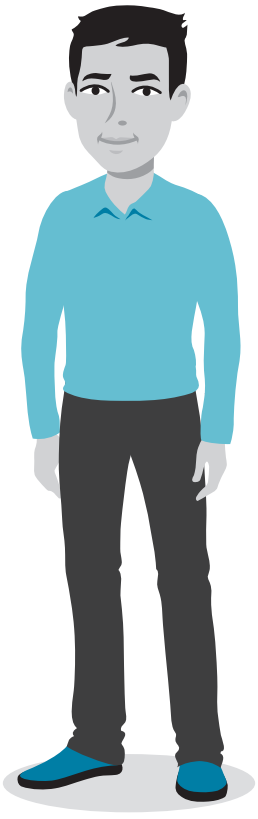
Next, let's investigate what happens when each side of an inequality is multiplied or divided by the same positive number.

Consider the inequality $3 < 6$.



1. Perform each operation to the numbers 3 and 6. Then, plot the new values on the number line. Finally, write a corresponding inequality statement.

Remember, the representation for division can include fraction notation.

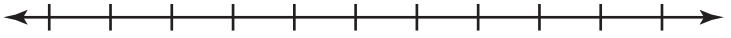


a. Multiply each number by $\frac{1}{2}$.



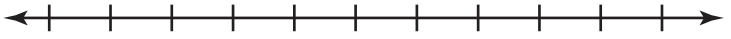
$3\left(\frac{1}{2}\right)$ ____ $6\left(\frac{1}{2}\right)$

b. Multiply each number by 2.



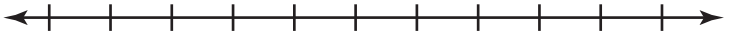
3(2) ____ 6(2)

c. Multiply each number by 3.



3(3) ____ 6(3)

d. Divide each number by $\frac{1}{2}$.



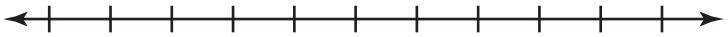
$3 \div \frac{1}{2}$ ____ $6 \div \frac{1}{2}$ or $\frac{3}{\frac{1}{2}}$ ____ $\frac{6}{\frac{1}{2}}$

e. Divide each number by 2.



$3 \div 2 \_\_\_\_ 6 \div 2$ or $\frac{3}{2} \_\_\_\_ \frac{6}{2}$

f. Divide each number by 3.



$3 \div 3 \_\_\_\_ 6 \div 3$ or $\frac{3}{3} \_\_\_\_ \frac{6}{3}$

2. When you multiply or divide each side of the inequality by the same positive number, what do you notice about the resulting inequality symbol?

Alyssa



No matter what positive number I multiply or divide by on both sides of the inequality, the relationship between the two sides of the inequality stays the same:

$$3 < 6$$
$$3(a) < 6(a)$$
$$\frac{3}{a} < \frac{6}{a}$$

3. Identify the constraints of the value a that makes Alyssa's claim correct.

4. Consider the inequality $2x < 6(2)$.

   a. Write an inequality to describe the possible values of x.

CC BY NC

b. What could you do to both sides of the original inequality to determine your answer to part (a)?

5. Suppose you have the inequality $2x < 6$. Determine the possible values of x. Explain your reasoning.

6. Alyssa is 3 times as old as her sister Natalia.

   For each question, write and solve an equation or inequality to describe Natalia's possible ages. Then, graph the solution set on the number line.

   a. How old will Natalia be when Alyssa is at least 27 years old?



   b. How old will Natalia be when Alyssa is younger than 30 years old?



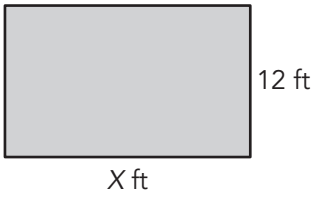
   c. How old will Natalia be when Alyssa is 42 years old?



You can model geometric concepts with inequalities.

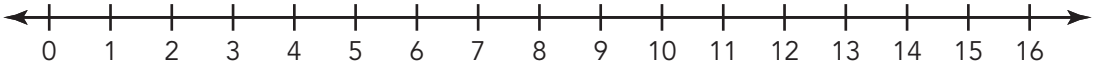


Trung wants to put a pool in his backyard. The width of the pool is 12 feet.

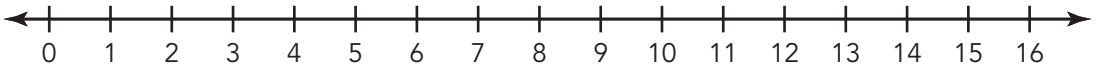


7. Write and solve an inequality that models each situation. Then, graph the solution set on the number line.

   a. Trung wants the area of the pool to be greater than 144 square feet. Determine the possible lengths of the side of the pool.



   b. Trung has at most 180 square feet of space available for the pool. Determine the possible lengths of the side of the pool.



   c. Do all the solutions in parts (a) and (b) make sense in terms of the problem situation?

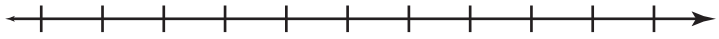
CC BY NC

8. Solve each inequality or equation and graph the solution set on the number line.

   a. $10x \geq 45$



   b. $\frac{x}{3} < 21$



   c. $3.25 = \frac{x}{6}$

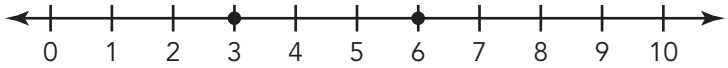


9. Choose one of the inequalities from Question 8 and write a real-world situation that can be modeled by the algebraic statement.

ACTIVITY
5.3 Multiplying and Dividing by Negative Numbers

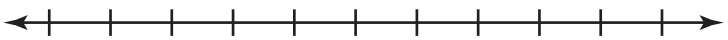
Finally, let's investigate what happens when each side of an inequality is multiplied or divided by the same negative number.

Consider the inequality $3 < 6$.



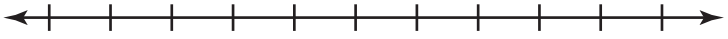
1. Perform each operation to the numbers 3 and 6. Then, plot the new values on the number line. Finally, write a corresponding inequality statement.

   a. Multiply each number by $-\frac{1}{2}$.

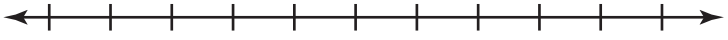
   $3\left(-\frac{1}{2}\right)$ ____ $6\left(-\frac{1}{2}\right)$

   b. Multiply each number by -2.

   $3(-2)$ ____ $6(-2)$

   c. Multiply each number by -3.

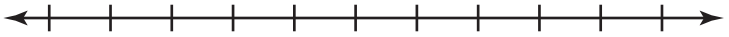
   $3(-3)$ ____ $6(-3)$

d. Divide each number by $-\frac{1}{2}$.



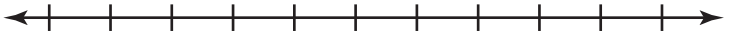
$3 \div \left(-\frac{1}{2}\right) \_\_\_\_ 6 \div \left(-\frac{1}{2}\right)$ or $\frac{3}{\left(-\frac{1}{2}\right)} \_\_\_\_ \frac{6}{\left(-\frac{1}{2}\right)}$

e. Divide each number by -2.



$3 \div (-2) \_\_\_\_ 6 \div (-2)$ or $\frac{3}{(-2)} \_\_\_\_ \frac{6}{(-2)}$

f. Divide each number by -3.



$3 \div (-3) \_\_\_\_ 6 \div (-3)$ or $\frac{3}{(-3)} \_\_\_\_ \frac{6}{(-3)}$

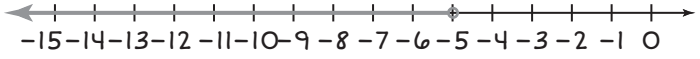
2. When you multiply the same negative number to each side of the inequality or divide the same negative number from each side of the inequality, what do you notice about the resulting inequality symbol?

3. Kaya and Matthew are trying to solve $-4x < 20$. Consider their solutions and explanations.

Matthew's Solution

$-4x < 20$

$x < -5$

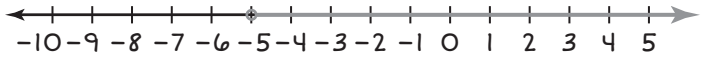


I divided both sides by -4 to solve the inequality.

Kaya's Solution

$-4x < 20$

$x > -5$



I know that when I multiply or divide any given number by a negative number, I have to pay attention to the sign of my answer. So, when I divide both sides of the inequality by − 4, the inequality sign should reverse.

a. Determine who is correct. List three values from each person's solution and verify that those solutions make the original inequality $-4x < 20$ true. What do you notice? Explain your reasoning.

Check for Matthew's solution. Check for Kaya's solution.

CC BY NC



b. Circle the correct solution and explanation and cross out the incorrect solution and explanation from Matthew's and Kaya's work.

4. Solve each inequality or equation and graph the solution set. When possible, list three values from each solution set and verify that each value makes the original equation or inequality true.

a. $-7x = -42$

b. $\frac{x}{3} \leq -4$

c. $-5x < 35$

d. $\frac{x}{-2} \geq 5$

Talk the Talk

Summarizing Inequalities

The **properties of inequalities** allow you to solve inequalities involving any numbers.

Properties of inequalities	For all numbers a, b, and c, . . .
Addition property of inequalities	If $a < b$, then $a + c < b + c$. If $a > b$, then $a + c > b + c$.
Subtraction property of inequalities	If $a < b$, then $a - c < b - c$. If $a > b$, then $a - c > b - c$.
Multiplication property of inequalities	If $a < b$, then $a \cdot c < b \cdot c$, for $c > 0$. If $a > b$, then $a \cdot c > b \cdot c$, for $c > 0$.
Division property of inequalities	If $a < b$, then $\frac{a}{c} < \frac{b}{c}$, for $c > 0$. If $a > b$, then $\frac{a}{c} > \frac{b}{c}$, for $c > 0$.

These properties also hold true for $\leq$ and $\geq$.

1. Write a paragraph to summarize your understanding of the properties of inequalities.

2. State the division property of inequalities for $c < 0$.

CC BY NC

3. Describe how to solve an inequality. How do you check to see whether a value is a solution to an inequality?

4. Write a real-world situation that can be modeled by each inequality. Be sure to define your variables.

   a. $15 > 3.25m$

   b. $20x \geq 85$

   c. $\frac{1}{2}s < 34$

Lesson 5 Assignment

Write

Explain how solving an inequality is similar to and different from solving an equation.

Remember

To *solve an inequality* means to determine what value or values will replace the variable to make the inequality true.

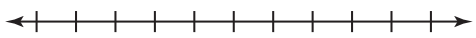
Practice

1. Match each inequality with the correct solution.

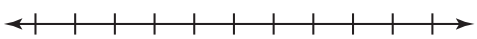
a. $x < -2$	i. $4x < 8$
b. $x < 2$	ii. $20 < 10x$
c. $x > -2$	iii. $-\frac{3}{2}x > 3$
d. $x > 2$	iv. $-8x < 16$

2. Solve each one-step inequality and graph the solution set on a number line.

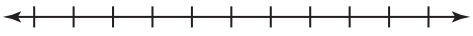
a. $x + 7 \geq 13$



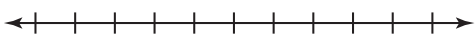
b. $-4 > x - 3$



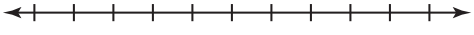
c. $\frac{x}{4} \leq \frac{5}{2}$



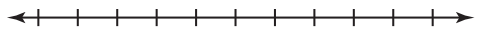
d. $18.3 > 6.1x$



e. $3 < \frac{x}{-8}$



f. $-10x \geq 45$



Lesson 5 Assignment

3. Ashley washes cars for $8 each. Ashley wants to attend a musical that costs $145.75.

   a. Write and solve an inequality to determine the minimum number of cars Ashley must wash to be able to buy the ticket to the musical.

   b. Is the answer to the question the same as the solution to the inequality? Explain.

4. Eduardo has $15 to spend at the gourmet candy store. He wants to buy gummy bears, which are $5.25 per pound. (Weights are measured to the nearest hundredth.) Write and solve an inequality to determine the maximum number of pounds of gummy bears Eduardo can buy.

Lesson 5 Assignment

Prepare

Write an inequality for each verbal statement.

1. x is less than 5.

2. 4 times g is no more than 9.

3. y is at least 2 more than x.

4. 3 less than the product of 4 and some number is greater than another number.

CC BY NC

TOPIC 2 SELF-REFLECTION

Name: _____

Equations and Inequalities

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Equations and Inequalities* topic by:

TOPIC 2: *Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
determining whether a given value makes one-variable, one-step equations or inequalities true.	○	○	○
writing a one-variable, one-step equation or inequality to represent constraints or conditions within problems.	○	○	○
recognizing that inequalities of the forms $x > c$, $x < c$, $x \geq c$, *and* $x \leq c$ have infinitely many solutions.	○	○	○
representing solutions for one-variable, one-step equations and inequalities on number lines.	○	○	○
modeling and solving one-variable, one-step equations using bar models.	○	○	○
using inverse operations to solve one-variable, one-step equations and inequalities.	○	○	○
recognizing that when solving one-step inequalities, the inequality symbol reverses when you multiply or divide both sides of an inequality by a negative number.	○	○	○

continued on the next page

CC BY NC

TOPIC 2 SELF-REFLECTION *continued*

TOPIC 2: *Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
writing and solving equations and inequalities for real-world mathematical problems containing one unknown.	○	○	○
writing a corresponding real-world problem for a given one-variable, one-step equation or inequality.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Equations and Inequalities* topic.

_____

_____

_____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

_____

_____

_____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

_____

_____

_____



TOPIC 2 SUMMARY

Equations and Inequalities Summary

LESSON 1 Reasoning with Equal Expressions

An **equation** is a statement of equality between two expressions. An equation can contain numbers, variables, or both in the same mathematical sentence. Equations may be always true, never true, or true for certain values of the variable.

A mathematical expression is a number or a combination of numbers and operations. An algebraic expression has at least one variable.

Consider the difference between an equation and an expression.

The expressions 8 and $(-2)^2 + 4$ are both numeric expressions. When you write an equal sign between two numeric expressions, you create a statement of equality. For example, $(-2)^2 + 4$ is an equation.

Consider $7x + 3 - 2x$ and -10. They are both expressions. However, $7x + 3 - 2x$ is an algebraic expression, and -10 is a numeric expression. When you put an equal sign between an algebraic and a numeric expression, you create a statement of equality. So, $7x + 2 - 2x = -10$ is also an equation.

You can write an equal sign between two algebraic expressions. The expressions $10x - 5$ and $2x + 35$ are both algebraic expressions. When you put an equal sign between two algebraic expressions, you create a statement of equality. So, $10x - 5 = 2x + 35$ is also an equation.

Both expressions and equations can also be expressed verbally, using words instead of symbols. For example, "seven is two more than a number" is a verbal equation. The statement, "three more than a number squared" is a verbal expression.

Always True	Never True	Sometimes True
$6 = 10 - 4$ $x = x$	$10 = 20$ $x = x + 2$	$x = 5$ $x + 2 = 12$

The **reflexive property of equality** says that when both sides of an equation look exactly the same, their values are equal.

NEW KEY TERMS

- equation [ecuación]
- reflexive property of equality [propiedad reflexiva de la igualdad]
- solution [solución]
- addition property of equality [propiedad de suma/adición de la igualdad]
- subtraction property of equality
- multiplication property of equality [propiedad de multiplicación de la igualdad]
- division property of equality [propiedad de división de la igualdad]
- symmetric property of equality [propiedad simétrica de la igualdad]
- zero property of multiplication [propiedad del cero de la multiplicación]
- identity property of multiplication [propiedad de identidad de la multiplicación]
- identity property of addition [propiedad de identidad de la suma/adición]
- graph of an inequality [gráfica de una desigualdad]
- solution set of an inequality

CC BY NC

- bar model [modelo de barras]
- one-step equation
- inverse operations [operación inversa]
- literal equation [ecuación literal]
- inequality [desigualdad]
- solve an inequality [resolver una desigualdad]
- properties of inequalities [propiedades de las desigualdades]

A **solution** to an equation is any value for the variable that makes the equation true.

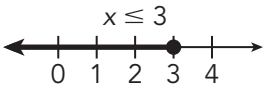
Properties of equality are logical rules that allow you to maintain balance and rewrite equations.

Properties of equality	For all numbers a, b, and c
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Symmetric property of equality	If $a = b$, then $b = a$.

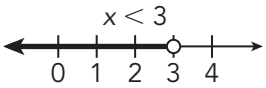
Equations that have an infinite number of solutions are equations that are true no matter what value you assign to the variable. These kinds of equations often describe important properties of numbers. For example:

- The **zero property of multiplication** states that the product of any number and 0 is 0: $x \cdot 0 = 0$.
- The **identity property of multiplication** states that the product of any number and 1 is the number: $x \cdot 1 = x$.
- The **identity property of addition** states that the sum of any number and 0 is the number: $x + 0 = x$.

You can use a number line to represent inequalities. **The graph of an inequality** in one variable is the set of all points on a number line that make the inequality true. The set of all points that make an inequality true is the **solution set of an inequality**.



The solution to any inequality can be represented on a number line by a ray, which begins at a starting point and goes on forever in one direction. A closed circle means that the starting point is part of the solution set of the inequality. An open circle means that the starting point is not part of the solution set of the inequality.



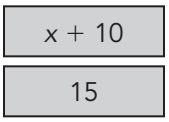
For example, the solution set of the inequality $x \leq 3$ is all numbers equal to or less than 3, and the solution set of the inequality $x < 3$ is all numbers less than 3.

LESSON 2

Solving One-Step Addition Equations

A **bar model** uses rectangular bars to represent known and unknown quantities.

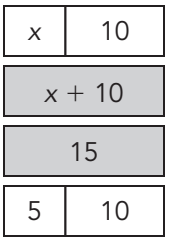
For example, the equation $x + 10 = 15$ states that for some value of x, the expression $x + 10$ is equal to 15. This can be represented using a bar model.



The expression $x + 10$ can be decomposed into a part representing x and a part representing 10. The number 15 can be decomposed in a similar way: $15 = 5 + 10$. The bar model demonstrates that these two equations are equivalent.

$$x + 10 = 15$$

$$x + 10 = 5 + 10$$



By examining the structure of the second equation, you can see that 5 is the value for x that makes this equation true.

A **one-step equation** is an equation that can be solved using only one operation. To solve a one-step addition equation, isolate the variable using number sense or *inverse operations*. **Inverse operations** are pairs of operations that reverse the effects of each other.

For example, solve the equation $h + 6 = 19$.

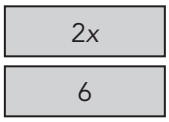
$h + 6 = 13 + 6$	Write equivalent expressions that mirror structure.
$h + 6 - 6 = 13 + 6 - 6$	Use the inverse operations to reverse the addition of 6 to h by creating a zero pair.
$h + 0 = 13 + 0$	Combine like terms and apply the additive identity property.
$h = 13$	

CC BY NC

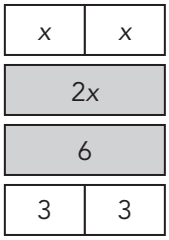
LESSON 3

Solving One-Step Multiplication Equations

You can also use bar models to reason about the solution to multiplication equations.

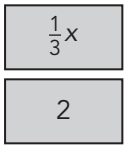


For example, the equation $2x = 6$ states that for some value of x, the expression $2x$ is equal to 6. You can decompose $2x$ by rewriting it as the equivalent expression $1x + 1x$, or $x + x$. To maintain equivalence, decompose 6 in a similar way. The bar model demonstrates that these two equations are equivalent.



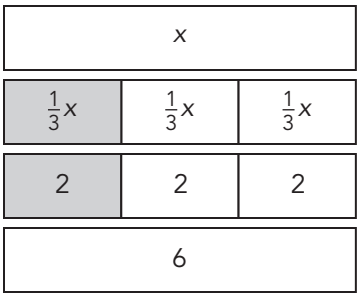
$$2x = 6$$
$$x + x = 3 + 3$$

By examining the structure of the second equation, you can see that $x = 3$.



Bar models can also be used to solve multiplication equations with fractional coefficients.

For example, represent $\frac{1}{3}x = 2$ as a bar model.



To solve this equation for x, compose 3 equally-sized parts to create the whole, x. To maintain equivalence, compose 3 equally-sized parts for the other expression, too. This structure allows you to see the value of x that makes the equation true: $x = 6$.

You can also use the inverse operation of multiplication to solve one-step multiplication equations.

For example, solve the equation $4r = -32$.

$$4r = -32$$

$4(1r) = 4(-8)$ Write equivalent expressions with similar structure.

$\frac{4(1r)}{4} = \frac{4(-8)}{4}$ Use inverse operations to reverse the multiplication of 4 and $1r$.

$1r = 1(-8)$ Perform division.

$r = -8$ Apply the identity property of multiplication.

LESSON 4

Solving Equations to Solve Problems

A literal equation is an equation in which the variables represent specific measures. You most often see literal equations when you study formulas. The formula for the area of a triangle, $A = \frac{1}{2}bh$, is a literal equation. The variables represent the measures of the area, base, and height of a triangle.

You can solve the literal equation for one of the variables, much like when you solve one-variable equations.

Consider the formula for the area of a triangle. Solve the equation for b, the base of the triangle.

$$A = \frac{1}{2}bh$$

$$(2)A = \frac{1}{2}bh(2).$$

$$\frac{2A}{h} = \frac{(bh)}{h}$$

$$\frac{2A}{h} = b$$

The base of the triangle, b, is equal to the area of the triangle multiplied by 2, then that product divided by h, the height of the triangle.

Now that you have this literal equation, when you are given the area of a triangle, A, and the height of the triangle, h, you can substitute those values into the equation to calculate b, the base of the triangle.

For example, determine the base of the triangle in which its height is 7 cm and area is 42 cm^2.

$$\frac{2A}{h} = b$$

$$\frac{2(42)}{7} = b$$

$$12 = b$$

The base of the triangle is 12 cm.

LESSON 5

Solving Inequalities with Inverse Operations

An **inequality** is any mathematical sentence that has an inequality symbol. To **solve an inequality** means to determine *solution set*, which includes all values of the variable that make the inequality statement true. Solving one-step inequalities is similar to solving one-step equations. Just be sure to follow the properties of inequalities when isolating the variable and determining the solution to the inequality.

The **properties of inequalities** allow you to solve inequalities involving any numbers. These properties also hold true for $\leq$ and $\geq$.

Properties of inequalities	For all numbers a, b, and c
Addition property of inequalities	If $a < b$, then $a + c < b + c$. If $a > b$, then $a + c > b + c$.
Subtraction property of inequalities	If $a < b$, then $a - c < b - c$. If $a > b$, then $a - c > b - c$.
Multiplication property of inequalities	If $a < b$, then $a \cdot c < b \cdot c$, for $c > 0$. If $a > b$, then $a \cdot c > b \cdot c$, for $c > 0$.
Division property of inequalities	If $a < b$, then $\frac{a}{c} < \frac{b}{c}$, for $c > 0$. If $a > b$, then $\frac{a}{c} > \frac{b}{c}$, for $c > 0$.

The inequality symbol remains the same when adding, subtracting, multiplying, or dividing both sides of an inequality by a positive number. However, the inequality symbol reverses when multiplying or dividing both sides of an inequality by a negative number.

For example, solve the following inequalities using the properties of inequalities.

$$x + 4 \geq -9$$
$$x + 4 - 4 \geq -9 - 4$$
$$x \geq -13$$

The inequality symbol does not reverse since it involves only adding to or subtracting from both sides of an inequality.

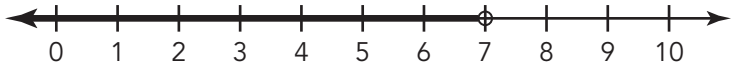
$$\frac{x}{8} < -7$$
$$(8)\frac{x}{8} < -7(8)$$
$$x < -56$$

The inequality symbol does not reverse since it involves multiplying or dividing both sides by a positive number.

$$-3x > -21$$
$$\frac{-3x}{-3} > \frac{-21}{-3}$$
$$x < 7$$

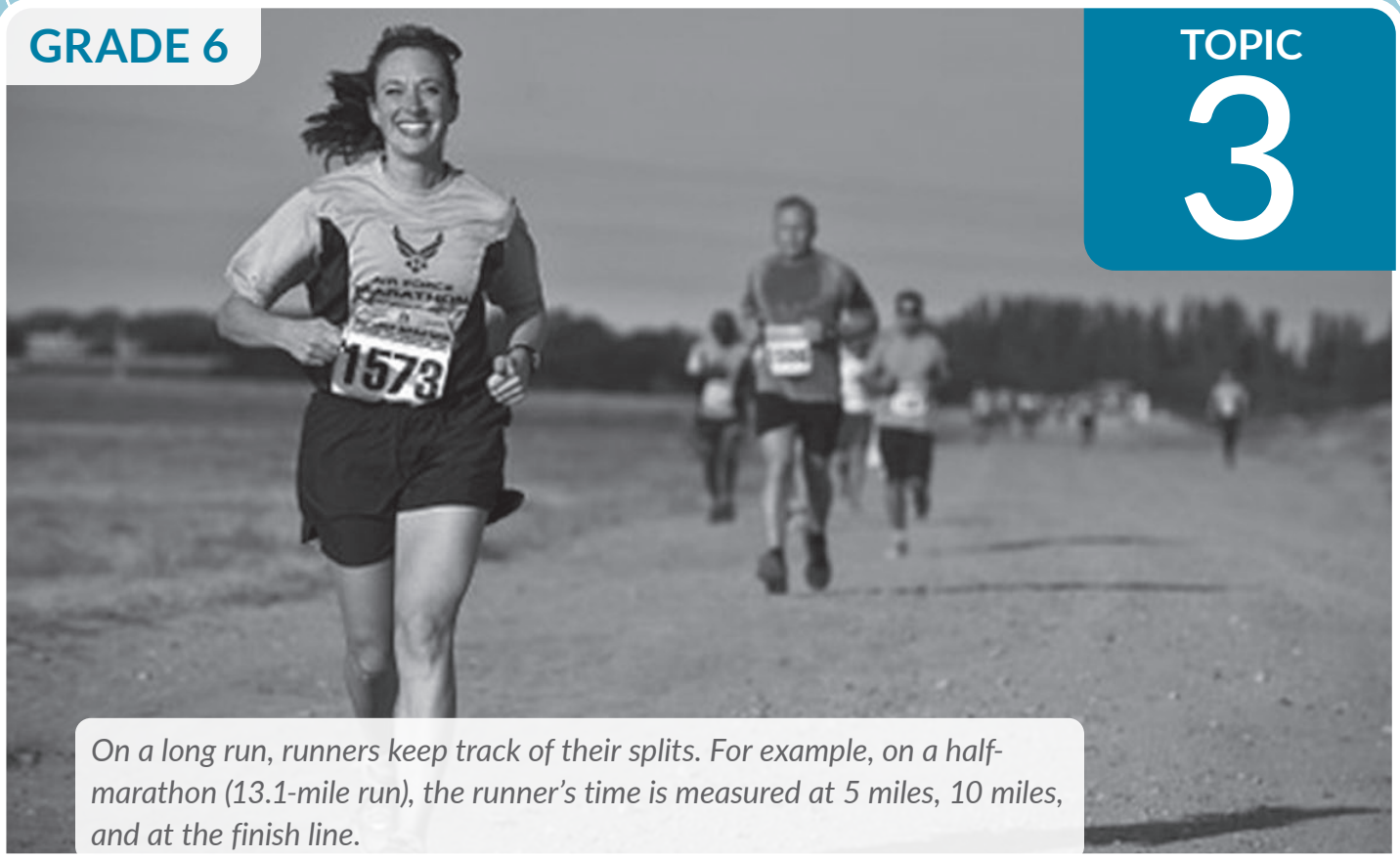
The inequality symbol does reverse since it involves multiplying or dividing both sides by a negative number.

The solution to any inequality can be represented on a number line by a ray in which its starting point is an open or closed circle. For example, the solution $x < 7$ is represented by this number line.



GRADE 6

TOPIC 3



On a long run, runners keep track of their splits. For example, on a half-marathon (13.1-mile run), the runner's time is measured at 5 miles, 10 miles, and at the finish line.

Graphing Quantitative Relationships

LESSON 1 Independent and Dependent Variables **883**

LESSON 2 Using Graphs to Solve Problems **909**

LESSON 3 Multiple Representations of Equations **923**

LESSON 4 Relating Distance, Rate, and Time **943**

LESSON 5 Problem Solving on the Coordinate Plane. **961**

1 Independent and Dependent Variables

OBJECTIVES

- Interpret information about a situation from a graphical representation.
- Identify the graphs of situations.
- Identify and use variables to define independent and dependent quantities in real-world problems.
- Write an equation to express a quantity that is the dependent variable in terms of another quantity, the independent variable.

NEW KEY TERMS

- dependent quantity
- independent quantity
- independent variable
- dependent variable

Throughout this course, you have analyzed quantities in a variety of ways. Often, the equation you write to represent variable quantities depends on the question you are answering.

How do you tell what variable quantity is the focus of a mathematical question?

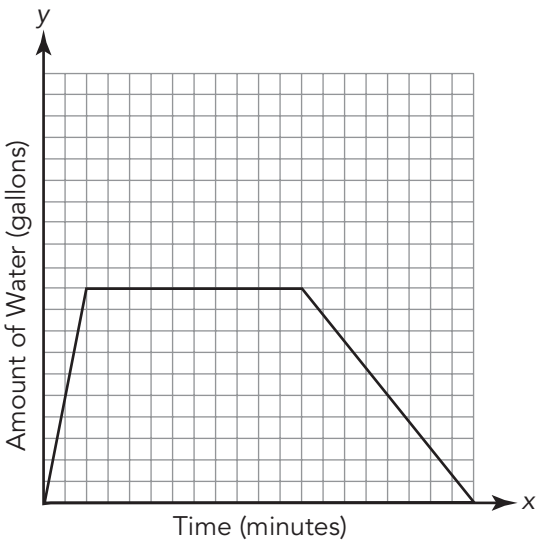
CC BY NC

Getting Started

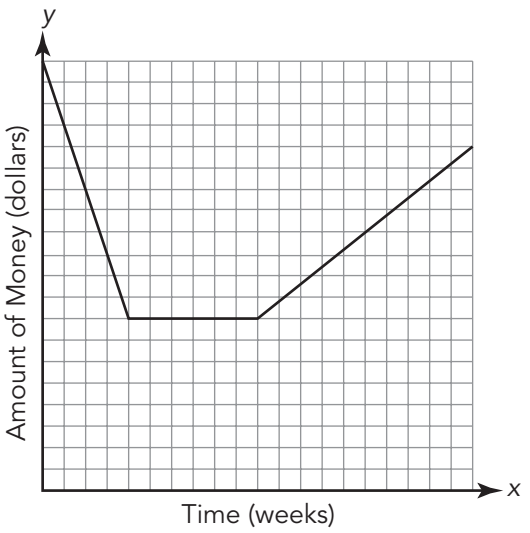
It's Not a Tall Tale!

Write a story to describe the situation that each graph represents.

1. The Water Level in the Bathtub



2. Money in Your Bank Account



ACTIVITY
1.1 Match and Analyze

In this activity, you will match a specific graph to a real-world problem situation.

Cut out the graphs and scenarios located at the end of the lesson.

1. Glue or tape each graph in the box with the appropriate scenario. Label the axes with appropriate quantities and units.

Ask Yourself . . .
Can others follow and understand your process and reasoning?

Scenario	Graph	Horizontal Axis	Vertical Axis

2. How did you determine which graph matched which scenario?

Even though only certain points make sense in the situation, you can draw a line to represent the shape of the data.

CC BY NC

Some graphs represent sets of isolated points. Often the coordinates of those points are counting numbers. In other graphs, each point on the graph, even those with fractional numbers as coordinates, represents a solution to the graphed scenario.

3. Which graphs have isolated points and which do not? How does the scenario inform you whether or not the graph will have isolated points?

4. Which graph(s) represent equivalent ratios? How does the scenario inform you that the graph(s) will display equivalent ratios?



Be sure to keep your graphs and scenarios. You will use them in the next activity.

ACTIVITY
1.2 Quantities That Change

When one quantity is determined by another in a real-world problem situation, it is said to be the **dependent quantity**. The quantity that is not determined by another is called the **independent quantity**. The variable that represents the independent quantity is called the **independent variable**, and the variable that represents the dependent quantity is called the **dependent variable**.

Consider the scenarios from the previous activity.

1. Use the Vending Machine scenario to answer each question.

   a. What two quantities are changing in this situation?

   b. Which quantity depends on, or is determined by, the other?

   c. Define variables for each quantity and label them appropriately as the independent and dependent variables.



2. Use the Fish Tank scenario to answer each question.

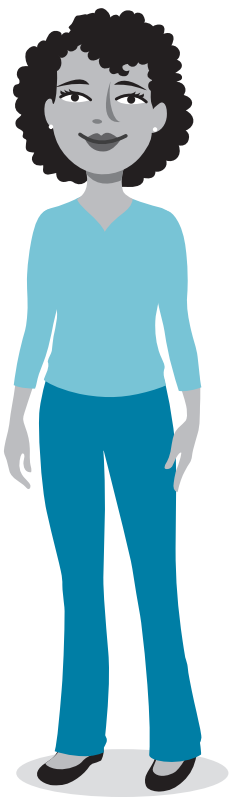
   a. What two quantities are changing in this situation?

   b. Which quantity is the independent quantity and which is the dependent quantity?

   c. The equation that represents the fish scenario is $w = 200 - 10t$. What do the variables w and t represent in this equation?

   d. What do you notice about which variable is isolated in the equation?

Examine the graphs. Do you see any connection between the independent and dependent variables and the graph?



3. Identify the independent quantity and the dependent quantity in each of the four remaining scenarios.

   a. Rainy Day

      Independent Quantity:

      Dependent Quantity:

   b. Toy Rocket

      Independent Quantity:

      Dependent Quantity:

   c. Pool Party

      Independent Quantity:

      Dependent Quantity:

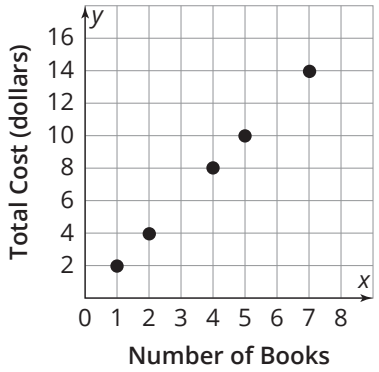
   d. T-Shirt Sales

      Independent Quantity:

      Dependent Quantity:

CC BY NC

4. The library is having a used book sale. Each book costs $2. The graph represents the sales from the first 5 customers. Determine which quantity represents the independent variable and which quantity represents the dependent variable. Then, list the independent values and dependent values of the graphed points.



ACTIVITY
1.3 Total Price and Profit

A store makes 20% profit on the total price of all the items they sell.

Analyze the situation.

1. Name the two quantities that are changing.

Profit is the extra money for selling items, over and above the cost of producing the items.

2. Describe which value depends on, or is determined by, the other.

Let t represent the total price of all items sold in dollars, and let p represent the *profit* in dollars.

3. Write an equation to represent the relationship between these variables.

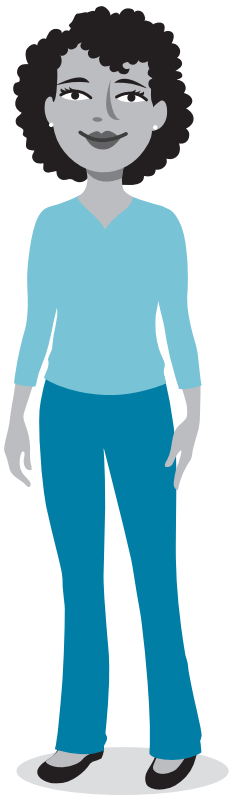
4. Identify the independent and dependent variables in this situation.

CC BY NC

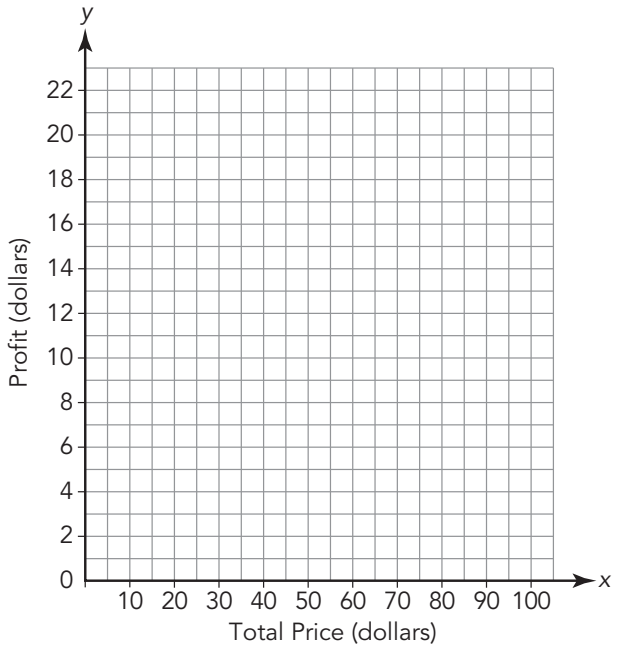
5. Complete the table.

	Independent Quantity	Dependent Quantity
Quantity Name		
Unit of Measure		
Variable		

Why do you think the axes are labeled with total price on the horizontal axis and profit on the vertical axis?



6. Use the table to complete the graph.



7. On which axis is the independent variable? On which axis is the dependent variable?

Talk the Talk

Create Your Own Story

1. Create a real-world situation to match the given table.

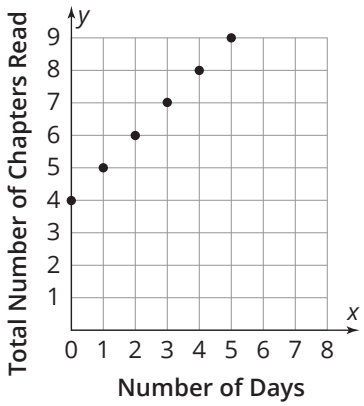
x	y
0	0
3	15
5	25
6	30
8	40

2. Identify the independent and dependent quantities in your scenario.

CC BY NC

Use the table or graph to identify the independent quantity and dependent quantity. Then, list the values for the independent quantity and the values for the dependent quantity that are given.

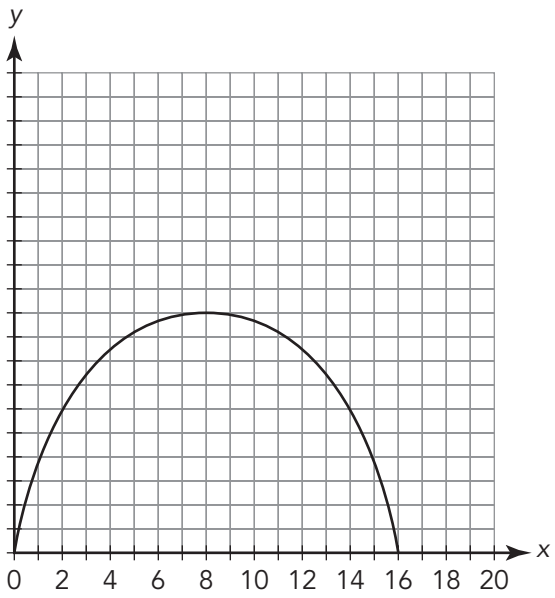
3. Chris has already read 4 chapters of his book. His teacher challenges him to read an additional chapter each day.



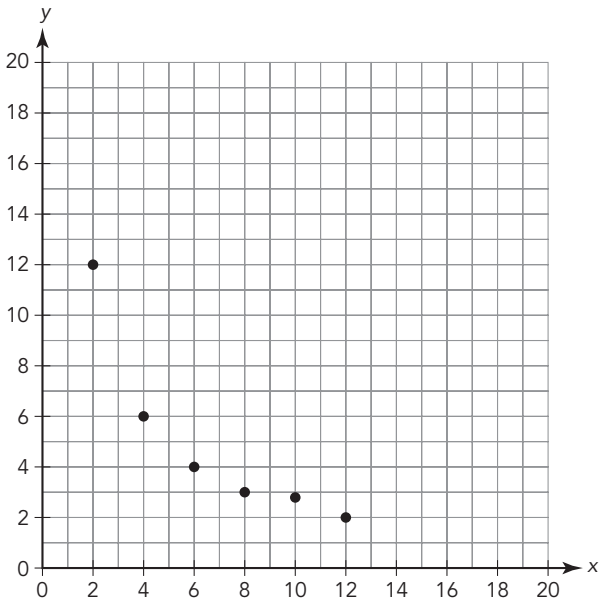
4. A bookstore sells graphic novels for $12 each.

Number of Graphic Novels	1	2	4	6	7	10
Total Cost (dollars)	12	24	48	72	84	120

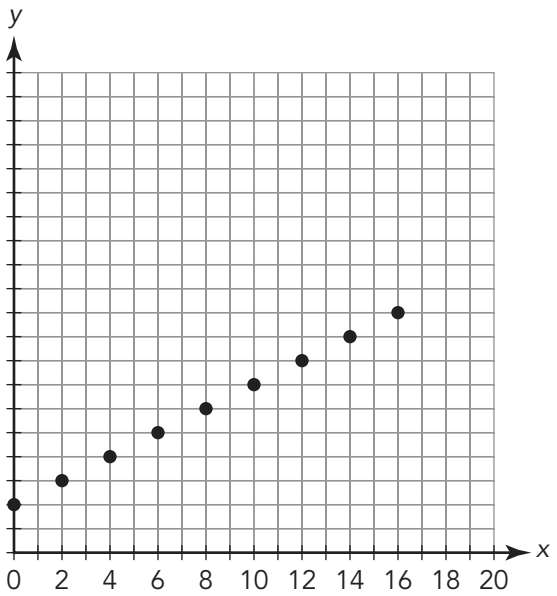
A


y
x
0 2 4 6 8 10 12 14 16 18 20

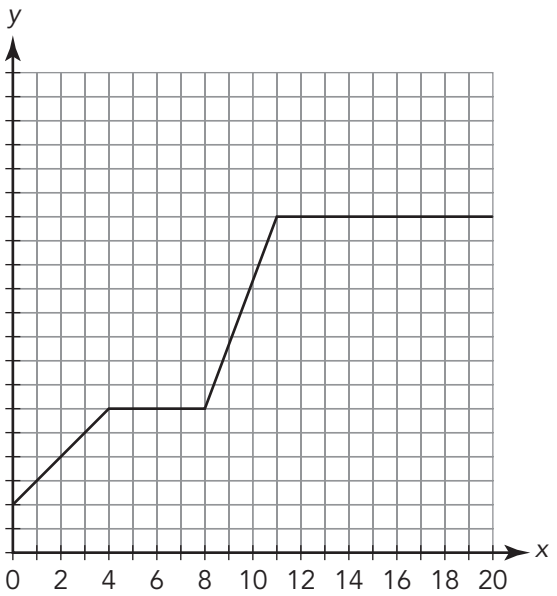
B


y
x
0 2 4 6 8 10 12 14 16 18 20

C


y
x
0 2 4 6 8 10 12 14 16 18 20

D

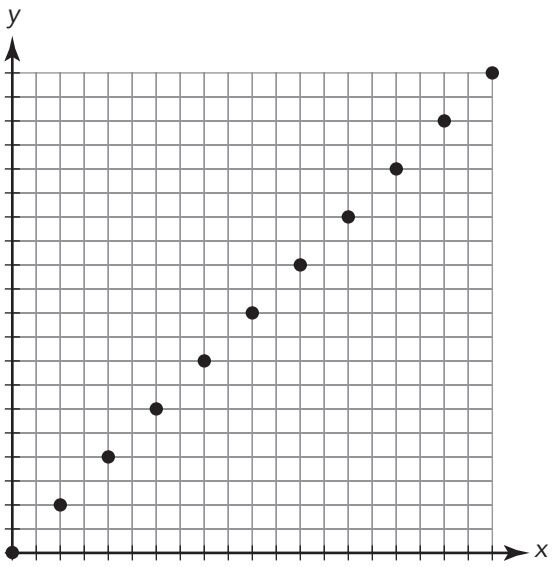

y
x
0 2 4 6 8 10 12 14 16 18 20

CC BY NC

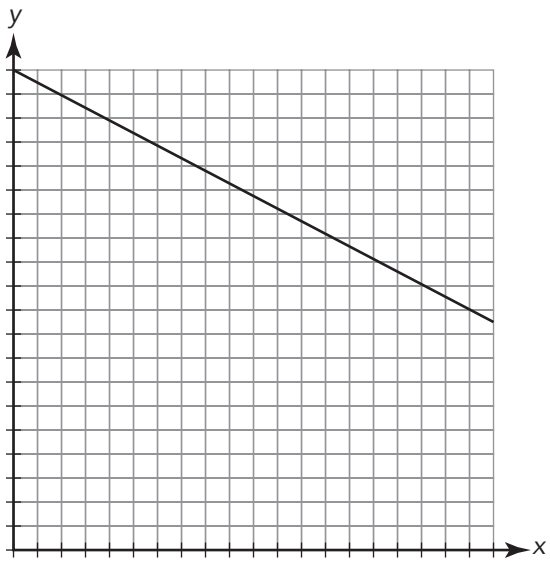
Why is this page blank?

So you can cut out the graphs on the other side.

E



F



Pool Party

You have cookies for your team pool party, but you don't know how many of your teammates will show up. How many cookies will each teammate receive when everyone receives the same number of cookies?

Why is this page blank?

So you can cut out the graphs and scenarios on the other side.

Fish Tank

You are draining a 200-gallon fish tank at a rate of 10 gallons per minute. How much water remains in the tank at a specific time?

Rainy Day

When you left home, the rain was falling at a steady rate. Then, it stopped raining for a few hours before a sudden downpour. Finally, it stopped raining. How many inches of rain had fallen at different points of the day?

Why is this page blank?

So you can cut out the scenarios on the other side

Toy Rocket

You launch a toy rocket into the air from the ground and observe its height through its entire flight. How many feet high was the rocket at a specific time after launch?

Vending Machine Snacks

The vending machine charges $2.00 for snacks. How many snacks can you buy for different amounts of money?

Why is this page blank?

So you can cut out the scenarios on the other side.

T-Shirt Sales

You buy T-Shirts to sell for your school. There is a $25 design charge plus the cost per T-Shirt. What is the total cost for different numbers of T-Shirts?

CC BY NC

Why is this page blank?

So you can cut out the scenario on the other side.

Lesson 1 Assignment

Write

Write the term that best completes each statement.

1. In a real-world problem situation, when a quantity does not depend on another quantity, it is called the _____.
   In the equation that models the problem situation, this quantity is represented by the _____.

2. In a real-world problem situation, when a quantity depends on another quantity, it is called the _____.
   In the equation that models the problem situation, this quantity is represented by the _____.

Remember

The mathematical or real-world question asked often determines which quantity depends on the other. When graphed, the independent quantity is plotted on the x-axis and the dependent quantity is plotted on the y-axis.

Practice

1. Determine the independent variable and the dependent variable in each given equation.

   a. You can use the equation $T = 75 - d$ to calculate the water temperature, T, at a depth, d, in a particular lake.

   b. You can use the equation $p = \frac{1}{3}b$ to calculate the individual profit, p, that each of three brothers made while a lemonade stand with a total profit, t.

Lesson 1 Assignment

2. An online ticket broker charges a flat service fee of $6.50 per ticket sold. You are interested in the total amount of money you must pay for a given number of tickets.

   a. Name the two quantities that are changing in this situation.

   b. Define variables for each quantity and identify which represents the independent quantity and which represents the dependent quantity.

   c. Write an equation for the relationship between the two variables.

3. Kaya is a runner. When she is training for a race, she averages 8 miles per hour. She is interested in how far she can run in a given number of hours.

   a. Define variables for each changing quantity and identify each as the independent or dependent variable.

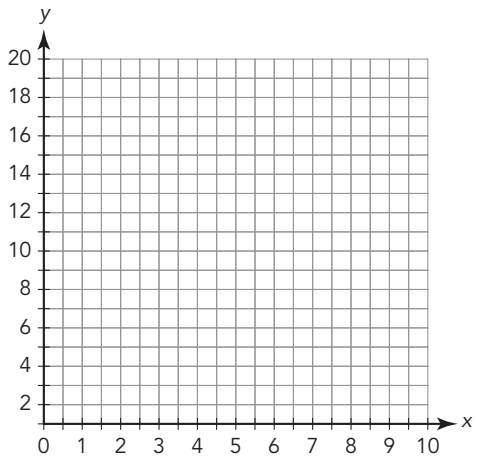
   b. Write an equation to represent this situation.

Lesson 1 Assignment

c. Use your equation to create a table of values for this situation.

	Independent Quantity	Dependent Quantity
Quantity Name		
Unit of Measure		
Variable		

d. Use your equation and table to create a graph. Remember to label your axes.



Lesson 1 Assignment

e. Explain how you knew which variable to graph on each axis.

Prepare

Which equations represent proportional relationships? Explain how you know.

1. $c = 2.5n$

2. $l = w + 25$

3. $d = \frac{1}{3}t$

4. $T = 100 - d$

2 Using Graphs to Solve Problems

OBJECTIVES

- Analyze the relationship between the independent and dependent variables in a graph and relate the variables to an equation.
- Use multiple representations to solve one-step real-world problems.
- Use an inequality of the form $x > c$ or $x < c$ to represent constraints when solving a real-world problem.

You have learned how to solve one-step equations using reasoning and the properties of equality.

How can you use graphs to solve one-step real-world problems?

