Bluebonnet Learning
Secondary Mathematics
EDITION 1

Grade 6

Volume 1

STUDENT EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

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Bluebonnet Learning

Secondary Mathematics

EDITION 1

Grade 6

Course Guide

GRADE 6

Welcome to the Course Guide for Secondary Mathematics, Grade 6

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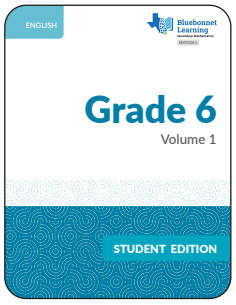
Course Table of Contents FM-31

Instructional Design

The instructional materials help you learn math in different ways. There are two types of resources: Learning Together and Learning Individually. These resources provide various learning experiences to develop your understanding of mathematics.

Learning Together

On Learning Together days, you spend time engaging in active learning to build mathematical understanding and confidence in sharing ideas, listening to one another, and learning together. The Student Edition is a consumable resource that contains the materials for each lesson.



STUDENT EDITION

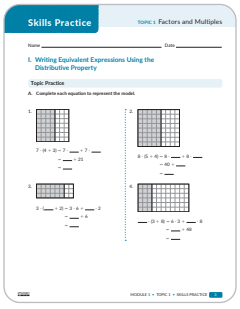
I am a record of your thinking, reasoning, and problem solving.

My lessons allow you to build new knowledge from prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

Learning Individually

On Learning Individually days, Skills Practice offers opportunities to engage with skills, concepts, and applications that you learn in each lesson. It also provides opportunities for interleaved practice, which encourages you to flexibly move between individual skills, enhancing connections between concepts to promote long-term learning. This resource will help you build proficiency in specific skills based on your individual academic needs as indicated by monitoring your progress throughout the course.



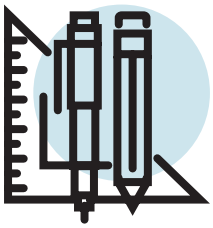
SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student edition.

My purpose is to provide problem sets for additional practice, enrichment, and extension.

Instructional Design

The instructional materials intentionally emphasize active learning and sense-making. Deep questions ensure you thoroughly understand the mathematical concepts. The instructional materials guide you to connect related ideas holistically, supporting the integration of your evolving mathematical understanding and developing proficiency with mathematical processes.



Intentional Mathematics Design

Mathematical Coherence: The path through the mathematics develops logically, building understanding by linking ideas within and across grades so you can learn concepts more deeply and apply what you've learned to more complex problems.

TEKS Process Standards: The instructional materials support your development of the TEKS mathematical process standards. They encourage you to experiment, think creatively, and test various strategies. These mathematical processes empower you to persevere when presented with complex real-world problems.

Multiple Representations: The instructional materials connect multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: The instructional materials focus on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

What principles guide the design and organization of the instructional materials?

Active Learning: Studies show that learning becomes more robust as instruction moves from entirely passive to fully interactive (Chi, 2009). Classroom activities require active learning as students engage by problem solving, explaining and justifying reasoning, classifying, investigate, and analyze peer work.

Discourse Through Collaborative Learning: Collaborative problem solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities intentionally promote active dialogue centered on structured activities.

Personalized Learning: Research has proven that problems that capture your interests are more likely to be taken seriously (Baker et. al, 2009). In each lesson, learning is linked to prior knowledge and experiences, creating valuable and authentic opportunities for you to build new understanding on the firm foundation of what you already know. You move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures.

Focus on Problem Solving: Solving problems is an essential life skill that you need to develop. The problem-solving model provides a structure to support you as you analyze and solve problems. It is a strategy you can continue to use as you solve problems in everyday life.

1 Writing Equivalent Expressions Using the Distributive Property

1 OBJECTIVES

- Write, read, and evaluate equivalent numeric expressions.
- Identify the adjacent side lengths of a rectangle as factors of the area value.
- Identify parts of an expression, such as the product and the factors.
- Write equivalent numeric expressions for the area of a rectangle by decomposing one side length into the sum of two or more numbers.
- Apply the distributive property to rewrite the product of two factors.

3 NEW KEY TERMS

- numeric expression
- equation
- distributive property

2

You know how to add, subtract, multiply, and divide numbers using different strategies. Taking apart numbers before you perform a mathematical operation can highlight important information or make calculations easier.

How can taking apart numbers help you to express number sentences in different ways?

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LESSON STRUCTURE

1 Objectives

Objectives are stated for each lesson to help you take ownership of the objectives.

2 Essential Question

Each lesson begins with a statement connecting what you have learned with a question to ponder. Return to this question at the end of this lesson to gauge your understanding.

3 New Key Terms

The new key terms for each lesson are identified to help you connect your everyday and mathematical language.

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4 Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

4 Getting Started

Ask Yourself . . .
How can you use the area of rectangles in everyday life?

Break It Down to Build It Up

Sofia is building a rectangular walkway up to her house. The width of the walkway is 5 feet and the length is 27 feet. She needs to calculate the area of the walkway to determine the amount of materials needed to build it.

1. Mark and label two different ways you could divide an area model to determine the area of the walkway.

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

2. Determine the areas of each of the subdivided parts of your models.

3. What is the total area of the walkway?

5

ACTIVITY 2.1

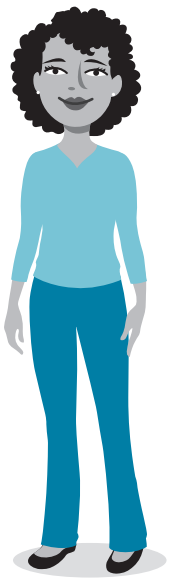
Prime Factors

You just determined the factors for a given number as well as the common factors that two numbers share.

In this activity, you will learn to determine the prime factors of a given number.

A factor tree is a way to organize the prime factors of a number. Choose any factor pair to get started.

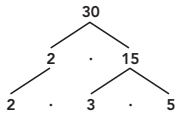
When you express any whole number as a product of primes, only primes, and nothing, you are determining the **prime factorization** of that number.



WORKED EXAMPLE

Use a factor tree to write the prime factorization for 30.

- Begin with the number 30.
- Pick any whole number factor pair of 30, other than 1 and 30.
- Draw a branch from 30 to each factor, 2 and 15.
- Since one of the factors is not prime, you are not finished.
- Use branches to write a factor pair for 15.
- Because 2, 3, and 5 are all prime, this factor tree is complete.



1. Use the factor tree to write the prime factorization of 30.

The factor tree in the Worked Example is not the only factor tree you can create for 30.

2. How many different factor trees are there for 30?

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5 Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about getting the answer. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

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6 Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

Talk the Talk 6

The Floor Is Yours

You can apply the distributive property to solve real-world problems. Consider the situation.

Diego is setting up the gym floor for an after-school program. He wants to include a rectangular area for playing volleyball and another for dodgeball. He also wants to have an area for kids who like to play board games or just sit and read. The gym floor is already 50 feet by 84 feet, or 4200 square feet.

1. Create a diagram to show how you would split up the gym floor. Represent your diagram using the distributive property and write an explanation for the areas assigned to each activity.

Lesson 1 Assignment

7 Write

Explain the distributive property in terms of composing and decomposing numbers.

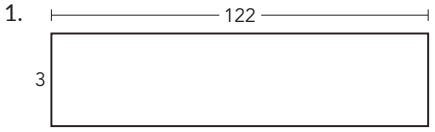
Remember 8

There are many ways to rewrite equivalent expressions using properties. The distributive property of multiplication over addition states that for any numbers a, b, and c, $a(b + c) = ab + ac$.

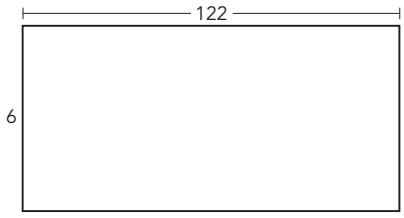
9 Practice

Decompose each rectangle into two or three smaller rectangles to demonstrate the distributive property. Then, write each area in the form $a(b + c) = ab + ac$.

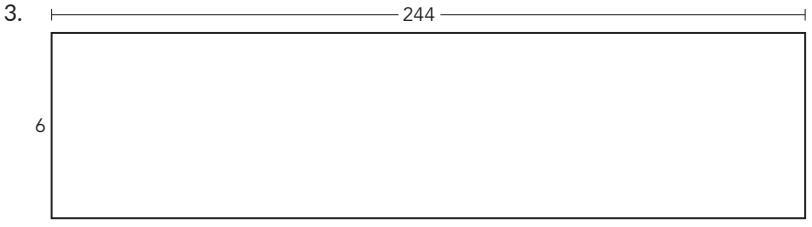
1.



2.



3.



ASSIGNMENT

7 Write

Reflect on your work and clarify your thinking.

8 Remember

Take note of the key concepts from the lesson.

9 Practice

Use the concepts learned in the lesson to solve problems.

Lesson 1 Assignment

4. $6(12 + 4)$

5. $10 + 4(2 + 20)$

6. $7(4 + 19)$

10 Prepare

1. In the array of numbers shown, circle the prime numbers, cross out the composite numbers, and use a box to identify any number that is neither prime nor composite.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

ASSIGNMENT

10 Prepare

Get ready for the next lesson.

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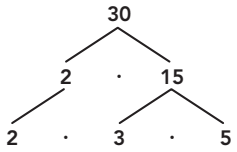
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Research-Based Strategies

WORKED EXAMPLE

Use a factor tree to write the prime factorization for 30.

- Begin with the number 30.
- Pick any whole number factor pair of 30, other than 1 and 30.
- Draw a branch from 30 to each factor, 2 and 15.
- Since one of the factors is not prime, you are not finished.
- Use branches to write a factor pair for 15.
- Because 2, 3, and 5 are all prime, this factor tree is complete.



WORKED EXAMPLE

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

Ask Yourself . . .

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

Jayden and Mateo want to determine the number of miles in 31,680 feet using unit analysis.

Mateo

$31{,}680\ \text{ft} \cdot \frac{1\ \text{mi}}{5280\ \text{ft}} = 6\ \text{mi}$

Jayden

$31{,}680\ \text{ft} \left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right) =$

$167{,}270{,}400\ \text{mi}$

Ask Yourself . . .

- Why is this method correct?
- Have I used this method before?

Ask Yourself . . .

- Where is the error?
- Why is it an error?
- How can I correct it?

THUMBS UP

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

THUMBS DOWN

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

WHO'S CORRECT?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine whether the work is correct or incorrect.



4. Jamal and Tiara are solving the equation $5 = \frac{p}{7}$.
   a. Jamal says that to solve $5 = \frac{p}{7}$, he would divide by 7. The value of p that makes the equation true is $\frac{5}{7}$. Tiara disagrees and says that they should divide by $\frac{1}{7}$ and the solution is 35. Who is correct?

   b. How can Jamal and Tiara check to see whose answer is correct?

Ask Yourself . . .

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

The Crew

The Crew is here to help you on your journey. Sometimes, they will remind you about things you already learned. Sometimes, they will ask you questions to help you think about different strategies. Sometimes, they will share fun facts. They are members of your group—someone you can rely on!



Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.



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TEKS Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I Can" expectations listed below align with the TEKS mathematical process standards and encourage students to develop their mathematical learning and understanding.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I CAN:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I CAN:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology, as appropriate; and techniques including mental math, estimation, and number sense, as appropriate, to solve problems.

I CAN:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language, as appropriate.

I CAN:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Create and use representations to organize, record, and communicate mathematical ideas.

I CAN:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

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Analyze mathematical relationships to connect and communicate mathematical ideas.

I CAN:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I CAN:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.



The Problem-Solving Model

Productive mathematical thinkers are problem solvers. These instructional materials include a problem-solving model to help you develop proficiency with the TEKS mathematical process standards. An opportunity to use the problem-solving model is present every time you see the problem-solving model icon.

The problem-solving model represents a strategy you can use to make sense of problems you must solve. The model provides questions you can use to guide your thinking and the graphic organizer provides a place for you to show and organizer your work.

Understanding the Problem-Solving Model



Notice | Wonder

Understand the situation by asking these questions.

- What do I notice?
- What do I wonder?
- How do I analyze the given information to identify what is important?
- Do I have enough information to formulate a plan and determine a solution?



Organize | Mathematize

Devise a plan for your mathematical approach by asking these questions.

- What mathematical relationships exist between this problem and similar problems I have solved?
- What plan or strategy can I use to solve this problem?
- How can I efficiently solve this problem?
- How can I organize, record, and communicate my mathematics?



Predict | Analyze

Carry out your plan to determine a solution. Then, ask yourself the following questions.

- Did I display my work using multiple representations?
- Did I explain my reasoning in terms of the problem situation?
- Did I communicate the strategy used to determine the solution?
- Did I justify my mathematical argument clearly using precise mathematical language?
- Can I use my mathematical reasoning to make any predictions?



Test | Interpret

Look back at your work and ask these questions.

- Does my solution clearly and completely answer the original question/problem?
- Is my solution reasonable?
- Does my solution make sense in terms of the problem situation?
- Can I solve the problem using a different strategy? Would another strategy be more efficient?
- Can I justify my solution?



Report

As you share your mathematical reasoning with others ask these questions.

- Did you use multiple representations to represent your mathematics?
- Did you justify your mathematical reasoning?
- Can others understand my process and solution?

The Problem-Solving Model Graphic Organizer



Understand the Problem



Devise a Plan



Carry Out the Plan



Look Back



Report

Academic Glossary

Knowing what these terms mean and using them will help you demonstrate proficiency with the TEKS mathematical process standards as you think, reason, and communicate your ideas.

Analyze

Definition

Study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

Explain Your Reasoning

Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself...

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

- Show your work
- Explain your calculation
- Justify
- Why or why not?

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Represent

Definition

Display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

- Predict
- Approximate
- Expect
- About how much?

Estimate

Definition

Make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Describe

Definition

Represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

What is Productive Struggle?

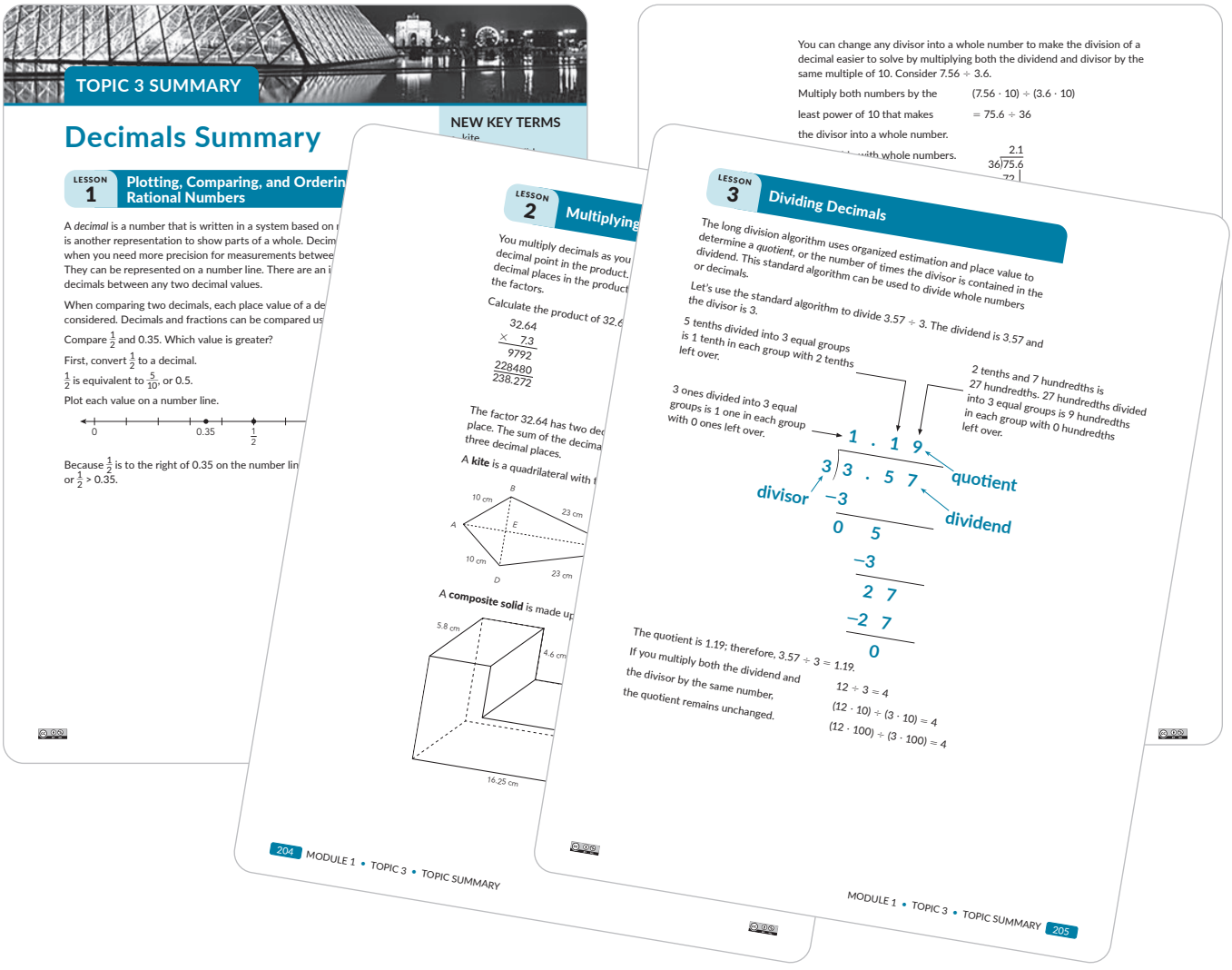
Imagine you are trying to beat a game or solve a puzzle. At some point, you are not sure what to do next. You don't give up; you keep trying repeatedly until you accomplish your goal. You call this process *productive struggle*. Productive struggle is when you choose to persist, think creatively, and try various strategies to accomplish your goal. Productive struggle supports you as you develop characteristics and habits that you will use in everyday life.

Things to do:	Things not to do:
• Persevere. • Think creatively. • Try different strategies. • Look for connections to other questions or ideas. • Ask questions that help you understand the problem. • Help your classmates without telling them the answers.	• Get discouraged. • Stop after trying your first attempt. • Focus on the final answer. • Think you have to make sense of the problem on your own.

This course will challenge you by providing you with opportunities to solve problems that apply the mathematics you know to the real-world. As you work through these problems, your first approach may not work. This is okay, even when your initial strategy does not work, you are learning. In addition, you will discover multiple ways to solve a problem. Looking at various strategies will help you evaluate the problem-solving process and identify an efficient approach. As you persist in problem-solving, you will better understand the math.

Topic Summary

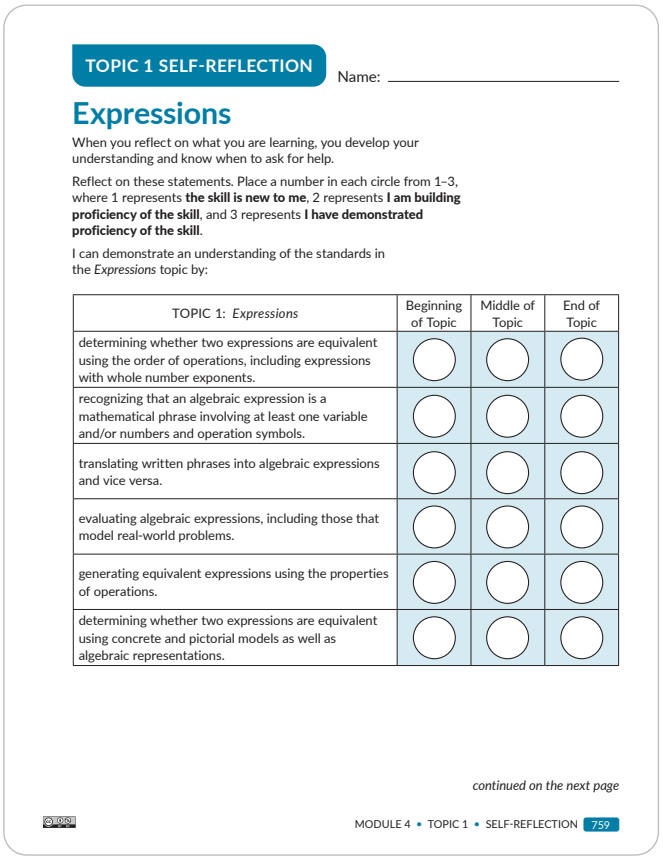
Each topic includes a Topic Summary. The Topic Summary contains a list of all new key terms addressed in the topic and a summary of each lesson, including worked examples and new key term definitions. Use the Topic Summary to review each lesson's major concepts and strategies as you complete assignments and/or share your learning outside of class.



Topic Self-Reflection

The Topic Self-Reflection, provided at the end of each topic, empowers you to develop confidence in your mathematical understanding and monitor your own learning processes. Taking the time for self-reflection helps you identify your strengths and where you want to focus your efforts to improve.

Use the Topic Self-Reflection throughout the topic to monitor your progress toward the mathematical goals for the topic.



TOPIC 1 SELF-REFLECTION

Name: _____

Expressions

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

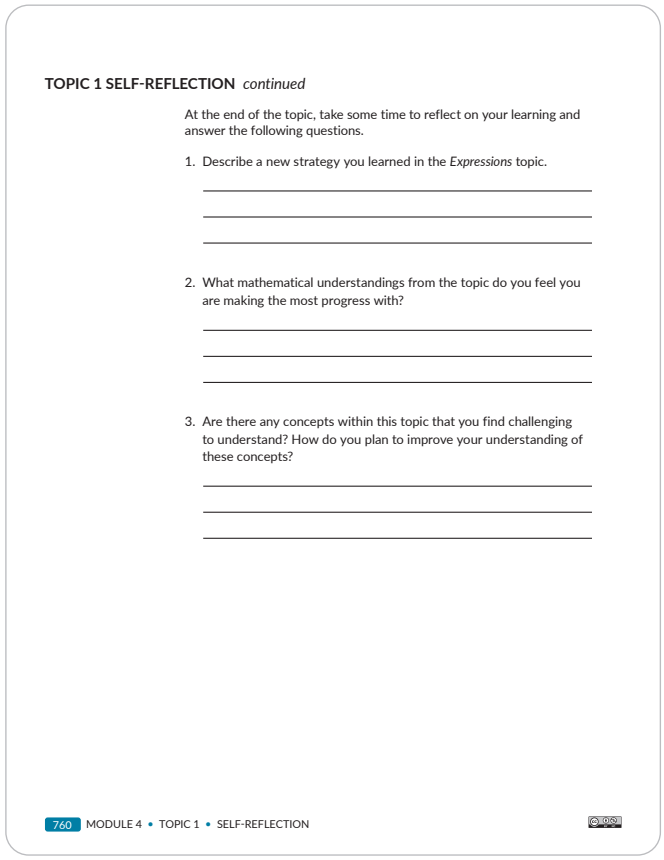
Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Expressions* topic by:

TOPIC 1: *Expressions*	Beginning of Topic	Middle of Topic	End of Topic
determining whether two expressions are equivalent using the order of operations, including expressions with whole number exponents.	○	○	○
recognizing that an algebraic expression is a mathematical phrase involving at least one variable and/or numbers and operation symbols.	○	○	○
translating written phrases into algebraic expressions and vice versa.	○	○	○
evaluating algebraic expressions, including those that model real-world problems.	○	○	○
generating equivalent expressions using the properties of operations.	○	○	○
determining whether two expressions are equivalent using concrete and pictorial models as well as algebraic representations.	○	○	○

continued on the next page

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TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Expressions* topic.

   _____

   _____

   _____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   _____

   _____

   _____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   _____

   _____

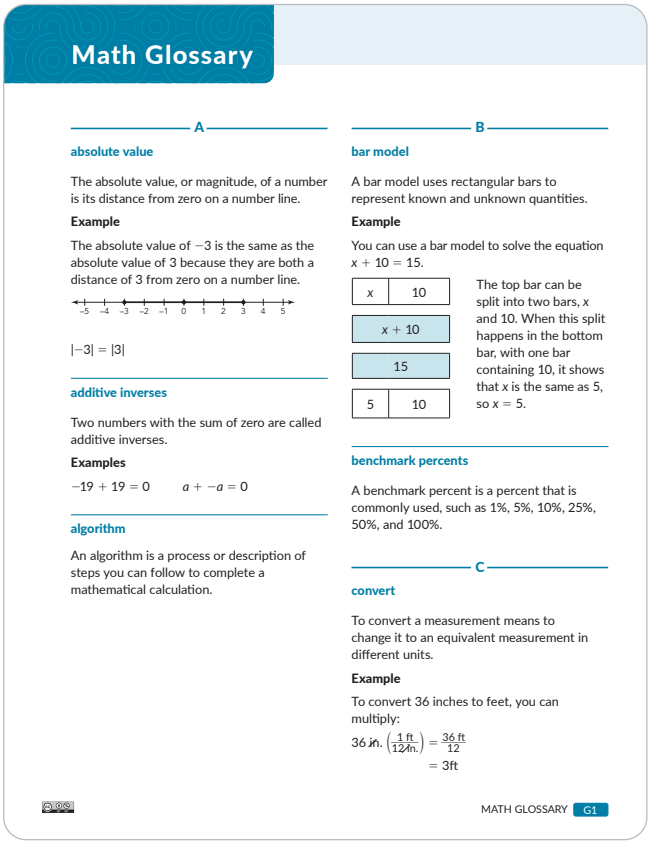
   _____

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Math Glossary

A course-specific math glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.



Math Glossary

A

absolute value

The absolute value, or magnitude, of a number is its distance from zero on a number line.

Example

The absolute value of -3 is the same as the absolute value of 3 because they are both a distance of 3 from zero on a number line.

$|-3| = |3|$

additive inverses

Two numbers with the sum of zero are called additive inverses.

Examples

$-19 + 19 = 0$ $a + -a = 0$

algorithm

An algorithm is a process or description of steps you can follow to complete a mathematical calculation.

B

bar model

A bar model uses rectangular bars to represent known and unknown quantities.

Example

You can use a bar model to solve the equation $x + 10 = 15$.

x	10
$x + 10$	
15	
5	10

The top bar can be split into two bars, x and 10. When this split happens in the bottom bar, with one bar containing 10, it shows that x is the same as 5, so $x = 5$.

benchmark percents

A benchmark percent is a percent that is commonly used, such as 1%, 5%, 10%, 25%, 50%, and 100%.

C

convert

To convert a measurement means to change it to an equivalent measurement in different units.

Example

To convert 36 inches to feet, you can multiply:

$36\ \text{in.} \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right) = \frac{36\ \text{ft}}{12}$

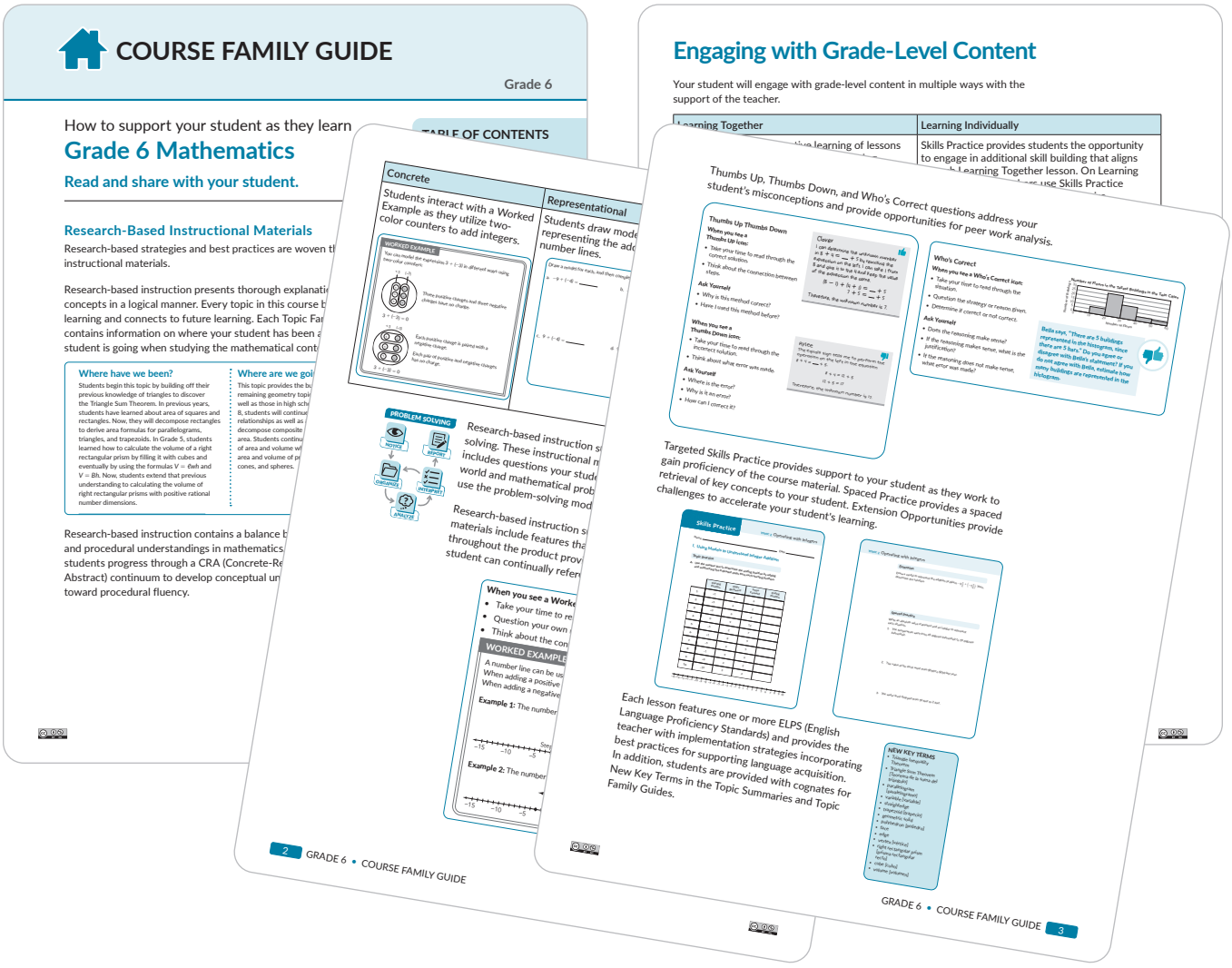
$= 3\text{ft}$

MATH GLOSSARY G1

Course Family Guide

The Course Family Guide provides you and your family with an overview of the course design. The guide details the resources available to support your learning, such as the Math Glossary, the Topic Family Guide, the Topic Self-Reflection, and the Topic Summaries.

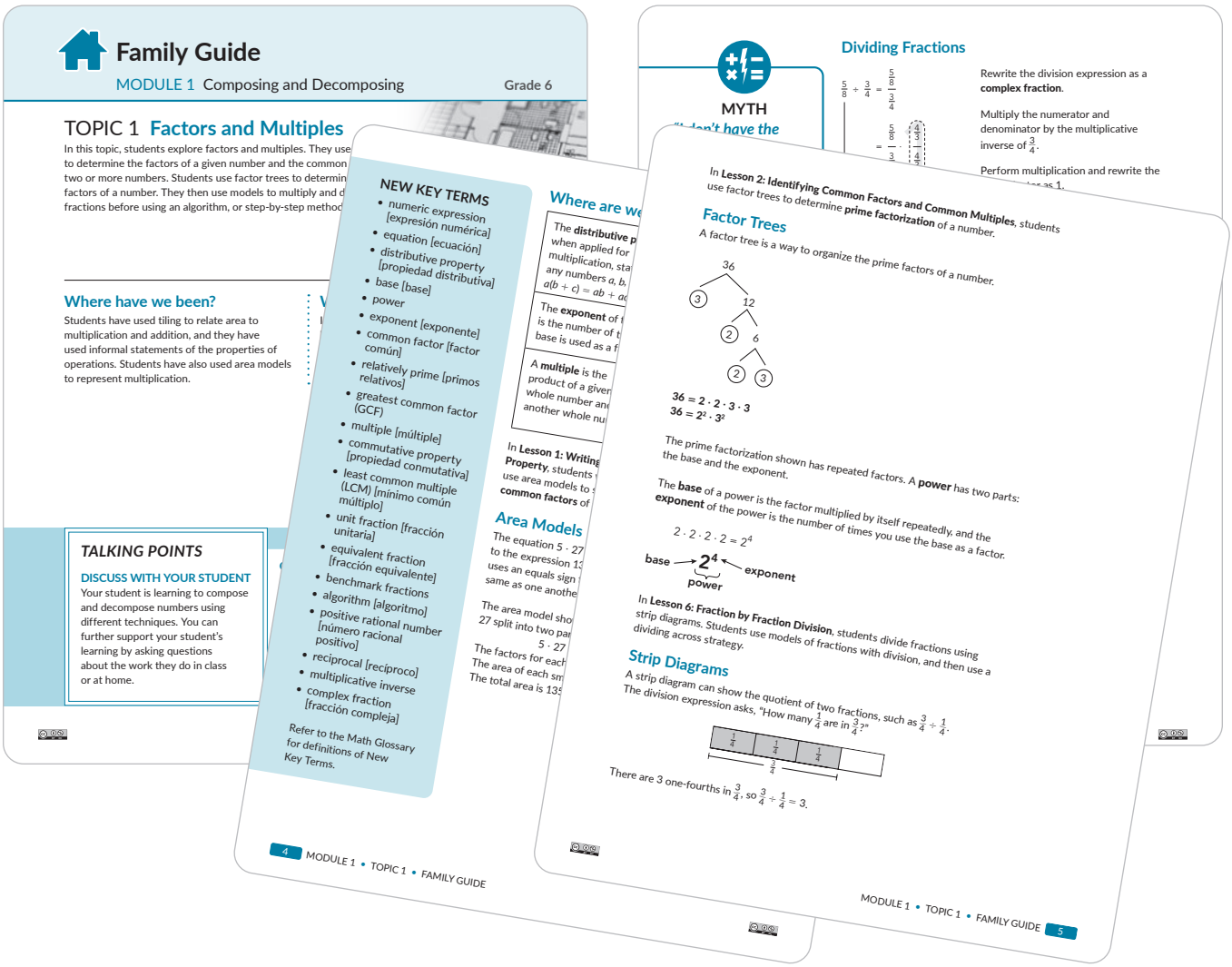
The purpose of the Course Family Guide is to bridge your learning in the classroom to your learning at home. The goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together.



Topic Family Guides

Each topic contains a Topic Family Guide that provides an overview of the math of the topic and answers the questions, "Where have we been?" and "Where are we going?" Additional components of the Topic Family Guide are an example of a math model or strategy taught in the topic, definitions of a few key terms, busting of a math myth, and questions family members can ask you to support your learning.

Learning outside of the classroom is crucial to student success at school. The Topic Family Guide serves to assist families in talking to students about the learning that is happening in the classroom.



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Math Glossary G-1

GRADE 6

MODULE 1

Composing and Decomposing

TOPIC 1 Factors and Multiples 3

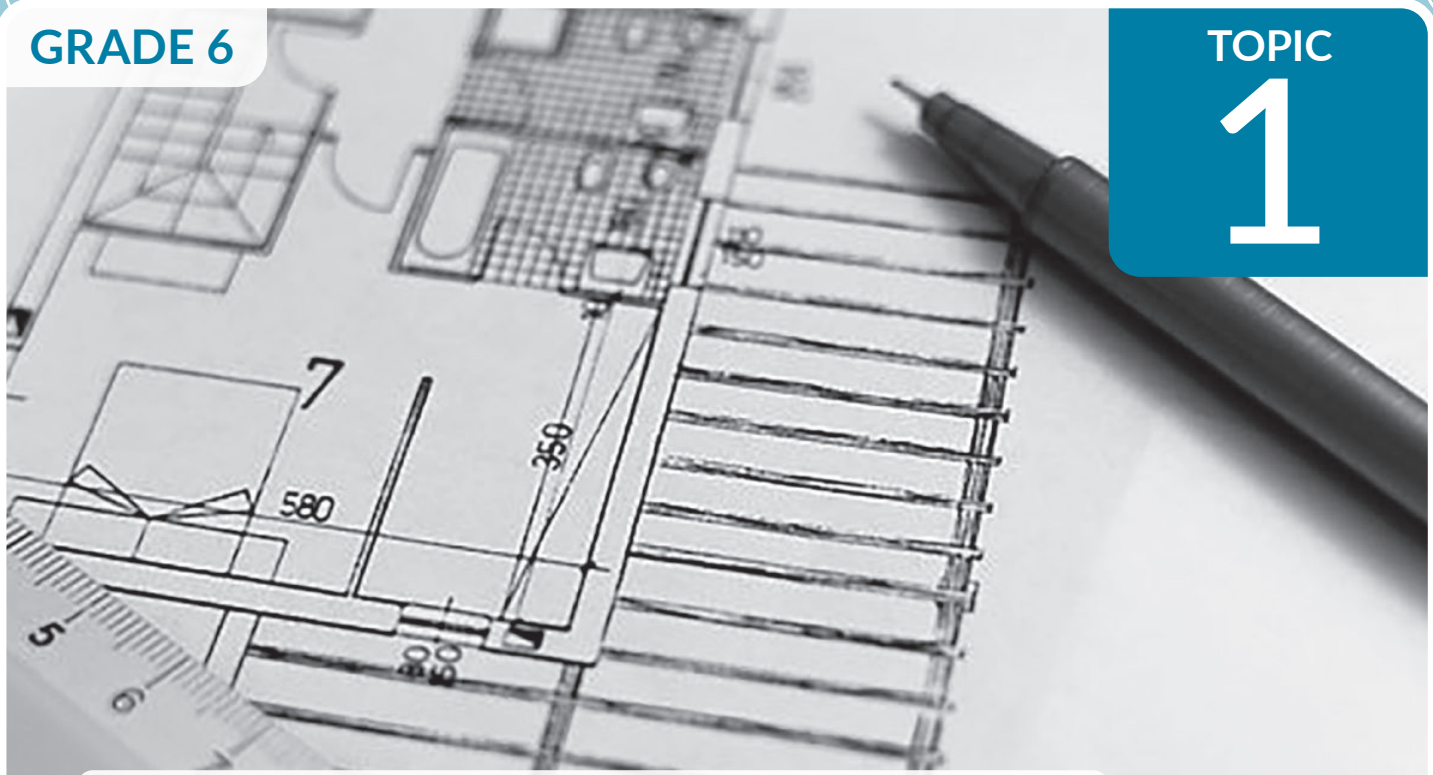
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GRADE 6

TOPIC 1



Architects, city planners, landscape gardeners, and others determine areas using measuring and multiplication.

Factors and Multiples

INTRODUCTION LESSON

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INTRODUCTION LESSON

Introduction to the Problem-Solving Model and Learning Resources

OBJECTIVES

- Establish a community of learners.
- Discover learning resources available for this course.
- Apply the problem-solving model to a real-life situation.

In previous math classes, you have analyzed patterns and relationships, learned about numbers and operations in base ten and fractions, measurement and data, and geometry.

What resources are available in this course to help you extend your mathematical thinking?

Getting Started

You Already Know a Lot

Each lesson in this book begins with a Getting Started that gives you the opportunity to use what you know about the world and what you have learned in previous math classes. You know a lot from a variety of learning experiences.

Ask Yourself . . .
How do your strategies change based on what you are learning and what you already know about it?

Think back to how you learn something new.

1. List three different skills that you recently learned. Then, describe why you wanted to learn that skill and the strategies that you used.

New Skill	Motivation to Learn the New Skill	Strategies I Used to Learn This Skill

One learning strategy is to talk with your peers. In this course, you will work with your classmates to solve problems, discuss strategies, and learn together.

Compare and discuss your list with a classmate.

Think About . . .
Listening well, cooperating with others, and appreciating different perspectives are essential life skills.

2. Which strategies do you have in common? Which strategies does your classmate have that you did not think of on your own?

Be prepared to share your list of learning strategies with the class.

ACTIVITY
1.1 Learning Resources



In this course, you will learn new math concepts by exploring and investigating ideas, reading, writing, and talking to your classmates. You will even learn by making mistakes with concepts you haven't mastered yet.

Let's practice exploring and investigating. You do not need to answer the question yet. You will solve the question as you work through the problem-solving model.

Ethan has a collection of 48 baseball cards. He decides to organize his cards into groups so that each group contains the same number of cards. **Describe** the ways in which Ethan can organize his cards.

The Academic Glossary is your guide as you engage with the kind of thinking you do as you are learning the content.

1. Locate the word **describe** in the Academic Glossary in the Course Guide. What questions should you ask yourself as you describe the ways Ethan can organize his cards?

2. What is a related word or phrase for the word **describe**?

The Academic Glossary provides definitions of terms you will see and use throughout the course as you think, reason, and communicate your ideas. The Math Glossary provides the definitions of the new key terms in each lesson.

The problem-solving model provides a structure to help you become a better problem-solver.

Notice and Wonder



The first step in modeling a situation mathematically is to understand the problem, gather information, notice patterns, and formulate mathematical questions about what you notice.

Read through the questions to ask yourself for the first step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

3. What do you notice about the number of cards Ethan has?

4. Why do you think the first step of the problem-solving model is important? How will it help you when you solve problems?



Organize and Mathematize

The second step in the problem-solving model is to devise a plan. When devising a plan, you will organize your information and begin to represent it using mathematical notation.

Read through the questions to ask yourself for the second step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

5. Describe your strategy for organizing the ways Ethan can group his cards.

6. Why do you think the second step of the problem-solving model process is important? How can you use the questions to ask yourself to help develop a strategy to solve the problem?



Predict and Analyze

The third step in the modeling process involves carrying out your plan. As you carry out the plan you will complete operations, analyze mathematical results, make predictions, and extend patterns.

Read through the questions to ask yourself for the third step of the problem-solving model. Use the Problem-Solving Model Graphic Organizer to answer the questions and show your thinking.

7. Use your strategy to determine the different ways Ethan can group his cards.

8. Ethan says 3 groups of 16 is the same as 16 groups of 3 because the total number of cards is the same. Do you agree with Ethan? Explain your reasoning.

9. How can the questions to ask for the third step help you communicate your mathematical thinking?

Test and Interpret



The fourth step in the modeling process is to look back, interpret your results, and test your mathematical predictions in the real world. When your predictions are incorrect, or your results are not reasonable, you can revisit your mathematical work and make adjustments—or start all over!

Read through the questions to ask yourself for the fourth step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

10. Ethan purchases 8 additional cards. Can you apply the same reasoning to determine the ways he can organize 56 cards into equal groups? Explain your reasoning.

11. How do the questions for the fourth step of the problem-solving model help you evaluate the reasonableness of your solution?



Report

The final step in the modeling process is to share your results.

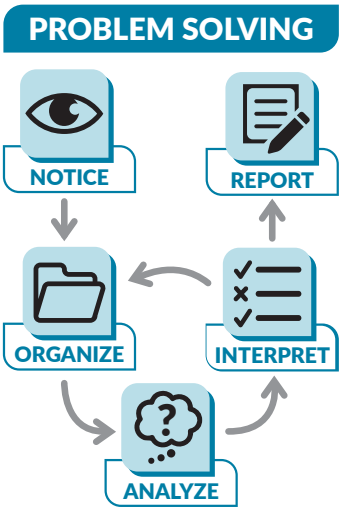
12. How does listening to others help you learn mathematics? Locate the TEKS mathematical process standards in the Course Guide.

13. Which TEKS mathematical process standard(s) did you use to determine the number of ways Ethan can organize his cards?

Talk the Talk

The Problem-Solving Model

In this lesson, you used a problem-solving model to solve a real-world problem. The basic steps of the problem-solving model are summarized in the diagram.



Summarize what is involved in each phase of this problem-solving model.

Notice and Wonder

Organize and Mathematize

Predict and Analyze

Test and Interpret

Report

You will see this symbol throughout the course to remind you to use the problem-solving model.

1 Writing Equivalent Expressions Using the Distributive Property

OBJECTIVES

- Write, read, and evaluate equivalent numeric expressions.
- Identify the adjacent side lengths of a rectangle as factors of the area value.
- Identify parts of an expression, such as the product and the factors.
- Write equivalent numeric expressions for the area of a rectangle by decomposing one side length into the sum of two or more numbers.
- Apply the distributive property to rewrite the product of two factors.

NEW KEY TERMS

- numeric expression
- equation
- distributive property

You know how to add, subtract, multiply, and divide numbers using different strategies. Taking apart numbers before you perform a mathematical operation can highlight important information or make calculations easier.

How can taking apart numbers help you to express number sentences in different ways?

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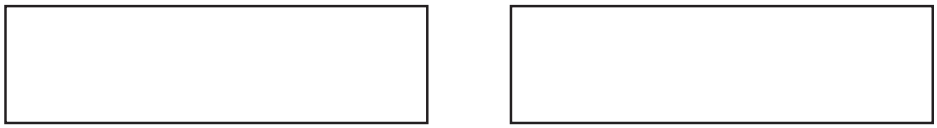
Break It Down to Build It Up

Ask Yourself . . .
How can you use the area of rectangles in everyday life?

Sofia is building a rectangular walkway up to her house. The width of the walkway is 5 feet, and the length is 27 feet. She needs to calculate the area of the walkway to determine the amount of materials needed to build it.

1. Mark and label two different ways you could divide an area model to determine the area of the walkway.

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?



2. Determine the areas of each of the subdivided parts of your models.

3. What is the total area of the walkway?

ACTIVITY
1.1 Connecting Area Models and the Distributive Property

The numeric expression of $5 \cdot 27$ represents the area of the walkway from the Getting Started. A **numeric expression** is a mathematical phrase that contains numbers and operations.

The equation $5 \cdot 27 = 135$ shows that the expression $5 \cdot 27$ is equal to the expression 135.

An **equation** is a mathematical sentence that uses an equals sign to show that two or more quantities are the same as one another.

They are many different ways to represent the operation of multiplication. 5×27, $5 \cdot 27$ and $(5)(27)$ all represent 5 times 27.

1. Reflect on the different ways you can rewrite the product of 5 and 27. Select one of your area models to complete the example.

How did you split the side length of 27?

$5 \cdot 27 = 5(\_\_\_\_\_ + \_\_\_\_\_)$

What are the factors of each smaller region?

$= (5 \cdot \_\_\_\_\_) + (5 \cdot \_\_\_\_\_)$

What is the area of each smaller region?

$= \_\_\_\_\_ + \_\_\_\_\_$

What is the total area?

$= \_\_\_\_\_$

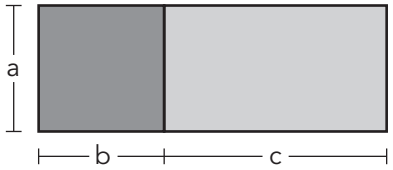
What are other ways you could split one of the factors and write a corresponding equation? What would the equation look like if you split one of the factors into more than two regions?



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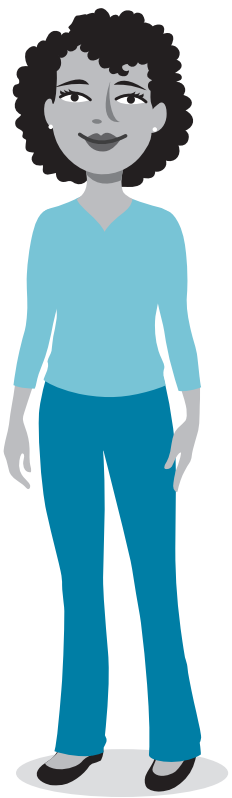
You just used the *distributive property!*

The **distributive property,** when applied for multiplication, states that for any numbers a, b, and c, the equation $a(b + c) = ab + ac$ is true.



2. Explain the distributive property using the area model shown.

You can also use grouping symbols to show that you need to multiply each set of factors before you add them, $(4 \cdot 2) + (4 \cdot 15)$.



WORKED EXAMPLE

Consider this example of the distributive property.

$4(2 + 15) = 4 \cdot 2 + 4 \cdot 15$

You can read and describe the expression $4(2 + 15)$ in different ways. For example, you can say:

- "four times the quantity of two plus fifteen",
- "four times the sum of two and fifteen", or
- "the product of four and the sum of two and fifteen".

You can describe the expression $4(2 + 15)$ as a product of two factors. The quantity $(2 + 15)$ is both a single factor and a sum of two terms.

3. Fill in the missing addend in each box that makes the equation true.

a. $7 (\_\_\_\_\_ + 10) = 21 + 70$

b. $3 (\_\_\_\_\_ + 15) = 36 + 45$

c. $8 (2 + \_\_\_\_\_) = 16 + 56$

d. $5 (6 + \_\_\_\_\_) = 30 + 45$

4. Rewrite a factor as the sum of two terms in each equation and use the distributive property to verify each product.

   a. $4 \cdot 17 = 68$

   b. $9 \cdot 34 = 306$

   c. $3 \cdot 29 = 87$

5. Identify each statement as true or false. If the statement is false, show how you could rewrite it to make it a true statement.

   a. True False $3(2 + 4) = 3 \cdot 2 + 4$

   b. True False $6(10 + 5) = 6 \cdot 10 + 6 \cdot 5$

   c. True False $7(20 + 8) = 7 + 20 \cdot 8$

   d. True False $4(5 + 10) = 20 + 10$

   e. True False $2(6 + 11) = 12 + 22$

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

Talk the Talk

The Floor Is Yours

You can apply the distributive property to solve real-world problems. Consider the situation.

Diego is setting up the gym floor for an after-school program. He wants to include a rectangular area for playing volleyball and another for dodgeball. He also wants to have an area for kids who like to play board games or just sit and read. The gym floor is already 50 feet by 84 feet, or 4200 square feet.

1. Create a diagram to show how you would split up the gym floor. Represent your diagram using the distributive property and write an explanation for the areas assigned to each activity.

Lesson 1 Assignment

Write

Explain the distributive property in terms of composing and decomposing numbers.

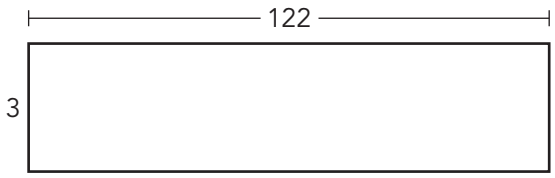
Remember

There are many ways to rewrite equivalent expressions using properties. The distributive property of multiplication over addition states that for any numbers a, b, and c, $a(b + c) = ab + ac$.

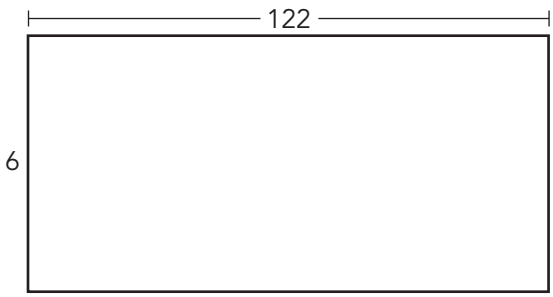
Practice

Decompose each rectangle into two or three smaller rectangles to demonstrate the distributive property. Then, write each area in the form $a(b + c) = ab + ac$.