CC BY NC

Getting Started

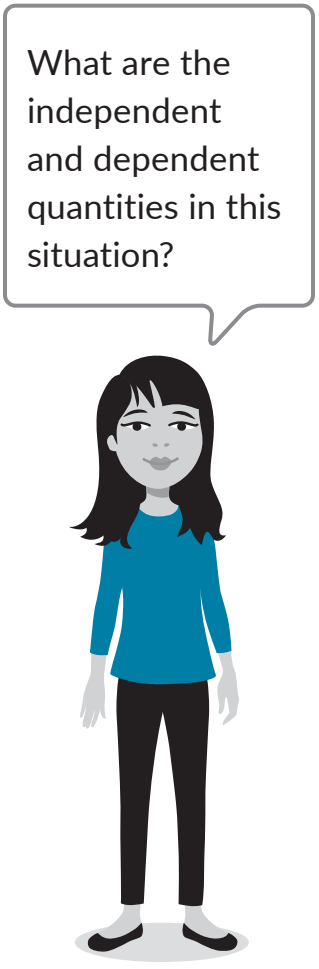
Selling Pretzels

Mario sells pretzels at the morning baseball and softball games held at the Community Center. At the end of the games, he reports the number of pretzels he sold and the total amount of money collected in a table.

Mario sold pretzels on three additional mornings, but he only reported either the number of pretzels sold or the dollar amount collected.

1. Complete the table.

Number of Pretzels Sold	Total Amount of Money Collected
10	12.50
15	18.75
16	
40	
32	40



2. What are the independent and dependent variables in the situation?

3. Write an equation to represent the relationship between the number of pretzels sold, x, and the amount of money collected in dollars, y.

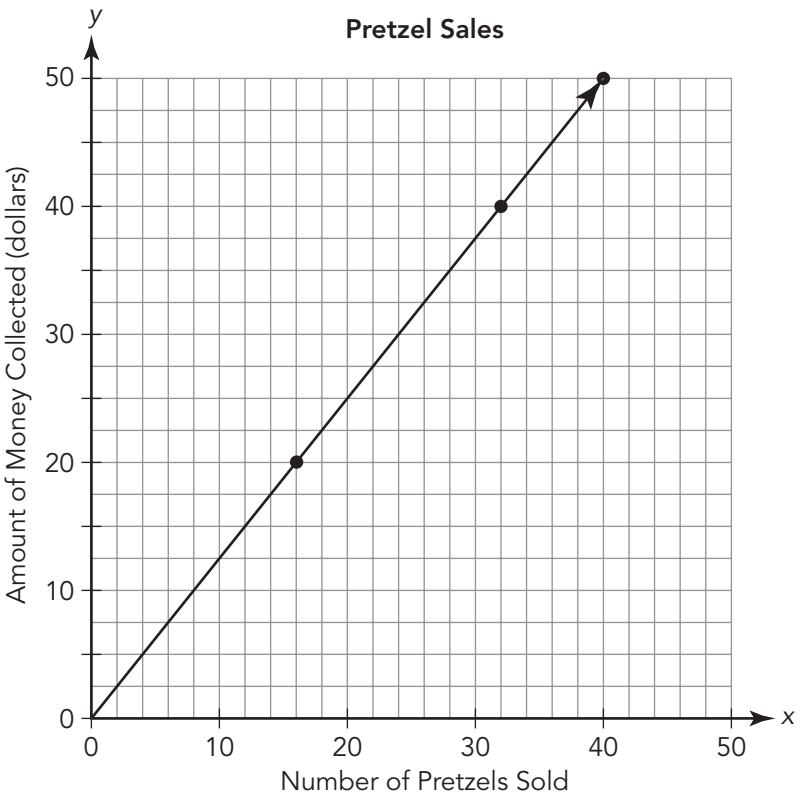
4. Does this situation represent a proportional relationship? Justify your answer.

ACTIVITY 2.1

Using a Graph to Determine Unknown Quantities

In the previous activity, you answered questions and wrote the equation $y = 1.25x$, where x represents the number of pretzels sold and y represents the amount of money collected in dollars.

The amount of money collected for the number of pretzels sold can be represented by points on the graph. The graph shows the ordered pairs corresponding to the three questions you answered about Mario selling pretzels. The equation corresponding to the graph is $y = 1.25x$.



1. Label the three ordered pairs shown on the graph.

2. What does each ordered pair represent?

3. Identify the unit rate in this situation. Plot and label it on the graph.

One way to analyze the relationship between ordered pairs displayed on a graph is to draw a line.

When you model a relationship with a line, it is up to you to consider each situation and interpret the meaning of the data values from the line drawn on the graph.

In some problem situations, when you draw a line, all the points will make sense. In other problem situations, like this one, not all the points on the line will make sense.

CC BY NC

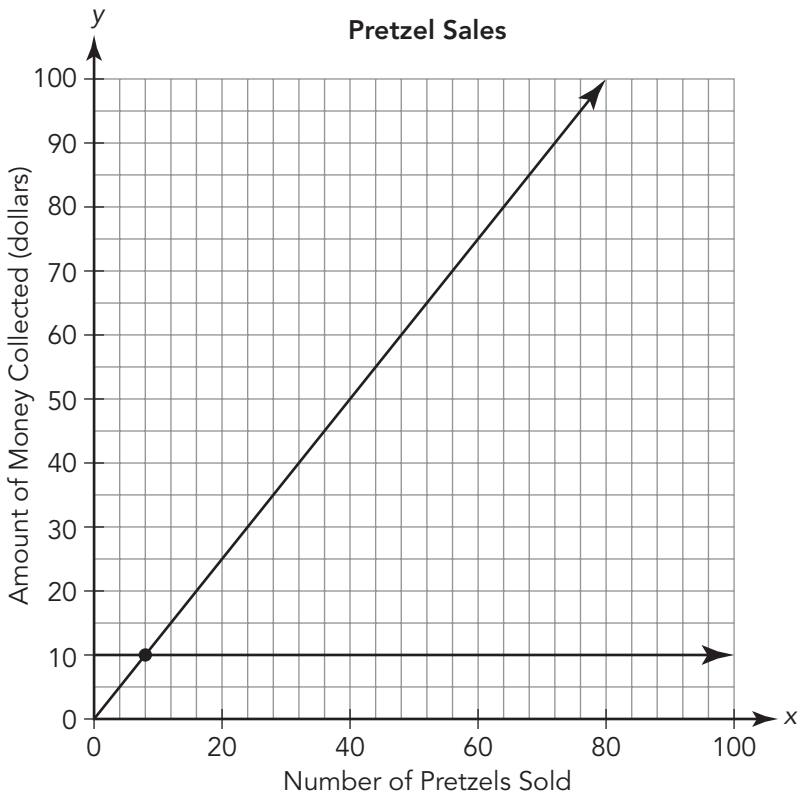
You can use a graph to determine an independent quantity given a dependent quantity.

Remember . . .
the solution to an equation is any value that makes the equation true. When you are given a graph, a solution is any point on that graph.

WORKED EXAMPLE

You can use the graph to determine how many pretzels Mario sold when he collected $10.

First, locate 10 on the *y*-axis and draw a horizontal line. This shows that $10 is the amount of money collected. The *x*-value of the point where your horizontal line intersects with the graph of $y = 1.25x$ is the number of pretzels sold for $10.



4. How many pretzels did Mario sell when he collected $10?

Use the graph in the Worked Example to answer each question.

5. How many pretzels did Mario sell when he collected:

   a. $25.00?

   b. $33.75?

   c. $75.00?

Ask Yourself . . .
Did you justify your mathematical reasoning?



6. Mario reported that on Saturday morning, he sold 13 pretzels and collected $16.25, and on Saturday afternoon, he sold 42 pretzels and collected $55.00.

   Do you think he reported accurately? Explain your reasoning.

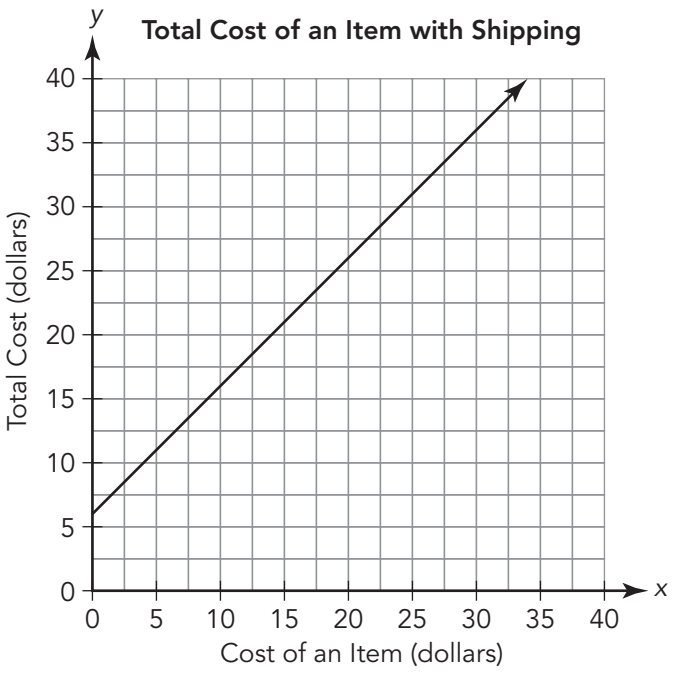
CC BY NC

ACTIVITY
2.2 Shipping Charges

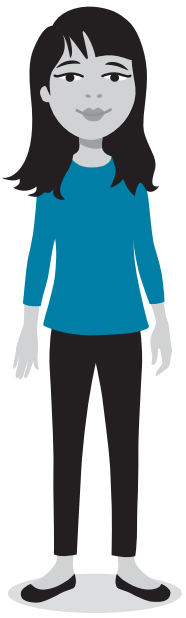
A *flat fee* is a one-time charge on each order, regardless of how many items are ordered.

An online site sells a single closeout item each day. The items and prices change daily. The company charges a flat fee of $6.00 for shipping.

The graph shown represents a model of this problem situation.



In this problem situation, do all the points on the line make sense?



1. Write an equation to represent the relationship between the cost of an item, x, and the total cost, y. Label the graph of the line with your equation.

2. Does this situation represent a proportional relationship? Justify your answer.

Use the graph to answer each question.

3. What is the total cost when the cost of the item is:

   a. $18.00?

   b. $25.00?

   c. $32.50?

   d. $75.00?

4. Chris was considering buying one of the daily closeout items that costs $23.99. He has a $25.00 gift card. Can he afford the total cost of the daily closeout item?

5. Suppose the flat fee for shipping changed to $6.80. How would the graph change? How would the equation change? Would that change the way you could use the graph to determine values?



Talk the Talk

Plus or Times?

1. Describe the similarities and differences between the pairs of equations.

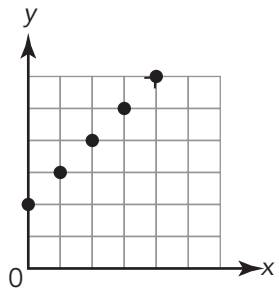
   a. $y = 5x$ and $y = x + 5$

   b. $y = \frac{1}{2}x$ and $y = \frac{1}{2} + x$

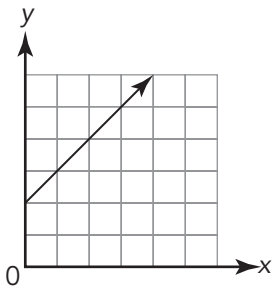
   c. $y = 4.95x$ and $y = x + 4.95$

2. Describe the similarities and differences among the graphs shown. Write an equation for each graph.

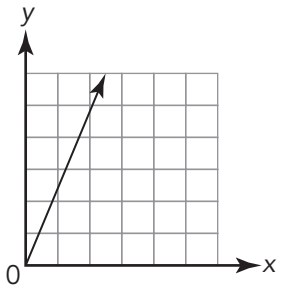
Graph A



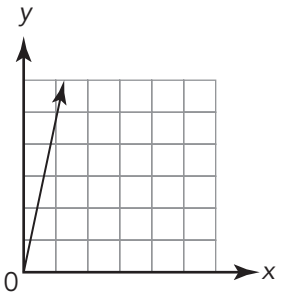
Graph B



Graph C



Graph D



3. Describe how you can use a graph to solve one-step equations.

CC BY NC

Lesson 2 Assignment

Write

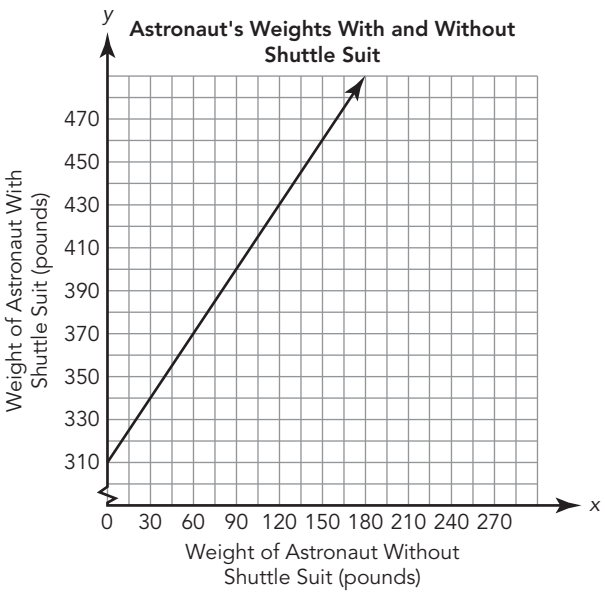
Describe the similarities and differences between the graphs of equations represented in the forms $y = kx$ and $y = x + b$, where b is any positive rational number and x and y are unknown quantities.

Remember

Graphs are powerful visual representations of how quantities are related. You can use a graph to estimate a solution. You can also formally solve equations to determine exact values.

Practice

A shuttle spacesuit, including the life support system, weighs about 310 pounds. The break in the y-axis represents the values from 0–300.



1. What does each ordered pair on the line represent?

2. Write an equation to represent the relationship shown in the graph.

3. Is this a proportional relationship? Justify your answer.

4. In this problem situation, do all the points on the line make sense? Explain your reasoning.

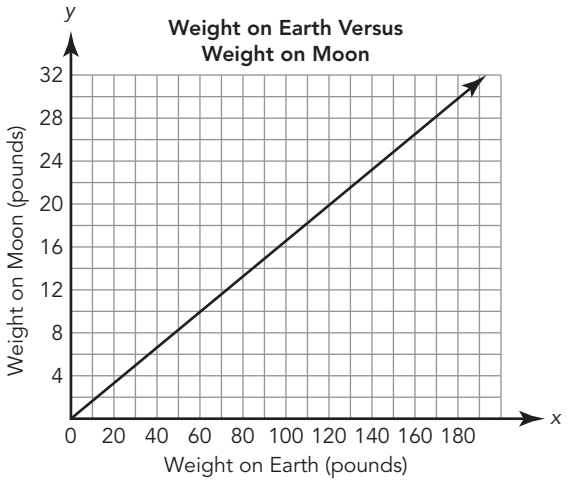
CC BY NC

Lesson 2 Assignment

5. Determine the weight of an astronaut without the shuttle suit, given the astronaut's weight while wearing the shuttle suit.
   a. 480 lbs
   b. 467 lbs
   c. 520 lbs

The gravitational pull of the Moon is not as great as that on Earth. In fact, when a person checks their weight on the Moon, it will be only $\frac{1}{6}$ of their weight on Earth.

6. What does each ordered pair on the line represent?



7. Write an equation to represent the relationship shown in the graph.

8. Is this a proportional relationship? Justify your answer.

9. In this problem situation, do all the points on the graph make sense?

10. Determine the weight of a person on Earth given his weight on the Moon.
    a. 12 lbs
    b. 21 lbs
    c. 36 lbs

Lesson 2 Assignment

Prepare

Identify two quantities in each situation. Then, state which quantity depends on the other.

1. Snow is falling at a rate of 3 inches per hour.

2. The outside temperature is increasing at an average of 10 degrees each day.

3. The car wash generated an average profit of $8.00 per car.

4. The income from the sale of movie tickets was $8.50 per person.

5. A dog groomer charges $35 for every dog.

CC BY NC

3 Multiple Representations of Equations

OBJECTIVES

- Write and solve equations that represent relationships given in tables, graphs, and situations.
- Identify independent and dependent quantities represented in tables, graphs, and scenarios.
- Analyze the relationship between the independent and dependent quantities in a situation using graphs, tables, and equations.

You have identified independent and dependent quantities in relationships and have expressed these relationships using equations.

How can you relate independent and dependent quantities in a variety of situations, using different representations?

CC BY NC

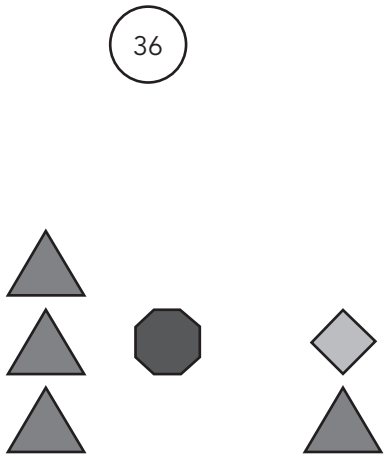
Getting Started

To the Equation-Mobile!

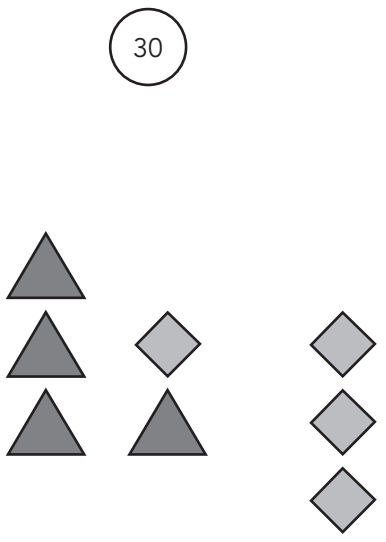
A mobile (MO-beel) is hanging art. It features all kinds of different objects suspended from string or wire. Balance is important to the visual effect of mobiles.

Determine what value each shape represents in each mobile.

1.



2.



ACTIVITY
3.1 Equations from Tables

A photo printing service charges the same price for each 3 in. by 4 in. picture print.

1. The table shows a few orders and the cost of each.

Number of Prints	Cost ($)
10	1.20
32	3.84
50	6.00
110	13.20

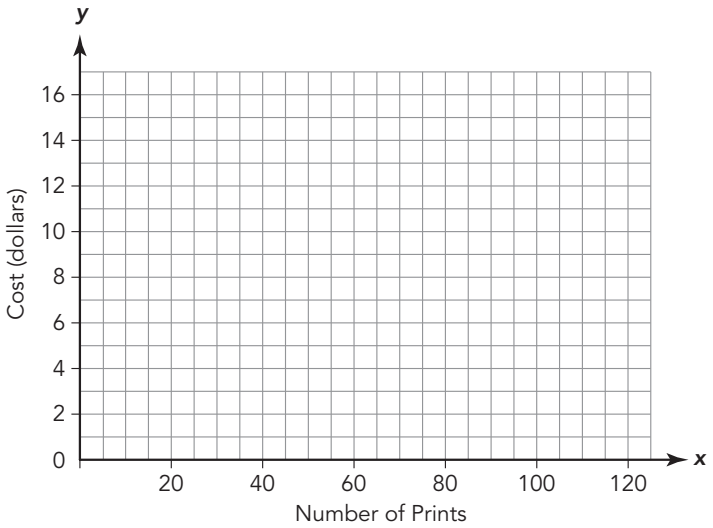
   a. What is the cost of one print? Explain how you determined the cost.

   b. Define variables for the quantities in this situation. Then write an equation that models the relationship between these quantities.

CC BY NC

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

c. Create a graph for this situation.



d. Which quantity is the independent quantity and which is the dependent quantity?. Explain your reasoning.

e. You can draw a line to show the shape of the graph. Do all the points on the line make sense in this scenario?

2. The table shows the cost of a particular item.

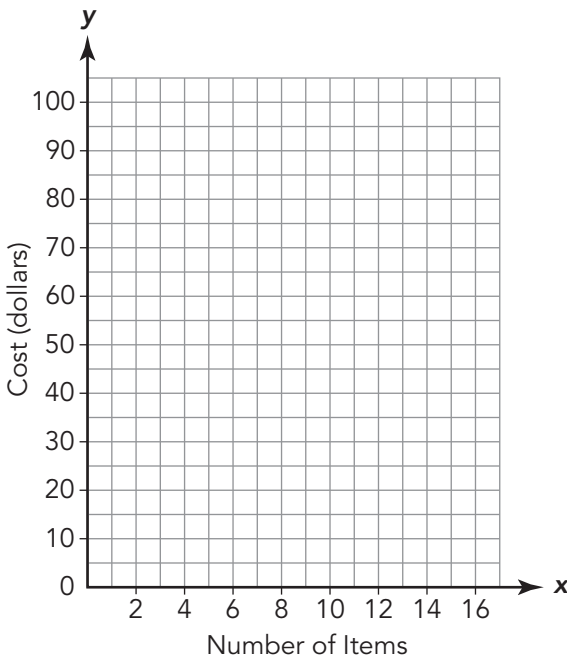
Number of Items	Cost ($)
1	6
8	48
16	96
5	30

a. Describe how the cost is related to the number of items.

b. Define variables for the quantities in this situation. Then write an equation that models the relationship between these quantities.

c. Explain which quantity is the independent quantity and which quantity is the dependent quantity.

d. Create a graph for this situation.



e. What do you notice about the shape of the graph? You can connect the points to see the shape.

CC BY NC

3. Read each situation and analyze the corresponding table of values. Identify the independent and dependent quantities in each. Then, write an equation that models the relationship between the quantities.

   a. The total profit made on cutting lawns and the profit each person made are represented in the table shown.

Total Profit Made ($)	Profit Each Person Made ($)
21.00	7.00
25.50	8.50
45.00	15.00

   b. The number of boxes of cookies sold and the total profit are represented in the table shown.

A table of values can be represented vertically or horizontally.

Boxes of Cookies Sold	Total Profit ($)
3	7.50
5	12.50
7	17.50

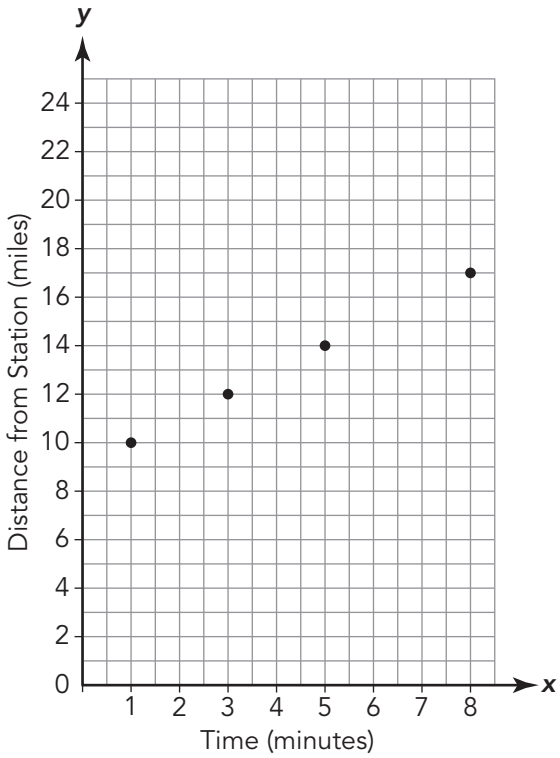
c. The number of tiles required to complete a job and the number of tiles ordered are represented in the table shown.

Number of Tiles Required	75	95	115
Number of Tiles Ordered	90	110	130

CC BY NC

ACTIVITY
3.2 Equations from Graphs

The graph shows the relationship between the distance of a train from the station and the time in minutes.



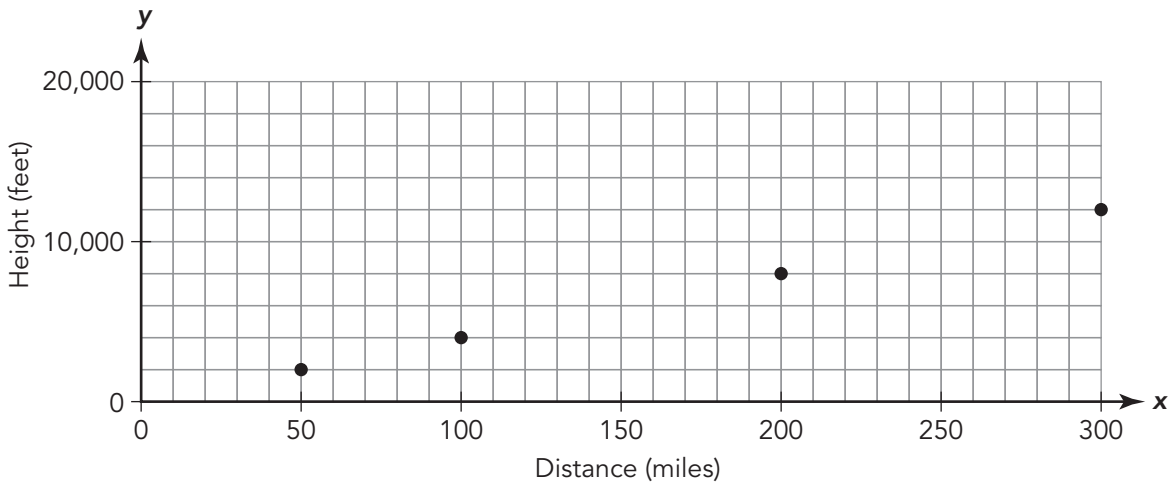
1. Complete the table using the points from the graph.

2. Define variables and write an equation to represent the relationship between the quantities.

3. How far is the train from the station after 20 minutes?

4. Connect the points to show the shape of the graph. Do all of the values on the line make sense in this situation?

The graph shows the relationship between the height of a plane and its distance from the airport in miles.



5. Complete the table using the points from the graph.

6. Define variables and write an equation to represent the relationship between the quantities.

7. What is the height of the plane at 150 miles?

8. Connect the points to show the shape of the graph. Do all of the values on the line make sense in this situation?

ACTIVITY
3.3 Equations from Scenarios

Paola got a job working at the local hardware store making $8.76 per hour.

1. Define variables and write an equation that models the relationship between the number of hours Paola worked and how much she earned.

2. How much would Paola earn when she worked the given times?

   a. 5 hours

   b. $2\frac{1}{2}$ hours

   c. 5 hours and 30 minutes

   d. 10 hours and 15 minutes

3. Use your equation to calculate the number of hours Paola worked given her total earnings.

   a. $210.24

   b. $293.46

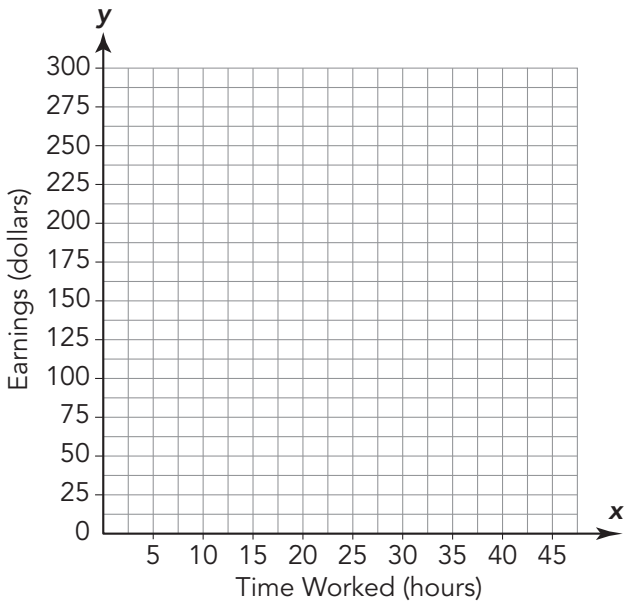
   c. $203.67

CC BY NC

4. Complete the table.

Time Worked (hours)	Earnings ($)
5	
2.5	
5.5	
10.25	
	210.24
	293.46
	203.67

5. Use the table to complete the graph.



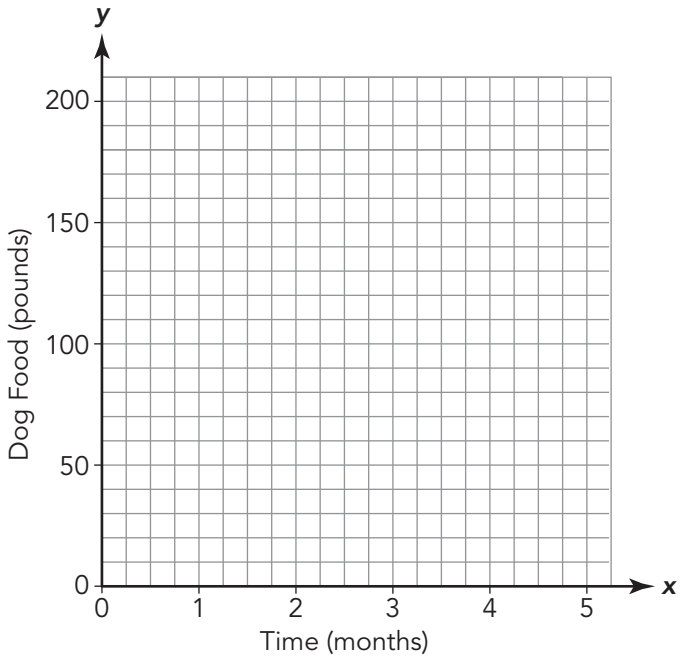
6. Connect the points to show the shape of the graph. Do all of the values on the line make sense in this situation?

7. Jaylen's dog eats an average of 40 pounds of dry dog food in one month.

   a. Define variables and write an equation to model the relationship between the number of pounds of dog food and the number of months.

   b. Complete the table and graph.

Time (months)	Dog Food (pounds)
0	
1	
2	
	120
	160
	200



   c. Connect the points to show the shape of the graph. Do all of the values on the line make sense in this situation?

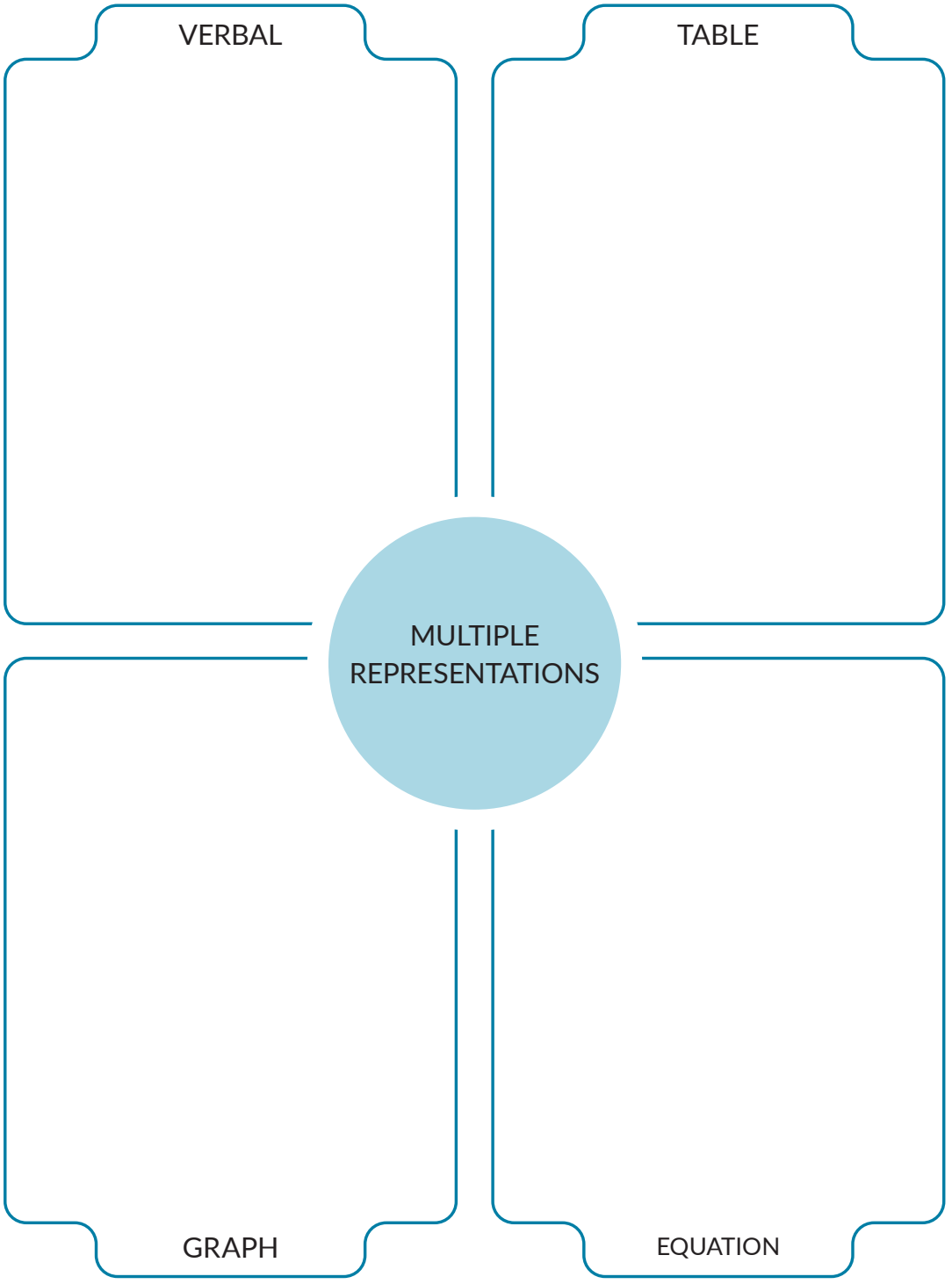
Talk the Talk

Let Us Organize the Ways

Multiple representations—including drawings or diagrams, verbal descriptions, tables, graphs, and equations—can be useful in analyzing and solving problems.

1. Complete the graphic organizer by describing the advantages of each representation.
   - Verbal
   - Table
   - Graph
   - Equation

Ask Yourself . . .
How can you organize and record your mathematical ideas?


VERBAL
TABLE
MULTIPLE REPRESENTATIONS
GRAPH
EQUATION

Lesson 3 Assignment

Write

Create an algebraic equation. Represent the equation using a word problem, a table, and a graph.

Remember

You can relate independent and dependent quantities using a table of values, a graph, or an equation.

Practice

1. Elena works at the local candy shop. The candy shop sells bulk candy by the pound. Customers place the candy they would like to buy in a plastic bucket and then Elena weighs it to determine how much the customer owes. Before calculating the price, Elena must subtract the weight of the plastic bucket. The candy bucket weighs 0.72 pound.

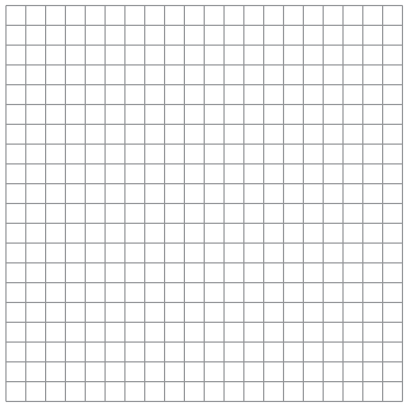
   a. Complete the table.

Total Weight (lbs)	Weight of Candy (lbs)
2.84	
3.00	
	0.71
0.98	
	1.71
	1.13

   b. Define variables and write an equation that models the relationship between the quantities in this situation.

Lesson 3 Assignment

c. Use the table to create a graph of the relationship.



d. Explain whether all points on the line make sense.

2. Elena is packaging some bulk candy for a sale. The price is $3.98 per pound.

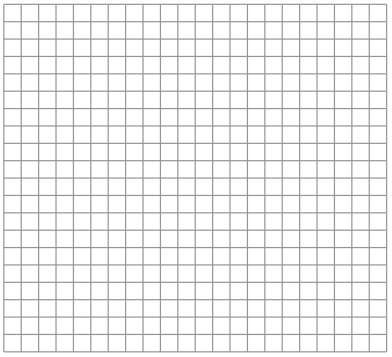
   a. Define variables and write an equation to model the relationship between the total cost and the weight of the candy.

Lesson 3 Assignment

b. Complete the table.

Weight of Candy (lbs)	Total Cost ($)
2.50	
3.20	
	4.98
1.97	
	9.47
	13.93

c. Use the table to create a graph of the relationship.



d. Explain whether it makes sense to connect the points on your graph.

CC BY NC

Lesson 3 Assignment

Prepare

1. Express 3 hours and 15 minutes as a decimal.

2. Express 3 hours and 15 minutes in terms of minutes.

3. Express 2.75 hours in terms of hours and minutes.

4. Express 2.75 hours in terms of minutes.

4 Relating Distance, Rate, and Time

OBJECTIVES

- Use multiple representations to solve one-step real-world and mathematical problems.
- Analyze the relationship between independent and dependent quantities using graphs, tables, and equations.
- Summarize the relationship between distance, rate, and time.

You have graphed and analyzed a variety of relationships between two quantities. Some quantities are often grouped together. One set of such quantities is distance, rate, and time.

What relationship exists between these quantities?

CC BY NC

Getting Started

Gearing Up for the Olympics

Tiara has her sights set on competing in the triathlon in the 2024 Summer Olympics. A triathlon includes three sports: swimming, cycling, and running. Tiara must build up her endurance to be able to complete all three events in quick succession!

As part of her training, Tiara will participate in a variety of triathlons over the next year. The table provides the distances for each leg of five different triathlons.

	Triathlon A	Triathlon B	Triathlon C	Triathlon D	Triathlon E
Swim	2.4 km	600 m	1.5 km	750 m	3.9 km
Cycle	29 km	15 km	40 km	20 km	180 km
Run	12.9 km	5 km	10 km	5 km	42 km



1. What is the total distance covered in each triathlon in kilometers?

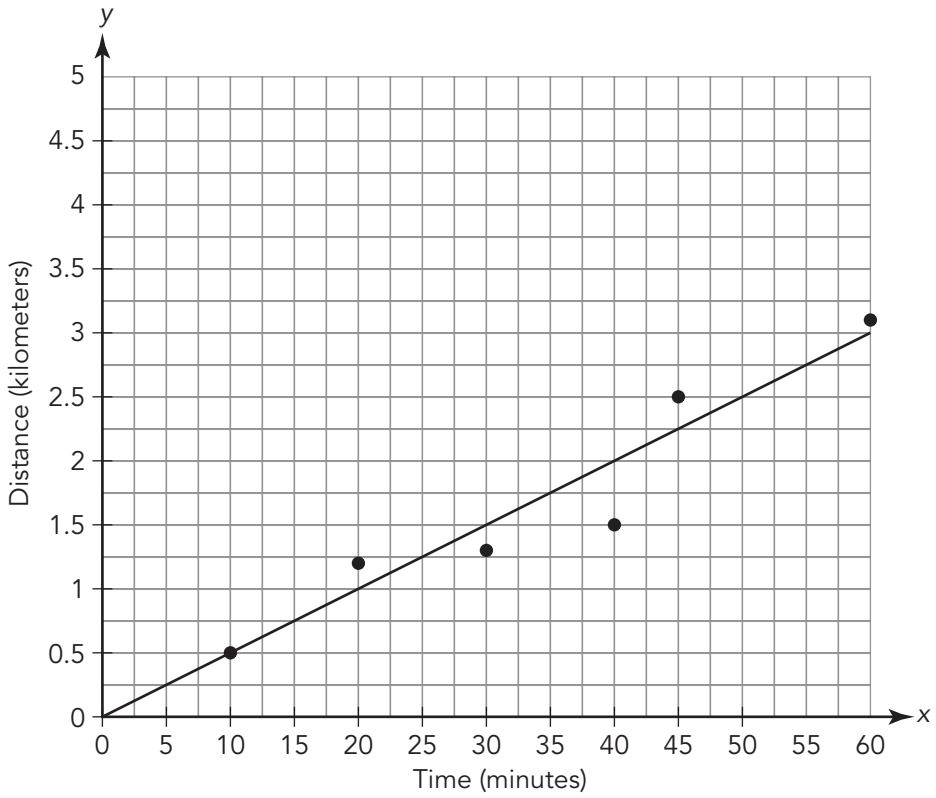
	Triathlon A	Triathlon B	Triathlon C	Triathlon D	Triathlon E
Total Kilometers					

2. Suppose Tiara completes all five triathlons in one year. How many meters will she swim during these competitions? How many meters will she cycle? How many meters will she run?

ACTIVITY
4.1 Swimming Rate

Swimming is the first leg of the triathlon, so Tiara has trained with a coach to improve her chance of getting off to a great start.

Tiara's coach plotted her times and distances from her last few training sessions. Based on the data, the coach drew in a line to represent an approximation of her average speed, or swimming rate.





1. Tiara's little sister thinks that all of Tiara's (time, distance) points should be on the line that her coach drew. Is she correct? Explain your reasoning.

2. Use the graph to determine Tiara's average swimming rate.

Tiara wants to know how long it will take her to complete the swimming segment of each triathlon. She decides to start with Triathlon C, which has a 1.5 km swim leg.

WORKED EXAMPLE

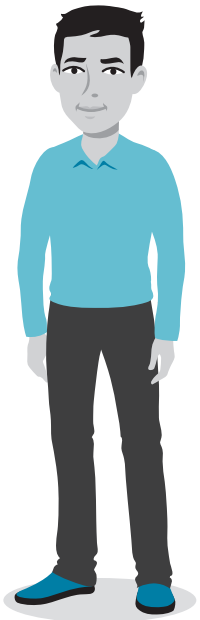
$$\frac{\text{distance}}{\text{time}} = \frac{1 \text{ km}}{20 \text{ minutes}}$$

$$\frac{1.5 \text{ km}}{\text{time}} = \frac{1 \text{ km}}{20 \text{ minutes}}$$

$$\frac{1.5 \text{ km}}{\text{time}} = \frac{1 \text{ km}}{20 \text{ minutes}} \quad \begin{matrix} \cdot 1.5 \\ \cdot 1.5 \end{matrix}$$

It should take Tiara 30 minutes to complete the swim segment of Triathlon C.

There is a proportional relationship between distance and time.



3. Assuming Tiara swims at her average rate, determine how long it should take her to complete the swimming segment of the four remaining triathlons.

a. Triathlon A

b. Triathlon B

c. Triathlon D

d. Triathlon E

4. Write an equation to represent the amount of time in minutes, t, required for Tiara to swim a given distance, d, in kilometers.

5. Write another equation to represent the distance, d, Tiara can swim for a given amount of time, t.

6. Tiara's coach surprised her with an entry into a secret triathlon, last weekend.

   a. When she swam at her average rate and completed the swim segment in 45 minutes, how long was the swim segment? Explain your reasoning.

   b. How could you use a different strategy to verify your answer?



You can use the equation $d = rt$, where d represents the distance traveled, r represents the rate of the distance traveled to the time, and t represents the time, to solve a variety of real-world problems.

ACTIVITY
4.2 Cycling Rate

Tiara cycles as a regular part of her training schedule. After each ride, she records her distances and times in a table.

Distance Biked (kilometers)	Time (hours)
5	$\frac{1}{4}$
35	$1\frac{3}{4}$
90	$4\frac{1}{2}$

1. Determine Tiara's cycling rate in minutes per kilometer and in kilometers per minute.



2. Tiara would like to predict how long it will take her to complete the cycling segment of each triathlon. She thinks she should use the *minutes per kilometer* rate but her sister says that she should use the *kilometers per minute* rate. Who's correct? Explain your reasoning.

3. Assuming she cycles at the same average rate, how long should it take Tiara to complete the cycling segment of each triathlon?

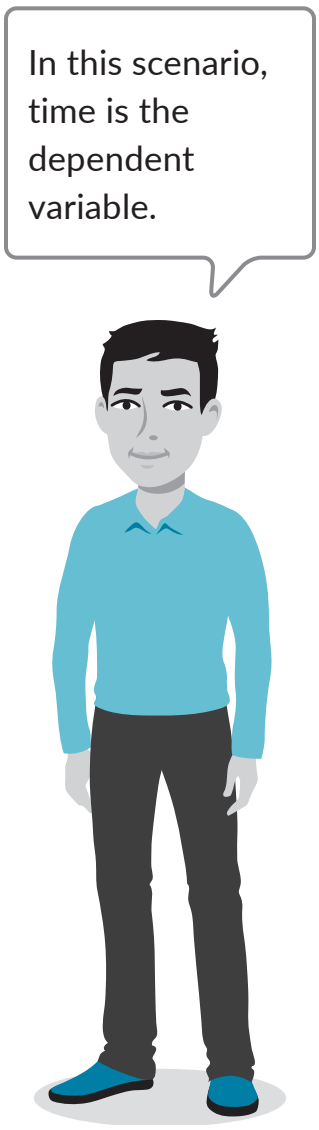
a. Triathlon A

b. Triathlon B

c. Triathlon C

d. Triathlon D

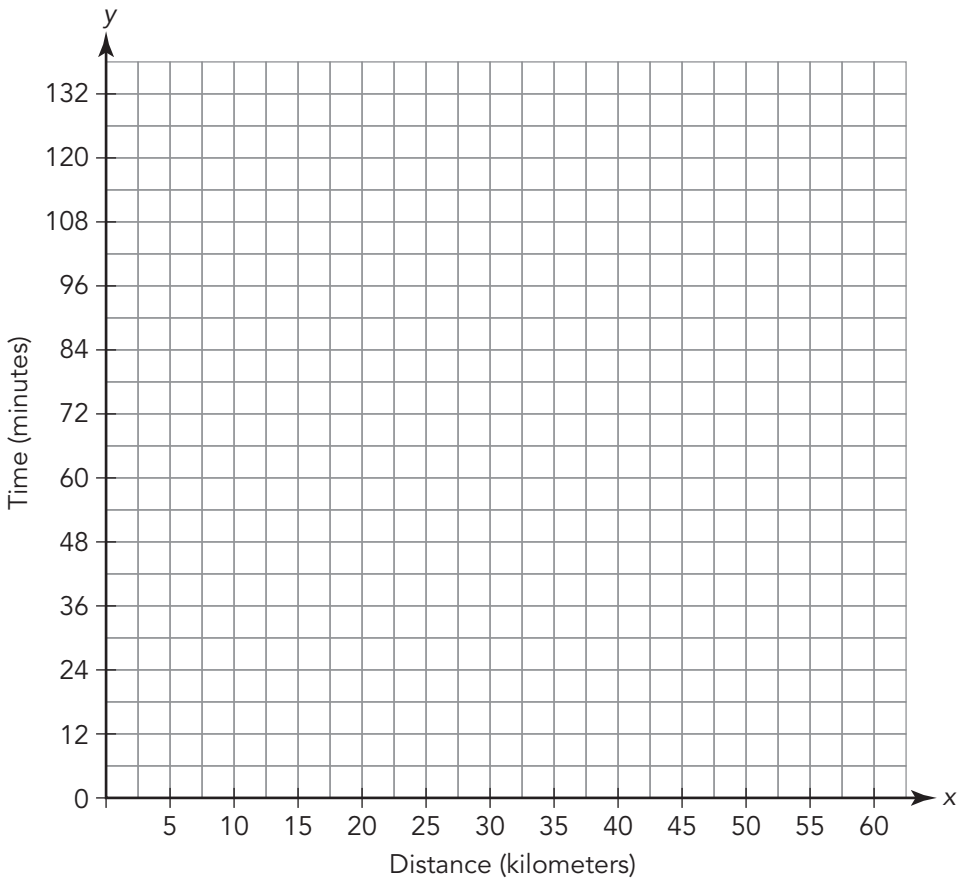
e. Triathlon E



4. Write an equation to determine the amount of time required for Tiara to cycle a given distance.

CC BY NC

5. Use your results to create a graph of the time in minutes that Tiara cycles versus the distance she cycles in kilometers. Connect the plotted points.



6. You cannot graph the time and distance for Triathlon E on this graph. Explain how you know it would be on the line when the graph is extended.

7. In the secret triathlon, the cycling segment is 35 kilometers. Use your graph to estimate how long it should take Tiara to complete this segment of the triathlon. Explain your strategy.

ACTIVITY
4.3 Running Rate

Tiara runs every day as part of her training routine. She averages 5 minutes per kilometer.

1. Write an equation to determine the amount of time required to run a given distance.

2. Use your equation to determine how long it will take Tiara to complete the running segment of each triathlon.

   a. Triathlon A

   b. Triathlon B

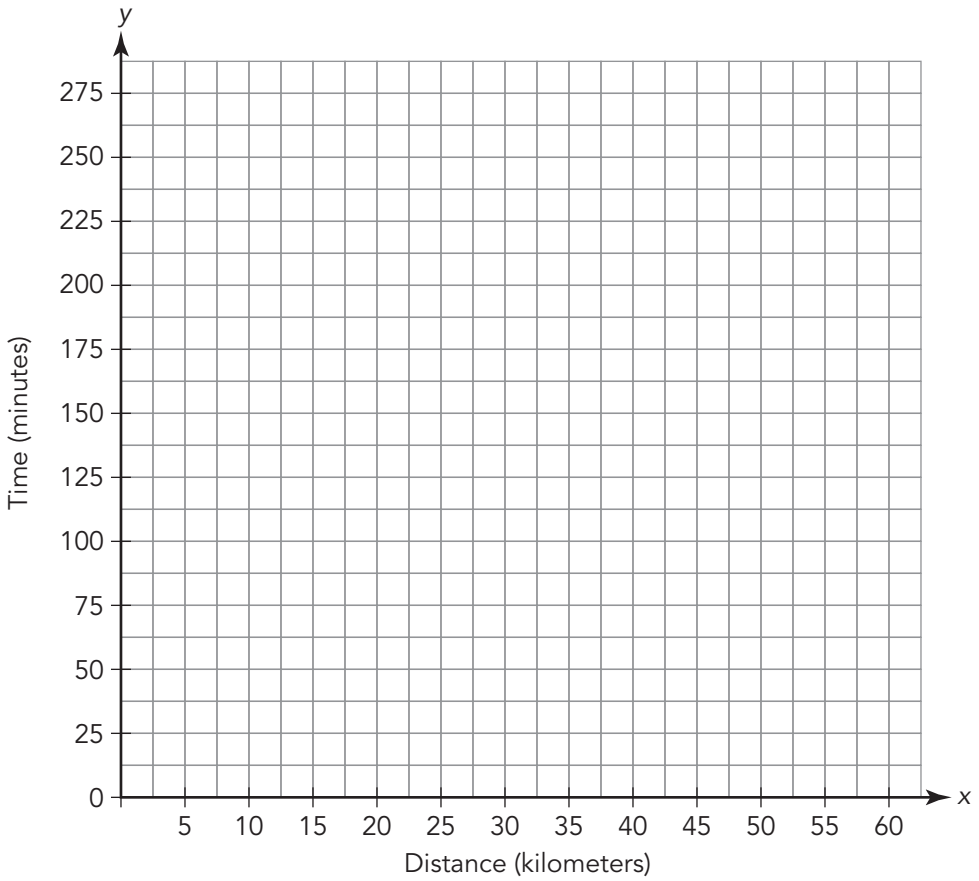
   c. Triathlon C

   d. Triathlon D

   e. Triathlon E

CC BY NC

3. Use your equation and the results from Question 2 to create a graph of the time, in minutes, that Tiara runs versus her distance, in kilometers. Connect the plotted points.