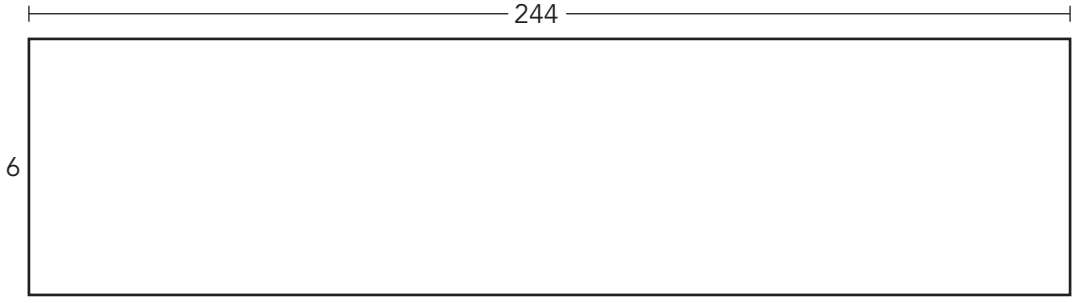
1.



2.



3.



Lesson 1 Assignment

4. $6(12 + 4)$

5. $10 + 4(2 + 20)$

6. $7(4 + 19)$

Prepare

1. In the array of numbers shown, circle the prime numbers, cross out the composite numbers, and use a box to identify any number that is neither prime nor composite.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

2 Identifying Common Factors and Common Multiples

OBJECTIVES

- Identify the factors and multiples of numbers, and the common factors and multiples of two whole numbers.
- Use powers and exponents to write the prime factorization of a number.
- Write and evaluate numeric expressions using the distributive property to model composing and decomposing the areas of rectangles.
- Rewrite the sum of two whole numbers with a common factor as a product using the distributive property.

NEW KEY TERMS

- common factor
- prime factorization
- base
- power
- exponent
- greatest common factor (GCF)
- relatively prime
- multiple
- commutative property
- least common multiple (LCM)

Just as you can compose and decompose shapes, you can compose and decompose numbers using factors and multiples.

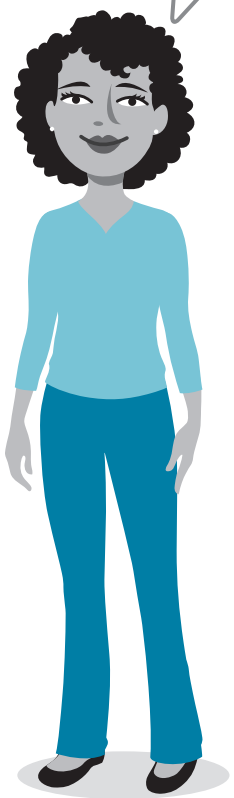
How can you use shapes to see relationships between pairs of numbers?

Getting Started

Ask Yourself . . .
How can I organize and record my mathematical ideas?

Ask Yourself . . .
What observations can I make?

How do you know if you have created all of the possible rectangles with the given area?



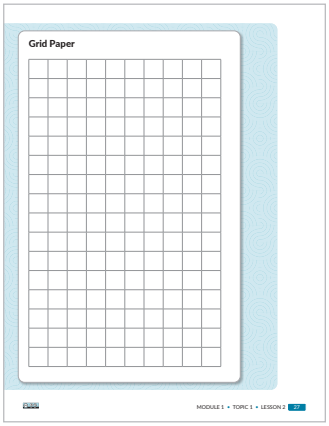
How Many Rectangles Can You Build?

Understanding the area of rectangles is helpful when learning about factors. A rectangular area model is one way to represent multiplication. You and your partner will create area models for the numbers 12 and 16.

Number assigned to me _____

Number assigned to my partner _____

Use the grid paper located at the end of the lesson to create and cut out as many unique rectangles as possible with the area of your assigned number.



1. Label each rectangle with its dimensions. List the dimensions of all of the rectangles that you created for your assigned number.

2. How did you represent factors and factor pairs in your rectangles?

3. List all of the factors of 12 and 16.

Together with your partner, combine one of your rectangles and one of your partner's rectangles to make a bigger rectangle. Use this method to create additional rectangles.

4. Complete the table with the information for each combined rectangle that you and your partner created.

Dimensions of Rectangle with an Area of 12	Dimensions of Rectangle with an Area of 16	Dimensions of the Combined Rectangle	Area of Combined Rectangle as a Sum of the Smaller Rectangles	Total Area of Combined Rectangle
$l \times w_1$	$l \times w_2$	$l(w_1 + w_2)$	$A_1 + A_2$	

5. How are the dimensions of the combined rectangle related to its total area?

6. For each combined rectangle you and your partner created, write a numeric expression that relates the dimensions of the combined rectangle to the sum of the areas of the smaller rectangles.

Consider any factors shared between your number and your partner's number.

7. How are the common factors represented in the combined rectangles that you and your partner created?

A **common factor** is a number that is a factor of two or more numbers.

8. How did you represent common factors in the numeric expressions that you and your partner wrote?

9. List the common factors of the two numbers.

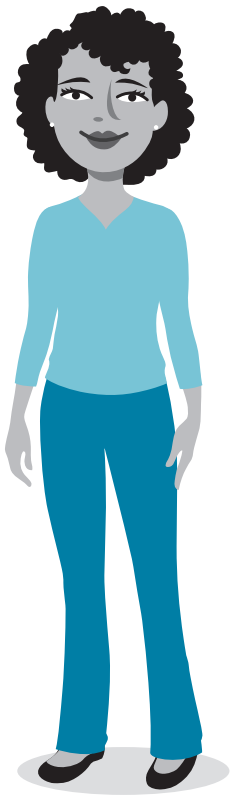
ACTIVITY
2.1 Prime Factors

You just determined the factors for a given number as well as the common factors that two numbers share.

In this activity, you will learn to determine the prime factors of a given number.

A factor tree is a way to organize the prime factors of a number. Choose any factor pair to get started.

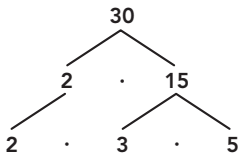
When you express any whole number as a product of primes, only primes, and nothing, you are determining the **prime factorization** of that number.



WORKED EXAMPLE

Use a factor tree to write the prime factorization for 30.

- Begin with the number 30.
- Pick any whole number factor pair of 30, other than 1 and 30.
- Draw a branch from 30 to each factor, 2 and 15.
- Since one of the factors is not prime, you are not finished.
- Use branches to write a factor pair for 15.
- Because 2, 3, and 5 are all prime, this factor tree is complete.



1. Use the factor tree to write the prime factorization of 30.

The factor tree in the Worked Example is not the only factor tree you can create for 30.

2. How many different factor trees are there for 30?

3. Construct a factor tree and write the prime factorization for each number.

   a. 24 b. 81 c. 96

You may have noticed that each prime factorization in Question 3 had repeat factors. You can represent repeated multiplication as a *power*. A **power** has two elements: the base and the exponent.

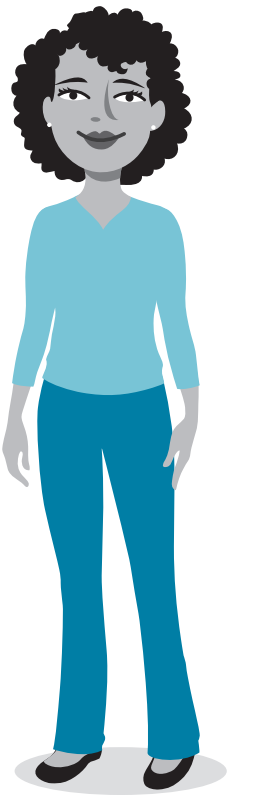
The **base** of a power is the factor multiplied by itself repeatedly, and the **exponent** of the power is the number of times you use the base as a factor.

$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$$

base → 2^4 ← exponent

power

You can read a power in different ways: "2 to the fourth power" or "2 raised to the fourth power."



4. Identify the base and exponent in each power. Then, write each power in words.

   a. 7^5 b. 4^8

5. Write the prime factorization for each number in Question 3 using powers.

   a. 24 b. 81 c. 96

ACTIVITY
2.2 Common Factors

Suppose you want to determine the common factors of 56 and 42, but you do not have grid paper or scissors to create rectangles. Is there another way?

WORKED EXAMPLE

One way to determine common factors is to use prime factorization. Start by writing each number as a product of its prime factors.

$56 = 2 \cdot 2 \cdot 2 \cdot 7$
$42 = 2 \cdot 3 \cdot 7$

Organize the prime factors into a table. Only list shared factors in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

The common factors of the two numbers are the numbers that are in both rows and the product of the numbers that are in both rows.

The common factors of 56 and 42 are 2, 7, and 14.

1. How do you know that 14 is a common factor of 56 and 42?

2. Why is there a space between 2 and 7 in the top row of the table?

3. Consider the numbers 54 and 84.

   a. Create a table of prime factors.

   b. Identify all of the common factors of 54 and 84.

   c. Of the common factors, which factor is the greatest?

The **greatest common factor (GCF)** is the largest factor two or more numbers have in common.

4. Rewrite each sum using the GCF and the distributive property.

   a. 56 + 42

   b. 54 + 84

Two numbers that do not have any common factors other than 1 are **relatively prime**.

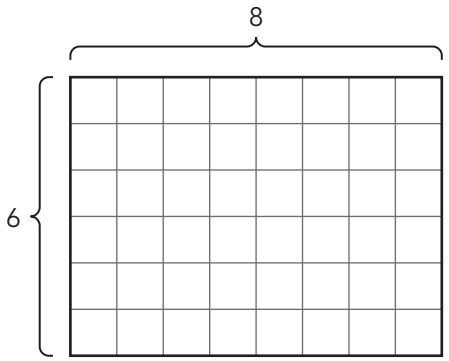
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ACTIVITY
2.3 Common Multiples

A **multiple** is the product of a given whole number and another whole number.

You can use rectangular arrays to determine multiples and common multiples.

Consider the area model for $6 \cdot 8 = 48$.



One way to think about the area model is to analyze the collection of columns. The addition of each new column creates a multiple of 6.

- The first column is a $6 \cdot 1$ rectangle representing the first multiple of 6, or 6.
- The first and second columns together are a $6 \cdot 2$ rectangle representing the second multiple of 6, or 12.
- The whole rectangle represents $6 \cdot 8$, or 48.

The **commutative property,** when applied for multiplication, states that for any numbers a and b, the product $a \cdot b$ is equal to the product $b \cdot a$.

1. List the first eight multiples of 6 by labeling each column of the area model.

Next, think about the area model as a collection of six rows. The first row alone creates an $8 \cdot 1$ rectangle, which represents the first multiple of 8, or 8. Including all rows, the $8 \cdot 6$ rectangle represents the sixth multiple of 8, or 48.

2. List the first six multiples of 8 by labeling each row of the area model.

While 48 is a multiple shared by both 6 and 8, it is not the **least common multiple (LCM)**. The LCM is the least multiple (other than zero) that two or more numbers have in common.

Analyze the multiples of 6 and 8 that you labeled on the area model.

3. Identify the least common multiple of 6 and 8.

   LCM(6, 8) = _____

   As demonstrated by the rectangular array, for any two whole numbers *a* and *b*, a common multiple is $a \cdot b$. However, this number may not be the *least* common multiple of *a* and *b*.

4. Determine the least common multiple of 6 and 9.

   a. List the first 9 multiples of 6.

   b. List the first 6 multiples of 9.

   c. What is the least common multiple of 6 and 9?

      LCM(6, 9) = _____

5. Determine the least common multiple of 7 and 8.

   LCM(7, 8) = _____

6. Using prime factorization, how can you determine whether the least common multiple of two numbers is the product of the two numbers or is less than the product of the two numbers?

ACTIVITY 2.4

Using Prime Factors to Determine the LCM

Suppose you want to determine the LCM of 56 and 42 without drawing an area model. Is there another way?

WORKED EXAMPLE

Organize the prime factors into a table. Only list shared factors in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

Determine the prime factors that the numbers share.

Shared prime factors: 2, 7

Determine the prime factors that the numbers *do not* share.

Non-shared prime factors: 2, 2, 3

The least common multiple of the two numbers is the product of their shared prime factors and non-shared prime factors.

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$

LCM(56, 42) = 168

1. For each pair of numbers, determine their product. Then, use a factor table to determine their LCM and their GCF.

   a. 12 and 10

b. 9 and 15

c. 9 and 10

d. 5 and 9

2. Write a sentence to describe the relationship between the product, GCF, and LCM.

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Talk the Talk

Bringing It Back Around

You have composed and decomposed numbers using factors and multiples.

Use the relationship between factors and multiples to answer each question.

1. Consider the sum $36 + 24$.

   a. Express the sum $36 + 24$ as many ways as possible as the product $a(b + c)$.

   b. How can you use factors to determine whether you have listed all possible products $a(b + c)$ that are equivalent to $36 + 24$?

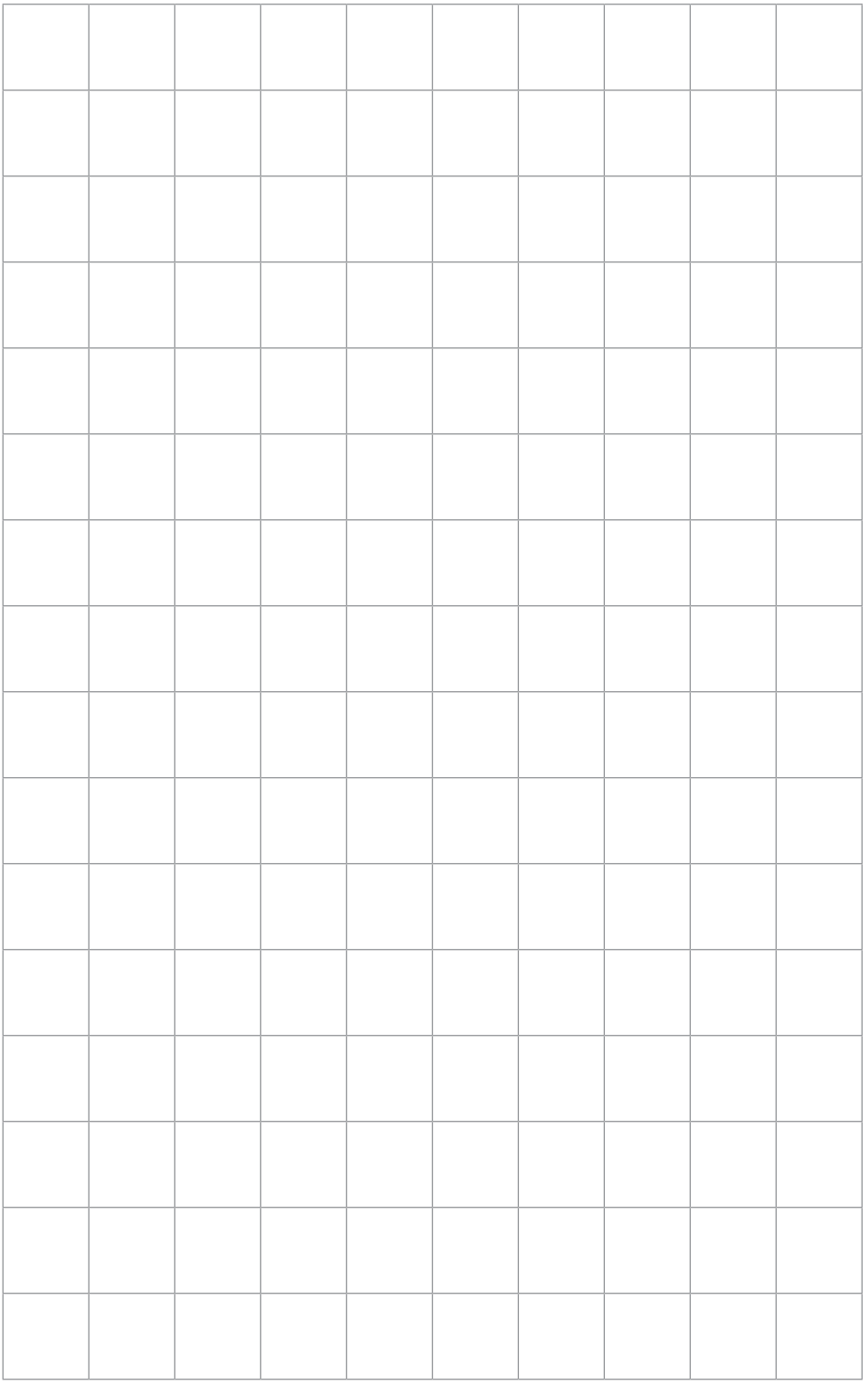
2. Can you always determine the greatest common factor of any two numbers? Explain your reasoning.

3. When the greatest common factor of two numbers is 1, what do you notice about the numbers?

4. Can you always determine the least common multiple of any two numbers? Explain your reasoning.

5. When the least common multiple of two numbers is the product of those numbers, what do you notice about the two numbers?

Grid Paper



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So you can cut out rectangles from the grid paper on the other side

Lesson 2 Assignment

Write

Select the word that makes the following statement true. Then, use complete sentences to explain your choice.

The LCM of two numbers is *(always, sometimes, never)* the product of the two numbers.

Remember

You can decompose numbers into a product of their prime factors. You can compose numbers into multiples. You can relate numbers using their greatest common factor and their least common multiple.

Practice

1. Construct a factor tree and write the prime factorization for each number.

a. 70

b. 90

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Lesson 2 Assignment

2. Consider the numbers 18 and 30.

   a. List all of the factors of 18.

   b. List all of the factors of 30.

   c. What factors do 18 and 30 have in common?

   d. What is the greatest common factor of 18 and 30?

3. Consider the numbers 54 and 72.

   a. Create a table of prime factors of 54 and 72.

   b. Identify the common factors of 54 and 72.

   c. Identify the greatest common factor of 54 and 72.

4. For each pair of numbers, determine the least common multiple and at least one other common multiple.

   a. 3 and 5

   b. 4 and 6

   c. 8 and 12

Lesson 2 Assignment

Prepare

Use reasoning to compare each pair of fractions.

1. If two people equally split one donut, how much of the donut does each person receive?

2. If three people equally split one donut, how much of the donut does each person receive?

3. If *n* people equally split one donut, how much of the donut does each person receive?

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3 Dividing a Whole into Fractional Parts

OBJECTIVES

- Create equal parts of a whole.
- Determine whether fractions are equal.

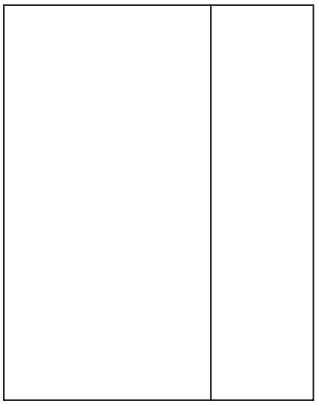
NEW KEY TERMS

- unit fraction
- equivalent fractions

You have used concrete models to determine factors and multiples. How can you use strip diagrams to represent and compare fractions with different denominators?

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Newspaper Column Preparation



You signed up to participate in the school newspaper club. During the first meeting, faculty advisors Ms. Williams and Ms. Perez showed everyone copies of last year's publication of the *Rocket*. The teachers have already planned out the sections for this year's *Rocket*.

Daniel volunteered to create the "Random Acts of Kindness" section of the school newspaper, the *Rocket*. The section will appear along the right side of the paper's back page. The newspaper is printed on $8\frac{1}{2}$-inch by 11-inch paper.

Daniel plans to put a box in each homeroom and ask students to nominate classmates for the monthly recognition of random acts of kindness. Students must write what nice act their nominee performed on a nomination slip.

In preparation for completing his section, help Daniel plan the layout of the column; do not worry about the top or bottom margin of the page.

1. To begin, cut eight strips of paper the length of a newspaper page. Remember, the *Rocket* is printed on $8\frac{1}{2}$-inch $\times$ 11-inch paper. Each strip of paper should be 1-inch wide. The strip represents one whole column. Do not fold the first strip, and label it as "1," to represent one whole.

The paper strips are provided for you at the end of this lesson.

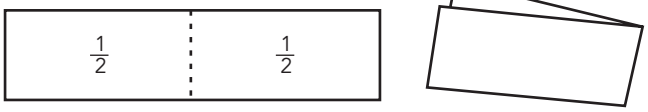


2. How many students can be recognized on a whole strip?

ACTIVITY
3.1 Building and Comparing Strip Diagrams

Consider the number of students that can be recognized in different columns.

1. Take one of your paper strips and fold it carefully in half to divide the strip into two equal parts, like the one shown. Label each folded part of the paper strip with the appropriate fraction and draw a line to mark your fold. How many students can be recognized in this column?



2. Take another paper strip and fold it carefully in half two times. Unfold and draw lines to mark your folds. Then, label each folded part of the paper strip with the appropriate fraction. How many students can be recognized in this column?

3. Take another paper strip and fold it in half three times. Be very careful to fold accurately. Unfold and draw lines to mark your folds. Then, label each folded part of the paper strip with the appropriate fraction. How many students can be recognized in this column?

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4. Take another paper strip and fold it very carefully in half, four times. Unfold and draw lines to mark your folds. Then, label each folded part of the paper strip with the appropriate fraction. How many students can be recognized in this column?

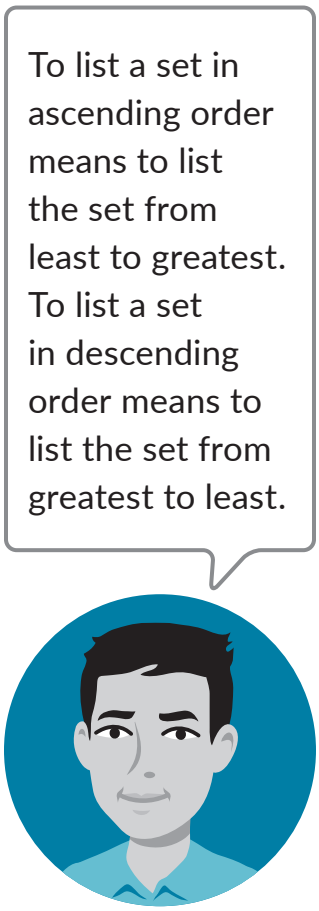
5. Take another paper strip and fold it carefully into three equal sections. Unfold and draw lines to mark your folds. Then, label each folded part of the paper strip with the appropriate fraction. How many students can be recognized in this column?

6. Take the next paper strip and fold it into thirds and then fold the strip in half. Unfold and draw lines to mark your folds. Then, label each folded part of the paper strip with the appropriate fraction. How many students can be recognized in this column?

7. Finally, take your last paper strip and fold it into thirds. Then, fold in half and then fold in half once more. Unfold and draw lines to mark your folds. Then, label each folded part of the paper strip with the appropriate fraction. How many students can be recognized in this column?

Arrange your strip diagrams in a column so that all of the left edges are lined up and the strip diagrams are ordered from the strip with the smallest parts to the strip with the largest parts.

8. As the number of students who can be recognized in the column increases, describe what happens to the space for each student.



A **unit fraction** is a fraction that has a numerator of 1 and a denominator that is a positive integer.

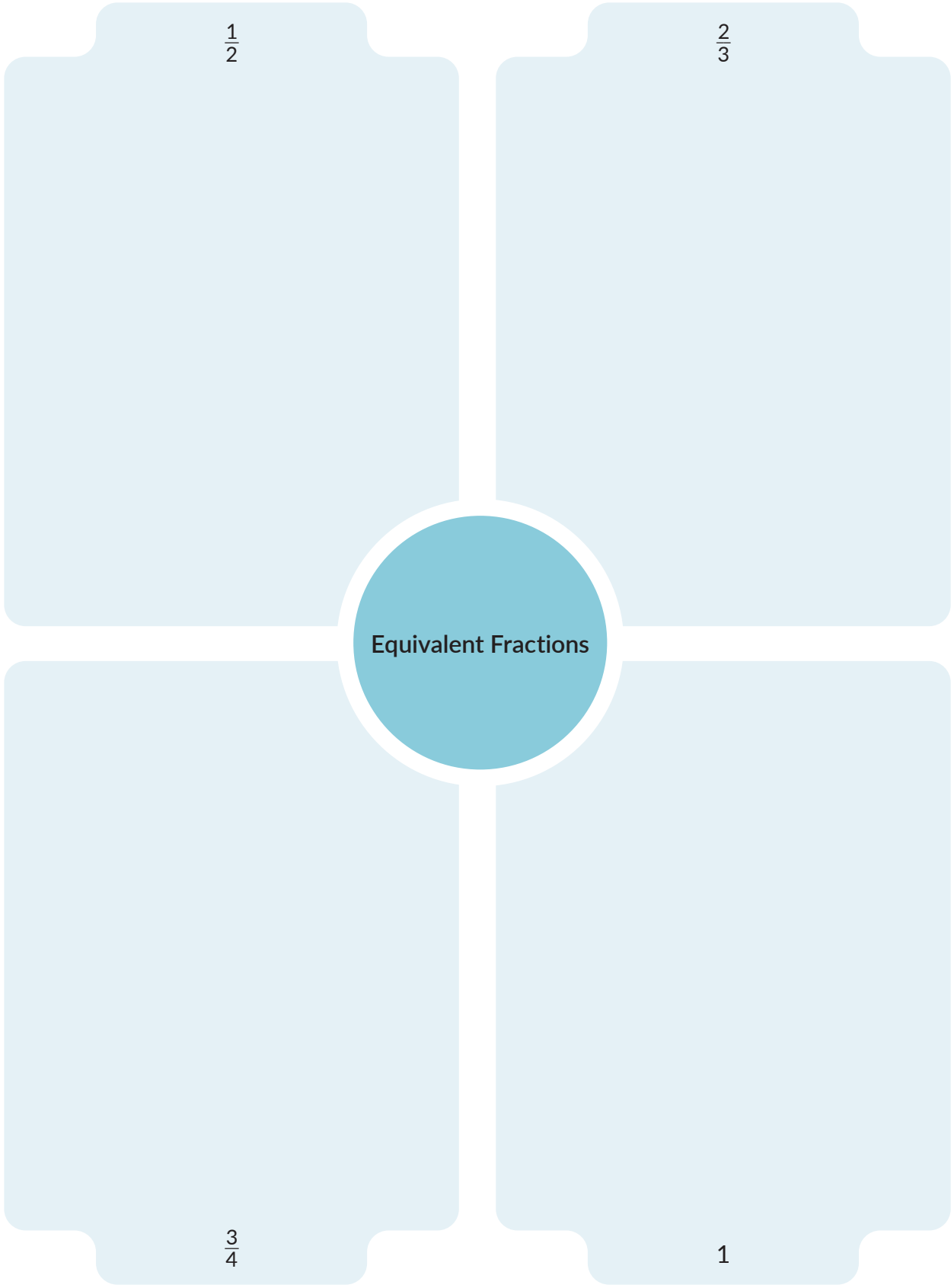
9. List the unit fractions for each strip diagram you created in ascending order.

10. Explain how understanding the size of a unit fraction helps you determine the size of the whole.

If you folded the paper strips carefully, you will notice that some of the folds line up with each other. Fractions that represent the same part-to-whole relationship are **equivalent fractions**.

11. Show that $\frac{1}{2}$ is equivalent to $\frac{6}{12}$. Draw on the strip diagrams to represent halves and twelfths. Then, shade the strip diagrams to represent $\frac{1}{2}$ and $\frac{6}{12}$.

12. Make a collection of equivalent fractions using your strip diagrams. Then, complete the graphic organizer by writing all the equivalent fractions for each.


$\frac{1}{2}$
$\frac{2}{3}$
Equivalent Fractions
$\frac{3}{4}$
1

Ask Yourself . . .
Did you justify your mathematical reasoning?

13. What do you notice in the collection of equivalent fractions? Give an example to justify your answer.

Talk the Talk

Numerate the Denomination

1. What do you notice about the numerator and denominator of the equivalent fractions?

2. What do you need to do to both the numerator and the denominator of a fraction in order to write another equivalent fraction?

Be prepared to share your solutions and methods.

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So you can cut out the paper strips on the other side

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Lesson 3 Assignment

Write

Describe how to use strip diagrams to compare two fractions with different denominators.

Remember

Fractions that represent the same part-to-whole relationship are equivalent fractions. With strip diagrams, equivalent fractions have folds that line up with each other.

Practice

Divide the strip diagram into equal parts to represent the fractional part provided. Then, shade the strip diagram to represent the given fraction.

1. tenths; $\frac{4}{10}$

2. sixths; $\frac{2}{6}$

3. eighths; $\frac{6}{8}$

Divide the strip diagrams into equal parts and shade to show that the given fractions are equivalent.

4. Show that $\frac{3}{4}$ is equivalent to $\frac{6}{8}$.

5. Show that $\frac{1}{4}$ is equivalent to $\frac{3}{12}$.

Divide the strip diagrams into equal parts and shade to determine the equivalent fraction.

6. How many 12ths are equivalent to $\frac{3}{4}$?

7. How many 10ths are equivalent to $\frac{1}{5}$?

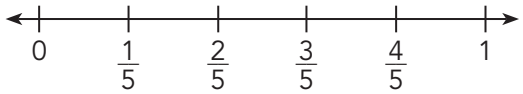
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Lesson 3 Assignment

Prepare

Use reasoning to compare each pair of fractions.

1. Locate $\frac{1}{2}$ on this number line.



2. How did you determine the location of $\frac{1}{2}$ on the number line?

4 Benchmark Fractions

OBJECTIVES

- Identify fractions and their equivalents on a 10 × 10 grid.
- Represent fractions on a number line.
- Estimate fractions by using benchmark fractions.
- Estimate sums of fractions using benchmark fractions.

NEW KEY TERM

- benchmark fractions

You have created strip diagrams and used them to compare numbers.

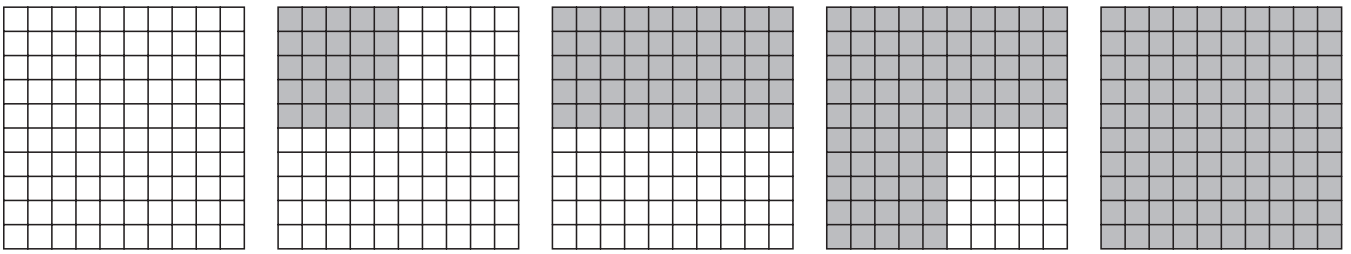
How can you use grids and number lines of common fractions like $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to estimate the value of other fractions?

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Getting Started

Shady Grids

Consider the grids shown.



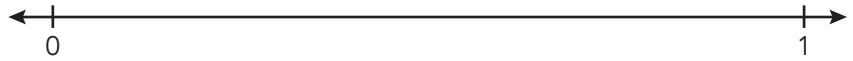
1. What fraction does each grid represent? Write a fraction under each grid.

2. How does each 10×10 grid representation change as you move from left to right? Explain your reasoning.

ACTIVITY
4.1 Graphing Strip Diagrams

In the previous lesson, you folded strips of paper to represent fractions. Consider how to translate those strip diagrams to number lines.

1. Label the number line to represent fourths.



   a. How does this number line connect with the grid representations in the Getting Started?

   b. Plot the fractions $\frac{1}{4}$ and $\frac{3}{4}$.

2. Label each number line to represent the fractional part provided and plot the given fractions.

   a. twelfths



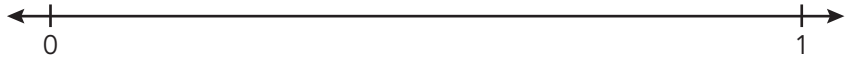
   • $\frac{5}{12}$ • $\frac{11}{12}$ • $\frac{1}{12}$ • $\frac{7}{12}$

   b. sixteenths



   • $\frac{10}{16}$ • $\frac{1}{16}$ • $\frac{7}{16}$ • $\frac{15}{16}$

   c. eighths



   • $\frac{5}{8}$ • $\frac{7}{8}$ • $\frac{1}{8}$ • $\frac{3}{8}$

As you get ready to label each number line, think about how you folded your strips in the Rocket Strips lesson. This will help you get the number line evenly spaced. To get fourths, you folded the strip in half first, so mark $\frac{2}{4}$ on the number line first. Then, mark $\frac{1}{4}$, and finally, mark $\frac{3}{4}$.

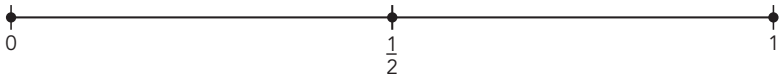


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ACTIVITY
4.2 Benchmark Fractions

Benchmark fractions are common fractions you can use to estimate the value of fractions.

Three common benchmark fractions are 0, $\frac{1}{2}$, and 1.

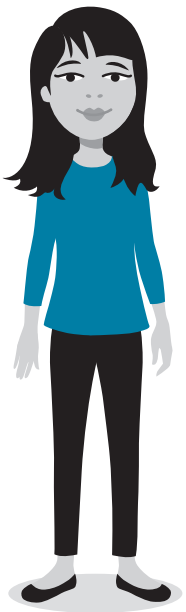


A fraction is close to 0 when the numerator is very small compared to the denominator.

A fraction is close to $\frac{1}{2}$ when the numerator is about half the size of the denominator.

A fraction is close to 1 when the numerator is very close in size to the denominator.

Even though you used four different number lines, how are the fractions you wrote for each question similar?



1. Use each number line you completed in Activity 4.1 to write a fraction that is:

   a. less than $\frac{1}{2}$

   b. exactly $\frac{1}{2}$

   c. greater than but not equal to $\frac{1}{2}$

   d. close to but not equal to 0

   e. exactly 1

   f. close to but not equal to 1

   g. exactly 0

2. Name the closest benchmark fraction for each fraction given.

   a. $\frac{4}{9}$ b. $\frac{8}{9}$ c. $\frac{6}{100}$

   d. $\frac{5}{67}$ e. $\frac{7}{15}$ f. $\frac{7}{12}$

   g. $\frac{5}{6}$ h. $\frac{14}{27}$ i. $\frac{12}{13}$

   h. $\frac{1}{17}$ k. $\frac{5}{11}$ l. $\frac{3}{7}$

3. Write the unknown numerator or denominator so that each fraction is close to but less than $\frac{1}{2}$.

a. $\frac{(\ \ )}{12}$ b. $\frac{(\ \ )}{27}$ c. $\frac{8}{(\ \ )}$

d. $\frac{7}{(\ \ )}$ e. $\frac{(\ \ )}{13}$ f. $\frac{9}{(\ \ )}$

4. Write the unknown numerator or denominator so that each fraction is close to but less than 1.

a. $\frac{(\ \ )}{17}$ b. $\frac{11}{(\ \ )}$ c. $\frac{(\ \ )}{8}$

d. $\frac{(\ \ )}{18}$ e. $\frac{13}{(\ \ )}$ f. $\frac{(\ \ )}{10}$

5. Use benchmark fractions and estimation to order the numbers from least to greatest.

$\frac{25}{24}, \frac{1}{2}, \frac{12}{16}, 1\frac{2}{3}, \frac{3}{24}, \frac{2}{8}$

6. A plumber installs a hot water tank in a new home using copper pipes. The fractions represent the diameter of the pipes the plumber uses in inches. Order the diameters from greatest to least using benchmark fractions and estimation.

$\frac{5}{4}, \frac{6}{8}, 1\frac{1}{2}, \frac{2}{4}, \frac{6}{6}, \frac{2}{16}$



7. Rewrite each expression using benchmark fractions. Then, estimate the sum. Explain your reasoning.

a. $\frac{8}{9} + \frac{6}{7}$ b. $\frac{1}{11} + \frac{8}{17}$

8. When three fractions that are greater than $\frac{1}{2}$ but less than 1 are added together, what can you say about their sum? Explain your reasoning.

9. When two fractions that are less than $\frac{1}{2}$ but greater than 0 are added together, what can you say about their sum? Explain your reasoning.

10. When seven fractions that are slightly less than 1 are added together, what can you say about their sum? Explain your reasoning.

Talk the Talk

Copycat!

The teacher asked each of her students to create a rectangular design and shade only a portion of their design. The table describes three students' designs.

Student	Unshaded portion	Shaded portion
Chloe	$\frac{1}{4}$	$\frac{3}{4}$
Minh	$\frac{2}{5}$	$\frac{3}{5}$
Mia	$\frac{3}{12}$	$\frac{9}{12}$

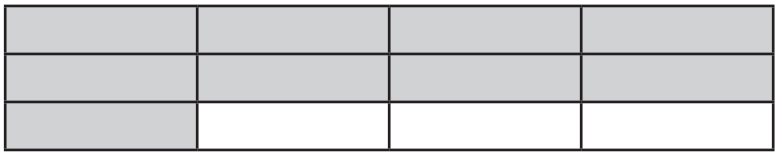
Chloe's design:



Minh's design:



Mia's design:



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When the students saw each other's designs, Chloe immediately accused Mia of being a copycat! Mia replied that she did no such thing. Mia said her design was much bigger than Chloe's and not even the same shape. Minh said she was staying out of this argument and would not take Chloe or Mia's side of the disagreement.

1. Do you think Mia copied Chloe's design? Why would Chloe accuse Mia of copying her design? Explain your reasoning.

2. Mia thought her design contained the largest portion of parts not shaded. Why does Mia think this way? Do you agree with her? Explain your reasoning.

Review the definition of **explain your reasoning** in the Academic Glossary.

Lesson 4 Assignment

Write

Describe how you could use benchmark fractions to order the fractions $\frac{3}{7}$, $\frac{4}{5}$, $\frac{5}{9}$, and $\frac{1}{8}$ from least to greatest.

Remember

You can use the benchmark fractions 0, $\frac{1}{2}$, and 1 to determine which fractions are greater than or less than other fractions and to estimate sums and differences.

Practice

Fill in the missing numerator or denominator so the fraction is close to but greater than $\frac{1}{2}$.

1. $\frac{\quad}{15}$
2. $\frac{\quad}{11}$
3. $\frac{7}{\quad}$
4. $\frac{19}{\quad}$
5. $\frac{\quad}{14}$
6. $\frac{\quad}{20}$

Fill in the missing numerator or denominator so the fraction is close to but less than 1.

7. $\frac{\quad}{9}$
8. $\frac{9}{\quad}$
9. $\frac{4}{\quad}$
10. $\frac{\quad}{8}$
11. $\frac{12}{\quad}$
12. $\frac{\quad}{13}$

13. Victoria walks five days each week. This week, she walked $\frac{7}{8}$ mile on Monday, $\frac{3}{5}$ mile on Tuesday, $\frac{4}{10}$ mile on Wednesday, $\frac{1}{10}$ mile on Thursday, and $\frac{9}{10}$ mile on Friday. Use benchmark fractions to estimate the total distance Victoria walked this week. Explain your reasoning.

Lesson 4 Assignment

14. The table shows the fraction of an hour that each student spent running laps at track practice one afternoon.

Student	Fraction of One Hour Spent Running Laps
Aaliyah	$\frac{1}{10}$
Juan	$\frac{2}{3}$
Jayden	$\frac{11}{12}$
Kai	$\frac{3}{4}$
Ava	$\frac{7}{15}$

a. Which benchmark fraction should Coach Nguyen use to help her determine which students ran laps for more than 30 minutes and which students ran laps for less than 30 minutes? Explain your reasoning.

b. Which students ran laps for less than 30 mnutes? Explain your reasoning.

c. Which students ran laps for more than 30 minutes? Explain your reasoning.

d. Did any of the students run laps for almost an hour? Explain your reasoning.

Lesson 4 Assignment

15. A school participates in a reading contest. The table shows each sixth-grade class' portion of the grade's total reading minutes. Use benchmark fractions to help you order the classes from the greatest number of reading minutes to the least number of reading minutes. Explain your reasoning.

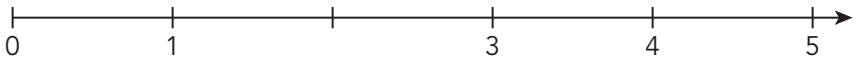
Class	Portion of Reading Minutes
Mr. Garcia	$\frac{5}{12}$
Ms. Johnson	$\frac{1}{18}$
Ms. Gonzalez	$\frac{4}{9}$
Mr. Park	$\frac{1}{12}$

16. What fractions between 0 and 1 other than $\frac{1}{2}$ might be useful benchmark fractions? Explain.

Lesson 4 Assignment

Prepare

Plot and label each value on the number line shown.



1. 2

2. $\frac{1}{4}$

3. $4\frac{1}{3}$

4. $\frac{7}{2}$

5 Multiplying Fractions

OBJECTIVES

- Connect an area model to the standard algorithm for multiplying two fractions.
- Multiply two fractions using the standard algorithm.
- Calculate the products of fractions in real-world and mathematical problems.

NEW KEY TERM

- algorithm

You have used area models to represent products of whole numbers.

How can you connect an area model to the procedure for multiplying two fractions?

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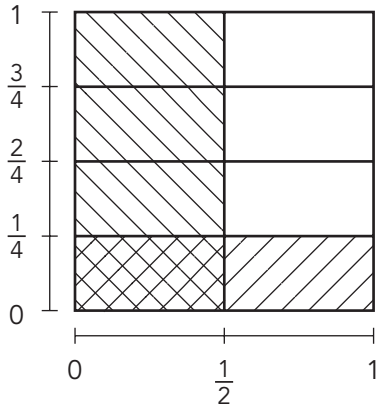
Getting Started

A Part of a Part

Previously, you used an area model to represent products, determine factors, and list multiples of given numbers. In the same way that area models represent whole number multiplication, area models can represent fraction multiplication.

Consider the expression $\frac{1}{4} \cdot \frac{1}{2}$ represented in the area model shown.

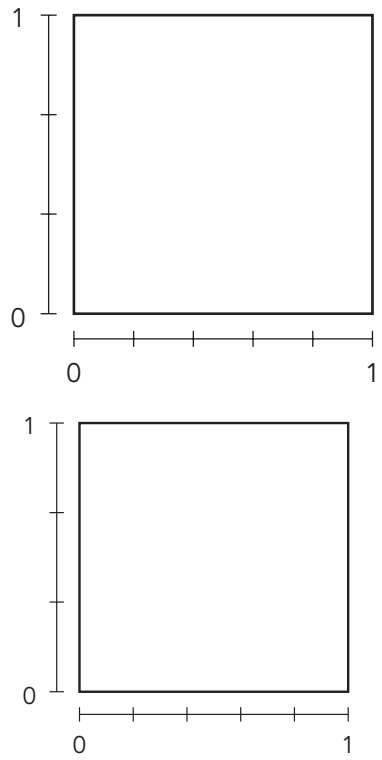
1. How are the factors $\frac{1}{4}$ and $\frac{1}{2}$ represented in the model?



2. What is the product of $\frac{1}{4} \cdot \frac{1}{2}$? Describe how the product is represented in the model.

Consider the expression $\frac{2}{3} \cdot \frac{3}{5}$.

3. Model the expression and determine the product.



4. Show how the *algorithm* for multiplying two fractions less than 1 gives the same product as the model.

An **algorithm** is a process or description of steps you can follow to complete a mathematical calculation.

PROBLEM SOLVING



Use the problem-solving model whenever you see this icon.

Review the steps and questions from the problem-solving model.
Ask Yourself: Did you make a plan to solve the problem?

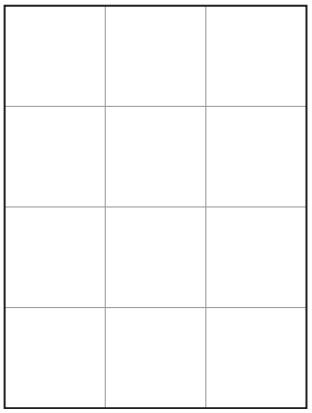
ACTIVITY 5.1

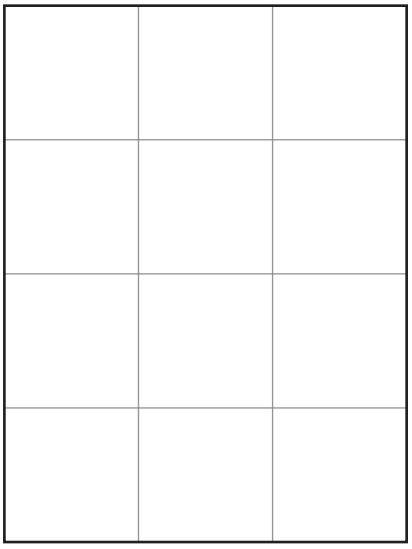
Using Area Models to Multiply Mixed Numbers

Isabella is tiling the top of a table that measures $3\frac{1}{2}$ feet by $2\frac{1}{2}$ feet. She has 12 ceramic tiles that each measure 1 foot by 1 foot.

Consider the 4×3 area model that represents the 12 tiles Isabella will use.

1. Create a model to represent the dimensions of the table.

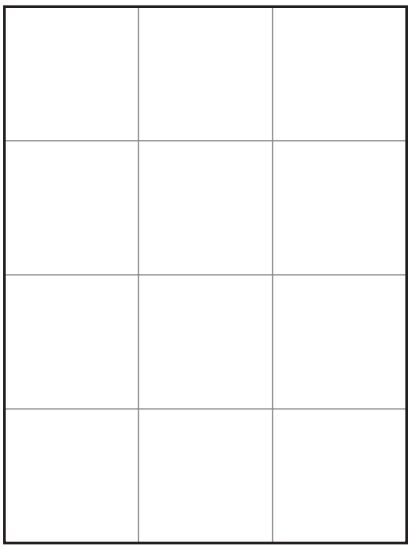


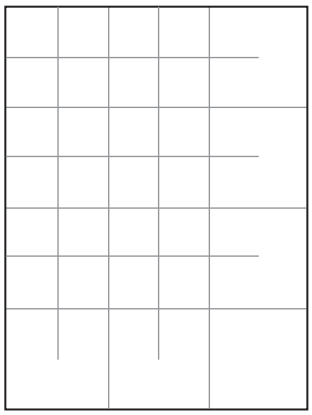


2. What portion of the tiles will Isabella use to cover the tabletop?

Suppose Isabella doesn't want to cut the 1 ft $\times$ 1 ft tiles. Instead, she wants to buy smaller sized square tiles that she can use to cover the entire tabletop.

3. Continue to divide the model into equal-sized tiles. Describe your model.





4. What size tiles should she buy?

5. How many tiles of that size does she need?

6. Show that the area represented in the model is the same as the product of the side lengths.

ACTIVITY
5.2 Multiplying Mixed Numbers

You can use the standard algorithm to multiply a whole number and a mixed number, a mixed number, and a fraction less than 1, or two mixed numbers when solving real-world and mathematical problems.

RECIPE

Hawaiian Trail Mix Extravaganza

$3\frac{3}{8}$ cups of macadamia nuts


$2\frac{1}{3}$ cups of almonds


$2\frac{1}{4}$ cups of pumpkin seeds


$1\frac{1}{3}$ cups of sunflower seeds


$3\frac{3}{8}$ cups of dried cherries


$2\frac{5}{6}$ cups of honey


$4\frac{5}{8}$ cups of popcorn


$4\frac{1}{2}$ cups of raisins


$\frac{3}{4}$ cups of corn syrup


$2\frac{3}{4}$ cups of granola

Feeds 12 People

The teachers at a local middle school decide to make trail mix for an upcoming field trip. Ms. Martinez shares her Hawaiian Trail Mix Extravaganza recipe with the other teachers. The recipe shown is for 1 batch.

1. Harper and Mateo determined the number of cups of almonds it will take to make 3 batches. How is Mateo's strategy different than Harper's?

Harper

$3 \cdot 2\frac{1}{3}$

$\frac{3}{1} \cdot \frac{7}{3} = \frac{21}{3}$

$= 7$

7 cups

Mateo

$3 \cdot 2\frac{1}{3}$

$\frac{\cancel{3}}{1} \cdot \frac{7}{\cancel{3}} = 7$

7 cups

2. Determine the number of cups of each ingredient it will take to make $4\frac{1}{2}$ batches. Show your work.

   a. corn syrup

   b. sunflower seeds

   c. pumpkin seeds

3. Calculate each product. Write your answer as a mixed number. Then, state whether the product is greater than or less than the first factor.

   a. $2\frac{1}{2} \cdot 3\frac{2}{5}$

   c. $1\frac{3}{4} \cdot \frac{2}{5}$

   e. $3\frac{3}{4} \cdot 2$

   b. $4\frac{1}{4} \cdot 2\frac{2}{3}$

   d. $1\frac{1}{2} \cdot \frac{5}{6}$

   f. $2\frac{5}{8} \cdot 3$

4. Without multiplying, determine whether each product is greater than or less than the first factor.

   a. $1\frac{3}{4} \cdot \frac{3}{8}$

   c. $\frac{7}{8} \cdot 4\frac{1}{5}$

   b. $\frac{5}{4} \cdot 3\frac{2}{3}$

   d. $\frac{3}{7} \cdot \frac{5}{12}$

Talk the Talk

Going in a General Direction

Look back at the factors and products in this lesson. What generalizations can you make about the multiplication of fractions?

1. Determine whether each statement is *always*, *sometimes*, or *never* true. Provide examples.

   a. When a fraction between 0 and 1 is multiplied by another fraction between 0 and 1, the product will be less than 1.

   b. When a fraction between 0 and 1 is multiplied by a mixed number, the product will be greater than 1.

2. Describe the algorithm for multiplying any two fractions or mixed numbers.

Lesson 5 Assignment

Write

Explain how to use an area model to represent the multiplication of two mixed numbers.

Remember

When multiplying two mixed numbers, rewrite each mixed number as a fraction greater than 1 before using the standard algorithm to calculate the product.

Practice

Read and solve each problem.

1. Sofia has an exercise routine that includes swimming, cycling, and running. The table shows the distances of each portion of her routine.

Exercise	Distance
Swimming	$\frac{2}{3}$ mile
Cycling	$10\frac{1}{2}$ miles
Running	$5\frac{3}{4}$ miles

   a. If Sofia completes her routine each day this week, how many miles would she swim this week?

   b. If Sofia completes her routine each day this week, how many miles would she cycle this week?

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Lesson 5 Assignment

2. Liam is putting new sod down in his rectangular yard. His yard measures $6\frac{1}{5}$ meters by $8\frac{1}{3}$ meters. What is the area of the yard?

Calculate each product. Show your work.

3. $3\frac{1}{2} \cdot 4\frac{5}{8}$

4. $12 \cdot 2\frac{2}{3}$

5. $7\frac{1}{5} \cdot 1\frac{5}{6}$

6. $2\frac{4}{5} \cdot 7$

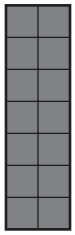
7. $8\frac{1}{3} \cdot 3\frac{2}{5}$

8. $5\frac{3}{4} \cdot 4\frac{1}{10}$

Prepare

Write the multiplication-division fact family for each rectangular array.

1. 

2. 

6 Fraction by Fraction Division

OBJECTIVES

- Model the division of fractions using bar models and number lines.
- Compute and interpret quotients of fractions and interpret remainders in real-world problems.
- Divide with mixed numbers.

NEW KEY TERMS

- positive rational number
- reciprocal
- multiplicative inverse
- complex fraction

You know how to multiply and divide with whole numbers and positive rational numbers.

How can you apply what you know to understand how to divide two fractions or mixed numbers?

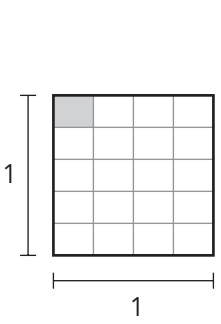
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Getting Started

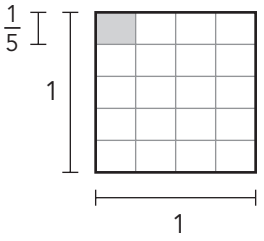
All in the Fact Family

You can write a multiplication-division fact family using fractions. Consider the unit square model divided into 5 equal rows and 4 equal columns.

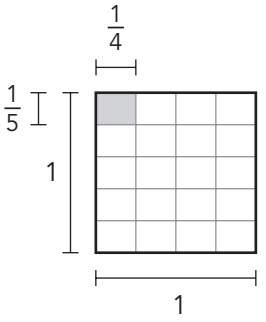
WORKED EXAMPLE



The shaded area represents the fraction $\frac{1}{20}$ because 1 rectangle is shaded of the 20 total rectangles.



The height of the shaded rectangle is $\frac{1}{5}$ of the height of the model.



The width of the shaded rectangle is $\frac{1}{4}$ of the width of the model.

So, the shaded area of the rectangle represents the product $\frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}$.

Fractions are part of a larger set of numbers called rational numbers. A *positive rational number* is a number you can write in the form $\frac{a}{b}$, where a and b are both whole numbers greater than zero.

1. Write a multiplication-division fact family for the model.

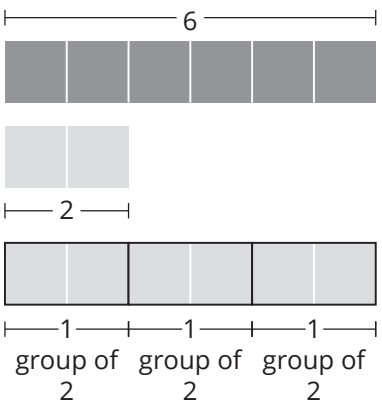
2. Describe how the model shows the division of fractions.

ACTIVITY
6.1 Whole Number ÷ Fraction

Division is the process of breaking a number up into an equal number of parts.

WORKED EXAMPLE

The model shows $6 \div 2$. The expression asks, "How many groups of two are in 6?"

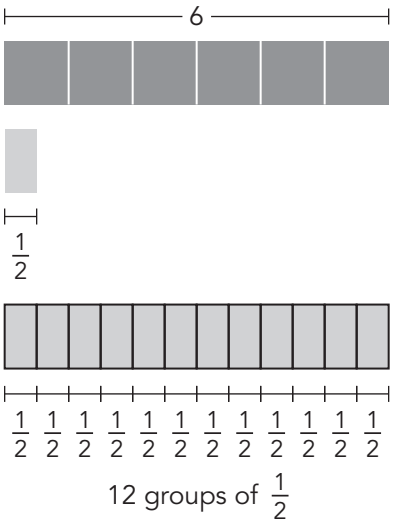


There are 3 groups of 2 in 6, so $6 \div 2 = 3$.

When you divide with fractions, you are asking the same question.

WORKED EXAMPLE

The model shows $6 \div \frac{1}{2}$.
The expression asks, "How many halves, or groups of $\frac{1}{2}$, are in 6?"



There are twelve $\frac{1}{2}$ parts in 6, so $6 \div \frac{1}{2} = 12$.

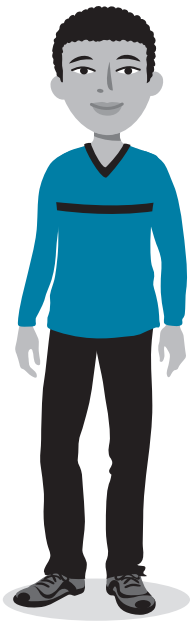
The *dividend* is the number divided into equal groups, the *divisor* is the number that divides the dividend, and the *quotient* is the result of the division.



1. Label the dividend, divisor, and quotient in each model.

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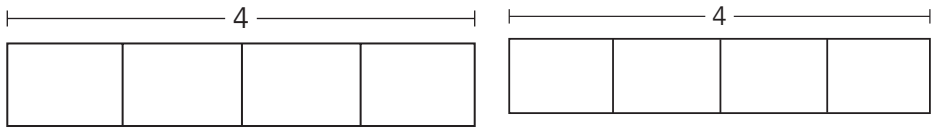
How can you use what you know about multiplication-division fact families to check your work?



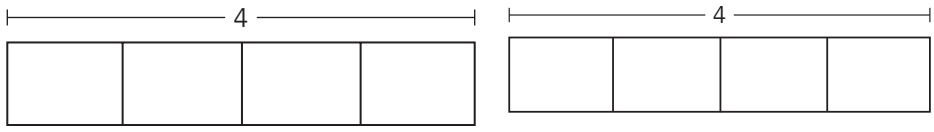
Consider each situation.
Suppose you are creating snack-sized portions from 4 cups of trail mix.

2. Determine the number of portions you can make when you use different measuring scoops. Draw a diagram and write the appropriate number sentence.

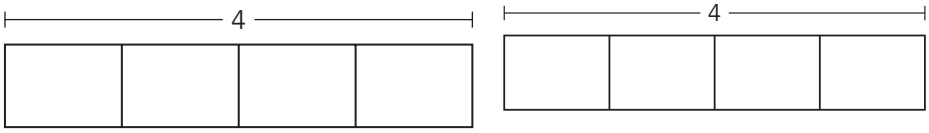
a. $\frac{1}{2}$ cup measuring scoop



b. $\frac{1}{4}$ cup measuring scoop



c. $\frac{1}{3}$ cup measuring scoop



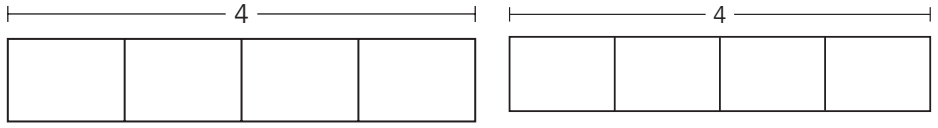
d. How did the number of portions change as the size of the measuring scoop changed?

e. Analyze the three number sentences you wrote. Consider the relationship between the dividend, the divisor, and the quotient in each model. What patterns do you notice? Explain your reasoning.

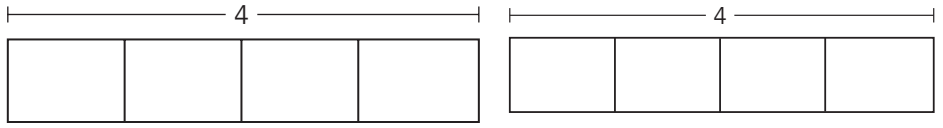
Suppose you are organizing games for a field day event that lasts 4 hours.

3. Determine how many games you can organize for the event when each game lasts the given amount of time. Draw a diagram and write the appropriate number sentence.

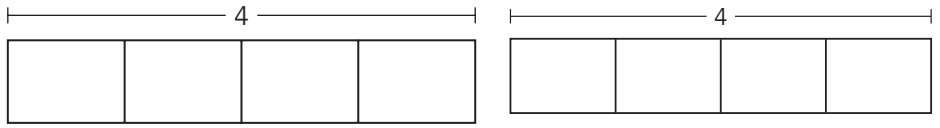
   a. $\frac{2}{3}$ hour



   b. $\frac{2}{5}$ hour



   c. $\frac{4}{5}$ hour



   d. Consider the relationship between the dividend, the divisor, and the quotient in each number sentence. How do these number sentences compare to the ones you wrote in Question 2? What patterns do you notice? Explain your reasoning.

Consider the relationships between the different division statements that you just modeled.

4. How is the quotient of $4 \div \frac{1}{3}$ related to the quotient of $4 \div \frac{2}{3}$? Explain your reasoning.

5. From a previous example, you know that $6 \div \frac{1}{2} = 12$. Use reasoning to determine each quotient.

   a. $6 \div \frac{1}{4}$ b. $6 \div \frac{1}{8}$ c. $6 \div \frac{1}{16}$



6. Aaliyah is throwing a small party. She has 4 pizzas and decides that everyone at her party should receive a serving size that is $\frac{3}{5}$ of a pizza. Aaliyah says she has $6\frac{2}{3}$ servings, but Elijah says she has $6\frac{2}{5}$ servings. Draw a diagram of the situation and solve for the quotient to determine who is correct. Explain why the other person is not correct.

7. Noah has 4 pizzas and decides that everyone at his party should receive a serving size that is $\frac{3}{4}$ of a pizza. Draw a diagram and determine the number of servings he will have for his party.

ACTIVITY
6.2 Fraction ÷ Fraction

PROBLEM SOLVING

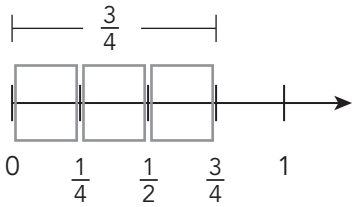


In this activity, you will use a number line model to determine a quotient when the dividend and divisor are both fractions.

You want to determine how many $\frac{1}{4}$ inch pieces are in $\frac{3}{4}$ inch.

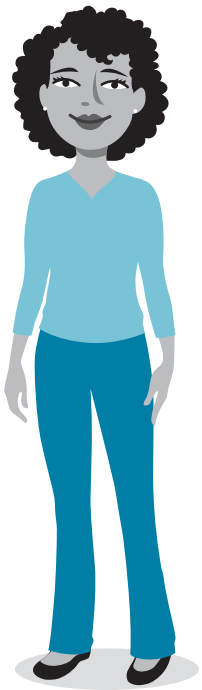
WORKED EXAMPLE

The model shows $\frac{3}{4} \div \frac{1}{4}$.



The division expression asks, "How many $\frac{1}{4}$s are in $\frac{3}{4}$?"

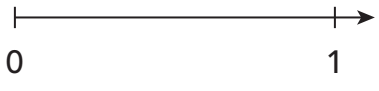
How can you partition the number line into equal-sized intervals?



1. What is the quotient $\frac{3}{4} \div \frac{1}{4}$ represented in the model shown in the Worked Example? Interpret the quotient in terms of the situation.

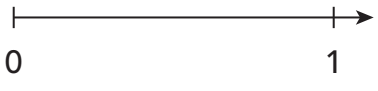
2. Write a question to describe what each division expression is asking. Then, draw a model to determine the quotient. Write a sentence to describe your answer.

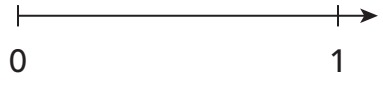
a. $\frac{1}{2} \div \frac{1}{8}$





b. $\frac{3}{4} \div \frac{3}{8}$





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Create a model to represent each situation and then answer the question.

3. Ethan has $\frac{2}{3}$ of a foot of ribbon. He needs to divide the ribbon into $\frac{1}{6}$ foot pieces. How many pieces can he cut from the ribbon?

4. Samuel places signposts along a trail that is $4\frac{1}{2}$ miles long. The signposts divide the trail into sections that are $\frac{3}{4}$ mile long. How many sections does Samuel create along the trail?

5. Ava has a board that is $3\frac{1}{3}$ feet long. She needs to cut the board into $\frac{5}{12}$ foot pieces. Into how many pieces can she cut the board?

ACTIVITY
6.3 From Model to Procedure

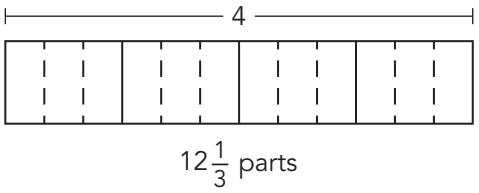
You have drawn models to answer, "How many groups of a certain size are in a number?"

In this activity, you will analyze a few models that you drew to develop an efficient way to perform division of fractions without a model.

Let's revisit the strategy to draw a model for $4 \div \frac{1}{3} = 12$ and $4 \div \frac{2}{3} = 6$.

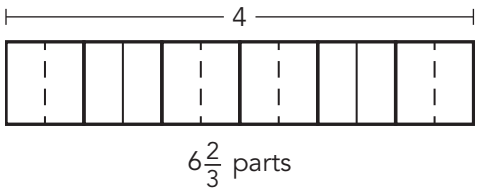
WORKED EXAMPLE

To create the model $4 \div \frac{1}{3}$, you partitioned 4 wholes to create twelve $\frac{1}{3}$ parts, so $4 \div \frac{1}{3} = 12$.



Notice there is one step to this process.
STEP 1 The denominator of the divisor is 3. Partition each whole into $\frac{1}{3}$ parts. This is the same as $4 \cdot 3 = 12$.

To create the model $4 \div \frac{2}{3}$, you partitioned 4 units to create twelve $\frac{1}{3}$ parts and then grouped 2 $\frac{1}{3}$ parts together, so $4 \div \frac{2}{3} = 6$.



Notice there are two steps to this process.
STEP 1 The denominator of the divisor is 3. Partition each whole into $\frac{1}{3}$ parts.
This is the same as $4 \cdot 3 = 12$.
STEP 2 The numerator of the divisor is 2. Group two of the $\frac{1}{3}$ parts together.
This is the same as $12 \div 2 = 6$.
The combination of steps is the same as $4 \cdot 3 \div 2 = 6$.

You can rewrite a fraction as division or division as a fraction: $\frac{a}{b} = a \div b$. So, you can rewrite the final number sentence, $4 \cdot 3 \div 2 = 6$, as $4 \cdot \frac{3}{2} = 6$.

1. Compare the resulting equivalent number sentences from the Worked Examples. What do you notice?
   - $4 \div \frac{1}{3} = 12$ and $4 \cdot 3 = 12$
   - $4 \div \frac{2}{3} = 6$ and $4 \cdot \frac{3}{2} = 6$

A **reciprocal**, or *multiplicative inverse*, is one of a pair of numbers whose product is 1. The **multiplicative inverse** of the number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{d} \cdot \frac{d}{c}$

You just developed a rule you can use to rewrite any division sentence as multiplication. You can rewrite any division sentence as multiplication by taking the reciprocal of the divisor.

2. Rewrite each division sentence as a multiplication sentence and then calculate the quotient.

   a. $5 \div \frac{1}{4}$

   b. $\frac{1}{8} \div 2$

   c. $\frac{4}{5} \div \frac{1}{3}$

   d. $\frac{3}{10} \div \frac{1}{3}$

3. Explain why Chloe is incorrect and provide the correct reciprocal.

Chloe



The reciprocal of $3\frac{8}{5}$ is $3\frac{5}{8}$.

Emily and Carlos rewrote the expression $6 \cdot 2 \div 3$ as only a multiplication sentence using a whole number and a fraction.

Emily rewrote the expression as $6 \cdot \frac{2}{3}$. Carlos rewrote the expression as $6 \cdot \frac{3}{2}$.



4. Who wrote the multiplication sentence correctly? Explain your reasoning.

Solve each problem. Show your work and be sure to label your answer.

5. When you have $5\frac{2}{3}$ pounds of trail mix, how many bags can you make so that each bag contains $1\frac{5}{6}$ pounds?

6. A cook in a restaurant made $47\frac{1}{2}$ cups of mashed potatoes. When there are $1\frac{1}{4}$ cups of mashed potatoes in a serving, how many servings did he make?

7. The hiking trail through Glacier Gorge in Rocky Mountain National Park is $9\frac{3}{5}$ miles round trip. When you hike $1\frac{3}{5}$ miles per hour, how many hours will the round trip take?

ACTIVITY
6.4 Different Strategies to Divide Numbers

You have learned that one way to divide two fractions is to rewrite the division problem as multiplication by the reciprocal of the divisor.

In this activity, you will consider two other strategies for dividing fractions.

WORKED EXAMPLE

In the same way that you can "multiply across," or multiply the numerators and multiply the denominators, to determine the product of two fractions. You can also "divide across" to determine the quotient of two fractions.

Determine the quotient $\frac{7}{8} \div \frac{1}{2}$.

Divide the numerators. Then, divide the denominators.

$$\frac{7}{8} \div \frac{1}{2} = \frac{7 \div 1}{8 \div 2}$$
$$= \frac{7}{4}$$

1. Divide across to determine each quotient.

   a. $\frac{3}{8} \div \frac{1}{4}$

   b. $\frac{4}{9} \div \frac{2}{3}$

   c. $\frac{2}{5} \div \frac{1}{5}$

   d. $\frac{1}{4} \div \frac{3}{4}$

2. Think about the structure of the fractional parts of each dividend and divisor. What was special about the numbers that allowed you to divide across?

Diego is trying to use the dividing across strategy to determine the quotient for the expression $\frac{6}{7} \div \frac{2}{3}$ but gets stuck.

$$\frac{6}{7} \div \frac{2}{3} = \frac{6 \div 2}{7 \div 3} = \frac{3}{7 \div 3}$$

Analyze Mia's reasoning to help Diego.

Mia

This is still a fraction, and I know that 3 divided by 3 is 1. So, the answer is $\frac{1}{7}$.

$$\frac{3^{1}}{7 \div 3^{1}} = \frac{1}{7}$$

3. Explain why Mia's reasoning is not correct.

Analyze Victoria's reasoning to help Diego.

Victoria

We know how to rewrite a division sentence as a fraction.

$$\frac{3}{7 \div 3} = \frac{\frac{3}{7}}{3}$$

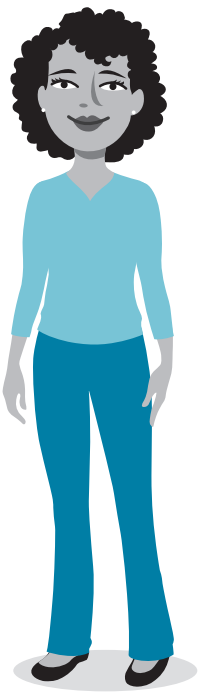
We also want the denominator to be 1, so we will need to use multiplicative inverses.

$$\frac{3}{7 \div 3} = \frac{\frac{3}{7}}{3} \cdot \frac{\frac{3}{7}}{\frac{3}{7}} = \frac{3 \cdot \frac{3}{7}}{1}$$

$$= \frac{9}{7}$$

So the answer is $\frac{9}{7}$.

Remember, the definition of division is that $a \div b = \frac{a}{b}$.



4. Explain how Victoria maintained equivalent fractions when she multiplied by $\frac{\frac{3}{7}}{\frac{3}{7}}$.

A **complex fraction** is a fraction that has a fraction in either the numerator, the denominator, or both the numerator and the denominator.

You can use multiplicative inverses to rewrite any division sentence as multiplication.

WORKED EXAMPLE

$\frac{5}{8} \div \frac{3}{4} = \frac{\frac{5}{8}}{\frac{3}{4}}$

Rewrite the division expression as a complex fraction.

$= \frac{\frac{5}{8}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}}$

Multiply the numerator and denominator by the multiplicative inverse of $\frac{3}{4}$.

$= \frac{\frac{5}{8} \cdot \frac{4}{3}}{\frac{3}{4} \cdot \frac{4}{3}} = \frac{\frac{5}{8} \cdot \frac{4}{3}}{1}$

Perform multiplication and rewrite the denominator as 1.

$= \frac{5}{\cancel{8}_{2}} \cdot \frac{\cancel{4}^{1}}{3} = \frac{5}{6}$

How can you, a savvy mathematician, look at the structure of a division problem and choose the most efficient strategy?



5. Use any method to determine each quotient.

a. $\frac{9}{10} \div \frac{1}{3}$

b. $\frac{5}{6} \div \frac{1}{3}$

c. $\frac{4}{9} \div \frac{1}{2}$

d. $\frac{3}{8} \div \frac{1}{2}$

e. $\frac{7}{9} \div \frac{3}{7}$

f. $\frac{5}{2} \div \frac{2}{3}$

Talk the Talk

Going (Almost) Numberless

Look back at the dividends, the divisors, and the quotients in this lesson. What generalizations can you make about the division of fractions?

1. Complete each statement with *greater than*, *less than*, or *the same as*.

   a. When you divide a quantity greater than 1 by a value between 0 and 1, the quotient will be _____ the original quantity.

   b. When you divide a quantity between 0 and 1 by a value greater than 1, the quotient will be _____ the original quantity.

   c. When you divide a quantity between 0 and 1 by a value between 0 and 1, the quotient will be _____ the original quantity.

2. Complete each statement with *always*, *sometimes*, or *never*.

   a. When you divide a mixed number by another mixed number, the quotient will _____ be greater than 1.

   b. If you mulitply a fraction between 0 and 1 by another fraction between 0 and 1, the product will _____ be less than 1.

   c. When you divide a whole number by a fraction between 0 and 1, the quotient will _____ be less than 1.

   d. When you multiply a fraction between 0 and 1 by a mixed number, the product will _____ be greater than 1.

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3. Consider the quotients of $\frac{5}{6} \div \frac{1}{2}$ and $\frac{5}{6} \div 2$. Describe how these quotients are different.

Lesson 6 Assignment

Write

Explain how to use a model to represent the division of two fractions.

Remember

One way to divide two fractions is to rewrite the division problem as multiplication by the reciprocal of the divisor.

Practice

1. Calculate each quotient.

   a. $\frac{2}{5} \div \frac{1}{3}$

   b. $\frac{7}{8} \div \frac{1}{4}$

   c. $\frac{3}{4} \div \frac{1}{6}$

   d. $\frac{15}{16} \div \frac{3}{4}$

   e. $\frac{7}{12} \div \frac{1}{3}$

   f. $1\frac{1}{8} \div \frac{5}{6}$

   g. $5\frac{3}{8} \div \frac{1}{4}$

   h. $7\frac{1}{3} \div 1\frac{2}{3}$

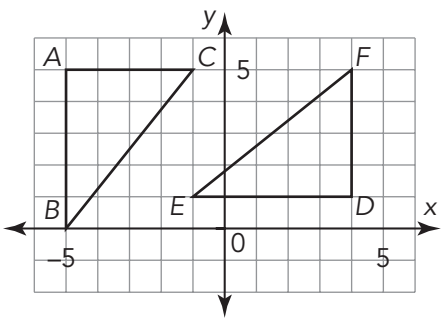
2. The top of a rectangular table has an area of 21 square feet. The width of the table is $3\frac{1}{2}$ feet. What is the length of the table?

Lesson 6 Assignment

3. The area of a rectangular pendant on a necklace is $\frac{3}{4}$ square inch. The height of the rectangle is $1\frac{1}{4}$ inches. What is the length of the base of the rectangle?

Prepare

Use the coordinate plane to determine each distance. Show your work.



1. What is the distance from point *F* to point *D*?

2. What is the distance from point *A* to point *B*?

3. What is the distance from point *C* to point *A*?

4. What is the distance from point *E* to point *D*?

TOPIC 1 SELF-REFLECTION

Name: _____

Factors and Multiples

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Factors and Multiples* topic by:

TOPIC 1: *Factors and Multiples*	Beginning of Topic	Middle of Topic	End of Topic
applying properties of operations to compose and decompose numbers and shapes to understand the relationship between factors and multiples.	○	○	○
creating equivalent expressions using the commutative and distributive properties.	○	○	○
identifying the factors of two whole numbers and determine the greatest common factor.	○	○	○
identifying the multiples of two whole numbers and determine the least common multiple.	○	○	○
generating equivalent numerical expressions using whole number exponents and prime factorization.	○	○	○
determining and using equivalent fractions to show equal parts of the same whole.	○	○	○
representing and comparing benchmark fractions using models, including number lines.	○	○	○
ordering positive rational numbers in mathematical and real-world contexts.	○	○	○

continued on the next page

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TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Factors and Multiples*	Beginning of Topic	Middle of Topic	End of Topic
computing products of fractions multiplied by whole numbers and fractions (including mixed numbers).			
determining whether a quantity is increased or decreased when multiplied by a fraction greater than one or less than one.			
computing quotients of fractions divided by whole numbers and fractions (including mixed numbers).			
interpreting quotients of fractions in real-world and mathematical problems.			
representing division of fractions and mixed numbers using visual models.			
recognizing that dividing by a rational number and multiplying by its reciprocal result in equivalent values.			
using an algorithm to fluently solve multiplication and division problems with fractions.			

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Factor and Multiples* topic.

   _____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   _____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   _____

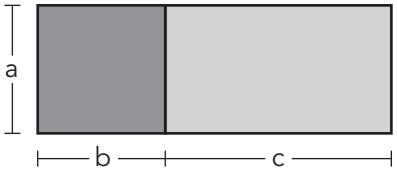
Factors and Multiples Summary

LESSON 1 Writing Equivalent Expressions Using the Distributive Property

A **numeric expression** is a mathematical phrase that contains numbers and operations. An **equation** is a mathematical sentence that uses an equals sign to show that two quantities are the same as one another.

The equation $5 \cdot 27 = 135$ shows that the expression $5 \cdot 27$ is equal to the expression 135.

There are many ways to rewrite equivalent expressions using properties of operations. The **distributive property**, when applied for multiplication, states that for any numbers a, b, and c, $a(b + c) = ab + ac$.



For example, you can use the distributive property to rewrite the expression $4(2 + 15)$.

$$4(2 + 15) = 4 \cdot 2 + 4 \cdot 15$$

You can read and describe the expression $4(2 + 15)$ in different ways.

For example, you can say:

- four times the quantity of two plus fifteen,
- four times the sum of two and fifteen, or
- the product of four and the sum of two and fifteen.

You can describe the expression $4(2 + 15)$ as a product of two factors. The quantity $(2 + 15)$ is both a single factor and a sum of two terms.

You can also use grouping symbols to show that you need to multiply each set of factors before you add them, $(4 \cdot 2) + (4 \cdot 15)$.

NEW KEY TERMS

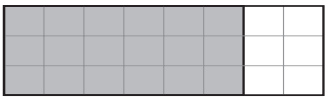
- numeric expression [expresión numérica]
- equation [ecuación]
- distributive property [propiedad distributiva]
- base [base]
- power
- exponent [exponente]
- common factor [factor común]
- prime factorization
- relatively prime [primos relativos]
- greatest common factor (GCF)
- multiple [múltiple]
- commutative property [propiedad conmutativa]
- least common multiple (LCM) [mínimo común múltiplo]
- unit fraction [fracción unitaria]
- equivalent fraction [fracción equivalente]
- benchmark fractions
- algorithm [algoritmo]
- positive rational number [número racional positivo]
- reciprocal [recíproco]
- multiplicative inverse
- complex fraction [fracción compleja]

LESSON 2

Identifying Common Factors and Common Multiples

Understanding the area of rectangles is helpful when learning about factors. A rectangular area model is one way to represent multiplication. You can determine the factors of a number by creating rectangles with a given area. You can combine rectangles with a shared side length, or *common factor*, to create larger rectangles. A **common factor** is a number that is a factor of two or more numbers.

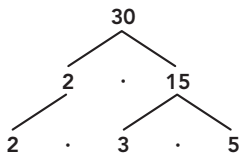
In this example, the common factor, or shared sided length, of the two smaller rectangles is three.



Dimensions of Rectangle with an Area of 18	Dimensions of Rectangle with an Area of 6	Dimensions of the Combined Rectangle	Area of the Combined Rectangle as a Sum of the Smaller Rectangles	Total Area of Combined Rectangle
3×6	3×2	$3(6 + 2)$	$18 + 6$	24

One way to determine the prime factors of a number is by creating a factor tree. You can use a factor tree to organize the prime factors of a given number.

The example shows the **prime factorization** for 30.

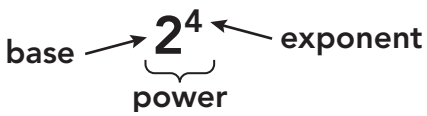


- Pick any whole number factor pair of 30, other than 1 and 30.
- Draw a branch from 30 to each factor, 2 and 15.
- Since both of the factors are not prime, you are not finished.
- Use branches to write a factor pair for 15.
- Because 2, 3, and 5 are all prime, this factor tree is complete.

The prime factors of 30 are 2, 3, and 5.

In some cases, the prime factorization has repeat factors. You can represent repeated multiplication as a power. A **power** has two elements: the base and the exponent. The **base** of a power is the factor multiplied by itself repeatedly, and the **exponent** of the power is the number of times you use the base as a factor.

$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$$



Previously, we used area models to determine common factors between numbers. Another way to determine common factors is to use prime factorization.

For example, you can use prime factorization to determine common factors of 56 and 42. Start by writing each number as a product of its prime factors.

$56 = 2 \cdot 2 \cdot 2 \cdot 7$
$42 = 2 \cdot 3 \cdot 7$

Organize the prime factors into a table, where only shared factors are listed in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

The common factors of the two numbers are the numbers that are in both rows and the product of the numbers that are in both rows.

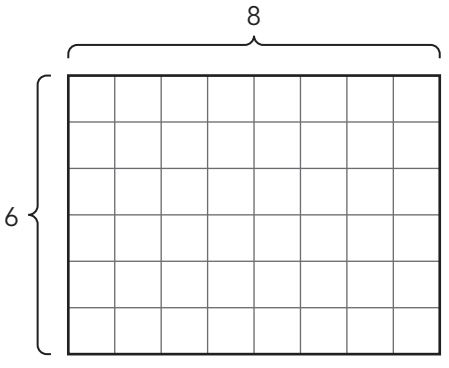
The common factors of 56 and 42 are 2, 7, and 14.

The **greatest common factor (GCF)** is the largest factor two or more numbers have in common. In the example of 56 and 42, the greatest common factor is 14. Two numbers that do not have any common factors other than 1 are **relatively prime**.

The **commutative property**, when applied for multiplication, states that for any numbers a and b, the product $a \cdot b$ is equal to the product $b \cdot a$.

You can use rectangles to determine multiples and common multiples. A **multiple** is the product of a given whole number and another whole number. One way to think about the area model is to analyze the collection of rows and columns in a rectangle.

In the example shown, the addition of each new row creates a multiple of 8. The addition of each new column creates a multiple of 6.



While 48 is a multiple shared by both 6 and 8, it is not the **least common multiple (LCM)**. The LCM is the least multiple (other than zero) that two or more numbers have in common.

You can use tables like the one above with 56 and 42 to determine the LCM. You can determine the prime factors that the numbers share and the prime factors that the numbers do not share. The least common multiple of the two numbers is the product of their shared prime factors and non-shared prime factors.

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$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168$

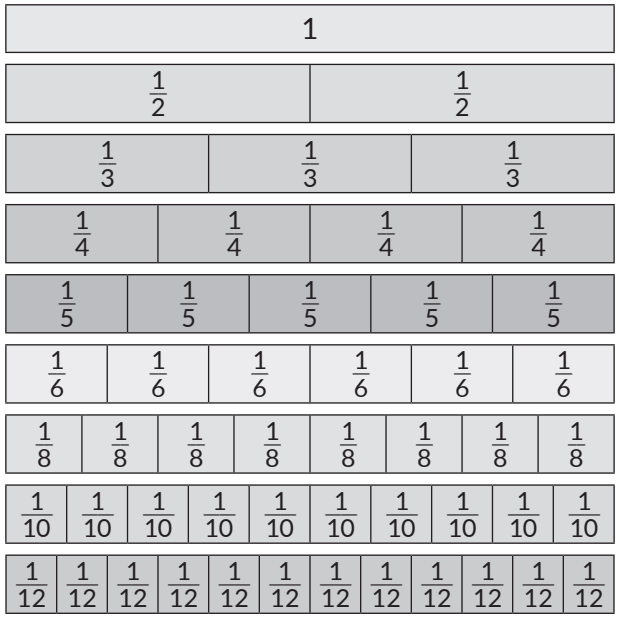
LCM(56, 42) = 168

You can compose and decompose numbers using factors and multiples.

LESSON 3 Dividing a Whole into Fractional Parts

A **unit fraction** is a fraction that has a numerator of 1 and a denominator that is a positive integer. Fractions that represent the same part-to-whole relationship are **equivalent fractions**.

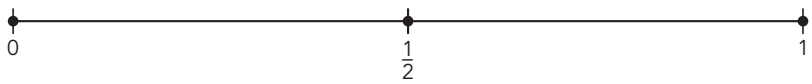
To list a set of fractions in ascending order means to list the set from least to greatest. To list a set of fractions in descending order means to list the set from greatest to least.



LESSON 4 Benchmark Fractions

Benchmark fractions are common fractions you can use to estimate the value of fractions.

Three common benchmark fractions are 0, $\frac{1}{2}$, and 1.



A fraction is close to 0 when the numerator is very small compared to the denominator.

A fraction is close to $\frac{1}{2}$ when the numerator is about half the size of the denominator.

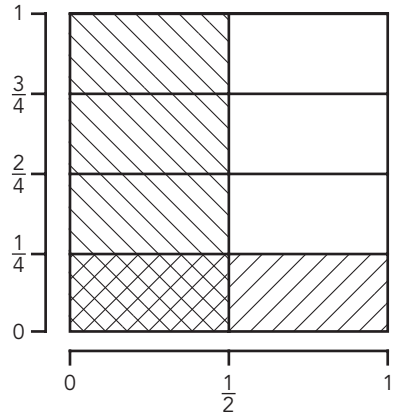
A fraction is close to 1 when the numerator is very close in size to the denominator.

Use benchmark fractions to help order fractions.

LESSON 5 Multiplying Fractions

Area models can be used to represent products, to determine factors, and to list multiples of given numbers. In the same way that area models represent whole number multiplication, area models can represent fraction multiplication.

The expression $\frac{1}{4} \cdot \frac{1}{2}$ is represented in the area model shown.



To multiply two fractions, you multiply across the numerators and denominators. The numerator represents the part of the whole, shown in the model as the number of overlapping individual rectangles. The denominator represents the total number of equal parts in the whole, shown in the model as the total number or individual rectangles.

An **algorithm** is a process or description of steps you can follow to complete a mathematical calculation. The algorithm for multiplying fractions is to multiply across the numerators and denominators.

For example, determine the product of $1\frac{3}{4} \cdot 2\frac{1}{2}$.

Rewrite the factors as improper fractions. $1\frac{3}{4} \cdot 2\frac{1}{2} = \frac{7}{4} \cdot \frac{5}{2}$

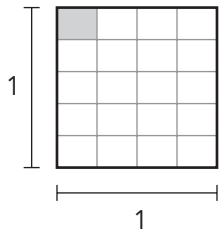
Multiply across numerators and denominators. $\frac{7}{4} \cdot \frac{5}{2} = \frac{35}{8}$

Rewrite the improper fraction as a mixed number. $\frac{35}{8} = 4\frac{3}{8}$

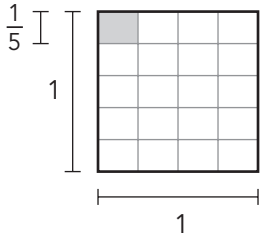
You can use the standard algorithm to multiply a whole number and a mixed number, a mixed number and a fraction less than one, or two mixed numbers when solving real-world and mathematical problems.

LESSON 6 Fraction by Fraction Division

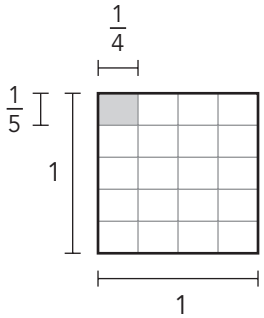
You can write a multiplication-division fact family using fractions. Consider the unit square model divided into 5 equal rows and 4 equal columns.



The shaded area represents the fraction $\frac{1}{20}$, because 1 rectangle is shaded of the 20 total rectangles.



The height of the shaded rectangle is $\frac{1}{5}$ of the height of the model.



The width of the shaded rectangle is $\frac{1}{4}$ of the width of the model.

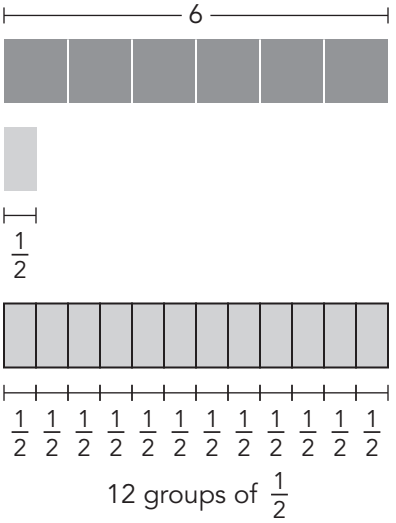
So, the shaded area of the rectangle represents the product $\frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}$.

Fractions are part of a larger set of numbers called rational numbers. A **positive rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are both whole numbers greater than 0.

Division is the process of breaking a number up into an equal number of parts. The expression $6 \div 2$ asks, "How many twos are in 6?" When you divide with fractions, you are asking the same question.

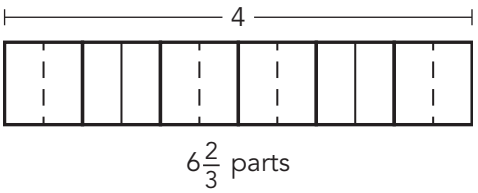
The model shows $6 \div \frac{1}{2}$. The expression asks, "How many halves, or groups of $\frac{1}{2}$, are in 6?"

There are twelve $\frac{1}{2}$ parts in 6, so $6 \div \frac{1}{2} = 12$.



You can use bar models or strip diagrams to solve division problems when you are dividing a whole number by a fraction that is not a unit fraction.

To create the model $4 \div \frac{2}{3}$, you partitioned 4 units to create twelve $\frac{1}{3}$-parts and then grouped two $\frac{1}{3}$-parts together, so $4 \div \frac{2}{3} = 6$.



Notice there are two steps to this process.

STEP 1 The denominator of the divisor is 3.

Partition each whole into $\frac{1}{3}$-parts.

This is the same as $4 \cdot 3 = 12$.

STEP 2 The numerator of the divisor is 2.

Group two of the $\frac{1}{3}$-parts together.

This is the same as $12 \div 2 = 6$.

The combination of steps is the same as $4 \cdot 3 \div 2 = 6$.

You can rewrite any division sentence as multiplication by taking the reciprocal of the divisor. A **reciprocal**, or **multiplicative inverse**, is one of a pair of numbers whose product is 1. The multiplicative inverse of the number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

For example, $\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \cdot \frac{16}{15} = \frac{2}{3}$

A **complex fraction** is a fraction that has a fraction in either the numerator, the denominator, or both the numerator and denominator. You can use multiplicative inverses to rewrite any division sentence as multiplication.

$\frac{5}{8} \div \frac{3}{4} = \frac{\frac{5}{8}}{\frac{3}{4}}$

Rewrite the division expression as a complex fraction.

$= \frac{\frac{5}{8}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}}$

Multiply the numerator and denominator by the multiplicative inverse of $\frac{3}{4}$.

$= \frac{\frac{5}{8} \cdot \frac{4}{3}}{\frac{3}{4} \cdot \frac{4}{3}} = \frac{\frac{5}{8} \cdot \frac{4}{3}}{1}$

Perform multiplication and rewrite the denominator as 1.

$= \frac{5}{\cancel{8}_{2}} \cdot \frac{\cancel{4}^{1}}{3} = \frac{5}{6}$

GRADE 6

TOPIC 2



Austin is Texas's fourth largest city with a population of 964,254 (2018 Census) and is home to the State Capitol Building, completed in 1888. The building opened as the seventh largest in the world, and still rises 14 feet higher than the National Capitol in Washington D.C.

Shapes and Solids

LESSON 1 Constructing Triangles Given Sides 97

LESSON 2 Triangle Sum Theorem 109

LESSON 3 Area of Triangles and Quadrilaterals 119

LESSON 4 Volume of Rectangular Prisms 139

1 Constructing Triangles Given Sides

OBJECTIVES

- Use patty paper to investigate triangles.
- Construct triangles from three angle measures or side lengths, identifying when the conditions determine a unique triangle, more than one triangle, or no triangle.

NEW KEY TERM

- Triangle Inequality theorem

You know how to draw a triangle.

Can you construct a specific triangle when you are given only two or three possible side lengths? Is there more than one possible triangle that you can construct?

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A counterexample is an example that shows the statement is not true.

Tri-, Tri-, and Tri- Again

Classify each statement as *always* or *sometimes* true about triangles.

- For each *always true* statement, explain your reasoning.
- For each *sometimes true* statement, provide an example and a counterexample.

1. The angles of a triangle have the same measure.

2. A triangle has three angles.



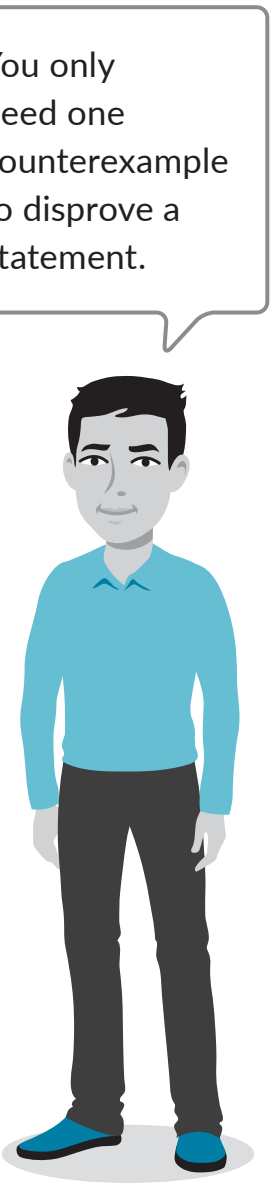
3. Two sides of a triangle have the same measure.

4. One angle of a triangle measures 90 degrees.

ACTIVITY
1.1 Pasta Triangles

Let's investigate the conditions necessary for forming a triangle with different side lengths.

1. Isabella says that when you know two segment lengths, you can form many different triangles. She claims she can also use any three segment lengths to form a triangle. Ethan does not agree. He thinks some combinations will not work. Who is correct?
   Use a counterexample to disprove the incorrect statement.



Ethan then claims that he can just look at the three lengths and know immediately whether they will work. Isabella is unsure. She decides to explore this for herself.

Help Isabella by working through the following investigation.

To begin, you will need a piece of strand pasta. Break the pasta at two random points so the strand is divided into three pieces.

- Try to form a triangle from your three pieces of pasta.
- Measure each of your three pieces of pasta in centimeters.
- Repeat the experiment with a new piece of pasta.

2. Record your measurements and the measurements of your group members in the table provided.

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3. Now, add to your table. Collect and record your classmates' measurements.

Piece 1 (cm)	Piece 2 (cm)	Piece 3 (cm)	Forms a Triangle? (yes or no)

4. Examine the lengths of the pasta pieces that did form a triangle. Compare them with the lengths of the pasta pieces that did not form a triangle. Make a conjecture about the conditions under which it is possible to form a triangle.

Ask Yourself . . .
Did you share your solution with others?

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ACTIVITY
1.2 A Triangle Given Three Segments

Is there a way you can always tell whether three side lengths will form a triangle?



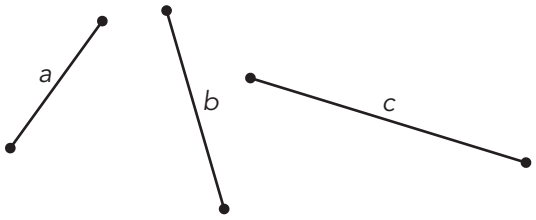
You and your partner should use different sets of segments for this investigation.

Let's continue to investigate Isabella's question and your conjecture with patty paper.

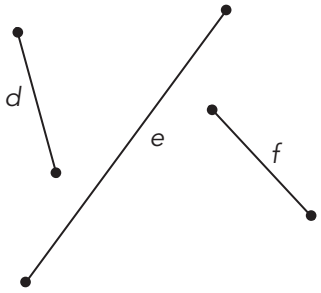
1. Trace each of the three segments onto its own sheet of patty paper.

   a. Overlay the sheets to determine whether you can create a triangle. When you can, record the triangle on its own sheet of patty paper.

Set 1:



Set 2:



   b. Now, create as many different triangles as you can, using the given segments as sides of a possible triangle. Use a different sheet of patty paper to record each unique triangle.

c. What do you notice? How many different triangles were you able to create?

How can you determine the measures of the angles in your triangles?



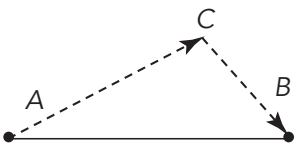
2. Use the patty paper examples from Set 1 and Set 2 to make a conjecture about when three segments can be used to create a triangle. Test your conjecture by creating additional triangles.

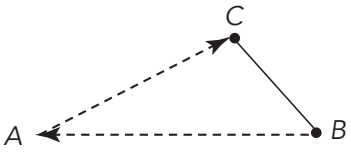
Triangles are *congruent* when all their corresponding angle measures and corresponding side lengths are the same.

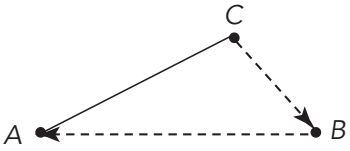
ACTIVITY
1.3 Triangle Inequality Theorem

In Euclidean geometry—the geometry of straight lines on flat planes—the shortest distance between two points is a straight line.

- Any distance $AC + CB$ will be greater than the distance AB.
- Any distance $BA + AC$ will be greater than or equal to the distance BC.
- Any distance $CB + BA$ will be greater than the distance AC.







1. How could you use this fact to test whether three line segments can form a triangle? Explain your reasoning.

2. Provide examples of line segments that cannot possibly form a triangle.

3. What would it mean for the distance $AC + CB$ to be equal to the distance AB? Would these three segments form a triangle?

4. Based on your observations, determine whether it is possible to form a triangle using segments with the given measurements. Explain your reasoning.

   a. 2 cm, 5.1 cm, 2.4 cm

   b. 9.2 cm, 7 cm, 1.9 cm

The rule that you have been using is known as the *Triangle Inequality theorem*. The **Triangle Inequality theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

A theorem is a mathematical rule that can be formally proven.

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Talk the Talk

None, One, or Many?

Determine whether the given information could be used to form a unique triangle, many different triangles, or no triangles. Explain your reasoning.

1. 3 in., 2.9 in., 5 in.

2. 112 mm, 300 mm

3. 5 yd, 10 yd, 21 yd

4. 8 ft, 9 ft, 11 ft

5. 13.8 km, 6.3 km, 7.5 km

Lesson 1 Assignment

Write

Describe the Triangle Inequality theorem in your own words.

Remember

When given two line segments, it is possible to construct an infinite number of triangles When given three line segments, it is possible to construct 0 triangles or a single unique triangle.

Practice

1. Determine whether the given side lengths could be used to form a unique triangle, many different triangles, or no triangles. Explain your reasoning.

   a. 300 mm, 190 mm

   b. 4 m, 5.1 m, 12.5 m

   c. 7.4 cm, 8.1 cm, 9.8 cm

   d. 12 ft, 7 ft, 14 ft

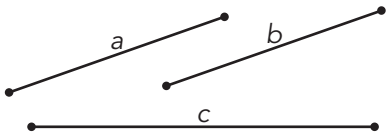
   e. 20.2 in., 11 in., 8.2 in.

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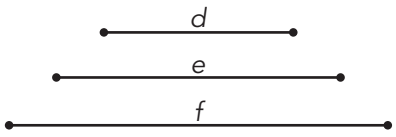
Lesson 1 Assignment

2. Analyze the given line segments. Determine whether the given information would create a unique triangle, multiple triangles, or no triangles. Then, use the information to construct a triangle, when possible.

a.



b.



Prepare

For each of the terms below, describe the angle measure and sketch an example.

1. Acute angle

2. Right angle

3. Obtuse angle

4. Straight angle

2 Triangle Sum Theorem

OBJECTIVES

- Establish the Triangle Sum theorem.
- Explore the relationship between the interior angle measures and the side lengths of a triangle.

NEW KEY TERM

- Triangle Sum theorem

You already know a lot about triangles.

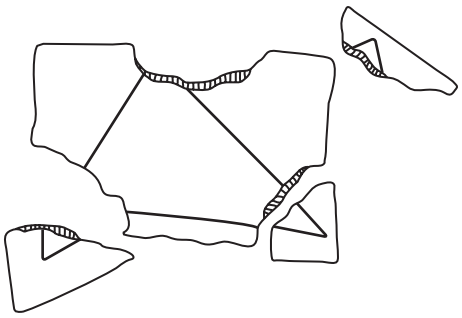
What special relationships exist among the interior angles of a triangle?

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Getting Started

Rip 'Em Up

Draw any triangle on a piece of patty paper. Tear off the triangle's three angles. Arrange the angles so that they are adjacent angles.



1. What do you notice about these angles? Write a conjecture about the sum of the three angles in a triangle.

2. Compare your angles and your conjecture with your classmates'. What do you notice?

ACTIVITY
2.1 Analyzing Angles and Sides

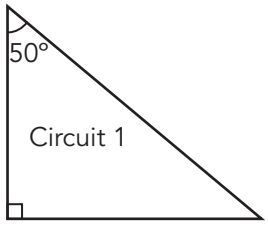
In the previous activity, what you noticed about the relationship between the three angles in a triangle is called The *Triangle Sum theorem*. The **Triangle Sum theorem** states that the sum of the measures of the interior angles of a triangle is 180°.

Samuel is organizing a criterium bike race. Criteriums consist of several laps around a closed circuit. Based on the city map provided to him, Samuel designs three different triangular circuits and presents scale drawings of them to the local cycling association for consideration.

1. Classify each circuit according to the type of triangle created.

2. Samuel needs to label each circuit with the measures of each angle and the lengths of each side. Work with one triangle at a time.

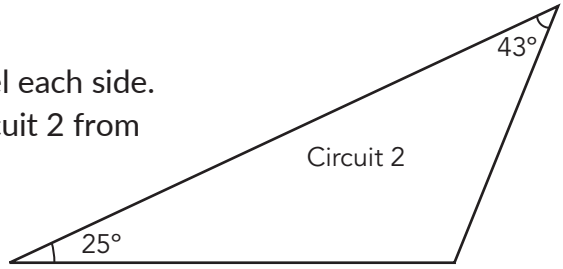
   a. Use the Triangle Sum theorem to determine the measure of the third angle for Circuit 1. Label the unknown angle measure. Then, use the table to list the angle measures for Circuit 1 from least to greatest.



   b. Measure the length of each side of Circuit 1. Label each side length. Then, use the table to list the side lengths for Circuit 1 from least to greatest.

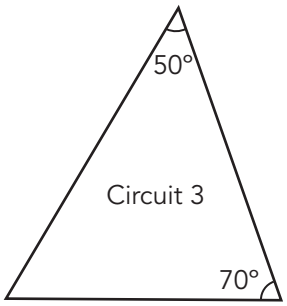
   c. Use the Triangle Sum theorem to determine the measure of the third angle for Circuit 2. Label the unknown angle measure. Then, use the table to list the angle measures for Circuit 2 from least to greatest.