Assume that Tiara runs at her average rate in this triathlon.

4. After competing in the secret triathlon, Tiara reports that it took her 87 minutes to complete the running segment. Use the graph to estimate the length of the running segment of this triathlon.

5. Rewrite the equation in Question 1 to determine the distance traveled for a given amount of time.

6. Use your new equation to determine the actual length of the running segment of the secret triathlon.

Talk the Talk

Reflecting on Triathlon Training

To analyze the three segments of the triathlons, you used the distances traveled, amount of time, and rates.

1. Record the two equations that you could use to describe each leg of the race.
   - determine the distance, given the time
   - determine the time, given the distance

	Determine Distance	Determine Time
Swim		
Cycle		
Run		

2. What do you notice about the coefficients in the equations? Why does this make sense?

CC BY NC

3. Write the rate, including units, represented in the *Determine Distance* equation for each segment of the race.

4. Explain why the ratios or rates listed in Question 3, and not their reciprocals, are the appropriate rates to use in determining distance.

Lesson 4 Assignment

Write

Suppose your work partner was absent today. Write at least three sentences that summarize the relationship between *distance* (d), *rate* (r), and *time* (t). Be sure to include multiple representations of the relationship (verbal statements, graphs, tables, and/or equations) in your summary.

Remember

The equation that relates distance, rate, and time is often written as $d = rt$.

Practice

1. An airplane takes off and climbs at a constant rate of 1400 feet per minute.

   a. Write an equation to model the relationship between the plane's altitude and the time in minutes.

   b. Complete the table.

Time (min)	Altitude (ft)
1	
1.5	
2	
2.5	

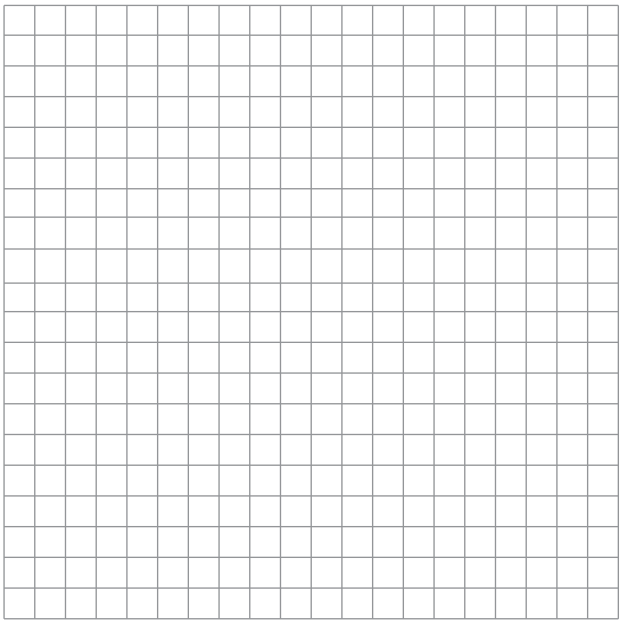
   c. Use the equation to determine how much time it takes for the plane to reach an altitude of 3 miles.

Lesson 4 Assignment

2. A helium balloon rises at a constant rate of 200 feet per minute.

   a. Write an equation to model the relationship between the balloon's altitude and the time in minutes.

   b. Graph the equation.



   c. Use your graph to determine how much time it takes for the balloon to reach an altitude of 700 feet.

Lesson 4 Assignment

3. A car travels on the interstate at a constant speed. The distances are recorded in a table.

Distance (miles)	Time (hours)
16.25	0.25
32.5	0.5
260	4
390	6

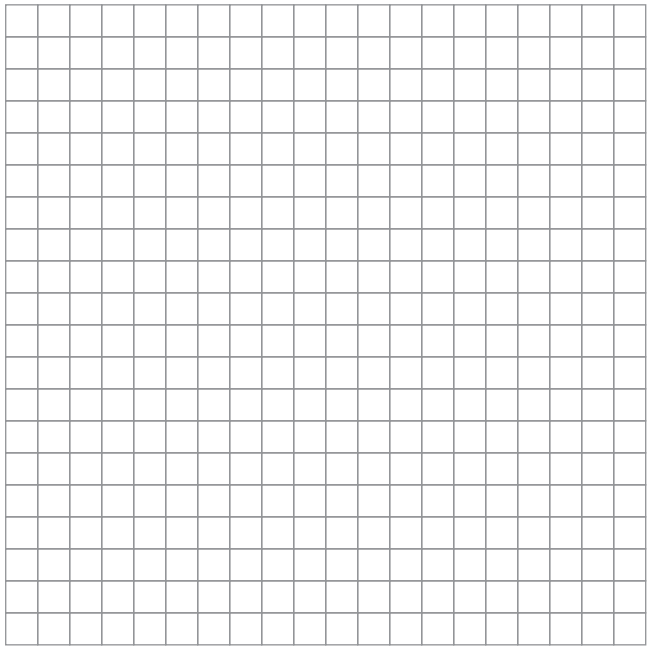
a. Determine the car's rate in miles per hour and in hours per mile.

b. Write an equation to determine the amount of time required to travel a given distance.

CC BY NC

Lesson 4 Assignment

c. Use the table to create a graph of the time versus the distance traveled.



d. Determine how many minutes it will take the car to travel 43 miles.

Lesson 4 Assignment

Prepare

Solve each equation.

1. $120 + h = 315$

2. $w - 17 = 38$

3. $\frac{c}{5} = 12$

4. $169 = 13w$

CC BY NC

5 Problem Solving on the Coordinate Plane

OBJECTIVES

- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.
- Interpret the meaning of points plotted on the coordinate plane.
- Use equations to solve real-world problems.
- Use graphs relating an independent and dependent quantity changing in relationship to one another to solve real-world problems.
- List advantages and disadvantages of different representations for solving real-world and mathematical problems on the coordinate plane.

Now that you understand how to plot points in all four quadrants of the coordinate plane, you can solve many more types of problems than you previously could.

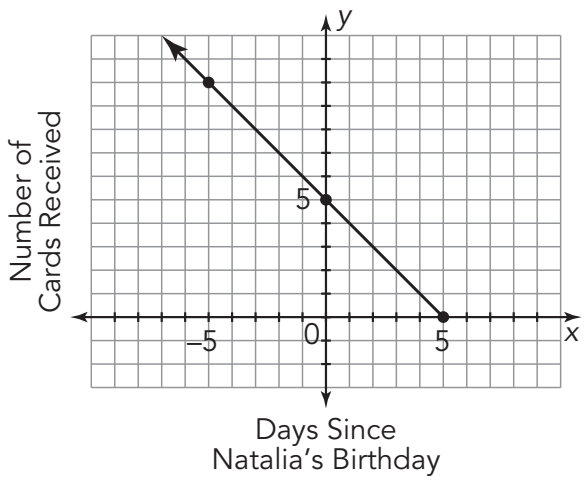
How can you use graphs and equations to solve problems?

CC BY NC

Getting Started

Natalia's Birthday

Analyze the graph.



1. Explain what you can determine about the situation from the graph.

2. What do the plotted points mean in terms of this situation?

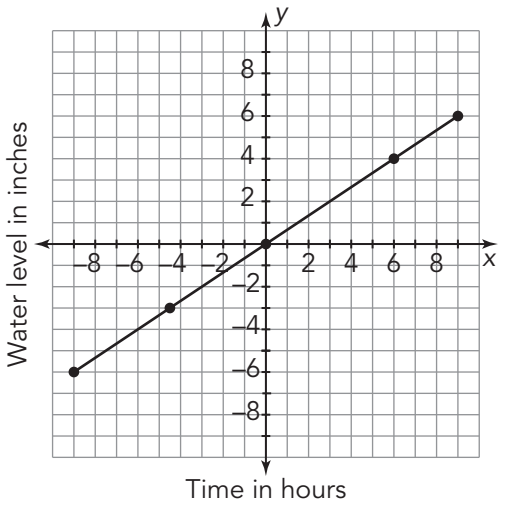
3. Do all of the values on the line make sense in terms of the situation?

4. Can you determine an equation for the graph?

CC BY NC

ACTIVITY
5.1 Pool Level

The graph shows the water level of a pool. The origin represents 3:00 PM and the desired water level. The *x*-axis represents the time difference in hours from 3:00 PM, and the *y*-axis represents the water level in inches, relative to the desired water level.



Ask Yourself . . .
How can you use graphs, tables, and equations to describe independent and dependent variables in everyday life?

1. Label the graph with the independent and dependent quantities and their units.

2. Create a table of values for the points plotted and describe the meaning of each.

x	*y*	Meaning

3. At what rate did the water go into the pool? Explain your reasoning.

4. Describe a situation that would match the graph.

5. Write an equation for this situation.

6. Why does the graph stop rather than continue infinitely?

7. Using any of your mathematical tools, determine the time when the pool was 3 inches above the desired fill level. Is your answer exact or approximate? Explain.

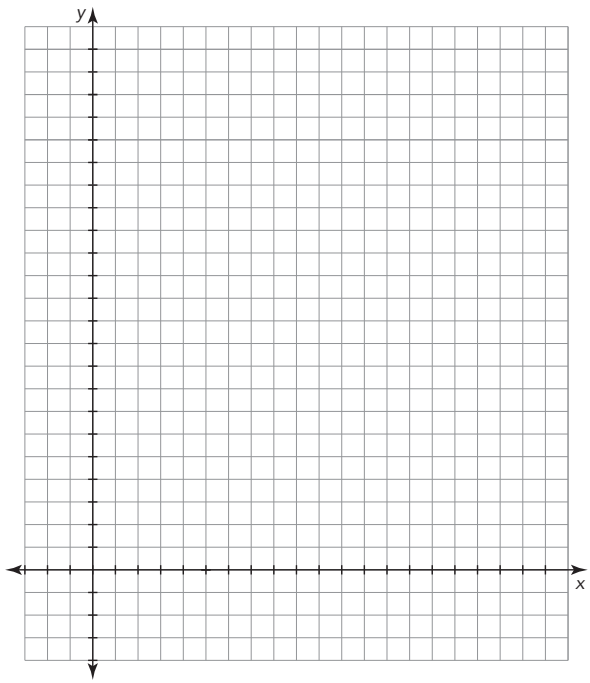
CC BY NC

ACTIVITY
5.2 Water in the Bucket

As part of a science project, Alejandro collected water in a bucket in his backyard and is studying the evaporation. Unfortunately, Alejandro is a bit forgetful and forgets to take measurements of the water every day. The first day he remembered was Sunday, which was 4 days AFTER the data collection was to begin. He collects the following data.

Days Since Sunday	Height of Water (inches)
0	27
5	22
7	20
12	15

1. Graph the data. Connect the data values with a line. Be sure to label your axes.



2. Assume the water evaporated at the same rate every day and use the graph to analyze the scenario. Plot the corresponding point and state the quadrant where the point is located.

   a. Determine the water level the day Alejandro was supposed to start data collection.

   b. When was the water level at a height of 30 inches?

   c. When was the water level at a height of 12 inches?

   d. When was the water level at a height of 5 inches?

   e. When was the water level at a height of 0 inches?

3. How can you use your graph to confirm your answers to Question 2 are correct?

4. Explain whether you should extend your graph into Quadrant IV. Do both negative x-values and y-values make sense in this problem situation?

CC BY NC

PROBLEM SOLVING



ACTIVITY
5.3 Broken Yardstick

Eduardo and Mariana need to measure some pictures so they can buy picture frames. They looked for something to use to measure the pictures but could find only a broken yardstick. The yardstick was missing the first $2\frac{1}{2}$ inches.

They both thought about how to use this yardstick.

Mariana said that all they had to do was measure the pictures and then subtract $2\frac{1}{2}$ inches from each measurement.

1. Is Mariana correct? Explain your reasoning.

2. They measured the first picture's length and width to be 11 inches and $9\frac{1}{2}$ inches. What are the actual length and width?

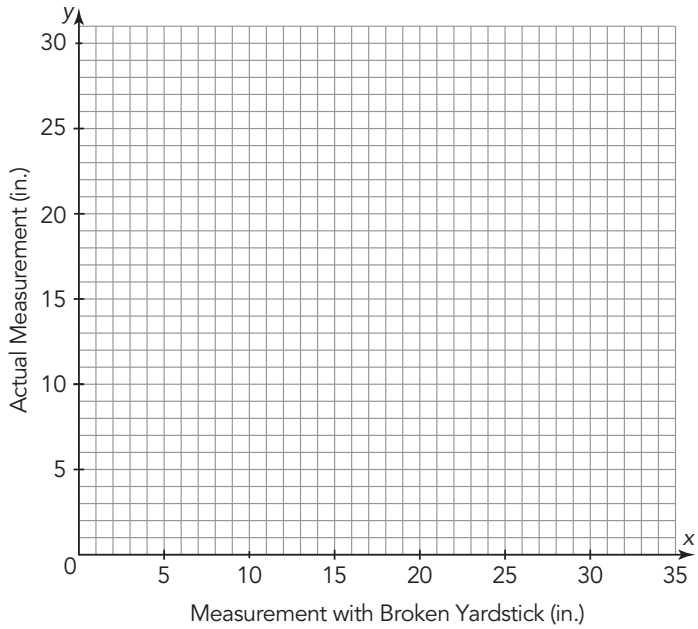
3. Define variables for a measurement with the broken yardstick and the actual measurement.

4. Write an equation that models the relationship between the variables.

5. Complete the table of values for the measurement on the yardstick and the actual measurement.

Measurement with Broken Yardstick (in.)	Actual Measurement (in.)
11	
$9\frac{1}{2}$	
$25\frac{3}{4}$	
21	
$18\frac{5}{8}$	
	12
	$29\frac{1}{8}$
	$6\frac{7}{8}$

6. Use the table to complete the graph of the actual measurements versus the measurement taken with the broken yardstick.

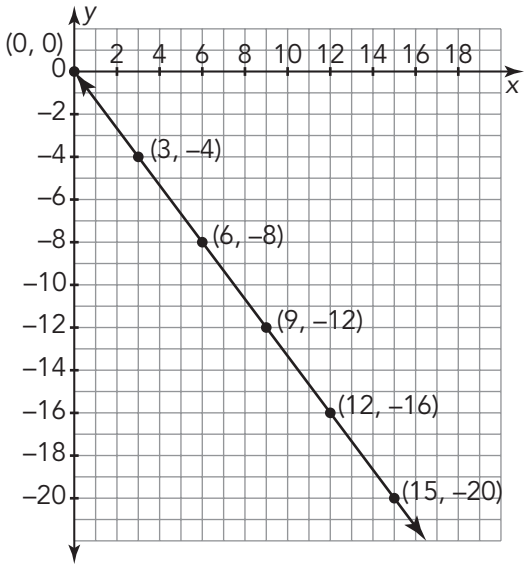


7. Would it make sense to connect the points on this graph? Explain why or why not.

Talk the Talk

Your Turn!

1. Create a situation that the graph can model.



- Be sure to determine the ratio, or rate, for how the variables change in relation to each other.
- Describe the meaning of each point on the graph.
- Define variables for the independent and dependent quantities based on your situation.
- Write an equation to represent the problem situation.

Lesson 5 Assignment

Write

Give an example of when you might want to use an equation to answer a question and another example of when you might want to use a graph.

Remember

Graphs, tables, equations, and scenarios provide different information and allow for various levels of accuracy when solving problems.

Practice

1. The gravitational pull of the Moon is not as great as that of Earth. In fact, when a person checks his weight on the Moon, it will be only $\frac{1}{6}$ of his weight on Earth.

   a. When a person weighs 186 pounds on Earth, how much will he weigh on the Moon? How many pounds different from his actual weight is that?

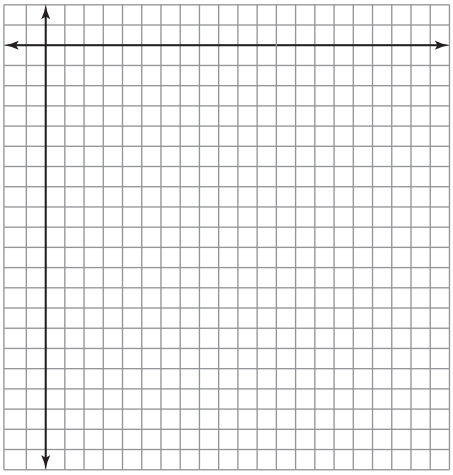
   b. Complete the table of values for a person's weight on Earth, weight on the Moon, and difference of the two weights. Use negative numbers when the weight is less than the person's earth weight.

Weight on Earth (lbs)	186	168		198		
Weight on Moon (lbs)			29		21	24
Weight Differential	−155					

CC BY NC

Lesson 5 Assignment

c. Graph the weight differential versus the weight on Earth. Be sure to label your axes.



2. Two runners were comparing distances as they ran down the sidewalk. Runner A gave Runner B a 3.5 meter head start. Runner A tracked their distance but Runner B did not. When they reached the end of the first block, Runner A had run 47 meters. After 2 blocks, Runner A had gone 155 meters.

   a. Define variables for measuring Runner A's distance and Runner B's distance.

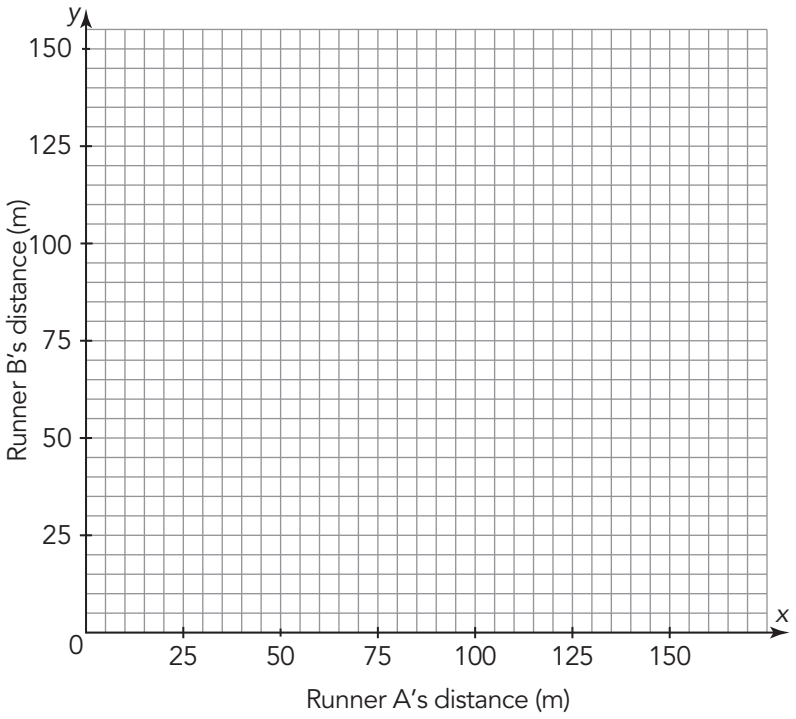
   b. Write an equation that models the relationship between the variables.

Lesson 5 Assignment

c. Complete the table of values.

Runner A's distance (meters)	Runner B's distance (meters)
47	
155	
13.3	
	11
	130.5
	37

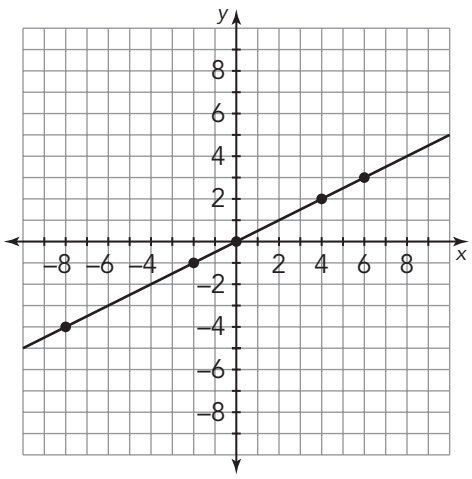
d. Use the table to complete the graph of Runner A's distance versus Runner B's distance.



Lesson 5 Assignment

e. Would it make sense to connect the points on this graph? Explain why or why not.

3. The following graph shows the average temperature in degrees Celsius in Fairbanks, Alaska for a three week period. The *x*-axis represents time in days from January 1, and the *y*-axis represents degrees Celsius.



a. Label the axes with the independent and dependent quantities and their units.

b. Create a table of values for the points on the graph and describe the meaning of each.

x	*y*	Meaning

Lesson 5 Assignment

c. At what rate did the temperature increase?

d. Define variables for the quantities that are changing, and write an equation for this situation.

4. Samantha put her dog on a wellness plan that includes daily exercise and a special type of dog food. She estimates her dog will lose $1\frac{1}{2}$ pounds per month on this plan.

   a. How many pounds does Samantha estimate her dog will lose in 2 months? In $8\frac{1}{2}$ months?

   b. Define variables for the independent and dependent quantities for this situation.

   c. Write an equation for this situation. (Because Samantha's dog is losing weight, the number of pounds the dog loses will be defined as a negative value.)

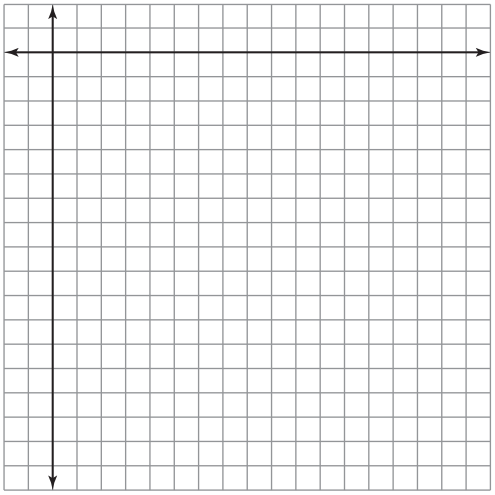
CC BY NC

Lesson 5 Assignment

d. Complete the table of values for the situation.

	Independent Quantity	Dependent Quantity
Quantity Name		
Unit of Measure		

e. Create a graph of the situation.



Lesson 5 Assignment

f. Explain what points in Quadrant I would mean for Samantha's dog.

Prepare

1. What is 10% of $56?

2. What is 7% of $25?

3. What is 25% of $1500?

CC BY NC

TOPIC 3 SELF-REFLECTION

Name: _____

Graphing Quantitative Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Graphing Quantitative Relationships* topic by:

TOPIC 3: *Graphing Quantitative Relationships*	Beginning of Topic	Middle of Topic	End of Topic
identifying independent and dependent quantities.	○	○	○
defining variables to represent two quantities in a real-world problem that vary in relationship to one another.	○	○	○
writing an equation to represent the relationship between independent and dependent quantities.	○	○	○
analyzing the relationship between the dependent variable and the independent variable using tables, graphs, and equations.	○	○	○
representing real-world problems using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$.	○	○	○

continued on the next page

CC BY NC

TOPIC 3 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Graphing Quantitative Relationships* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?



TOPIC 3 SUMMARY

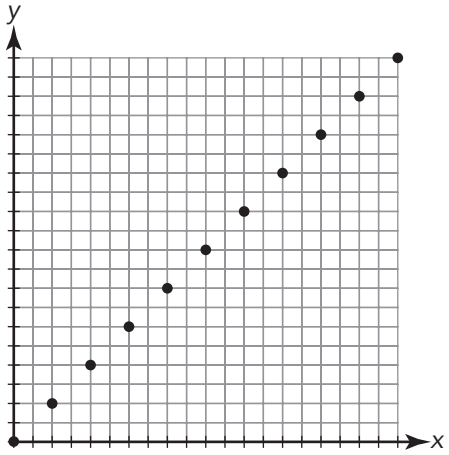
Graphing Quantitative Relationships Summary

NEW KEY TERMS

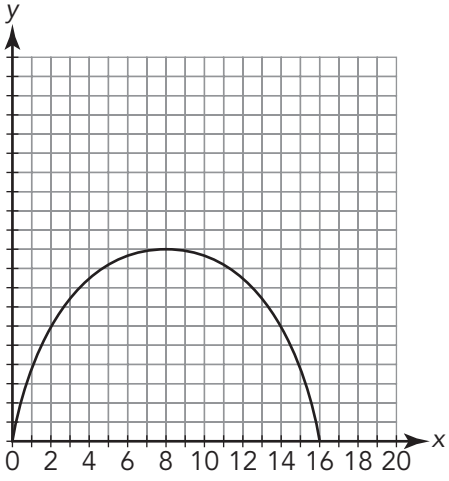
- dependent quantity [cantidad dependiente]
- independent quantity [cantidad independiente]
- independent variable [variable independiente]
- dependent variable [variable dependiente]

LESSON 1 Independent and Dependent Variables

Some graphs represent real-world situations that are sets of isolated points.



In other graphs, each point on the graph, even those with fractional numbers as coordinates, represents a solution to the situation.

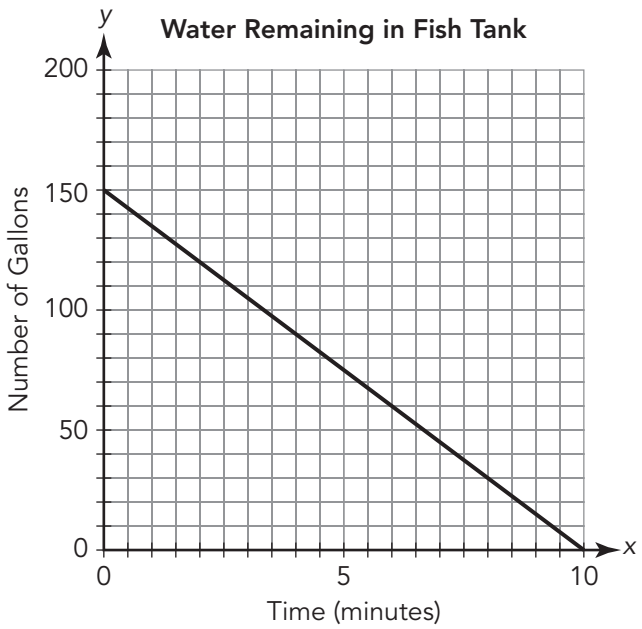


When the situation is represented by isolated points, you can connect the points to model the scenario, see the shape of the graph, and read more points on the graph, but you need to remember that all values on the line may not make sense in terms of the situation.

CC BY NC

When one quantity is determined by another in a real-world problem situation, it is said to be the **dependent quantity**. When one quantity does not depend on another, it is called the **independent quantity**. The variable that represents the independent quantity is called the **independent variable**, and the variable that represents the dependent quantity is called the **dependent variable**.

For example, suppose you are draining a 150-gallon fish tank at a rate of 15 gallons per minute. How much water remains in the tank at a specific time?



In this scenario, the independent quantity is time in minutes, and the dependent quantity is the number of gallons of water in the fish tank. The equation that represents the scenario is $w = 125 - 15t$. The independent variable is t, which represents the number of minutes, and the dependent variable is w, which represents the gallons of water in the tank.

Note that the independent quantity is plotted on the x-axis, and the dependent quantity is plotted on the y-axis.

LESSON 2

Using Graphs to Solve Problems

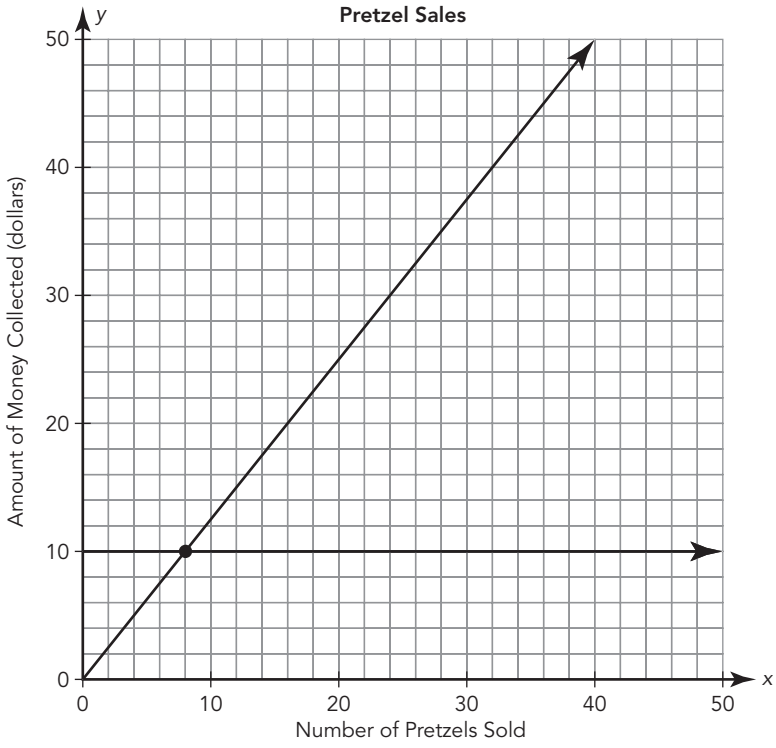
You can use a graph to determine an independent quantity when given a dependent quantity.

For example, Mario sells pretzels for $1.25 each morning at the baseball and softball games that are held at the Community Center. The amount of money collected for the number of pretzels sold can be represented by points on the graph. The equation corresponding to the graph is $y = 1.25x$. You can use the graph to determine how many pretzels Mario sold if he collected $10.

First, locate 10 on the *y*-axis and draw a horizontal line. This shows that $10 is the amount of money collected. The *x*-value of the point where your horizontal line intersects with the graph of $y = 1.25x$ is the number of pretzels sold for $10.

In this graph, a solution to the equation represented by the graphed line is any point on that line. Mario sold 8 pretzels when he collected $10.

In some problem situations, when you model a relationship with a line, not all of the points on the line will make sense. It is up to you to interpret the meaning of data values from the line drawn on a graph for each situation.



LESSON 3

Multiple Representations of Equations

You can write an equation from a relationship given in a table.

For example, the number of tiles required to complete a job and the number of tiles ordered are represented in the table shown.

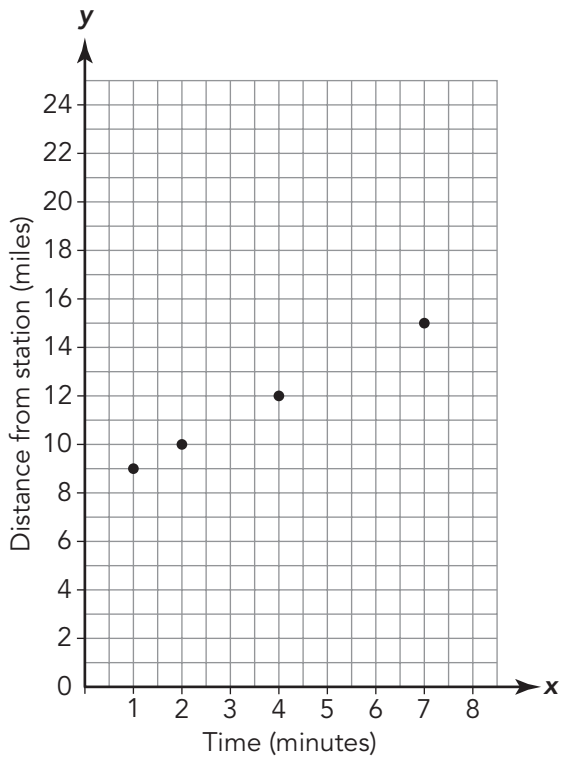
Number of Tiles Required	60	75	100
Number of Tiles Ordered	80	95	120

The independent quantity is the number of tiles required to complete a job, and the dependent quantity is the number of tiles ordered. By analyzing the table, you can see that the number of tiles ordered is always 20 more than the number of tiles required. An equation that models this relationship is $y = x + 20$.

You can write an equation from a relationship represented in a graph.

For example, the graph shows the relationship between the distance of a train from the station and the time in minutes. You can use the points from the graph to complete the table of values.

Time (minutes)	Distance (miles)
1	9
2	10
4	12
7	15



If t represents the time in minutes and d represents the distance from the station in miles, then the equation $d = t + 8$ represents the relationship between the quantities.

You can write an equation from a scenario.

For example, Ashley got a job working at the post office making $10.25 per hour.

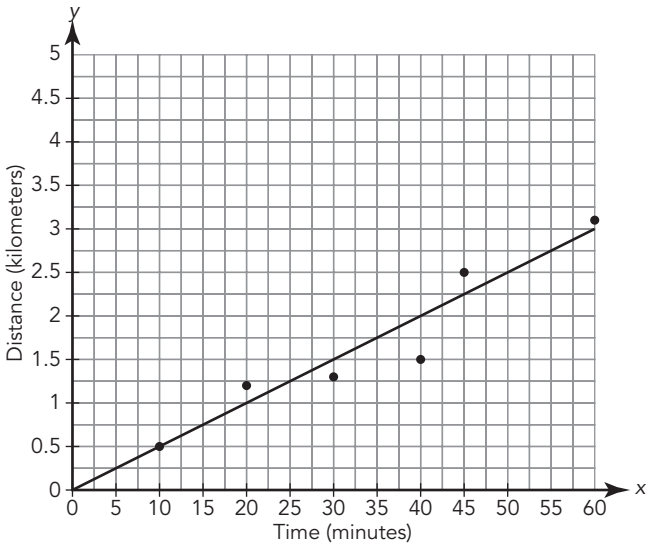
You can write an equation that models the relationship between the number of hours Ashley worked and the amount of money she earned. Let a represent the amount Ashley earned, and let h represent the number of hours she worked. The equation is $a = 10.25h$.

LESSON 4 Relating Distance, Rate, and Time

The equation that relates distance, rate, and time is often written as $d = rt$.

For example, Tiara is training for a triathlon. Tiara's coach plotted her times and distances from her last few swimming training sessions. Based on the data, the coach drew in a line to represent an approximation of her average speed, or swimming rate.

Tiara's swimming speed is a unit rate. There is a proportional relationship between distance and time.



Tiara wants to know how long it will take her to complete the swimming segment of the Olympic Style triathlon, which has a 1.5 km swim leg.

$$\frac{\text{distance}}{\text{time}} = \frac{1 \text{ km}}{20 \text{ minutes}}$$

$$\frac{1.5 \text{ km}}{\text{time}} = \frac{1 \text{ km}}{20 \text{ minutes}}$$

$$\frac{1.5 \text{ km}}{\text{time}} = \frac{1 \text{ km}}{20 \text{ minutes}} \quad \begin{matrix} \cdot 1.5 \\ \cdot 1.5 \end{matrix}$$

It should take Tiara 30 minutes to complete the swimming segment of the triathlon.

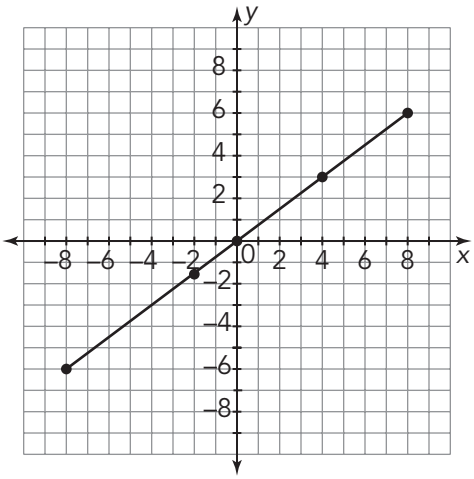
CC BY NC

LESSON 5

Problem Solving on the Coordinate Plane

Graphs, tables, equations, and scenarios provide various information and allow for different levels of accuracy when solving problems.

For example, the graph shows the water level of a pool. The origin represents 3:00 PM and the desired water level. The *x*-axis represents the time difference, in hours from 3:00 PM, and the *y*-axis represents the water level in inches, relative to the desired water level.



You can create a table of values for the points plotted and describe the meaning of each.

x	*y*	Meaning
−8	−6	At 7:00 AM, the water level is 6 inches below the desired water level.
−2	$-1\frac{1}{2}$	At 1:00 PM, the water level is $1\frac{1}{2}$ inches below the desired water level.
0	0	At 3:00 PM, the water is at the desired water level.
4	3	At 7:00 PM, the water level is 3 inches above the desired water level.
8	6	At 11:00 PM, the water level is 6 inches above the desired water level.

You can use the graph to determine that the water went into the pool at a rate of $\frac{3}{4}$ inch per hour. An equation that represents this situation would by $y = \frac{3}{4}x$.

GRADE 6

TOPIC 4



The more education or training a person receives, the greater their earning potential.

Financial Literacy: Accounts, Credit, and Careers

LESSON 1 Checking Accounts 989

LESSON 2 Debit Cards vs. Credit Cards 1009

LESSON 3 Understanding Credit Reports 1025

LESSON 4 Career Exploration 1039

LESSON 5 Paying for College 1055

1 Checking Accounts

OBJECTIVES

- Analyze the features of a checking account.
- Practice writing checks.
- Balance a checkbook.
- Determine money earned on an interest-bearing checking account.
- Compare checking account and debit card options.

NEW KEY TERMS

- checking account
- check
- statement
- account balance
- deposit
- withdrawal
- debit
- transfer
- overdraft
- Annual Percentage Yield (APY)

You know how to add and subtract money, and how to calculate a percent of a given amount.

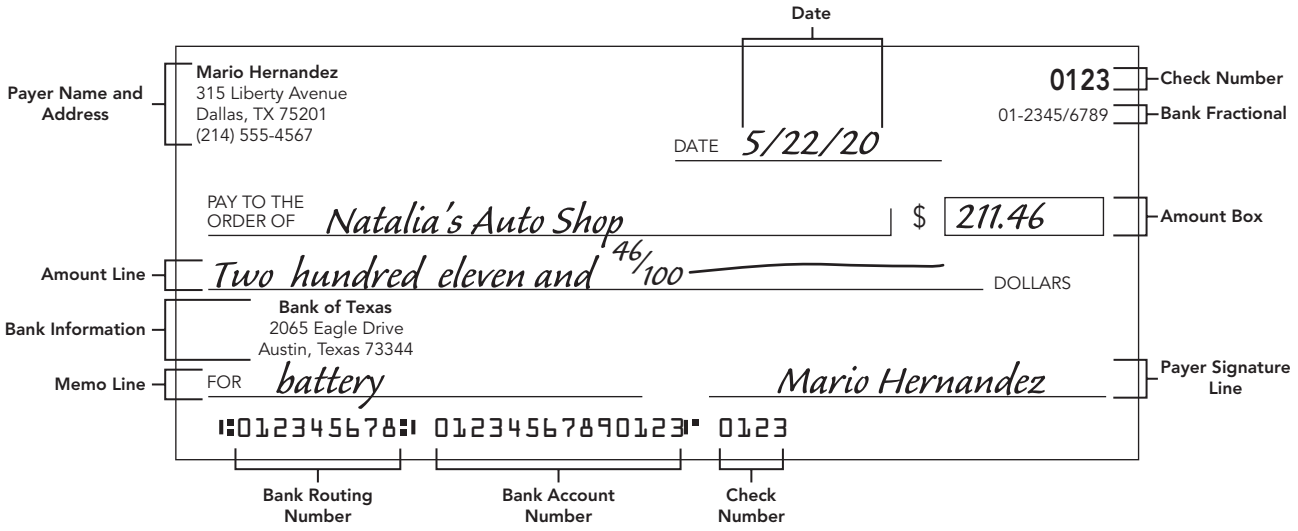
How can you apply these skills to manage your money and compare checking account options?

CC BY NC

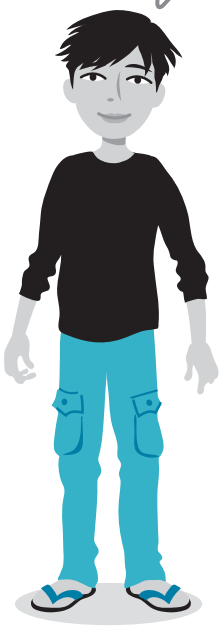
Getting Started

Check This Out!

Analyze the check that was written to pay a bill.



What would happen if someone wrote $217 in the box, but they really meant $211?



1. Who was the check written to? Can you identify the purpose of the check? Why or why not?

2. What two places on the check show the amount of the check? Why do you think the amount of the check is written in two different places in two different ways?

3. Who wrote the check? What is the person's address?

4. What is the bank account number? What is the check number? What do these two numbers mean?

5. When was the check written? Why do you think the date is important?

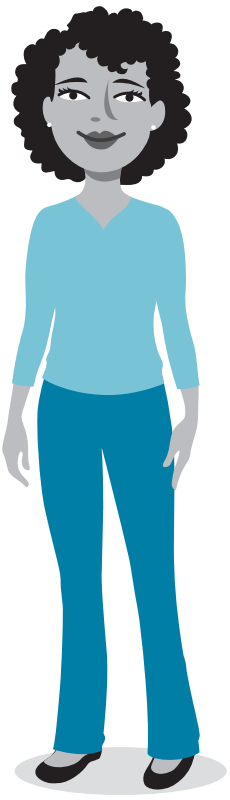
CC BY NC

ACTIVITY
1.1 Checks and Checking Accounts

On payday, most people put their money into a *checking account* at the bank. A **checking account** allows customers to safely store money in the bank and write *checks*. A **check** is a written order to a bank to pay a specific amount of money to a person or company out of your checking account. A key benefit of using a checking account is the ease with which it allows people to save money, withdraw it, and pay their bills.

Many people pay regular bills like water, electric, and mortgage payments online, but checks are still sometimes used for paying bills.

What information do you think would be important to include on the FOR line of this check?



1. Mario ordered tennis equipment today from Sports & Fitness, Inc. The total bill is $214.08. Complete the check to pay for the tennis equipment.

Mario Hernandez
315 Liberty Avenue
Dallas, TX 75201
(214) 555-4567

0124
01-2345/6789

DATE

PAY TO THE ORDER OF $

DOLLARS

Bank of Texas
2065 Eagle Drive
Austin, Texas 73344

FOR

⑈012345678⑈ 01234567890123⑈ 0124

2. Mario has account 3698742 with the American Heating Company. His heating bill is $170.08 for the month. Complete the check to pay the bill.

Mario Hernandez
315 Liberty Avenue
Dallas, TX 75201
(214) 555-4567

0125
01-2345/6789

DATE

PAY TO THE ORDER OF $

DOLLARS

Bank of Texas
2065 Eagle Drive
Austin, Texas 73344

FOR

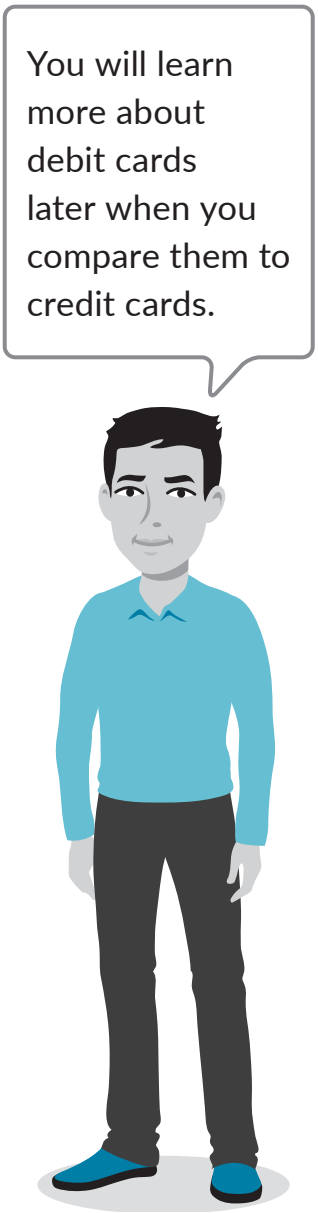
⑈012345678⑈ 01234567890123⑈ 0125

Most banks send their customers a *statement* every month. A **statement** is a summary of the **account balance**, which is the amount of money in the account at a given time. A statement also includes every transaction that occurred throughout the month. The transactions may include *deposits*, *withdrawals*, *debits*, and *transfers*.

- A **deposit** is money put into the account. Deposits may come in the form of cash, checks, or transfers. Paychecks are often deposited directly into a checking account.
- A **withdrawal** occurs when money is taken out of the account, usually from a check, online payment, or Automated Teller Machine (ATM).
- A **debit** occurs when money is taken out of the account automatically through the use of a debit card. Debit cards offered by different banks sometimes have different features and costs.
- A **transfer** occurs when money is moved from one account to another.

3. Samantha wrote down her payments and deposits for the month of November in her checkbook register. Analyze Samantha's checkbook register.

Check	Date	Transaction Description	Payment/ Withdrawal	Deposit	Balance
103	Nov 1	Electric Company	$72.50		$1227.50
	Nov 1	ATM withdrawal	$80.00		
	Nov 2	ATM deposit		$50.00	
	Nov 3	ATM deposit		$40.00	
	Nov 3	Running Shoes	$120.00		
	Nov 18	Only Fresh Grocery	$182.25		
104	Nov 27	Rent payment	$700.00		
	Nov 27	Transfer from Savings		$500.00	
105	Nov 29	American Heating Co.	$98.40		
	Nov 30	Direct Deposit		$1205.60	

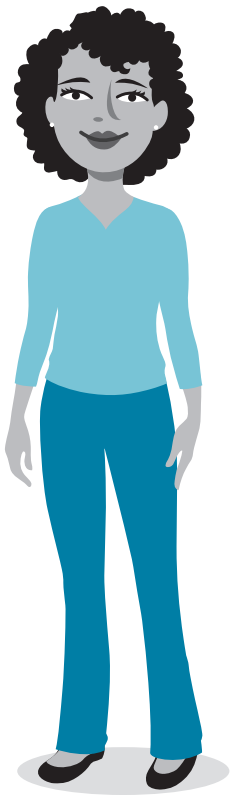


a. Determine the balance in Samantha's checking account after each transaction. Write the balance amounts in the register.

b. Did Samantha earn more money than she spent in the month of November? Describe two ways to answer this question.

c. What were Samantha's two greatest expenses during November?

Overdrafts may also prevent you from receiving a good credit score. You will learn more about credit scores later.



An **overdraft** occurs when money is withdrawn from a bank account and the balance goes below zero. It is important to prevent an overdraft from occurring because most banks charge a fee every time this happens.

4. Trung receives her statement from the bank in early December. After comparing the statement to her own check register, she thinks the bank made an error. Compare Trung's check register to the bank's statement.

Trung's Check Register

	Nov 18	Groceries	$182.35		$995.14
104	Nov 27	Mortgage payment	$1135.47		−$140.33
	Nov 30	Transfer from Savings		$500.00	$359.67
105	Nov 29	American Heating Co.	$98.45		$261.22

Bank Statement

	Nov 18	Groceries	$182.35		$995.14
104	Nov 27	Mortgage payment	$1135.47		−$140.33
	Nov 29	Overdraft Fee	$40.00		−$180.33
	Nov 30	Transfer from Savings		$500.00	$319.67
105	Nov 29	American Heating Co.	$98.45		$221.22

a. Do you believe the bank made an error? Explain your reasoning.

b. Describe the reason for the difference in the balance.



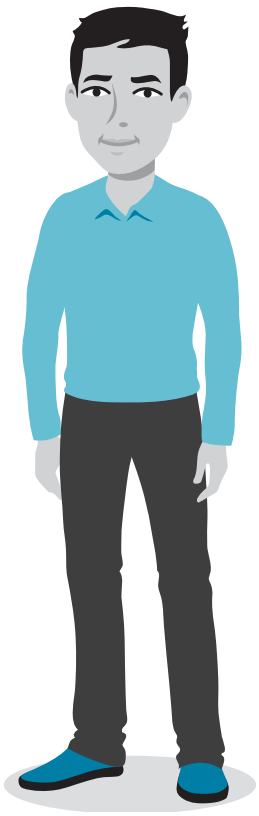
ACTIVITY
1.2 Comparing Checking Accounts

Banks may offer different monthly fees, or they may require a minimum deposit to open an account. **Annual Percentage Yield** (**APY**) is a percentage that is paid to customers based on the account balance in an account for a year.

1. A bank offers a checking account that pays 0.2% APY. A minimum deposit of $500.00 is required to open the account. Alex opens an account and maintains an average balance of $800. How much money will he earn from the account over the course of a year?

   Analyze Andrew and Paola's calculations.

The formula $I = prt$ is often used, where I is the amount earned from the account, p is the principal (amount of $), r is rate, and t is time. For this problem, the time is one year.



Andrew

The 0.2% APY means that he will earn 20 cents for every $100 he keeps in the account. So, $\frac{0.2}{100} = \frac{x}{800}$. Scaling up by a factor of 8, $\frac{0.2 \times 8}{100 \times 8} = \frac{1.6}{800}$, I determined that $x = 1.6$. Therefore, he will earn $1.60.

Paola

To determine the 0.2% APY, I can convert to the decimal 0.002 and then multiply this rate by 800: $800.00 \times 0.002 = 1.60$. He will earn $1.60 for the year.

a. Explain why Andrew and Paola are correct.

b. Which method do you prefer? Explain your reasoning.

2. Determine the amount earned in each account over the course of one year.



   a. An account with an average balance of $600 that pays 0.3% APY

   b. An account with an average balance of $500, 0% APY, and $5.00 per month in fees

3. Elena maintained a balance of $1200 in her checking account and earned $1.50 from the account last year. Determine the APY. Show all of your work and explain your reasoning.

CC BY NC

4. Mariana is interested in opening a checking account at a local bank. She estimates that she will be able to keep about $7500 in her account throughout the year. She is deciding among three banks. Their account advertisements are shown.

Texas Interest Bank
- 0.3% APY
- $1.00/month fee
- $7000 minimum average balance (additional fees assessed for lower balances)

Lone Star of Texas Bank
- 0.2% APY
- $0.50/month fee
- $5000 minimum average balance (additional fees assessed for lower balances)

People's Choice Bank
- 0.1% APY
- No monthly fees
- No minimum average balance

What advice would you give Mariana in choosing an account? Show all of your work and explain your reasoning.

To use a debit card, you must have a checking account open with a bank. Remember, since money is taken directly out of the checking account when using a debit card, it is important to track how often you use it and how much money is in your account to avoid overdraft fees. In addition to the monthly fees charged for the checking account, banks may charge overdraft fees and fees to use the debit card at another financial institution's Automatic Teller Machine (ATM).

5. Ashley is interested in opening a checking account at a local bank. She plans to use a debit card to make purchases and withdraw money from her account at ATMs. She will keep about $8500 in her account. She uses an ATM from other financial institutions once a month to withdraw cash. She is considering three different banks. Their account features, including the charges for using debit cards at another institution's ATM, are shown.

Bluebonnet Bank	**Friend Bank**	**Texas Central Bank**
• 0.4% APY • $1.25/month fee • $1.50 fee for using the debit card at another institution's ATM	• 0.3% APY • $0.75/month fee • $1.25 fee for using the debit card at another institution's ATM	• 0.2% APY • $3.00/month fee • No fee for using the debit card at another institution's ATM

What advice do you have for Ashley about choosing an account? Explain your reasoning.

Talk the Talk

Put 'Em in Check

PROBLEM SOLVING



1. The table lists Alejandro's recent checking account transactions.

Transaction Description	Amount
Electric Bill	$68.55
Rent	$750.00
Gift Check from Aunt Tiara	$30.00
Pair of boots	$63.98
Pay	$1185.80
Cable Bill	$83.92
Charity Donation	$25.00
ATM withdrawal	$150.00

a. Enter each amount into the correct column in the check register.

b. Calculate the balance when Alejandro has $1567.25 before the given transactions.

Check	Date	Transaction Description	Payment/ Withdrawal	Deposit	Balance
107	Dec 3	Electric Bill			
108	Dec 3	Rent			
	Dec 6	Gift Check from Aunt Tiara			
109	Dec 10	Macy's – pair of boots			
	Dec 16	Pay			
110	Dec 19	Cable Bill			
111	Dec 19	Charity Donation			
	Dec 22	ATM withdrawal			

2. On average, Alejandro has a balance of $1500 in his checking account. He earns a 0.2% APY. His bank charges a fee of $2.00 per month. What amount of money does Alejandro earn (or lose) each year by having his checking account at this bank?

Ask Yourself . . .
When you carried out your plan, did it work? Why or why not? When your initial plan does not work, what should you do?

Lesson 1 Assignment

Write

Describe the information that is included in a monthly bank statement for a checking account.

Remember

A *checking account* allows customers to safely store money in the bank and write checks. A key benefit of using a checking account is the ease with which it allows people to save money, withdraw it, and pay their bills.

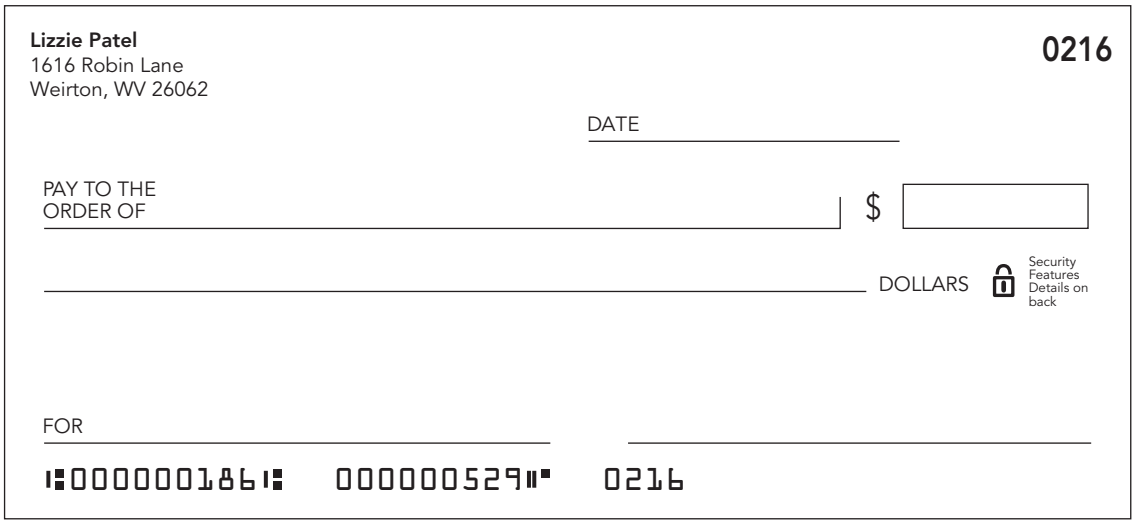
Practice

1. A list of Lizzie's checking account transactions for the month of April is shown.

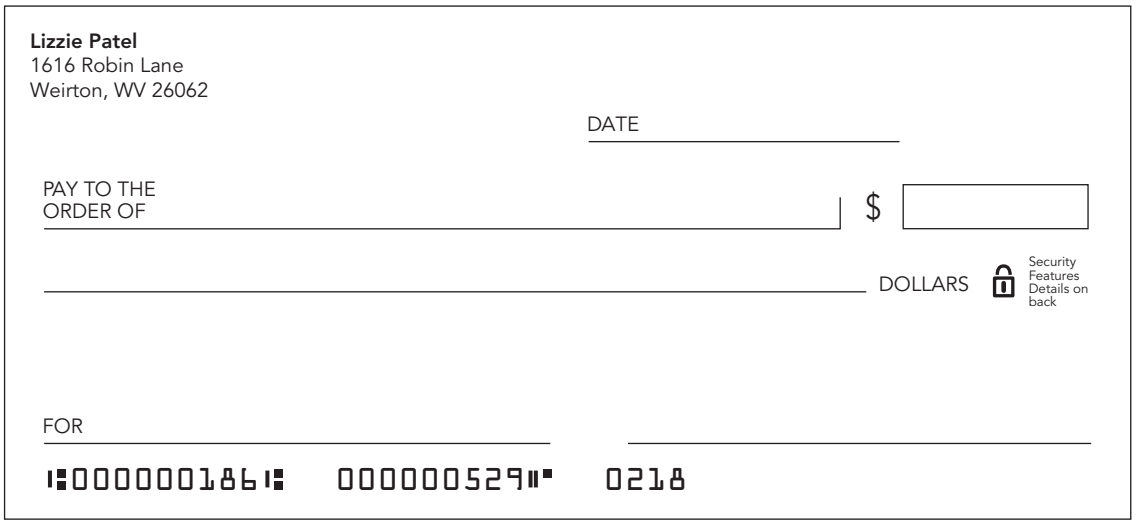
Date	Check Number	Description	Amount
4/1/20		Direct Deposit	$1815.45
4/1/20	215	Home Lender (Mortgage)	$1250.00
4/2/20	216	Motion Electric (Bill)	$47.85
4/4/20	217	Boxcar Cable Company (Bill)	$98.18
4/5/20		Grass Country Groceries	$109.18
4/8/20		Refund for March purchase	$45.32
4/10/20		Pizzaria Mysterious	$24.78
4/15/20		Direct Deposit	$1815.45
4/15/20	218	Charity Productions (Donation)	$245.00
4/25/20		Automatic Transfer from Savings	$200.00
4/27/20	219	ClearClean Water (Bill)	$25.18
4/28/20		ATM deposit	$125.00
4/30/20		Classic Movie House	$34.25

Lesson 1 Assignment

a. Complete the check based on the given information.



b. Complete the check based on the given information.



Lesson 1 Assignment

c. Complete Lizzie's checkbook register using the information given on page 1 to determine her balance at the end of the month. Prior to the given transactions, Lizzie has a balance of $1882.96.

Check Number	Date	Description	Payment/ Withdrawal	Deposit	Balance
		Direct Deposit			
215		Home Lender (Mortgage)			
216		Motion Electric (Bill)			
217		Boxcar Company (Bill)			
		Grass Country Groceries			
		Refund for March purchase			
		Pizzaria Mysterious			
		Direct Deposit			
218		Charity Productions (Donation)			
		Automatic Transfer from Savings			
219		ClearClean Water (Bill)			
		ATM deposit			
		Classic Movie House			

Lesson 1 Assignment

2. Eduardo keeps an average balance of $8000 in his bank account.

   a. Eduardo's current bank, American Interest Bank, pays a 0.25% APY on his earnings with a $0.50 fee per month. How much money does his account earn each year with American Interest Bank?

   b. Eduardo is thinking about changing banks. He has narrowed his options down to two other banks. The information on these two banks and his current bank is shown.

American Interest Bank	Better Banking Services	One Nation Financial
• 0.25% APY • $0.50 monthly fee • $7000 minimum average balance (additional fees assessed for lower balances)	• 0.3% APY • $1.50 monthly fee • $8000 minimum average balance (additional fees assessed for lower balances)	• 0.1% APY • no monthly fee • no minimum average balance

   Write a letter to Eduardo giving him your recommendation on which bank he should use.

Lesson 1 Assignment

3. Alyssa keeps an average balance of $7775 in her checking account. She is thinking about changing banks. She plans to use her debit card frequently. Alyssa pulls money out of the ATM at the local grocery store weekly and is charged a fee from her bank each time. The information on the three banks Alyssa is considering is shown.

American Best Bank	Dollar Sense Bank	National Bank
• 0.5% APY • $2.25 monthly fees • Additional ATM fee $0.50 for each withdrawal	• 0.3% APY • $2.00 monthly fees • No additional ATM withdrawal fee	• 0.75% APY • $1.00 monthly fees • Additional ATM fee $0.75 for each withdrawal

Alyssa is going to use one of these financial institutions for at least a full year. Which one should she choose?

Lesson 1 Assignment

Prepare

Solve these percentage problems.

1. What is 20% of $80?

2. $24 is what percent of $80?

3. $80 is 10% of what number?

2 Debit Cards vs. Credit Cards

OBJECTIVES

- Analyze the features of a debit card.
- Analyze the features of a credit card.
- Distinguish between debit cards and credit cards.

NEW KEY TERMS

- debit card
- credit card
- interest [interés]
- interest rate

You have already learned about checks and how to monitor your balance in a checking account.

What are other ways that you can pay for things besides cash? What should you consider when deciding which form of payment is best?

Getting Started

No Cash? No Problem!

1. Chris just bought a laptop computer, but he didn't hand the store clerk any cash. What are three possible ways Chris may have paid for the item?

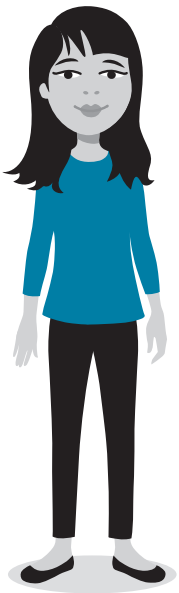
2. State at least two possible reasons why Chris may have decided not to pay with cash.

ACTIVITY
2.1 Comparing Debit and Credit Cards

Many people enjoy using **debit cards** and **credit cards**. Both are plastic cards that allow shoppers to pay for purchases without having the cash on hand. The two types of cards look similar, but they are very different. To use them responsibly, it is important to understand what they are as well as the advantages and disadvantages of each type of card. The table below outlines key characteristics of each card.

Debit Card	Credit Card
A debit card is a card that allows a bank's customer to make purchases using money from their account. • Issued by the bank • Generally given to all bank customers when they open a checking account • Funds are taken in full directly from the customer's checking account • Protected by a security code (PIN, or Personal Identification Number) • Because money is immediately taken out, recovering funds due to fraud or theft may be difficult • Spending is limited by the amount of money in your account	A credit card is a card that allows a person to borrow a certain amount of money and pay the borrowed money back at a later time. • Customer applies for a credit card through a financial company • Money is loaned to the person to cover expenses • A bill for all purchases charged is sent at the end of the month • Additional money, based on a percentage of the amount charged, is added to the bill when not paid in full at the end of each month • Protected by the customer's signature • Customers are protected from fraud or theft • Often offer rewards for their use • Each card has a limit to the amount that a customer can borrow

Credit cards have advantages but only when used responsibly. Credit card purchases should be paid off in full at the end of the month. When they are not paid in full, you will end up paying a lot more for your purchases!



1. Determine whether each statement describes a debit card, credit card, both, or neither. Explain your reasoning.

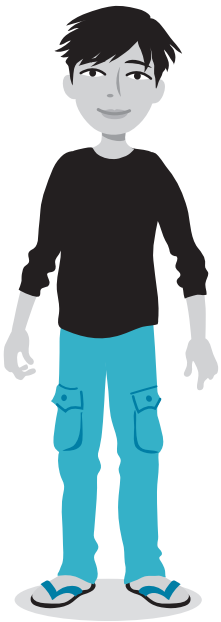
   a. This card allows you to spend more money than you have in your bank account at a given time.

   b. When using this card, money is taken immediately out of your bank account.

   c. This card allows you to purchase products online.

   d. With this card, you may end up paying more for a product than the list price.

When do you think it would be most appropriate to use a credit card?



Many credit cards offer reward programs. Loyal customers are given certain "gifts," which may be frequent flyer miles, free hotel stays, or money back on purchases made with the cards. These rewards are enticing to customers, but the costs far outweigh the benefits when the card is not paid off at the end of each month!

2. Jorge is traveling soon and wants to get a credit card. He considers the two credit card options described. Each has a reward program.

Investor's Choice	Venture Limited
• 1.5% cash back on all purchases • No annual fee • $5000 limit	• 2.0% cash back on all purchases • $40 annual fee • $5000 limit

He plans on using the card for travel expenses totaling approximately $4500.00. Which card would you recommend for Jorge? Show all of your work and explain your reasoning.

3. Jorge does not pay his monthly credit card bill regularly. He ends up paying an additional 18% on his travel fees.

   a. How much additional money does Jorge pay on his travel fees?

   b. Do the benefits from either card outweigh the additional amount paid? Show all of your work and explain your reasoning.

4. Summarize the similarities and differences between credit cards and debit cards.

ACTIVITY
2.2 Credit Cards

PROBLEM SOLVING

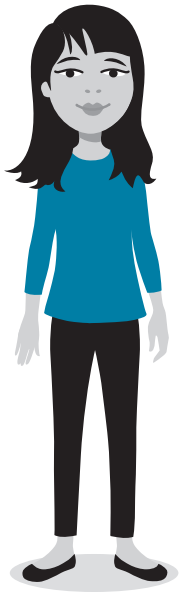


A major convenience of credit cards is that they allow customers to purchase products now, but pay for them later. Credit card companies also allow customers to pay a minimum amount every month, usually a percentage of the balance. This amount is often much less than what is owed on the purchase. Credit cards allow you to have a product now, but pay for it gradually over time. Unfortunately, this convenience is exactly what can make credit cards risky.

When the borrower doesn't make the entire payment at the end of each month, *interest* is added to the cost of the item. **Interest** is an additional charge for borrowing money, usually a percentage of the amount borrowed. The **interest rate** is the percentage of the balance on the credit card that is added to the cost of the purchase.

1. When you pay the minimum payment of $15.00 per month to a credit card company over two and a half years for a $290 purchase, how much money is spent on interest?

Credit card debt is much more complicated than just calculating interest. The questions here are simplified to give you a general understanding of how credit cards work.



2. Jaylen didn't have money for a game system, so he bought it with his credit card. He paid for it monthly over the course of a year. The $350.00 game system ended up costing $429.

   a. How much interest did he pay on the game system purchase?

   b. What percentage of the game system price did he pay in interest?

CC BY NC

Minimum payments are typically a small percentage of the balance on the credit card. This percentage varies depending on the card, but it is usually between 2% and 5%. Minimum payments are usually due monthly.

3. Calculate the minimum payment for each situation.

   a. a $400 balance with a 2% minimum payment

   b. a $1200 balance with a 3.5% minimum payment

4. What is the required percentage for the minimum payment when the balance is $4000 and the minimum payment is $100?



Credit cards generally charge between 15% and 22% interest per year. You can approximate the amount of the monthly minimum credit card payment that goes toward interest alone by multiplying the current balance by the interest rate and dividing by 12.

$$\text{interest each month} = \frac{\text{balance} \cdot \text{interest rate}}{12}$$

5. Suppose the interest rate on the credit card in Question 3 is 15%. Answer each question. Show all of your work.

   a. How much of the minimum payment is being paid in interest each month?

   b. How much of the minimum payment is being paid toward the balance each month?

6.

> **Jamal**
> My credit card has a balance of $100 and a minimum payment of $10. So, I'll have it paid off in 10 months.

Explain why Jamal is incorrect. What advice would you give Jamal?

CC BY NC

Talk the Talk

Pick a Card . . . Any Card

1. Determine whether a credit card, debit card, both, or neither is appropriate for each situation. Explain your reasoning.

   a. Josh has only $20 in his checking account on the 14th of the month but gets paid on the 15th. He needs a new pair of glasses that will cost $159.99. He purchases them and pays for them the next day when he gets his paycheck of $1250.00.

   b. Alyssa is traveling to Corpus Christi for the weekend and doesn't want to bring a lot of cash on vacation.

   c. Matthew buys a bicycle for $200.00. He has the money but doesn't want to carry that much cash with him to the bike shop.

   d. Jaylen wants to buy the latest game system but doesn't have enough money in his checking account.

2. Describe a situation when a consumer might choose to use a credit card and not incur any additional interest or fees on top of the original purchase price.

3. Describe a situation in which a consumer could use a credit card and incur additional interest or fees on top of the original purchase price.

4. Describe the advantages and disadvantages of debit cards.

5. Describe the advantages and disadvantages of credit cards.

CC BY NC

Lesson 2 Assignment

Write

What are some of the advantages and disadvantages to using a credit card for purchases?

Remember

Both credit and debit cards allow people to pay for purchases without using cash. Using a debit card pulls money directly from the customer's checking account, while using a credit card is paying with funds that are loaned by a financial company. Additional money, based on a percentage of the amount charged, is added to the bill when not paid in full at the end of each month.

Practice

Identify whether each statement is describing a debit card, a credit card, both, or neither. Explain your reasoning.

1. This card allows you to purchase goods from a store.

2. This card is issued by the bank when a customer opens a checking account.

3. This card is protected by the customer's signature.

4. This card has a limit on spending.

5. This card may offer rewards for its use.

Lesson 2 Assignment

6. This card may make it more difficult to recover funds due to fraud or theft.

Lizzie is renovating her kitchen. She will be making a lot of purchases from a number of different stores throughout the renovation process. She has $2000 in her checking account for this project as well as a credit card with a $5000 limit.

7. One of the first projects Lizzie is doing is replacing the appliances for her kitchen. She determines the cost will be approximately $1750 for a new stove, refrigerator, and dishwasher. How would you recommend Lizzie pay for these purchases?

8. Lizzie decides to pay $2050 for construction costs using her credit card. Her card currently has a rewards program which pays her 1.5% cash back on all purchases. How much cash back will she get with this purchase?

Lesson 2 Assignment

9. Lizzie charges the cost for the plumber to her credit card. The cost for the plumber is $850. Unfortunately she did not make her payments at the end of the month and ended up paying an extra 15% interest on these charges. What was the total amount she paid for the plumber?

10. After the renovations are finished, Lizzie charged a total of $4835 to her credit card. By the time she finished paying off the charges, she has spent $5380. What percentage of the renovation price did she pay in interest?

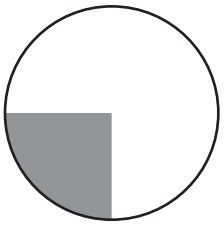
CC BY NC

Lesson 2 Assignment

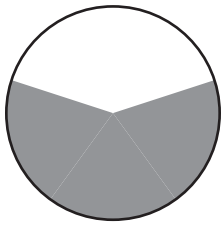
Prepare

Estimate the shaded part of each figure as a percent.

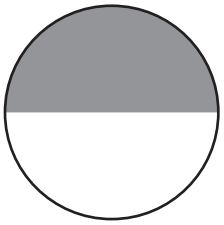
1.



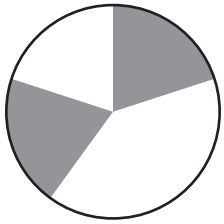
2.



3.



4.



3 Understanding Credit Reports

OBJECTIVES

- Describe the features of a credit report and how long the information is retained.
- Describe the importance of a good credit score and a positive credit history.
- Determine the significance of a credit report to borrowers and lenders.

NEW KEY TERMS

- credit report
- credit history [historial de crédito]
- credit score
- circle graph [gráfica circular]

You have learned about using credit cards to pay for purchases.

How does your credit history impact your long-term financial options?

Getting Started

Know Your History

When you buy a car, a house, take out a loan, apply for a credit card, or even apply for a job, the lender or employer may look at your *credit report*. A **credit report** is a detailed listing of an individual's credit history.

1. What information do you think is in a credit report?

2. Why do you think lenders look at a potential borrower's credit report?

ACTIVITY
3.1 Credit Reports and Scores

Your credit history includes a record of all loans, credit card debt, late payments, and so on. Similar to the grades that you get in school, a credit report has a *credit score* attached to it. A **credit score** is a number used by lenders to rate how likely it is that a person will repay their debts. This score can determine whether you qualify for a loan, the amount of money that you can borrow, and the interest rate for the loan.

Your credit report can have a huge impact on the quality of your life. What exactly is in this report? What is a good score? What can you do to get a good score? Let's research these questions.

1. Determine whether each piece of information is included in a credit report. Write each item in the appropriate box.

A. Job/Career	B. Car loans	C. Religion
D. Amount of money owed to credit cards	E. Number of credit cards you have	F. Whether you are married or single
G. Payments made with cash	H. Home Value	I. Debit card use
J. Employment history	K. Race	L. Social Security number
M. Current address	N. Previous address	O. Missed payments on a loan



Information Included in a Credit Report	Information Not Included in a Credit Report

2. Identify any additional information that is included in a credit report.

3. What is the range of credit scores? What scores are considered to be good?

4. Why is it important to have a good score?

5. What can you do to make sure you receive a good credit score?

6. Why is a person's credit score important to a bank or financial group that loans money to people?

7. How long is the information in a person's credit report kept on file?

8. Decide whether each statement is true or false. Explain your reasoning.

   a. Late payments can affect a credit score.

   b. A credit score of 800 is generally considered to be good.

   c. High credit card debt does not lower a credit score as long as payments are made on time.

CC BY NC

d. A person with a low credit score often ends up paying higher interest rates on loans.

e. A consumer can obtain a free copy of their credit report each year.

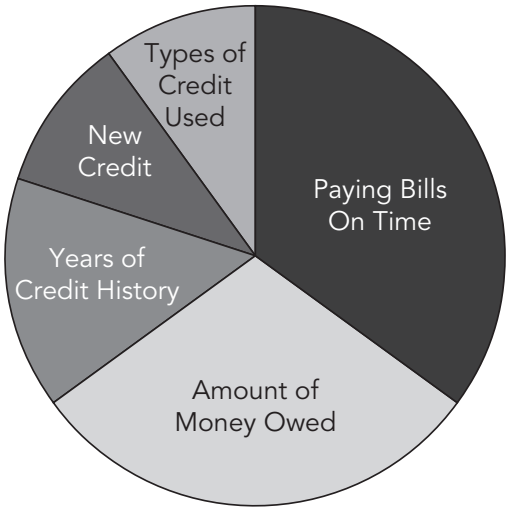
f. A late payment made 3 years ago will no longer affect your credit score.

9. Many car advertisements include a low interest rate for "well qualified buyers." Explain what this phrase means.

ACTIVITY
3.2 Credit Score Factors

A **circle graph** is a circular chart divided into pieces. Each piece shows the relative size of each value. The circle graph below provides a breakdown of the importance of the information in a credit report when determining a credit score.

Credit Score Factors



1. Analyze the circle graph.

   a. Identify the two most significant factors in establishing good credit.

   b. Identify the two least important factors in establishing good credit.



2. Estimate the percentage of the circle graph for each category.

3. The five items listed are included in Jaylen's credit report. Rank the items from 1–5 in terms of their importance in determining his credit score.

_____ Jaylen still owes $100,000 on his house, car, and credit cards.

_____ Jaylen opened one new credit card this year.

_____ Jaylen has been making payments toward his loans for the last 15 years.

_____ Jaylen has paid all of his bills on time. No record of late payments.

_____ Jaylen has several credit cards, an installment loan for his car, and a mortgage.

4. When Jaylen bought his first car 7 years ago, he stopped making payments on the loan and the car was repossessed. Will this still impact his credit report today? Explain your reasoning.

Talk the Talk

What's the Score?

1. Give the top 2 factors that affect a credit score.

2. Give 3 reasons why a credit score is important.

Lesson 3 Assignment

Write

In your own words, describe what a credit score is.

Remember

A *credit report* is a detailed listing of an individual's credit history, along with a credit score. A *credit score* is a number used by lenders to rate how likely someone will repay their debts. A good credit score is obtained by paying bills on time and avoiding too much debt. Credit scores determine whether someone qualifies for a loan, the amount of money they can borrow, and the interest rate of the loan.

Practice

Identify whether each piece of information is included in a credit report.

1. Current and previous addresses
2. Marital status
3. Job/career
4. Number of credit cards
5. Religion
6. Social Security number
7. Current loans
8. Debit and cash payment history

Lesson 3 Assignment

Jamal is hoping to get a loan to buy a house. The bank first needs to run a credit report to determine whether he is high-risk or low-risk. Jamal's financial history is as follows:

- He has a steady job and makes enough money to pay his bills each month.
- He currently lives in an apartment and makes his rent payment each month.
- He has school loans that he is paying off. He makes the minimum payment each month on time and has never missed a payment.
- He balances his checkbook and is aware of how much money he has in his account each month.
- He has a loan for his new car. He makes the minimum payment each month on time and has never missed a payment.
- He has 3 credit cards open. He uses one only for emergencies, one only for travel, and one for day-to-day needs. He is usually able to pay off the cards each month, but some months, he may only be able to make the minimum payment. He then pays it off the next month.
- He tries to make most of his purchases with cash when he is able to.
- He is married, and his wife also has a job.

9. Identify the information in Jamal's financial history that will be considered while determining his credit score.

Lesson 3 Assignment

10. Identify the information in Jamal's financial history that will not be considered while determining his credit score.

11. Do you think the bank will give Jamal his loan? Would you consider him high-risk or low-risk? Explain your reasoning.

Prepare

Matthew makes $8.25 per hour working at the local library.

1. At that rate, how much does he make in an 8-hour work day?

2. How much does he make in a 5-day work week?

3. How much will he make in a year if he works five days each week? There are 52 weeks in a year.

CC BY NC

4 Career Exploration

OBJECTIVES

- Compare the education and training required for various careers.
- Analyze the impact that education has on career possibilities.

NEW KEY TERMS

- post-secondary education
- associate's degree
- undergraduate degree
- master's degree [maestría]
- vocational school [escuela vocacional]
- student loan

You have likely been asked, "What do you want to be when you grow up?"

What do you need to consider with regard to your education to achieve your career goals?

Getting Started

Career Day

1. What type of career would you like to have when you are an adult?

2. Why is that career appealing to you?

3. Do you know anyone that has this career? What have they shared with you about their job?

4. What education do you need to have the career you chose?

5. What are reasons people choose the career path that they do?

ACTIVITY
4.1

Post-Secondary Education and Earning Potential

One way to think about your education is as an investment. In general, the more education or training you receive, the greater your earning potential. A certain career may require one, two, four, or even eight years of *post-secondary education*.

Post-secondary education refers to education beyond high school. An **associate's degree** is a degree earned in a one- or two-year program in which students learn a specific skill or trade. An **undergraduate degree** is a four-year degree from a college or university. A **master's degree** is a one- or two-year degree beyond an undergraduate degree.

There are many reasons to earn a college degree. It can provide a sense of accomplishment and allow you the opportunity to pursue a career in a field that interests you. Schooling provides large financial benefits as well.

Consider each fact from a recent study to answer Questions 1 and 2.

Education Level Earned	Financial Benefit
Master's Degree	A person with a master's degree earns approximately 30% more annually than a person with only an undergraduate degree.
Undergraduate Degree	A person with an undergraduate degree earns 84% more over a lifetime than a person with just a high school diploma.
High School Graduate	A high school graduate earns $350,000 more over a lifetime than a person without a high school diploma.



CC BY NC

1. Josh has an undergraduate degree and earns $51,000 per year. He considers going back to school for a master's degree. He calculates his anticipated salary increase.

Josh

I determined 30% of $51,000 by setting up two equal ratios and solving for x.

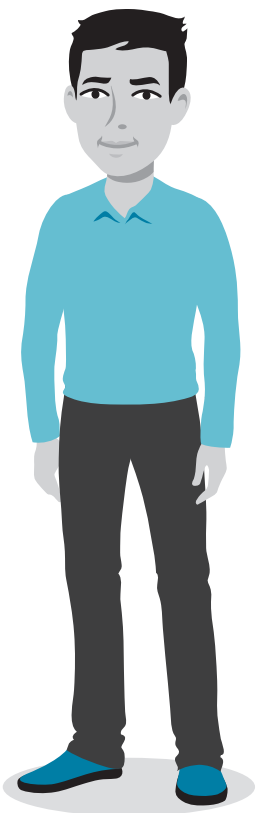
$$\frac{30}{100} = \frac{51{,}000}{x}$$

$51{,}000 \div 30 = 1700$

$1700 \cdot 100 = 170{,}000$

I might earn $170,000 more with a master's degree.

Remember, the data presented are averages. The salaries of specific individuals can vary greatly depending on many different factors.



Determine the error in Josh's reasoning and correct his work.

2. Over the course of a lifetime, the average person with only a high school diploma can expect to earn 1.2 million dollars.

   a. How much money can a person with an undergraduate degree expect to earn over the course of a lifetime?

b. How much money can a person with a master's degree expect to earn over the course of a lifetime?

3. According to a recent study, 63% of jobs in America require some sort of post-secondary education.

   a. What percent of jobs require only a high school education or less?

   b. Out of 2000 total jobs, how many require a post-secondary education? Show all of your work and explain your reasoning.

CC BY NC

PROBLEM SOLVING



ACTIVITY 4.2

Calculating Total Earnings

In the previous problem, you analyzed the financial impact that education can have on your life. In general, the higher degree you attain, the more money you will make.

1. Consider Mario's career path. Mario chose a career in construction.

Mario

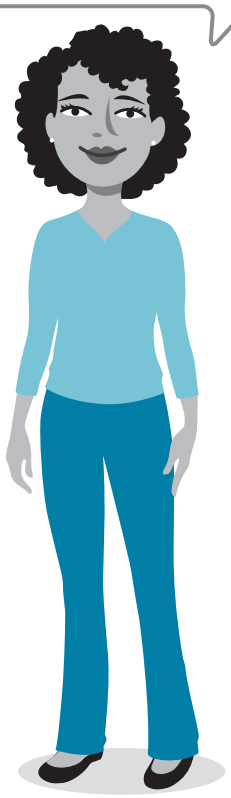
- Received paid, on-the-job training
- Started working at 18
- Average Salary
  Ages 18–27: $25,000
  Ages 28–42: $40,000
  Ages 43–65: $59,000

a. Determine Mario's total earnings from the ages of 18–27. Show all of your work.

b. Determine Mario's total earnings from the ages of 28–42. Show all of your work.

c. At the age of 28, Mario earned his undergraduate degree and entered management. Determine the percent increase in his average salary from the ages of 18–27 to the ages of 28–42. Show all of your work and explain your reasoning.

The salary for any career varies by job experience and location. Research job postings in your area for a more accurate reflection of the potential salary for a career you are interested in.



2. Natalia studied at Rice University for 4 years and became a computer software engineer.

Natalia

- Spent $50,000 each year for 4 years to attend college
- Started college at 18
- Started working at 22
- Average Salary
  Ages 22–28: $52,000
  Ages 29–42: $68,000
  Ages 43–65: $85,000

a. Determine Natalia's total earnings at the age of 20.

b. Determine Natalia's total earnings at the age of 40.



3. Compare Mario's and Natalia's career paths.

a. Determine Mario's total earnings at the age of 20.

b. Compare Mario's and Natalia's total earnings at the age of 20.

c. Determine Mario's total earnings at age 40.

d. Compare Mario's and Natalia's total earnings at the age of 40.

e. Describe the difference between the total earnings of Mario and Natalia over time?

4. The table shows the average annual salary for four alternative jobs that Mario and Natalia could choose.

Job	Average Annual Salary (dollars)
Project Manager	72,363
College Professor	52,180
Architect	54,239
Computer Software Trainer	60,725

Based on this information, how much more would Mario make in 10 years, time if he chose to be an architect instead of a college professor?

Higher education usually leads to increased earnings, but college and *vocational school* can be very expensive. A **vocational school** is a school in which students are taught the skills needed to perform a particular job. Many students take out *student loans* to finance their education. A **student loan** is money borrowed to pay for college or trade school.

A general rule of thumb for student loans is to borrow less than your anticipated annual salary. In other words, if you expect to make $40,000 per year upon graduation, borrow less than this amount to pay for school.

5. Consider Andrew's and Kaya's career paths. Andrew attended community college before transferring to a larger university to earn an accounting degree. Kaya took out a large number of student loans over four years to attend a prestigious university, where she also earned an accounting degree.

Andrew	**Kaya**
• Earned grants, scholarships, and took out no student loans • Starting salary of $50,000 • Earns a $1,000 raise each year	• Pays $4,700 per year in student loans for 20 years • Starting salary of $52,500 • Earns a $1,200 raise each year

Was Kaya's decision to take out loans to attend a very prestigious university a sound financial decision? Show all of your work and explain your reasoning.

Talk the Talk

Stacking Paper

This table is a simplified display showing that the more education or training a person receives, the greater their earning potential.

Complete this table by starting at the bottom with the lowest level of education earning $10 per hour.

Calculate the hourly, weekly, yearly and lifetime pay for each educational level based upon the information provided in the hourly pay column.

	Educational Level	Hourly Pay	Weekly Pay (for 40 hours per week)	Yearly Pay (for 52 weeks per year)	Lifetime Pay (for 40 years)
1.	Master's Degree	Earns 30% more than a person with an undergraduate degree. (Round to the nearest cent.)			
2.	Undergraduate Degree	Earns 84% more than a high school graduate.			
3.	High School Graduate	Makes $350,000 more over a lifetime than a person who did not earn a high school diploma. For 40 hours per week, for 52 weeks, for 40 years, this is about $4 more per hour!			
4.	No High School Diploma	Makes $10 per hour.			

5. Between what two educational levels is there the greatest increase in income?

6. About how many times more does a person with a master's degree make than a person without a high school diploma over the course of a lifetime?

Lesson 4 Assignment

Write

What are the different factors that someone should consider when making decisions about their future career? Include discussions of education, salary, long-term success, interests, and strengths.

Remember

Certain careers may require one, two, four, or even eight years of post-secondary education. For the most part, the higher degree you attain, the more money you will make.

Practice

Three siblings each took a different career path. The earning history for each sibling until the age of 35 is shown.

Samantha

- Entered college at age 18
- Took out loans for college totaling $60,000
- Earned her undergraduate degree at age 23
- Entered the workforce at age 24 as a zookeeper
- Ages 24–26 earned $38,000 per year
- Ages 27–30 earned $42,000 per year
- Ages 31–33 earned $45,000 per year
- At age 33 she earned her master's degree paid for by her employer
- Ages 34–35 earned $50,000 per year

Jaylen

- Graduated high school at age 18 and entered the workforce as a delivery driver
- Ages 18–22 earned $18,500 per year
- Ages 23–25 earned $20,000 per year
- Ages 26–30 earned $23,000 per year
- At age 30 he went to trade school and took out loans totaling $9000
- Ages 32–35 earned $42,000 per year

Tiara

- Entered college at age 18 on a partial scholarship
- Took out loans for college totaling $30,000
- Earned her undergraduate degree at age 23
- Entered Graduate school at age 23
- Took out loans for graduate school totaling $60,000
- Earned her master's degree at age 27
- Entered the workforce at age 27 as a social worker
- Ages 27–30 earned $58,000 per year
- Ages 31–32 earned $62,000 per year
- Ages 33–35 earned $67,000 per year

CC BY NC

Lesson 4 Assignment

1. Determine which sibling earned the most over his or her career so far.

2. Which sibling do you think made the best decision in terms of his or her education and career?

Lesson 4 Assignment

Prepare

Calculate each value. Round your answer to the nearest tenth, if necessary.

1. What is 15% of 80?

2. What is 12% of 140?

3. 42 is what percent of 70?

4. 10 is what percent of 25?

5 Paying for College

OBJECTIVES

- Explore various methods of paying for post-secondary education.
- Determine the differences among grants, scholarships, loans, and work-study programs.

NEW KEY TERMS

- tuition
- grant
- scholarship
- work-study program
- public school [escuela pública]
- private school [escuela privada]

You have thought about what you need to do after high school to meet your career goals. What options are available to pay for post-secondary education?

CC BY NC

Getting Started

Are the Rumors True?!

Do you know someone who is in college now or recently graduated from college? You may have heard talk about the high cost of college tuition and the financial assistance needed to pay tuition.

Before we begin this lesson on paying for college, see what you already know about this topic.

Read each statement below, and then write T for "true" and F for "false" next to each statement.

1. _____ When selecting a college, tuition is the only factor to consider.

2. _____ Students must pay the entire cost of college tuition.

3. _____ Private college tuition is generally much more than public university tuition.

4. _____ Many students take out student loans to help them pay their college tuition.

5. _____ A scholarship is always a full-scholarship, meaning it pays the entire cost of a student's college tuition.

6. _____ Academic and athletic scholarships are the only types of scholarships available to help pay college tuition.

7. _____ Grants are a form of financial aid that do not have to be paid back after graduation from college.

8. _____ Some grants are from the federal government.

9. _____ A work-study job is a job that a college offers a student to help them earn money to pay their college tuition.

10. _____ All students who attend the same public university in a given state pay the same tuition, regardless of what state they are a resident of.

ACTIVITY 5.1

Paying for Your Post-Secondary Education

Tuition is the fee paid to receive instruction at a school. Tuition may be paid in a variety of ways, through personal savings, student loans, *grants*, *scholarships*, or *work-study programs*.

- A **grant** is money awarded to a person from the government or school to help pay tuition.
- A **scholarship** is money awarded to a person from the college or a private institution to help a student pay for school. Scholarships may be academic, athletic, based on financial need, or awarded from private organizations for a variety of reasons.
- A **work-study program** is a plan set up by the college that allows a student to work while in school to earn money.

Scholarships and grants do not have to be repaid as long as the student meets any obligations required by them. For example, an academic scholarship may require that the student maintain a B average, or an athletic scholarship may require participation in a sport. Colleges may offer different financial aid packages to attract students. With tuition varying so greatly from school to school, it's important to be able to mathematically compare different options to make a good decision.



1. Suppose you start saving for college now, and you could save $50 per week starting today.

   a. Without interest, how much would you have for college in 6 years?

   b. How much would you have to save per week to have $10,000 in 5 years?

CC BY NC

PROBLEM SOLVING



ACTIVITY 5.2

Choosing a Post-Secondary Institution

1. Paola has a B+ average and plays softball for her high school team. She receives offers from Texas A&M Galveston and Texas Tech. Both schools have quality programs that interest her. She studies the financial aid packages given by each school. The approximate cost for tuition in 2023 for both schools is shown.

Texas A&M Galveston	Texas Tech
Tuition: $12,050 per year **Athletic Scholarship:** 30% off tuition each year **Work-study:** $2500 per year	**Tuition:** $11,800 per year **Academic Scholarship:** $4000 per year **Work-study:** $1200 per year

   a. Which school offers more money in financial aid? Show all of your work and explain your reasoning.



   b. Trung argues that Texas Tech offers the better scholarship deal. Alejandro disagrees, claiming that Texas A&M offers the better scholarship. Who is correct? Explain your reasoning.

2. Elena has a 3.0 grade point average. She is a member of the volleyball team and participates in the service club. She and her family estimate that they will be able to afford $5000 per year for college. She applies to the University of Texas at Austin and receives financial aid information from the school. The tuition rate for 2022-2023 is shown.

The University of Texas at Austin
Tuition: $11,698 per year
Scholarship: $1,600 athletic scholarship for playing on the volleyball team
Grants: $750 per semester
Work-study: 40 hours per semester at $10/hr

a. Can she currently afford to attend school at UT Austin without additional money? Show all of your work and explain your reasoning.

b. What advice would you give Elena?



ACTIVITY 5.3

Public and Private Colleges and Universities

Colleges and universities are either *public* or *private*. A **public school** receives funding from the state government to offset the costs of running the university. A **private school** does not receive funding from the state. Private schools raise funds from tuition and donations. The tuition for public universities is generally much less than for private schools. Additionally, the cost of public schools in a given state is generally much less for residents of the state than for non-residents. Simply put, when you live in Texas, you will pay much less to attend a Texas public university than people who move to Texas from another state to attend a Texas university.

1. Eduardo and Chris analyze tuition costs for Midwestern State University and Lamar University to determine which school is the better deal for Texas residents. For the 2022-2023 school year tuition for Midwestern State University was $10,233 for residents and $12,183 for non-residents. Tuition for Lamar University for the 2022-2023 school year was $8,690 for residents and $18,482 for non-residents.



Eduardo

Lamar is a better deal for in-state residents because it's only $8690 when you live in Texas, compared to $10,233 for Midwestern.



Chris

Midwestern is the better deal. The ratio of in-state to out-of-state tuition, $\frac{10,233}{12,183}$, is less compared to $\frac{8690}{18482}$ for Midwestern.

Who is correct? Explain your reasoning.

2. For the 2022-2023 school year, Texas residents paid only about 43% of the out-of-state tuition rate at The University of Texas Pan American. Tuition for non-Texas residents is $17,230. How much do Texas residents pay? Show all of your work and explain your reasoning.

3. In 2021, the average Texas resident paid about $8000 for undergraduate tuition at Texas public colleges, compared to about $24,000 for non-residents.

   a. Determine the percentage of tuition that residents pay compared to non-residents per year. Show all of your work and explain your reasoning.

   b. What percentage do residents pay compared to non-residents over 4 years? Show all of your work and explain your reasoning.

Talk the Talk

Time to Reflect

1. Describe things to consider when choosing a public or private school.

2. Describe various ways that you can finance your college education.

Lesson 5 Assignment

Write

A family friend is a senior in high school and is choosing between two universities: Local College and Math Institute of America. She says, "I am going to go to Local College because the tuition there is less than Math Institute of America." Write a letter to your family friend explaining everything that she should consider before making her final decision.

Remember

Tuition for post-secondary education can be paid using personal savings, student loans, grants, scholarships, or work-study programs. Colleges might also offer financial aid packages to students, which can subsidize the costs for school.

Practice

Mariana is graduating high school in a few years and is looking into colleges. She currently plays on her high school volleyball team, volunteers every weekend at a nursing home, and works part-time at a local grocery store. She also maintains a 3.3 grade point average. Mariana lives in Arizona and is looking at the 2022-2023 tuition rates at the colleges shown.

Arizona State University	University of Nevada, Reno	Purdue University (Indiana)
Tuition **Residents:** $11,618 **Non-residents:** $30,592 **Financial Aid** **Scholarship:** Athletic: $5000 per year **Work-study:** $1750 per year	**Tuition** **Residents:** $8947 **Non-residents:** $24,501 **Financial Aid** **Scholarship:** Academic: 30% of tuition **Work-study:** $1500 per year	**Tuition** **Residents:** $9992 **Non-residents:** $28,794 **Financial Aid** **Scholarship:** Academic: 35% of tuition **Work-study:** $2000 per year

1. Mariana is determining the cost of tuition at Arizona State University.

   a. Determine the yearly cost of tuition if Mariana receives the athletic scholarship but does not participate in the work-study program.

b. Determine the yearly cost of tuition if Mariana participates in the work-study program but does not accept the athletic scholarship.

2. Mariana is determining the cost of tuition at Purdue University in Indiana.

   a. Determine the yearly cost of tuition Mariana has to pay without financial aid. Explain your reasoning.

   b. Suppose Mariana receives the academic scholarship but does not participate in the work-study program. Determine Mariana yearly cost of tuition.

Lesson 5 Assignment

3. Mariana really wants to go to one of the out-of-state colleges. However, she knows she can only attend one of these colleges if tuition is $16,000 or less. Suppose Mariana receives all the financial aid offered. Will she be able to attend one, both, or neither of these schools?

Lesson 5 Assignment

Prepare

Ms. Davis asked the 25 sixth-graders in her class, "How many pets do typical 6th-graders in our class have?" Ms. Davis summarized the responses in the table.

Number of Students	Number of Pets
8	0
10	1
4	2
2	3
1	38

1. About how many pets does each sixth-grader in Ms. Davis's class own? How did you make your decision?

TOPIC 4 SELF-REFLECTION

Name: _____

Financial Literacy: Accounts, Credit, and Careers

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Financial Literacy: Accounts, Credit and Careers* topic by:

TOPIC 4: *Financial Literacy: Accounts, Credit, and Careers*	Beginning of Topic	Middle of Topic	End of Topic
comparing checking account and debit card options from different financial institutions.	○	○	○
distinguishing between debit cards and credit cards.	○	○	○
balancing a check register that includes deposits, withdrawals, transfers, and overdrafts.	○	○	○
describing the information in a credit report and how credit scores are determined.	○	○	○
describing how borrowers and lenders use credit reports.	○	○	○
explaining why it is important to establish a positive credit history.	○	○	○

continued on the next page

CC BY NC

TOPIC 4 SELF-REFLECTION *continued*

TOPIC 4: *Financial Literacy: Accounts, Credit, and Careers*	Beginning of Topic	Middle of Topic	End of Topic
describing various methods to pay for post-secondary education, including savings, grants, scholarships, student loans, and work-study programs.	○	○	○
comparing the annual salary of several occupations requiring various levels of post-secondary education or vocational training.	○	○	○
determining the effects of different annual salaries on lifetime income.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Financial Literacy: Accounts, Credit, and Careers* topic.

_____

_____

_____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

_____

_____

_____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

_____

_____

_____



TOPIC 4 SUMMARY

Financial Literacy: Accounts, Credit, and Careers Summary

LESSON 1 Checking Accounts

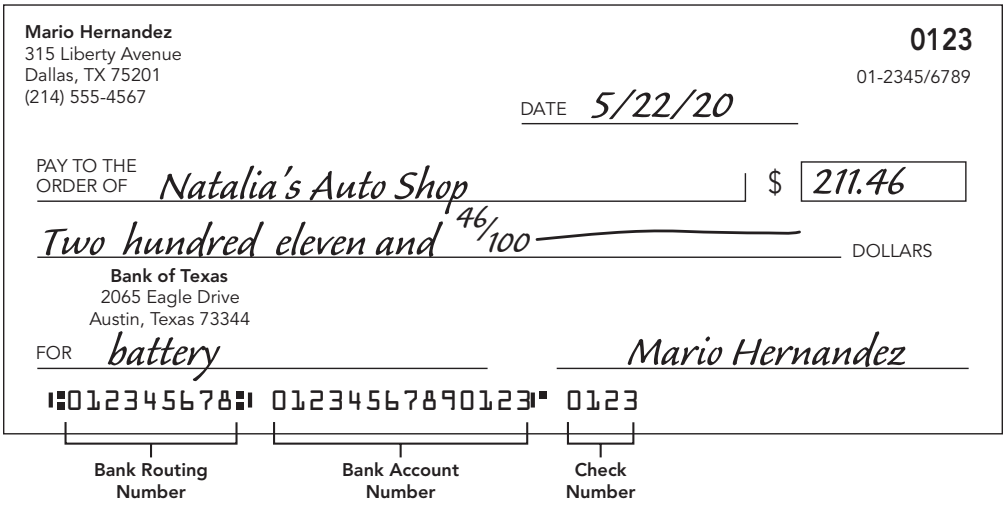
A **checking account** allows customers to safely store money in the bank and write checks. A **check** is a written order to a bank to pay a specific amount of money to a person or company out of a checking account. A check must contain certain information, such as the name of the person writing the check, the person or company receiving the check, the checking account number, the date the check was written, the amount of the check in numerical and word form, and the signature of the person writing the check.

NEW KEY TERMS

- checking account
- check [cheque]
- statement
- account balance
- deposit [depósito]
- withdrawal
- debit [débito]
- transfer [transferencia]
- overdraft
- Annual Percentage Yield (APY)
- debit card
- credit card
- interest [interés]
- interest rate
- credit report
- credit history [historial de crédito]
- credit score
- circle graph [gráfica circular]
- post-secondary education
- associate's degree
- undergraduate degree
- master's degree [maestría]
- vocational school [escuela vocacional]
- student loan
- tuition
- grant
- scholarship
- work-study program
- public school [escuela pública]
- private school [escuela privada]

CC BY NC

For example, Mario Hernandez is writing a check to pay for a battery he purchased at Natalia's Auto Shop. He wrote the check on May 22, 2020. The total cost of the helmet is $211.46.