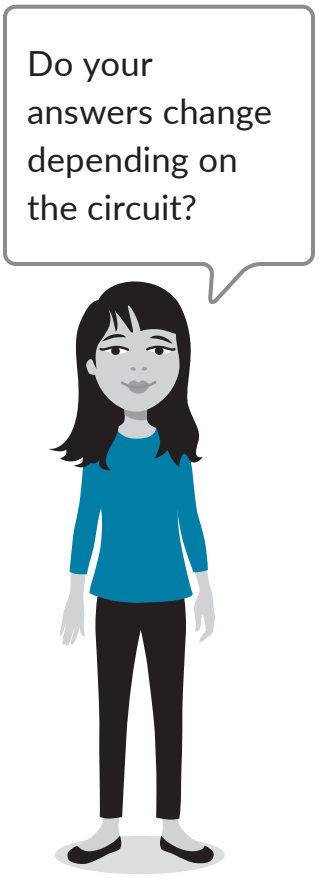
   d. Measure the length of each side of Circuit 2. Label each side. Then, use the table to list the side lengths for Circuit 2 from least to greatest.



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e. Use the Triangle Sum theorem to determine the measure of the third angle for Circuit 3. Label the unknown angle measure. Then, use the table to list the angle measures for Circuit 3 from least to greatest.





f. Measure the length of each side of Circuit 3. Label each side length in the diagram. Then, use the table to list the side lengths for Circuit 3 from least to greatest.

	Circuit 1	Circuit 2	Circuit 3
Angle Measures			
Side Lengths			

The sharper the angles on a race course, the more difficult the course is for cyclists to navigate.

3. Perform the following tasks for each circuit.

   a. Describe what you notice about the location of the angle with the least measure and the location of the shortest side.



   b. Describe what you notice about the location of the angle with the greatest measure and the location of the longest side.

4. Harper, the president of the local cycling association, presents a fourth circuit for consideration. The measures of two of the interior angles of the triangle are 57° and 61°. Determine the measure of the third angle, then describe the location of each side with respect to the measures of the opposite interior angles without drawing or measuring any part of the triangle.

   a. Measure of the third angle

   b. Longest side of the triangle

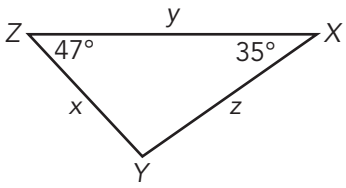
   c. Shortest side of the triangle

When two angles of a triangle have equal measures, what does that mean about the relationship between the sides opposite the angles?

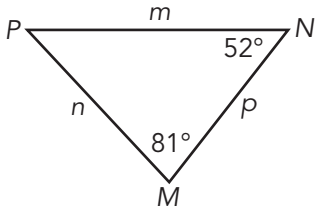


5. List the side lengths from shortest to longest for each diagram.

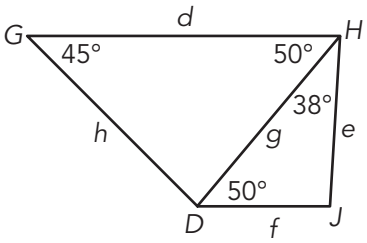
   a.



   b.



   c.



6. Determine the measure of each unknown angle in Question 6.

   a. m∠*Y*

   b. m∠*P*

   c. m∠*GDH*

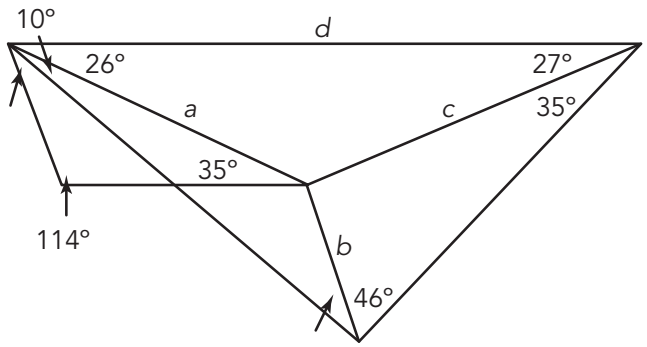
   d. m∠*DJH*

The measure of angle *A* in degrees can be written as m∠*A*. For example, you can write the measure of ∠*X* in Question 6 as m∠*X* = 35°.

Talk the Talk

So Many Angles!

1. Consider the diagram shown.



a. Determine the measures of the eight unknown angle measures inside the figure.

b. List the labeled side lengths in order from least to greatest.

Lesson 2 Assignment

Write

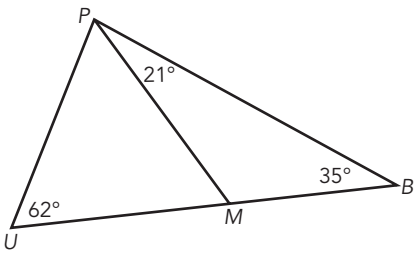
Describe the connection between the measure of an interior angle in a triangle and the length of the side of the triangle opposite that angle.

Remember

The sum of the measures of the interior angles of a triangle is 180°.

Practice

1. Use the figure shown to answer each question.



   a. Explain how you can use the Triangle Sum theorem to calculate $\angle UPB$.

   b. Calculate the measure of $\angle UPB$.

   c. Explain how you can use the Triangle Sum theorem to calculate the measure of $\angle PMU$.

Lesson 2 Assignment

d. Calculate the measure of $\angle PMU$.

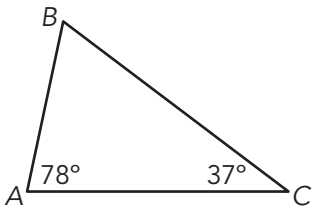
e. List the sides of $\triangle PMB$ in order from shortest to longest.
Explain how you determined your answer.

f. List the sides of $\triangle PUB$ in order from shortest to longest.
Explain how you determined your answer.

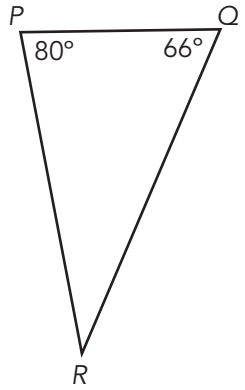
Lesson 2 Assignment

2. Determine the measure of the unknown angle in each triangle.

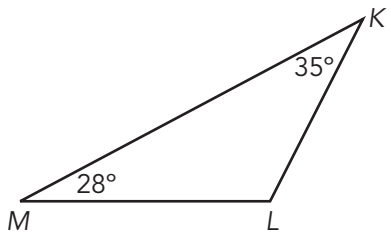
a.



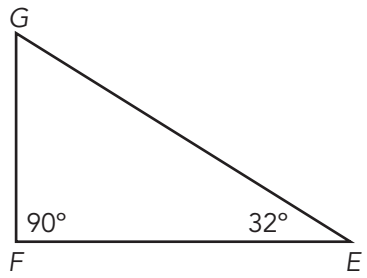
b.



c.

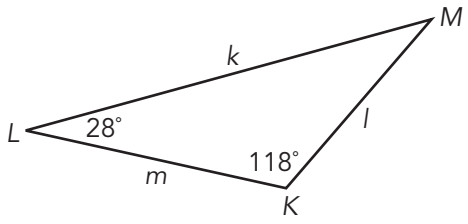


d.

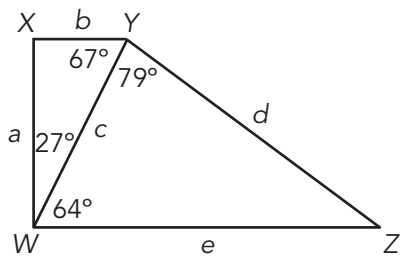


3. List the side lengths from shortest to longest for each diagram.

a.



b.

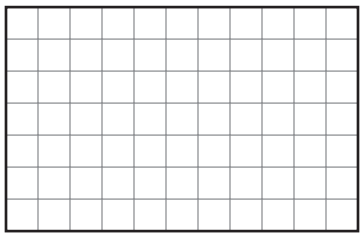


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Lesson 2 Assignment

Prepare

Write 3 different expressions to describe the total area of the rectangle shown.



3 Area of Triangles and Quadrilaterals

OBJECTIVES

- State and compare the attributes of different figures.
- Explain that the area of a parallelogram is the same as the area of a rectangle with the same base length and height.
- Derive the formulas for the areas of triangles, parallelograms, and trapezoids by composing or decomposing the various figures into rectangles and triangles.
- Solve real-world and mathematical problems by composing and decomposing figures.

NEW KEY TERMS

- parallelogram
- variable
- straightedge
- trapezoid

You can take a figure apart and put it back together in a different way without changing its area.

How can you compose and decompose rectangles to reason about the areas of common figures?

Getting Started

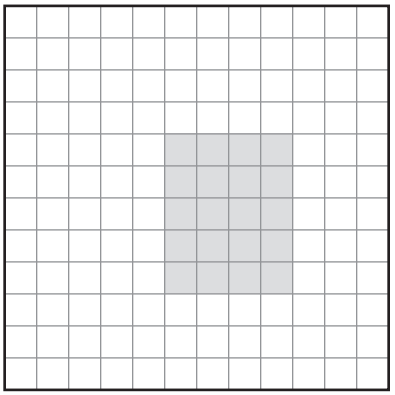
In the 20s

Consider each two-dimensional figure.

Take Note . . .
An *attribute* is a characteristic to describe a figure.

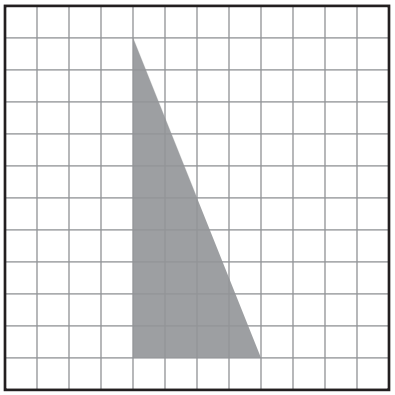
1. Name each figure and describe the attributes.

a.



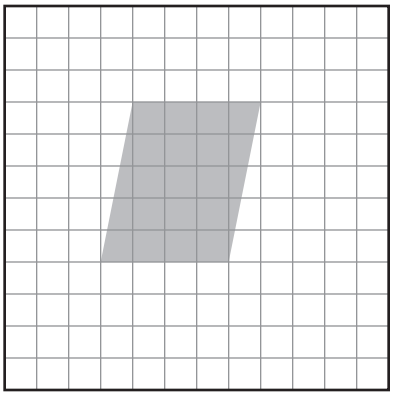
Name: _____
Attributes:

b.



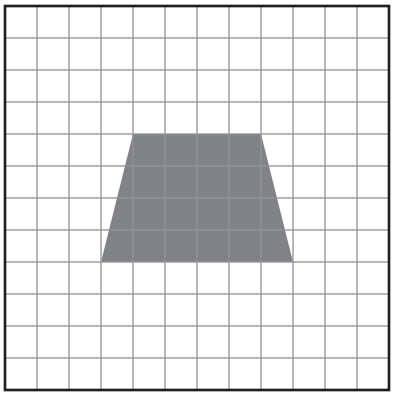
Name: _____
Attributes:

c.



Name: _____
Attributes:

d.



Name: _____
Attributes:

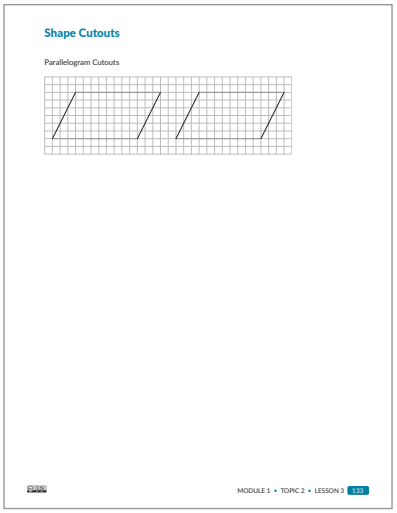
2. Each shaded figure shown has an area of exactly 20 square units. Show how you know.

ACTIVITY 3.1

Investigating the Area of a Parallelogram

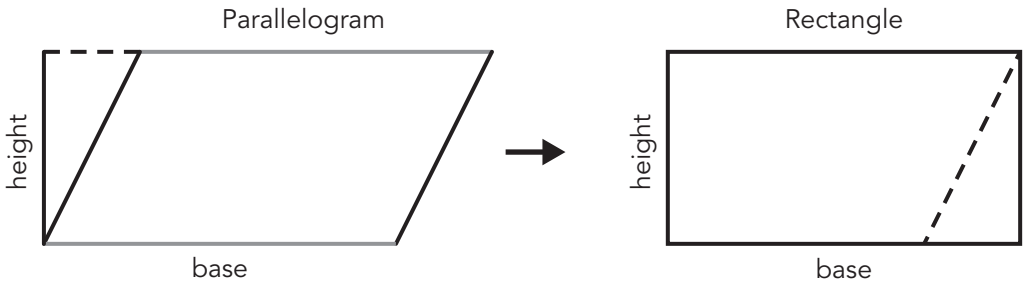
In this activity, you will investigate the area of a *parallelogram* using what you know about the area of a rectangle. A **parallelogram** is a four-sided figure with two pairs of parallel sides and opposite sides that are equal in length.

Cut out a parallelogram from the grid located at the end of the lesson.



Remember . . .
A rectangle is a special type of parallelogram.

1. Cut your parallelogram into pieces so that you can reassemble it to form a rectangle. Tape your rectangle in the space provided.



In a parallelogram, you can label any of the four sides as the base. The height, represented by a line segment, is the perpendicular distance from a base to its opposite side.

2. Label the base and height of the parallelogram and rectangle.

Take Note . . .
The right angle symbol indicates that the lines are perpendicular.

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Compare the attributes of the parallelogram and the rectangle you composed.

3. How does the height of the parallelogram relate to the height of the rectangle? How does the length of the base of the parallelogram relate to the length of the base of the rectangle? Explain your reasoning.

Ask Yourself . . .
When you write a sentence to **explain your reasoning**, be sure to express a complete idea. If you cover up the question, does your sentence make sense?

4. Describe the relationship between the areas of a parallelogram and rectangle that have the same base and height.

5. Use the terms *base* and *height* to describe how to calculate the area of a parallelogram.

Take Note . . .
A **variable** is a letter used to represent a number.

When you want to represent a quantity that varies or changes, you can use a *variable*. The use of variables helps you write formulas to express relationships.

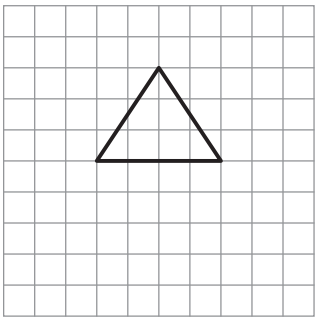
6. Write the formulas to calculate the areas of a parallelogram and a rectangle. Use b to represent the length of the base and h to represent the height.

ACTIVITY
3.2 Investigating the Area of a Triangle

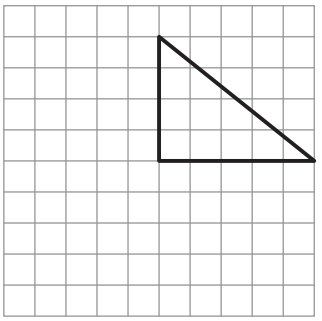
In this activity, you will investigate the area of a triangle using what you know about the area of a parallelogram.

Consider each triangle shown.

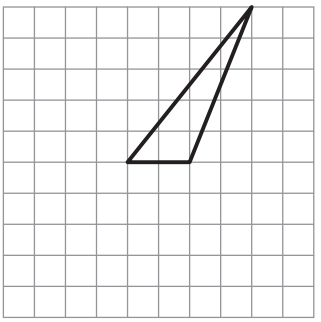
Triangle 1



Triangle 2



Triangle 3



1. Use a separate piece of patty paper to trace each triangle.

   a. Rotate the patty paper to create a parallelogram composed of two identical triangles.

   b. Draw the parallelogram you created for each triangle on your patty paper and label its base and height.

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2. For each triangle, compose a second parallelogram using a different side of the triangle.

3. Determine the area of each parallelogram you created.

4. How does the area of each triangle relate to the area of the parallelogram?

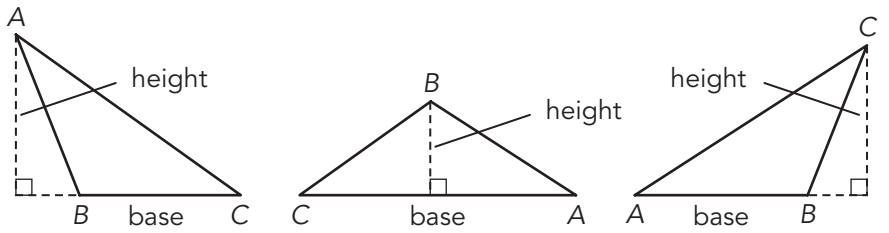
Remember . . .
The formula for the area of a parallelogram is $A = bh$.

5. Write a formula to calculate the area of a triangle using the formula for the area of a parallelogram. Use b to represent the length of the base and h to represent the height.

WORKED EXAMPLE

As with the base of a parallelogram, the base of a triangle can be any of its sides. The height of a triangle, represented by a line segment, is the perpendicular distance from a vertex to the line containing the base.

Triangle *ABC* is shown in three different positions.

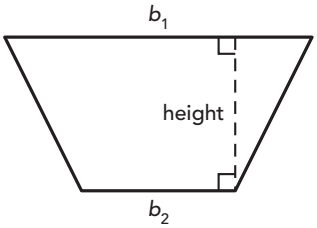


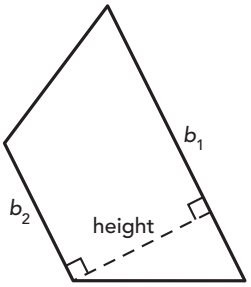
6. Analyze the Worked Example. What general statement can you make about determining the area of a triangle? Explain your reasoning.

ACTIVITY
3.3 Investigating the Area of a Trapezoid

You have seen that decomposing and composing can help you think about shapes differently to determine their areas. In this activity, you will use the same strategy to determine the formula for calculating the area of a *trapezoid*.

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The two bases of a trapezoid are parallel to each other. The other two sides are called the *legs* of the trapezoid. A height of a trapezoid is the length of a line segment drawn perpendicular from one base to the other.



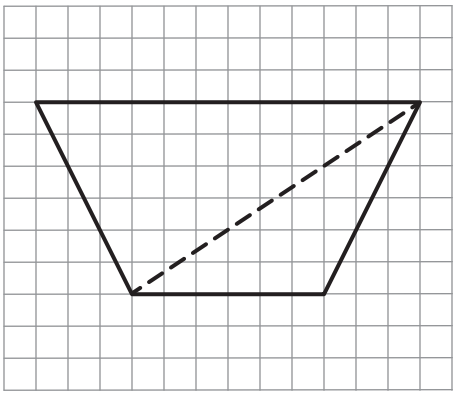


Take Note . . .
The variable b represents a base, but a trapezoid has two bases. So, we use subscripts to distinguish between the two different bases: b_1 and b_2 are not equal in length.

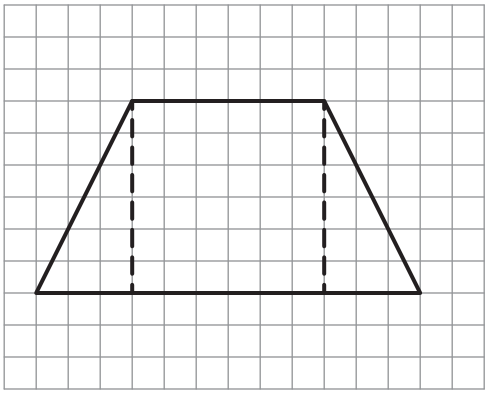
Sofia, Diego, and Daniel are discussing how to determine the area of the same trapezoid.



Sofia
I can determine the area of the trapezoid by decomposing it into two triangles.

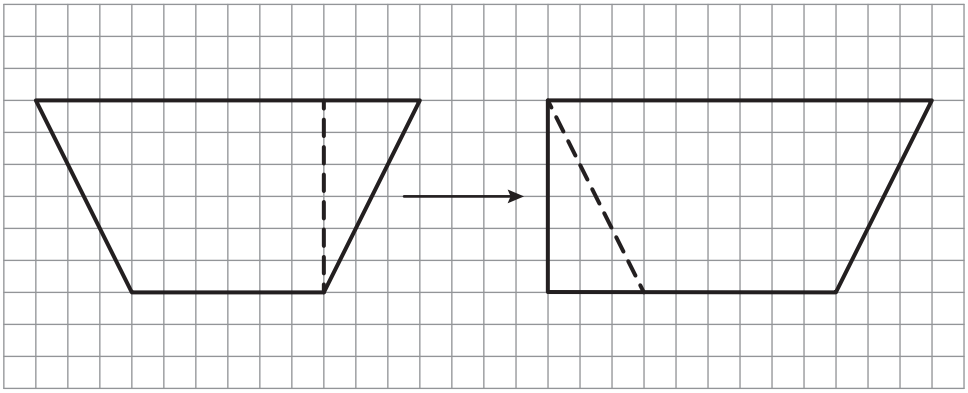


Diego
I can determine the area of the trapezoid by decomposing it into two triangles and a rectangle.



Daniel

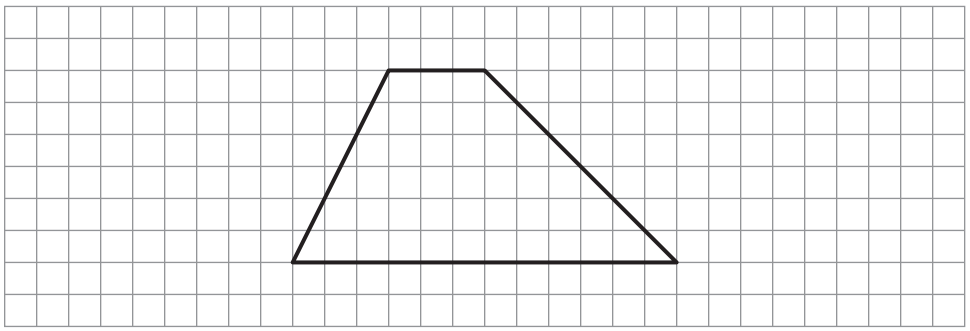
I can determine the area of the trapezoid by cutting off a triangle from one side of the trapezoid and rearranging the shape into a rectangle.



1. Calculate the area of the trapezoid using each method.

You can use what you know about determining the area of a parallelogram to determine the formula for the area of a trapezoid.

2. To determine the exact area of the trapezoid shown, compose two trapezoids into a parallelogram. Show how you can determine the area of the trapezoid.



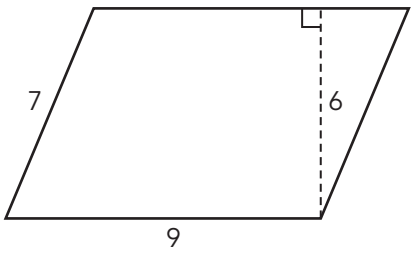
Take Note . . .
When you trace or draw a figure, use a straightedge. A **straightedge** is a tool to draw straight lines.

3. Describe how to calculate the area of any trapezoid in terms of the two bases and the height.

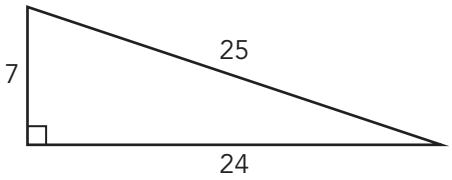
ACTIVITY
3.4 Calculating Areas of Figures

Calculate the area of each figure.

1.



2.



Which dimensions do you need to determine the area?

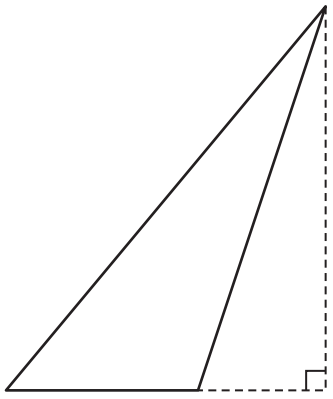


Use a ruler to measure the dimensions of each figure to the nearest $\frac{1}{2}$ inch. Then, determine the area of each figure.

3.



4.



Determine the unknown length in each figure.

5. A parallelogram has an area of 63 square units. The height of the parallelogram is 7 units. What is the base length of the parallelogram?

6. A triangle has an area of 24 square units. The base length of the triangle is 4 units. What is the height of the triangle?



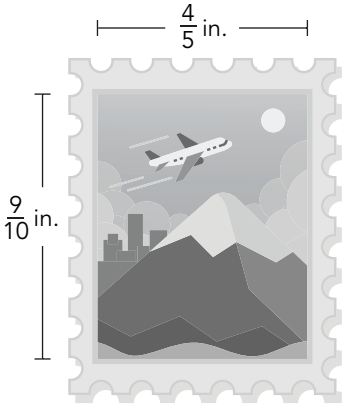
Solve each area problem.

7. Chloe has enough material to build a rectangular dance floor with an area of 200 square feet. The greatest width the dance floor can be is $12\frac{1}{2}$ feet. What would be the length of this dance floor?

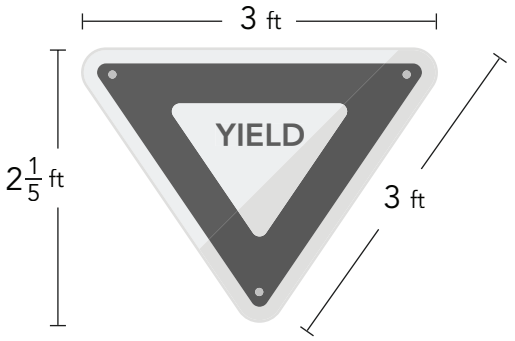
Remember . . .
Use a straightedge to draw your figures.

8. Determine the area of each object.

a.



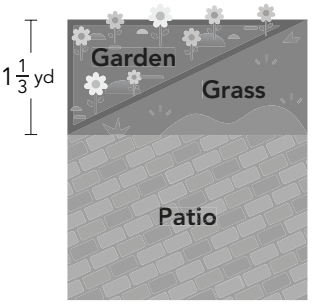
b.



PROBLEM SOLVING



9. Mia has enough garden soil to make a garden bed with an area of 6 square yards. She wants to build the garden bed in the corner of her yard so it is the shape of a right triangle. Because of the location of her patio, one leg of the triangle must be $1\frac{1}{3}$ yards long. Determine the length of the other leg of the triangle.



Talk the Talk

Figure 'Em Out!

You have decomposed and composed parallelograms in this lesson to derive the formulas for the area of a parallelogram, a triangle, and a trapezoid.

1. Draw each figure and then label a base and height. Next, write the formula to calculate the area of each. Use *A* for the area, *b* for the length of the base, and *h* for the height.

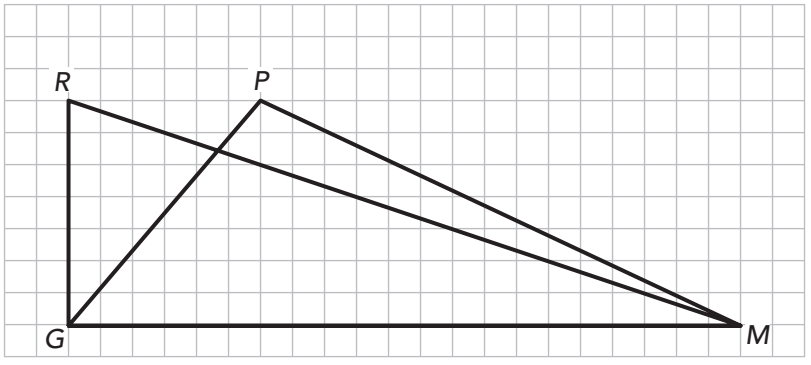
   a. Parallelogram

   b. Triangle

   c. Trapezoid

Consider $\triangle RGM$ and $\triangle PGM$.

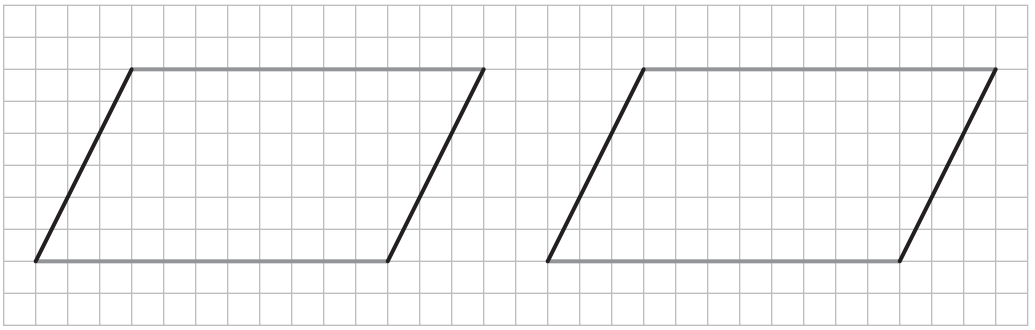
2. Without performing any calculations, determine which triangle has the greater area. Write a sentence to explain your reasoning.



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Shape Cutouts

Parallelogram Cutouts



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Why is this page blank?

So you can cut out the parallelograms on the other side

Lesson 3 Assignment

Write

Define each term in your own words.

1. Height of a parallelogram
2. Height of a triangle

Remember

To determine the area of a parallelogram, a triangle, or a trapezoid, compose or decompose it into one or more figures.

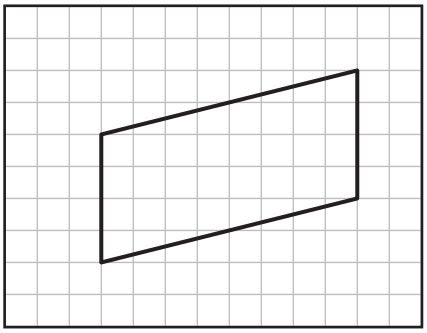
Area of a parallelogram $= bh$

Area of a triangle $= \frac{1}{2}bh$

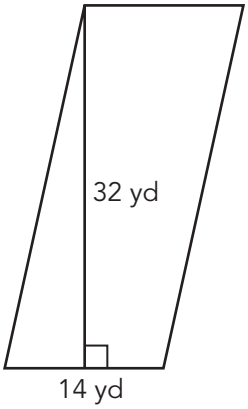
Area of a trapezoid $= \frac{1}{2}(b_1 + b_2)h$

Practice

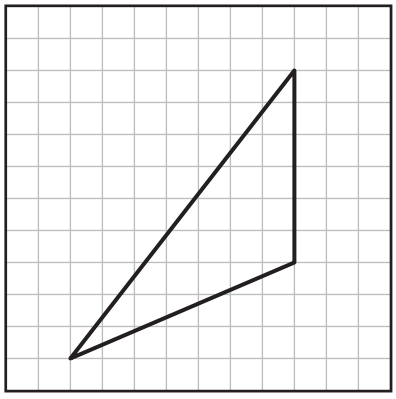
1. Identify a base and the corresponding height for the given parallelogram. Then, determine its area.



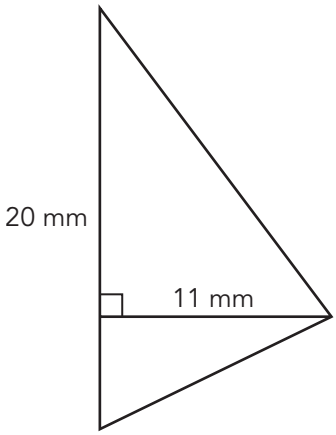
2. Calculate the area of the parallelogram.



3. Identify a base and the corresponding height for the given triangle. Then, determine its area.



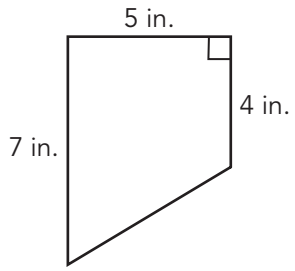
4. Calculate the area of the triangle.



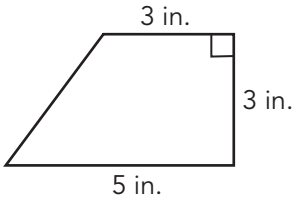
Lesson 3 Assignment

5. Juan cut pictures in the shapes shown to place into his scrapbook. What is the area of each picture?

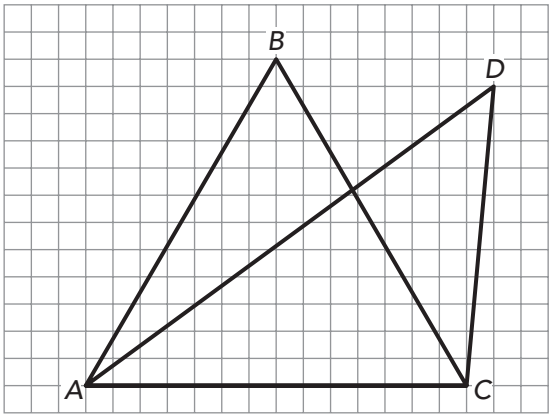
a.



b.



6. Without performing any calculations, determine which triangle has the greater area. Write a sentence to explain your reasoning.



Lesson 3 Assignment

Prepare

Determine each least common multiple.

1. LCM(2, 10)

2. LCM(3, 8)

3. LCM(6, 14)

4. LCM(10, 15)

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4 Volume of Rectangular Prisms

OBJECTIVES

- Determine the volume of right rectangular prisms with fractional edge lengths.
- Connect the volume formulas $V = \ell wh$ and $V = Bh$.

NEW KEY TERMS

- volume
- geometric solid
- polyhedron
- face
- edge
- vertex
- right rectangular prism
- cube

You know about three-dimensional figures such as cubes and other rectangular prisms. You also know how to operate with positive rational numbers.

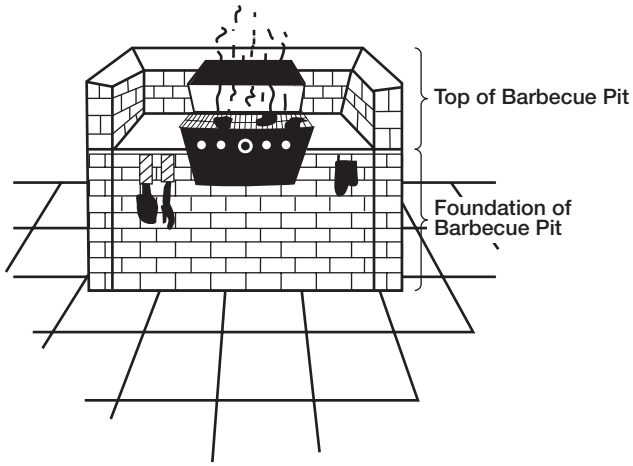
How can you calculate the measurements of any rectangular prism, even one with fractional edge lengths?

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Getting Started

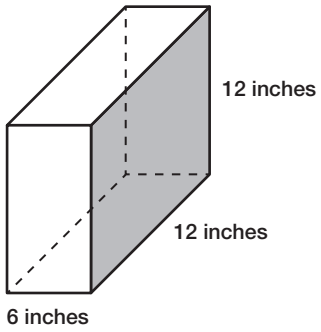
Brick by Brick

A landscaping company is installing a brick barbecue pit for a client on their backyard patio.



Remember . . .
Volume is the amount of three dimensional space occupied by an object. You measure the volume of an object in cubic units.

The bricks for the barbecue pit are 12 inches tall, 12 inches long, and 6 inches wide.

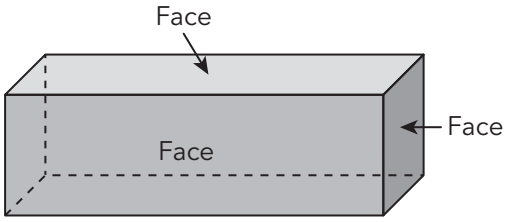


1. Write an equation to determine the volume of one brick in inches. Then, determine the volume.

2. Write an equation to determine the volume of one brick in feet. Then, determine the volume.

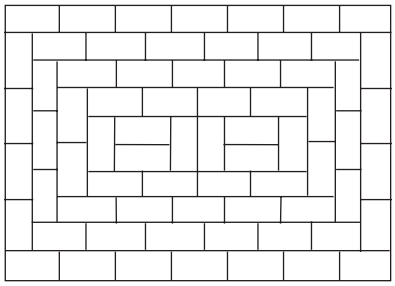
ACTIVITY
4.1 Connecting Volume Formulas

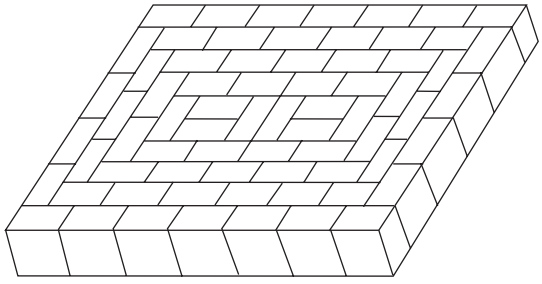
Recall that a polygon is a closed figure formed by three or more line segments. A **geometric solid** is a bounded three-dimensional geometric figure. A **polyhedron** is a three-dimensional solid figure made up of polygons. A **face** is one of the polygons that makes up a polyhedron. An **edge i**s the intersection of two faces of a three-dimensional figure. The point where multiple edges meet is known as a **vertex** of a three-dimensional figure.



The brick from the Getting Started is a *right rectangular prism*. A **right rectangular prism** is a polyhedron with three pairs of congruent and parallel rectangular faces.

The foundation of the barbecue pit is a rectangular prism that is 4 feet tall. The bricks are staggered so that the structure is solid. Consider the overhead and side view of the first layer of bricks.





1. What is the height of one layer of bricks, in feet?

2. How many layers are needed to complete the foundation of the barbecue pit?

3. Use the dimensions of 1 brick and the side view of the foundation to determine the length and width of the foundation.

4. Use the height, length, and width of the barbecue pit foundation to determine the volume of the barbecue pit foundation.

5. Determine the number of bricks needed for one layer of the foundation.

6. Determine the number of bricks needed to complete the entire foundation of the barbecue pit.

Consider the formulas for volume of a prism and area of a rectangle:

$$V = \ell \cdot w \cdot h$$
$$A = \ell \cdot w$$

When you use B to represent the area of the base of a rectangular prism, then you can rewrite the formula for area: $B = \ell \cdot w$.

Using both of these formulas, you can rewrite the formula for the volume of a rectangular prism as $V = B \cdot h$, where V represents the volume, B represents the area of the base, and h represents the height.

7. Write an equation and calculate the volume of the barbeque pit foundation using the formula $V = \ell wh$.

8. Write an equation and calculate the volume of the barbeque pit foundation using the formula $V = Bh$.

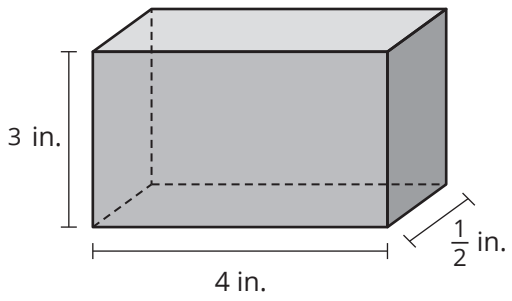
One advantage of using the formula $V = Bh$ is you can use this formula to calculate the volume of many different geometric solids. However, the formula for calculating the value of B will change depending on the base shape of the solid.

ACTIVITY
4.2 Solving Volume Problems

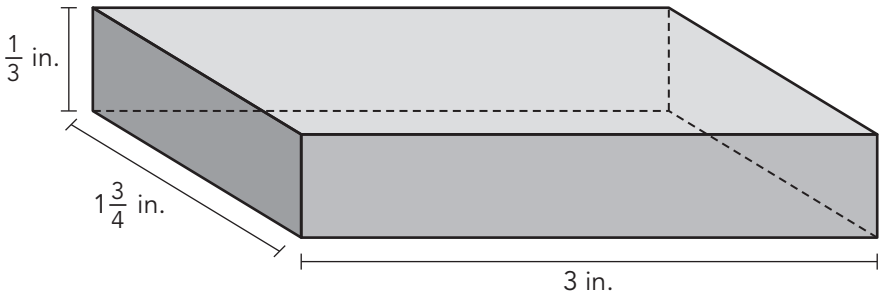
Use either of the volume formulas for a right rectangular prism to write an equation and answer each question.

1. Calculate the volume of each right rectangular prism.

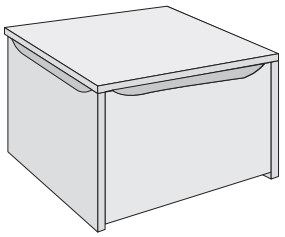
a.



b.



2. A carpenter is building a large toy chest. The chest is in the shape of a rectangular prism with square bases. The area of the base of the toy chest is 9 square feet. The height of the chest is $2\frac{2}{3}$ feet.

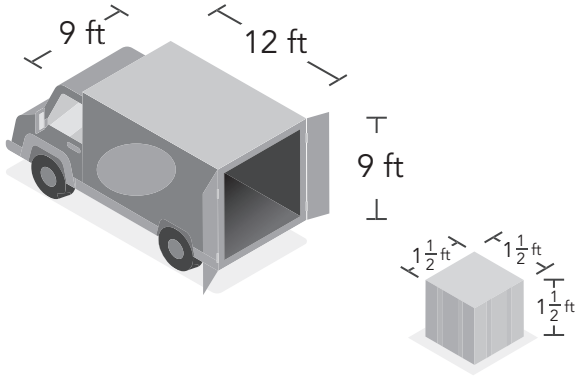


What is the volume of the toy chest?

Remember, a **cube** is a polyhedron that has congruent squares as faces.

3. Jayden packs a moving truck with cube-shaped boxes that have side lengths of $1\frac{1}{2}$ feet. The back of the truck is a rectangular prism with the dimensions $7\frac{1}{2}$ feet by 15 feet by $7\frac{1}{2}$ feet.

   a. What is the volume of each box?



   b. Calculate the volume of the back of the moving truck.

   c. Determine the number of boxes that will completely fill the back of the truck.

Talk the Talk

Fractionally Full

Solve each problem. Show your work.

1. Determine the volume of a right rectangular prism with dimensions $2\frac{1}{2}$ feet $\times$ 1 foot $\times$ $\frac{1}{2}$ foot.

2. Minh makes earrings and packages them in cubic boxes that measure $\frac{1}{6}$ foot wide. What is the volume of each box?

3. The school athletic director has a storage closet that is $4\frac{1}{2}$ feet long, $2\frac{2}{3}$ feet deep, and 6 feet tall.

   a. She wants to put carpet in the closet. How much carpeting will she need?

   b. What is the volume of the closet?

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Lesson 4 Assignment

Write

Suppose a rectangular prism has fractional edge lengths. Describe how you can determine the volume of the rectangular prism.

Remember

The volume of a rectangular prism is a product of its length, width, and height.

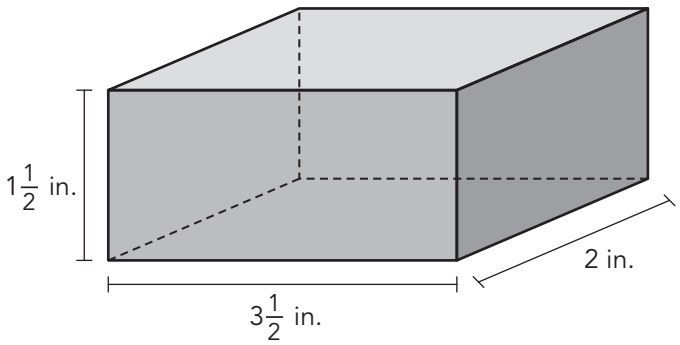
$$V = l \cdot w \cdot h$$

or

$$V = B \cdot h$$

Practice

1. Determine the volume of the prism shown.

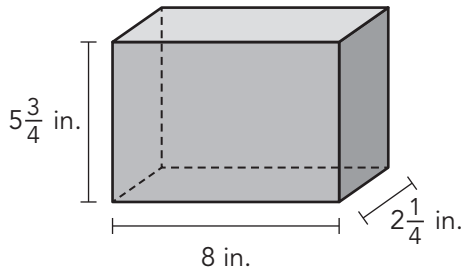


2. A rectangular prism has a base with an area of 22 square feet and a height of $2\frac{3}{4}$ feet. Determine the volume of the rectangular prism.

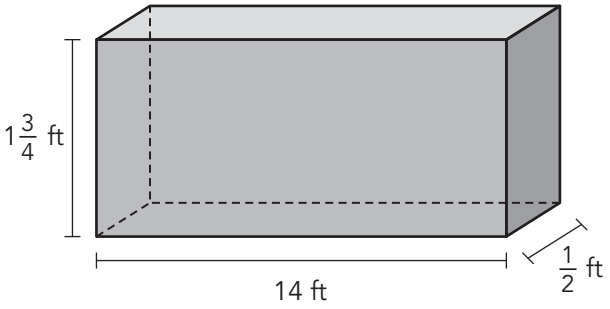
Lesson 4 Assignment

3. Calculate the volume of each right rectangular prism. Round your answer to the nearest hundredth, when necessary.

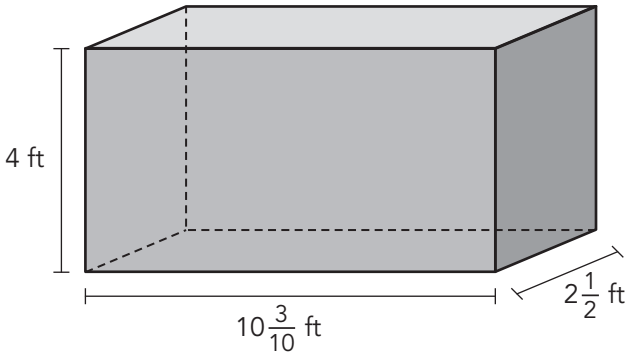
a.



b.



c.



Lesson 4 Assignment

Prepare

Write each decimal in expanded form.

1. 240.12

2. 5.06

TOPIC 2 SELF-REFLECTION

Name: _____

Shapes and Solids

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Shapes and Solids* topic by:

TOPIC 2: *Shapes and Solids*	Beginning of Topic	Middle of Topic	End of Topic
determining whether three given line segments will construct a triangle.	○	○	○
informally proving that the sum of the interior angles of a triangle is 180°.	○	○	○
determining the relationship between the lengths of the sides of a triangle and the measures of the angles.	○	○	○
recognizing and knowing how to compose and decompose shapes into triangles and rectangles.	○	○	○
applying the techniques of composing and/ or decomposing to determine the area of triangles, and special quadrilaterals to solve mathematical and real-world problems.	○	○	○
discussing, developing, and justifying formulas for triangles and parallelograms.	○	○	○
applying volume formulas for right rectangular prisms to solve real-world and mathematical problems involving rectangular prisms with fractional edge lengths.	○	○	○

continued on the next page

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TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Shapes and Solids* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?



Shapes and Solids Summary

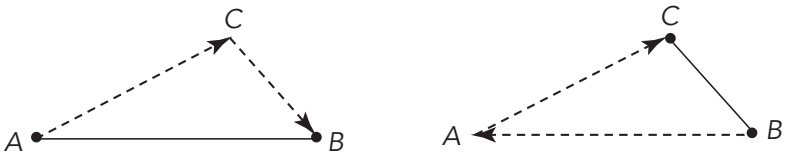
LESSON 1 Constructing Triangles Given Sides

Triangles are congruent when all of their corresponding angle measures and corresponding side lengths are the same. When given information can be used to construct congruent triangles, the information is said to define a unique triangle.

The **Triangle Inequality Theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AC + CB > AB$

$BA + AC > BC$



When given two line segments, it is possible to construct an infinite number of triangles. When given three line segments, it is possible to either construct a unique triangle or no triangle.

LESSON 2 Triangle Sum Theorem

The **Triangle Sum theorem** states that the sum of the measures of the interior angles of a triangle is 180°. The longest side of a triangle is opposite the interior angle with the greatest measure, and the shortest side is opposite the interior angle with the least measure.

You can use the Triangle Sum theorem to determine the measure of the third angle of a triangle when two angle measures of the same triangle are given.

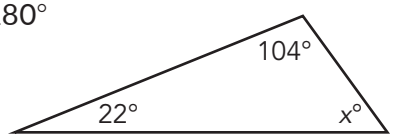
$x + 22 + 104 = 180$

$x + 126 = 180$

$x = 54$

The measure of the third angle in this triangle is 54°.

$22° + 104° + 54° = 180°$



NEW KEY TERMS

- Triangle Inequality theorem
- Triangle Sum theorem [Teorema de la suma del triángulo]
- parallelogram [paralelogramo]
- variable [variable]
- straightedge
- trapezoid [trapecio]
- geometric solid
- polyhedron [poliedro]
- face
- edge
- vertex [vértice]
- right rectangular prism [prisma rectangular recto]
- cube [cubo]
- volume [volumen]

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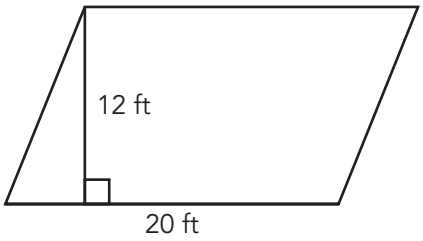
LESSON 3

Area of Triangles and Quadrilaterals

A **parallelogram** is a four-sided figure with two pairs of parallel sides with each pair equal in length.

In a parallelogram, the height is the perpendicular distance from the base to the opposite side. The area of a parallelogram is equal to $b \cdot h$, where the variable b represents the base and h represents the height. A **variable** is a letter used to represent a number.

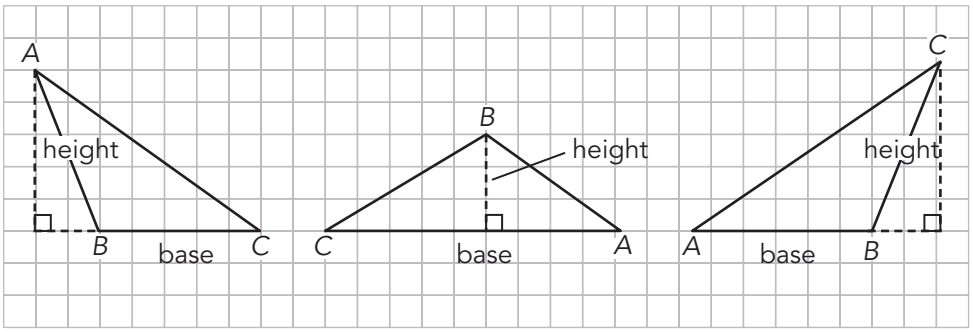
For example, in this parallelogram, the base, b, is 20 feet and the height, h, is 12 feet.



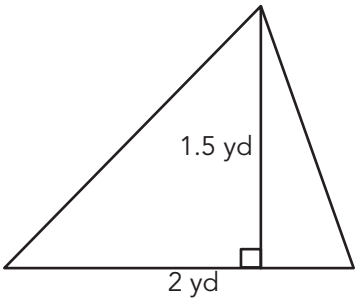
$$\begin{aligned}\text{Area of a parallelogram} &= bh \\ &= (20)(12) \\ &= 240 \text{ square feet}\end{aligned}$$

The area of a triangle is equal to $\frac{1}{2}bh$. The base of a triangle can be any of its sides.

The height of a triangle is the length of a line segment drawn from a vertex of the triangle to the opposite side so that it forms a right angle with the opposite side.



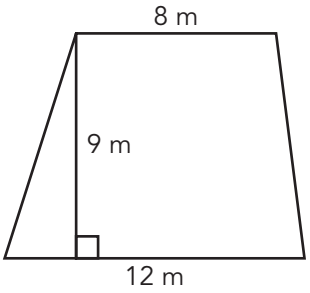
For example, in this triangle, the base, b, is equal to 2 yards, and the height, h, is equal to 1.5 yards.