Most banks send their customers a statement every month. A **statement** is a summary of the **account balance**, which is the amount of money in the account at any given time. A statement also includes every transaction that occurred throughout the month. The transactions may include deposits, withdrawals, debits, and transfers.

- A **deposit** is money put into the account. Deposits may come in the form of cash, checks, or transfers. Paychecks are often deposited directly into a checking account.
- A **withdrawal** occurs when money is taken out of the account, usually from a check, online payment, or Automated Teller Machine (ATM).
- A **debit** occurs when money is taken out of the account automatically through the use of a debit card. Debit cards offered by different banks can have different features and costs.
- A **transfer** occurs when money is moved from one account to another.

For example, Kaya keeps track of all transactions made during the month of May. Her account balance is $2106.97 at the end of the month.

Check	Date	Transaction Description	Payment/ Withdrawal	Deposit	Balance
	May 3	Water company bill payment	$72.50		$1227.50
	May 10	ATM withdrawal	$80.00		$1147.50
	May 5	Direct deposit		$1440.00	$2587.50
	May 18	Grocery withdrawal	$182.35		$2405.15
104	May 27	Rent payment	$700.00		$1705.15
	May 27	Transfer from savings		$500.00	$2205.15
	May 30	Cable TV bill	$98.18		$2106.97

An **overdraft** occurs when money is withdrawn from a bank account and the balance goes below zero. It is important to prevent an overdraft from occurring because most banks charge a fee every time this happens.

Banks may charge different monthly fees, or they may require a minimum deposit to open an account. **Annual Percentage Yield (APY)** is a percentage that is paid to customers based on the account balance in an account for a year.

For example, Ashley opens a checking account with an average monthly balance of $600. The account has a 2.5% APY.

$(600)(0.025) = 15$

Ashley will earn $15 on this checking account over the course of the year.

LESSON 2

Debit Cards vs. Credit Cards

Many people enjoy using **debit cards** and **credit cards**. Both are plastic cards that allow shoppers to pay for purchases without having the cash on hand. The two types of cards look similar, but they are very different. To use them responsibly, it is important to understand what they are as well as the advantages and disadvantages of each type of card. The table outlines key characteristics of each card.

Debit Card	Credit Card
A debit card is a card that allows a bank's customer to pay for goods using money from their account. • Issued by the bank • Generally given to all bank customers when they open a checking account • Funds are taken in full directly from the customer's checking account • Protected by a security code (PIN, or Personal Identification Number) • Because money is immediately taken out, recovering funds due to fraud or theft may be difficult • Spending is limited by the amount of money in your account	A credit card is a card that allows a person to borrow a certain amount of money and pay the borrowed money back at a later time. • Customer applies for a credit card through a financial company • Money is loaned to the person to cover expenses • A bill for all purchases charged is sent at the end of the month • Additional money, based on a percentage of the amount charged, is added to the bill if not paid in full at the end of each month • Protected by the customer's signature • Customers are protected from fraud or theft • Often offer rewards for their use • Each card has a limit to the amount that a customer can borrow

Many credit cards offer reward programs. Loyal customers are given certain "gifts," which may be frequent flyer miles, free hotel stays, or money back on purchases made with the cards. These rewards are enticing to customers, but the costs far outweigh the benefits if the card is not paid off at the end of each month!

For example, Tiara uses her credit card for purchases because she gets 1.5% cash back on all purchases. This past month, she put $3300 in charges on her credit card.

$(3300)(0.015) = 49.50$

Tiara will get $49.50 cash back as a reward for her purchases.

Credit cards allow a consumer to have a product now, but pay for it gradually over time. If the borrower doesn't make the entire payment at the end of each month, interest is added to the cost of the item. **Interest** is an additional charge for borrowing money, usually a percentage of the amount borrowed. The **interest rate** is a percentage of the balance on the credit card that is added to the balance.

For example, Jorge bought a stereo system for $640 and put the purchase on his credit card. He paid off the purchase over several months and the stereo system ended up costing him $730.

$730 - 640 = 90$

Jorge paid $90 in interest.

$$r = \frac{90}{640}$$

$$\approx 0.14$$

Jorge paid approximately 14% in interest.

Credit cards generally charge between 15% and 22% interest per year. You can approximate the amount of the monthly minimum credit card payment that goes toward interest alone by multiplying the current balance by the interest rate and dividing by 12.

$$\text{interest each month} = \frac{\text{balance} \cdot \text{interest rate}}{12}$$

LESSON 3

Understanding Credit Reports

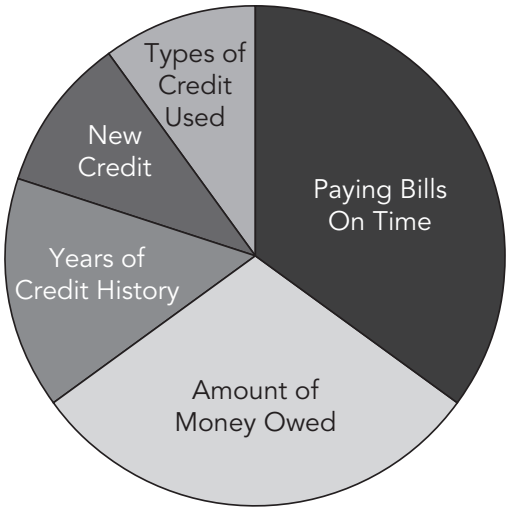
When you buy a car, a house, take out a loan, apply for a credit card, or even apply for a job, the lender or employer may look at your credit report. A **credit report** is a detailed listing of an individual's credit history. Your **credit history** includes a record of all loans, credit card debt, late payments, and so on. Similar to the grades that you get in school, a credit report has a score attached to it.

A **credit score** is a number used by lenders to rate how likely it is that a person will repay his or her debts. Credit scores range from 300 to 850. Generally, scores that are above 700 are good. This score can determine whether a person qualifies for a loan, the amount of money that they can borrow, and the interest rate for a loan. Credit reporting agencies usually keep your credit history for 7 to 10 years. It is important to lenders that you have a good credit score. It's also important for you to get the best interest rates when you borrow money.

For example, Lizzie is hoping to get a loan to open a new business. She currently has two credit cards open. She is paying off her car and her mortgage. She makes her monthly payments on time and has never missed a payment.

Lizzie will likely be considered low-risk and receive a good credit score. She will probably receive the loan she is requesting.

Card Credit Factors



A **circle graph** is a circular chart divided into pieces. Each piece shows the relative size of each value. The circle graph shown provides a breakdown of the importance of the information used by the major credit bureaus to determine a credit score.

Paying your bills on time is the most important factor in determining good credit, because it is represented by the biggest piece of the circle graph.

LESSON 4

Career Exploration

One way to think about your education is as an investment. In general, the more education or training you receive, the greater your earning potential. A certain career may require one, two, four, or even eight years of post-secondary education.

Post-secondary education is education beyond high school. An **associate's degree** is a one- or two-year degree in which students learn a specific skill or trade. An **undergraduate degree** is a four-year degree from a college or university. A **master's degree** is a one- or two-year degree beyond an undergraduate degree. There are many reasons to earn a college degree. A degree can provide a sense of accomplishment and allow you the opportunity to pursue a career in a field that interests you. Schooling provides large financial benefits as well. Each of these degrees can have a different earning potential.

- A person with a high-school diploma earns $350,000 more over their lifetime than a person who does not earn their high-school diploma.
- A person with an undergraduate degree earns approximately 84% more over their lifetime than a person with a high-school diploma.
- A person with a master's degree earns approximately 30% more annually than a person with an undergraduate degree.

For example, Alyssa has her undergraduate degree and currently earns $45,000 each year. She is going back to school to get her master's degree.

$(45{,}000)(0.30) = 13{,}500$

$45{,}000 + 13{,}500 = 58{,}500$

Alyssa might earn $58,500 each year after she receives her master's degree.

Higher education usually leads to increased earnings, but college and vocational school can be very expensive. A **vocational school** is a school in which students are taught the skills needed to perform a particular job. Many students take out student loans to finance their education. A **student loan** is money borrowed to pay for college or trade school.

A general rule of thumb for student loans is to borrow less than your anticipated annual salary. In other words, if you expect to make $40,000 per year upon graduation, borrow less than this amount to pay for school.

CC BY NC

LESSON 5

Paying for College

Tuition is a fee paid to receive instruction at a school. Tuition may be paid in various ways, through personal savings, student loans, grants, scholarships, or work-study programs.

- A **grant** is money awarded to a person from the government or school to help pay tuition.
- A **scholarship** is money awarded to a person from the college or a private institution to help a student pay for school. Scholarships may be academic, athletic, or awarded from private organizations for a variety of reasons.
- A **work-study program** is a plan set up by the college that allows a student to work while in school to earn money.

Scholarships and grants do not have to be repaid, provided the student meets any obligations required by them. For example, an academic scholarship may require that the student maintain a B average, or an athletic scholarship may require participation in a sport. Colleges may offer different financial aid packages to attract students. With tuition varying so greatly from school to school, it's important to be able to mathematically compare different options to make a good decision.

For example, Natalia currently has a 3.8 grade average, participates in drama club, and volunteers often at a local homeless shelter. She is interested in attending college at University of South Carolina.

$(23{,}817)(0.15) = 3572.55$

$23{,}817 - 3572.55 = 20{,}244.45$

$20{,}244.45 - ((2)(500)) = 19{,}244.45$

University of South Carolina

Tuition: $23,817
Academic Scholarship: 15% of tuition
Grants: $500 per semester

Natalia would pay $19,244.45 per year if she received all the financial aid offered.

Colleges and universities are either public or private. A **public school** receives funding from the state government to offset the costs of running the university. A **private school** does not receive funding from the state. They raise funds from tuition and donations. The tuition to public universities is generally much less than the tuition to private schools. Additionally, the cost of public school in a given state is generally much less for residents of the state than for non-residents.

For example, residents of Texas who attend Lamar University pay about 42% of the tuition non-residents pay. Non-residents pay $20,574 per year to attend Lamar University.

$$\frac{42}{100} = \frac{x}{20{,}574}$$

$$20{,}574 \div 100 = 205.74$$

$$(205.74)(42) = 8{,}641.08$$

Residents of Texas pay about $8,641.08 per year in tuition at Lamar University.

GRADE 6

MODULE 5

Describing Variability of Quantities

TOPIC 1 The Statistical Process 1081

TOPIC 2 Numerical Summaries of Data 1155

CC BY NC

GRADE 6

TOPIC 1



On average, one out of every 25 sheep has black wool. A quick way to estimate the size of a flock of sheep is to count the black sheep and multiply by 25.

The Statistical Process

LESSON 1 Understanding the Statistical Process **1083**

LESSON 2 Analyzing Numerical Data Displays **1103**

LESSON 3 Using Histograms to Display Data **1127**

1 Understanding the Statistical Process

OBJECTIVES

- Recognize and design statistical questions and anticipate variability in data related to the question.
- Differentiate between surveys, observational studies, and experiments.
- Describe the four stages of the statistical process.
- Discuss the different types of data that can be collected, displayed, and analyzed.
- Analyze and interpret bar graphs and circle graphs.

NEW KEY TERMS

- variability
- data
- statistical question
- statistical process
- categorical data
- quantitative data
- discrete data
- continuous data
- population
- sample
- survey
- observational study
- experiment
- bar graph
- frequency
- frequency table
- relative frequency table
- mode

You have been solving mathematical problems throughout this course. Now, you will study statistical problems.

How are mathematics and statistics similar and different?

CC BY NC

Getting Started

Statistical or Not, That Is the Question

In statistics, **variability** means that the value of the attribute being studied can change from one person or thing to another.

Data are categories, numbers, or observations gathered in response to a statistical question.

Have you ever wondered, "How much money do professional athletes make?" or, "How long are the books assigned to sixth-graders?" These are statistical questions. When you seek out the answer to your question, you engage in the statistical process.

Cut out the survey question cards provided at the end of the lesson. Read each question and sort them into as many groups as you like. There must be more than one group, and there must be at least two questions per group.

1. Record your groups and the questions in each group.

2. Which questions from your sort result in data with variability? Which result in data without variability?

Keep your questions. You will need them later in the lesson.



In this module, you will begin your formal study of statistics and the statistical process. Statistics is a problem-solving process, also called an *investigative process*, because the heart of statistics is about determining a possible answer to a question that has *variability*.

Statistical problem solving begins with a *statistical question*.
A **statistical question** is a question that anticipates an answer based on data with variability.

ACTIVITY
1.1 Formulating Statistical Questions

The **statistical process** has four components:

- Formulating a statistical question.
- Collecting appropriate data.
- Analyzing the data graphically and numerically.
- Interpreting the results of the analysis.

This lesson provides an overview of the statistical process, but you will continue to use the process throughout your study of statistics. Statistics is about posing interesting questions that you want to answer about varying attributes.

Ask Yourself . . .
How does the statistical process compare to the problem-solving model?

Analyze the questions posed by Bianca and Rajan.

Catalina



"What clubs am I in?"
"How many students are in the Chess Club?"

James



"What clubs do my classmates belong to?"
"How many members do the clubs at my school have?"

1. Explain why Catalina's questions are not statistical questions but James's are.

2. What kinds of answers do you expect from James's questions?

CC BY NC

Discrete data are quantitative data that can only take certain values. The data can be counted but not measured.

Answering a statistical question requires collecting variable data. You will learn about two types of data: *categorical data* and *quantitative data*. **Categorical data,** or *qualitative data*, are data for which each piece of data fits into exactly one of several different groups or categories. **Quantitative data,** or *numerical data*, are data for which each piece of data can be placed on a numerical scale and compared. There are two types of qualitative data: *continuous data* and *discrete data*.

3. Would the answers to James's questions be categorical or quantitative?

Continuous data are quantitative data that take any value. The data can be measured but not counted.

4. Gather the statistical questions from the *Statistical* or *Not* activity. Which questions have categorical answers and which have quantitative answers?

5. For each question, determine whether it is a statistical question. When it is not, rewrite it as a statistical question. Then, state whether the data would be categorical or quantitative.

   a. How many text messages did you send and receive yesterday?

   b. What are the most popular school mascots?

c. How much time did you spend watching TV or playing video games last weekend?

Some data, like area codes, are numbers, but are not quantitative variables. This data serves as a label, or category.

d. How many hours do 6th graders sleep each night?

e. What is your favorite sport?

6. Write at least 2 additional statistical questions that you would be interested in answering. State whether the data would be categorical or quantitative.

Ask Yourself. . .
What would you like to know? Don't worry about the answer to the question, if there even is one.

CC BY NC

ACTIVITY
1.2 Methods of Data Collection

For this activity, let's consider the topic of school lunches.

1. Write three statistical questions that you can ask about school lunches.

   a.

   b.

   c.

The second component of the statistical process is to collect the data to answer the statistical question.

A statistical question can be answered by collecting data from an entire *population* or, more commonly, from a *sample* of the population. A **population** is an entire set of items from which data are collected. A **sample** is a selection from a population.

For example, to answer the question, "How tall are 6th-graders?" using the population of all 6th-graders, you would need to determine the heights of every 6th-grader in the world. However, you could choose to answer the question by collecting data from a sample of 6th-graders—the 6th-graders at your school. This would not be a random sample.

Three common methods of data collection are *surveys*, *observational studies*, and *experiments*. In a **survey**, people are asked one or more questions. Similarly, in an **observational study**, the researcher (you!) collects data by observing the variable of interest. In an **experiment**, the researcher imposes a condition and observes the results.

You could conduct an experiment to investigate whether 6th-graders perform better on an assessment when they read a textbook or watch a video about the material. You would randomly assign half the students to read the text and half the students to watch the video. All students would be given the same assessment. You would compare the scores of the students in the two groups.

2. For each statistical question you wrote in Question 1, identify the population and sample of interest.

   a.

   b.

   c.

3. Do you think a survey, observational study, or experiment would be the best way to collect the data to answer your statistical questions? Explain your reasoning.

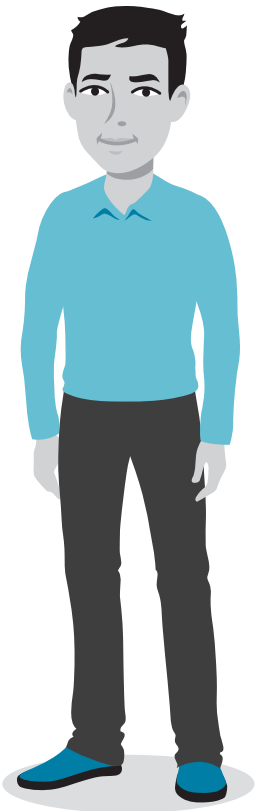
   a.

   b.

   c.

Ask Yourself . . .
Which tool or strategy is most efficient?

When you have completed a science project, you have probably conducted an experiment.



ACTIVITY 1.3

Analyzing and Interpreting Data

Formulating a statistical question is Part 1 of the statistical process.

Suppose you are interested in characteristics of sixth-graders at your school.

1. Formulate three categorical statistical questions and survey your class to obtain a sample.

Collecting data is Part 2 of the statistical process.

Analyzing the collected data is Part 3 of the statistical process.

In the statistical process, after you collect the data, it is time to analyze and interpret the results. Analysis includes selecting the most appropriate graphical display and numerical summaries for your question and your method of data collection.

You already have experience displaying and summarizing categorical data using bar graphs and circle graphs.

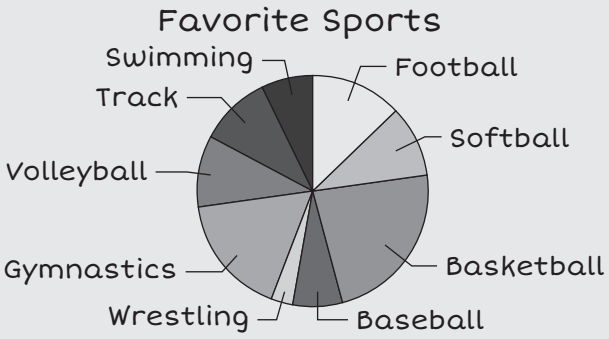
A **bar graph** displays categorical data using either horizontal or vertical bars on a graph. The height or length of each bar indicates the value for that category.

A circle graph, often called a *pie chart*, displays categorical data using sectors, or "wedges," of a circle. It shows how parts of the whole relate to the whole and how parts of the whole relate to the other parts. The area of each sector corresponds to the percentage of the part with respect to the whole.

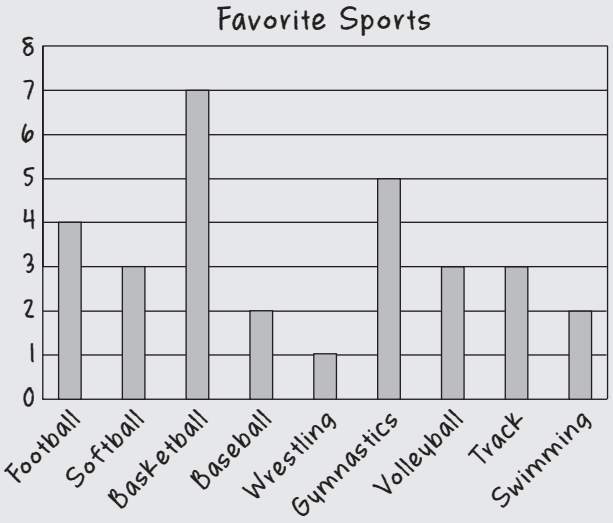
Linh and Ricardo were interested in the favorite sports of 6th-graders. They surveyed their class of 30 students. Then, they displayed their class's data in different ways. Analyze each graph.

Ask Yourself . . .
How can you use data displays in everyday life?

Linh



Ricardo



2. How are the graphs similar? How are they different?

In order to create the graphs, Linh and Ricardo determined the *frequency* of each response. A **frequency** is the number of times an item or number occurs in a data set. They then recorded the frequencies in a *frequency table*. A **frequency table** is a table used to organize data according to how many times a data value occurs. Once the frequency is known, you can determine the *mode*. The **mode** is the value or values that occur most frequently in a data set.

You can extend the frequency table to display *relative frequencies* also. A **relative frequency table** displays the ratio or percent of occurrences within a category to the total of the category. To determine the ratio of each category, set up a fraction with the part in the numerator and the whole in the denominator. Then, to determine the percent, divide the numerator by the denominator.

Sport	Frequency (f)	Relative Frequency
Football	4	$\frac{4}{30} \approx 0.133 = 13.3\%$
Softball	3	
Basketball	7	
Baseball	2	
Wrestling	1	
Gymnastics	5	
Volleyball	3	
Track	3	
Swimming	2	

Interpreting the data or drawing conclusions to describe the data distribution is Part 4 of the statistical process.

3. Complete the relative frequency column in the table.

4. Use the table and graph to answer each question. Explain your reasoning.

   a. What conclusions can you make about the most popular sport in Linh and Ricardo's class?

   b. Which category represents the mode? Explain your reasoning.

Would your conclusions be different had you changed the order of the categories?



5. Compile your class's responses to the three survey questions you asked in Question 1. Record the frequency of each response in a table.

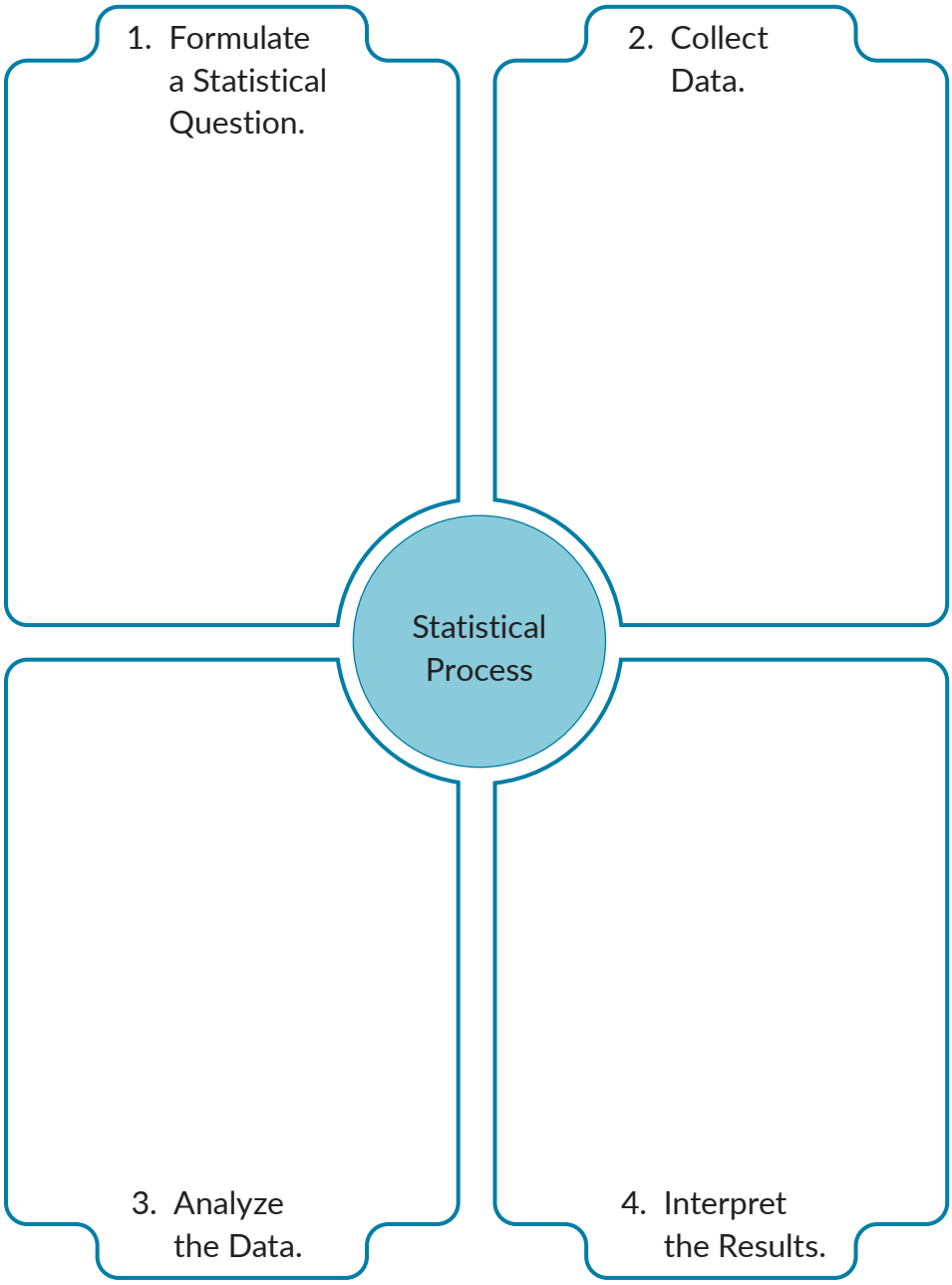
6. Create a graphical display for your assigned survey question. What conclusions can you make about your class based on your graph?

CC BY NC

Talk the Talk

(Graphically) Organizing the Process

Complete the graphic organizer for the statistical process. In each section, summarize what you know about the component and provide examples.



Survey Questions

A What is my favorite sport?	**B** How many TVs are in my house?	**C** What is the most popular favorite color in the school?
D How far do I travel to school?	**E** What grade did I earn on my last math test?	**F** How many siblings do I have?
G How many siblings do 6th-graders have?	**H** How tall are 6th-graders?	**I** How many TVs are in a U.S. household?
J What is my favorite color?	**K** How many clubs are at my school?	**L** How tall am I?
M What kinds of sports do 6th-graders prefer?	**N** How far do the students at my school travel to get to school each day?	**O** What grades were earned on the last math test?

CC BY NC

Why is this page blank?

So you can cut out the questions on the other side

Lesson 1 Assignment

Write

Match each definition to its corresponding term.

1. An entire set of items from which data can be selected
2. The information that is collected from an experiment, study, or survey
3. A question that anticipates variability
4. Imposing a condition to test a specific result
5. A method for collecting information by asking one or more questions
6. A method for collecting information by observing a phenomenon in action
7. A subset of a population
8. The value of an attribute, or quality, being studied can change from one person or thing to another
9. Data for which each piece of data fits into exactly one of several different groups or categories
10. Data that can be placed on a numerical scale and compared which can consist of discrete or continuous variables
11. A graph that shows how parts of the whole relate to the whole and how parts relate to other parts
12. A way of displaying categorical data by using either horizontal or vertical bars, so that the height or length of the bars indicates the value for that category
13. The number of times an item or number occurs in a data set
14. The observation or value that occurs the most

a. Data

b. Experiment

c. Bar graph

d. Variability

e. Statistical question

f. Categorical data

g. Circle graph

h. Survey

i. Observational study

j. Population

k. Sample

l. Frequency

m. Mode

n. Quantitative data

CC BY NC

Lesson 1 Assignment

Remember

There are four components to the statistical process:

- Formulate a statistical question.
- Collect data.
- Analyze the data using graphical displays and numerical summaries.
- Interpret the results in terms of the original statistical question and context.

Practice

1. Determine whether each given question is a statistical question. When the situation is not a statistical question, rewrite it to make it a statistical question.

   a. How many people in the class like to play video games?

   b. Is pizza your favorite food?

   c. What time do you go to bed on school nights?

Lesson 1 Assignment

2. Determine whether a survey, observational study, or experiment would be the best way to answer each given statistical question.

   a. How many of the students in your class ate breakfast this morning?

   b. Which students in your school can run a 40-meter sprint the fastest?

   c. How many students in your class can type at least 30 words per minute?

   d. How many students in your class ride the bus to school each day?

3. Determine whether each set of given data are categorical or quantitative. When the data are quantitative, determine whether they are discrete or continuous.

   a. Each student in your math class records their height.

   b. The members of the Horse Club list the types of horses they have.

CC BY NC

Lesson 1 Assignment

c. The members of the Horse Club list the numbers of horses they each have.

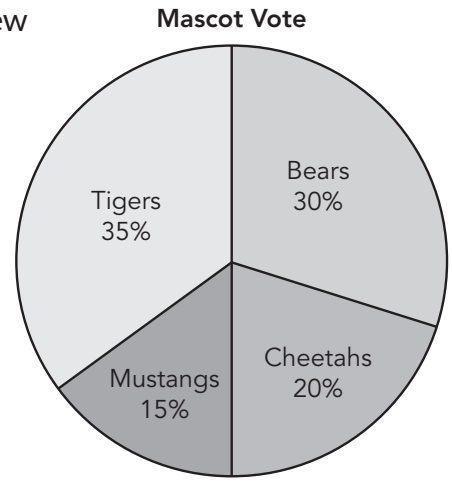
4. Lauren claims that apple juice tastes better than orange juice. Jackson claims that there is no difference between the 2 types of fruit juice. Lauren and Jackson would like to determine the answer to the following question: Do more 6th-graders prefer apple juice or orange juice?

   a. Is this a statistical question? Explain your reasoning.

   b. Explain how they can answer this question with an experiment.

Lesson 1 Assignment

5. The circle graph shows the results of the vote for the new school mascot.



   a. Suppose 400 students voted. How many students voted for the Cheetahs?

   b. Create a bar graph to display the information, in terms of frequency, of each mascot.

   c. What conclusions about the question of what mascot students want to adopt can you make based on the graphs?

CC BY NC

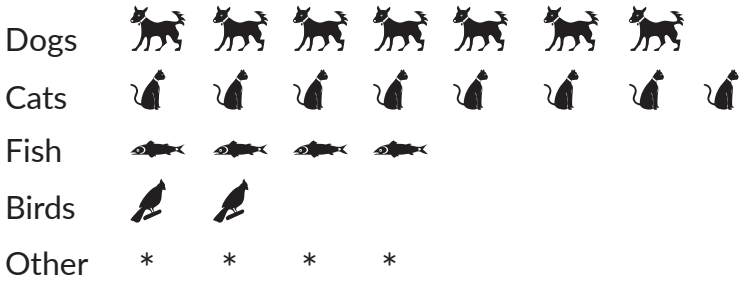
Lesson 1 Assignment

Prepare

Mr. Flores surveyed his class and asked them what types of pets they own. Analyze the pictograph that shows the results of his survey.

Pets Owned by Students in Mr. Flores's Class

(each symbol represents 1 student)



1. How many students in Mr. Flores's class own dogs?

2. How many students own fish or birds?

3. Can you tell by looking at the pictograph how many students own pets? Why or why not?

2 Analyzing Numerical Data Displays

OBJECTIVES

- Create and interpret dot plots.
- Create and interpret stem-and-leaf plots.
- Describe the center, spread, and overall shape of a data distribution.

NEW KEY TERMS

- dot plot
- distribution
- symmetric
- skewed right
- skewed left
- clusters
- gaps
- peaks
- outliers
- stem-and-leaf plot
- key

You know how to use picture graphs, bar graphs, and dot plots to display categorical and numerical data.

What additional plots can you use to display and analyze numerical data?

CC BY NC

Getting Started

Rock-Climbing Competition

Ms. Lopez poses the question, "Which grade has the fastest average rock-climbing time when each student is given one attempt?"

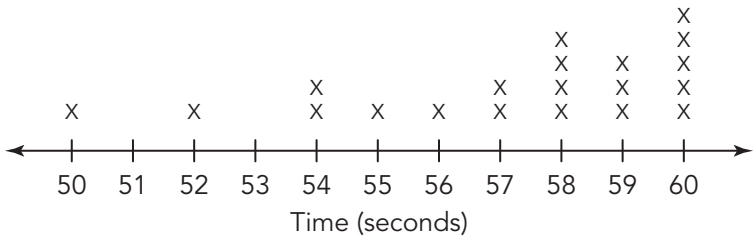
She selects one class from each grade level, times each student as they climb the rock wall, and records the times. Then, she creates data displays for each class.

Sixth-Grade Completion Times (seconds):
60, 50, 58, 59, 60, 54, 55, 58, 59, 60, 52, 54, 56, 57, 57, 58, 60, 60, 59, 58

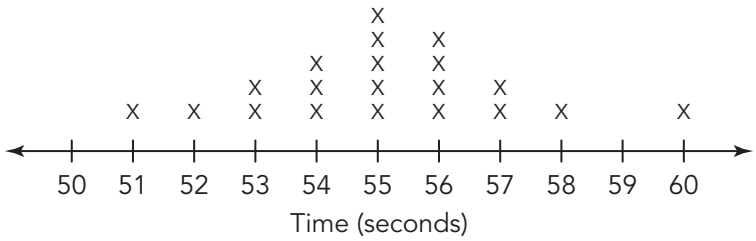
Seventh-Grade Completion Times (seconds):
51, 52, 53, 53, 54, 54, 54, 55, 55, 55, 55, 55, 56, 56, 56, 56, 57, 57, 58, 60

Eighth-Grade Completion Times (seconds):
48, 54, 53, 50, 54, 52, 48, 49, 51, 54, 53, 48, 50, 50, 49, 51, 51, 52, 53

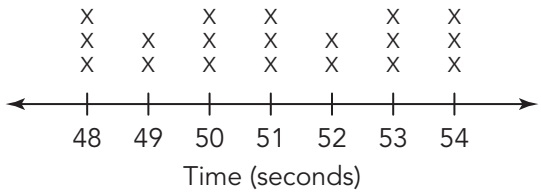
Rock-Climbing Times (6th Grade)



Rock-Climbing Times (7th Grade)



Rock-Climbing Times (8th Grade)



1. How are these data displays similar? How are they different?

2. What can you observe from a data display that you cannot see from looking at the numerical data?

ACTIVITY
2.1 Creating and Analyzing Dot Plots

The 2022 Winter Olympics were held in Beijing, China. While watching the Olympics, Nakota and Alejandra decided to pose statistical questions about the Games.

1. Nakota asked, "How many medals did the United States win? How many of those were gold?" Alejandra thought a better set of questions would be, "What is the typical number of medals won? What is the typical number of gold medals won by a country?" Who's correct? Explain your reasoning.

The table at the end of the lesson lists the number of gold medals and the total medals won by all medal-winning countries for the 2022 Winter Olympics.

2. Analyze the data shown in the table.

   a. What conclusions can you make about the numbers of total medals won at the 2022 Winter Olympics?

   b. Are the data in the table categorical or quantitative? Explain your reasoning.



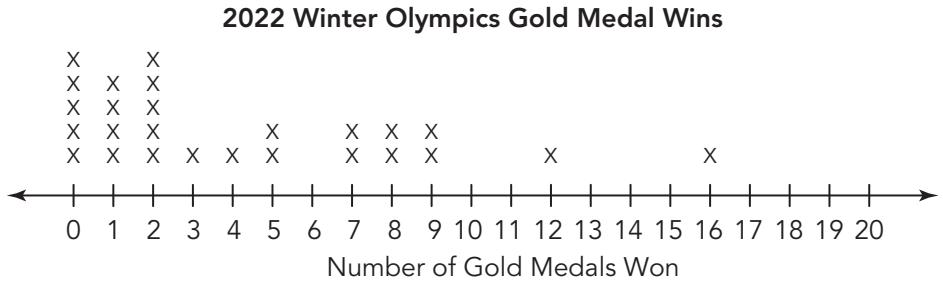
c. Are the data in the table discrete or continuous? Explain your reasoning.

One way to describe a set of quantitative data is by drawing a graphical display of the data.

A **dot plot** is a data display that shows discrete data on a number line with dots, Xs, or other symbols. Dot plots help organize and display a small number of data points.

WORKED EXAMPLE

This dot plot shows the gold-medal data. The number line represents the number of gold medals. Each X above a number represents the number of countries that won that many gold medals.



3. Use the dot plot to answer each question.

   a. What do the two Xs above the number 8 represent?

   b. What do the five Xs above the number 0 represent?

   c. Why are there no Xs above the number 6?

d. Determine the number of medal-winning countries shown in the data. Explain your strategy.

Let's create a dot plot to display the total number of medals won, as listed in the 2022 Winter Olympics data table.



4. Make a plan for creating your dot plot.

   a. What will you name your dot plot?

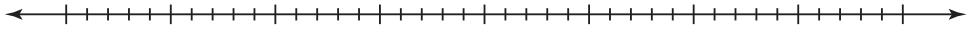
   b. What numbers will begin and end your number line? Why did you select these numbers?

   c. What interval will you use on your number line? Why did you select this interval?

CC BY NC

5. Create your dot plot displaying the data for the total medals won at the 2022 Winter Olympics.

Ask Yourself . . .
When creating your dot plot, think about what symbol you could use to represent a medal other than a dot or x



6. Write a brief summary to report the results of your data analysis back to Alejandra and Nakota to help answer their questions about gold medals and all medals won at the 2022 Winter Olympics.

ACTIVITY
2.2 Describing Distributions

When you analyze a graphical representation of numeric data, you can look at its shape, center, and spread to draw conclusions.

- What is the overall shape of the graph? Does it have any interesting patterns?
- Where is the approximate middle, or center, of the graph?
- What does the graph tell me about how spread out the data values are?

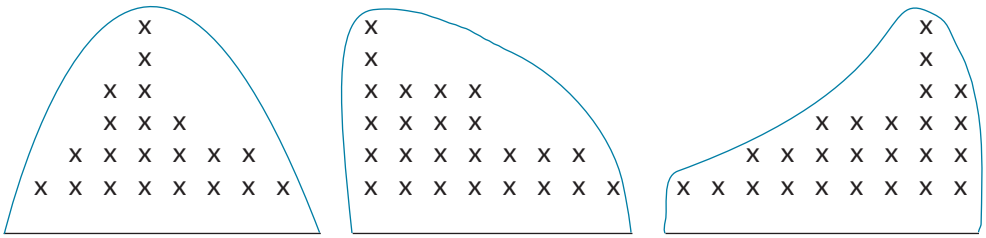
These questions are part of analyzing data.



The overall shape of a graph is called the *distribution* of data. A **distribution** is the way in which the data are spread out.

The shape of the distribution can reveal a lot of information about data. There are many different distributions, but the most common are *symmetric*, *skewed right*, and *skewed left*.

Shapes of Typical Distributions of Graphical Displays of Data



symmetric

- The left and right halves of the graph are mirror images of each other.
- The peak is in the middle because there are many data values in the center.

skewed right

- The peak of the data is to the left side of the graph.
- There are only a few data points to the right side of the graph.

skewed left

- The peak of the data is to the right side of the graph.
- There are only a few data points to the left side of the graph.

A peak is usually the value with the greatest frequency, or one of the values with the greatest frequency, and is often surrounded by data values with other large numbers of data points.



1. Hailey says that the dot plot shown in the previous activity for the number of gold medals won is skewed right. Do you agree with her statement? Explain your reasoning.

Ask Yourself . . .
Did you justify your mathematical reasoning?

Examine the dot plot you created for the total number of medals won by medal-winning countries.

2. What is the distribution of the dot plot? Explain what this means in terms of the total number of medals won.

When analyzing a graphical display of data, you can also look for any interesting patterns. Some of these patterns include:

- **clusters:** areas where data are grouped close together
- **gaps:** areas where there are no data
- **peaks:** values that contain more data points than the values on either side of it
- **outliers:** values that lie a large distance from the other data. Outliers usually accompany gaps in data.

Examine the dot plot you analyzed for the number of gold medals won by medal-winning countries.

3. Identify any clusters, gaps, peaks, or outliers. Explain what each means in terms of the number of gold medals won.

Gaps usually span multiple possible data values.

Examine the dot plot you created for the total number of medals won by medal-winning countries.

4. Identify any clusters, gaps, peaks, or outliers. Explain what each means in terms of the total number of medals won.

Another common shape for a data distribution is a uniform distribution. This occurs when each data value appears the same number of times in the set.

Refer back to the dot plots from the *Rock-Climbing Competition* activity at the beginning of the lesson.

5. Describe the shape of each dot plot, including its overall shape and any relevant patterns.

ACTIVITY
2.3 Stem-and-Leaf Plots

At the 2022 Winter Olympics, 91 countries competed in the events, but only 29 won medals. By contrast, the 2020 Summer Olympics in Tokyo, Japan, had 206 countries compete in the events. Athletes from 93 countries won medals, but only 46 won at least 5 medals.

The table at the end of the lesson lists the total number of medals won by the top-performing countries in the 2020 Summer Olympics.

1. What comparisons can you make between the number of medals won at the 2022 Winter Olympics and the number of medals won at the 2020 Summer Olympics?

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

2. Do you think using a dot plot would be a good way to organize and analyze the data in the Summer Olympics table? Explain your reasoning.

A numerical data display that can easily display data sets with a larger range of data values would be helpful to plot the 2020 Summer Olympic data. A **stem-and-leaf plot** is a graphical method used to represent ordered numerical data. Once the data is ordered, the stems and leaves are determined. Typically, the stem is all of the digits in a number except the rightmost digit, which is the leaf. However, this can vary so stem-and-leaf plots have a *key* to explain how the data is represented.

A **key** explains how each data set is represented in a data display.

A stem-and-leaf plot displaying the number of medals won in the 2020 Summer Olympics is shown.

3. Use the stem-and-leaf plot to answer each question.

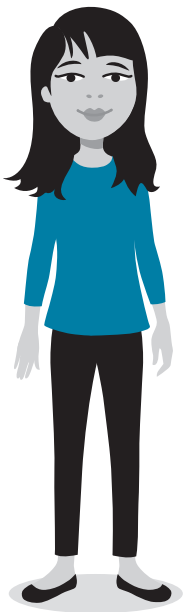
Total Medals Won by Countries
2020 Summer Olympics

0	1 2 2 3 4 4 4 4 5 5 7 7 7 7 8 8 8 9 9 9
1	0 1 1 3 3 4 5 7 9
2	0 0 0 1 4
3	3 6 7
4	0 6
5	8
6	4
7	1
8	9
9	
10	
11	3

Key: 4|6 = 46 medals won

a. Describe what you notice about the stem-and-leaf plot.

The first column in the plot represents the stems. The numbers after the vertical line are the leaves.



b. What does 7|1 mean in the stem-and-leaf plot?

c. What does 0|5 mean?

d. How many stems are in the stem-and-leaf plot?

4. Analyze the stems and leaves in the stem-and-leaf plot.

   a. How many leaves are in the stem-and-leaf plot? Why are there that many leaves?

   b. Why would a stem have more than one leaf?

   c. Why do some stems have no leaves?

   d. Why do some stems have the same leaf repeated?



e. Joey claims that he should write 0s as leaves after the stems 9 and 10 to show that there are no countries that have an amount of medals in the 90s or 100s. Is Joey correct? Explain your reasoning.

5. What is the most common number of medals won? How can you determine this from the stem-and-leaf plot?

To see the distribution more clearly, rotate the stem-and-leaf plot so the stems resemble a horizontal number line.

6. Describe the distribution and any interesting patterns you notice in the stem-and-leaf plot. Interpret your findings in terms of the number of medals won in the 2020 Summer Olympics.

CC BY NC

ACTIVITY 2.4

Creating and Analyzing Stem-and-Leaf Plots

PROBLEM SOLVING



Brianna is collecting and analyzing data on the age of the U.S. presidents when they take office. She writes the question: "At what age do presidents take office?"

1. Explain why Brianna's question is a statistical question.

To answer her question, Brianna collected data on the ages when 45 former presidents of the United States were first inaugurated. Her data are presented in the table at the end of the lesson.

To analyze the data, let's create a stem-and-leaf plot of the former presidents' ages at inauguration.

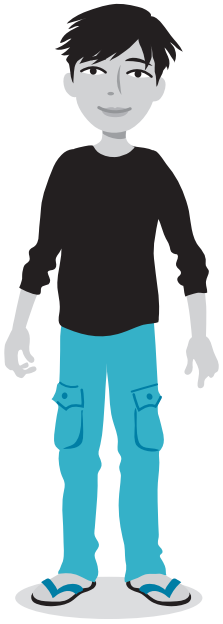
2. Make a plan for creating the stem-and-leaf plot.

   a. What will you choose for your stems? Why did you choose those numbers?

   b. How many leaves will you have in your stem-and-leaf plot? Explain your reasoning.

   c. Create a key for your stem-and-leaf plot. Why is this needed?

Put the data in ascending order, from least to greatest, before filling in the leaves.



3. Create a stem-and-leaf plot to display the age at which each president was inaugurated.

4. Describe the distribution of the ages of presidents at their inaugurations.

5. According to the Constitution, the minimum age to become president of the United States is 35 years old. How is this requirement reflected in your stem-and-leaf plot?

6. What was the most common age for presidents to be inaugurated? Explain using your stem-and-leaf plot.

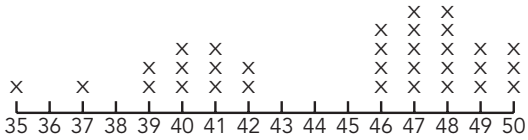
CC BY NC

Talk the Talk

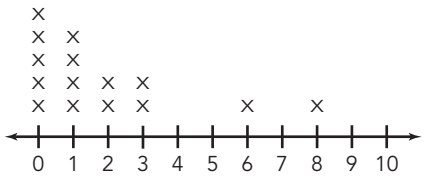
Peaks, Gaps, and Clusters . . . Oh, My!

Consider the six data displays shown.

Plot A



Plot B

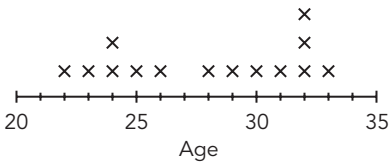


Plot C

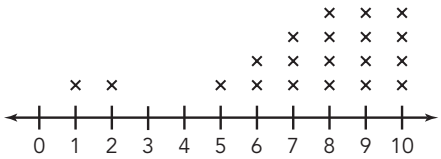
1	9
2	5 5 9
3	0 2 2 2 2 4 4 5 6 6 6 9 9 9
4	0 0 1 1 2 4 5 6 6 7 7 8 9
5	1 1 1 2 3 4 4 6 7
6	9

Key: 5|6 = 56

Plot D



Plot E



Plot F

3	2
4	
5	
6	1 4 4
7	2 5 7 9
8	0 2 2 5 6 8
9	0 0 1 2 3 3 5

Key: 3|2 = 3.2

1. Which plot or plots illustrate each graphical feature?

   a. Cluster(s) b. Gap(s) c. Outlier(s) d. Skewness

2. Select a symmetric distribution and explain how you can make it skewed left or skewed right.

Use with Activity 2.1, Creating and Analyzing Dot Plots

2022 Winter Olympics Medal Wins					
Country	Gold Medals	Total Medals	Country	Gold Medals	Total Medals
Russia	5	32	Slovenia	2	7
United States	9	25	Japan	3	18
Norway	16	37	Italy	2	17
Canada	4	26	Belarus	0	2
Netherlands	8	17	Poland	0	1
Germany	12	27	Finland	2	8
Austria	7	18	Great Britain	1	2
France	5	14	Ukraine	0	1
Sweden	8	18	Latvia	0	1
Switzerland	7	15	Australia	1	4
China	9	15	Slovakia	1	2
South Korea	2	9	New Zealand	2	3
Czech Republic	1	2	Estonia	0	1

Use with Activity 2.3, Stem-and-Leaf Plots.

Total Number of Medals Won by Top-Performing Countries, 2020 Summer Olympics			
Country	Medals Won	Country	Medals Won
United States	113	Czech Republic	11
China	89	Kenya	10
Russia	71	Sweden	9
Great Britain	64	Serbia	9
Japan	58	Jamaica	9
Australia	46	Croatia	8
Italy	40	Kazakhstan	8
Germany	37	Georgia	8
Netherlands	36	Belarus	7
France	33	Iran	7
Canada	24	Azerbaijan	7
Brazil	21	Belgium	7
South Korea	20	Uzbekistan	5
New Zealand	20	Colombia	5
Hungary	20	Ethiopia	4
Ukraine	19	Greece	4
Spain	17	Romania	4
Cuba	15	Mexico	4
Poland	14	South Africa	3
Turkey	13	Thailand	2
Switzerland	13	Malaysia	2
Denmark	11	North Macedonia	1

Use with Activity 2.4, Creating and Analyzing Stem-and-Leaf Plots.

President	Age at First Inauguration	President	Age at First Inauguration
Washington	57	Harrison, B.	55
Adams, J.	61	McKinley	54
Jefferson	57	Roosevelt, T.	42
Madison	57	Taft	51
Monroe	58	Wilson	56
Adams, J.Q.	57	Harding	55
Jackson	61	Coolidge	51
Van Buren	54	Hoover	54
Harrison, W.H.	68	Roosevelt, F.D.	51
Tyler	51	Truman	60
Polk	49	Eisenhower	62
Taylor	64	Kennedy	43
Fillmore	50	Johnson, L.B.	55
Pierce	48	Nixon	56
Buchanan	65	Ford	61
Lincoln	52	Carter	52
Johnson, A.	56	Reagan	69
Grant	46	Bush, G.H.W.	64
Hayes	54	Clinton	46
Garfield	49	Bush, G.W.	54
Arthur	50	Obama	47
Cleveland	47	Trump	70
		Biden	78

CC BY NC

Use with the Assignment.

Season	Wins	Losses	Season	Wins	Losses
2015–2016	48	34	1991–1992	38	44
2014–2015	60	22	1990–1991	43	39
2013–2014	38	44	1989–1990	41	41
2012–2013	44	38	1988–1989	52	30
2011–2012	40	26	1987–1988	50	32
2010–2011	44	38	1986–1987	57	25
2009–2010	53	29	1985–1986	50	32
2008–2009	47	35	1984–1985	34	48
2007–2008	37	45	1983–1984	40	42
2006–2007	30	52	1982–1983	43	39
2005–2006	26	56	1981–1982	42	40
2004–2005	13	69	1980–1981	31	51
2003–2004	28	54	1979–1980	50	32
2002–2003	35	47	1978–1979	46	36
2001–2002	33	49	1977–1978	41	41
2000–2001	25	57	1976–1977	31	51
1999–2000	28	54	1975–1976	29	53
1998–1999	31	19	1974–1975	31	51
1997–1998	50	32	1973–1974	35	47
1996–1997	56	26	1972–1973	46	36
1995–1996	46	36	1971–1972	36	46
1994–1995	42	40	1970–1971	36	46
1993–1994	57	25	1969–1970	48	34
1992–1993	43	39	1968–1969	48	34

Lesson 2 Assignment

Write

Write a definition for each term in your own words.

1. dot plot
2. distribution
3. symmetric
4. skewed right
5. skewed left
6. clusters
7. gaps
8. peaks
9. outliers
10. stem-and-leaf plot

Remember

Data sets have distributions that can be described according to their shape.

Dot plots are ideal for small data sets.

Stem-and-leaf plots are ideal for moderately sized data sets, especially when you need to see the actual data values.

Practice

The data table at the end of the lesson shows the number of wins and losses a basketball team has had in 48 seasons.

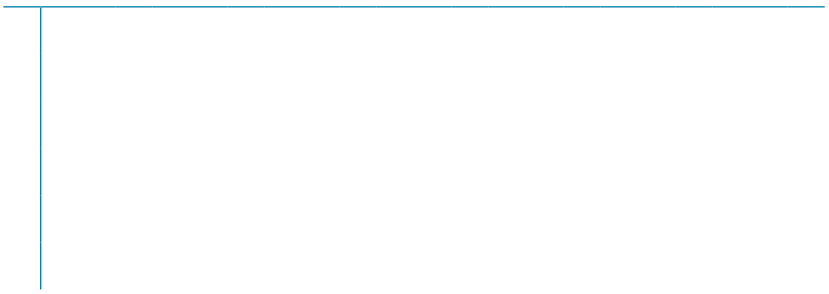
1. Create a dot plot or a stem-and-leaf plot for the number of wins by the basketball team. Be sure to name your plot and provide a key, when necessary.

CC BY NC

Lesson 2 Assignment

2. Describe the distribution of the data. Include any specific graphical features or patterns. Explain what your answer means in terms of the number of wins by the basketball team.

3. Create a dot plot or a stem-and-leaf plot for the number of losses by the basketball team. Be sure to name your plot and provide a key, when necessary.



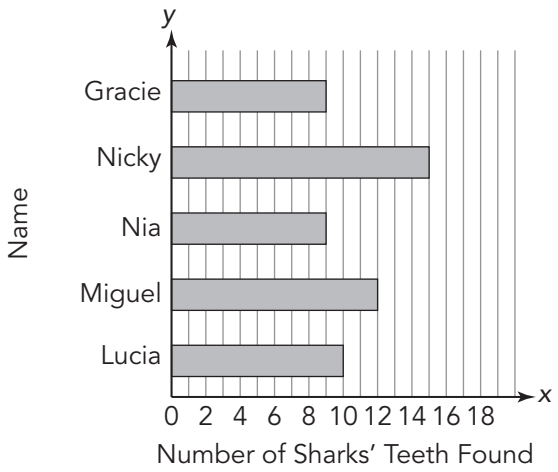
4. Describe the distribution of the data. Include any specific graphical features or patterns. Explain what your answer means in terms of the number of losses by the basketball team.

Lesson 2 Assignment

5. Propose a win–loss record for an upcoming season that would result in a change in the overall distribution of both plots.

Prepare

Use the bar graph to answer each question.



1. Who found the most sharks' teeth? How many did that person find?

2. What total number of sharks' teeth did the friends find?

CC BY NC

3 Using Histograms to Display Data

OBJECTIVES

- Display and interpret numerical data in histograms.
- Compare data displays.

NEW KEY TERM

- histogram

You have used dot plots and stem-and-leaf plots, which are good for small data sets.

How can you display data sets with a larger number of observations?

CC BY NC

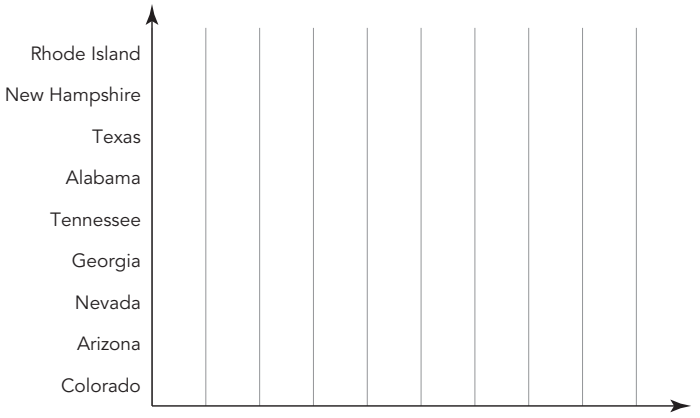
Getting Started

State Parks

There are over 6000 state parks in the United States. The table shows how many state parks there are in each of the states listed.

State	Number of Parks
Colorado	42
Arizona	34
Nevada	24
Georgia	49
Tennessee	56
Alabama	21
Texas	89
New Hampshire	93
Rhode Island	13

1. Create a bar graph using the data in the table.



2. How would the bar graph change when the states are listed in alphabetical order?



3. Suppose you wanted to graph state parks according to the region of the country. How would your bar graph be different?

The remainder of this lesson is about histograms, which look similar to bar graphs.

CC BY NC

ACTIVITY
3.1 Histograms

The graph shown is a *histogram*.

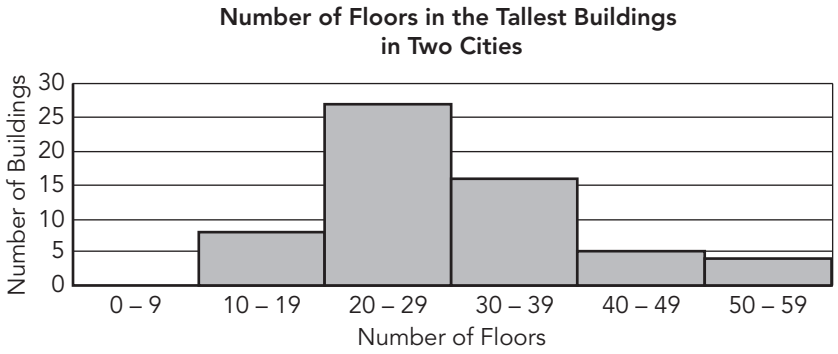
A **histogram** is a graphical way to display quantitative or numerical data using vertical bars.

The width of a bar in a histogram represents an interval of data and is often referred to as a *bin* or range.

The height of the bar indicates the frequency, or the number of data values included in any given bin.

Consider two other cities that are both home to flourishing downtowns with tall buildings.

1. Look at the graph below.



   a. How is this graph different from the bar graphs you have used previously?

   b. What information does the histogram display? Describe the data represented in the histogram shown. Look at the title and the labels on the axes.

   c. Are the data represented in the histogram discrete or continuous? Explain your reasoning.

2. Describe the distribution of the data in terms of the overall shape and the existence of peaks, clusters, and gaps.

The first vertical bar in the histogram represents 8 buildings that have at least 10 floors, but fewer than 20 floors.

3. Let's think about how the bars are displayed in the histogram.

   a. How many ranges are shown?

   b. Are all the ranges the same size?

   c. What does the height of each bar represent?

What statistical question might have led to the collection of this data?



4. Describe the number of floors included in each of the remaining ranges shown on the horizontal axis.

   a. 2nd range: interval 20–29

   b. 3rd range: interval 30–39

   c. 4th range: interval 40–49

   d. 5th range: interval 50–59

In the second range, the numbers 20 and 29 are called the *bounds* of the range. What are the bounds of the 5th range?



5. When a new building is constructed with 20 floors, which range would change? How would it change?

6. Juliana says, "There are 5 buildings represented in the histogram since there are 5 bars." Do you agree or disagree with Juliana's statement? When you do not agree with Juliana, estimate how many buildings are represented in the histogram.

Dot plots show individual data values. Histograms display grouped data.

7. Can you determine how many buildings have 31 floors? Explain your reasoning.

The *range* is the difference between the greatest and the least data value.



8. Is it possible to determine the number of buildings that have more than 35 floors from the histogram? Why or why not?

9. Is it possible to determine the range of the data set from the histogram? Why or why not?

To create a histogram, you will usually organize into a frequency table. The frequency table for the number of floors in the two cities' 60 tallest buildings is shown.

10. Complete the frequency table for the number of floors in the two cities' tallest buildings.

Floor Intervals	Frequency (f)
10–19	
20–29	
30–39	
40–49	
50–59	

Remember that a *frequency table* is a table used to organize data according to how many times a data value occurs.

11. Compare the histogram to the frequency table.

Remember, you can also extend the frequency table to display relative frequencies.

12. The relative frequency for the 10–19 floor interval is $\frac{8}{60} \approx 0.133$, which means about 13.3% of the tallest buildings in the two cities have at least 10 floors but less than 20 floors. Complete the relative frequencies for each floor interval. Round decimals to the nearest thousandth.

Floor Intervals	Frequency (f)	Relative Frequency
10–19	8	$\frac{8}{60} \approx 0.133 = 13.3\%$
20–29	27	
30–39	16	
40–49	5	
50–59	4	

13. Write a brief summary to report the results of your data analysis about the number of floors in the two cities' tallest buildings.

ACTIVITY
3.2 Creating and Analyzing Histograms

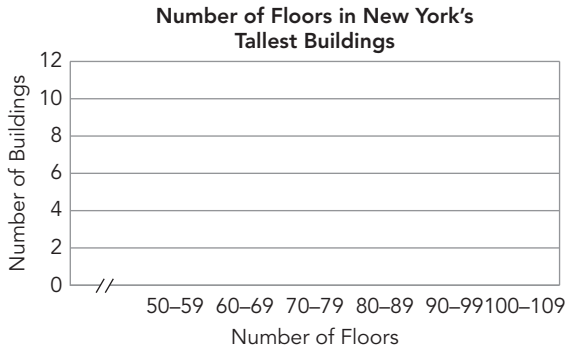
New York City has over 7000 tall buildings and is home to the ninth tallest building in the United States, the Empire State Building, which is 381 meters in height. Not to be outdone, Chicago is home to the Willis Tower, formerly known as Sears Tower. It stands an impressive 442 meters in height. So how do these big cities stack up to each other? Are there any similarities or differences in the number of floors each city's 20 tallest buildings have?

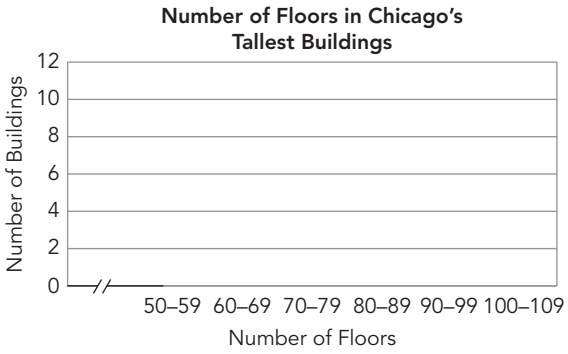
The tables listing each city's 20 tallest buildings are provided at the end of the lesson. Use the tables to create frequency tables and histograms for each city's tallest buildings.

1. Complete the frequency tables for the number of floors in each city's 20 tallest buildings. Then, complete the histograms. Make sure that you name your tables and histograms.

Number of Floors	Frequency (f)

Number of Floors	Frequency (f)





Remember . . .
Similar to the breaks on the axes of coordinate planes to best display points and graphs, histograms may have breaks in their axes to better display the data.

Ask Yourself . . .
How can you organize and record your mathematical ideas?

2. What is similar about the two histograms? What are the differences between the two histograms?

3. Use what you know about the distributions and patterns of a graphical display to describe what the histograms say about the number of floors in each city's 20 tallest buildings.

ACTIVITY
3.3 Creating a Histogram with Continuous Data

Each year, the Empire State Building Run-Up (ESBRU) challenges runners to race up its stairs. You surveyed runners about their times at the end of the Run-Up. The results are shown in the table.

Amount of Time to Complete the ESBRU (minutes)					
10.4	11.25	15.76	9.81	12.05	18.2
10.52	13.73	13.01	12.75	14.99	11.24
15.0	15.57	16.6	14.8	13.35	12.22

1. Is the data in the table discrete or continuous?
   Explain your reasoning.

Xavier and Ben decide to make a histogram for the data set.
The ranges they each want to use for the histogram are shown.

Ben

9–10

11–12

13–14

15–16

17–18

Xavier

9–10.9

11–12.9

13–14.9

15–16.9

17–18.9

CC BY NC

2. Explain why both Ben's and Xavier's ranges are incorrect. Use a data value from the table to explain.

3. Give an example of the ranges Ben and Xavier can use to include all the data.

Remember, if a data value lies on one of the bounds, it should go in the range to the right of that bound.

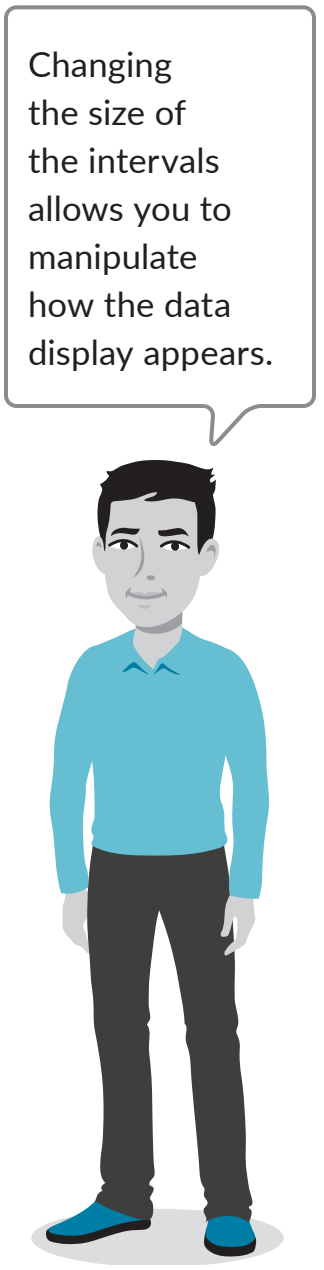
4. Create a frequency table and histogram for the amount of time to complete the ESBRU.



5. Create a second frequency table and histogram for the amount of time to complete the ESBRU. Use a different range width than you used in Question 3. What do you notice?







CC BY NC

6. What conclusions can you make about the amount of time it takes to complete the Empire State Building Run-Up? Use what you know about distributions and patterns of graphical displays.

Talk the Talk

Which Plot Is Best?

Throughout this topic, you have created and analyzed a variety of numerical data displays.

1. List at least one advantage and one disadvantage of using each type of plot to display numerical data.

	Advantage (or Use)	Disadvantage (or Limitation)
Dot plot		
Stem-and-leaf plot		
Histogram		

CC BY NC

Use with Activity 3.2, Creating and Analyzing Histograms.

New York City's 20 Tallest Buildings	
Name of Building	Number of Floors
One World Trade Center,	104
Central Park Tower,	99
111 West 57th,	85
One Vanderbilt,	73
432 Park Avenue,	85
270 Park Avenue,	60
30 Hudson Yards,	73
Empire State Building,	103
Bank of America Tower,	55
3 World Trade Center,	80
The Brooklyn Tower,	74
53W53,	77
Chrysler Building,	77
The New York Times Building,	52
The Spiral,	66
One57,	75
35 Hudson Yards,	72
One Manhattan West,	67
50 Hudson Yards,	58
4 World Trade Center,	72

Chicago's 20 Tallest Buildings	
Name of Building	Number of Floors
Willis Tower,	108
Trump International Hotel and Tower,	98
St. Regis,	101
Aon Center,	83
875 North Michigan Ave.,	100
Franklin Center,	61
Two Prudential Plaza,	64
One Chicago East Tower,	78
311 South Wacker Drive,	65
NEMA,	76
900 North Michigan,	66
Aqua,	82
Water Tower Place,	74
Chase Tower,	60
Park Tower,	67
One Bennett Park,	69
Salesforce Tower,	60
The Legacy at Millennium Park,	73
110 North Wacker,	51
1000M,	73

Lesson 3 Assignment

Write

Write a definition for each term in your own words.

1. *histogram*

2. *relative frequency table*

Remember

Histograms are used when the data are numerical. Numerical data can be represented continuously in intervals.

The intervals in a histogram must all be the same size. The width of the bar represents the interval. The height of the bar indicates the frequency of values in the interval.

Practice

Antonio's scores for the first 20 times he played a card game are listed.

50, 199, 246, 356, 89, 210, 391, 325, 273, 260, 100, 172, 123, 167, 194, 172, 23, 426, 75, 239

1. Create a frequency table and a histogram to display Jeremy's scores. Be sure to name your histogram.

CC BY NC

Lesson 3 Assignment

2. Add a column for relative frequency to the table that you created in Question 1.

3. Describe the distribution of the data. Include any specific graphical features or patterns. Explain what your answer means in terms of Antonio's scores.

Lesson 3 Assignment

4. Create a second frequency table and histogram to provide a different view of the data distribution.

CC BY NC

Lesson 3 Assignment

Prepare

Simplify each numeric expression.

1. $(13 + 17) \div 2$

2. $(29 + 36) \div 2$

3. $(48 + 9) \div 2$

4. $(27 + 31) \div 2$

TOPIC 1 SELF-REFLECTION

Name: _____

The Statistical Process

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill.**

I can demonstrate an understanding of the standards in *The Statistical Process* topic by:

TOPIC 1: *The Statistical Process*	Beginning of Topic	Middle of Topic	End of Topic
identifying the four components of the statistical process.	○	○	○
explaining what it means to collect data with and without variability.	○	○	○
recognizing and generating statistical questions.	○	○	○
describing the data being collected, including how it was measured and its units of measurement.	○	○	○
describing the shape, center, and spread of the distribution of a data set.	○	○	○
representing numerical data graphically with dot plots, stem-and-leaf plots, and histograms.	○	○	○
interpreting numerical data summarized in dot plots, stem-and-leaf plots, and histograms.	○	○	○
summarizing numerical data sets with respect to their contexts.	○	○	○

continued on the next page

CC BY NC

TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *The Statistical Process*	Beginning of Topic	Middle of Topic	End of Topic
summarizing categorical data sets with numerical and graphical summaries, including mode and relative frequency.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in *The Statistical Process* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?



TOPIC 1 SUMMARY

The Statistical Process Summary

LESSON 1 Understanding the Statistical Process

Statistical problem solving begins with a *statistical question*. A **statistical question** is a question that anticipates an answer based on *data* that vary. **Data** are categories, numbers, or observations gathered in response to a statistical question. Statistics is a problem-solving process because it is about determining a possible answer to a question that has *variability*. In statistics, **variability** means that the value of the attribute being studied can change from one person or thing to another.

The **statistical process** has four components:

1. Formulate a statistical question.

   The statistical question posed should result in data with variability.
   Example (data with variability): How many members do the clubs at my school have?
   Non-example (data without variability): How many students are in the Chess Club?

2. Collect appropriate data.

   Two types of variable data that can be collected are *categorical data* and *quantitative data*. **Categorical data**, or *qualitative data*, are data for which each piece of data fits into exactly one of several different groups or categories. **Quantitative data**, or *numerical data*, are data for which each piece of data can be placed on a numerical scale and compared.

   There are two types of quantitative data: *discrete data* and *continuous data*. **Discrete data** are quantitative data that can only take certain values. The data are counted not measured. Continuous data are quantitative data that take any value. The data are measured not counted.

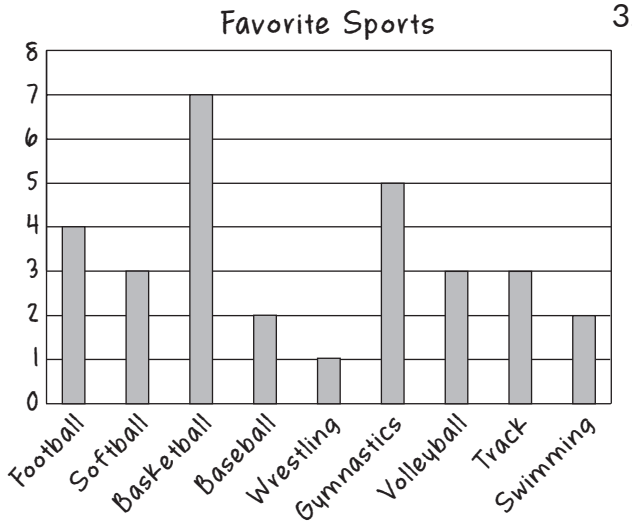
   A statistical question can be answered by collecting data from an entire population or, more commonly, from a *sample* of the *population*. A **population** is an entire set of items from which data are collected. A **sample** is a selection from a population.

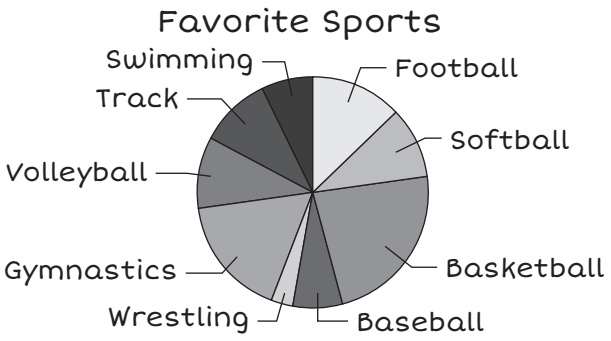
NEW KEY TERMS

- statistical question
- data [datos]
- variability [variabilidad]
- statistical process [proceso estadístico]
- categorical data [datos categóricos]
- quantitative data [datos cuantitativos]
- discrete data [dato discreta/discontinua]
- continuous data [dato continua]
- population [población]
- sample
- survey
- observational study [estudio observacional]
- experiment [experimento]
- bar graph [gráfica de barras]
- frequency [frecuencia]
- frequency table [tabla de frecuencia]
- mode [moda]
- relative frequency table [tabla de frecuencia relativa]
- dot plot [diagrama de puntos]
- distribution [distribución]
- symmetric [simétrico]
- skewed right
- skewed left
- clusters
- gaps
- peaks [picos]
- outliers
- stem-and-leaf plot
- key
- histogram [histograma]

CC BY NC

Three common methods of data collection are *surveys*, *observational studies*, and *experiments*. In a **survey**, people are asked one or more questions. Similarly, in an **observational study**, the researcher collects data by observing the variable of interest. In an **experiment**, the researcher imposes a condition and observes the results.





3. Analyze the data graphically and numerically.

After you collect the data, it is time to analyze and interpret the results. Analysis includes selecting the most appropriate graphical display and numerical summaries for your question and your method of data collection.

A **bar graph** displays categorical data using horizontal or vertical bars on a graph. The height or length of each bar indicates the value for that category.

A *circle graph*, often called a *pie chart*, displays categorical data using sectors, or "wedges," of a circle. It shows how parts of the whole relate to the whole and how parts of the whole relate to the other parts. The area of each sector corresponds to the percentage of the part with respect to the whole.

To create graphs, you can determine the *frequency* of each response to a statistical question. A **frequency** is the number of times an item or number occurs in a data set. You can record the frequencies in a **frequency table**, which is a table used to organize data according to how many times data values within a given range of values occur. Once the frequency is known, you can determine the *mode*. The **mode** is the value or values that occur most frequently in a data set. You can extend the frequency table to display relative frequencies. A **relative frequency table** displays the ratio or percent of occurrences within a category to the total of the category. To determine the ratio of each category, set up a fraction with the part in the numerator and the whole in the denominator. Then, to determine the percent, divide the numerator by the denominator.

For example, in the table shown, football has a relative frequency of $\frac{4}{30} \approx 0.133 \approx 13.3\%$.

4. Interpret the results of the analysis.

   You can use your analysis to make conclusions about the data. For example, from the graphs and table on the previous page, you can conclude that basketball is the most popular sport among the population surveyed.

Sport	Frequency (f)
Football	4
Softball	3
Basketball	7
Baseball	2
Wrestling	1
Gymnastics	5
Volleyball	3
Track	3
Swimming	2

LESSON 2

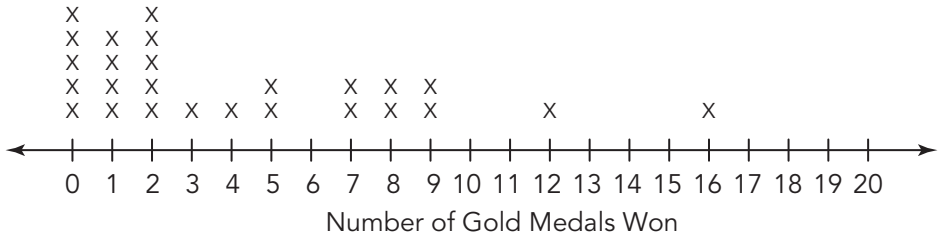
Analyzing Numerical Data Displays

One way to describe a set of quantitative data is by drawing a graphical display of the data.

A **dot plot** is a data display that shows discrete data on a number line with dots, Xs, or other symbols. Dot plots help organize and display a small number of data points.

In this example of a dot plot, the number line represents the number of gold medals won by countries in the 2022 Winter Olympics. Each X above a number represents a country that won that many gold medals.

2022 Winter Olympics Gold Medal Wins



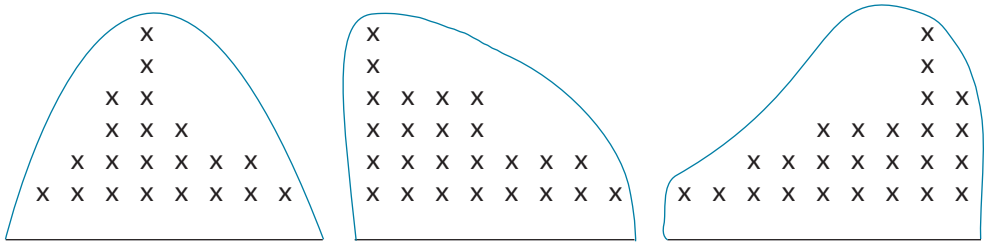
When you analyze a graphical representation of numeric data, you can look at its shape, center, and spread to draw conclusions.

The overall shape of a graph is called the distribution of data. A **distribution** is the way in which the data are spread out. The shape of the distribution can reveal a lot of information about data. There are many different distributions, but the most common are **symmetric**, **skewed right**, and **skewed left**.

When analyzing a graphical display of data, you can also look for any interesting patterns. Some of these patterns include:

- **clusters:** areas where data are grouped close together
- **gaps:** areas where there are no data
- **peaks:** values that contain more data points than the values on either side of them
- **outliers:** values that lie a large distance from the other data; outliers usually accompany gaps in data.

Shapes of Typical Distribution of Graphical Displays of Data



symmetric

- The left and right halves of the graph are mirror images of each other.
- The peak is in the middle because there are many data values in the center.

skewed right

- The peak of the data is to the left side of the graph.
- There are only a few data points to the right side of the graph.

skewed left

- The peak of the data is to the right side of the graph.
- There are only a few data points to the left side of the graph.

A **stem-and-leaf plot** is a graphical method used to represent ordered numerical data sets with a larger range of data values. Once the data is ordered, the stem and leaves are determined. Typically, the stem is all the digits in a number except the rightmost digit, which is the leaf.

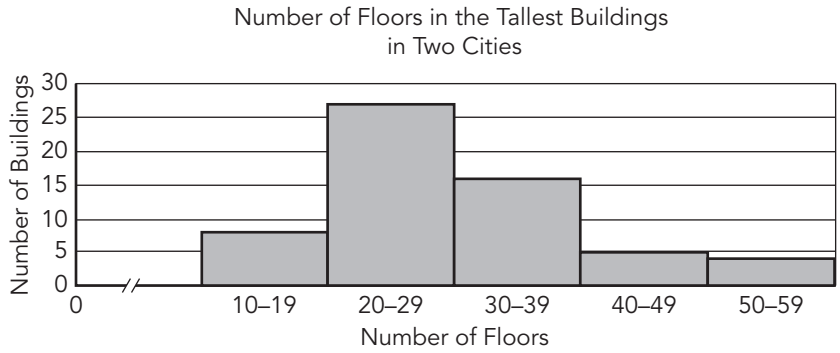
Total Medals Won by Countries
2020 Summer Olympics

0	1 2 2 3 4 4 4 4 5 5 7 7 7 7 8 8 8 9 9 9
1	0 1 1 3 3 4 5 7 9
2	0 0 0 1 4
3	3 6 7
4	0 6
5	8
6	4
7	1
8	9
9	
10	
11	3

Key: 4|6 = 46 medals won

LESSON 3 Using Histograms to Display Data

A **histogram** is a graphical way to display quantitative or numerical data using vertical bars. The numerical data are represented continuously with intervals. The intervals in a histogram must all be the same size. The width of a bar in a histogram represents the interval of data, and is often referred to as a *range*. The height of the bar indicates the frequency, or the number of data values, in the interval or range.



Dot plots show individual data are values. Histograms display grouped data. For example, you cannot determine from the histogram how many buildings have exactly 21 floors or more than 45 floors.

To create a histogram, data are usually organized into a frequency table.

For example, this frequency table displays the data represented by the histogram above. The relative frequency of the number of floors in the tallest building in the two cities having between 10–19 floors can be determined.

$\frac{8}{60} \approx 0.133 = 13.3\%$

About 13.3% of the tallest buildings in the two cities have between 10–19 floors.

Floor Intervals	Frequency (f)
10–20	8
20–30	27
30–40	16
40–50	5
50–60	4

CC BY NC

GRADE 6

TOPIC 2



The size of vertebrates varies widely, from blue whales at the large end to tiny frogs at the small end.

Numerical Summaries of Data

LESSON 1 Analyzing Data Using Measures of Center **1157**

LESSON 2 Displaying the Five-Number Summary **1177**

LESSON 3 Collecting, Displaying, and Analyzing Data **1199**

CC BY NC

1 Analyzing Data Using Measures of Center

OBJECTIVES

- Define the three measures of center: *mode*, *median*, and *mean*.
- Recognize that a measure of center for a numerical data set is a single value that summarizes all of its values.
- Give quantitative measures of center for a data set, including mean and/or median, and interpret the mode, median, and mean for a data set.

NEW KEY TERMS

- measure of center
- mode
- median
- balance point
- mean

You have analyzed, created, and interpreted data displays, such as dot plots, stem-and-leaf plots, and histograms. You have described shapes and patterns in distributions of data displays.

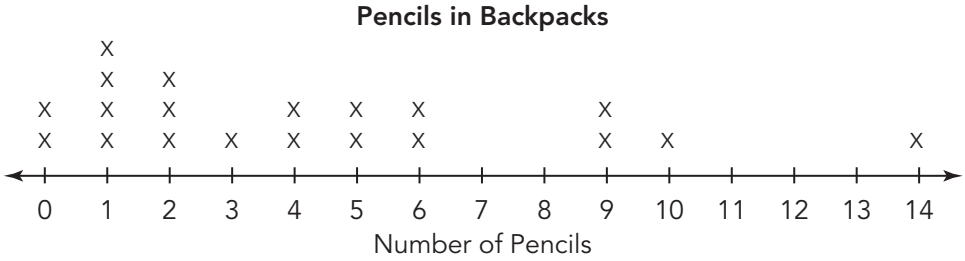
How can you describe a numerical data set as a single value?

Getting Started

Describing Data

Analyze each display. Identify the most typical value and estimate the middle value in each.

1.

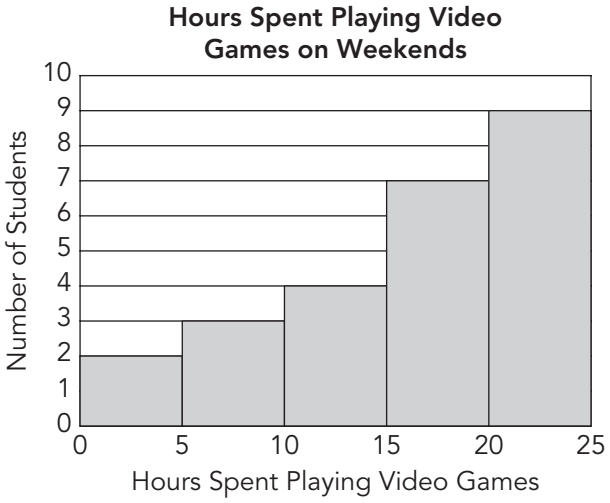


2. **Ages of U.S. First Ladies (20th Century)**

Stem	Leaf
3	1
4	0 3 4 5 7 8 9
5	0 2 4 6 6 6 9
6	0 0 3



3.



ACTIVITY
1.1 Measures of Center

When you analyze a set of data, you often want to describe it numerically. One way to numerically describe a data set is to use a *measure of center*.

There are three measures that describe how a data set is centered: the *mean*, the *median*, and the *mode*.

The **mode** is the data value or values that occur most frequently in a data set.

The **median** is the middle number in a data set when the values are placed in order from least to greatest or greatest to least. When a data set has an odd number of data values, you can determine which number is exactly in the middle of the data set. When there is an even number of data values, the median is calculated by adding the two middle numbers and dividing by 2.

A **measure of center** tells you how the data values are clustered, or where the "center" of a graph of the data is located.

A data set can have more than one mode.

1. The local middle school basketball team has a chance to be in the league playoffs. Coach Flores must determine which of the following three players should get more playing time in the first playoff game. In the past six games, Catalina scored 12, 12, 6, 26, 4, and 12 points. Linh scored 3, 2, 8, 17, 10, and 20 points. Lauren scored 15, 12, 13, 10, 8, and 14 points.

   a. Determine the mode for the number of points that each player scored.

   b. Determine the median number of points that each player scored.

c. Which of these two measures of center, mode or median, would be better for Coach Flores to use in making her decision? Explain your reasoning.

James says that the median is 10 for the data set 5, 6, 10, 4, and 9.



2. Explain what James did incorrectly to determine that the median was 10. Then, determine the correct median.

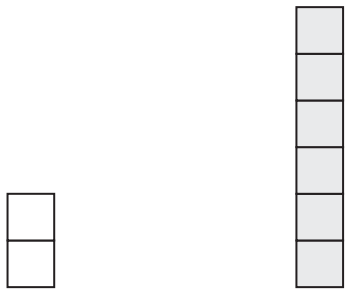
ACTIVITY 1.2

A Third Measure: Mean as Fair Share

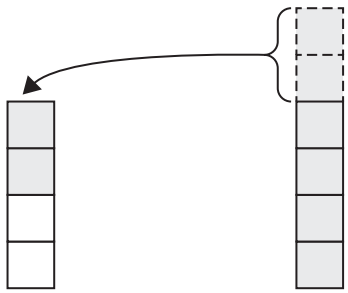
There is a third measure of center that can describe the values in a data set. This measure of center is called the *mean* and is based on leveling off or creating fair shares.

WORKED EXAMPLE

Analyze the two stacks of cubes.

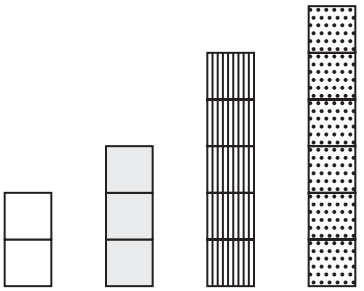


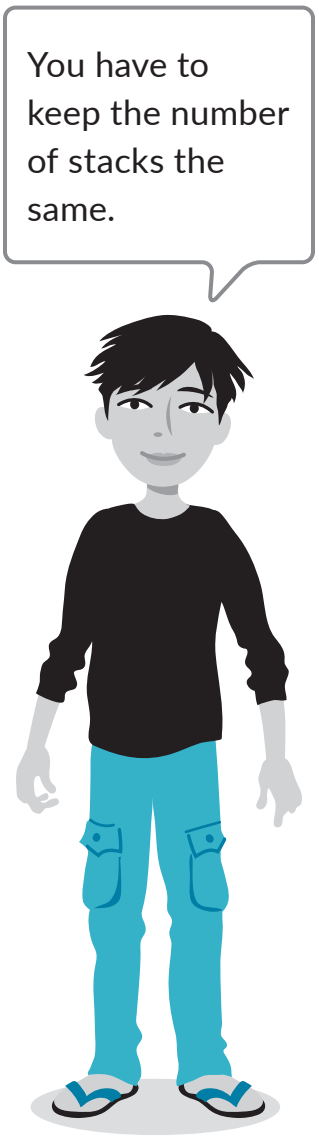
Supposing that you were to create two equal stacks of cubes, you would subtract two cubes from the greater stack and add the two cubes to the lesser stack. In doing so you have created two equal stacks.



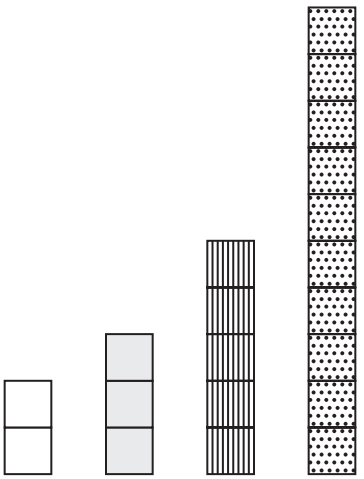
1. Analyze each stack of cubes shown. Create four equal stacks of cubes. Record what operations you performed.

   a. 2, 3, 5, 6





   b. 2, 3, 5, 10



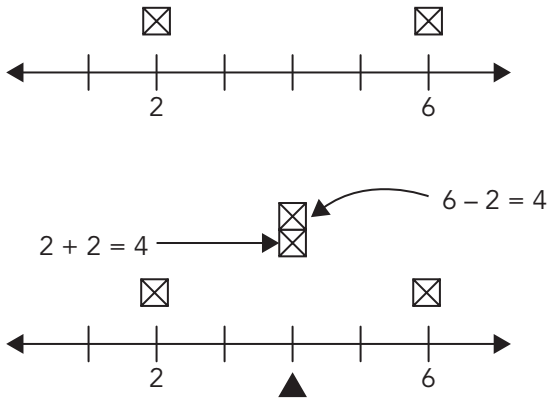
2. Compare your results from parts (a) and (b). How do the number of cubes in each equal stack in part (b) compare to the number of cubes in each equal stack in part (a)? Explain why this happened.

ACTIVITY
1.3 A Third Measure: Mean as Balancing

In the previous activity, data values were represented by stacks of cubes. You rearranged the stacks to create equal stacks, or fair shares. You can also represent quantities on a number line and create a *balance point*.

WORKED EXAMPLE

Consider the data set: 2, 6.



The value 2 was moved to the right from 2 to 4. To maintain balance, 6 was moved 2 to the left from 6 to 4. The balance point is 4.

When you are attempting to create a balance on a number line, when you move a value to the right a certain amount, you must also move a value to the left the same amount. You can move a data value to the left and right as much as you like, as long as you do the opposite to another data value. You can start however you like.

1. What do you think the ▲ represents?

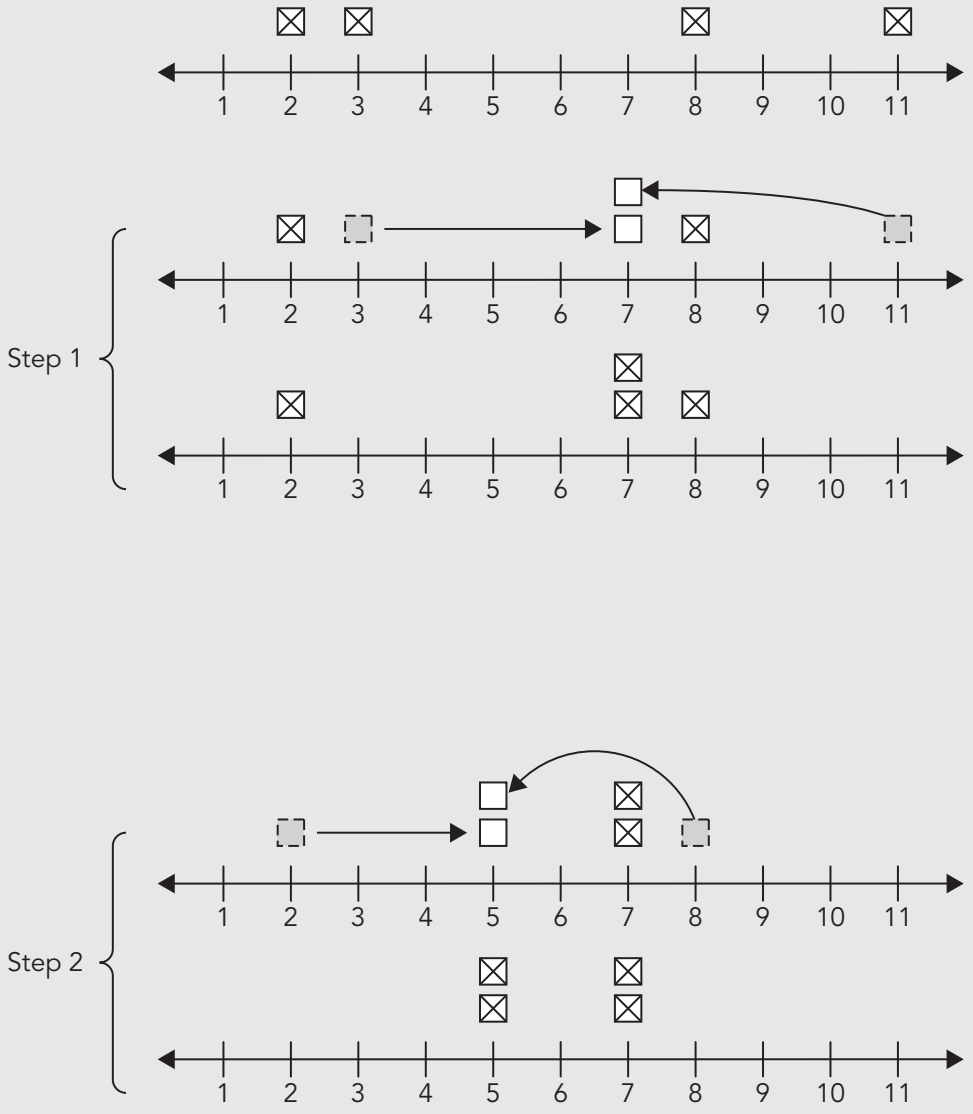
When you have all the points at the same value, the number line is balanced. The value where the number line is balanced is called the **balance point**.

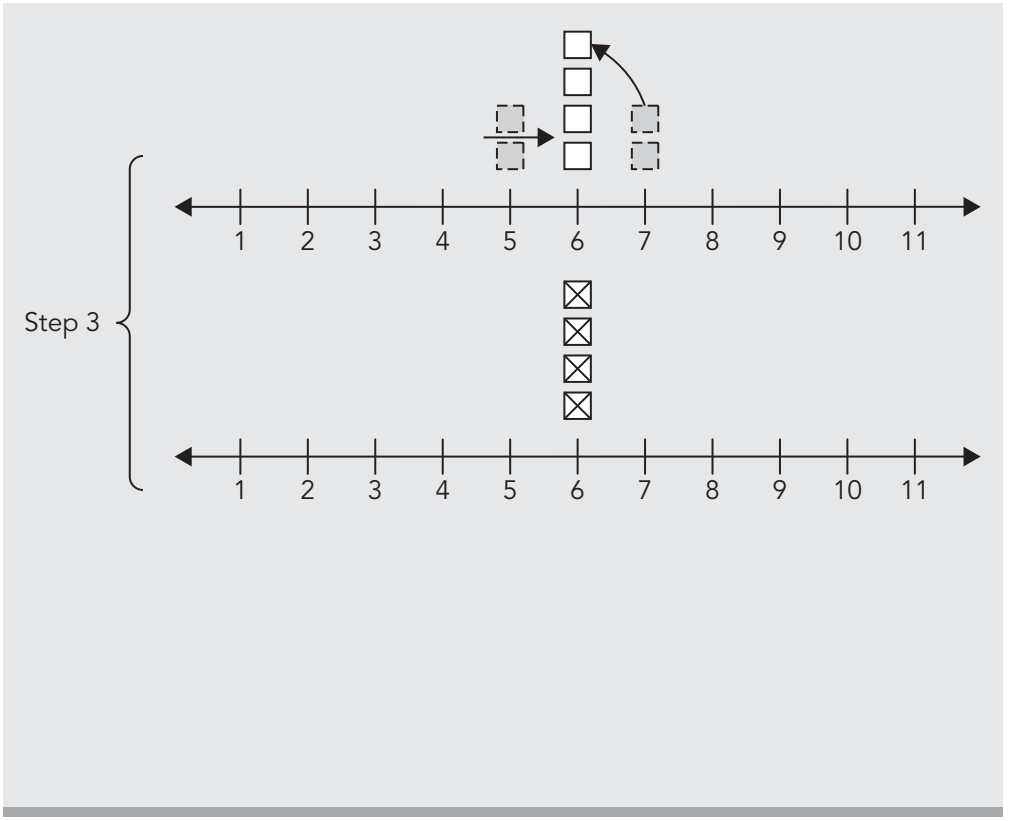
You can also determine the balance point of a number line with more than two data points.

2. Nakota determined the balance point of the data set. Record the operations she used on the blank lines in each step. Label the balance point in Step 3.

Nakota's Method

Data set: 2, 3, 8, 11.

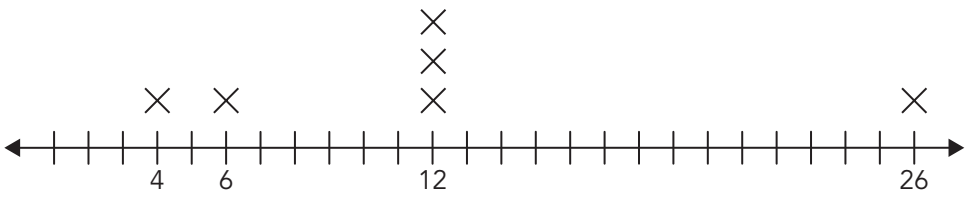




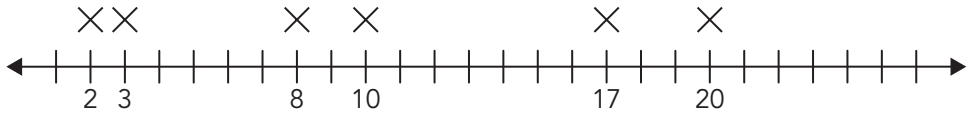
Recall the data sets for the number of points each player scored for the local middle school basketball team.

3. For each data set, determine the balance point on the number lines shown. Record the steps you used to determine the balance point.

   a. Catalina
   Data set: 12, 12, 6, 26, 4, 12



b. Linh
Data set: 3, 2, 8, 17, 10, 20



The balance point can also be called the *mean*. The **mean** is the arithmetic average of the numbers in a data set.

WORKED EXAMPLE

The mean is calculated by adding all of the values in the data set and dividing the sum by the number of values.

The mean of the data set for the points Catalina scored is calculated by:

Step 1: $12 + 12 + 6 + 26 + 4 + 12 = 72$

Step 2: $\frac{72}{6} = 12$

You can verify that the mean is 12 because the balance point of the data set is 12.

The expression $72 \div 6$ takes the 72 total points and divides them evenly into 6 games, for 12 points in each game.

4. Calculate the mean number of points scored by each player.

a. Linh
Data set: 3, 2, 8, 17, 10, 20

b. Lauren
Data set: 15, 12, 13, 10, 8, 14

ACTIVITY
1.4 In the Middle

A corporation is awarding grants to local schools to purchase fitness equipment. The principal at a local middle school would like to submit an application for the grant. When awarded the money, the school would like to add fitness equipment to the gym.

Before she submits the application, the principal wants to understand how much time the students at the middle school spend exercising each weekday. She decides to give a survey to 15 anonymous students. The results are shown.

0 minutes	40 minutes	60 minutes	30 minutes	60 minutes
10 minutes	45 minutes	30 minutes	150 minutes	90 minutes
30 minutes	120 minutes	60 minutes	0 minutes	20 minutes

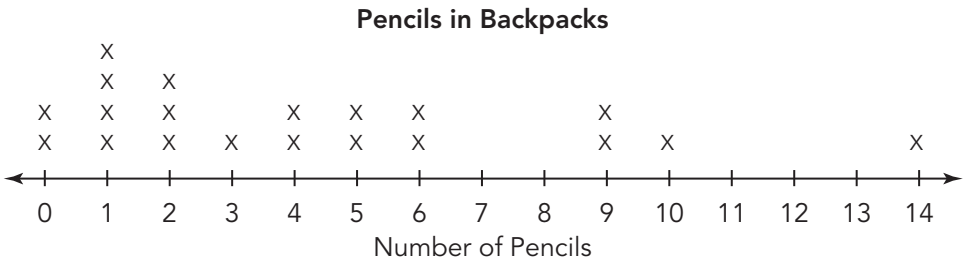
1. Identify the statistical question posed in this situation.
   Create a display from the survey data. Calculate and interpret each measure of center. Then, write a summary statement based on your findings.

Statistical Process
- Formulate Question
- Collect Data
- Graph and Analyze
- Interpret

Ask Yourself . . .
How can you organize and record your mathematical ideas?

Let's further analyze the data displays from the Getting Started.

2. Pencils in Backpacks



a. How many backpacks were used to collect the data?

b. Describe the distribution of the data.

c. What is the mode of the data? Interpret the mode in terms of the problem situation.

3. Ages of U.S. Female Astronauts

In 1992, Astronaut Mae Jemison became the first African-American woman to travel to space.