$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2}bh \\ &= \frac{1}{2}(2)(1.5) \\ &= 1.5 \text{ square yards}\end{aligned}$$

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. Trapezoids have two bases, often labeled b_1 and b_2. The bases are parallel to each other. The height is the perpendicular distance between the bases. The area of a trapezoid is equal to $\frac{1}{2}(b_1 + b_2)h$.

For example, in this trapezoid, one of the bases is 8 meters and the other base is 12 meters. The height, h, of the trapezoid is 9 meters.



$$\begin{aligned}\text{Area of trapezoid} &= \frac{1}{2}(b_1 + b_2)h \\ &= \frac{1}{2}(8 + 12)(9) \\ &= \frac{1}{2}(20)(9) \\ &= 90 \text{ square meters}\end{aligned}$$

LESSON 4

Volume of Rectangular Prisms

A *polygon* is a closed figure formed by three or more line segments. A **geometric solid** is a bounded three-dimensional geometric figure. A **polyhedron** is a three-dimensional solid figure made up of polygons; each of these polygons is called a **face**. An **edge** is the intersection of two faces, and a **vertex** is the point where the edges meet.

For example, Figure A is a **right rectangular prism**, which is a polyhedron with three pairs of congruent and parallel rectangular faces.

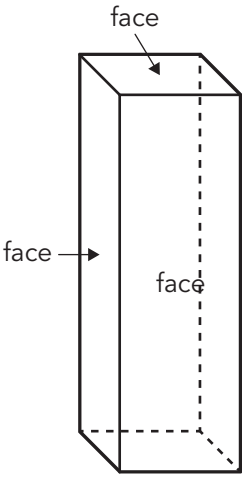


Figure A

Figure B is a **cube**, which is a polyhedron that has six congruent squares as faces.

Volume is the amount of space occupied by an object. The volume of an object is measured in cubic units.

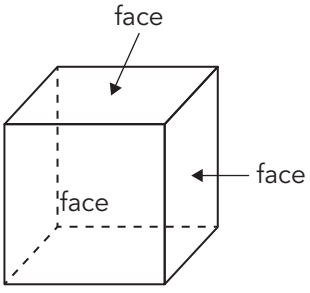
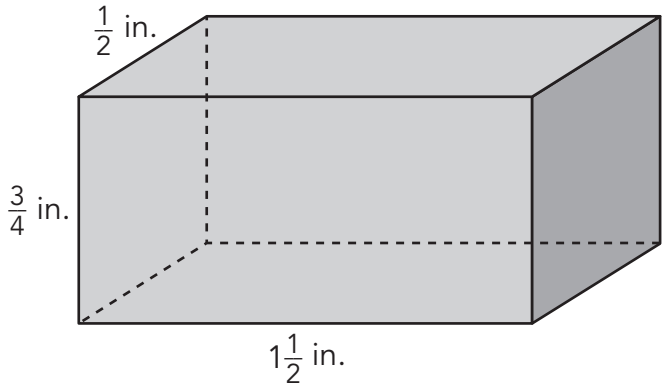


Figure B

The volume of a rectangular prism is the product of its length, width, and height: $V = l \cdot w \cdot h$

You can use the formula $V = Bh$ to calculate the volume of any prism. However, the formula for calculating the value of B will change depending on the shape of the base. In a rectangular prism, $B = l \cdot w$.

For example, consider the right rectangular prism with the given dimensions.



$V = Bh = (l \cdot w) \cdot h$

$1\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{4} = \frac{9}{16}$

The volume of the right rectangular prism is $\frac{9}{16}$ cubic inch.

GRADE 6

TOPIC 3



The Louvre Pyramid in Paris was designed by Chinese-American architect I.M. Pei.

Decimals

LESSON 1 Plotting, Comparing, and Ordering Rational Numbers 159

LESSON 2 Multiplying Decimals 169

LESSON 3 Dividing Decimals 181

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1 Plotting, Comparing, and Ordering Rational Numbers

OBJECTIVES

- Plot decimals on a number line.
- Compare and order decimal values.
- Compare fractions and decimals using a number line.

You know how to represent a number in more than one way by decomposing it.

How can you compare numbers represented in different ways?

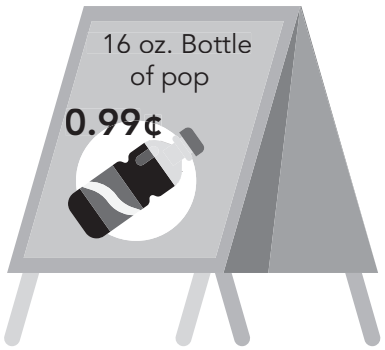
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Getting Started

Representations and Misrepresentations

Recall that you can write decimals to represent fractions with a denominator of 10 or a power of 10 (such as 100, 1000, 10,000, and so on). You can use decimals to write amounts of money.

1. Ethan saw this advertisement outside a convenience store. He went in and handed the cashier a penny to pay for his bottle of pop. The cashier told him that it wasn't enough money. Who's correct? Explain your reasoning.



2. Consider the representations shown for the cost of each item. Circle the correct representations. If a representation is not correct, state why not.

   a. Item B costs one dollar and fifty cents.
      - $1\frac{1}{2}$ dollars
      - $\$1\frac{1}{2}$ cents
      - $1.50
      - 1.50¢

   b. Item C costs one dollar.
      - $1
      - 100 cents
      - $1.00
      - $1.00¢

ACTIVITY
1.1 Using a Number Line to Compare Decimals

Decimals are valuable when you need more precision for measurements between whole numbers.

The decimal point in a number separates the whole part of the number from the fractional part. The digits following the decimal point represent a value less than 1.

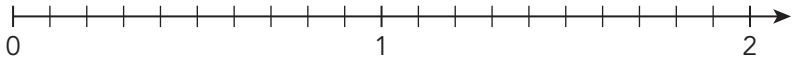
Let's use a number line to represent decimals.

Your teacher will assign students to participate in the activity. Be sure to record what happens on the number line.

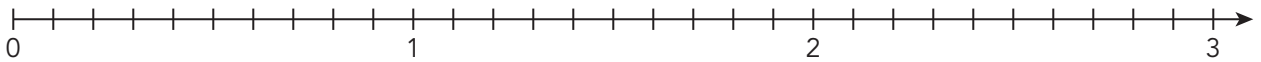
Remember . . .

You can write a rational number in the form $\frac{a}{b}$ where b is not equal to 0. If you can write a decimal as a fraction, then it is a rational number.

1. Plot and label the point where each student stands on the number line.



2. Plot a point on the number line to represent each decimal.



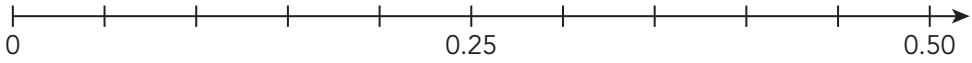
a. 0.25 b. 1.38 c. 0.5 d. 2.3

e. 0.56 f. 2.92 g. 1.75 h. 1.1

3. Which number has the greatest value? The least value?

4. How can you use a number line to compare numbers?

5. Analyze the number line.



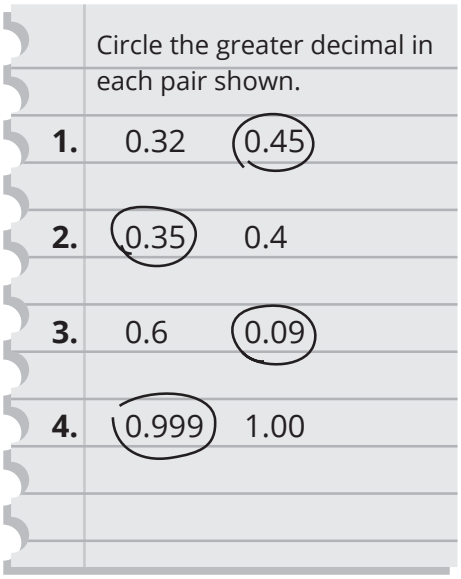
   a. Label each tick mark on the number line with the decimal value it represents. Then, plot points to represent 0.3 and 0.4.

   b. Plot and label four points that represent decimals between 0.3 and 0.4.

6. Write four decimals between 0.45 and 0.46.

7. Write a rule that states how to determine when a decimal is greater than another decimal.

You are babysitting your neighbor Isabella, who is in the fourth grade.
Isabella is doing her homework and asks you whether her work is correct.



8. Explain to Isabella whether her homework is correct and instruct her on how to correct any incorrect answers.

9. Explain to Harper why she is incorrect.

Harper

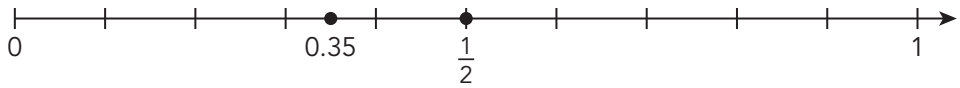
I can tell how large each decimal is by looking at its last digit. If the last digit of one decimal is greater than the last digit of a second decimal, then the first decimal is greater.

You can compare fractions and decimals using a number line.

WORKED EXAMPLE

Consider $\frac{1}{2}$ and 0.35. Which value is greater?

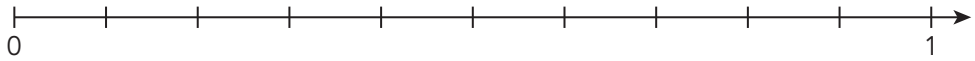
Plot each value on a number line.



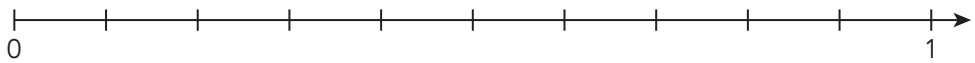
Because $\frac{1}{2}$ is to the right of 0.35 on the number line, $\frac{1}{2}$ is greater than 0.35, or $0.35 < \frac{1}{2}$.

10. Plot each pair of values on a number line. Then, use >, <, or = to compare each pair of numbers.

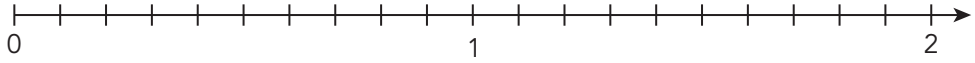
a. $\frac{9}{10}$ _____ 0.71



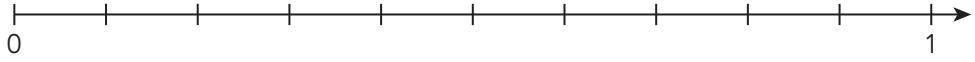
b. 0.15 _____ $\frac{1}{4}$



c. $1\frac{3}{4}$ _____ 0.67



d. $\frac{1}{5}$ _____ 0.2



Talk the Talk

Be Fast on Your Feet

You can compare decimals to solve real-world problems.

Sofia and Minh are judges at the sixth-grade track meet. They recorded these results for the 200-meter dash. The runners' names and their times are shown.

Runner	Time (seconds)
Chloe	26.98
Mia	25.23
Daniel	25.16
Juan	25.3
Jayden	27.78

1. Sofia claims Juan is first because 25.3 is the fastest time. Minh insists Daniel is the winner because 25.16 is the fastest time. Who's correct? Explain your reasoning.

2. Identify which runner won each medal or ribbon.

   Gold Medal/First Place _____

   Silver Medal/Second Place _____

   Bronze Medal/Third Place _____

   Ribbon/Fourth Place _____

   Ribbon/Fifth Place _____

Ask Yourself . . .
Did you make a plan to solve the problem?

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Lesson 1 Assignment

Write

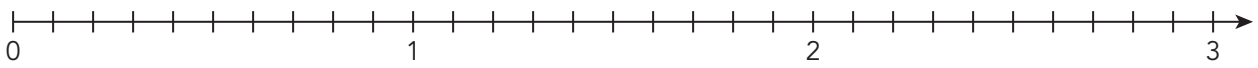
Use your own words to define *rational number*.

Remember

To determine whether one decimal is greater than another, line up the decimals using place value and compare the digits from left to right until one digit is greater than the other. The decimal with the greater digit is the greater number.

Practice

1. Plot and label each decimal on the number line shown.



a. 1.8 b. 0.62 c. 2.25 d. 0.05 e. 1.37 f. 2.9

2. Write four decimals between 0.82 and 0.83.

3. Order the decimals from greatest to least.

516.84	512.78	516.24
512.79	512.04	513.64
513.98	516.09	513.99

4. Compare the values in each pair using $>$, $<$, or $=$.

a. $\frac{7}{10}$ _____ 0.48

b. $\frac{2}{5}$ _____ 0.25

c. 0.04 _____ $\frac{1}{25}$

d. 1.56 _____ $1\frac{2}{3}$

Lesson 1 Assignment

5. Victoria's dog had eight puppies. She recorded the weight of each puppy at birth. List the weights of the puppies in ounces from least to greatest.

13.6 oz	13.09 oz
12.88 oz	12.96 oz
13.75 oz	14.1 oz
12.47 oz	13.32 oz

Prepare

Round each decimal to the nearest ten and then to the nearest tenth.

1. 19.54

2. 11.96

3. 124.35

4. 245.339

5. 10.888

6. 1258.445

2 Multiplying Decimals

OBJECTIVES

- Estimate the product of decimals.
- Analyze patterns of products of decimals.
- Multiply decimals by whole numbers.
- Use the standard algorithm to multiply two decimals.
- Solve real-world and mathematical problems involving area and volume.

NEW KEY TERMS

- kite
- composite solid

You know how to multiply multi-digit whole numbers.

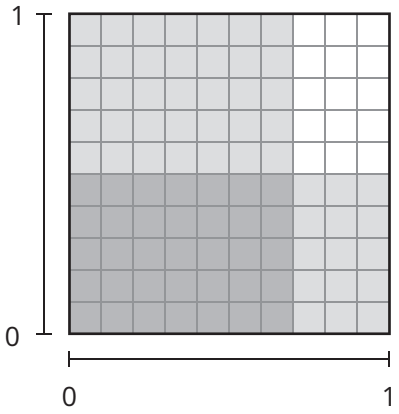
How can you use place value to determine the position of a decimal point in the product of decimals?

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Doing Some Re-Modeling

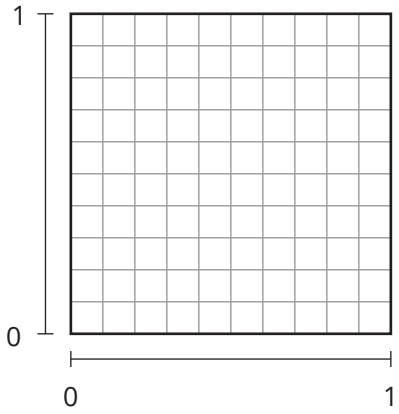
You previously used an area model to show the product of two fractions. An area model can also represent the product of two decimals.

Consider the area model shown.



1. Write the multiplication problem represented by the area model.

2. Use the hundredths grid shown to model the expression $0.2 \cdot 0.8$ and determine the product.



Ask Yourself . . .
How can you organize and record your mathematical ideas?



3. Mateo says that the product of any two decimals that are less than 1 will always be less than 1. Liam says that the product can be greater than 1. Who's correct? Use the area model to explain your reasoning.

ACTIVITY
2.1 Decimal Products

Elijah needs to calculate the area of the floor in a hallway of his house so he can buy new carpeting. The floor is 32.75 feet long and 7.5 feet wide.

Elijah said, "I use estimation to help place the decimal point correctly in the product."

WORKED EXAMPLE

The area of the floor is 32.75 feet · 7.5 feet.

He estimates his two numbers.

- 32.75 is close to 30.
- 7.5 is close to 8.
- $30 \cdot 8 = 240$

So he knows his product is close to 240 but larger since he rounded down. Next, he calculates the product of 32.75 · 7.5.

$$\begin{array}{r} 32.75 \\ \cdot \quad 7.5 \\ \hline 16375 \\ +\ 229250 \\ \hline 245.625 \end{array}$$

Elijah knows the product will be close to, but greater than, 240, so he must place the decimal point after the 5. The area of the floor is 245.625 square feet.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

1. Try Elijah's estimation strategy to determine the correct decimal place in each multiplication problem.

   a. $52.6 \cdot 0.83 = 43658$

   b. $7.9 \cdot 0.6 = 474$

   c. $0.94 \cdot 24.9 = 23406$

Take Note . . .
Multiply decimals as you would with whole numbers. Then, place the decimal point in the product.

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2. Aaliyah thought that using a pattern would help her understand how to calculate the product in a decimal multiplication problem.

   a. Complete the table.

Problem	Product	Problem	Product	Problem	Product
$32 \cdot 100$		$3.2 \cdot 100$		$0.32 \cdot 100$	
$32 \cdot 10$		$3.2 \cdot 10$		$0.32 \cdot 10$	
$32 \cdot 1$		$3.2 \cdot 1$		$0.32 \cdot 1$	
$32 \cdot 0.1$		$3.2 \cdot 0.1$		$0.32 \cdot 0.1$	
$32 \cdot 0.01$		$3.2 \cdot 0.01$		$0.32 \cdot 0.01$	
$32 \cdot 0.001$		$3.2 \cdot 0.001$		$0.32 \cdot 0.001$	

   b. Describe any patterns that you notice.

3. Use the fact that $26 \cdot 31 = 806$ to calculate each product. Do not use your calculator.

   a. $2.6 \cdot 31$ b. $2.6 \cdot 3.1$ c. $0.26 \cdot 3.1$ d. $2.6 \cdot 0.31$

   e. $0.26 \cdot 31$ f. $2.6 \cdot 0.031$ g. $0.026 \cdot 0.31$ h. $0.26 \cdot 0.31$

4. Look at the pattern in Question 3. How can you tell without multiplying which expressions will have the same product?

5. Consider Kai's statement.

> **Kai**
>
> Write a zero at the end of a number to multiply it by 10 and two zeros at the end to multiply it by 100. So, $3.2 \cdot 10 = 3.20$ and $3.2 \cdot 100 = 3.200$.

a. Explain to Kai why her reasoning is incorrect.

b. Explain to Kai the similarities and differences between these number sentences.

$32 \cdot 10 = 320$ $3.2 \cdot 10 = 32$ $32 \cdot 100 = 3200$ $3.2 \cdot 100 = 320$

When multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in the factors.

6. Consider Ava's statement.

> **Ava**
>
> I multiplied $3.65 \cdot 4.22$ on my calculator and got 15.403. There should be four decimal places in the product, so it must actually be 1.5403.

a. Explain to Ava why the calculator is correct.

b. How could the estimate of $3.65 \cdot 4.22$ have helped Ava understand?



ACTIVITY
2.2 Solving Problems with Decimals

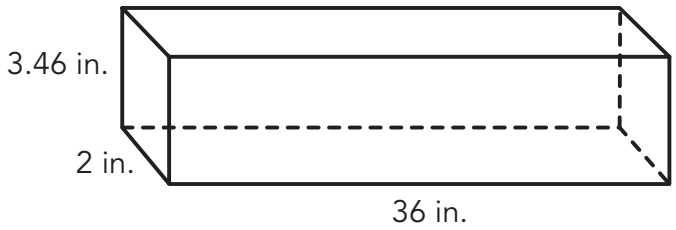
Use the algorithms for operating with decimals to solve each problem.

1. A water bottle holds 16.9 fluid ounces of water. Samuel drinks 8 bottles of water each day. How many fluid ounces of water does Samuel drink each day?

2. Noah is buying 2.5 pounds of sliced turkey from the deli. The sliced turkey sells for $6.72 a pound. How much will Noah pay for the sliced turkey?

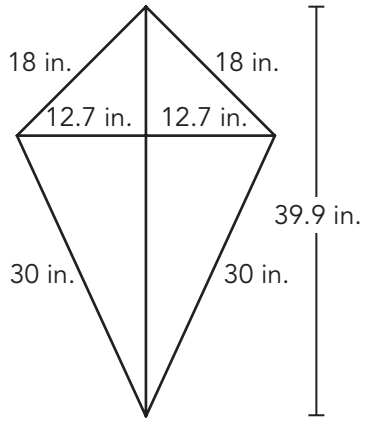
3. One inch is equal to about 2.54 centimeters. About how many centimeters long is a rope that is 7.625 feet long?

4. A poster is rolled up and mailed in a cardboard box in the shape of a rectangular prism.



The rolled-up poster takes up 113.04 cubic inches of space. Calculate the amount of space left in the cardboard box after the poster is placed inside it.

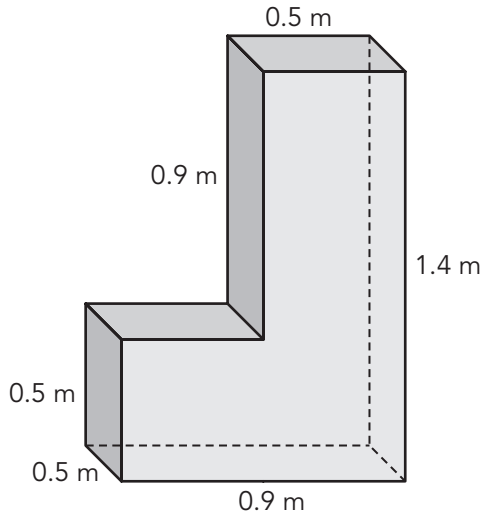
5. Emma is designing her own kite. She uses two wooden sticks as a frame to hold the material. The longer stick measures 39.9 inches and divides the shorter stick into equal lengths of 12.7 inches where they cross. Determine the area of the material on Emma's kites.



Take Note . . .

In mathematics, a **kite** is a quadrilateral with two pairs of consecutive congruent sides.

6. Your class is building a concrete chair for a community garden. A drawing of the chair design is shown. Calculate the amount of concrete needed to build the chair.



Take Note . . .

The chair is a *composite solid*. A **composite solid** is made up of more than one geometric solid.

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Talk the Talk

Disappearing Decimals

Use the patterns of decimal products learned in this lesson to complete this activity.

You did your homework on multiplying decimals, but when you were walking to school, it started raining and some of the decimal points got washed away.

1. Insert decimal points to make each multiplication sentence correct.

   a. $368 \cdot 526 = 19.3568$

   b. $8962 \cdot 9121 = 81.742402$

   c. $75.6 \cdot 98.75 = 7465500$

   d. $152 \cdot 152 = 231.04$

   e. $5875 \cdot 2569 = 0.15092875$

   f. $94.05 \cdot 6.27 = 5896935$

Ask Yourself . . .
What observations can you make?

Lesson 2 Assignment

Write

Describe a way to use estimation to help place the decimal point correctly in the product of two decimals.

Remember

When multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in the factors.

Practice

1. Insert decimal points into each multiplication equation to make the equation true.

   a. $25.7 \cdot 3.4 = 8738$

   b. $4157 \cdot 321 = 13.34397$

   c. $135.4 \cdot 15.04 = 2036416$

   d. $256 \cdot 1111 = 284.416$

2. Estimate each product to the nearest whole number. Then, calculate each product.

   a. $3.9 \cdot 1.4$

   b. $6.18 \cdot 9.4$

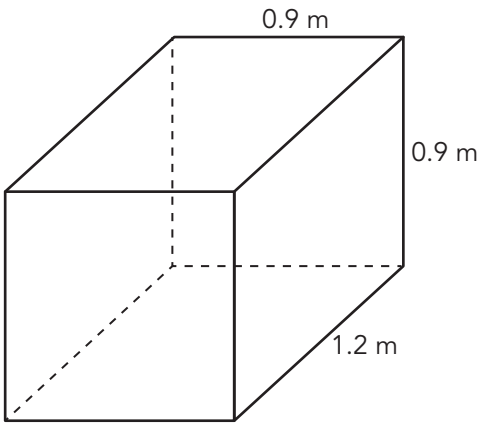
   c. $4.07 \cdot 7.25$

   d. $5.216 \cdot 2.87$

Lesson 2 Assignment

3. Use the algorithms for operating with decimals to solve each question.

a.



b. William is buying 2.6 pounds of apples from the market. The apples sell for $1.75 per pound. How much will William pay for the apples?

Lesson 2 Assignment

Prepare

Write a fact family for each division equation.

1. $72 \div 8 = 9$

2. $84 \div 6 = 14$

3. $2464 \div 308 = 8$

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3 Dividing Decimals

OBJECTIVES

- Estimate quotients.
- Use the standard algorithm to divide decimals.
- Write fractions as decimals.

NEW KEY TERMS

- terminating decimal
- repeating decimal

You know how to use place value to multiply a number by a power of 10. How can you use this knowledge to determine the quotients of decimals?

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Getting Started

Take Note . . .
Quotients can be whole numbers, decimals, or fractions.

Just the Facts

You previously learned that division helps you determine how many times one number contains another number. In other words, division is determining the quotient given a dividend and a divisor.

$$\text{divisor})\overline{\text{dividend}}\quad \text{quotient}$$

or

$$(\text{quotient})(\text{divisor}) = (\text{dividend})$$

Remember . . .
Multiplication and division are inverse operations.

Consider the expression $48 \div 6$.

1. Diego evaluated the expression using the following strategy.

> **Diego**
>
> I know that $6 \cdot 8 = 48$, so $48 \div 6 = 8$.

   a. Describe the strategy John used to evaluate the expression.

   b. Identify the dividend, the divisor, and the quotient in the statement $48 \div 6 = 8$.

   c. Which values in the statement $48 \div 6 = 8$ can you switch and the statement is still true? Explain your reasoning.

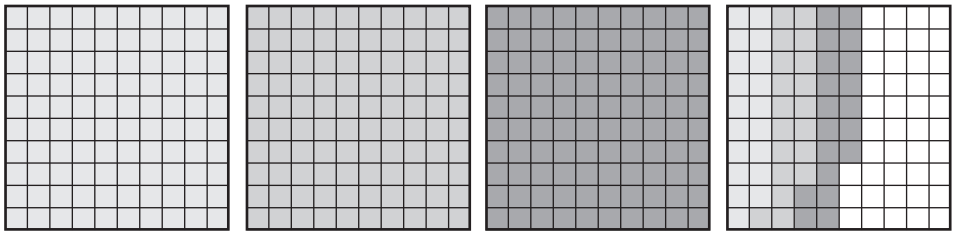
2. Is Diego's strategy reasonable for evaluating any division expression? Explain your reasoning.

ACTIVITY
3.1 Decimal Division

You can use hundredths grids to model dividing decimals.
Consider the quotient 3.57 ÷ 3.

WORKED EXAMPLE

Step 1 Represent 3.57 using hundredths grids.



Step 2 Divide the shaded model into three equal groups.

One whole grid and 19 small squares are in each group.
So, 3.57 ÷ 3 = 1.19.

You can also use a standard algorithm to divide 3.57 ÷ 3.

WORKED EXAMPLE

Step 2: 5 tenths divided into 3 equal groups is 1 tenth in each group with 2 tenths left over.

Step 3: 2 tenths and 7 hundredths is 27 hundredths. 27 hundredths divided into 3 equal groups is 9 hundredths in each group with 0 hundredths left over.

Step 1: 3 ones divided into 3 equal groups is 1 in each group with 0 ones left over.

$$\begin{array}{r} 1.19 \\ 3\overline{)3.57} \\ -\ 3\downarrow \\ \hline 0\ 5 \\ -\ 3\downarrow \\ \hline 27 \\ -\ 27 \\ \hline 0 \end{array}$$

Quotient
Dividend
Divisor

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1. Compare the two Worked Examples.

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

a. Describe how the hundredths grid model represents different parts of the standard algorithm.

b. Why does the standard algorithm show subtracting 3 from the 3 ones in the dividend?

c. What does the 05 represent in the standard algorithm?

d. What does 27 − 27 represent in the standard algorithm? Use the hundredths grid model to help you explain.

Take Note . . .
The quotient remains the same because you are multiplying by a form of 1.

If you multiply both the dividend and the divisor by the same number, the quotient remains unchanged.

$12 \div 3 = 4$

$(12 \cdot 10) \div (3 \cdot 10) = 4$

$(12 \cdot 100) \div (3 \cdot 100) = 4$

You can use this information to change any divisor into a whole number to make the division of a decimal easier to solve.

WORKED EXAMPLE

Consider $7.56 \div 3.6$

Multiply both numbers by the least power of 10 that makes the divisor into a whole number.

$(7.56 \cdot 10) \div (3.6 \cdot 10)$
$= 75.6 \div 36$

Then, divide with whole numbers.

$$\begin{array}{r} 2.1 \\ 36\overline{)75.6} \\ -\,72\downarrow \\ \hline 36 \\ -\,36 \\ \hline 0 \end{array}$$

2. Rewrite each division problem so the divisor is a whole number. Explain how you determined your answer.

   a. $48 \div 8.6$

   b. $59.5 \div 0.17$

   c. $6.2 \div 0.02$

You have seen how to divide decimals by whole numbers. Let's think about how to divide decimals by decimals.

3. Look at these division problems.

   $7\overline{)56}$ $70\overline{)560}$ $700\overline{)5600}$ $7000\overline{)56{,}000}$

   a. How are the divisors and dividends in the last three problems related to the first problem?

b. Calculate all four quotients. What do you notice about them?

c. What happens to the quotient when you multiply the dividend and divisor by the same number?

4. Which of the division expressions shown have the same quotient as 475 ÷ 25? How do you know?

Remember . . .

The definition of division is $a \div b = \frac{a}{b}$. Therefore, you can use what you already know about equivalent fractions to determine which expressions have the same quotient as 475 ÷ 25.

a. 4.75 ÷ 0.25

b. 47.5 ÷ 0.025

c. 0.475 ÷ 0.25

d. 0.0475 ÷ 0.0025

5. Calculate each quotient.

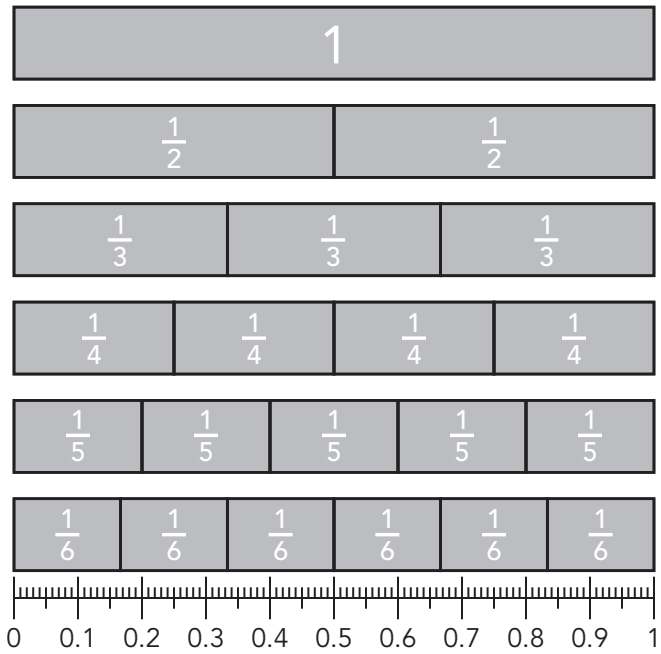
a. 74.4 ÷ 0.12

b. 66.22 ÷ 2.2

ACTIVITY
3.2 Fraction and Decimal Equivalents

You can use strip diagrams to determine equivalent decimals.

1. Use a straightedge and the chart to determine the decimal that is equal to or approximately equal to the fractions given. Round the decimal to the nearest hundredth.



a. $\frac{1}{3}$ = _____ b. $\frac{3}{5}$ = _____ c. $\frac{1}{4}$ = _____

d. $\frac{1}{5}$ = _____ e. $\frac{4}{6}$ = _____ f. $\frac{1}{6}$ = _____

You can also change a fraction to a decimal by using the meaning of the fraction bar.

WORKED EXAMPLE

$\frac{3}{8}$ means 3 divided by 8.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -\,24 \\ \hline 60 \\ -\,56 \\ \hline 40 \\ -\,40 \\ \hline 0 \end{array}$$

So $\frac{3}{8} = 0.375$.

WORKED EXAMPLE

$\frac{2}{3}$ means 2 divided by 3.

$$\begin{array}{r} 0.666 \\ 3\overline{)2.000} \\ -\,18 \\ \hline 20 \\ -\,18 \\ \hline 20 \\ -\,18 \\ \hline 2 \end{array}$$

So $\frac{2}{3} = 0.6\overline{6}$.

This decimal, 0.375, is called a **terminating decimal** because there is a remainder of 0. So, the denominator divides evenly into the numerator.

This decimal, $0.6\overline{6}$, is called a *repeating decimal*. A **repeating decimal** is a decimal in which a digit or a group of digits repeats without end. So, the denominator does not divide evenly into the numerator.

Take Note . . .

The bar over the 6 means that the 6 repeats without ending.

2. Convert each fraction to a decimal written to the nearest thousandths place. Identify whether the decimal is terminating or repeating.

a. $\frac{1}{8}$ b. $\frac{5}{8}$ c. $\frac{2}{3}$

d. $\frac{13}{25}$ e. $\frac{1}{6}$ f. $\frac{7}{20}$

3. Use a calculator to write the first 10 digits of the decimal for each fraction. Do not round the answers.

   a. $\frac{1}{7}$ b. $\frac{2}{7}$

   c. $\frac{3}{7}$ d. $\frac{4}{7}$

4. Consider the decimals you wrote in Question 3.

   a. What pattern do you notice in the decimals' values?

   b. Use the pattern to write the first ten digits of the decimal equivalent of $\frac{5}{7}$. Explain your reasoning.

5. Write the first 4 digits of the decimal for each fraction. Do not round the answers.

   a. $\frac{1}{9}$ b. $\frac{2}{9}$ c. $\frac{3}{9}$

   d. $\frac{4}{9}$ e. $\frac{5}{9}$ f. $\frac{6}{9}$

6. Consider the decimals you wrote in Question 5.

   a. What pattern do you notice in the decimal values?

   b. Use the pattern to write the first 4 digits of the decimal equivalents of $\frac{7}{9}$ and $\frac{8}{9}$.

PROBLEM SOLVING

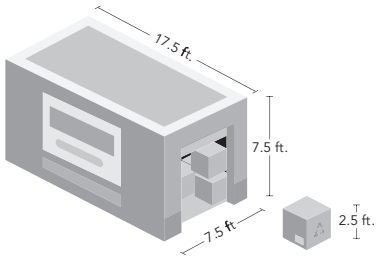


ACTIVITY 3.3

Solving Problems Using Decimal Division

Solve each problem without the use of a calculator.

1. A portable self-storage container is in the shape of a rectangular prism. Mia is packing it with cube-shaped boxes that each have a side length of 2.5 feet.



What is the greatest number of boxes that Mia can fit into the storage container?

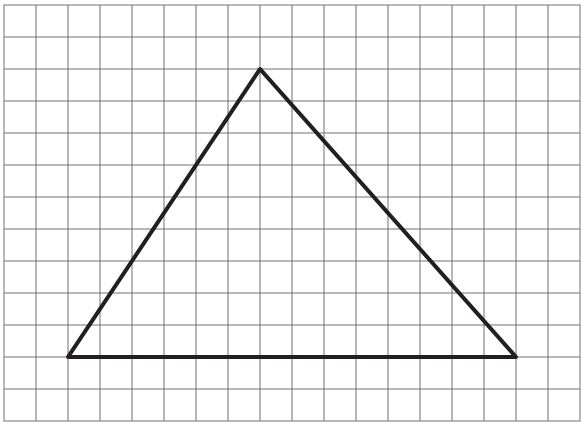
2. Calculate the side length of a square with the given perimeter.

a. 36.45 inches

b. 768 feet

3. The weight of one serving of uncooked rice is 1.6 ounces. How many servings are there in a 20-ounce package of uncooked rice?

4. The triangle shown on the grid represents a sailboat racecourse. Each square on the grid represents 0.1 mile by 0.1 mile.



The race organizers want any boats anchored within the perimeter of the course to have at least 0.002 square mile of space to prevent overcrowding. What is the maximum number of boats that can anchor within the perimeter of the racecourse?

5. Isabella uses a loaf pan to make cornbread. The pan is 8.5 inches long, 4.5 inches wide, and 2.5 inches deep.

   a. The pan has a volume of approximately 6.6 cups. What is the approximate volume of each cup in cubic inches? Estimate and then calculate your answer. Show your work.

   b. The cornbread Isabella makes fills only half the depth of the loaf pan. How much cornbread does Isabella make? Give your answer in cups and cubic inches.

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PROBLEM SOLVING



ACTIVITY 3.4

Parts of a Dollar

Ethan has just pulled his coat out of the closet for the first time since last winter. When he puts his hands in the pockets, he discovers some spare change.

1. In the right-hand coat pocket, Ethan finds 9 pennies, 8 nickels, and 2 dimes. Write the answer to each question in both fraction and decimal form. Show all your work.

   a. What fraction of a dollar does Ethan have in pennies?

   b. What fraction of a dollar does Ethan have in nickels?

   c. What fraction of a dollar does Ethan have in dimes?

   d. What is the total amount of money that Ethan found in his right-hand pocket?

2. In the left-hand pocket of his coat, Ethan finds nickels, dimes, and quarters. He calculates the fraction of a dollar he has in terms of each coin.

   Write the answer to each question in decimal form. Show all your work.

   a. Ethan has $\frac{3}{20}$ of a dollar in nickels. How much money does he have in nickels?

   b. Ethan has $\frac{2}{5}$ of a dollar in dimes. How much money does he have in dimes?

   c. Ethan has $\frac{1}{4}$ of a dollar in quarters. How much money does he have in quarters?

   d. What is the total amount of money Ethan found in his left-hand pocket?

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Talk the Talk

It's Great to Estimate!

Recall that estimation is a helpful strategy when operating with decimals to make sense of your solutions.

1. Estimate the quotients for each expression shown. Make sure to show your work.

   a. $7.5 \div 0.8$ b. $98.3 \div 23$ c. $99.2 \div 1.6$

2. Divide each problem in Question 1 using your calculator. Round your answers to the nearest thousandth. Compare your estimates to the actual calculations.

3. Daniel started his homework and did the first two problems.

   $1.3 \div 0.25 = 0.52$

   $39.6 \div 0.11 = 36$

   Juan said immediately that his answers were wrong. How did Juan know there was a mistake just by looking at the two problems and not doing any calculations?

4. The width of a rectangle is 8.3 inches, and the area is of the rectangle is 236.55 square inches. What is the length of the rectangle in inches?

Lesson 3 Assignment

Write

Describe what is meant by the operation of division using the terms *dividend*, *divisor*, and *quotient*.

Remember

In a division problem, if you multiply the dividend and the divisor by the same number, the quotient remains the same.

Practice

1. A local superstore had a surplus of 10,000 pencil packs and decided to donate them to area schools. If there are 27 schools and the store wants to donate the same number of packs to each school, how many pencil packs will each school receive?

2. Maria is baking cookies. She has 10 cups of flour to use. Maria will use $\frac{3}{4}$ of a cup of flour for each batch of cookies. How many batches of cookies can Maria make?

Lesson 3 Assignment

3. A group of 6 friends paid a total of $141.90 for tickets to a trampoline park. Each friend paid the same amount for a ticket. How much did each friend pay for a ticket to the trampoline park?

Lesson 3 Assignment

Prepare

Write a fraction to represent each situation.

1. The number of students wearing glasses in your math class compared to the number of students in the class

2. The number of students not wearing glasses in your math class compared to the number of students in the class

3. The number of students in your math class that are absent today compared to the total number of students in the class

4. The number of students in your math class that are in attendance today compared to the total number of students in your class

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TOPIC 3 SELF-REFLECTION

Name: _____

Decimals

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Decimals* topic by:

TOPIC 3: *Decimals*	Beginning of Topic	Middle of Topic	End of Topic
locating the value of a decimal on a number line	○	○	○
comparing decimals to the thousandths using $>$, $<$, and $=$	○	○	○
fluently adding, subtracting, multiplying, and dividing multi-digit decimals using the standard algorithm for each operation with accuracy	○	○	○
dividing multi-digit decimals using the standard algorithm with accuracy	○	○	○

continued on the next page

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TOPIC 3 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Decimals* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?



Decimals Summary

NEW KEY TERMS

- kite
- composite solid
- terminating decimal [decimal terminal]
- repeating decimal [decimal repetitivo]

LESSON 1 Plotting, Comparing, and Ordering Rational Numbers

A *decimal* is a number that is written in a system based on multiples of 10 and is another representation to show parts of a whole. Decimals are valuable when you need more precision for measurements between whole numbers. They can be represented on a number line. There are an infinite number of decimals between any two decimal values.

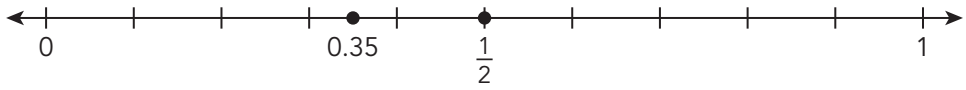
When comparing two decimals, each place value of a decimal needs to be considered. Decimals and fractions can be compared using a number line.

Compare $\frac{1}{2}$ and 0.35. Which value is greater?

First, convert $\frac{1}{2}$ to a decimal.

$\frac{1}{2}$ is equivalent to $\frac{5}{10}$, or 0.5.

Plot each value on a number line.



Because $\frac{1}{2}$ is to the right of 0.35 on the number line, $\frac{1}{2}$ is greater than 0.35, or $\frac{1}{2} > 0.35$.

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LESSON 2

Multiplying Decimals

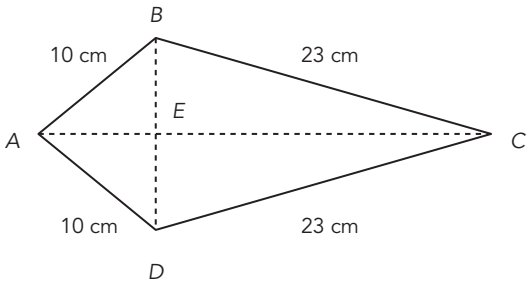
You multiply decimals as you would with whole numbers before placing the decimal point in the product. When multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in the factors.

Calculate the product of $32.64 \cdot 7.3$.

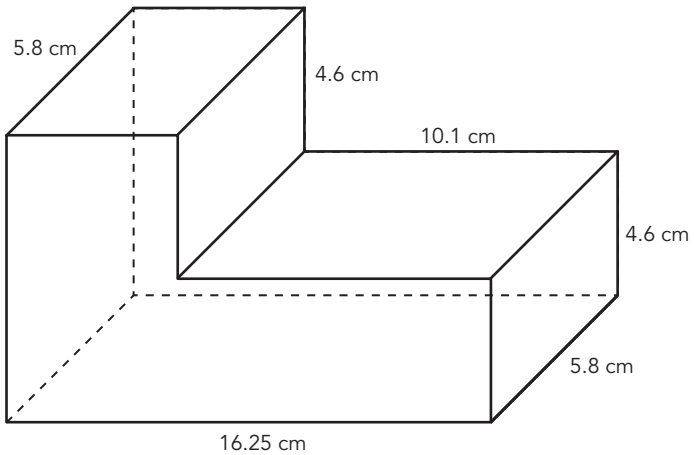
$$\begin{array}{r} 32.64 \\ \times \quad 7.3 \\ \hline 9792 \\ \underline{228480} \\ 238.272 \end{array}$$

The factor 32.64 has two decimal places, and the factor 7.3 has one decimal place. The sum of the decimal places in the factors is 3, so the product has three decimal places.

A **kite** is a quadrilateral with two pairs of consecutive congruent sides.



A **composite solid** is made up of more than one geometric solid.



LESSON 3

Dividing Decimals

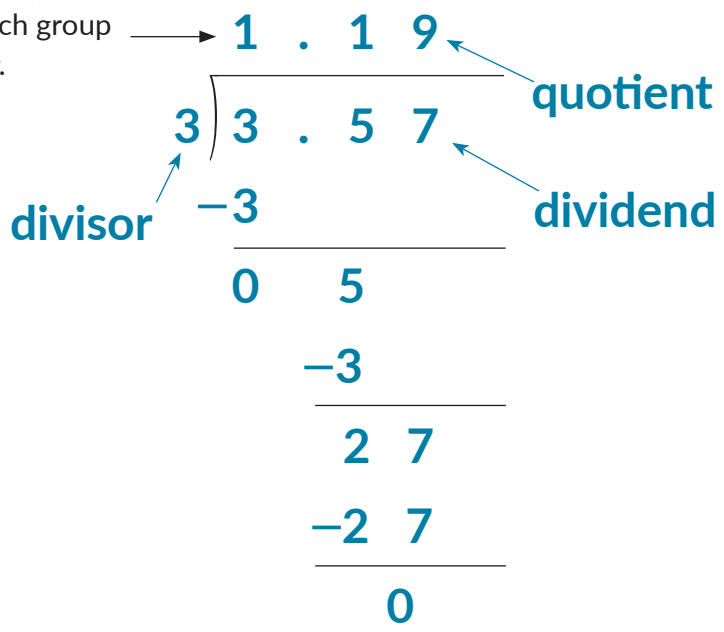
The long division algorithm uses organized estimation and place value to determine a *quotient*, or the number of times the divisor is contained in the dividend. This standard algorithm can be used to divide whole numbers or decimals.

Let's use the standard algorithm to divide 3.57 ÷ 3. The dividend is 3.57, and the divisor is 3.

5 tenths divided into 3 equal groups is 1 tenth in each group with 2 tenths left over.

2 tenths and 7 hundredths is 27 hundredths. 27 hundredths divided into 3 equal groups is 9 hundredths in each group with 0 hundredths left over.

3 ones divided into 3 equal groups is 1 one in each group with 0 ones left over.



The quotient is 1.19; therefore, 3.57 ÷ 3 = 1.19.

If you multiply both the dividend and the divisor by the same number, the quotient remains unchanged.

12 ÷ 3 = 4

(12 · 10) ÷ (3 · 10) = 4

(12 · 100) ÷ (3 · 100) = 4

You can change any divisor into a whole number to make the division of a decimal easier to solve by multiplying both the dividend and divisor by the same multiple of 10. Consider $7.56 \div 3.6$.

Multiply both numbers by the least power of 10 that makes the divisor into a whole number.

$$(7.56 \cdot 10) \div (3.6 \cdot 10)$$
$$= 75.6 \div 36$$

Then, divide with whole numbers.

$$\begin{array}{r} 2.1 \\ 36\overline{)75.6} \\ -72 \\ \hline 36 \\ -36 \\ \hline 0 \end{array}$$

You can also change a fraction to a decimal by using the meaning of the fraction bar.

$\frac{3}{8}$ means 3 divided by 8.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -24 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

so $\frac{3}{8} = 0.375$.

$\frac{2}{3}$ means 2 divided by 3.

$$\begin{array}{r} 0.666 \\ 3\overline{)2.000} \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

so $\frac{2}{3} = 0.\overline{6}$.

When the denominator of a fraction divides evenly into the numerator, the resulting decimal is a **terminating decimal** because it has digits that end. From the example above, 0.375 is a terminating decimal.

When the denominator of a fraction does not divide evenly into the numerator, the resulting decimal is a **repeating decimal** because a digit or group of digits repeats without end. From the example above, $0.\overline{6}$ is a repeating decimal.

GRADE 6

MODULE 2

Relating Quantities

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GRADE 6

TOPIC 1



Artists mix paints in specific ratios to produce different colors. Graphic designers and web developers use these ratios too. They can specify a color with an RGB value: a specific mix of red, green, and blue.

Ratios and Rates

LESSON 1 Introduction to Ratio and Ratio Reasoning 211

LESSON 2 Comparing Ratios and Rates to Solve Problems 229

LESSON 3 Determining Equivalent Ratios and Rates 241

LESSON 4 Using Tables to Represent Equivalent Ratios and Rates 263

LESSON 5 Graphs of Ratios and Rates 279

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1 Introduction to Ratio and Ratio Reasoning

OBJECTIVES

- Distinguish between additive and multiplicative relationships between two quantities.
- Understand the concept of a ratio: a ratio represents a multiplicative comparison between two quantities that may contain the same or different units.
- Write ratios in different forms and use ratio language to represent relationships between two quantities.
- Distinguish between part-to-part and part-to-whole ratios.
- Understand that fractions are part-to-whole ratios between two quantities.
- Understand that percents are part-to-whole ratios between a quantity and 100.

NEW KEY TERMS

- additive reasoning
- multiplicative reasoning
- ratio
- percent

In previous courses, you made many comparisons using addition and subtraction. You answered questions like, "When Lucas has 9 apples and Mason has 12 apples, who has more apples?"

Is there another way to compare values?

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Getting Started

Predict the Score

The Crusaders and the Blue Jays just finished the first half of their basketball game.

	Halftime Score	Final Score
Crusaders	30	?
Blue Jays	20	?

1. Predict the final score. Explain your reasoning.

ACTIVITY 1.1

Additive and Multiplicative Reasoning

Adriana and Nahimana each predicted the final score of a basketball game between the Crusaders and the Blue Jays.

1. Analyze each prediction.

   a. Describe the reasoning that Adriana and Nahimana used to make each statement.

Adriana



	Halftime Score	Final Score
Crusaders	30	60
Blue Jays	20	40

I think the final score will be double the score at halftime.

Nahimana

	Halftime Score	Final Score
Crusaders	30	50
Blue Jays	20	40

I think the Crusaders will play hard enough to stay 10 points ahead of the Blue Jays.

   b. Which team had a better second half in each prediction?

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One of the students used *additive reasoning* to make her comparison and the other used *multiplicative reasoning*. **Additive reasoning** focuses on the use of addition and subtraction for comparisons. **Multiplicative reasoning** focuses on the use of multiplication and division.

2. Which student used additive reasoning and which used multiplicative reasoning?

Valentina and her nephew Sebastian share the same birthday. They were both born on March 4.

Valentina: "Today I'm 40 years old, and you're 10. I'm 4 times as old as you are!"

Sebastian: "Wow, you're old!"

Valentina: "Yeah, but in 5 years, I'll be 45, and you'll be 15. Then I will only be three times as old as you."

Sebastian: "I'm catching up to you!"

Valentina: "And 15 years after that, I'll be 60 and you'll be 30. Then I'll only be twice as old as you!"

Sebastian: "In enough time, I'll be older than you, Aunt Valentina!"



3. Is Valentina correct about how their ages change? Is Sebastian correct in thinking that he will eventually be older than his aunt?

4. The table represents the different statements from this problem situation. Let V represent Valentina's age and S represent Sebastian's age.

   a. Complete the last column by identifying each relationship as either additive or multiplicative.

Verbal	Numeric	Relationship	
Today I'm 40 years old, and you're 10.	$V = 40, S = 10$	$V = S + 30$	
I'm 4 times as old as you are!	$V = 40, S = 10$	$V = 4S$	
Yeah, but in 5 years, I'll be 45, and you'll be 15.	$V = 45, S = 15$	$V = S + 30$	
Then I will only be three times as old as you.	$V = 45, S = 15$	$V = 3S$	
And 15 years after that, I'll be 60 and you'll be 30.	$V = 60, S = 30$	$V = S + 30$	
Then I'll only be twice as old as you!	$V = 60, S = 30$	$V = 2S$	

   b. At any point in this age scenario, which relationship does not change?

5. Write an equation to represent the relationship for each table. Then, determine whether each table represents an additive or multiplicative relationship.

   a.