Ages of U.S. Female Astronauts (data as of March 2024)

Stem	Leaf
3	6 6 7
4	1 5 5 7
5	2 7 8 8
6	1 4 7 7
7	2 2 4

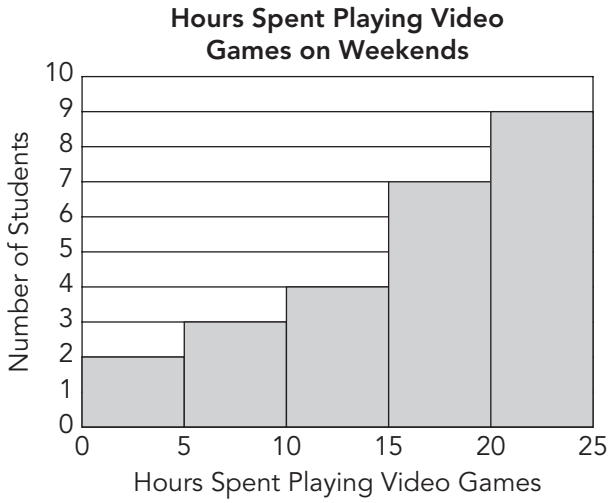
Key: 3|6 means 36 years.

a. How many female astronauts were between the ages of 40 and 59?

b. Describe the distribution of the data.

c. What is the median of the data? Interpret the median in terms of the problem situation.

4. Hours Spent Playing Video Games on Weekends



a. How many students spend between 5 and 19 hours playing video games?

b. Describe the distribution of the data.

c. How does the mean of the data compare to the mode of the data?

Talk the Talk

Describing a Numerical Data Set as a Single Value

In this lesson you learned about three measures of center: mode, median, and mean.

1. Describe how you can use each measure of center to describe a data set.

2. What are the most important differences between each of the measures of center?

3. Which is your favorite measure of center? Why?

Lesson 1 Assignment

Write

Choose a word to best complete each sentence.

measure of center	mode	median
mean	balance point	

1. A _____ for a numerical data set summarizes all of its values with a single number.

2. The _____ is the arithmetic average of the numbers in a data set.

3. When you have all the points on a number line at the same value after moving data values, this value is called the _____.

4. The _____ is the middle number in a data set when the values are placed in order from least to greatest.

5. The _____ is the data value or values that occur most frequently in a data set.

Remember

There are three measures of center: mode, median, and mean. Measures of center are numerical ways of determining where the center of data is located.

Practice

1. Determine the mode and median for each data set. What does each measure of center tell you about the data set?

   a. The heights of each of your classmates in inches are 62, 58, 67, 68, 68, 72, 66, 65, 60, 61, 64, 67, and 64.

CC BY NC

Lesson 1 Assignment

b. Alejandra made golf putts from distances of 7 feet, 15 feet, 8 feet, 9.5 feet, and 11 feet from the hole.

c. Everyone in your class reaches into their pockets to see how much change they have. The amounts in cents are 15, 48, 92, 72, 50, 75, 70, 18, 85, 95, 42, 25, 63, 59, 87, 13, 55, 75, 99, and 25.

2. Ms. Lopez's math class has had 5 quizzes, each worth 10 points. Ricardo and his friends, Hailey and Jackson, are determining who did the best on the quizzes. Their scores are:

   Ricardo: 3, 9, 9, 9, 10
   Hailey: 6, 7, 7, 10, 10
   Jackson: 6, 7, 8, 9, 10

   a. According to the mode, who did the best on the quizzes? Is the mode a good way to determine who did best on the quizzes? Why or why not?

Lesson 1 Assignment

b. According to the median, who did the best on the quizzes? Is the median a good way to determine who did best on the quizzes? Why or why not?

c. Determine the mean score by leveling off. Show diagrams and record your operations.

d. Determine each student's mean by determining the balance point. Show your steps on a number line and record your operations.

e. Calculate each student's mean score.

f. Who would you say did best on the quizzes? Explain your choice.

Lesson 1 Assignment

3. The rate at which crickets chirp is affected by the temperature. In fact, you can estimate the outside temperature by counting cricket chirps. As a homework assignment, Mr. Tanaka asks each of his students to count the number of chirps they hear in 15 seconds at 8:00 PM. The results are shown.

   36, 37, 41, 39, 35, 39, 35, 39, 42, 37, 40, 35, 36, 37, 42, 35, 37, 37, 38, 42, 41, 37, 41

   a. Determine the mode for the number of chirps heard in 15 seconds.

   b. What does the mode tell you about the number of chirps heard in 15 seconds?

   c. Determine the median number of chirps heard in 15 seconds.

   d. What does the median tell you about the number of chirps heard in 15 seconds?

   e. Calculate the mean number of chirps heard by the students in 15 seconds.

   f. What does the mean tell you about the number of chirps heard in 15 seconds?

Lesson 1 Assignment

4. An estimate of the temperature outside can be calculated by adding 40 to the number of cricket chirps you hear in 15 seconds. Chelsea used a sample of six estimated temperatures based on cricket chirps heard in 15 seconds:

   76, 74, 74, 76, 73, 77

   a. Use a number line to determine the balance point to estimate the mean temperature based on the number of chirps. Then, describe the steps you took to determine the balance point.

   b. Calculate the mean from the data values. How does it compare to your answer from part (a)?

CC BY NC

Lesson 1 Assignment

Prepare

The scores from an English quiz are displayed in the stem-and-leaf plot.

Stem	Leaf
4	0
5	8
6	1
7	2
8	3 4 5
9	2 3

Key
5|8 = 58%

1. What is the median of the data?

2. What does the median describe in the problem situation?

2 Displaying the Five-Number Summary

OBJECTIVES

- Give quantitative measures of variation, including interquartile range (IQR), and interpret the range, quartiles, and IQR as measures of variation for a data set.
- Calculate and interpret the five-number summary as a measure of variation for a data set.
- Display numerical data in box plots.
- Describe an overall pattern of data with respect to the context in which the data were gathered.

NEW KEY TERMS

- measures of variation
- range
- quartile
- interquartile range (IQR)
- box plot

Mean, median, and mode are used to describe measures of center for a data set. Other characteristics are also important, such as how much the data varies from that center.

How can you use mathematics to describe the variation in a data set?

CC BY NC

Getting Started

Human Box Plot

Your teacher will provide you with an index card and a penny. On the index card, write the date imprinted on the penny.

1. Consider the data from the class and predict the shape of the data set. Do you think that it will be skewed right, skewed left, or symmetrical? Do you think that there will be any clusters or gaps?

On your teacher's signal, line up with your index card from oldest date to the most recent date. As a class, discuss how to determine the following measures.

2. Complete the table for your data set.

Minimum	Lower Quartile (Q1)	Median	Upper Quartile (Q3)	Maximum

3. Use what you learned in this activity to write descriptions of each term.

a. minimum

b. lower quartile (Q1)

c. median

d. upper quartile (Q3)

e. maximum

CC BY NC

ACTIVITY
2.1 The Five-Number Summary

Given the collection of pennies provided by your teacher, line up the pennies from the earliest imprinted date to the most recent.

1. Describe the variation of the data.

A **measure of variation** describes the spread of data values. One measure of variation is the *range*. The **range** is the difference between the maximum and minimum values of a data set.

2. Identify the minimum and maximum values of the data set.

3. Calculate the range for the penny data.

Another set of values that helps to describe variation in a data set is a *quartile*. When data in a set is arranged in order, **quartiles** are the numbers that split data into quarters (or fourths).

Quartiles are often denoted by the letter Q, followed by a number that indicates which fourth it represents. Since the median is the second quartile, it could be denoted Q2. The other quartiles are Q1 and Q3.

4. How do you calculate the quartiles for a data set?

5. What is the minimum number of quartiles you must determine to divide the data into fourths?

In your workspace, divide your pennies into quartiles, leaving a space for each.

6. Identify the value of each quartile for the penny data in your group.

a. Q1

b. Q2

c. Q3

CC BY NC

7. For each quartile, identify what percent of the data is below the quartile and what percent of the data is above the quartile.

   a. Q1

   b. Q2

   c. Q3

8. What percent of the data is between Q1 and Q3?

The **interquartile range**, abbreviated **IQR**, is the difference between the third quartile, Q3, and the first quartile, Q1. The IQR indicates the range of the middle 50 percent of the data.

9. What do you think the "middle 50 percent" means?

10. Do you think it is possible for two sets of data to have the same range, but different IQRs? Explain your reasoning.

11. What is the IQR for your penny data?

To summarize and describe the spread of the data values, you can use the *five-number summary*. The five-number summary includes these five values from a data set:

- Minimum: the least value in the data set
- Q1: the first quartile
- Median: the median value of the data set
- Q3: the third quartile
- Maximum: the greatest value in a data set.

12. Determine the five-number summary and IQR for each data set. Explain the process you used to calculate the values and what they tell you about the data.



a. 24, 32, 16, 18, 30, 20

b. 200, 150, 260, 180, 300, 240, 280

Ask Yourself . . .
Can others follow and understand your process and reasoning?

CC BY NC

ACTIVITY 2.2

Box Plots

A box plot is sometimes referred to as a box-and-whisker plot as a way to remember the shape of the graph.

There is a special type of graph that displays the variation in a data set. A **box plot** is a graph that displays the five-number summary of a data set.

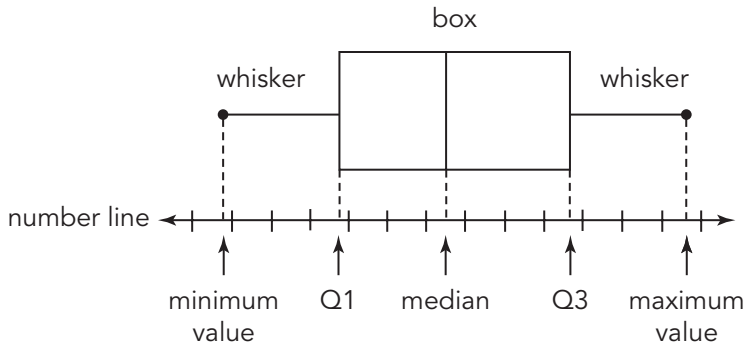
Examine the box plot shown.

WORKED EXAMPLE

Recall that the five-number summary consists of:

- minimum value in the data set
- Q1
- median
- Q3
- maximum value in the data set

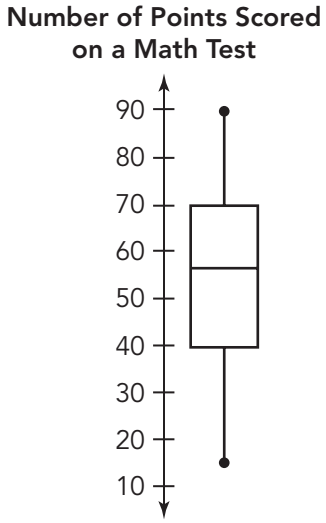
Parts of a Box Plot



1. What does the "box" in the box plot represent? What do the "whiskers" represent?

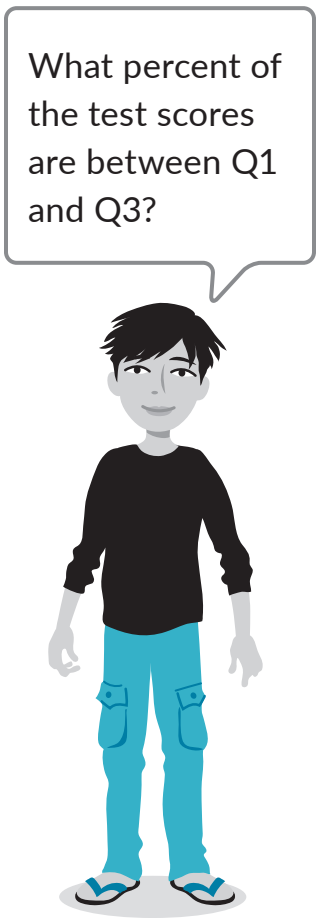
Box plots can be represented horizontally and vertically.

2. Use the box plot to answer each question.



a. Identify the given values for the points scored on the math test. Then, explain what these values tell you about the scores on the test.

- Minimum:
- Median:
- Maximum:
- Q1:
- Q3:
- Range:



b. Determine the IQR for the test scores. Then, explain what the IQR represents in this problem situation.

c. How many students took the math test? Explain your answer.



d. Brianna says the median should be at 50, because it is in the middle of the number line. Do you agree with Brianna's claim? Explain how you determined your answer.

e. Joey claims that more students scored between 15 and 40 than between 70 and 90, because the lower whisker is longer than the upper whisker. Do you agree or disagree with Joey? Explain your reasoning.

You can describe the distribution of a box plot in the same way you described the shapes of stem-and-leaf plots or histograms.

3. How would you describe the distribution of the box plot? Why?

4. Use the box plot to determine whether each statement is true or false. Explain your reasoning.

   a. One-half of the students scored between 40 and 70 points.

   b. Three-fourths of the students scored between 56 and 90 points.

   c. The number of students who scored between 15 and 40 points is equal to the number students who scored between 70 and 90 points.

CC BY NC

5. Create a box plot to represent the penny data. First, recall the values of the five-number summary.

a. Minimum:

b. Q1:

c. Median:

d. Q3:

e. Maximum:

Building a Box Plot

- Label the number line. Include the maximum and minimum values.
- Place dots above the minimum and maximum values.
- Place vertical lines above the median and quartile values. Draw the box around them.
- Draw lines, or whiskers, to connect the box to the minimum and maximum values.
- Name the box plot.

6. Use the number line shown to complete the box plot.



a. Describe the distribution of the box plot.

b. How do the lengths of the whiskers compare? Explain your reasoning.

7. Think about the line of pennies that you created when you ordered them by imprinted dates and separated them into quartiles.

   a. How many pennies were below Q1? How many pennies were above Q3?

   b. Do these values match the length of the whiskers on your box plot?

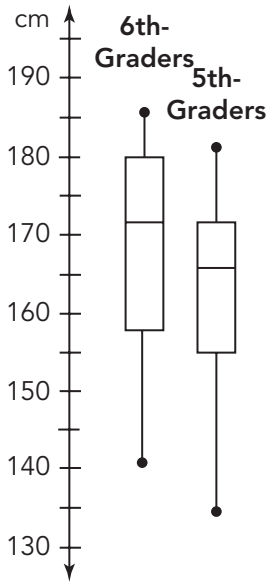
8. Summarize what you can determine about the penny data by examining the box plot.

CC BY NC

Talk the Talk

Build a Box

1. Analyze the box plots shown. Determine whether each statement is true or false, and provide a reason for your decision.



a. On average, the 5th-graders are taller.

b. The range of heights is greater for the 6th-graders.

c. One-half of the 5th-graders are over 165 cm tall.

d. One-half of the 6th-graders are over 172 cm tall.

e. The shortest person is a 6th-grader.

f. The tallest person is a 6th-grader.

2. Use the given information to determine a five-number summary and construct a box plot. Is your data set the only possible solution? Why or why not?

- The data set has a range of 30.
- The maximum value is 50.
- The IQR is 10.
- The median is closer to Q1 than to Q3.



Min.: _____

Q1: _____

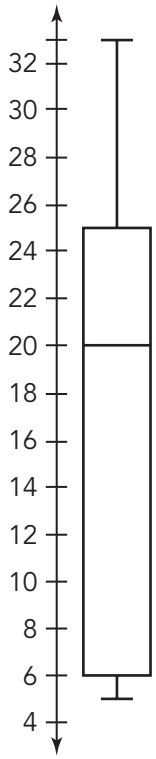
Median: _____

Q3: _____

Max.: _____

CC BY NC

3. Create a data set to represent the box plot shown. Is your data set the only possible solution? Why or why not?



a. Data set with 11 numbers.

b. Data set with 8 numbers.

Lesson 2 Assignment

Write

Write the term that best completes each statement.

range	quartiles	interquartile range	five-number summary

1. The _____ is the difference between the first quartile and the third quartile.
2. The _____ for a set of data is the difference between the maximum and minimum values.
3. _____ are values that divide a data set into four equal parts once the data are arranged in ascending order.
4. A(n) _____ lists the minimum and maximum values, the median, and the quartiles for a set of data.

Remember

To summarize and describe the spread of data values, you can use a five-number summary. A five-number summary includes five values from a data set:

- Minimum: the least value in the data set
- Q1: the first quartile, or the median of the lower half of data
- Median: the middle value of the data set
- Q3: the third quartile, or the median of the upper half of data
- Maximum: the greatest value in a data set.

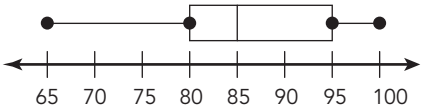
The representation of a five-number summary is called a box plot.

CC BY NC

Lesson 2 Assignment

Practice

1. The box plot shows the distribution of scores on a history quiz.



   a. Identify the median of the data and interpret its meaning.

   b. Identify the range of the data and interpret its meaning.

2. Answer each question using the data set: 0, 5, 5, 15, 30, 30, 45, 50, 50, 60, 75, 110, 140, 240, 330.

   a. Sketch a box plot.

   b. What is the median for the data set?

   c. What is Q3 for the data set?

Lesson 2 Assignment

3. Answer each question using the data set: 10, 10, 10, 10, 35, 75, 90, 95, 100, 175, 420, 490, 515, 515, 790.

   a. Sketch a box plot.



   b. What do you notice about this box plot?

   c. What is the median for the data set?

4. The residents of Summersville, West Virginia are concerned about people speeding through their town on U.S. Route 19. The police decide to monitor the speed of the cars that pass through the town at various times during the day. The data show the recorded speeds in miles per hour of 23 cars at 7:30 AM on one Wednesday morning. 73, 68, 72, 61, 51, 68, 70, 53, 72, 71, 46, 51, 55, 53, 65, 57, 65, 57, 58, 68, 61, 48, 83

   a. What is the range of data?

Lesson 2 Assignment

b. Construct a box plot of the data.



c. Interpret each number in the five-number summary.

d. What does the IQR value tell you about the speeds of the cars?

e. When the speed limit through the town is 50 miles per hour, should the residents be concerned based on this data?

Lesson 2 Assignment

5. San Francisco, California and Richmond, Virginia are located at about the same latitude on opposite sides of the United States. The table shows the amount of rainfall each city gets on average each month.

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
San Francisco, CA	4.4 in.	3.3 in.	3.1 in.	1.4 in.	0.3 in.	0.1 in.	0.0 in.	0.1 in.	0.3 in.	1.3 in.	2.9 in.	3.1 in.
Richmond, VA	3.3 in.	3.3 in.	3.6 in.	3.0 in.	3.8 in.	3.6 in.	5.0 in.	4.4 in.	3.3 in.	3.5 in.	3.3 in.	3.3 in.

a. Calculate and interpret the range of rainfall for each city.

b. Construct box plots for the average rainfall for each city on the same graph.



Lesson 2 Assignment

c. Compare the rainfall in San Francisco and Richmond. Describe the shape of each box plot.

d. Interpret the IQR for the rainfall in each city.

Prepare

Calculate each part using the percent equation.

1. 35% of 60 is what number?

2. 25% of 132 is what number?

3. 5% of 40 is what number?

4. 15% of 80 is what number?

3 Collecting, Displaying, and Analyzing Data

OBJECTIVES

- Discuss the different types of data that can be collected, displayed, and analyzed.
- Determine how to select an appropriate graph to display different types of data.
- Organize data into single, double, or stacked bar graphs.
- Analyze data and interpret results from single, double, or stacked bar graphs and percent bar graphs.

NEW KEY TERMS

- double bar graph
- stacked bar graph
- percent bar graph

You have represented data using histograms and box plots.

What are other ways to represent data, and how do you select the most appropriate display for the information?

CC BY NC

Getting Started

Categorical and Quantitative Data

Analyze each question shown.

- What is your approximate height in inches?
- What color is your hair?
  Brown _____ Blond _____ Black _____ Red _____ Other _____
- Do you own a cell phone?
  Yes _____ No _____
- About how many text messages do you send in one day?
- About how much time (in minutes) do you spend exercising every weeknight?
- What is your favorite type of television show to watch?
  Comedy _____ Sports _____ Reality _____ Drama _____
  Educational _____ News _____ Music _____ Other _____

Remember. . .
Categorical data, or qualitative data, are data for which each piece of data fits into exactly one of several different groups or categories. Quantitative data, or numerical data, are data for which each piece of data can be placed on a numerical scale.

1. Determine whether each of the data collected are categorical or quantitative.

   a. height

   b. hair color

   c. own a cell phone

   d. number of text messages

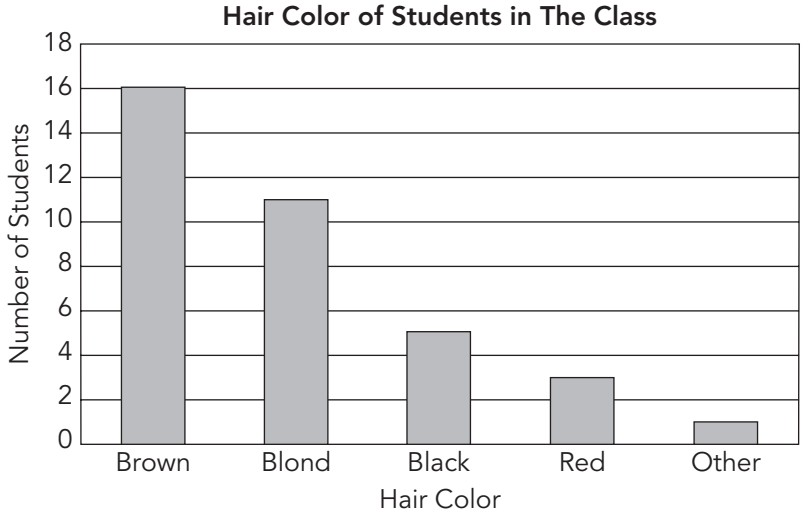
   e. time you spend exercising

   f. favorite type of TV show

ACTIVITY
3.1 Bar Graphs

A bar graph is a way of displaying categorical data by using either horizontal or vertical bars on a graph. The height or length of each bar indicates the value for that category.

1. Suppose the bar graph shown displays the results of the student hair color survey question you analyzed.



   a. Name one conclusion you can make about the students' hair color in the class.

   b. Will your conclusions change when you change the order of the categories? Why or why not?

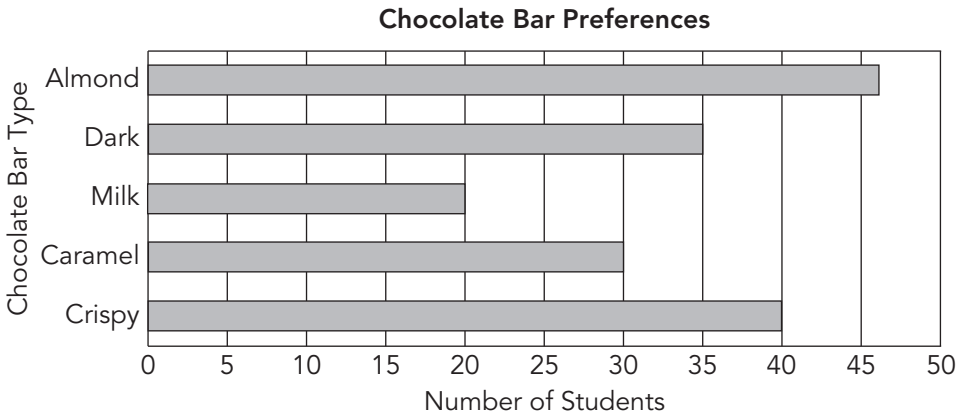
Remember. . .
The mode is the data value or values that occur most frequently in a data set.

c. How many students have blond hair?

d. How many students have either black or red hair?

e. What is the mode? Interpret the mode in terms of the problem situation.

2. The graph shown is an example of a bar graph where the bars are horizontal.



a. Which chocolate bar type is least preferred by students, according to the graph?

b. Which chocolate bar type is most preferred by students, according to the graph?

c. How many students prefer almond chocolate bars? How did you calculate your answer from the graph?

d. What interval does this bar graph use to record the number of students on the bar graph?



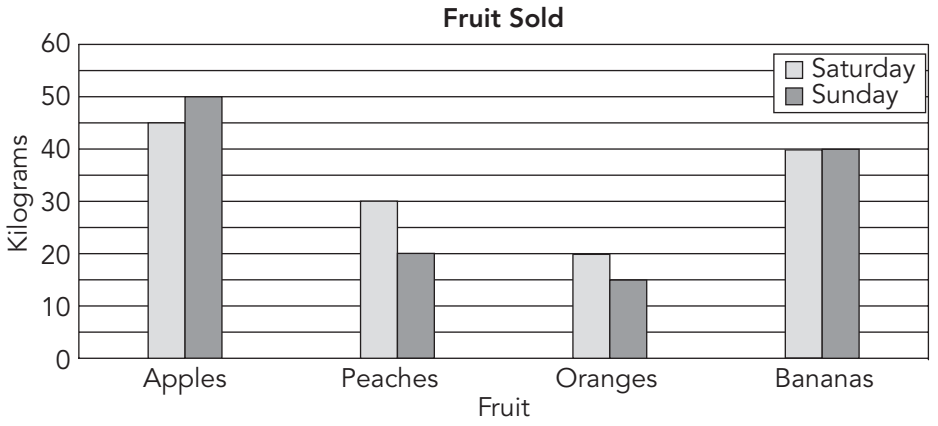
CC BY NC

ACTIVITY
3.2 Double and Stacked Bar Graphs

Could I show three data sets for each category?



Nicky owns a fruit stand at a local farmers market that he only opens on Saturdays and Sundays. Nicky recorded the amount of fruit he sold last weekend.



Nicky displayed his data in a *double bar graph*. A **double bar graph** is used when each category contains two different data sets. The bars may be vertical or horizontal. A key explains how each data set is represented in the graph. The two bars representing the same category are side by side. Space is used to separate the categories.

You can show three or more data sets as long as you show how each data set is represented!



1. Use the double bar graph to answer each question.

   a. What does the key tell you?

   b. How many kilograms of apples did Nicky sell on Sunday?

c. Which fruit showed the greatest difference in sales over the two days?

d. What was the difference between Saturday's sales and Sunday's sales of your answer to part (c)?

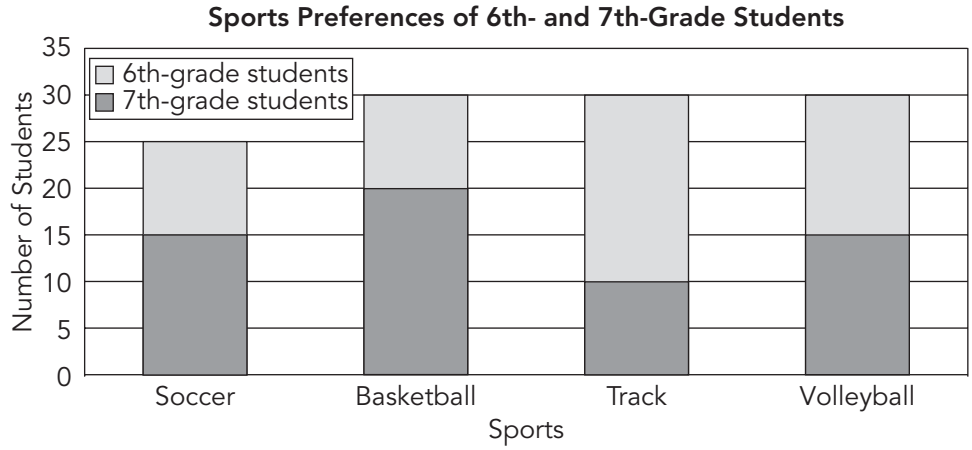
e. Which fruit showed the least difference in sales over the two days?

f. What was the difference between Saturday's sales and Sunday's sales of your answer to part (e)?

CC BY NC

2. When is a double bar graph a better graph choice than a single bar graph?

A survey asked sixth- and seventh-grade students in an after-school program which sport they preferred. Their answers are recorded in the graph.



The "Sports Preferences of 6th- and 7th-Grade Students" display is a *stacked bar graph*. A **stacked bar graph** is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole.

3. Use the stacked bar graph to answer each question.

   a. What does the key indicate?

b. How many total students prefer basketball rather than the other choices?

c. How many sixth-grade students and how many seventh-grade students prefer basketball?

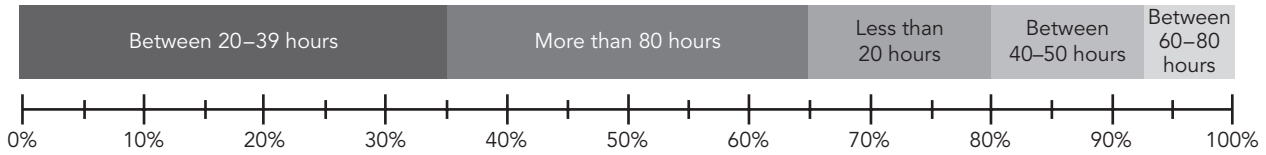
d. Which sport do the sixth-grade students and the seventh-grade students in the after-school program prefer equally? How can you tell?

CC BY NC

ACTIVITY
3.3 Percent Bar Graphs

Another type of graph used to display categorical data is a *percent bar graph*. A **percent bar graph** shows a single bar that represents 100%, with sections that represent the frequency of each category. You can display a percent bar graph horizontally or vertically.

In 2005, the *Volunteer Spotlights Magazine* surveyed over 20,000 students in Grades 6 through 12. One question on the survey asked: How much time do you spend volunteering during the year? The survey had 5 responses to choose from: less than 20 hours, between 20 and 39 hours, between 40 and 50 hours, between 60 and 80 hours, and more than 80 hours. The results are shown in the percent bar graph.



1. Determine the percent of values in each category.

2. How many hours per year do half as many students volunteer as those who volunteer more than 80 hours per year?

You can determine the mode by identifying the category that represents the largest percent of responses.

3. Which category represents the mode?

4. Use the percent bar graph to answer each question.

   a. Your classmate concludes that more students volunteer between 60 and 80 hours than between 20 and 39 hours. Is their conclusion correct? Explain your reasoning.

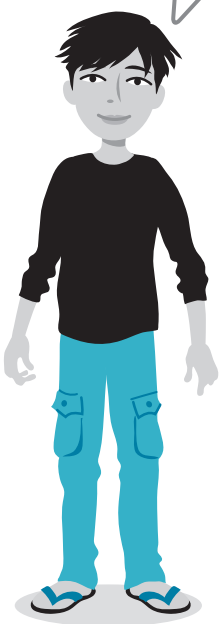
   b. When 20,000 students responded to the survey, how many students volunteer between 20 and 39 hours per year?

Remember that interpreting the data or drawing conclusions to describe the data distribution is part of the statistical process.

5. Kara decides to create a circle graph to represent the data in the percent bar graph. How will the circle graph compare to the percent bar graph?

Ask Yourself . . .
Did you justify your mathematical reasoning?

Do I need to multiply the percent by the number of students to figure this out?



6. Another question on the survey asked students, "What kind of volunteering do you do?" The results are:
   Volunteer through school: 64%
   Volunteer through religious affiliation: 50%
   Volunteer through community groups: 47%
   Other: 45%
   I don't volunteer: 4%
   Would a percent bar graph be an appropriate graph for the data? Why or why not?

Talk the Talk

Data On Display

Categorical data are a data set for which each piece of data fits into exactly one of several different groups or categories. Categorical data are also called *qualitative data*. Some examples of categorical data are hair color, favorite color, and birth month.

1. Describe how a bar graph displays categorical data.

2. Describe how a percent bar graph displays categorical data.

Lesson 3 Assignment

Write

Your school plans to give a survey to all of the students. Your principal asks for the different types of data that the school can collect from the survey questions. Write a note to your principal explaining the two different types of data, and give at least three examples of questions that the school survey could ask for each.

Remember

You can describe data as being categorical or quantitative. You can also refer to categorical data as qualitative, and you can display categorical data in tables, bar graphs, and percent bar graphs.

Practice

1. There are over 3100 public skate parks in the United States. The table shows how many skate parks there are in each of the states listed.

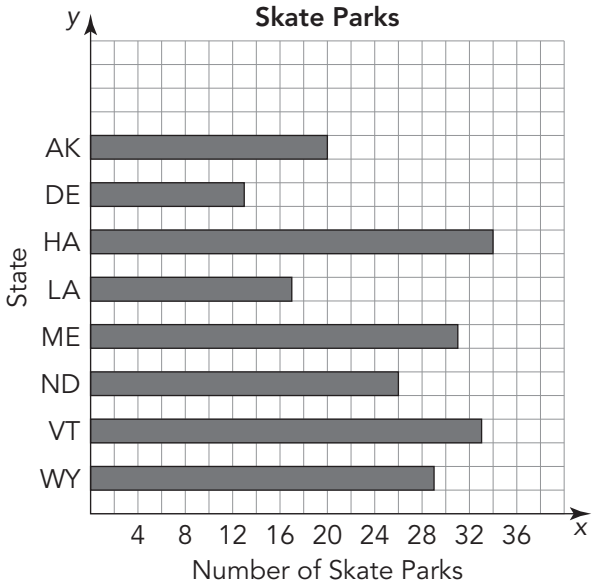
State	Number of Skate Parks
Alaska	20
Delaware	13
Hawaii	34
Louisiana	17
Maine	31
North Dakota	26
Vermont	33
Wyoming	29

   a. Is this categorical or quantitative data? Explain your reasoning.

CC BY NC

Lesson 3 Assignment

This bar graph uses the data represented in the table.



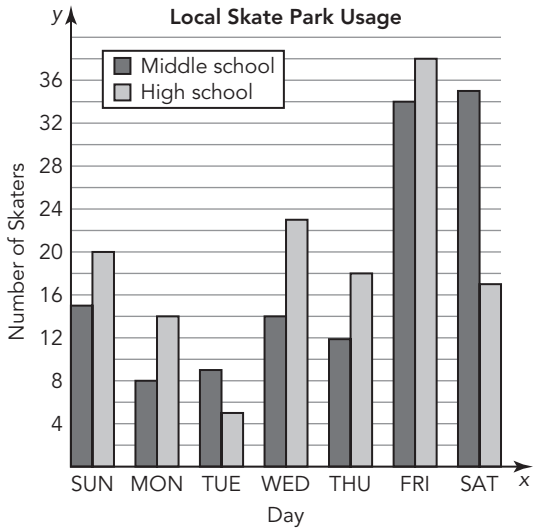
b. What do the bars represent?

c. Which state has the greatest number of skate parks? How many does it have?

d. Which state has the fewest skate parks? How many does it have?

Lesson 3 Assignment

2. The double bar graph shows the number of skaters who used the local skate park in one week. Use the double bar graph to answer the questions.



a. What does the key tell you?

b. What interval does this graph use to record the number of skaters on the bar graph?

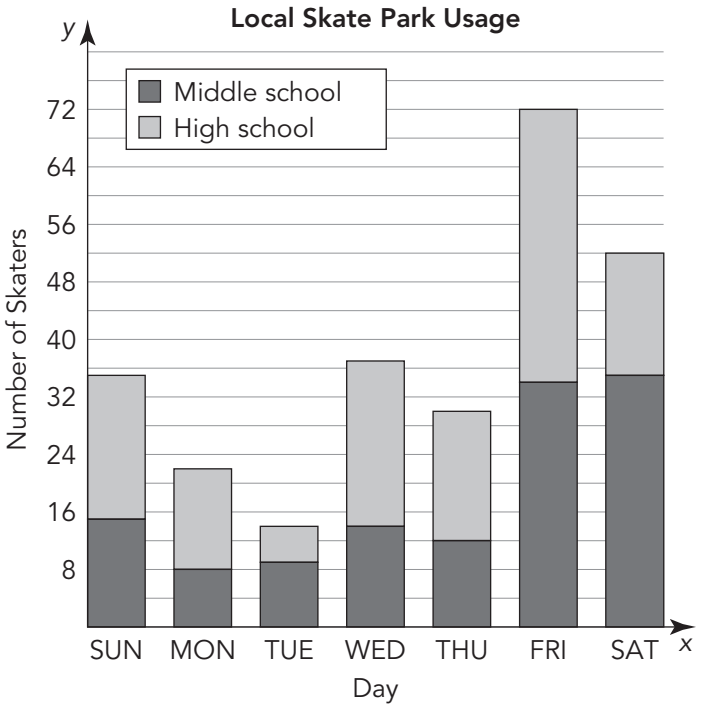
c. How many middle school skaters used the park during the week? Explain how you determined your answer.

Lesson 3 Assignment

d. What is the difference between the number of high school skaters that used the park on Friday and the number of high school skaters that used it on Saturday? Show your work.

e. On how many days were there more high school skaters using the park than middle school skaters? Explain how you determined your answer.

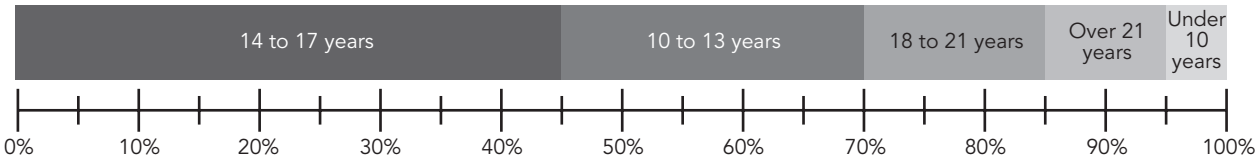
This stacked bar graph uses the same data as the double bar graph.



Lesson 3 Assignment

f. The city would like to close the park one day a week for maintenance. Which day would you recommend they close the park based on the data shown in the bar graph? Explain your reasoning.

3. The city would like to know the ages of the skaters using a different skate park. They survey all of the skaters that use the park in one week. The results are shown in the percent bar graph.



a. What is the percent of values for each category?

b. When 512 skaters were surveyed, how many are 10 to 13 years old? Show your work.

c. Which category represents the mode? Explain your reasoning.

Lesson 3 Assignment

d. What conclusions can you draw from the percent bar graph, the percent of values in each category, and the mode?

TOPIC 2 SELF-REFLECTION

Name: _____

Numerical Summaries of Data

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Numerical Summaries of Data* topic by:

TOPIC 2: *Numerical Summaries of Data*	Beginning of Topic	Middle of Topic	End of Topic
summarizing numerical data with measures of center (e.g., mean, median, and mode) that describe the shape, center, and spread of the data distribution.	○	○	○
summarizing numerical data with measures of variation (e.g., range and interquartile range) that describe how the data values vary or are spread out.	○	○	○
calculating the five-number summary and interquartile range (IQR) of a data set.	○	○	○
creating a box plot to display a set of numerical data.	○	○	○
using the graphical representation of data to describe the shape, center, and spread of the data distribution.	○	○	○
summarizing categorical data with numerical and graphical summaries, including the mode, the percent of values in each category (relative frequency), and the percent bar graph, and use these summaries to describe data distribution.	○	○	○

continued on the next page

CC BY NC

TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Numerical Summaries of Data* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?



TOPIC 2 SUMMARY

Numerical Summaries of Data Summary

LESSON 1

Analyzing Data Using Measures of Center

When analyzing a set of data, it is often useful to describe it numerically. One way to numerically describe a data set is to use a *measure of center*. A **measure of center** tells you how the data values are clustered, or where the center of a graph of the data is located. There are three measures that describe how a data set is centered: the *mode*, the *median*, and the *mean*.

The **mode** is the data value or values that occur most frequently in a data set. A data set can have more than one mode or no mode. For example, the mode of the data set 12, 6, 12, 26, 4, and 12 is 12.

The **median** is the middle number in a data set when the values are placed in order from least to greatest or greatest to least. When a data set has an odd number of data values, you can determine which number is exactly in the middle of the data set. If there is an even number of data values, then the median is calculated by adding the two middle numbers and dividing by 2.

For example, the median of the data set 15, 12, 13, 10, 8, and 14 is 12.5.

The third measure of center is based on leveling off or creating fair shares. For example, if you have a stack of two cubes and a stack of six cubes, you can rearrange the stacks to create two equal stacks of four cubes each.

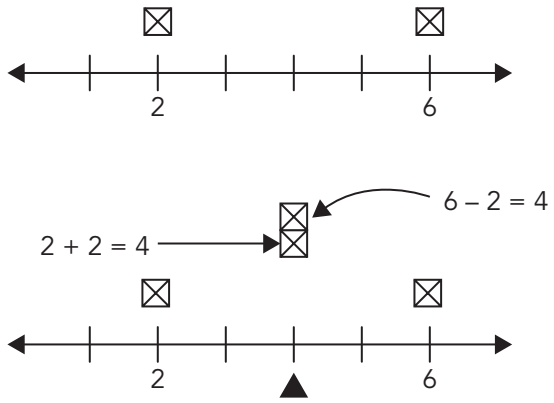
You can also represent quantities on a number line and create a *balance point*. When you have all the points at the same value, the number line is balanced. The value at which the number line is balanced is called the **balance point**.

NEW KEY TERMS

- measure of center [medida de tendencia central]
- mode [moda]
- median [mediana]
- balance point
- mean [media]
- measure of variation [medida de variabilidad]
- range [rango]
- quartile [cuartil]
- interquartile range (IQR) [rango intercuartílico]
- box plot
- frequency [frecuencia]
- double bar graph [gráfica de barras dobles]
- stacked bar graph
- percent bar graph [gráfica de barra de porcentaje]

CC BY NC

For example, consider the data set 2, 6.



The value 2 was moved to the right from 2 to 4. To maintain balance, 6 was moved to the left from 6 to 4. The balance point is 4. The balance point can also be called the *mean*. The **mean** is the arithmetic average of the numbers in a data set.

For example, determine the mean of the data set: 12, 12, 6, 26, 4, 12.

The mean is calculated by adding all the values in the data set and dividing the sum by the number of values.

Step 1: $12 + 12 + 6 + 26 + 4 + 12 = 72$

Step 2: $\frac{72}{6} = 12$

You can verify that the mean is 12, because the balance point of the data set is 12.

LESSON 2 Displaying the Five-Number Summary

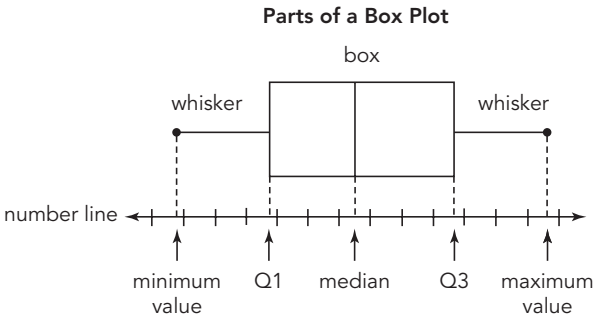
A **measure of variation** describes the spread of data values. One measure of variation is the *range*. The **range** is the difference between the maximum and minimum values of a data set.

For example, the range of the data set 200, 150, 260, 180, 300, 240, and 280 is $300 - 150 = 150$.

Another set of values that helps to describe variation in a data set is a **quartile**. When data in a set are arranged in order, quartiles are the numbers that split data into quarters (or fourths). Quartiles are often denoted by the letter Q, followed by a number that indicates which fourth it represents. Since the median is the second quartile, it could be denoted Q2. The other quartiles are Q1 and Q3. The **interquartile range**, abbreviated **IQR**, is the difference between the third quartile, Q3, and the first quartile, Q1. The IQR indicates the range of the middle 50 percent of the data.

To summarize and describe the spread of the data values, you can use the five-number summary. The five-number summary includes these five values from a data set:

- Minimum: the least value in the data set
- Q1: the first quartile
- Median: the middle value in the data set
- Q3: the third quartile
- Maximum: the greatest value in a data set.

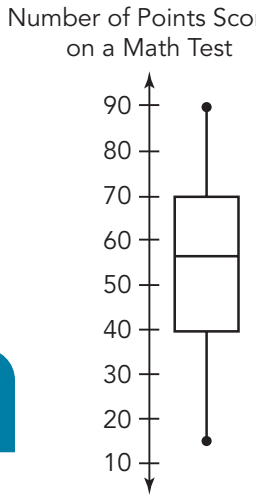


For the data set 24, 32, 16, 18, 30, and 20, the minimum is 16, Q1 is 18, the median is 22, Q3 is 30, and the maximum is 32.

A **box plot** is a graph that displays the five-number summary of a data set.

Box plots can be represented vertically, as well as horizontally.

For example, in this box plot, the minimum of the data set is 15, Q1 is 40, the median of the data set is 56, Q3 is 70, and the maximum of the data set is 90.

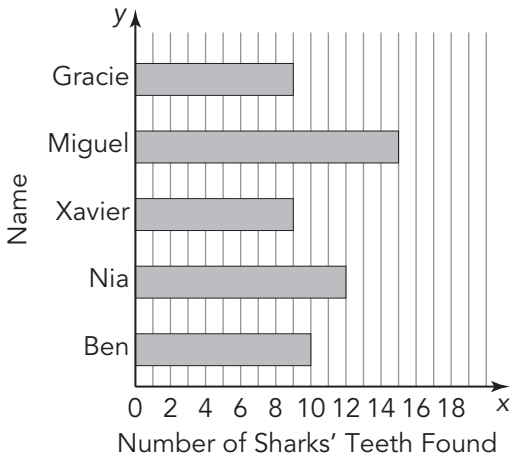


LESSON 3 Collecting, Displaying, and Analyzing Data

When you are collecting, displaying, and analyzing data, the data are one of two types: *categorical data* or *quantitative data*.

When displaying data, categorical data graphs are different from quantitative data graphs. Categorical data are often displayed in *bar graphs*.

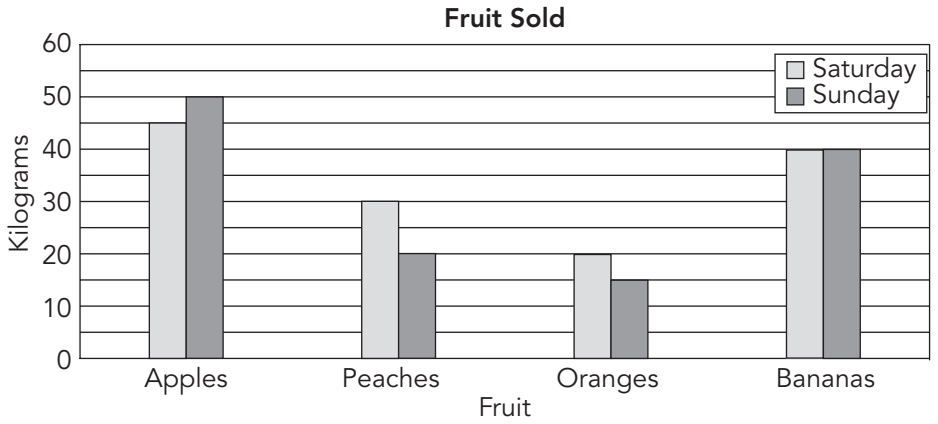
A bar graph is a way of displaying categorical data by using either horizontal or vertical bars on a graph. The height or length of each bar indicates the value for that category.



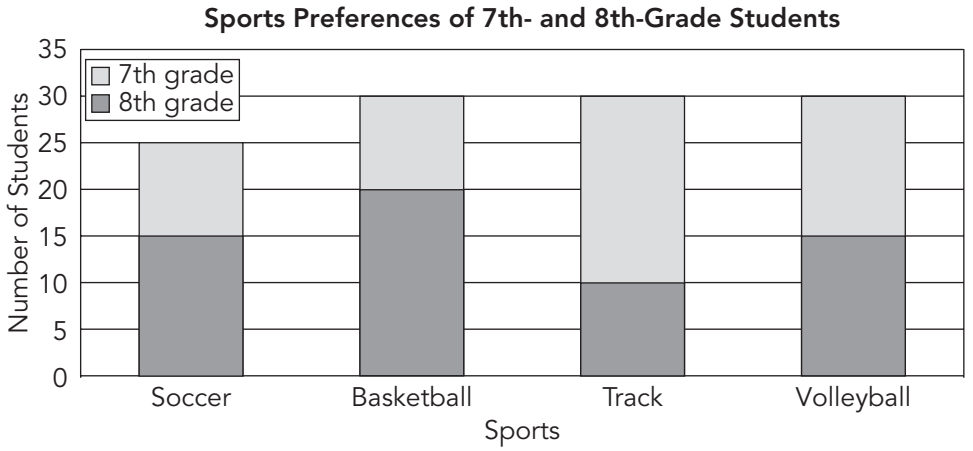
Each bar represents a total for the whole category, but still shows how many data pieces make up each group within the entire category. For the bar graph above, the number of sharks' teeth found by each person is 9, 15, 9, 12, and 10. The mode is 9, because it is the number that appears most frequently.

CC BY NC

A **double bar graph** is used when each category contains two different data sets. The bars may be vertical or horizontal. A key explains how each data set is represented by a color or a pattern in the graph. The two bars representing the same category are side by side. Space is used to separate the categories.

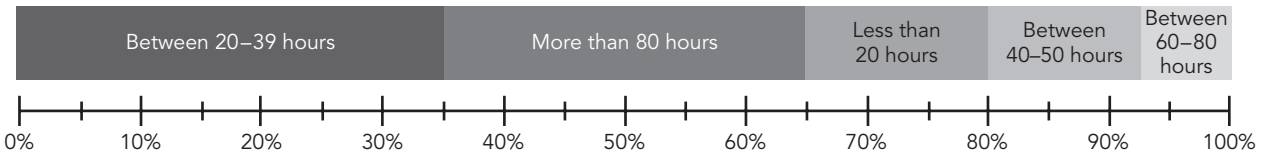


A **stacked bar graph** is a graph that stacks the frequencies of two different groups for a given category on top of one another, so that you can compare the parts to the whole. Each bar represents a total for the whole category, but still shows how many data pieces make up each group within the entire category.



Another type of graph used to display categorical data is a *percent bar graph*. A **percent bar graph** shows a single bar that represents 100%, with sections that represent the frequency of each category. You can display a percent bar graph horizontally or vertically.

For example, in 2005, the Volunteer Spotlights Magazine surveyed over 20,000 students in Grades 6 through 12. One question on the survey asked: How much time do you spend volunteering during the year? The results are shown in the percent bar graph.



Math Glossary

A

absolute value

The absolute value, or magnitude, of a number is its distance from zero on a number line.

Example

The absolute value of -3 is the same as the absolute value of 3 because they are both a distance of 3 from zero on a number line.



$|-3| = |3|$

account balance

Account balance is the amount of money in an account at a given time.

addition property of equality

The addition property of equality states that if two values a and b are equal when you add the same value c to each, the sums are equal.

Examples

$12 = 12$ and $12 + 7 = 12 + 7$

If $a = b$, then $a + c = b + c$.

additive inverses

Two numbers with the sum of zero are called additive inverses.

Examples

$-19 + 19 = 0$ $\quad$ $a + -a = 0$

additive reasoning

Additive reasoning focuses on the use of addition and subtraction for comparisons.

Example

Vicki is 40 years old, and Ben is 10 years old. In 5 years, Vicki will be 45 and Ben will be 15. Vicki will always be 30 years older than Ben. This is additive reasoning.

algebraic expression

An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Examples

a $\quad$ $2a + b$ $\quad$ xy $\quad$ $\frac{4}{p}$ $\quad$ z^2

algorithm

An algorithm is a process or description of steps you can follow to complete a mathematical calculation.

Annual Percentage Yield (APY)

Annual Percentage Yield (APY) is a percentage that is paid to customers based on the account balance in an account for a year.

Example

Yvonne opens a checking account with an average monthly balance of $600. The account has a 2.5% APY.

(600)(0.025) = 15

Yvonne will earn $15 on this checking account over the course of the year.

CC BY NC

associate's degree

An associate's degree is a degree earned in a one- or two-year program in which students learn a specific skill or trade.

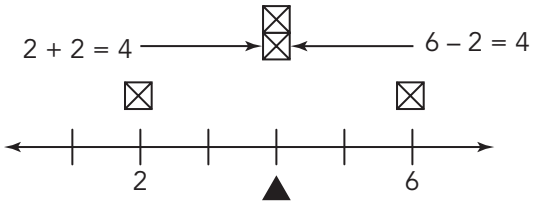
B

balance point

When you have all the points on a number line at the same value, the number line is balanced. The value where the number line is balanced is called the balance point.

Example

Consider the data set: 2, 6.

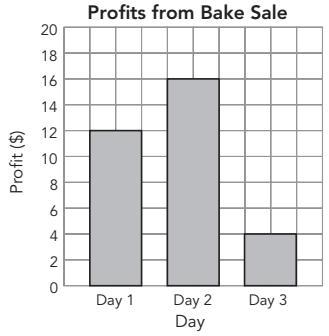


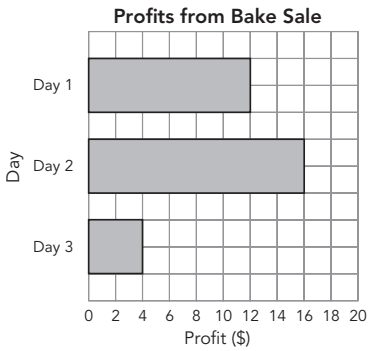
The balance point is 4.

bar graph

A bar graph displays categorical data using either horizontal or vertical bars on a graph. The height or length of each bar indicates the value for that category.

Examples



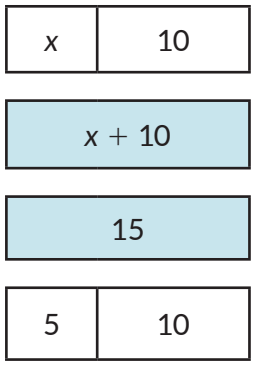


bar model

A bar model uses rectangular bars to represent known and unknown quantities.

Example

You can use a bar model to solve the equation $x + 10 = 15$.



The top bar can be split into two bars, x and 10. When this split happens in the bottom bar, with one bar containing 10, it shows that x is the same as 5, so $x = 5$.

base

The base of a power is the factor that is multiplied repeatedly in the power.

Examples

$2^3 = 2 \cdot 2 \cdot 2 = 8$ $8^0 = 1$

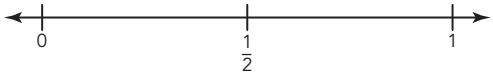
base base

benchmark fractions

Benchmark fractions are common fractions you can use to estimate the value of fractions.

Example

The numbers 0, $\frac{1}{2}$, and 1 are some benchmark fractions.



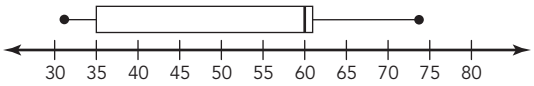
benchmark percents

A benchmark percent is a percent that is commonly used, such as 1%, 5%, 10%, 25%, $33\frac{1}{3}$%, 50%, and 100%.

box plot

A box-and-whisker plot, or just box plot, is a graph that displays the five-number summary of a data set: the median, the upper and lower quartiles (Q1 and Q3), and the minimum and maximum values.

Example



Data: 32, 35, 35, 53, 55, 60, 60, 61, 61, 74, 74

Minimum = 32

Q1 = 35

Median = 60

Q3 = 61

Maximum = 74

C

categorical data

Categorical data, or qualitative data, are data for which each piece of data fits into exactly one of several different groups or categories.

Examples

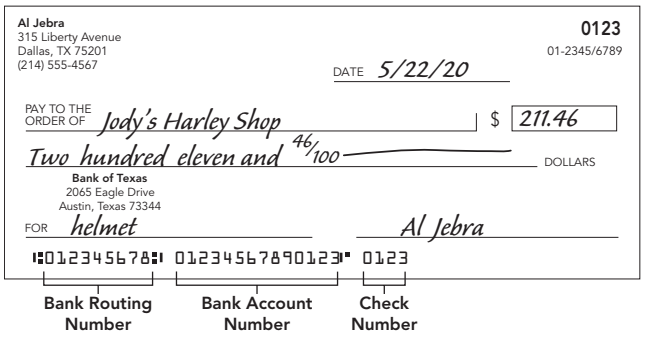
Animals: lions, tigers, bears, etc.

Colors: blue, green, red, etc.

check

A check is a written order to a bank to pay a specific amount of money to a person or company out of your checking account.

Example



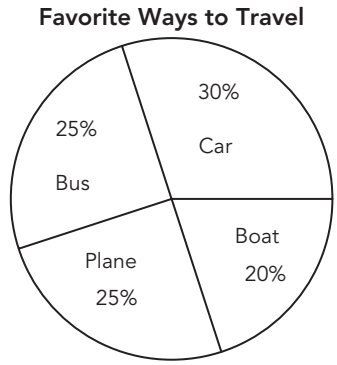
checking account

A checking account allows customers to safely store money in the bank and write checks.

circle graph

A circle graph, often called a *pie chart*, displays categorical data using sectors, or "wedges," of a circle.

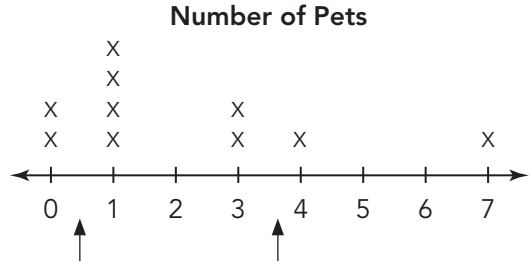
Example



clusters

Clusters are areas of the graph where data are grouped close together.

Example



There are clusters of data from 0 to 1 and from 3 to 4.

coefficient

A number that is multiplied by a variable in an algebraic expression is called a coefficient.

Examples

$14x$ $\frac{1}{3}(g)$ $w + 2.5$

coefficient

The coefficient is 1 even though it is not shown.

CC BY NC

common factor

A common factor is a number that is a factor of two or more numbers.

Example

Factors of 60: **1**, **2**, **3**, **4**, 5, **6**, 10, **12**, 15, 20, 30, 60

Factors of 24: 1, **2**, **3**, **4**, **6**, 8, **12**, 24

Common factors of 60 and 24: 1, 2, 3, 4, 6, and 12

commutative property of multiplication

The commutative property of multiplication states that for any numbers a and b, the product $a \cdot b$ is equal to the product $b \cdot a$.

Examples

$$\begin{array}{r} 29 \\ \cdot\ 3 \\ \hline 87 \end{array} = \begin{array}{r} 3 \\ \cdot\ 29 \\ \hline 27 \\ +\ 60 \\ \hline 87 \end{array} \qquad \underbrace{\frac{1}{5} \cdot \frac{2}{3}}_{\frac{2}{15}} = \underbrace{\frac{2}{3} \cdot \frac{1}{5}}_{\frac{2}{15}}$$

complex fraction

A complex fraction is a fraction that has a fraction in either the numerator, the denominator, or both the numerator and denominator.

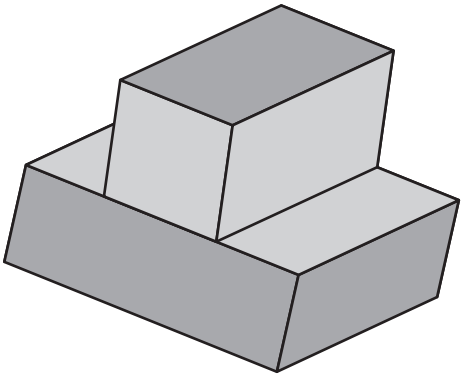
Examples

$\frac{\frac{3}{4}}{3}$, $\frac{7}{\frac{1}{2}}$ and $\frac{\frac{1}{4}}{\frac{2}{3}}$ are all complex fractions.

composite solid

A composite solid is made up of more than one geometric solid.

Example



continuous data

When quantitative data are a measurement of something and can have values that are between two counting numbers, the data are called continuous data.

convert

To convert a measurement means to change it to an equivalent measurement in different units.

Example

To convert 36 inches to feet, you can multiply:

$$36 \text{ in.} \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = \frac{36 \text{ ft}}{12}$$
$$= 3\text{ft}$$

credit card

A card that allows a person to borrow a certain amount of money and pay the borrowed money back at a later time is a credit card.

credit history

Credit history includes a record of all loans, credit card debt, late payments, etc.

credit report

A credit report is a detailed listing of an individual's credit history.

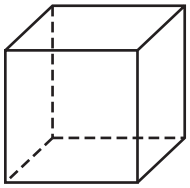
credit score

A credit score is a number used by lenders to rate how likely it is that a person will repay their debts.

cube

A cube is a polyhedron that has congruent squares as faces.

Example



D

data

Data are categories, numbers, or observations gathered in response to a statistical question.

Examples

Favorite foods of sixth graders
Heights of different animals at the zoo

debit

A debit occurs when money is taken out of the account automatically through the use of a debit card.

debit card

A debit card is a card that allows a bank's customer to pay for goods using money from their account.

density property

The density property states that between any two rational numbers, there is another rational number.

dependent quantity

The dependent quantity is the quantity that depends on another in a problem situation.

Example

Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

$\text{number of gallons} \cdot \frac{\text{miles}}{\text{gallon}} = \text{miles traveled}$

The dependent quantity is the total miles traveled. The number of miles traveled depends on the gallons of fuel.

dependent variable

The variable that represents the dependent quantity is called the dependent variable.

Example

Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

$\text{number of gallons} \cdot \frac{\text{miles}}{\text{gallon}} = \text{miles traveled}$

$$g \cdot m = t$$

The dependent quantity is the total miles traveled. Since t represents total miles traveled in the equation, t is the dependent variable.

deposit

A deposit is money put into the account. Deposits may come in the form of cash, checks, or transfers.

discrete data

Discrete data can only take on specific, individual values. Common examples of discrete data include the number of students in a class or shoe sizes.

distribution

The overall shape of a graph is called the distribution of data. A distribution is the way in which the data are spread out.

distributive property

The distributive property states that for any numbers a, b, and c, $a(b + c) = ab + ac$.

Example

$$\begin{aligned} 4(2 + 15) &= 4 \cdot 2 + 4 \cdot 15 \\ &= 8 + 60 \\ &= 68 \end{aligned}$$

division property of equality

The division property of equality states that when you divide equal values a and b by the same value c and $c \neq 0$, the quotients are equal.

Example

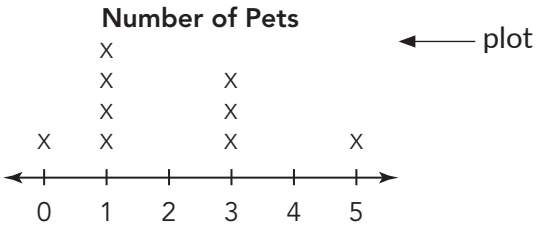
$12 = 12$ and $12 \div 7 = 12 \div 7$

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

dot plot

A dot plot (sometimes called a *line plot*) is a data display that shows discrete data on a number line with dots, Xs, or other symbols.

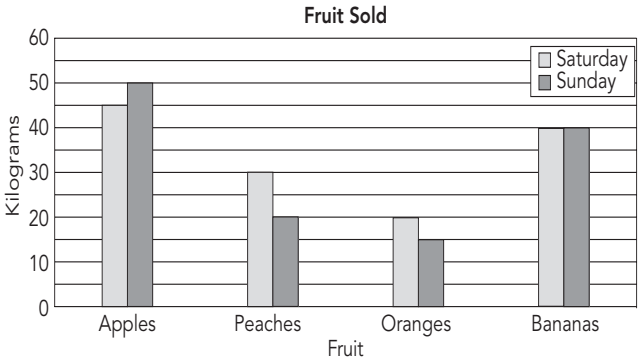
Example



double bar graph

A double bar graph is used when each category contains two different data sets. The bars may be vertical or horizontal.

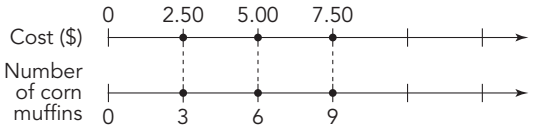
Example



double number line

A double number line is a model that is made up of two number lines used together to represent the ratio between two quantities.

Example

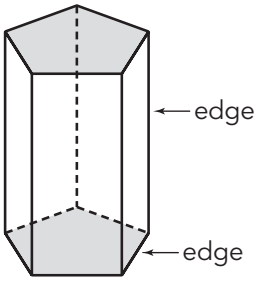


E

edge

An edge is the intersection of two faces of a three-dimensional figure.

Example



ellipsis

An ellipsis is a set of three periods used to represent infinity in a number set.

Example

$\{..., -2, -1, 0, 1, 2, ...\}$

ellipsis ellipsis

equation

An equation is a mathematical sentence that uses an equals sign to show that two quantities are the same as one another.

Examples

$$y = 2x + 4$$
$$6 = 3 + 3$$
$$2(8) = 26 - 10$$
$$\frac{1}{4} \cdot 4 = \frac{8}{4} - \frac{4}{4}$$

equivalent expressions

Two algebraic expressions are equivalent expressions if, when any values are substituted for variables, the results are equal.

Example

$$(x + 10) + (6x - 5) = 7x + 5$$
$$12 + 7 = 14 + 5$$
$$19 = 19$$

equivalent fractions

Fractions that represent the same part-to-whole relationship are equivalent fractions.

equivalent ratios

Equivalent ratios are ratios that represent the same part-to-part or part-to-whole relationship.

evaluate an algebraic expression

To evaluate an algebraic expression means to determine the value of the expression for a given value of each variable.

Example

Evaluate the expression $= \frac{(4x + (2^3 - y)}{p}$ for $x = 2.5$, $y = 8$, and $p = 2$.

- First replace the variables with numbers: $\frac{4(2.5) + (2^3 - 8)}{2}$
- Then calculate the value of the expression: $\frac{10 + 0}{2} = \frac{10}{2} = 5$

evaluate a numeric expression

To evaluate a numeric expression means to rewrite the expression as a single numeric value.

Example

$$19 - 4 \cdot 3$$
$$19 - 12$$
$$7$$

experiment (statistics)

An experiment is one method of collecting data in which a researcher imposes a condition and observes the results.

Example

A researcher conducts an experiment to investigate if 6th-graders perform better on an assessment if they read a textbook or watch a video about the material. The researcher randomly assigns half the students to read the text and half the students to watch the video. All students would be given the same assessment, and the scores of the students in the two groups would be compared.

exponent

The exponent of the power is the number of times the base is used as a factor.

Examples

$2^3 = 2 \cdot 2 \cdot 2$

exponent

$8^4 = 8 \cdot 8 \cdot 8 \cdot 8$

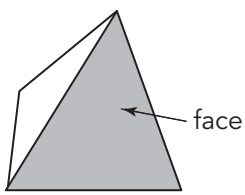
exponent

F

face

A face is one of the polygons that makes up a polyhedron.

Example



CC BY NC

frequency

A frequency is the number of times an item or number occurs in a data set.

Number Rolled	Tally	Frequency
2	𝍸 II	7

The number 2 was rolled 7 times, so its frequency was 7.

frequency table

A table used to organize data according to how many times a data value occurs is called a frequency table.

Example

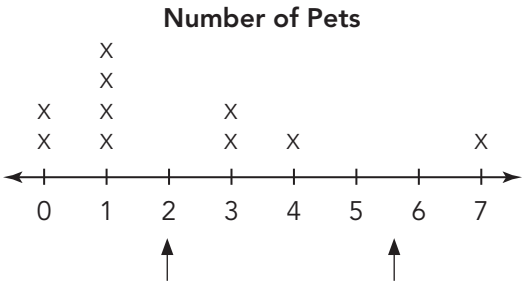
Number of Roller Coasters at Major Theme Parks	
Number of Roller Coasters	Frequency (f)
7–9	6
10–12	4
13–15	7
16–18	2

G

gaps

Gaps are areas of the graph where there are no data.

Example

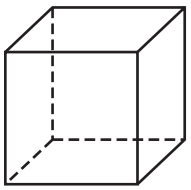


There are gaps between 1 and 3 and between 4 and 7.

geometric solid

A geometric solid is a bounded three-dimensional geometric figure.

Example



grant

A grant is money awarded to a person from the government or school to help pay tuition.

graph of an inequality

The graph of an inequality in one variable is the set of all points on a number line that make the inequality true.

Example



$x \leq 3$

greatest common factor (GCF)

The greatest common factor, or GCF, is the largest factor two or more numbers have in common.

Example

Factors of 16: **1**, **2**, **4**, 8, 16

Factors of 12: **1**, **2**, 3, **4**, 6, 12

Common factors: 1, 2, 4

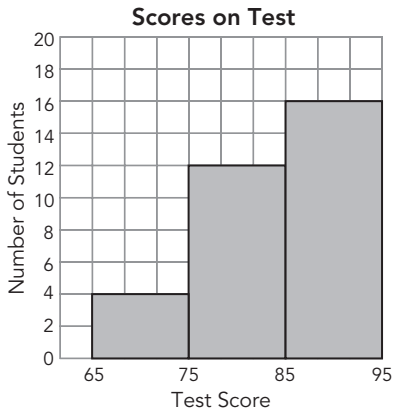
Greatest common factor: 4

H

histogram

A histogram is a graphical way to display quantitative or numerical data using vertical bars. The width of a bar represents an interval of data and is often referred to as a *bin*. The height of the bar indicates the frequency, or the number of data values included in any given bin.

Example



I

identity property of addition

The identity property of addition states that the sum of any number and 0 is the number.

Examples

$6 + 0 = 6$ $\quad \frac{3}{4} + 0 = \frac{3}{4}$

$5^2 + 0 = 5^2$ $\quad 0.125 + 0 = 0.125$

identity property of multiplication

The identity property of multiplication states that the product of any number and 1 is the number.

Examples

$6 \cdot 1 = 6$ $\quad \frac{3}{4} \cdot 1 = \frac{3}{4}$

$5^2 \cdot 1 = 5^2$ $\quad 0.125(1) = 0.125$

independent quantity

The independent quantity is the quantity the dependent quantity depends on.

Example

Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

$$\text{number of gallons} \cdot \frac{\text{miles}}{\text{gallon}} = \text{miles traveled}$$

The independent quantity is the number of gallons. The other quantity (miles traveled) is dependent upon this quantity.

independent variable

The variable that represents the independent quantity is called the independent variable.

Example

Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

$$\text{number of gallons} \cdot \frac{\text{miles}}{\text{gallon}} = \text{miles traveled}$$

$$g \cdot m = t$$

The independent quantity is the number of gallons. Since g represents the number of gallons in the equation, g is the independent variable.

inequality

An inequality is any mathematical sentence that has an inequality symbol.

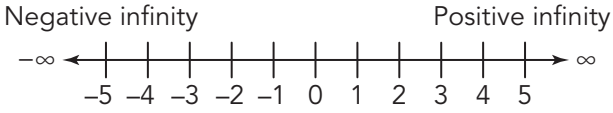
Examples

$8 > 2$ $\quad a \leq b$ $\quad 6.051 > 6.009$ $\quad 2x + 4 \geq 16$

infinity

Infinity, represented by the symbol ∞, means a quantity with no end or bound.

Example



integers

Integers are the set of whole numbers with their opposites.

Example

The set of integers can be represented as {... −3, −2, −1, 0, 1, 2, 3, ...}

CC BY NC

interest

Interest is an additional charge for borrowing money, usually a percentage of the amount borrowed.

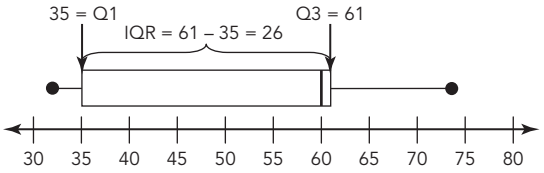
interest rate

The interest rate is the percentage of the balance on the credit card that is added to the cost of the purchase.

interquartile range (IQR)

The interquartile range, abbreviated IQR, is the difference between the third quartile, Q3, and the first quartile, Q1. The IQR indicates the range of the middle 50 percent of the data.

Example



inverse operations

Inverse operations are pairs of operations that reverse the effects of each other.

Examples

Addition and subtraction are inverse operations: $351 + 25 - 25 = 351$.

Multiplication and division are inverse operations: $351 \cdot 25 \div 25 = 351$.

K

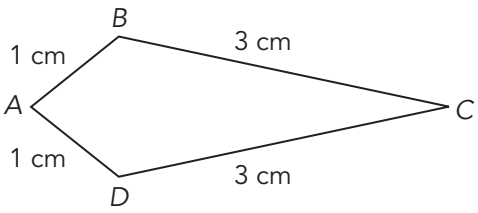
key

A key explains how each data set is represented by a color or a pattern in the graph.

kite

A kite is a quadrilateral with two pairs of consecutive congruent sides where opposite sides are not congruent.

Example



L

least common multiple (LCM)

The least common multiple, or LCM, is the smallest multiple (other than zero) that two or more numbers have in common.

Example

Multiples of 60: 60, **120**, 180, **240**, 300, 360, 420, 480 . . .

Multiples of 24: 24, 48, 72, 96, **120**, 144, 168, 192, 216, **240** . . .

Some common multiples of 60 and 24: 120, 240 . . .

Least common multiple of 60 and 24: 120

like terms

In an algebraic expression, like terms are two or more terms that have the same variable raised to the same power.

Examples

like terms

$4x + 3p + x + 2 = 5x + 3p + 2$

like terms

$24a^2 + 2a - 9a^2 = 13a^2 + 2a$

no like terms

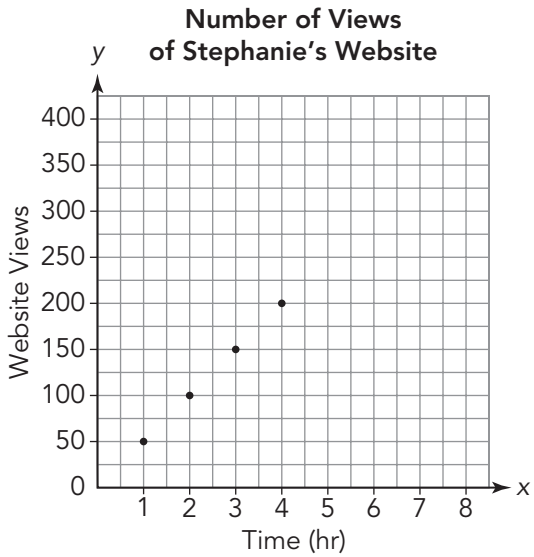
$m + m^2 - x + x^3$

linear relationship

When a set of points graphed on a coordinate plane forms a straight line, a linear relationship exists.

Example

The points graphed show a linear relationship.



literal equation

A literal equation is an equation in which the variables represent specific measures.

Examples

$A = lw$ $A = \frac{1}{2}bh$ $d = rt$

M

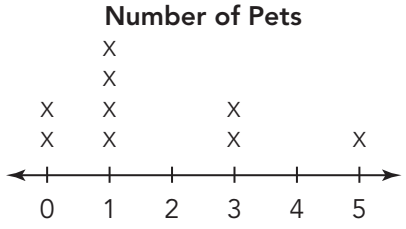
master's degree

A master's degree is a one- or two-year degree beyond an undergraduate degree.

mean

The mean is the arithmetic average of the numbers in a data set.

Example



$$\text{Mean} = \frac{0+0+1+1+1+1+3+3+5}{9}$$
$$= \frac{15}{9} = 1\frac{2}{3} \text{ pets}$$

measure of center

A measure of center tells you how the data values are clustered, or where the "center" of a graph of the data is located.

Examples

Mean, median, and mode are each a measure of center for data.

measure of variation

A measure of variation describes the spread of data values.

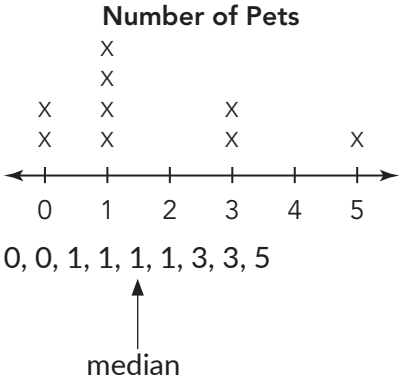
Example

Range is a measure of variation for data.

median

The median is the middle number in a data set when the values are placed in order from least to greatest or greatest to least.

Example



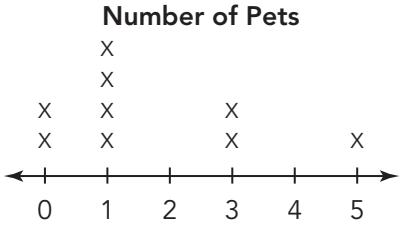
0, 0, 1, 1, 1, 1, 3, 3, 5

median

mode

The mode is the value or values that occur most frequently in a data set.

Example



0, 0, 1, 1, 1, 1, 3, 3, 5

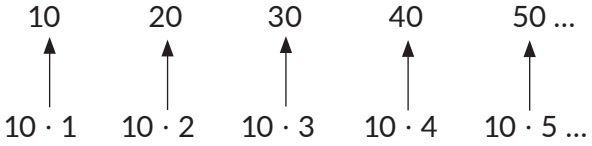
The mode of the data is 1.

multiple

A multiple is the product of a given whole number and another whole number.

Example

Multiples of 10:



multiplication property of equality

The multiplication property of equality states that if two values a and b are equal, when you multiply each by the same value c, the products are equal.

Example

$12 = 12$ and $12(7) = 12(7)$

If $a = b$, then $ac = bc$.

multiplicative inverse

The multiplicative inverse of a number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers. The product of any nonzero number and its multiplicative inverse is 1.

Examples

The multiplicative inverse of $\frac{3}{7}$ is $\frac{7}{3}: \frac{3}{7} \cdot \frac{7}{3} = \frac{21}{21} = 1$

The multiplicative inverse of 5 is $\frac{1}{5}: \frac{5}{1} \cdot \frac{1}{5} = \frac{5}{5} = 1$

multiplicative reasoning

Multiplicative reasoning focuses on the use of multiplication and division.

Example

Vicki is 40 years old, and Ben is 10 years old. Vicki is 4 times as old as Ben. In 5 years, Vicki will be 3 times as old as Ben.

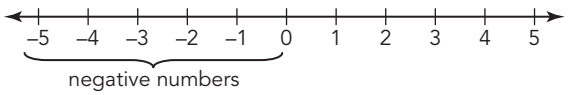
This is multiplicative reasoning.

N

negative numbers

The values to the left of zero on a number line are called negative numbers.

Example



numeric expression

A numeric expression is a mathematical phrase that contains numbers and operations.

Example

$5 \cdot 4 - 9$

O

observational study

An observational study is one method of collecting data in which a researcher collects data by observing the variable of interest.

Example

A researcher is interested in whether or not more men or women prefer a certain store. The researcher observes the number of men and women who visit the store over a number of hours and compares the values of the two groups.

one-step equation

A one-step equation is an equation that can be solved using only one operation.

order of operations

The order of operations is a set of rules that ensures the same result every time an expression is evaluated.

Example

$44 + (6 - 5) - 2 \cdot 75 \div 5^1$ Parentheses

$44 + 1 - 2 \cdot 75 \div 5^1$ Exponents

$44 + 1 - 2 \cdot 75 \div 5$ Multiplication and Division (from left to right)

$44 + 1 - 150 \div 5$

$44 + 1 - 30$ Addition and Subtraction (from left to right)

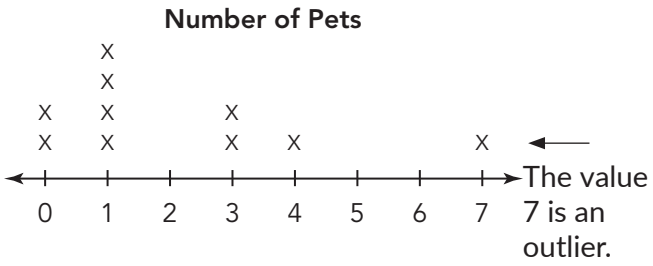
$45 - 30$

15

outliers

Outliers are data values that lie a large distance from the other data in a graph. Outliers usually accompany gaps in data.

Example



overdraft

An overdraft occurs when money is withdrawn from a bank account and the balance goes below zero. Most banks will charge a fee every time this happens.

P

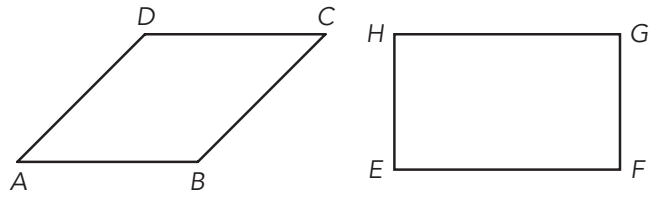
parallelogram

A parallelogram is a four-sided figure with two pairs of parallel sides and opposite sides that are equal in length.

Examples

In parallelogram *ABCD*, opposite sides *AB* and *CD* are parallel and equal in length; opposite sides *AD* and *BC* are parallel and equal in length.

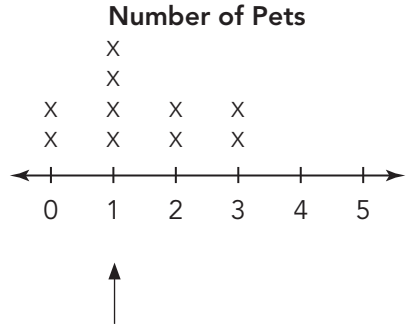
In parallelogram *EFGH*, opposite sides *EF* and *GH* are parallel and equal in length; opposite sides *FG* and *EH* are parallel and equal in length.



peaks

Peaks are values on a graph that contain more data points than the values on either side of it.

Example



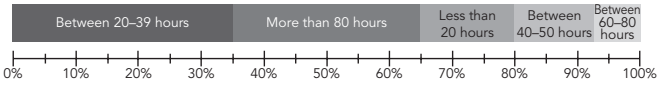
percent

A percent is a part-to-whole ratio where the whole is equal to 100. Percent is another name for hundredths. The percent symbol "%" means "per 100," or "out of 100."

percent bar graph

A percent bar graph shows a single bar that represents 100%, with sections that represent the frequency of each category. You can display a percent bar graph horizontally or vertically.

Example



perfect cube

A perfect cube is the cube of an integer.

Example

64 is a perfect cube: $4 \cdot 4 \cdot 4 = 64$

perfect square

A perfect square is the square of an integer.

Examples

9 is a perfect square: $3 \cdot 3 = 9$

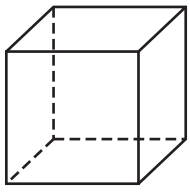
25 is a perfect square: $5 \cdot 5 = 25$

polyhedron

A polyhedron is a three-dimensional solid figure that has polygons as faces.

Example

A cube is a polyhedron. It has six square faces.



population

A population is an entire set of items from which data are collected.

Example

If you wanted to determine the average height of the students at your school, the number of students at the school would be the population.

post-secondary education

Post-secondary education refers to education beyond high school.

power

To represent repeated multiplication, use a power. A power has two elements: the base and the exponent.

Example

base → 6^2 ← exponent

power

private school

Colleges and universities are either public or private. A private school does not receive funding from the state. They raise funds from tuition and donations.

properties of inequalities

The properties of inequalities allow you to solve inequalities involving any numbers.

Examples

- addition property of inequalities
  If $a < b$, then $a + c < b + c$.
  If $a > b$, then $a + c > b + c$.
- subtraction property of inequalities
  If $a < b$, then $a - c < b - c$.
  If $a > b$, then $a - c > b - c$.
- multiplication property of inequalities
  If $a < b$, then $a \cdot c < b \cdot c$, for $c > 0$.
  If $a > b$, then $a \cdot c > b \cdot c$, for $c > 0$.
  If $a < b$, then $a \cdot c > b \cdot c$, for $c < 0$.
  If $a > b$, then $a \cdot c < b \cdot c$, for $c < 0$.
- division property of inequalities
  If $a < b$, then $\frac{a}{c} < \frac{b}{c}$, for $c > 0$.
  If $a > b$, then $\frac{a}{c} > \frac{b}{c}$, for $c > 0$.
  If $a < b$, then $\frac{a}{c} > \frac{b}{c}$, for $c < 0$.
  If $a > b$, then $\frac{a}{c} < \frac{b}{c}$, for $c < 0$.

proportion

A proportion is an equation that states that two ratios are equal.

Example

$\frac{1}{2} = \frac{4.5}{9}$

public school

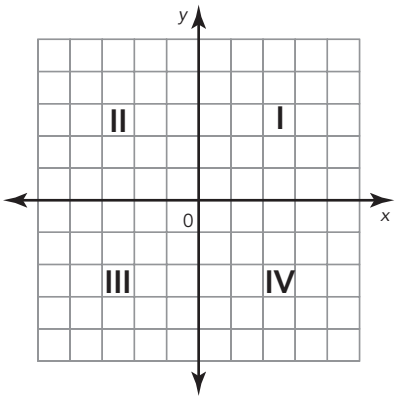
Colleges and universities are either public or private. A public school receives funding from the state government to offset the costs of running the university. The tuition to public universities is generally much less than the tuition to private schools.

Q

quadrants

The *x*- and *y*-axes divide the coordinate plane into four regions called quadrants. These quadrants are numbered with Roman numerals from one (I) to four (IV), starting in the upper right-hand quadrant and moving counterclockwise.

Example



quantitative data

Quantitative data, or numerical data, are data for which each piece of data can be placed on a numerical scale and compared.

Examples

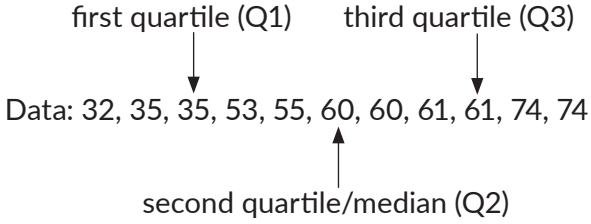
The zoo has 4 lions, 3 tigers, and 6 bears.

In 2006, Los Angeles had a population of about 3,849,378. In the same year, Atlanta had a population of about 429,500.

quartiles

Quartiles are a set of values that describe variation in a data set. When data in a set are arranged in order, quartiles are the numbers that split data into quarters (or fourths).

Example

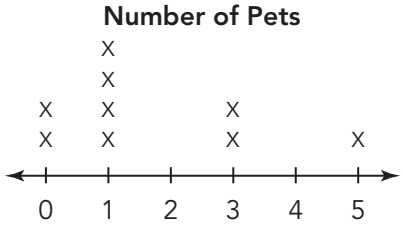


R

range

The range is the difference between the maximum and minimum values of a data set.

Example



0, 0, 1, 1, 1, 1, 3, 3, 5

$5 - 0 = 5$

The range of the data is 5.

rate

A rate is a comparison by division between two quantities that have different units.

Example

The speed of 60 miles in two hours is a rate:

$\frac{60 \text{ mi}}{2 \text{ h}} = \frac{30 \text{ mi}}{1 \text{ h}}$

ratio

A ratio is a multiplicative comparison between two quantities that may contain the same or different units.

Examples



The ratio of stars to circles is $\frac{3}{2}$, or 3:2, or 3 to 2.

The ratio of circles to stars is $\frac{2}{3}$, or 2:3, or 2 to 3.

rational numbers

Rational numbers are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples

-4, $\frac{1}{2}$, $\frac{2}{3}$, 0.67, and $\frac{22}{7}$ are examples of rational numbers.

reciprocal

The reciprocal of a number is also known as the multiplicative inverse of the number. (See *multiplicative inverse*.)

Examples

The reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$: $\frac{3}{7} \cdot \frac{7}{3} = \frac{21}{21} = 1$

The reciprocal of 5 is $\frac{1}{5}$: $\frac{5}{1} \cdot \frac{1}{5} = \frac{5}{5} = 1$

reflexive property of equality

The reflexive property of equality says that when both sides of an equation look exactly the same, their values are equal.

Examples

$7 = 7$

$a = a$

relative frequency table

A relative frequency table displays the ratio or percent of occurrences within a category to the total of the category.

Example

Test Scores for Mr. Watson's Math Test		
Test Score	Frequency	Relative Frequency
61–70	4	$\frac{4}{28} \approx 0.143 = 14.3\%$
71–80	7	$\frac{7}{28} = 0.25 = 25\%$
81–90	9	$\frac{9}{28} \approx 0.321 = 32.1\%$
91–100	8	$\frac{8}{28} \approx 0.286 = 28.6\%$

relatively prime

Two numbers that do not have any common factors other than 1 are called relatively prime.

Example

Positive whole number pairs that have a difference of 1 (4 and 5, 10 and 11, 15 and 16) are always relatively prime.

repeating decimal

A repeating decimal is a decimal in which a digit, or a group of digits, repeat(s) infinitely. Repeating decimals are rational numbers.

Examples

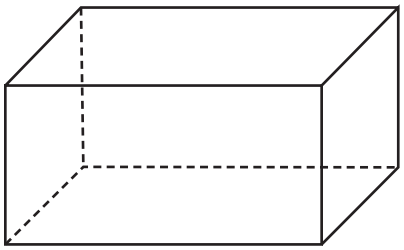
$\frac{1}{9} = 0.111\ldots$ $\frac{7}{12} = 0.58333\ldots$

$\frac{22}{7} = 3.142857142857\ldots$

right rectangular prism

A right rectangular prism is a polyhedron with three pairs of congruent and parallel rectangular faces.

Example



S

sample

A sample is a selection from a population.

Example

If you wanted to determine the average height of the students in your school, you could choose a certain number of students and measure their heights. The heights of the students in this group would be your sample.

scale factor

The scale factor is the factor that you multiply or divide both parts of a ratio by to scale up or scale down to an equivalent ratio.

scaling down

Scaling down means to divide both parts of the ratio by the same factor greater than 1, or multiply both parts of the ratio by the same factor less than 1.

Example

$$\frac{3}{6} = \frac{1}{2} \quad (\div 3 \text{ above and below})$$

scaling up

Scaling up means to multiply both parts of a ratio by the same factor greater than 1.

Example

$$\frac{1}{2} = \frac{3}{6} \quad (\cdot 3 \text{ above and below})$$

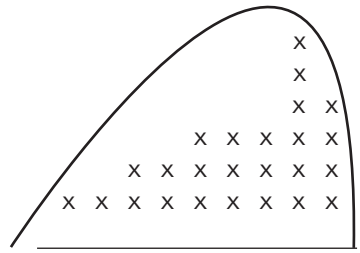
scholarship

A scholarship is money awarded to a person from the college or private institution to help a student pay for school. Scholarships may be academic, athletic, based on financial need, or awarded from private organizations for a variety of reasons.

skewed left distribution

In a skewed left distribution of data, the peak of the data is to the right side of the graph. There are only a few data points to the left side of the graph.

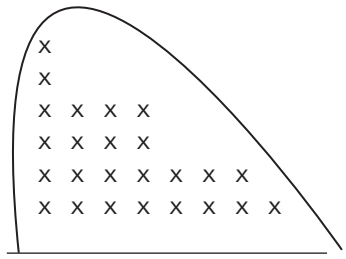
Example



skewed right distribution

In a skewed right distribution of data, the peak of the data is to the left side of the graph. There are only a few data points to the right side of the graph.

Example



CC BY NC

solution

A solution to an equation is any value for a variable that makes the equation true.

Example

The solution to the equation $2x + 4 = 8$ is $x = 2$.

solution set of the inequality

The set of all points that make an inequality true is the solution set of the inequality.

Examples

$x < 7$

The solution set for $x \geq 7$ is all the numbers greater than or equal to 7.

solve an inequality

To solve an inequality means to determine the values of the variable that make the inequality true.

Example

$2x + 4 \geq 16$

$2x \geq 12$

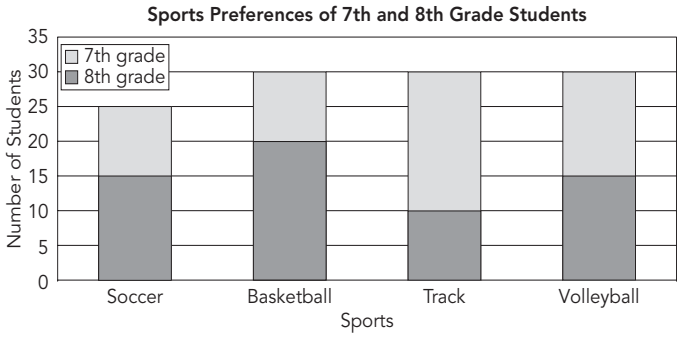
$x \geq 6$

Any value for x that is greater than or equal to 6 will make the inequality true.

stacked bar graph

A stacked bar graph is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole.

Example



statement

A statement is a summary of the account balance. A statement includes every transaction that occurred throughout the month including deposits, withdrawals, debits, and transfers.

statistical process

The statistical process has four components:

- Formulating a statistical question
- Collecting appropriate data
- Analyzing the data graphically and numerically
- Interpreting the results of the analysis

statistical question

A statistical question is a question that anticipates an answer based on data that vary.

Example

"What sport is the most popular in your school?" is a statistical question because it anticipates that the answers will vary since not everyone at your school is likely to have the same favorite sport.

"How many students are in Chess Club?" is NOT a statistical question because there is only one answer to the question.

stem-and-leaf plot

A stem-and-leaf plot is a graphical method used to represent ordered numerical data. Once the data are ordered, the stem and leaves are determined. Typically, the stem is all the digits in a number except the rightmost digit, which is the leaf.

Example

Books Read in Mr. Brown's Class

0	3, 6
1	0, 1, 5
2	
3	9, 9
4	0, 0, 0

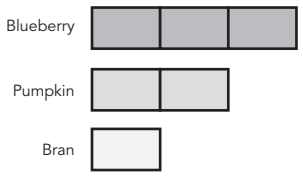
Key: 1 | 0 = 10

strip diagram

A strip diagram illustrates number relationships by using rectangles to represent ratio parts.

Example

A bakery sells packs of muffins in the ratio of 3 blueberry muffins : 2 pumpkin muffins : 1 bran muffin. The strip diagram represents the ratio of each type of muffin.



student loan

A student loan is money borrowed in order to pay for college or trade school.

subtraction property of equality

The subtraction property of equality states that when you subtract the same value c from equal values a and b, the differences are equal.

Examples

$12 = 12$ and $12 - 7 = 12 - 7$

If $a = b$, then $a - c = b - c$.

survey

A survey is one method of collecting data in which people are asked one or more questions.

Example

A restaurant may ask its customers to complete a survey with the following question:

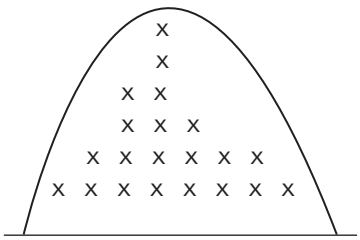
On a scale of 1–10, with 1 meaning "poor" and 10 meaning "excellent," how would you rate the food you ate?

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

symmetric distribution

In a symmetric distribution of data, the left and right halves of the graph are mirror images of each other. The peak is in the middle because there are many data values in the center.

Example



CC BY NC

symmetric property of equality

The symmetric property of equality states that if $a = b$, then $b = a$.

Example

$x = 3$ is the same as $3 = x$.

T

term

A term of an algebraic expression is a number, variable, or product of numbers and variables.

Example

The expression has four terms.

$3y + 5xy + \frac{1}{2}x + 6$

terms

terminating decimal

A terminating decimal has a finite number of digits, meaning that after a finite number of decimal places, all following decimal places have a value of 0. Terminating decimals are rational numbers.

Examples

$\frac{9}{10} = 0.9$ $\frac{15}{8} = 1.875$ $\frac{193}{16} = 12.0625$

transfer

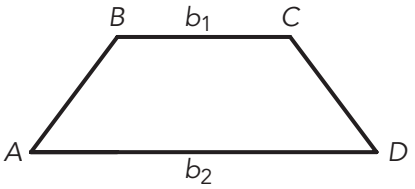
A transfer occurs when money is moved from one account to another.

trapezoid

A trapezoid is a quadrilateral with two bases that are parallel to each other, often labeled b_1 and b_2.

Example

Quadrilateral *ABCD* is a trapezoid. Side *BC* is parallel to side *AD*.



Triangle Inequality theorem

The Triangle Inequality theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Triangle Sum theorem

The Triangle Sum theorem states that the sum of the measures of the interior angles of a triangle is 180°.

tuition

Tuition is the fee paid in order to receive instruction at a school.

U

undergraduate degree

An undergraduate degree is a four-year degree from a college or university.

unit cube

A unit cube is a cube in which its sides are all 1 unit long.

unit fraction

A unit fraction is a fraction that has a numerator of 1 and a denominator that is a positive integer.

unit rate

A unit rate is a comparison of two different measurements in which the numerator or denominator has a value of one unit.

Example

The speed 60 miles in 2 hours can be written as a unit rate:

$\frac{60 \text{ mi}}{2 \text{ h}} = \frac{30 \text{ mi}}{1 \text{ h}}$.

The unit rate is 30 miles per hour.

unit rate of change

The unit rate of change describes the amount the dependent variable changes for every unit the independent variable changes.

V

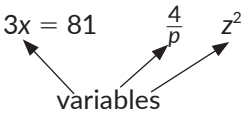
variability

In statistics, variability means that the value of the attribute being studied can change from one person or thing to another.

variable

A variable is a letter or symbol that is used to represent a number.

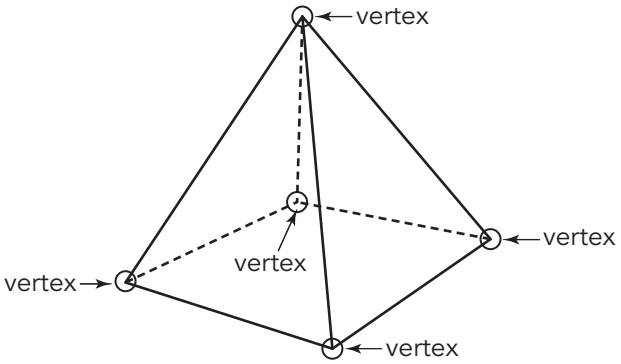
Examples



vertex

A vertex of a polyhedron is a point at which three or more of its edges meet.

Example



vocational school

A vocational school is a school in which students are taught the skills needed to perform a particular job.

volume

Volume is the amount of space occupied by an object. Volume is measured in cubic units.

W

withdrawal

A withdrawal occurs when money is taken out of an account, usually from a check, online payment, or Automated Teller Machine (ATM).

work-study program

A work-study program is a plan set up by the college that allows a student to work while in school to earn money.

Z

zero pair

A positive counter and a negative counter together make a zero pair, since the total value of the pair is zero.

Example

zero property of multiplication

The zero property of multiplication states that the product of any number and 0 is 0.

Examples

$6 \cdot 0 = 0$

$\frac{3}{4} \cdot 0 = 0$

$5^2 \cdot 0 = 0$

$0.125(0)\ 0 = 0$

ISBN: 978-1-970197-16-7

© 2024 Texas Education Agency. Portions of this work are adapted, with permission, from the originals created by and copyright © 2021 Carnegie Learning, Inc.

This work is licensed under a Creative Commons Attribution-Non-Commerial-4.0 International License.

You are free:

to Share—to copy, distribute, and transmit the work
to Remix—to adapt the work

Under the following conditions:

Attribution—You must attribute any adaptations of the work in the following manner:

> This work is based on original works of the Texas Education Agency and Carnegie Learning, Inc. This work is made available under a Creative Commons Attribution-Non-Commercial-4.0 International License. This does not in any way imply endorsement by those authors of this work.

NonCommercial—You may not use this work for commercial purposes.

With the understanding that:

For any reuse or distribution, you must make clear to others the license terms of this work. The best way to do this is with a link to this web page:

> https://creativecommons.org/licenses/by-nc/4.0/

Trademarks and trade names are shown in this book strictly for illustrative and educational purposes and are the property of their respective owners. References herein should not be regarded as affecting the validity of said trademarks and trade names.

Printed in the USA

Images

www.pixabay.com