Number of Roses, *r*	1	2	3	12
Number of Carnations, *c*	2	4	6	24

   b.

Number of Daffodils, *f*	1	2	3	12
Number of Daisies, *s*	3	4	5	14

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PROBLEM SOLVING

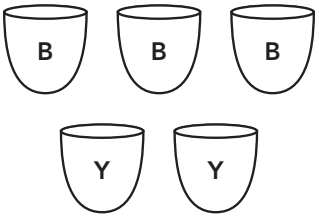


ACTIVITY 1.2

Comparing Quantities

The school colors at Riverview Middle School are a shade of bluish-green and white. The art teacher, Mr. Park, knows that to get the correct color of bluish-green it takes 3 parts blue paint to every 2 parts yellow paint.

There are different ways to think about this relationship and make comparisons. One way is to draw a picture or model.



From the model, you can make comparisons of the different quantities.

- blue parts to yellow parts
- yellow parts to blue parts
- blue parts to total parts
- yellow parts to total parts

Each comparison is called a *ratio*. A **ratio** is a multiplicative comparison between two quantities that may contain the same or different units. The first two comparisons are part-to-part ratios because you are comparing the individual quantities. The last two comparisons are part-to-whole ratios because you are comparing one of the parts (either blue or yellow) to the total number of parts.

So you are never in doubt what a ratio represents . . . label all quantities with the units of measure!



Suppose Mr. Park needs 2 parts blue paint and 5 parts yellow paint to make green paint.

1. Compare the quantities of blue and yellow paint in Mr. Park's mixture by writing all possible ratios for each type.

   a. part-to-part ratios

   b. part-to-whole ratios

   What is the difference between the part-to-part ratios that you wrote?

   What is the difference between the part-to-whole ratios that you wrote?

ACTIVITY
1.3 Ratio Hunt

Ratios can be found all around you, even in your classroom! Just consider two different quantities.

For example, how many students in your class are wearing sneakers? How many students in your class are wearing another type of shoe?

1. Use a ratio to describe the relationship given.

   a. Write a part-to-part ratio comparing the number of students wearing sneakers to the number of students wearing a different type of shoe.

   b. Write a part-to-part ratio comparing the number of students wearing a shoe other than sneakers to the number of students wearing sneakers.

   c. Write a part-to-whole ratio comparing the number of students wearing sneakers to the total number of students in the class.

   d. Write a part-to-whole ratio comparing the number of students wearing a type of shoe other than sneakers to the total number of students in the class.

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Let's go on a Ratio Hunt!

2. Search around your classroom for at least 3 pairs of quantities to compare. For each pair:
   - Identify the two quantities that are being compared using ratios.
   - Write all possible part-to-part and/or part-to-whole comparisons of the quantities.
   - Identify each ratio as part-to-part or as part-to-whole.
   - Be prepared to share your treasures from the Ratio Hunt with the class.

   a. Quantities being compared:
   Ratio(s):

   b. Quantities being compared:
   Ratio(s):

   c. Quantities being compared:
   Ratio(s):

ACTIVITY
1.4 The Meanings of Ratios

A local middle school is adopting a new nickname. They have narrowed their search to the following two names: Tigers or Lions. To choose a nickname, they conducted a school-wide survey and tallied all the votes.

Each homeroom analyzed the results of the school-wide survey and reported the results in a different way.

Homeroom 6A	Homeroom 6B
The votes for Tigers outnumbered the votes for Lions by a ratio of 240 to 160.	There were 80 more votes for Tigers than Lions.
Homeroom 7A	**Homeroom 7B**
The votes for Tigers outnumbered votes for Lions by a ratio of 3 to 2.	3 out of 5 votes were for Tigers.

1. Describe the meaning of each statement. Then, identify which describe ratios, and, if so, whether the ratios are part-to-part or part-to-whole ratios.

Fractional form simply means writing the relationship in the form $\frac{a}{b}$. Just because a ratio looks like a fraction does not mean it is representing a part-to-whole comparison.

WORKED EXAMPLE

Let's consider the results reported by Homeroom 7A:
"The votes for Tigers outnumbered votes for Lions by a ratio of 3 to 2."

This comparison is an example of a part-to-part ratio expressed in words. There are two other ways you can express this part-to-part ratio.

With a Colon

3 votes for Tigers : 2 votes for Lions

In Fractional Form

$$\frac{\text{3 votes for Tigers}}{\text{2 votes for Lions}}$$

Next, let's consider the results of the student vote as reported by Homeroom 7B: "3 out of 5 votes were for Tigers."

2. Complete the part-to-whole and part-to-part ratios written in words. Then, write each ratio with a colon and in fractional form. Label all quantities.

Part-to-Whole Ratio

In Words	With a Colon	In Fractional Form
3 out of 5 votes were for Tigers.		
____ out of 5 votes were for Lions.		

Part-to-Part Ratio

In Words	With a Colon	In Fractional Form
____ votes for Tigers for every 2 votes for Lions.		
2 votes for Lions for every ____ votes for Tigers.		

Finally, let's consider the results of the survey as reported by Homeroom 6A: "The votes for Tigers outnumbered the votes for Lions by a ratio of 240 to 160."

3. Complete the part-to-whole and part-to-part ratios written in words. Then write each ratio with a colon and in fractional form. Label all quantities.

Part-to-Whole Ratio

In Words	With a Colon	In Fractional Form
____ votes out of ____ votes were for Tigers.		
____ votes out of ____ votes were for Lions.		

Part-to-Part Ratio

In Words	With a Colon	In Fractional Form
____ votes for Tigers for every 160 votes for Lions.		
____ votes for Lions for every 240 votes for Tigers.		

4. Based on the survey, which mascot name was preferred?

ACTIVITY
1.5 Special Types of Ratios

Representing a ratio in fractional form, $\frac{a}{b}$, does not have the same meaning as saying a ratio is also a fraction. This is one reason units are critical when writing ratios.

Consider each statement.

- There is an 80 percent chance of rain tomorrow.
- He ate $\frac{2}{5}$ of the cake.
- Sales tax in Greenmont is 7 percent.
- Three-fourths of the class is absent.

The situations described are examples of special types of ratios: fractions and percents.

Notice that when you write a ratio using the total number of parts, you are also writing a fraction. A fraction can be used as a ratio that shows a part-to-whole relationship.

Ratios

part : part **part : whole**

$\frac{\text{part}}{\text{part}}$ $\frac{\text{part}}{\text{whole}}$

Fraction

A **percent** is a part-to-whole ratio where the whole is equal to 100. *Percent* is another name for hundredths. The percent symbol "%" means "per 100," or "out of 100." Therefore:

WORKED EXAMPLE

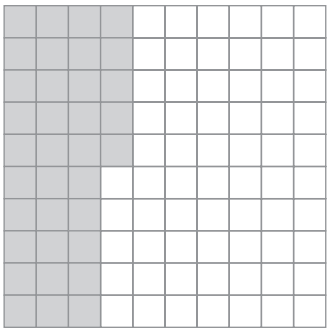
35% means 35 out of 100.

35% as a fraction is $\frac{35}{100}$.

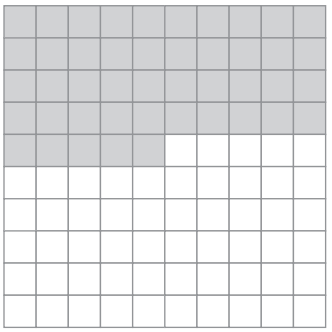
35% as a decimal is 0.35.

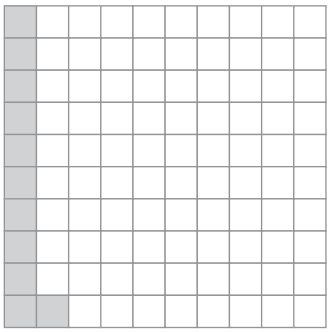
35% as a ratio is 35 to 100, or 35 : 100.

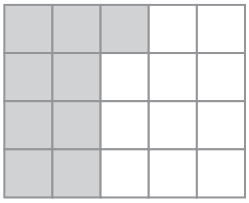
You can shade 35 of the 100 squares on the hundredths grid to represent 35%.

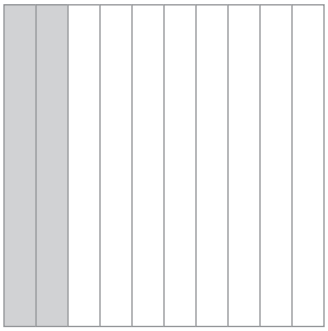


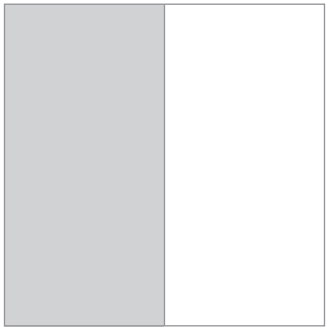
1. Each grid represents a whole. Write a fraction and a percent to represent the shaded part of each grid.

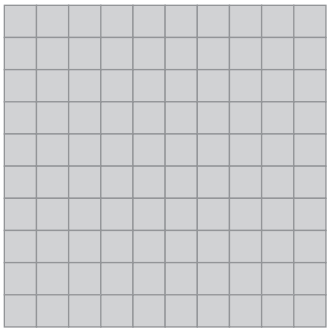
a. 

b. 

c. 

d. 

e. 

f. 

Talk the Talk

Writing and Classifying Ratios

There are several ways to compare two quantities and write ratios.

	Ratios	
With a Colon	part : part	part : whole
In Fractional Form	$\frac{\text{part}}{\text{part}}$	$\frac{\text{part}}{\text{whole}}$
		Fraction

1. Consider the statement: There are s sixth-grade band members and t total number of sixth-graders.

   a. Write a part-to-whole ratio using colon notation.

   b. Write a part-to-part ratio using colon notation.

2. A survey of sixth-graders with pets revealed that c students prefer cats and d students prefer dogs.

   a. How would you compare these two statements using part-to-part ratios?

   b. How would you compare these two statements using part-to-whole ratios?

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Lesson 1 Assignment

Write

Describe two ratios in the real world. Write about at least one part-to-whole ratio and one part-to-part ratio.

Remember

A ratio is a multiplicative comparison between two quantities that may contain the same or different units.

A part-to-whole ratio compares a part of a whole to the total number of parts.

A part-to-part ratio compares parts.

A part-to-whole ratio is a fraction.

A percent is a fraction in which the denominator is 100.

Practice

The Martinez brothers just joined a movie club that provides them with free movies based on a list that they pre-select. The boys work together to pick the first 10 movies for their list, each brother adding to the list based on their favorite type of movie. Gabriel puts 5 sports movies on the list, Diego chooses 3 war movies, and Logan adds 2 comedies.

Write the ratio in colon and in fractional form to express each relationship.

1. sports movies to war movies

2. comedies to total movies

3. war movies to comedies

4. sports movies to total movies

5. comedies to sports movies

6. war movies to total movies

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Lesson 1 Assignment

Prepare

Use reasoning to compare each pair of fractions.

1. $\frac{6}{7}$ and $\frac{8}{9}$

2. $\frac{7}{13}$ and $\frac{5}{11}$

3. $\frac{4}{5}$ and $\frac{4}{3}$

2 Comparing Ratios and Rates to Solve Problems

OBJECTIVES

- Apply qualitative ratio and rate reasoning to compare quantities in real-world and mathematical problems.
- Apply quantitative ratio and rate reasoning to compare quantities in real-world and mathematical problems.
- Define and use rates and rate reasoning to solve ratio and rate problems.
- Compare and order part-to-part and part-to-whole ratios and rates represented verbally, pictorially, and numerically.

NEW KEY TERM

- rate

You know how to write a ratio as a comparison of two quantities.

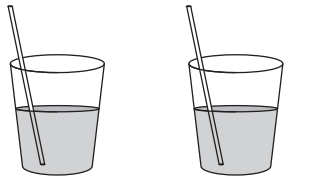
How can you compare two ratios to make decisions in real-world situations?

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Getting Started

Lemony-er Lemonade

Emily's glass of lemonade has a weaker tasting lemon flavor than Sarah's glass of lemonade. The shaded portion in each glass represents an amount of lemonade.



Emily's Glass Sarah's Glass

1. When Sarah and Emily add one teaspoon of lemon mix to each of their glasses, which glass will contain the lemonade with the stronger lemon flavor? Explain your reasoning.

ACTIVITY
2.1 Qualitative Comparisons

In this activity you will compare two quantities without measuring or counting quantities. When you reason like this, it is called *qualitative reasoning*.

1. The shaded portion in each glass represents an amount of lemonade. Answer each question and explain your reasoning.

   a. Angelina's glass of lemonade has a weaker tasting lemon flavor than Avery's glass of lemonade. When Angelina adds two fluid ounces of water to her glass and Avery adds one teaspoon of lemon mix to his glass, which glass will contain the lemonade with the stronger lemon flavor?


Angelina's Glass


Avery's Glass

   b. Parker and Carlos have glasses of lemonade that taste the same. When Parker and Carlos add one teaspoon of lemon mix to each of their glasses, which glass will contain the lemonade with the stronger lemon flavor?


Parker's Glass


Carlos's Glass

   c. Isaiah's glass of lemonade has a stronger tasting lemon flavor than Mei's glass of lemonade. When Mei adds one teaspoon of lemon mix to her glass and Isaiah adds one fluid ounce of water to his glass, which glass will contain the lemonade with the stronger lemon flavor?


Isaiah's Glass


Mei's Glass

2. Choose the correct statement to complete each sentence and explain your reasoning. When the answer cannot be determined, explain why not.

   a. When Lucas plans to use four more tablespoons of orange mix today than he used yesterday to make the same amount of orange sports beverage, his orange sports beverage today would have:

      - a stronger tasting orange flavor.
      - a weaker tasting orange flavor.
      - a mix that has the same strength of orange taste as yesterday.

   b. Mason and Camilla each made a pitcher of orange sports beverage. Camilla's pitcher is larger than Mason's pitcher. Camilla used more orange mix than Mason. Mason's orange sports beverage has:

      - a stronger tasting orange flavor.
      - a weaker tasting orange flavor.
      - a mix that has the same strength of orange taste as Camilla's sports beverage.

   c. When a race car travels more laps in less time than it did yesterday, its speed would be:

      - slower.
      - exactly the same.
      - faster.

ACTIVITY
2.2 Comparing Comparisons

A **rate** is a comparison by division between two quantities that have different units. The 6th-grade students are making hot chocolate to sell at the Winter Carnival. Each homeroom (HR) suggested a different recipe.

HR 6A

2 cups milk
3 T cocoa powder

HR 6B

5 cups milk
8 T cocoa powder

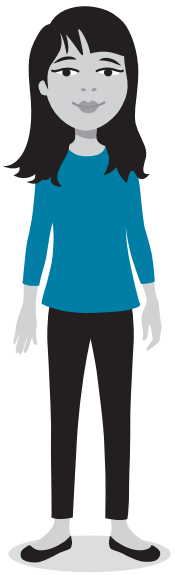
HR 6C

3 cups milk
4 T cocoa powder

HR 6D

4 cups milk
7 T cocoa powder

The "T" in each recipe stands for Tablespoon!



1. Consider the given recipes to answer each question.

   a. Use reasoning to determine which recipe has the strongest chocolate taste and which recipe has the weakest chocolate taste.

   b. Show how you reasoned about rates to order the recipes. Identify the rates that you used as part-to-part or part-to-whole.

   c. Create a poster to explain your answer and strategy to the class. Prepare to share!

Ask Yourself . . .
What units can you use to compare the amount of milk to the amount of cocoa powder?

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ACTIVITY
2.3 Ordering Part-to-Part and Part-to-Whole Rates

Suppose your class is in charge of providing punch at the upcoming open house. The Parent-Teacher Association bought lemon-lime soda and pineapple juice to combine for the punch, but they did not tell your class how much of each to use. Your classmates submitted suggestions for how to make the tastiest punch.

The shading, or lack of shading, of each cup represents the difference in the type of concentration.

Cut out the Punch Rate Cards at the end of the lesson. Order the cards from the least lemon-lime concentration to the most lemon-lime concentration. When you think more than one card describes the same ratio of lemon-lime soda and pineapple juice, group those cards together.



lemon-lime soda



pineapple juice

1. Describe the strategies you used to sort and order the cards.

Order from Least to Greatest	Part-to-Part	Part-to-Whole	Pictorial Representation

Talk the Talk

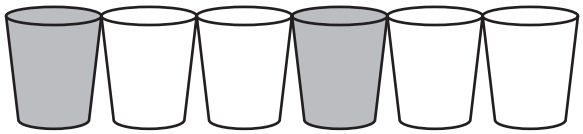
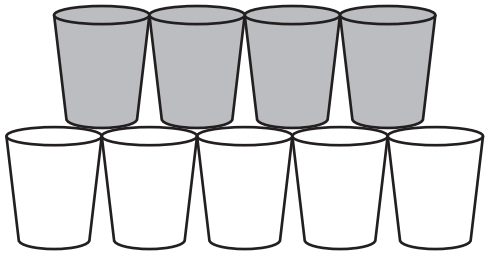
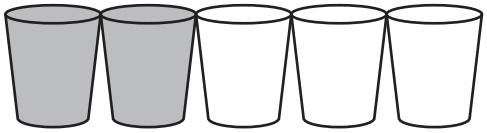
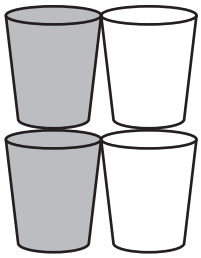
Put Me In, Coach

A soccer team has been awarded a penalty shot at the end of a tie game. If they make the penalty shot, they will win the league championship. The coach is considering three players to take the penalty. Jasmine has taken 4 penalty shots this season and has made 3 of them. Luna has taken 6 penalty shots and made 4. Kayla has taken 3 penalty shots and made 2.

1. Which player would you recommend take the penalty shot? Why?

Ask Yourself . . .
How can you organize and record your mathematical ideas?

Punch Rate Cards

A For every lemon-lime soda, there is a pineapple juice.	**B** 
C One-fourth of the punch is lemon-lime soda.	**D** 
E Half of the mixture is pineapple juice.	**F** 
G Lemon-lime soda : Pineapple juice = 4 : 5	**H** 
I For every lemon-lime soda, there are two pineapple juices.	**J** For every lemon-lime soda, there are $1\frac{1}{2}$ pineapple juices.
K Pineapple juice : lemon-lime soda = 3 : 1	**L** Three-fifths of the punch is pineapple juice.

Why is this page blank?

So you can cut out the Punch Rate Cards on the other side.

Lesson 2 Assignment

Write

Write two recipes for hot chocolate, each with a different rate of chocolate mix to water or milk. Describe how the two recipes are similar and different.

Remember

A ratio is a multiplicative comparison between two quantities that may contain the same or different units.
A rate is a comparison of two quantities with different units.

Practice

Nahimana is making fruit punch using fruit juice and ginger ale. She tries different combinations to get the mixture just right. When the ratio of fruit juice to ginger ale is too high, the punch is too fruity; When the ratio is too low, the punch is too gingery.

For each attempt, write a ratio Nahimana can try next time.

1. She tried 16 cups of fruit juice and 4 cups of ginger ale. That was too fruity.
2. She tried 10 cups of fruit juice and 8 cups of ginger ale. That was too gingery.
3. She tried 10 cups of fruit juice and 1 cup of ginger ale. That was too fruity.
4. She tried 8 cups of fruit juice and 4 cups of ginger ale. That was a little too gingery.

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Lesson 2 Assignment

5. Based on Nahimana's attempts in Questions 1–4, what might be a good ratio of fruit punch to ginger ale? Explain your thinking.

Prepare

Choose the correct statement to complete each sentence and explain your reasoning.

1. When the manager at Sweets-a-Plenty Bakery decides how many bakers are needed to bake muffins for a given day, she needs to consider the total number of muffins needed for the day.

   a. Making fewer muffins with more bakers will take:

      - less time.
      - an equal amount of time.
      - more time.

   b. Making more muffins in a shorter amount of time requires:

      - fewer workers.
      - an equal amount of workers.
      - more workers.

3 Determining Equivalent Ratios and Rates

OBJECTIVES

- Use drawings to model and determine equivalent ratios and rates.
- Reason about strip diagrams to model and determine equivalent ratios.
- Use scaling up and scaling down to determine equivalent ratios/rates.
- Use double number lines to solve real-world problems involving ratios and rates.

NEW KEY TERMS

- equivalent ratios
- strip diagram
- proportion
- scaling up
- scale factor
- scaling down
- double number line

Informally comparing ratios/rates, or qualitatively comparing ratios/rates, is important. However, there are many instances when you need to make more specific comparisons.

How can you use equivalent ratios/rates to compare relationships more precisely?

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Which Has More?

Consider the given representations to answer each question. Explain your reasoning.

1. Which dinner order has more pizza?

Order 1

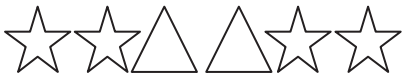


Order 2

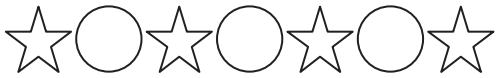


2. Which pattern has more stars?

Pattern 1



Pattern 2



3. Which pile of laundry has more shirts?

Pile 1



Pile 2



4. Which type of reasoning did you use for each question—additive or multiplicative? Explain why.

ACTIVITY
3.1 Using Drawings to Model Equivalent Ratios and Rates

Adriana and her friends are going hiking. Adriana invites her friends to meet at her house for a quick breakfast before heading out on their hike. Adriana wants to offer muffins to her friends.

1. She knows that one muffin combo has four muffins that can feed four people.

   a. Draw a model showing the relationship between the muffin combo and the number of people it will feed.

   b. When Adriana invites 6 friends, how many muffin combos will she need? Draw a model to show how many muffin combos she will need and explain your answer.

   c. When Adriana has $2\frac{3}{4}$ muffin combos, how many friends can she feed? Draw a model to show how many friends she can feed and explain your answer.

How do your models show a relationship between two quantities?

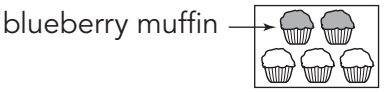


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Ask Yourself . . .
Is comparing 2 out of every 5 muffins are blueberry a ratio or rate?

Let's consider a different variety pack.

In one muffin variety pack, two out of every five muffins are blueberry.



2. Draw a model to answer each question. Explain your reasoning.

   a. How many muffins are blueberry muffins when there are a total of 25 muffins?

   b. How many muffins are blueberry muffins when there are a total of 35 muffins?

I think I see a pattern. Do you see it?



   c. How many total muffins are there when 8 muffins are blueberry?

As you solved these problems, you determined *equivalent ratios*. **Equivalent ratios** are ratios that represent the same part-to-part or part-to-whole relationship.

ACTIVITY
3.2 Strip Diagrams

The local bakery sells muffins in variety packs of blueberry, pumpkin, and bran muffins. They always sell the muffins in the ratio of 3 blueberry muffins : 2 pumpkin muffins : 1 bran muffin.

1. Write the ratio that expresses each relationship. Identify each as a part-to-part or a part-to-whole ratio.

   a. number of blueberry muffins to total number of muffins

   b. number of pumpkin muffins to total number of muffins

   c. number of bran muffins to total number of muffins

   d. number of blueberry muffins to number of pumpkin muffins

   e. number of bran muffins to number of pumpkin muffins

   f. number of blueberry muffins to number of bran muffins

Don't forget to label each quantity with the unit of measure!



You can represent a ratio by drawing the objects themselves, or you can use a *strip diagram*. A **strip diagram** illustrates number relationships by using rectangles to represent ratio parts. This strip diagram represents the ratio of the number of each type of muffin.



2. What does each small rectangle represent in the given strip diagram?

Strip diagrams provide a visual representation of ratios, but you can also use them to solve problems.

Remember. . .
In this scenario the ratio of muffins in each variety pack is always 3 blueberry muffins : 2 pumpkin muffins : 1 bran muffin.

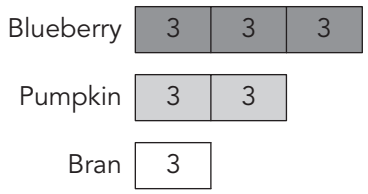
WORKED EXAMPLE

Suppose you purchase an 18-pack of muffins. How many blueberry, pumpkin, and bran muffins will you purchase?

There are 6 muffins represented in the strip diagram, and you want 18 total muffins that are in the same ratio.

Therefore, to determine how many muffins you need to maintain the same ratio, you can divide 18 by 6.
$18 \div 6 = 3$

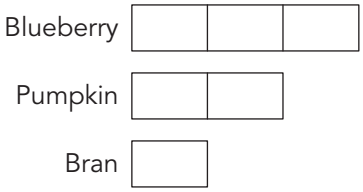
Therefore, each rectangle will represent 3 muffins.



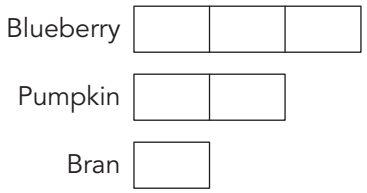
From the strip diagram, you can see that there are 9 blueberry muffins, 6 pumpkin muffins, and 3 bran muffins.

3. Is the ratio 9 : 6 : 3 equivalent to 3 : 2 : 1? Explain how you know.

4. Suppose you purchase a 36-pack of muffins. Use the strip diagram to illustrate how many blueberry, pumpkin, and bran muffins you will receive.



5. Suppose you wanted 20 pumpkin muffins in your variety pack. How many total muffins will be in your variety pack? Complete the strip diagram to determine the answer.



6. The table shows the number of muffins in specific sized variety packs. Complete just the unknown cells in the columns for the 6-pack and 36-pack of muffins.

Total Number of Muffins	6	12	18	24	36
Number of Blueberry Muffins			9		
Number of Pumpkin Muffins			6		
Number of Bran Muffins			3		

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7. Analyze the completed columns in the table.

   a. What do you notice about the numbers?

   b. How could you have determined the number of each type of muffin in the 18-pack without using the strip diagram?

   c. How could you have determined the number of each type of muffin in the 36-pack without using the strip diagram?

   d. Use what you noticed about the numbers in the table to complete the remaining columns for the number of each type of muffin in a 12-pack and in a 24-pack of muffins. Explain your strategy.

ACTIVITY
3.3 Rates and Proportions

One of the rounds at the Math Quiz Bowl tournament is a speed round. A team of four students will represent their middle school in the speed round of the Math Quiz Bowl. One student of the team will compete to solve as many problems as possible in 20 minutes. The results from this week's practice are recorded in the table.

Student	Number of Correctly Solved Problems in a Specified Time
Mei	4 problems correct in 5 minutes
Angelina	7 problems correct in 10 minutes
Sebastian	1 problem correct in 2 minutes
Gabriel	3 problems correct in 4 minutes

1. Explain how Valentina's reasoning and Emily's reasoning about who should compete in the speed round are incorrect.

Valentina



Angelina should definitely compete in the speed round because she correctly solved the most problems.

Emily



It took Angelina the longest time to complete her problems. She should not compete in the speed round.

Remember. . .

A rate is a comparison by division between two quantities that have different units. A ratio is a multiplicative comparison between two quantities that may contain the same or different units.

Each quantity in the table is a *rate*. The rate for each student in this situation is the number of problems solved per amount of time.

WORKED EXAMPLE

Mei's rate is 4 problems correct per 5 minutes. You can write this rate as:

$$\frac{4 \text{ problems correct}}{5 \text{ minutes}}$$

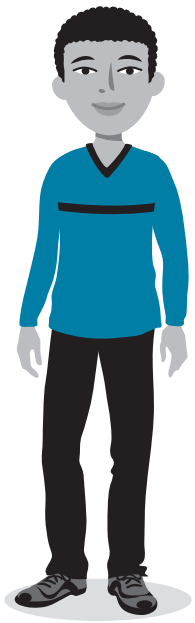
2. Write the rates for the other three team members.

a. Angelina

b. Sebastian

c. Gabriel

It's important to line up the units when writing equal ratios.



When two ratios or rates are equivalent to each other, you can write them as a *proportion*. A **proportion** is an equation that states that two ratios are equal. In a proportion, the quantities composing each part of the ratio/rate have the same multiplicative relationship between them.

WORKED EXAMPLE

For example, you know that Mei got four problems correct per 5 minutes. So, you can predict how many problems she could answer correctly in 20 minutes.

$$\frac{\text{problems correct}}{\text{minutes}} \rightarrow \frac{4}{5} \overset{\cdot 4}{=} \frac{16}{20}$$

Mei can probably answer 16 problems correctly in 20 minutes.

When you change one ratio/rate to an equivalent ratio/rate with larger numbers, you are *scaling up* the ratio. **Scaling up** means you multiply both parts of the ratio/rate by the same factor greater than 1. The factor that you multiply both parts of the ratio by is known as the **scale factor**.

3. Use the definition of a ratio to verify that $\frac{4}{5}$ is equivalent to $\frac{16}{20}$.

Remember, one way to represent a ratio/rate is in fractional form. It doesn't matter which quantity is in the numerator or denominator; it matters that the unit of measure is consistent among the ratios/rates.

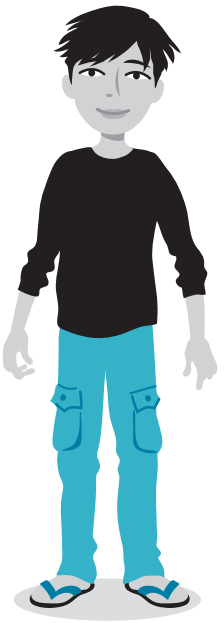
WORKED EXAMPLE

You can write the proportion in a different way.

$$\frac{\text{minutes}}{\text{problems correct}} \rightarrow \frac{5}{4} = \frac{20}{16} \quad (\cdot 4 \text{ on top}, \cdot 4 \text{ on bottom})$$

In this example, the scale factor is 4.

This is the same strategy you used in Grades 4 and 5 to write equivalent fractions.



4. Determine the number of problems each student can probably solve in 20 minutes. Explain the scaling up you used to determine the equivalent ratio.

   Angelina Sebastian Gabriel

5. Which team member is the fastest? Who would you pick to compete? Explain your reasoning.

ACTIVITY
3.4 Scaling Up and Scaling Down

Remember. . .
The definition of division, $a \div b = a \cdot \frac{1}{b}$.

When you change a ratio/rate to an equivalent ratio/rate with smaller numbers, you are *scaling down* the ratio/rate. **Scaling down** means you divide both parts of the ratio/rate by the same scale factor greater than 1, or multiply both parts of the ratio/rate by same scale factor less than 1. Scaling down a ratio/rate often makes it easier to understand.

1. Scale each ratio up or down to determine the unknown quantity. Identify the operation and scale factor that you used to scale up or down each ratio.

Ask Yourself . . .
How do you know whether these relationships are ratios?

a. $\frac{2 \text{ blueberry muffins}}{5 \text{ total muffins}} = \frac{20 \text{ blueberry muffins}}{? \text{ total muffins}}$

b. $\frac{12 \text{ red marbles}}{36 \text{ blue marbles}} = \frac{? \text{ red marbles}}{9 \text{ blue marbles}}$

c. $\frac{? \text{ goldfish}}{100 \text{ total fish}} = \frac{8 \text{ goldfish}}{20 \text{ total fish}}$

d. $\frac{6 \text{ parts blue paint}}{5 \text{ parts yellow paint}} = \frac{90 \text{ parts blue paint}}{? \text{ parts yellow paint}}$

e. $\frac{24 \text{ pumpkin muffins}}{144 \text{ total muffins}} = \frac{? \text{ pumpkin muffins}}{18 \text{ total muffins}}$

2. Scale up or down each rate to determine the unknown quantity. Identify the operation and scale factor that you used to scale down each rate.

a. $\frac{3 \text{ people}}{9 \text{ pizzas}} = \frac{?}{3 \text{ pizzas}}$

Ask Yourself . . .
How do you know whether these relationships are rates?

b. $\frac{2 \text{ hoagies}}{6 \text{ people}} = \frac{50 \text{ hoagies}}{?}$

c. $\frac{100 \text{ track shirts}}{25 \text{ people}} = \frac{?}{5 \text{ people}}$

d. $\frac{3 \text{ tickets}}{\$26.25} = \frac{1 \text{ ticket}}{?}$

e. $\frac{4 \text{ hours}}{240 \text{ miles}} = \frac{12 \text{ hours}}{?}$

f. $\frac{20 \text{ hours of work}}{\$240} = \frac{5 \text{ hours of work}}{?}$

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ACTIVITY
3.5 Double Number Lines

An interval is the amount of space between two tick marks on a number line.

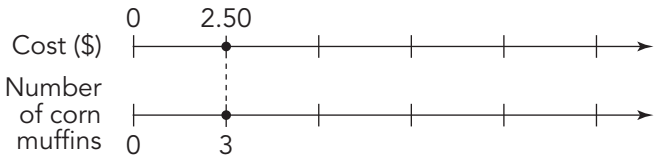
You know several strategies to determine the relationship between two quantities: drawing models, building strip diagrams, and scaling up or down. You can also use a *double number line* to visualize these relationships. A **double number line** is a model that is made up of two number lines used together to represent the ratio or rate between two quantities. The intervals on each number line maintain the same ratio or rate.

A local bakery offers two types of muffins—corn or cinnamon raisin. It costs the bakery $2.50 to make 3 corn muffins.

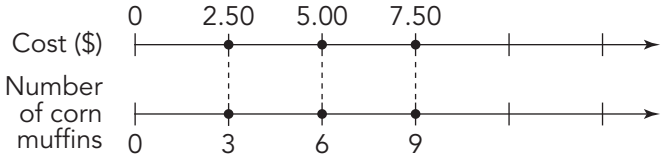
The scale for each number line is different, but the interval is the same for both lines.

WORKED EXAMPLE

The rate $2.50 : 3 corn muffins is shown on the double number line.



You can see other equivalent rates of cost : number of corn muffins by continuing to label each interval.

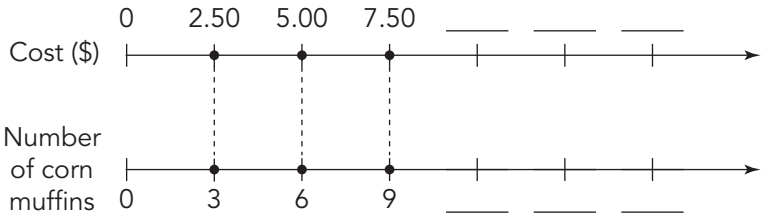


1. State the two new rates of cost : number of corn muffins shown on the second double number line.

2. Describe the interval represented on each number line.

3. Use the double number line to determine equivalent rates.

   a. Plot the new quantities. Explain your calculations.



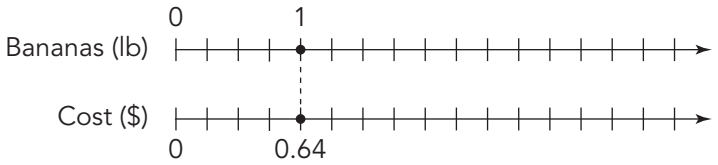
   b. What is the cost of making 12 corn muffins?

   c. What is the cost of making 15 corn muffins?

   d. What is the cost of making 18 corn muffins?

   e. Describe any patterns you notice between the cost and the number of corn muffins made.

4. One pound of bananas costs $0.64. Use the double number line to determine the cost for each quantity of bananas.

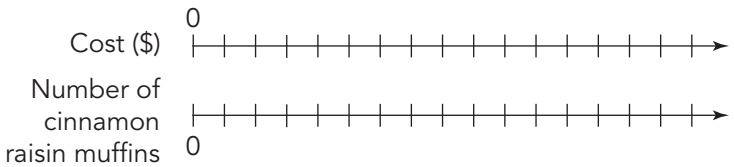


a. $2\frac{1}{2}$ pounds

b. $\frac{1}{2}$ pound

c. 2 pounds

5. The cost for the bakery to make 4 cinnamon raisin muffins is $3.20. Use the double number line to determine equivalent ratios and answer each question. Explain your calculations.

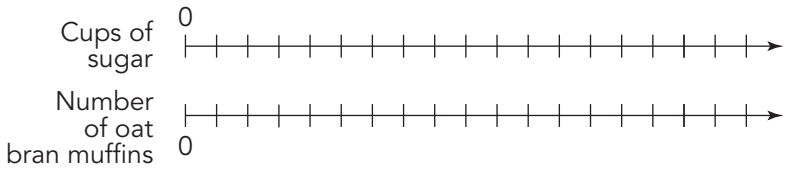


a. What is the cost to make 8 cinnamon raisin muffins?

b. How many cinnamon raisin muffins are made for $12.80?

c. What is the cost of making 12 cinnamon raisin muffins?

6. It takes 1 cup of sugar to make 12 oat bran muffins. Use the double number line to determine equivalent rates and answer each question. Explain your calculations.



a. Plot the given rate on the double number line.

b. How many oat bran muffins can be made using $\frac{1}{2}$ cup of sugar? $\frac{2}{3}$ cup of sugar? $1\frac{1}{2}$ cups of sugar?

c. How many cups of sugar are needed to make 3 muffins? 15 muffins? 9 muffins?

Talk the Talk

Make a Choice



Answer each question by using pictures, a strip diagram, a proportion, or a double number line. Show all of your work and explain why you chose your strategy.

1. A T-shirt store keeps 7 white T-shirts on the shelves for every 3 purple T-shirts on the shelves.

   a. How many white T-shirts are on the shelves when there are 15 purple T-shirts on the shelves?

   b. How many purple T-shirts are on the shelves when there are 49 white T-shirts on the shelves?

   c. How many white shirts are on the shelves when there are 40 total shirts (purple and white) on the shelves?

2. A grocery store advertises 4 pounds of apples for $6.00.

   a. What is the cost for 3 pounds of apples?

   b. What is the cost for 1 pound of apples?

   c. How many pounds of apples can you purchase with $40.00?

Circle the question that your teacher has asked you to present to the class. Write at least 3 sentences to tell your classmates how you completed the work.

Lesson 3 Assignment

Write

Compare and contrast strip diagrams and double number line models for representing ratio and rate relationships. Use an example in your description.

Remember

Equivalent ratios are ratios that represent the same part-to-part or part-to-whole relationship.

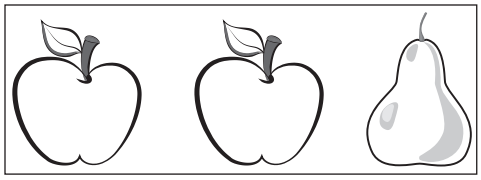
A proportion is an equation that states that two ratios or rates are equal. In a proportion, the quantities composing each part of the ratio/rate have the same multiplicative relationship between them.

Scaling up means you multiply both parts of the ratio/rate by the same scale factor greater than 1.

Scaling down means you divide both parts of the ratio/rate by the same scale factor greater than one, or multiply both parts of the ratio by the same scale factor less than 1.

Practice

1. Ms. Williams is putting together bags of fruit that contain 1 pear for every 2 apples. For each ratio given, create a picture model. Then, calculate the answer from your model and explain your reasoning.



   a. How many apples are in the bag if there are a total of 9 pieces of fruit?

   b. How many apples are in the bag if there are a total of 15 pieces of fruit?

   c. How many pieces of fruit are there if there are 8 apples in the bag?

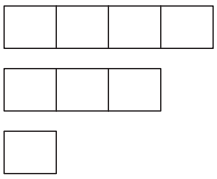
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Lesson 3 Assignment

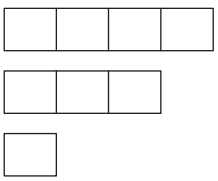
2. When creating playlists for dances, DJ Park likes to maintain a ratio of 4 hip-hop songs : 3 country songs : 1 slow song.

   a. Create a strip diagram to represent this ratio.

   b. Suppose DJ Park has 40 songs on his playlist. Use the strip diagram to illustrate how many hip-hop, country, and slow songs are on the playlist.



   c. Suppose DJ Park wants to put 36 hip-hop songs on the playlist. How many total songs will be on the playlist? Use a strip diagram to determine the answer.



Lesson 3 Assignment

3. Scale up or scale down to determine the unknown quantity. Identify the operation and scale factor that you used.

a. $\frac{2 \text{ teachers}}{26 \text{ students}} = \frac{8 \text{ teachers}}{?}$

b. $\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{?}{18 \text{ feet}}$

c. $\frac{\$39{,}000}{1 \text{ year}} = \frac{?}{3 \text{ years}}$

d. $\frac{18 \text{ pencils}}{1 \text{ box}} = \frac{108 \text{ pencils}}{?}$

e. $\frac{\$40}{15 \text{ gallons}} = \frac{?}{3 \text{ gallons}}$

f. $\frac{1200 \text{ boxes}}{9 \text{ truckloads}} = \frac{?}{3 \text{ truckloads}}$

g. $\frac{280 \text{ beats}}{4 \text{ seconds}} = \frac{70 \text{ beats}}{?}$

h. $\frac{520 \text{ cm}}{5.2 \text{ m}} = \frac{260 \text{ cm}}{?}$

4. A mason is a person who builds structures with bricks, stone, cement block, or tile. A mason usually uses mortar to hold the bricks together. A general rule of thumb in masonry is that $2\frac{1}{2}$ bags of mortar are needed for every 100 cement blocks.

   a. Complete a double number line to determine the amount of mortar needed for each quantity of blocks.

Lesson 3 Assignment

b. How many bags of mortar will a mason need for 350 blocks?

c. How many bags of mortar will a mason need for 50 blocks?

d. With $12\frac{1}{2}$ bags of mortar, how many blocks can the mason lay?

Prepare

It takes 1 cup of milk to make a batch of 8 pancakes.

1. How many cups of milk does it take to make 16 pancakes?

2. How many cups of milk does it take to make 4 pancakes?

3. How many pancakes can be made with 4 cups of milk?

4 Using Tables to Represent Equivalent Ratios and Rates

OBJECTIVES

- Create and reason about tables of equivalent ratios and rates.
- Use known values in a table to determine equivalent ratios and rates.
- Solve problems by reasoning about graphs, diagrams, and tables of equivalent ratios or rates.

You have created equivalent ratios and rates using pictures, strip diagrams, double number lines, and scaling up or scaling down.

Are there other strategies you can use to determine equivalent ratios and rates?

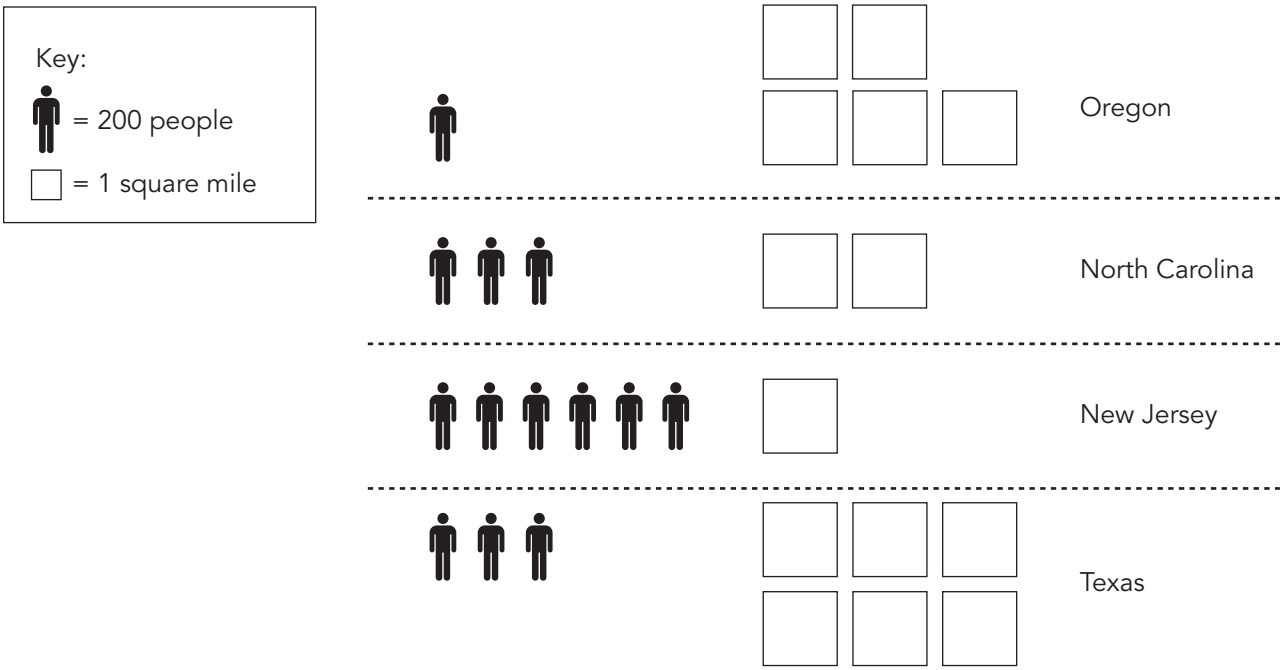
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Getting Started

I'm Your Density

Population density is a rate that compares people to square miles. The graph shown gives the approximate population density of four U.S. states in 2015.

1. Which of the states shown has the greatest population density? Which state has the least population density? Explain what this means in your own words.



2. What is the population density of your state or your city? How does this compare with other states or cities?

ACTIVITY
4.1 Introduction to Ratio Tables

Gravity is a natural force that attracts objects to each other. Gravity is the pull toward the center of an object like the Earth, a planet, or the moon. Your weight on Earth is the measure of the amount of gravitational attraction exerted by Earth on you. The moon has a weaker gravitational force than Earth.

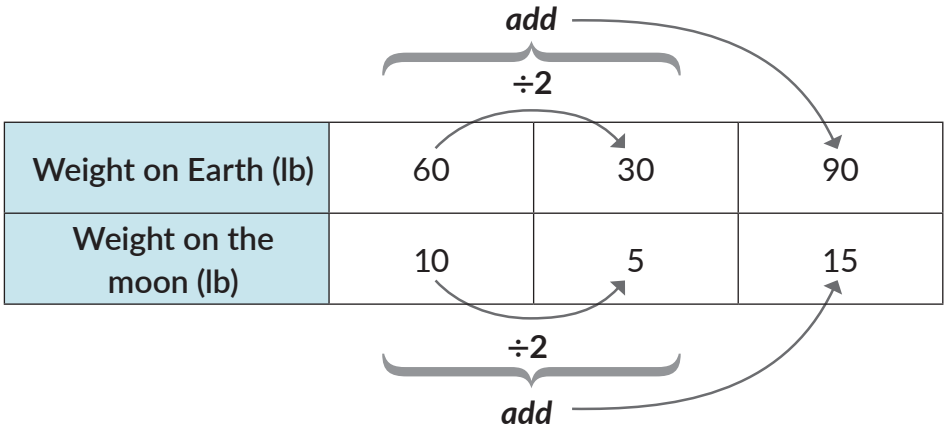
The ratio of weight on Earth : weight on the moon is approximately 60 lb : 10 lb.

You can use ratio tables to show how two quantities are related. Ratio tables are another way to organize information.

WORKED EXAMPLE

The table represents three equivalent ratios of weight on Earth (lb) : weight on the moon (lb).

The ratio of 60 lb on Earth : 10 lb on the moon is given.





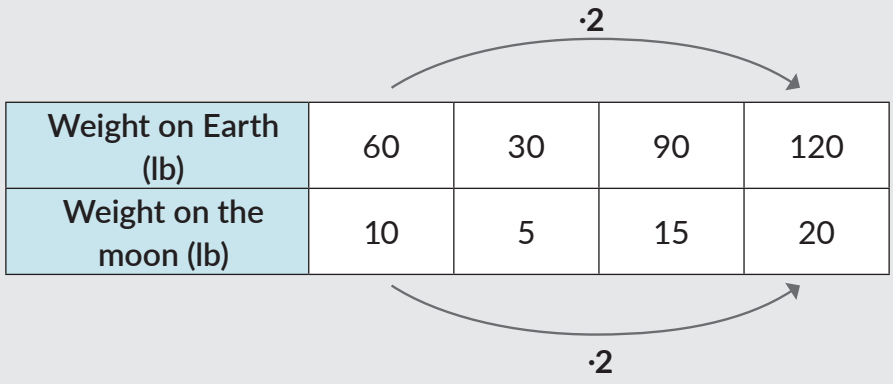
1. Verify that adding the two existing equivalent ratios, 60 lb on Earth : 10 lb on the moon and 30 lb on Earth : 5 lb on the moon, produces the equivalent ratio 90 lb on Earth : 15 lb on the moon by analyzing the quotient of each ratio.
   What do you notice?

2. Can you show a different strategy to determine the ratio of 90 lb on Earth : 15 lb on the moon?

3. Diego, Sarah, Camilla, and Logan each determined the weight of a 120-lb person on the moon.
   a. Compare Diego's and Sarah's strategies.

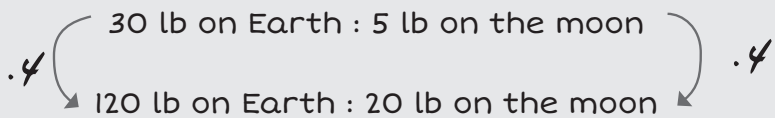
Diego

I can scale 60 up to 120 by multiplying by 2, so then I must also multiply 10 by 2 to get 20.



Sarah

I also got the ratio of 120 lb on Earth : 20 lb on the moon.



b. Explain Camilla reasoning. Then, verify the ratio 120 lb on Earth : 20 lb on the moon is a correct equivalent ratio.

Camilla



I used the weights for a 30-lb person and a 90-lb person to obtain the weight of a 120-lb person.



Weight on Earth (lb)	60	30	90	120
Weight on the moon (lb)	10	5	15	20



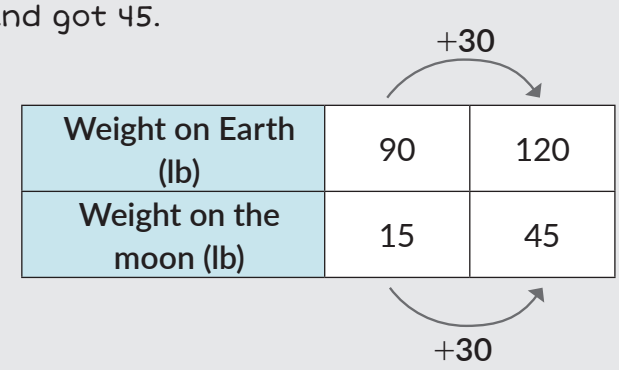
So that means 120 lb on Earth : 20 lb on the moon.

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c. Explain why Logan's reasoning is not correct.

Logan

The difference between 90 and 120 is 30, so I just added 30 to 15 and got 45.



I got the ratio of 120 lb on Earth : 45 lb on the moon.

4. Camilla said, "I see another equivalent ratio when I look at the way Sarah showed her work."
   30 lb on Earth : 5 lb on the moon
   120 lb on Earth : 20 lb on the moon
   150 lb on Earth : 25 lb on the moon

Is Camilla correct? Explain her reasoning.

5. Use the table to show a different calculation for the ratio of 150 lb on Earth : 25 lb on the moon. Explain your reasoning.

Weight on Earth (lb)	60	30	90	120	150
Weight on the moon (lb)	10	5	15	20	25

6. Use the ratio 100 lb on Earth : 38 lb on Mars to complete the ratio table.

Weight on Earth (lb)	100	50		200	25
Weight on Mars (lb)	38		57		

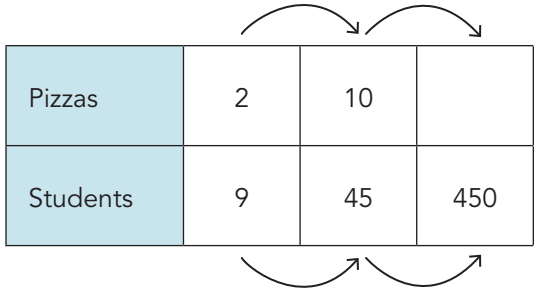
7. Use the table to determine how much a person weighing 125 lb on Earth would weigh on Mars. Explain your reasoning.

ACTIVITY
4.2 Using Equivalent Rate Tables

You can also represent rate relationships in tables.

The 6th grade is having a pizza party tomorrow. Jasmine is in charge of ordering the pizza for 450 students. The pizza parlor said two pizzas will serve 9 students. Jasmine made a rate table to help her determine how many pizzas to order for 450 students.

1. Explain Jasmine's strategy and determine the number of pizzas needed.



Pizzas	2	10	
Students	9	45	450

2. Complete the table to show the number of pizzas to order given the number of students. Explain your calculations.

Pizzas	2	10						
Students	9	45	450	135	270	225	900	1350

3. Use your table of values to answer each question. Explain your calculations.

   a. How many students will 12 pizzas feed?

   b. How many students will 20 pizzas feed?

   c. How many students will 90 pizzas feed?



ACTIVITY
4.3 Parts and Wholes in Ratio Tables

The school colors at a middle school are a shade of bluish-green and white. The art teacher, Mr. Park, needs to mix different quantities of the green paint for several school projects. It takes 3 parts blue paint to 2 parts yellow paint to create the bluish-green color. Sarah needs 5 total pints of the bluish-green paint, so she used 3 pints of blue paint and 2 pints of yellow paint.

Mr. Park thought that the art students needed a table to help determine the correct amount of each color of paint for different projects—both large and small.

1. Complete the table with the correct amounts. Explain your reasoning.

Amount of Bluish Green Paint Needed	5 pints	15 pints			
Amount of Yellow Paint	2 pints		8 pints		
Amount of Blue Paint	3 pints		12 pints	18 pints	1.5 pints

2. Examine Luna's answer. Explain what is wrong with her thinking.

Luna

When I want 15 pints of bluish-green paint, I will need to add 10 to the original 5 total parts of bluish-green to get 15. So, I should also add 10 to each of the other numbers to get 12 pints of yellow and 13 pints of blue.



Carlos said, "The table is helpful, but it cannot list every amount we might need for every painting project. I think when we multiply $\frac{2}{5}$ times the total amount of bluish-green paint we need, we can determine the amount of yellow paint needed. When we multiply $\frac{3}{5}$ times the total amount of bluish-green paint we need, we can determine the amount of blue paint needed."

3. What do you think about Carlos's method? Is he correct or incorrect? Explain your reasoning.

Parker said, "I am thinking about this in a different way. The amount of blue paint is always $1\frac{1}{2}$ times as much as the amount of yellow paint."

4. Is she correct in her thinking? Explain your reasoning.

Avery said, "My thinking is related to Parker's. The amount of yellow paint is $\frac{2}{3}$ of the amount of blue paint."

5. Is Avery correct? Explain your reasoning.

6. How does Avery's thinking relate to Parker's thinking?

Talk the Talk

Lollipop Recipe

Consider the recipe for making one batch of lollipops.

2 cups granulated sugar
$\frac{2}{3}$ cup light corn syrup
$\frac{3}{4}$ cup water
$\frac{1}{4}$ teaspoon flavoring oil

1. The table represents the ratio of ingredients used to make lollipops. Complete the ratio table. Explain your calculations.

Number of Batches	1	2	5	10
Sugar (c)				
Corn Syrup (c)				
Water (c)				
Flavoring Oil (tsp)				

2. For each number of batches, describe how you can use addition to determine the amount of each ingredient needed.

   a. 3 batches

   b. 7 batches

3. For each number of batches, describe how you can use subtraction to determine the amount of each ingredient needed.

   a. 3 batches

   b. 7 batches

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Lesson 4 Assignment

Write

Describe how to use addition with ratio and rate tables to create equivalent ratios and rates. Use examples in your explanation.

Remember

You can use a table to represent, organize, and determine equivalent ratios and rates. You can use addition and multiplication to create equivalent ratios and rates.

Practice

Each table represents the ratio of yellow daffodils to white daffodils for different garden displays. Complete each ratio table. Explain your calculations.

1.

Yellow daffodils	9	36	45	
White daffodils	15			90

2.

Yellow daffodils	7		28	
White daffodils	6	12		42

3.

Yellow daffodils	32			16
White daffodils		48	6	12

4.

Yellow daffodils	5	1		9
White daffodils		3	30	

Lesson 4 Assignment

5.

Yellow daffodils		105	84	21
White daffodils	20	60		

6.

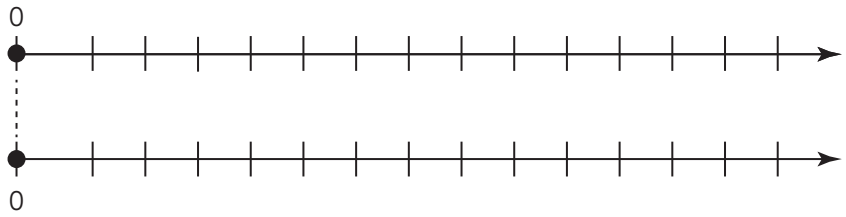
Yellow daffodils	55	22	77	
White daffodils	25	10		5

Prepare

A tree grows 3 feet per year.

1. Write a rate to represent the amount of growth in feet : the number of months.

2. Create a double number line that describes the growth of the tree every 12 months over a 48-month period.



5 Graphs of Ratios and Rates

OBJECTIVES

- Plot ratios and equivalent ratios on a coordinate plane.
- Read equivalent ratios from graphs.
- Use ratio reasoning to determine equivalent ratios from graphs.
- Recognize the graphical representation of equivalent ratios.

NEW KEY TERM

- linear relationship

So far, you have used scaling up or scaling down, tables, strip diagrams, pictures, and double number lines to determine equivalent ratios and rates.

How can you plot pairs of values on a coordinate plane and determine equivalent ratios and rates?

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Getting Started

Growing Rectangles

Consider a rectangle with a short side length of 2 units and a long side length of 3 units.

- In the first table, add the indicated number of units to both the long and short sides of the original rectangle.
- In the second table, multiply each original side length by the given value.
- For each rectangle, determine the ratio of the long side length : short side length.

1.

	Original	+2 units	+3 units	+4 units
Long side	3			
Short side	2			
Ratio	3 : 2			

2.

	Original	×2 units	×3 units	×4 units
Long side	3			
Short side	2			
Ratio	3 : 2			

3. What do you notice about the ratios for rectangles formed by adding to the sides of the rectangle?

4. What do you notice about the ratios for rectangles formed by multiplying the sides of the rectangle by a given value?



ACTIVITY
5.1 Analyzing Rectangle Ratios

You have 2 copies of Rectangle A. You need both for Question 6.

Analyze the rectangles at the end of the lesson.

1. Cut out each rectangle and sort into at least two piles. Share your sorts and your criteria.

Kayla's Group

	Short	Long	Ratio
A			
C			
E			
F			
G			
J			

2. Determine the side lengths of each rectangle. Label each rectangle with the length of its short side and the length of its long side.

3. Kayla grouped together Rectangles A, C, E, F, G, and J. What do you think was her reasoning?

4. Isaiah's sort was similar to Kayla's but he included Rectangle A with Rectangles B, D, H, I, and K. What do you think was his reasoning?

Isaiah's Group

	Short	Long	Ratio
A			
B			
D			
H			
I			
K			

5. Complete the table for Kayla's Group and Isaiah's Group. Write the ratios in fractional form, comparing the length of the short side to the length of the long side. Compare the ratios in each table. What do you notice?

6. Stack each group of rectangles with the smallest rectangle on top so that their longer sides are horizontal and their lower left corners align. What do you notice?

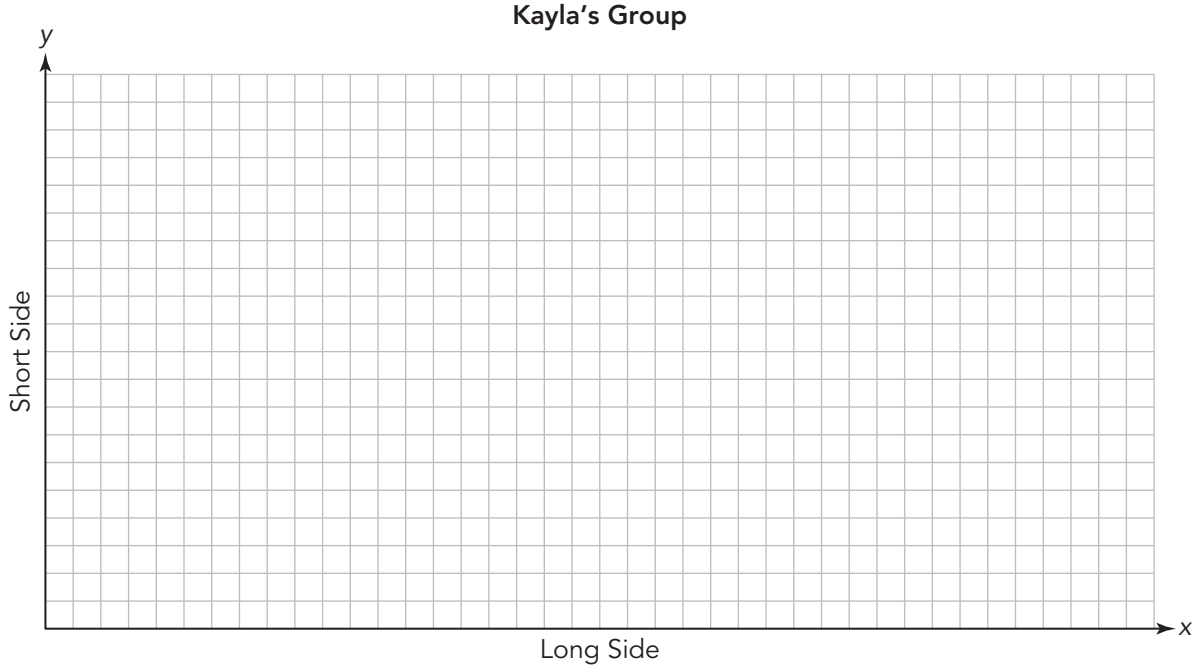
   a. Kayla's Group

   b. Isaiah's Group

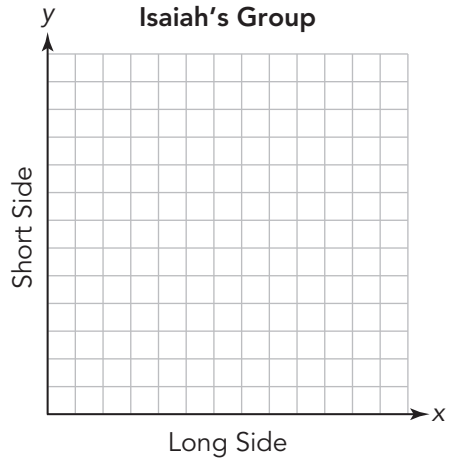
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7. Attach each set of stacked rectangles to the appropriate coordinate grid, with the lower left corner of the rectangles at the origin of the grid.

Kayla's Group



Isaiah's Group



8. Label the coordinates of the upper right corner of each rectangle. What do you notice about the coordinates of each rectangle?

9. Draw a line through the labeled points on each graph. What do you notice about which ordered pairs each line passes through?

10. Whose group represents multiplicative reasoning? Whose group represents additive reasoning? Explain your reasoning.

Just as equivalent ratios can be represented using tables and double number lines, they can also be represented on the coordinate plane. The ratio $\frac{y}{x}$ is plotted as the ordered pair (x, y). When you connect the points that represent the equivalent ratios, you form a straight line that passes through the origin, such as with Kayla's Group. In contrast, non-equivalent ratios are those represented by points that do not create a straight line through the origin, like Isaiah's Group.

When a set of points graphed on a coordinate plane forms a straight line, a **linear relationship** exists.

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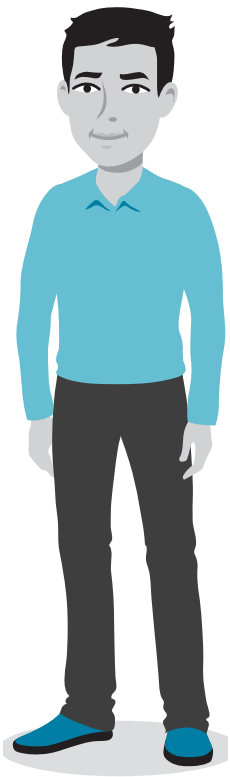
ACTIVITY
5.2 Graphing Equivalent Rates

Let's investigate how you can use a graph to determine other equivalent ratios and rates and see how all the representations are connected.

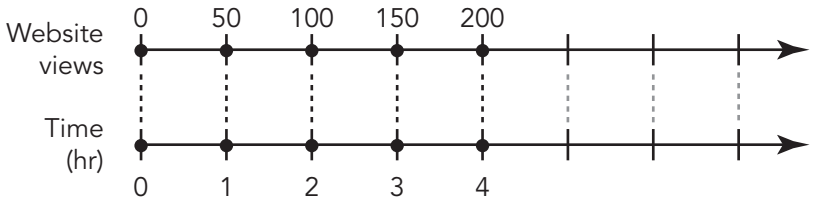
Nahimana runs a website for a local sports team that gets 50 views every hour. The table shows the rate time : website views.

Website Views	50	100	150	200
Time (hr)	1	2	3	4

Compare the labels on the double number line and the labels on the *x*- and *y*-axes. What do you notice?

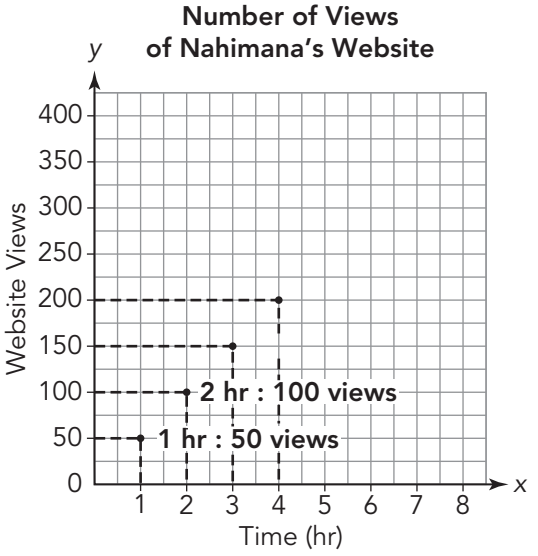


The double number line shown represents the same data.



You can also represent equivalent ratios and rates on a coordinate plane.

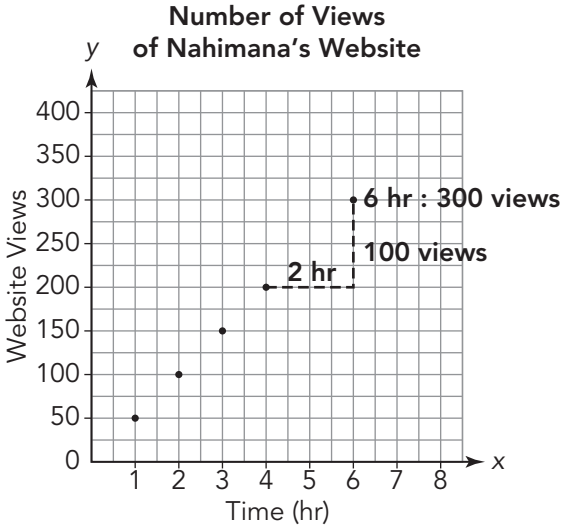
1. Label the remaining rates on the graph.



WORKED EXAMPLE

Consider the question: How many views will Nahimana's website have in 6 hours?

You know 4 different equivalent rates from the original graph. The graph shows how to use the two rates, 2 hr : 100 views and 4 hr : 200 views, to determine the equivalent rate 6 hr : 300 views.



Nahimana's website will have 300 views in 6 hours.

You have used various strategies to determine equivalent ratios and rates:

- drawing pictures
- strip diagrams
- scaling up or down
- ratio tables
- double number lines

2. Describe how to determine how many views Nahimana's website will have in 7 hours given each representation.

   a. using the graph

   b. using the table

   c. using the double number lines

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So, you are comparing time and views of a website. Do fractional values make sense?



One way to analyze the relationship between equivalent ratios and rates displayed on a graph is to draw a line to connect the points. You can also extend the line to make predictions of other equivalent ratios. Sometimes, all of the points on the line make sense. Other times when you draw a line, not all the points on the line make sense.

3. Draw a line through all the points you plotted on your graph. Do all the points on the line you drew make sense in this problem situation? Why or why not?

4. How do all the representations—tables, double number lines and graphs—show equivalent rates or ratios? How are they similar? Describe some of the advantages of each representation.

ACTIVITY
5.3 Using Rate Graphs to Solve Problems

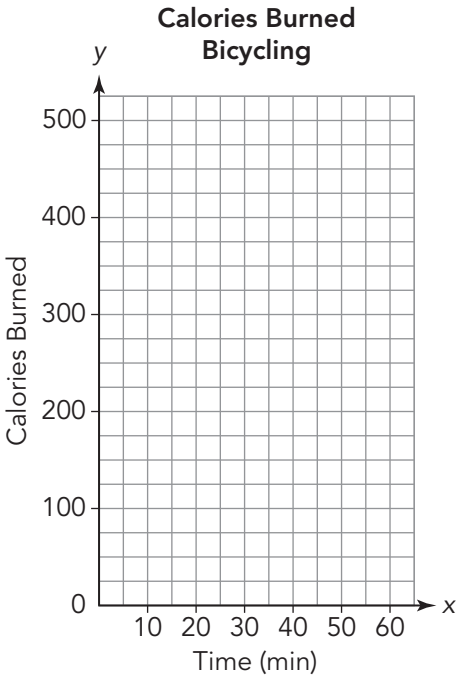
PROBLEM SOLVING



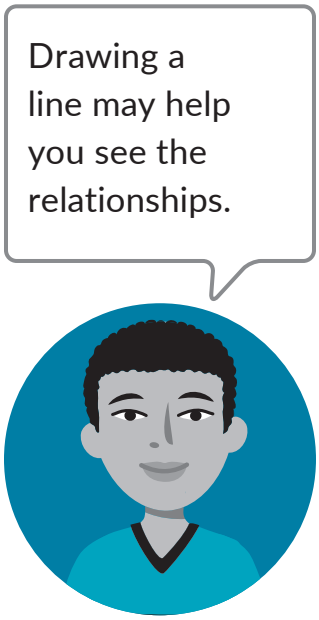
Lucas burns 225 calories for every 30 minutes he rides his bike.

1. Complete the table to chart the number of calories burned for different amounts of time. Then, plot the table of values on the graph.

Calories Burned				
Time (min)	30	10	60	50



2. Use your graph to answer each question.
   a. How many minutes would Lucas have to bike to burn 150 calories?



b. How many calories can he burn when he bikes for 25 minutes?

3. How was the graph helpful? Were there any limitations when using the graph to determine values?

Talk the Talk

To Graph or Not to Graph

Review the definition of **analyze** in the Academic Glossary.

Go back and examine all the graphs in this lesson.

1. What is similar between all of the graphs?

2. What is different between all the graphs?

3. Describe how you can use a line to analyze equivalent ratios or rates. What are the benefits and limitations of using a graph to display and interpret ratios or rates?

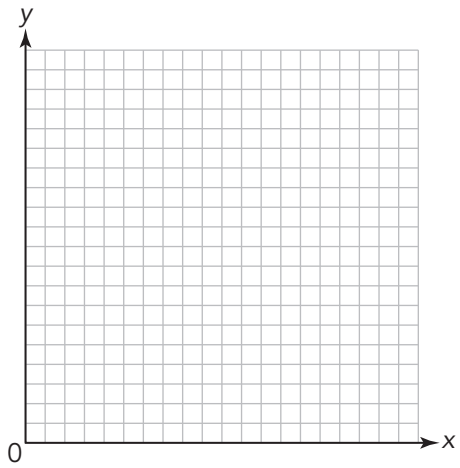
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4. Complete the graphic organizer to demonstrate your understanding of ratios.

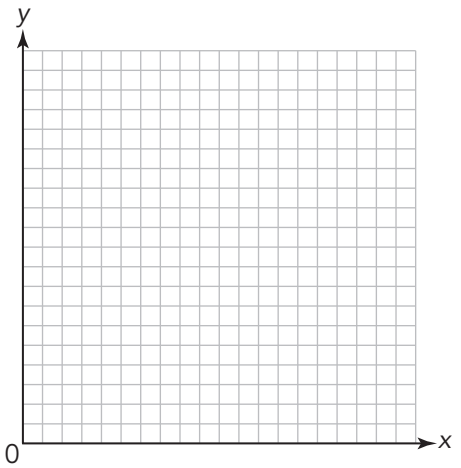
DEFINITION

CHARACTERISTICS

RATIO

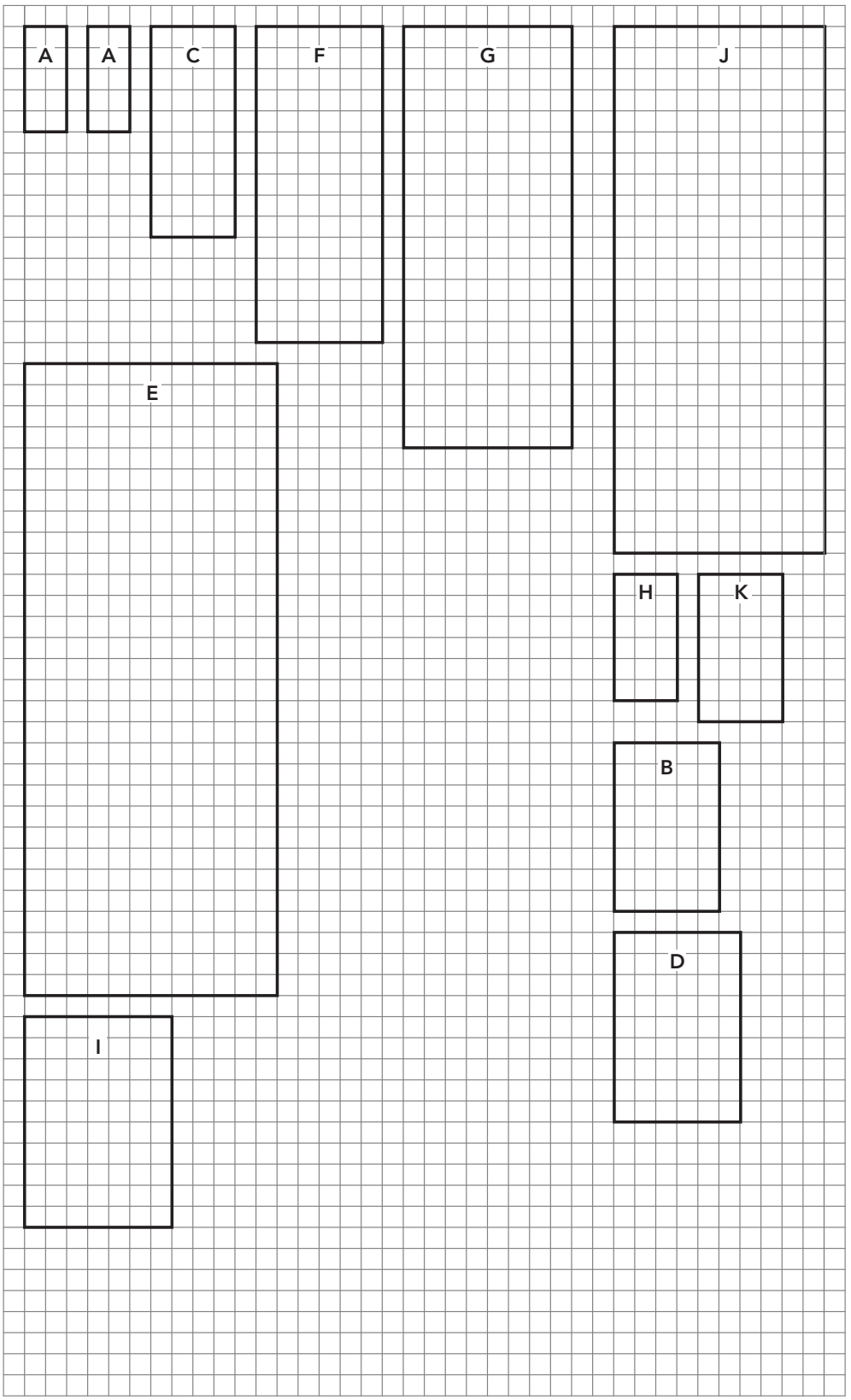


EXAMPLE



NON-EXAMPLE

Rectangle Cutouts



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Why is this page blank?

So you can cut out the rectangles on the other side.

Lesson 5 Assignment

Write

Compare the graph of a ratio or rate relationship with the graph of a relationship that is not represented by a ratio or rate. How are they similar and different? Use an example to explain.

Remember

Just as equivalent ratios and rates can be represented using tables and double number lines, they can also be represented in the coordinate plane. The ratio or rate $\frac{y}{x}$ is plotted as the ordered pair (x, y). When you connect the points that represent the equivalent ratios or rates, you form a straight line that passes through the origin. In contrast, non-equivalent ratios/rates are those represented by points that cannot be connected by a straight line through the origin.

Practice

Create a graph to represent the values shown in each ratio table. Then, make a statement about a scenario that might produce the information in each table.

1.

White Flowers	1	2	4	5
Pink Flowers	3	6	12	15

2.

Mystery Books	1	3	5	7
Total Books	25	75	125	175

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Lesson 5 Assignment

Create a graph to represent the values shown in each rate table. Then, make a statement about a scenario that might produce the information in each table.

3.

Time (minutes)	15	30	45	60
Calories	80	160	240	320

4.

Time (seconds)	1	10	15	20
Data (Mb)	10	100	150	200

Lesson 5 Assignment

5.

Time (minutes)	15	30	45	60
Distance (miles)	1.5	3	4.5	6

6.

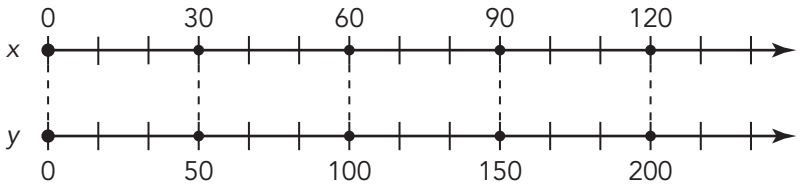
Time (minutes)	1	5	6	10
Height (feet)	6	30	36	60

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Lesson 5 Assignment

Prepare

1. Use the double number line to create a rate table.



x					
y					

2. Create a scenario that fits the data on the double number line and rate table. What rate is associated with your scenario?

6 Using and Comparing Ratio and Rate Representations

OBJECTIVES

- Use graphs to compare ratios and rates.
- Read and interpret ratios and rates from graphs, double number lines, and tables.
- Use ratio and rate reasoning and multiple ratio models to solve problems.
- Compare representations of additive and multiplicative relationships.

You have used a variety of tools to determine equivalent ratios and rates.

How can you compare the different representations as you solve ratio and rate problems?

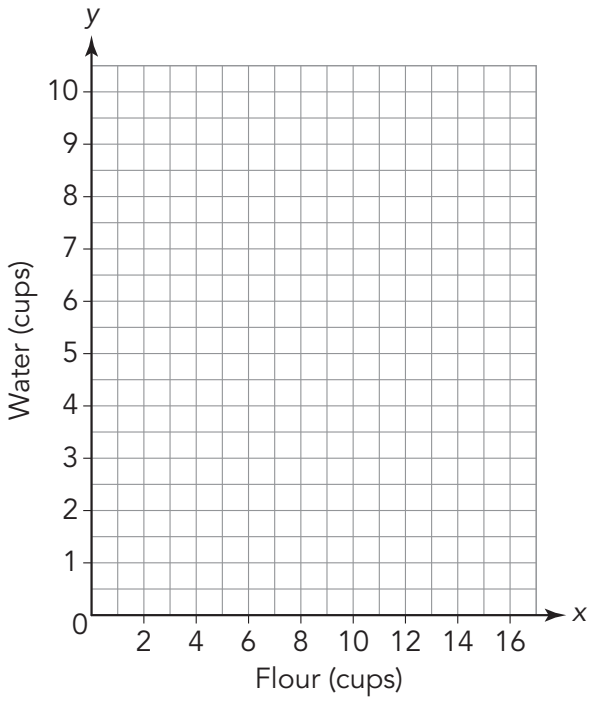
Getting Started

Just-Right Ratios

Avery's dad is trying to make his own bread. But each time he tries, the bread is either too dry because it has too much flour or too runny because it has too much water.

Flour (cups)	Water (cups)	Dry/Runny
11	4	dry
3	5	runny
6	2	dry
10	9	runny
8	8	runny
10	4	dry
10	5	dry
12	9	runny
15	8	dry
5	4	runny

1. Use Xs to graph each attempt that was too dry. Use Os to graph attempts that were too runny.



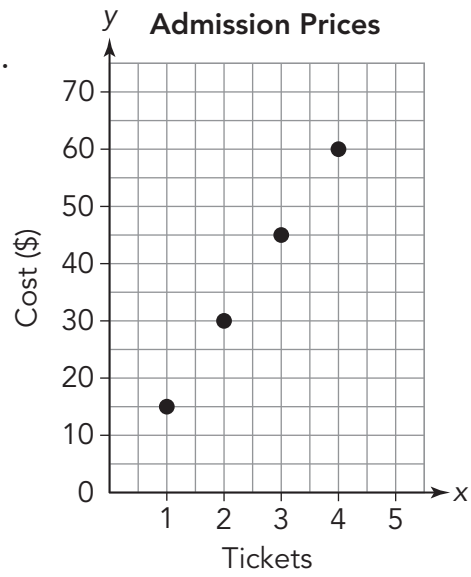
2. Estimate a ratio that is "just right" and graph the ratio. Explain your reasoning.

3. Compare your graph with your classmates' graphs. Did you all create the same graphs?

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ACTIVITY
6.1 Comparing Rate Graphs

The adult ticket price for admission into a regional amusement park is $15. The table and graph show the rate number of adult tickets : cost.



Adult Tickets	1	2	3	4
Cost ($)	15	30	45	60

The amusement park has different charges for students and preschool children. Student tickets are $10. Preschool tickets are $5.

1. Complete each table.

Student Tickets	1	2	3	4
Cost ($)				

Preschool Tickets	1	2	3	4
Cost ($)				

2. Plot each set of equivalent rates on the graph. Use a △ for the student tickets : cost rates and a □ for preschool tickets : cost rates.

3. Draw three separate lines through the points that represent each rate. What do you notice?

4. Do all the points on the line you drew make sense in this problem situation? Why or why not?

5. How can you tell by looking at the three lines which cost to ticket rate is the highest and the lowest?

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PROBLEM SOLVING

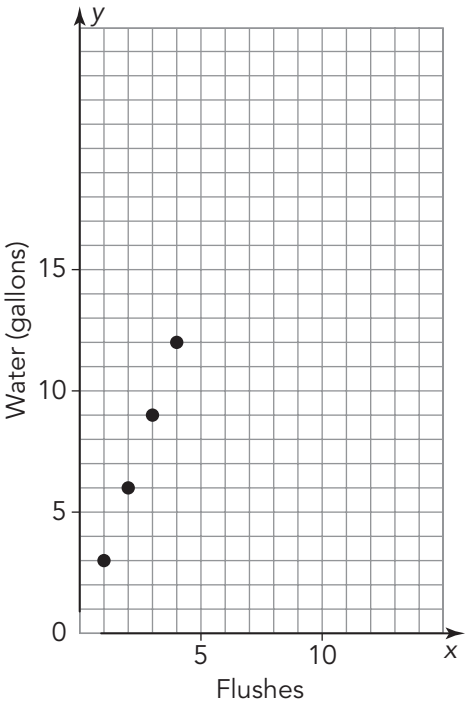


ACTIVITY 6.2

Choosing a Strategy to Solve Rate Problems

You know different ways to think about ratios and rates. So, you can use different strategies to solve problems.

1. The graph shown represents the number of gallons of water used for the number of times a toilet flushes.



   a. Write each point on the graph as the rate of gallons of water used : number of flushes.

   b. What do you notice about each rate?

   c. How many gallons of does it take to flush the toilet 8 times? Explain the method you used.

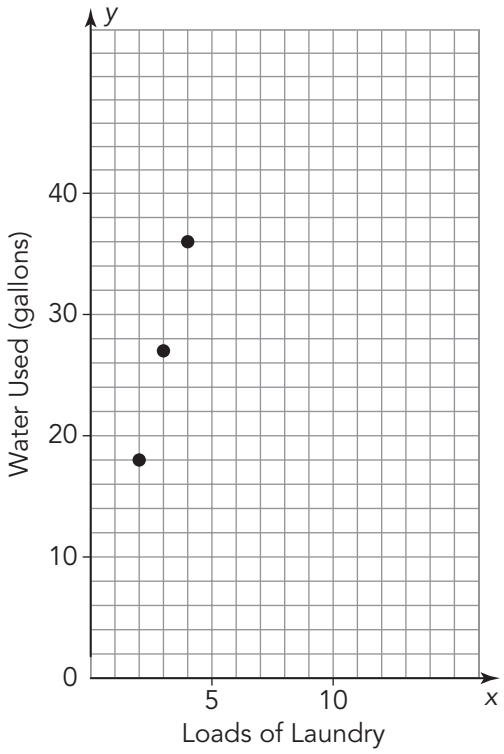
How do you know this graph represents equivalent rates?



   d. How many times must people flush the toilet to use 18 gallons of water? Explain the method you used.

   e. Did you use the same method to answer each question? When you did not, why?

2. The graph shown represents the number of gallons of water used for the number of loads of laundry washed.



   a. Write each point on the graph as the rate of gallons of water used : number of loads of laundry.

   b. What do you notice about each rate?

   c. How many gallons of water does it take to wash 7 loads of laundry? Explain the method you used.

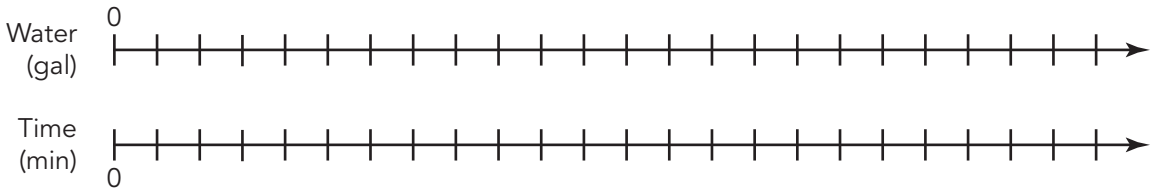
   d. How many loads of laundry can people do when they use 45 gallons of water? Explain the method you used.

   e. Did you use the same method to answer each question? When you did not, why?

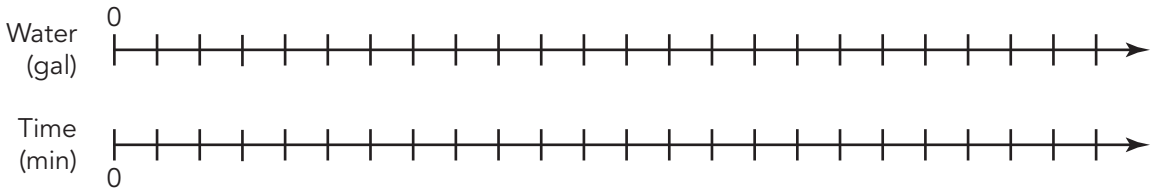
ACTIVITY
6.3 Comparing Rates with Double Number Lines

Showerheads come in various styles and allow different rates of water to flow. The rate gallons of water : time is different for three different showerhead models.

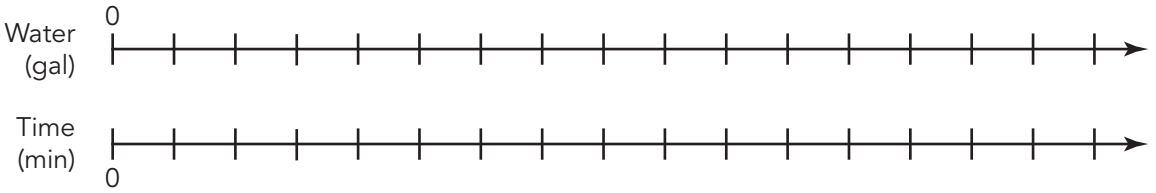
The first showerhead uses 20 gallons of water for every 5 minutes.



A second showerhead model uses 25 gallons of water for every 10 minutes.



A third showerhead model uses 8 gallons of water for every 5 minutes.



1. Which of the three showerheads used the least amount of water per minute?

Ask Yourself . . .
Did you justify your mathematical reasoning?

2. Explain your reasoning using double number lines.

ACTIVITY
6.4 Additive and Multiplicative Representations

Two different jogging situations are given on the next two pages, along with a diagram showing the current relationship between the joggers.

1. At the end of the lesson, there are diagrams, equations, graphs, and verbal statements that each match one of the situations. Cut them out and tape them in their appropriate location. Then explain why each representation describes that relationship between the two joggers.

   a. Choose the diagram that shows the relationship between the joggers after 5 minutes.

   b. Choose the equation that represents the relationship between the two joggers.

   c. Choose the graph that models the relationship between the two joggers.

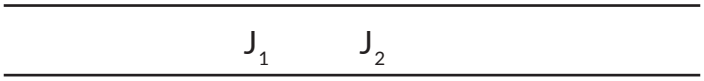
   d. Choose the type of relationship that exists between the two joggers.

In a proportion, the quantities composing each part of the ratio or rate have the same multiplicative relationship between them.

A *multiplicative relationship* is also known as a *proportional relationship*.

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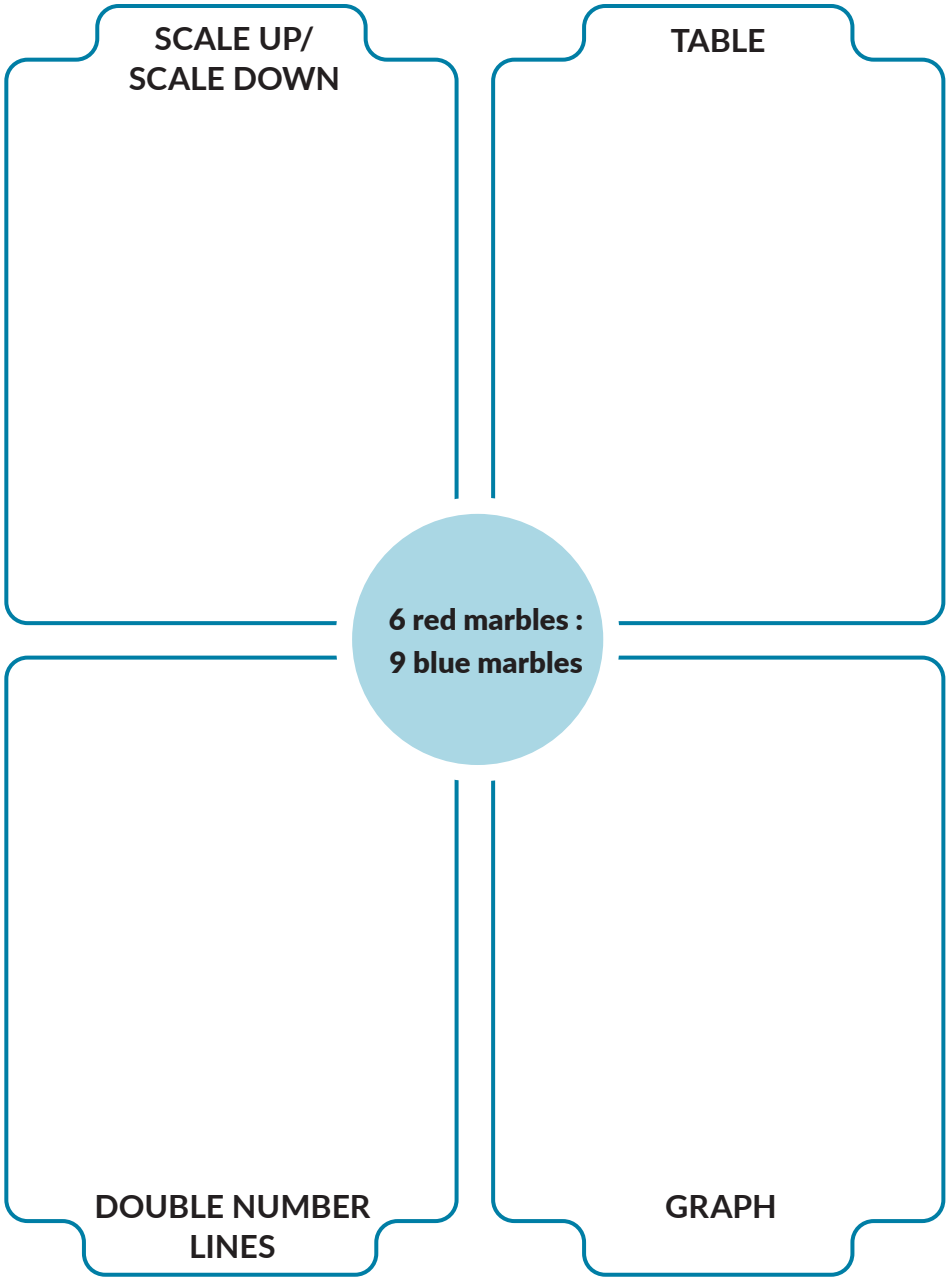
Two joggers are running at the same speed.
Diagram of the current position of the two joggers. J_1 J_2
Diagram of the two joggers after 5 minutes. Explanation:
Equation Explanation:
Graph Explanation:
Verbal Statement Explanation:

Jogger 2 runs twice as fast as Jogger 1.
Diagram of the current position of the two joggers 
Diagram of the two joggers after 5 minutes. Explanation:
Equation Explanation:
Graph Explanation:
Verbal Statement Explanation:

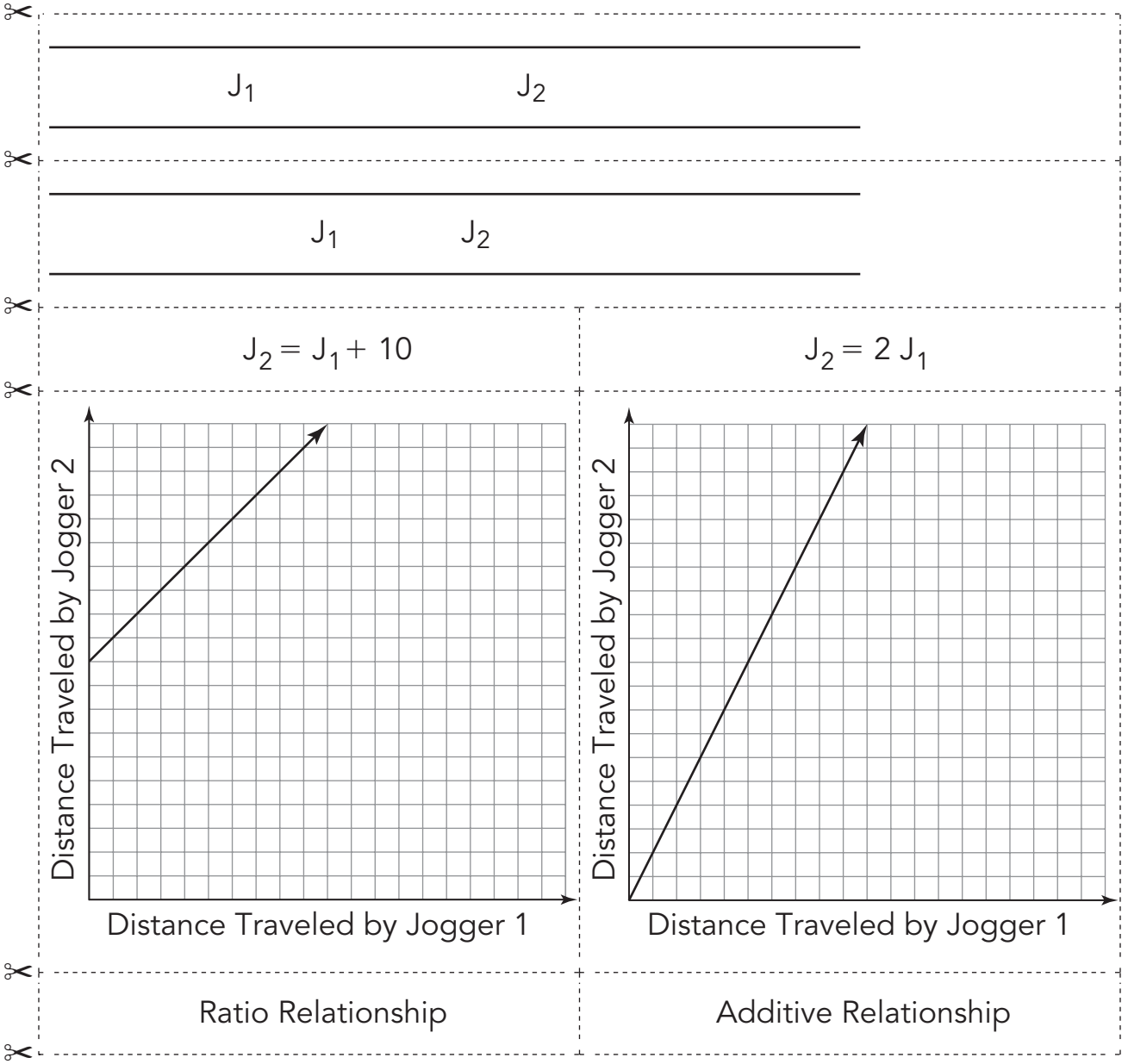
Talk the Talk

In Goes the Kitchen Sink

You have the ratio 6 red marbles : 9 blue marbles. For each model in the graphic organizer, write two ratios equivalent to the given ratio: one with numbers greater than the given and one with numbers less than the given. Show how you can use each model to determine the equivalent ratios.



Cutout for Activity 6.4



Why is this page blank?

So you can cut out the content on the other side.

Lesson 6 Assignment

Write

Describe the advantages and disadvantages of using double number lines, strip diagrams, equations, tables, and graphs to write, represent, and compare ratios and rates.

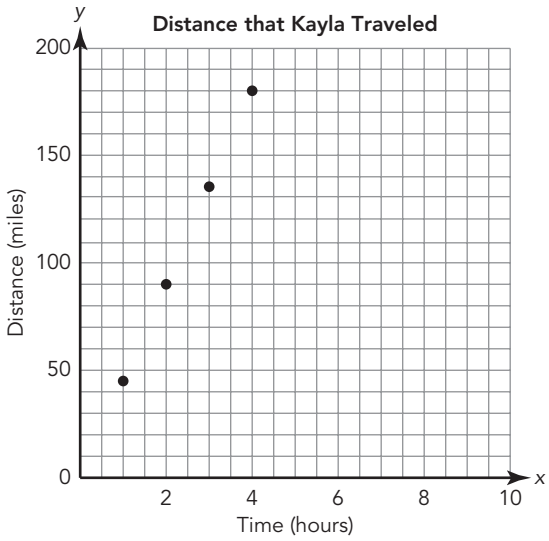
Remember

You can use a number of different models, like graphs, tables, double number lines, and strip diagrams, to analyze ratio and rate relationships and solve problems.

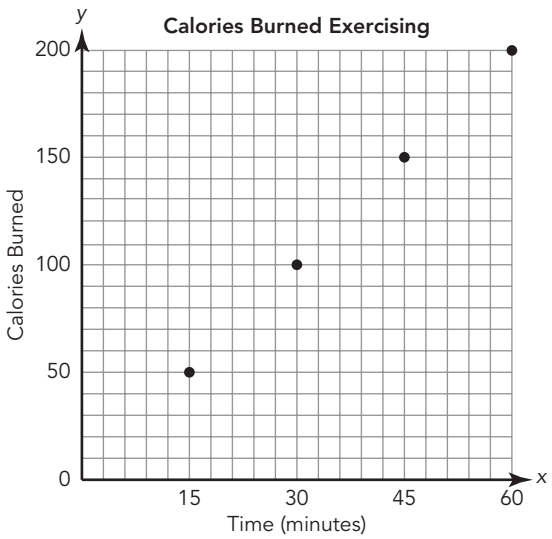
Practice

1. Use a graph to answer each question.

   a. Kayla is driving to the mountains for a summer camping trip. She is traveling at a constant rate of 45 miles per hour. The graph shows the ratio time : distance. How far has Kayla traveled after 4 hours?



   b. Diego is exercising. The graph shows the ratio calories burned : time for Diego. How many calories did Diego burn in 30 minutes?



Lesson 6 Assignment

2. A recipe calls for 2 eggs for every 5 cups of milk. When making the recipe, how many eggs should a person use when they use 20 cups of milk? Draw a double number line to answer the question.

3. Gabriel is in charge of making lunch at a summer camp. He knows that 3 tuna casseroles will serve 15 campers. How many tuna casseroles should Gabriel make to serve 35 campers?

Casseroles	1	3		
Campers		15	30	35

Prepare

Rewrite each fraction as an equivalent fraction with a denominator of 100.

1. $\frac{1}{10}$

2. $\frac{2}{5}$

3. $\frac{3}{20}$

4. $\frac{24}{40}$

TOPIC 1 SELF-REFLECTION

Name: _____

Ratios and Rates

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Ratios and Rates* topic by:

TOPIC 1: *Ratios and Rates*	Beginning of Topic	Middle of Topic	End of Topic
distinguishing between additive and multiplicative relationships between quantities.	○	○	○
writing and labeling ratios and rates in words, with a colon, and in fractional form.	○	○	○
explaining why order matters when writing a ratio or rate.	○	○	○
writing equivalent forms of ratios and rates.	○	○	○
comparing two qualitative and quantitative quantities with ratios and rates.	○	○	○
determining when ratios or rates represent part-to-whole relationships or part-to-part relationships.	○	○	○
using drawings, a double number line, and scaling up or down to generate equivalent ratios and rates.	○	○	○
making a table of equivalent ratios and rates using whole numbers.	○	○	○

continued on the next page

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TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Ratios*	Beginning of Topic	Middle of Topic	End of Topic
determining the unknown value in a table of equivalent ratios or rates.	○	○	○
plotting pairs of values that represent equivalent ratios or rates on the coordinate plane.	○	○	○
recognizing graphs of equivalent ratios and rates as those forming a straight line that passes through the origin.	○	○	○
solving a variety of real-world and mathematical problems involving ratio and rate through reasoning with tables, strip diagrams, double number lines, graphs, and equations.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Ratios and Rates* topic.

   _____

   _____

   _____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   _____

   _____

   _____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   _____

   _____

   _____

Ratios and Rates Summary

NEW KEY TERMS

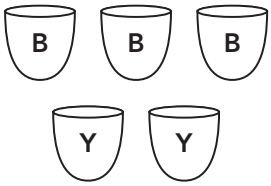
- additive reasoning [razonamiento aditivo]
- multiplicative reasoning [razonamiento multiplicativo]
- ratio
- percent [porcentaje]
- equivalent ratios
- strip diagram
- rate
- proportion [proporción]
- scaling up
- scale factor [factor de escala]
- scaling down
- double number line
- linear relationship [relación lineal]

LESSON 1 Introduction to Ratio and Rate Reasoning

Additive reasoning focuses on the use of addition and subtraction for comparisons. **Multiplicative reasoning** focuses on the use of multiplication and division for comparisons.

A **ratio** is a multiplicative comparison between two quantities that may contain the same or different units.

For example, an art teacher knows it takes 3 parts blue paint to every 2 parts yellow paint to create a certain shade of bluish-green. This is represented by the model shown.



A part-to-whole ratio compares a part of a whole to the total number of parts. 3 to 5 is a part-to-whole ratio comparing blue parts to total parts.	A part-to-part ratio compares individual quantities. 2 to 3 is a part-to-part ratio comparing yellow parts to blue parts.
With a Colon 3 blue parts : 5 total parts	**With a Colon** 2 yellow parts : 3 blue parts
In Fractional Form $\frac{\text{3 blue parts}}{\text{5 total parts}}$	**In Fractional Form** $\frac{\text{2 yellow parts}}{\text{3 blue parts}}$

Fractional form simply means writing the relationship in the form $\frac{a}{b}$. Just because a ratio looks like a fraction does not mean it represents a part-to-whole comparison. Only a part-to-whole ratio is a fraction.

A **percent** is a part-to-whole ratio where the whole is equal to 100. The percent symbol (%) means "per 100," or "out of 100."

35% means 35 out of 100.

35% as a fraction is $\frac{35}{100}$.

35% as a decimal is 0.35.

35% as a ratio is 35 to 100, or 35 : 100.

LESSON 2 Comparing Ratios and Rates to Solve Problems

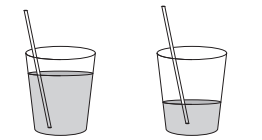
A **rate** is comparison by division between two quantities that have different units.

For example, Luna can answer 4 problems correctly in five minutes.

This rate is $\frac{4 \text{ problems correct}}{5 \text{ minutes}}$.

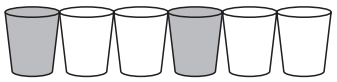
Ratios and rates are sometimes less than, greater than, or equal to another ratio or rate.

For example, the shaded portion in each glass represents an amount of lemonade. Suppose Parker and Carlos have glasses of lemonade that taste the same. If one teaspoon of lemonade mix is added to each glass, Carlos's glass will now contain lemonade with a stronger lemon flavor. The ratio of lemon mix to lemonade is greater in Carlos's glass because he had less lemonade in the glass to begin with.

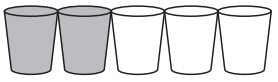


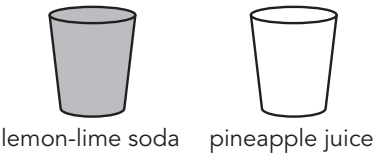
The cups represent different ratios of lemon-lime soda to pineapple juice in two different punches.

Punch A



Punch B





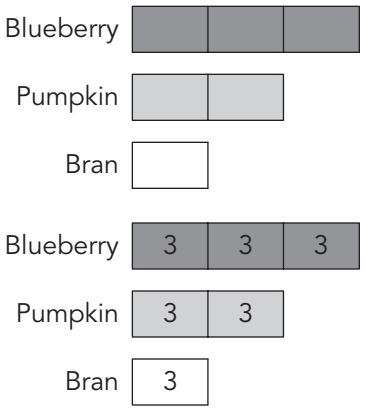
The concentration of lemon-lime soda in Punch A is $\frac{2}{6}$. The concentration of lemon-lime soda in Punch B is $\frac{2}{5}$. Punch B has a greater concentration of lemon-lime soda.

LESSON 3

Determining Equivalent Ratios and Rates

Equivalent ratios are ratios that represent the same part-to-part or part-to-whole relationship. You can use a strip diagram to help determine equivalent ratios. A **strip diagram** illustrates number relationships by using rectangles to represent ratio parts.

For example, the ratio of muffins in a variety pack is 3 blueberry muffins : 2 pumpkin muffins : 1 bran muffin and is represented by the strip diagram shown.



To determine how many of each type of muffin are in an 18-pack of muffins, you need to maintain the same ratio. Since there are 6 muffins represented in the strip diagram, divide 18 by 6.

Since 18 ÷ 6 = 3, each rectangle will represent 3 muffins. The ratio of muffins in an 18-pack will be 9 blueberry muffins : 6 pumpkin muffins : 3 bran muffins.

When two ratios or rates are equivalent to each other, you can write them as a proportion. A **proportion** is an equation that states that two ratios or rates are equal. In a proportion, the quantities composing each part of the ratio or rate have the same multiplicative relationship between them.

You can predict how many problems Luna could answer correctly in 20 minutes.

$$\frac{\text{problems correct}}{\text{minutes}} \rightarrow \frac{4}{5} = \frac{16}{20} \leftarrow \frac{\text{problems correct}}{\text{minutes}}$$

$\cdot 4$ above and $\cdot 4$ below

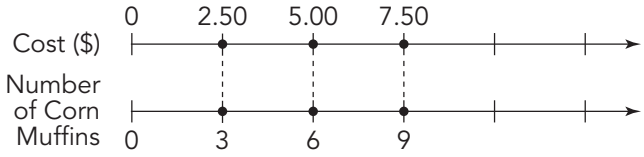
Luna can probably answer 16 problems correctly in 20 minutes.

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When you change one ratio or rate to an equivalent ratio or rate with larger numbers, you are scaling up. **Scaling up** means you multiply both parts of the ratio or rate by the same factor greater than 1. The factor that you multiply both parts of the ratio or rate by is known as the **scale factor**. When you change a ratio or rate to an equivalent ratio or rate with smaller numbers, you are scaling down. **Scaling down** means you divide both parts of the ratio or rate by the same scale factor greater than one or multiply both parts of the ratio or rate by the same scale factor less than one.

A **double number line** is a model that is made up of two number lines used together to represent the ratio or rate between two quantities. The intervals on each number line maintain the same ratio or rate.

For example, the rate \$2.50 : 3 corn muffins is shown on the double number line. You can see other equivalent rates of *cost* : *number of corn muffins* by continuing to label each interval.

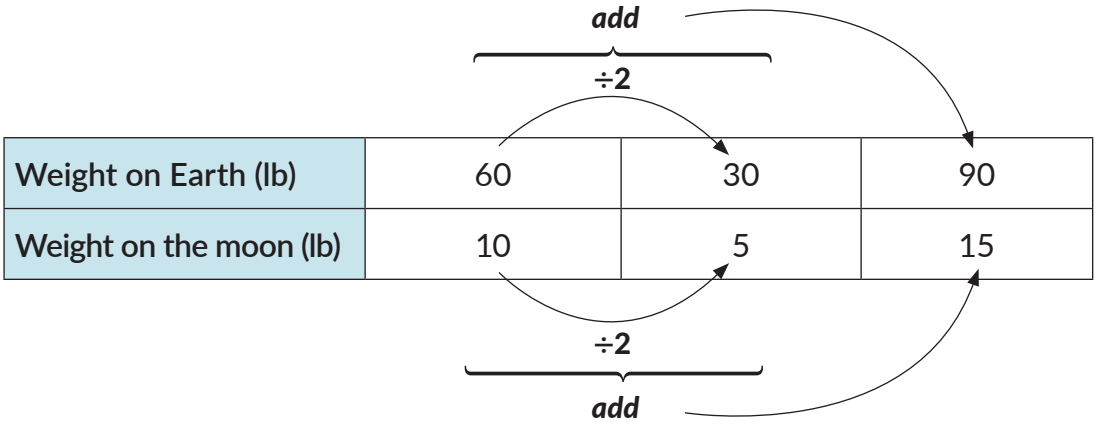


LESSON 4 Using Tables to Represent Equivalent Ratios and Rates

You can use ratio and rate tables to show how two quantities are related. Ratio and rate tables are another way to organize information.

You can use a table to represent, organize, and determine equivalent ratios and rates. You can use addition and multiplication to create equivalent ratios and rates.

For example, the table shown represents three equivalent ratios of *weight on Earth* (*lb*) : *weight on the moon* (*lb*). The ratio of 60 lb on Earth : 10 lb on the moon is given. One equivalent ratio was determined by dividing the original ratio by 2. Another was determined by adding two equivalent ratios.



LESSON 5

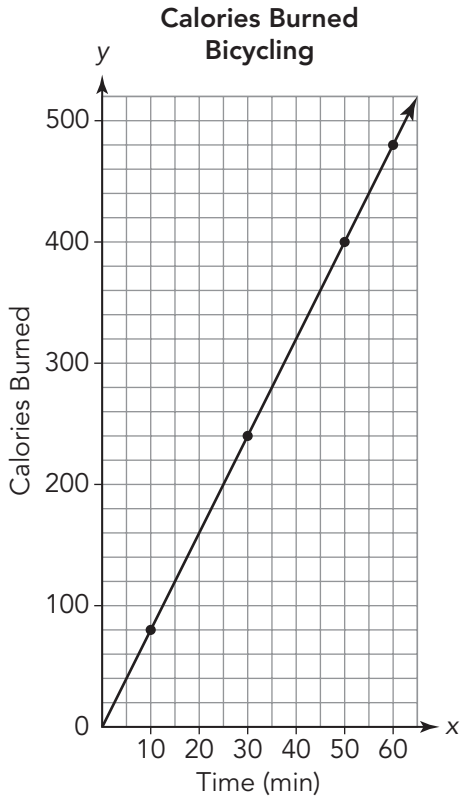
Graphs of Ratios and Rates

You can represent equivalent ratios or rates on the coordinate plane. The ratio or rate $\frac{y}{x}$ is plotted as the ordered pair (x, y). When you connect the points that represent equivalent ratios or rates, you form a straight line that passes through the origin. In contrast, non-equivalent ratios and rates are those represented by points that do not create a straight line through the origin. When a set of points graphed on a coordinate plane forms a straight line, a **linear relationship** exists.

For example, the table charts the numbers of calories Adriana burns for different amounts of time.

Calories Burned	240	80	480	400
Time (min)	30	10	60	50

The values are plotted on the graph.



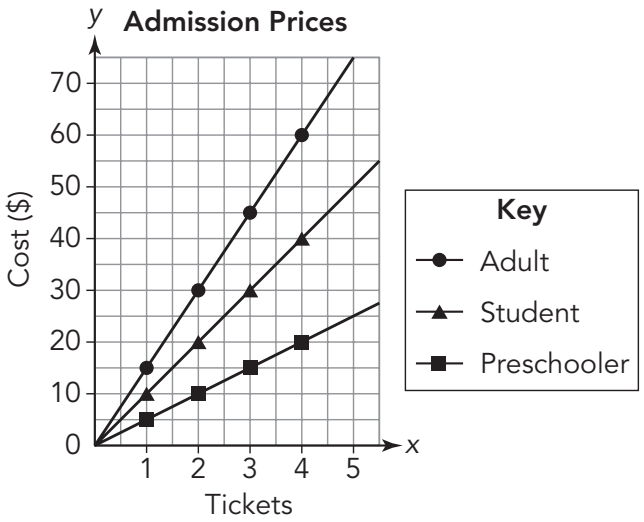
The graph shows that Adriana burns 200 calories after bicycling for 25 minutes and that it takes between 35 and 40 minutes of bicycling for her to burn 300 calories.

LESSON 6

Using and Comparing Ratio and Rate Representations

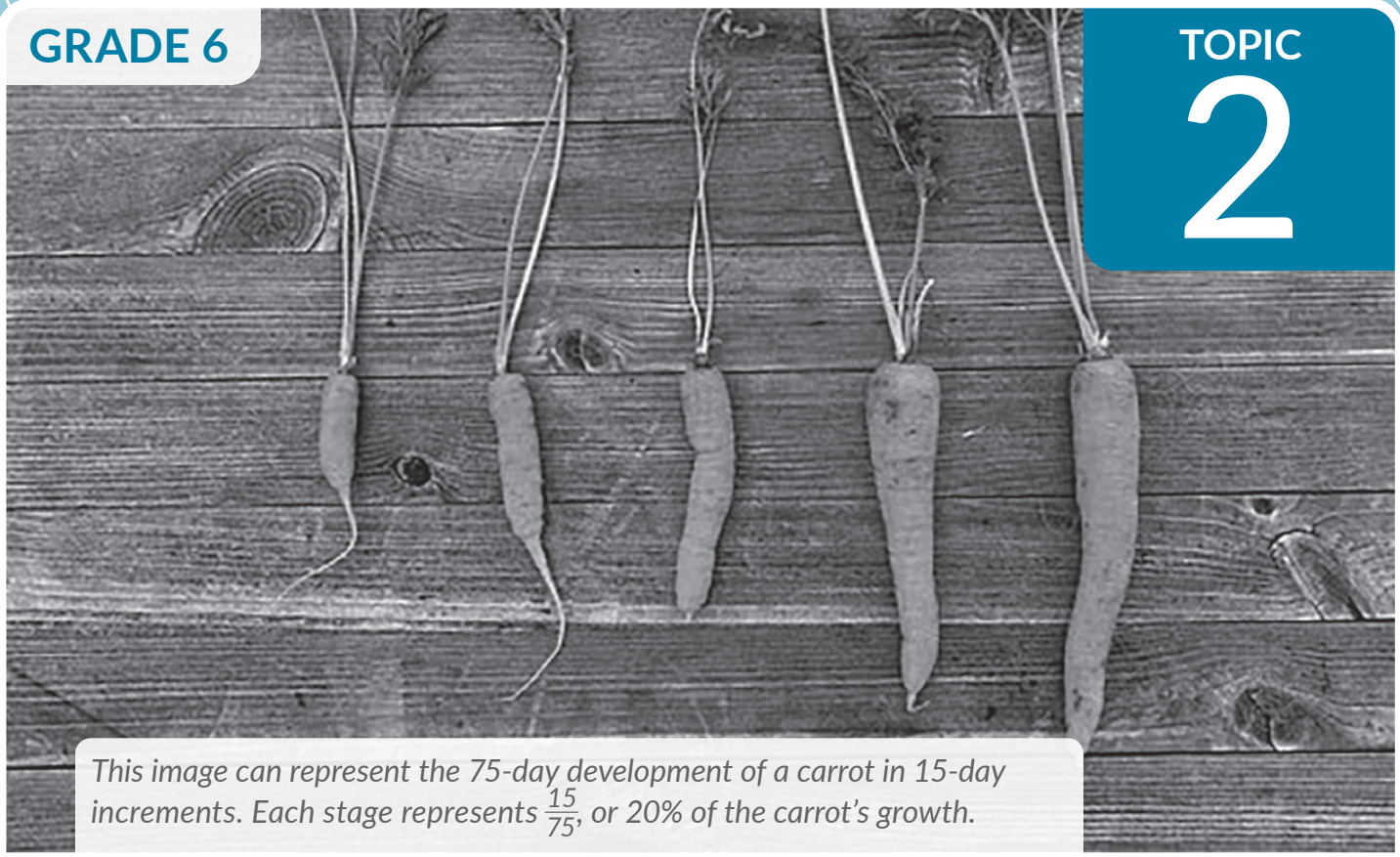
You can use a number of different models, like graphs, tables, double number lines, and strip diagrams, to analyze ratio and rate relationships and to solve problems.

For example, by comparing the graphed lines that represent each rate of *number of tickets* : *cost*, you can tell that the *cost* to *ticket* rate is the greatest for adults because it has the steepest line. Likewise, the *cost* to *ticket* rate is the lowest for the preschoolers because it has the least steep line.



GRADE 6

TOPIC 2



This image can represent the 75-day development of a carrot in 15-day increments. Each stage represents $\frac{15}{75}$, or 20% of the carrot's growth.

Percents

LESSON 1 Percent, Fraction, and Decimal Equivalence **325**

LESSON 2 Using Estimation and Benchmark Percents **347**

LESSON 3 Determining the Part and the Whole in Percent Problems . . **363**

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1 Percent, Fraction, and Decimal Equivalence

OBJECTIVES

- Write equivalent fractions, decimals, and percents.
- Model percents as rates per 100 on a hundredths grid.
- Explain the similarities and differences among percents, fractions, and decimals.

You have learned that percents are special types of ratios. You also know that fractions can be written as decimals.

How are percents like another special type of ratio—fractions? How are percents like decimals?

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Getting Started

One and the Same

Percents are everywhere! Write one or two sentences to explain the meaning of each statement.

1. Big Sale! 25% discount on all regularly priced items.



2. The star of the high school basketball team makes 80 percent of her free throws.

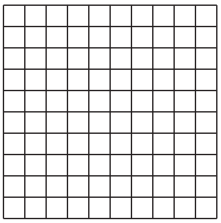
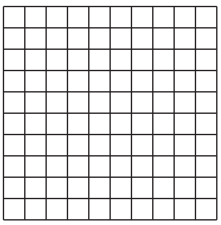
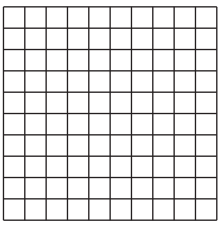
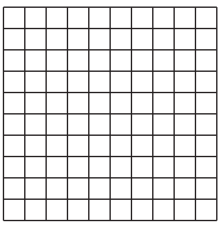
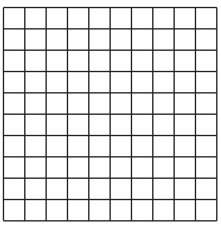
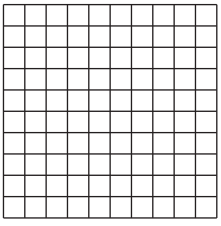
3. I scored an 80% on the 20-question test.

ACTIVITY
1.1 Determining Equivalences

The sixth-grade class is planning a field trip to Philadelphia. To decide which historical site they will visit, the 100 sixth-graders completed a survey.

1. The results of the survey are provided in the table. Complete the Ratio, Fraction, Decimal, and Grid columns with these representations of the survey results:
   - a ratio using colon notation
   - a fraction in lowest terms
   - a decimal
   - a shaded grid
   - an equivalent percent

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	Ratio	Fraction	Decimal	Grid	Percent
Which excursion would you like to take while in Philadelphia?					
35 of the students chose the Liberty Bell.					
22 of the students chose Independence Hall.					
30 of the students chose the National Constitution Center.					
13 of the students chose the Betsy Ross House.					
0 of the students chose Reading Terminal Market.					
Are you planning on going on the trip?					
100 of the students responded *Yes*.					

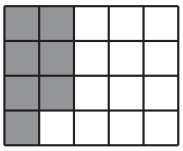
WORKED EXAMPLE

You can use equivalent fractions to represent the fraction $\frac{35}{100}$ in a 5×4 grid.

Dividing the numerator and denominator of $\frac{35}{100}$ by the same factor creates an equivalent fraction.

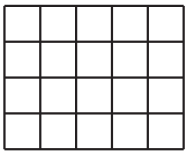
$$\frac{35 \div 5}{100 \div 5} = \frac{7}{20}$$

You can represent this in a 5×4 grid by shading 7 of the 20 squares in the grid.

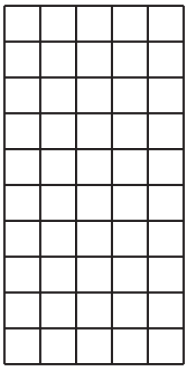


A 10×10 grid has 10 rows and 10 columns. A 5×4 grid has 5 rows and 4 columns. The x symbol is read as the word *by*.

2. Shade the model to represent the fraction $\frac{15}{100}$.



3. Shade the model to represent the fraction $\frac{22}{100}$.

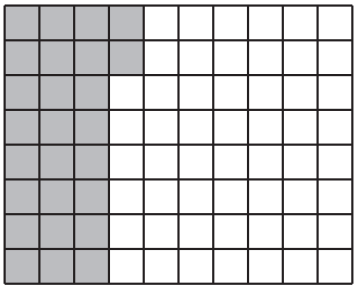


WORKED EXAMPLE

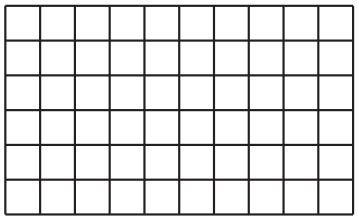
Sometimes, you need to write more than one equivalent fraction to represent a fraction on a grid.

There are 80 squares in the grid. Using equivalent fractions, we can represent $\frac{40}{100}$ on the grid.

$\frac{40}{100} = \frac{8}{20} = \frac{32}{80}$.



4. Shade the model to represent the fraction $\frac{35}{100}$.



Recall that a percent can be a special part-to-whole ratio with a whole of 100. You can also think of a percent as a fraction in which the denominator is 100.
Percents, fractions, and decimals can be used interchangeably.

WORKED EXAMPLE

You can write 15 out of 100 as the fraction $\frac{15}{100}$ or $\frac{3}{20}$.

Written as a decimal, 15 out of 100 is 0.15.

Because percent means "out of 100," 15 out of 100 can also be written as 15%.

The percent symbol (%) means "per 100" or "out of 100."

5. Express each of the ratios in the survey as a percent in the last column of the table.

6. Write a summary of the results of the student survey using percents.

7. Look at the percents and the decimals you wrote for Questions 1 and 5 to determine a pattern. Use this pattern to describe how you can write any percent as a decimal.

Remember, a percent tells you how many hundredths.



8. Write each percent as a decimal.

a. 80%

b. 3%

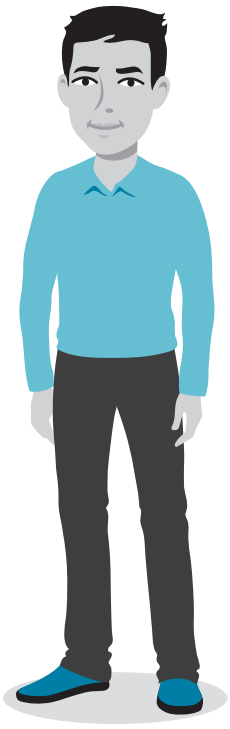
9. Write each decimal as a percent.

a. 0.4

b. 1.52

When the denominator is a factor of 100, you can scale up the fraction to write it as a percent. When the denominator is not a factor of 100, you can divide the numerator by the denominator to write the fraction as a decimal, which you can then write as a percent.

Use the scaling up method when the denominator is a factor of 100.



WORKED EXAMPLE

Write $\frac{5}{8}$ as a percent.

Divide the numerator by the denominator to write the fraction as a decimal.

$$
\begin{array}{r}
0.625 \\
8\overline{)5.000} \\
-\ 48 \\
\hline
20 \\
-\ 16 \\
\hline
40 \\
-\ 40 \\
\hline
0
\end{array}
$$

Then, Write the decimal as a percent.

$0.625 = 62.5\%$ So, $\frac{5}{8} = 62.5\%$

10. Write each fraction as a percent.

a. $\frac{4}{5}$

b. $\frac{3}{8}$

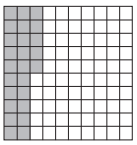
You can use fractions, decimals, and percents to represent real-world problems, including problems involving money.

WORKED EXAMPLE

Sebastian had lunch at a café with a friend. He paid $20.00 for their meals and gave the server a $5.00 tip.

Represent the amount Sebastian tipped as a fraction, decimal, a shaded grid, and an equivalent percent of the cost of the meals.

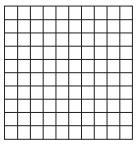
Fraction	Decimal	Percent
$\frac{\$5.00}{\$20.00} = \frac{1}{4}$	$\frac{\$5.00}{\$20.00} = \frac{(5 \cdot 5)}{(20 \cdot 5)} = \frac{25}{100} = 0.25$	$\frac{\$5.00}{\$20.00} = \frac{(5 \cdot 5)}{(20 \cdot 5)} = \frac{25}{100} = 25\%$



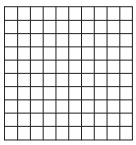
The amount Sebastian tipped the server can be represented as $\frac{1}{4}$, 0.25, or 25% of the cost of the meals.

11. Represent each situation using a fraction, decimal, and a percent. Use the grid to model the situation when needed.

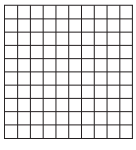
    a. Adriana had $25.00 in her wallet. She spent $13.00. What fraction, decimal, and percent of the money did she spend?



    b. A recipe for 35 cups of mixed fruit punch requires 7 cups of apple juice. What fraction, decimal, and percent of the total amount of mixed fruit punch is apple juice?



    c. Angelina went to the movie theater. She spent $12.50 on snacks and spent $20.00 total at the theater. What fraction, decimal, and percent of the total she spent was on snacks?



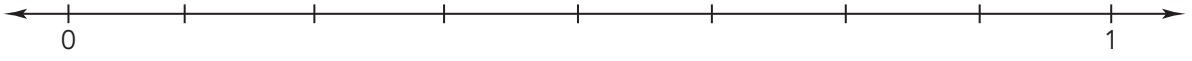
12. Label each mark on the number line with a fraction, decimal, and percent. Make sure your fractions are in lowest terms.

a.



Fraction	0	$\frac{1}{3}$	_____	1
Decimal	0.0	_____	$0.6\overline{6}$	1.0
Percent	0%	_____	_____	100%

b.



Fraction	0	_____	_____	_____	$\frac{1}{2}$	_____	_____	$\frac{7}{8}$	1
Decimal	0.0	0.125	_____	_____	_____	0.625	0.75	_____	1.0
Percent	0%	_____	25%	37.5%	_____	_____	_____	_____	100%

Remember . . .

The bar over a number in a decimal means the number repeats without ending.

ACTIVITY
1.2 Reasoning with Ratio and Percent

On Saturday, Mei won 3 out of 4 of her tennis matches at the Redstone Tournament. On Sunday, she won 1 out of 4 of her matches at the Mesa Tennis Tournament.

Each student summarized Mei's record over the weekend.

Lucas



Mei won 100% of her matches!

$\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

Kayla



Mei won 50% of her matches!

$$\frac{\text{3 matches won}}{\text{4 matches played on Sat}} + \frac{\text{1 match won}}{\text{4 matches played on Sun}} = \frac{\text{4 matches won}}{\text{8 total number of matches played}}$$

Mason



Mei won 4 out of 8 matches played.

$$\frac{\begin{array}{l}\text{3 matches won : 4 matches played on Saturday} \\ \text{1 match won : 4 matches played on Sunday}\end{array}}{\text{4 matches won : 8 total number of matches played}}$$

1. What is wrong with Lucas's reasoning?

Remember . . .

You can write both part-to-part and part-to-whole ratios in fractional form and in colon notation.

Labeling ratios with units is important.

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2. How did Kayla make her reasoning explicit?

3. What is the same about Kayla's and Mason's reasoning? What is different?

4. Why do Kayla's and Mason's answers make sense?

ACTIVITY
1.3

Matching Percents, Fractions, and Decimals

It's time to play The Percentage Match Game. In this game, you will use your knowledge of percents, fractions, and decimals.

Rules of the Game:

- For this 2-person game, 1 person needs to cut out the cards located at the end of the lesson.
- Lay out all the cards face down.
- The first player chooses any card. That player then turns over another card to see whether it is an equivalent match. When the values on the two cards are equivalent, then the match is put into the player's pile. The first player then picks again and repeats the process until they do not find a match.
- When the first player does not have an equivalent match, turn the cards back over. It is the second player's turn. The second player now follows the same process for picking and matching cards.
- Continue taking turns until all possible matches are made.
- The player with the greater number of correct equivalent matches wins the game.

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Talk the Talk

Comparing Representations

Percents, fractions, and decimals can be used interchangeably. The chart shows some common equivalent fractions, decimals, and percents.

Common Equivalent Fractions, Decimals, and Percents									
Fraction	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
Decimal	0.2	0.25	$0.\overline{3}$	0.4	0.5	0.6	$0.\overline{6}$	0.75	0.8
Percent	20%	25%	$33\frac{1}{3}\%$	40%	50%	60%	$66\frac{2}{3}\%$	75%	80%

1. How are percents similar to decimals? How are percents and decimals different?

2. How are percents similar to fractions? How are percents and fractions different?

3. How are percents similar to ratios? How are percents and ratios different?

$\frac{3}{5}$	$\frac{3}{10}$	$\frac{6}{10}$	30%
0.6	$\frac{1}{3}$	60%	33%
$\frac{1}{8}$	$\frac{1}{3}$	12.5%	$0.\overline{3}$
$\frac{1}{10}$	$\frac{1}{2}$	1%	50%
0.1	$\frac{2}{3}$	10%	$33\frac{1}{3}\%$
$\frac{1}{5}$	$\frac{3}{4}$	$\frac{2}{10}$	$\frac{6}{8}$
$\frac{1}{4}$	0.75	$\frac{2}{8}$	75%

Why is this page blank?

So you can cut out the fractions, decimals, and percents on the other side.

Lesson 1 Assignment

Write

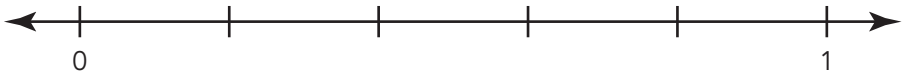
Define *percent* in your own words. Then, describe how to write fractions and decimals as percents.

Remember

A percent can be used to represent a part-to-whole relationship with a whole of 100. The symbol "%" means "out of 100."

Practice

1. Label each mark on the number line with a fraction, decimal, and percent. Make sure your fractions are in lowest terms.



Fraction	0	$\frac{1}{5}$	____	$\frac{3}{5}$	____	1
Decimal	0.0	____	0.4	____	____	1.0
Percent	0%	____	____	____	80%	100%

2. The table shows the portion of sixth-graders at your school who have a particular number of siblings. Complete the table by representing each portion as a part-to-whole ratio, a fraction, a decimal, and a percent. Make sure your ratios and fractions are in lowest terms.

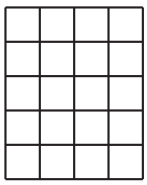
Number of Siblings	Ratio	Fraction	Decimal	Percent
0		$\frac{3}{20}$		
1				20%
2	3 : 8			
3			0.24	
4 or more		$\frac{7}{200}$		

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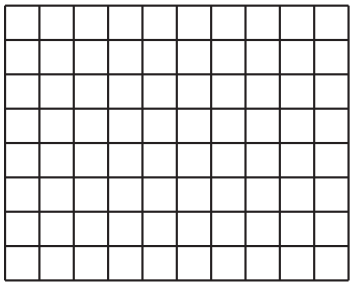
Lesson 1 Assignment

3. Shade the model to represent the given percentage.

   a. 50%



   b. 65%



4. Gabriel has $60.00 in his bank account. He spends $15.00 on a gift for a friend. What fraction, decimal, and percent of his money did Gabriel spend.

Lesson 1 Assignment

Prepare

1. $\frac{1}{10} \cdot 350$

2. $\frac{1}{100} \cdot 350$

3. $\frac{1}{10} \cdot 670$

4. $\frac{1}{100} \cdot 670$

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2 Using Estimation and Benchmark Percents

OBJECTIVES

- Order fractions, decimals, and percents.
- Estimate the percent of a quantity shaded in a model.
- Use benchmark percents to calculate common percents of quantities.
- Estimate percents using benchmarks.

NEW KEY TERM

- benchmark percents

You have used reasoning to calculate areas, volumes, decimal and fractional values, and equivalent ratios.

How can you use reasoning to solve percent problems?

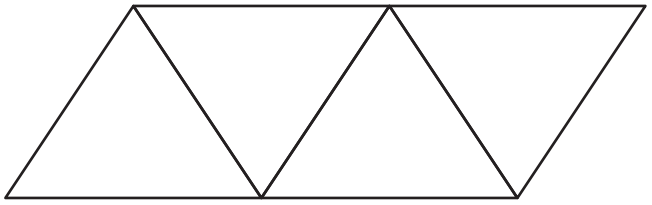
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Getting Started

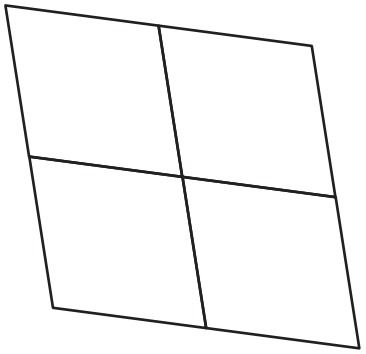
Representing Percents

1. Shade each image to represent the percent given.

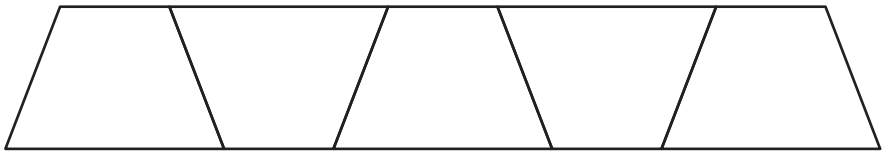
   a. 25%



   b. 50%



   c. 80%



ACTIVITY 2.1

Ordering Fractions, Decimals, and Percents

Each student has a note card that contains a number expressed as a fraction, decimal, or percent.

As a class, order the set of numbers from least to greatest.

1. Explain the strategies that your class used to order the numbers.

Think About . . .

All of the different ways to express your number.



Logan and Carlos received the numbers $0.0\overline{6}$ and 0.1% but they disagreed on which was larger. Logan says that $0.0\overline{6}$ is less than 0.1, so $0.0\overline{6}$ is less than 0.1%. Carlos says that since 0.1% is the same as 0.001 and 0.001 is less than $0.0\overline{6}$, 0.1% is less than $0.0\overline{6}$.

2. Who is correct? Explain your reasoning.

3. Order the numbers from least to greatest.

   0.99, $\frac{1}{9}$, $\frac{17}{20}$, 95%, 25%, $1\frac{1}{8}$, 70%, 4.3%, 1.12, 0.64, $\frac{11}{9}$

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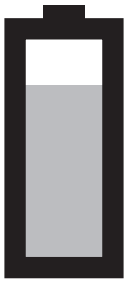
ACTIVITY
2.2 Estimating Percents from Pictures

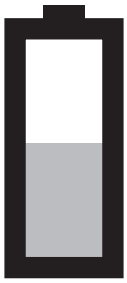
You know that 100% means one, or the whole, and 50% means half. You can estimate a lot of percents when using a visual model.

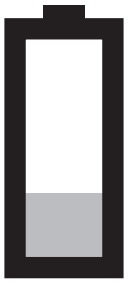
A laptop computer uses an icon of a battery on the toolbar to show how much power is left in the battery. When you glance at the icon, you can get a good estimate of how much battery life remains before you need to recharge the battery.

1. Estimate how much battery power remains by writing the percent under each battery icon.

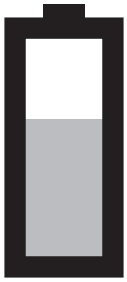


a. 

b. 

c. 

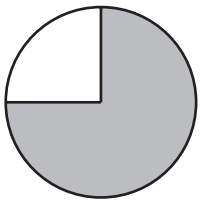
d. 

e. 

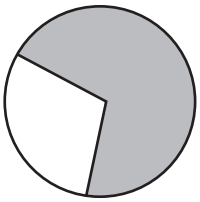
f. 

2. Estimate the shaded part of each circle shown, and write it as a percent.

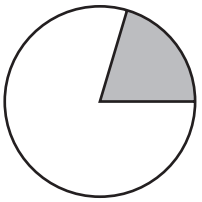
a. 

b. 

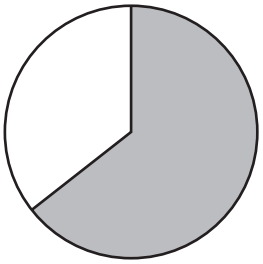
c. 

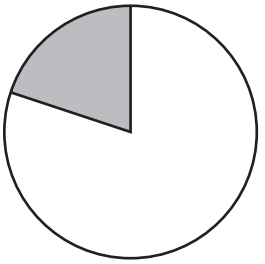
d. 

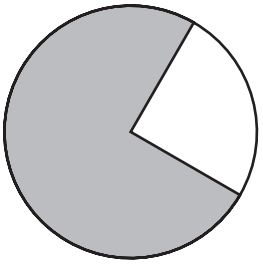
e. 

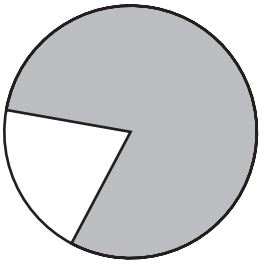
f. 

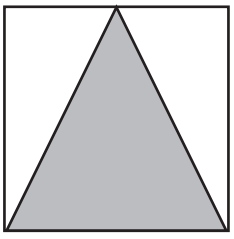
3. Estimate the shaded part of each model, and write it as a fraction, a decimal, and a percent. Write the fraction in lowest terms.

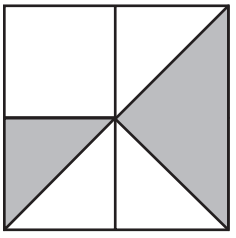
a. 

b. 

c. 

d. 

e. 

f. 



4. Describe the strategies that you used to make your estimations.

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ACTIVITY
2.3 Benchmark Percents

A benchmark percent is a percent that is commonly used, such as 1%, 5%, 10%, 25%, $33\frac{1}{3}$%, 50%, and 100%. With fractions and decimals you can use benchmarks to make estimations. With percents, however, you can use benchmarks to calculate any whole percent of a number.

100%

50%	50%

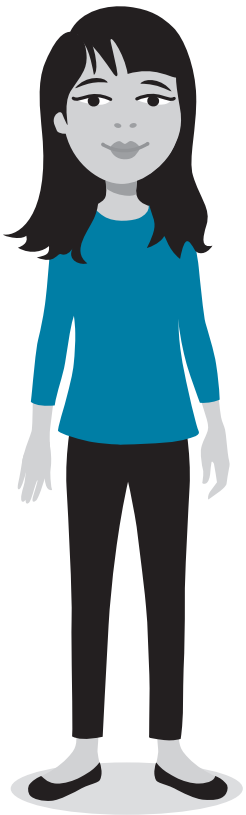
$33\frac{1}{3}$%	$33\frac{1}{3}$%	$33\frac{1}{3}$%

25%	25%	25%	25%

20%	20%	20%	20%	20%

10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

Remember, you worked with the benchmark fractions of 0, $\frac{1}{2}$, and 1.



1. Use the strip diagram to state each relationship.

   a. How is 50% related to 100%?

   b. How is $33\frac{1}{3}$% related to 100%?

   c. How is 25% related to 100%? How is 25% related to 50%?

   d. How is 10% related to 100%? How is 10% related to 50%?

2. Continue the pattern from the strip diagram to state each relationship.

Remember . . .
That 1% = 0.01.

a. How is 5% related to 10%?

b. How is 1% related to 10%? How is 1% related to 5%?

3. Use the benchmark percents to determine each value when 600 is 100%.

a. 50%

b. 25%

c. 10%

d. 5%

e. 1%

f. $33\frac{1}{3}\%$

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4. Use technology to determine the percent of each number.

a. 1% of 28 =

b. 10% of 28 =

c. 1% of 234 =

d. 10% of 234 =

e. 1% of 0.85 =

f. 10% of 0.85 =

g. 1% of 5.86 =

h. 10% of 5.86 =

i. 1% of 98.72 =

j. 10% of 98.72 =

k. 1% of 1085.2 =

l. 10% of 1085.2 =

5. What patterns do you notice in your answers in Question 4?

6. Write a rule to calculate 1% of any number.

7. Write a rule to calculate 10% of any number.

8. Use the patterns you recognized in Question 4 to calculate each value.

   a. 10% of 45.21

   b. 1% of 45.21

   c. 10% of 0.72

   d. 1% of 0.72

   e. 10% of 2854

   f. 1% of 2854

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ACTIVITY
2.4 Determining Percents using Benchmarks

Deciding how much tip to leave a server at a restaurant is one way that people use percents in the real world.



Diego eats at the Eat and Talk Restaurant and decides to leave a 15% tip. Diego says, "I can easily calculate 10% of any number, and then calculate half of that, which is equal to 5%. I can then add those two percent values together to get a sum of 15%."

1. Is Diego's method reasonable?

2. How much should he leave for a tip of 15% on $16.00?

3. What is 15% of each restaurant check total given? Explain how you calculated your answer. Round to the nearest hundredth when necessary.

   a. $24.00

   b. $32.56

   c. $47.00

You can determine any whole percent of a number by using 10%, 5%, and 1%.

4. How can you use 10%, 5%, and/or 1% to determine each percent given? Explain your reasoning.

   a. 18%

   b. 25%

   c. 37%

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5. Calculate each value using 1%, 5%, and 10%.

a. 27% of $84

b. 43% of $116

c. 98% of $389

d. 77% of $1400

e. 12% of $1248

PROBLEM SOLVING



6. About 12% of the United States population is left-handed. Use this estimate to determine about how many left-handed students there would be for each class of the given size.

a. 150 students

b. 200 students

c. 375 students

Talk the Talk

Brain Weights

A chimpanzee's brain weight can be compared to the brain weight of other mammals. Assume that the weight of an average chimpanzee's brain is 400 grams. The table provides the average brain weight of various mammals as a percent of a chimp's brain weight.

	Lion	Sheep	Cat	Rabbit	Bear
Average Brain Weight as a Percentage of a Chimp's Brain Weight	60%	35%	7%	2.5%	119%
Average Brain Weight (grams)					

1. Order from least to greatest the brain weights of the mammals in the table, along with the Chimpanzee, based on percents.

2. Use benchmarks to determine the average brain weights for each animal. Show your work.

3. Does the order of the percents match the order of the brain weights? Why or why not?

Lesson 2 Assignment

Write

Explain how to use *benchmark* percents to order and estimate the value of other percents.

Remember

Benchmarks percents—1%, 5%, 10%, 25%, $33\frac{1}{3}$%, 50% and 100%—can be used to perform mental estimation and calculation of percents. Values of benchmark percents can be added and subtracted to calculate the value of other percents.

Practice

The students at a middle school sold various products for a fall fundraiser. The table shows the percent of profit the school earned and the total amount sold for each type of product.

Product	Percent Profit	Amount Sold
Candy	65%	$6400
Wrapping paper	40%	$1200
Stationery	50%	$900
Calendars	25%	$3120

1. Use benchmark percents to calculate the amount of profit the school earned on the sale of each product.

   a. Candy

   b. Wrapping paper

   c. Stationary

   d. Calendars

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Lesson 2 Assignment

2. Suppose that the students also sold $4500 worth of pens and pencils, which earned a 42% profit. Calculate the profit the school earned on pens and pencils.

3. Order the numbers from least to greatest. $\frac{7}{8}$, 100%, 0.25, $\frac{2}{3}$, 0.75, 30%, $\frac{1}{3}$, 60%, 0.5, $\frac{5}{4}$

Prepare

Complete each equivalent fraction.

1. $\frac{6}{8} = \frac{\square}{12}$

2. $\frac{\square}{16} = \frac{15}{40}$

3. $\frac{6}{15} = \frac{\square}{10}$

3 Determining the Part and the Whole in Percent Problems

OBJECTIVES

- Use ratio and rate reasoning to solve percent problems involving determining the part, the percent, and the whole.
- Solve percent problems involving determining the part, given the whole and the percent.
- Use ratio reasoning to estimate the value of the whole in percent problems.
- Solve percent problems involving determining the whole, given the part and the percent.

You have learned how to use benchmarks to determine the percent of a number.

But what happens when you only know the part and the percent? How can you use your knowledge of percents to determine the whole amount?

Getting Started

The Big Picture

When you study problems in terms of ratios, like percents, it is important to think about the whole as well as the parts. The whole is not always 100 or 1. And when the whole changes, this changes the percent!



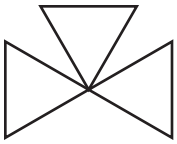
1. Consider the picture of triangles.

   a. When the picture shown is 30% of the triangles, draw 100% of the triangles.



   b. The given rectangle represents 25% of the whole figure. Draw a rectangle that represents 75% of the whole figure.

2. The figure shown represents 75% of the whole figure.



a. Draw 25% of the figure.

b. Draw 100% of the figure.

ACTIVITY
3.1 Calculating a Percent of a Quantity

Mr. Park, the sixth-grade math teacher, asked the class to determine 25% of 44. Five different student responses are shown.

Valentina



Since 25% of 44 means multiplying $\frac{25}{100}$ times the quantity, I used the fraction method.

$\frac{25}{100} = \frac{1}{4}$.

Then, I multiplied $\frac{1}{4} \cdot 44 = 11$.

Isaiah



I like decimals much better than fractions.

$\frac{25}{100} = 0.25$

$0.25 \cdot 44 = 11$

Lucas



25% is easy to do in my head. 50% of 44 is 22.

25% is $\frac{1}{2}$ of 50%, so 25% of 44 is $\frac{1}{2}$ of 22, which is equal to 11.

Avery



Since 25% is the same as $\frac{1}{4}$, I just divided by four.

$44 \div 4 = 11$

EMILY



I PREFER TO USE THE BENCHMARKS OF 10% AND 5%.

10% OF 44 = 4.4.

20% IS 2 • 10% = 2 • 4.4 = 8.8.

5% IS HALF OF 10% = 2.2.

THEREFORE, 20% + 5% = 8.8 + 2.2 = 11.0

1. Discuss each student method used.

   a. When is Valentina's method most efficient to use?

   b. When is Isaiah's method most efficient to use?

   c. When is Lucas's method most efficient to use?

   d. When is Avery's method most efficient to use?

   e. When is Emily's method most efficient to use?

A more efficient method is one that requires fewer steps or simpler steps to determine an answer.

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Sarah said, "All the methods are correct, and everyone got the correct answer, but what would have happened when Mr. Park gave us the problem 32% of 732?"

- Valentina said, "My fraction method is not as easy this time."
  $\frac{\cancel{32}^{8}}{\cancel{100}_{25}} \cdot \frac{732}{1} = \frac{5856}{25} = 234.24$
- Isaiah said,
  "$32\% = 0.32$
  $0.32 \cdot 732 = 234.24$
  My method is not any more difficult this time."
- Lucas said, "I can still estimate . . ., but my answer will be close, not exact. 32% is close to $\frac{1}{3}$, and $\frac{1}{3}$ of 732 is 244."
- Avery said, "I don't have an easy fraction to use for 32%, so my method works only for certain percents."
- Emily said, "I can still use my method."
  $32\% = 10\% + 10\% + 10\% + 1\% + 1\%$

  10% of $732 = 73.2$ 1% of $732 = 7.32$

  $73.2(3) = 219.6$ $7.32(2) = 14.64$

  $219.6 + 14.64 = 234.24$

2. Which method do you prefer with this particular percent of a quantity problem? Explain your thinking.

3. Determine the percent of each quantity.

a. 7% of 80

b. 75% of 240

c. 150% of 27

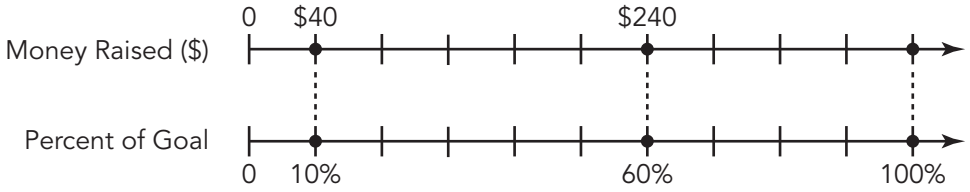
d. 12.5% of 64

ACTIVITY
3.2 Determining Parts and Wholes with Double Number Lines

Camilla is in charge of designing a way to keep a running total of the amount of money raised by her homeroom for the Food Bank project. As of today, her homeroom has raised $240, which is 60% of their goal.

WORKED EXAMPLE

Camilla decided to use a double number line to record the money raised and the percent of the goal raised.



The bottom number line represents the percent of the homeroom goal. The top number line represents the amount of money raised.

Camilla's homeroom has raised $240, which is 60% of the goal.

1. How did Camilla determine the value that corresponds to 10%?

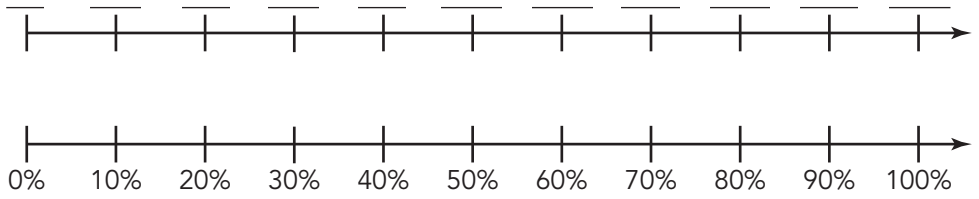
2. When $240 is 60% of the homeroom goal, what is 100% of the goal? Explain how you determined the answer.

3. Which way of reporting is more informative: the amount of money raised, or the percent of money raised? Explain your thinking.

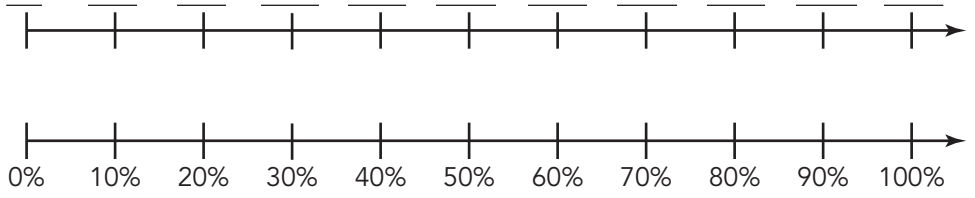
4. Complete each double number line to represent the goals of the other sixth-grade homerooms using the information from the table. Write the equivalent dollar amount for each percent shown.

Homeroom	6A	6B	6C
60% of Goal	240	144	
100% of Goal	400		360

a. Homeroom 6B



b. Homeroom 6C

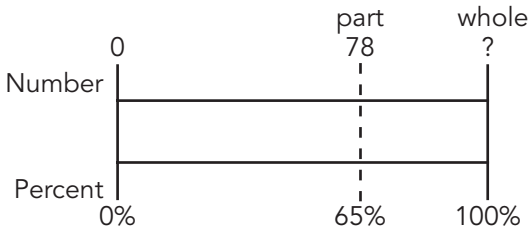


ACTIVITY
3.3 Wholes in Proportions

You can use proportions to determine the whole in percent problems.

WORKED EXAMPLE

Parker is told that 65% of the students, or 78 students, prefer pizza for lunch according to a recent survey. He wants to know how many students were surveyed. He drew the model shown to visualize the problem.



He then wrote the proportion and determined that 120 students were surveyed.

$$\frac{\text{part}}{\text{whole}} \quad \frac{78}{?} = \frac{65}{100}$$

$$\frac{65}{100} \overset{\div 5}{=} \frac{13}{20} \overset{\cdot 6}{=} \frac{78}{?}$$

$$\frac{78}{120} = \frac{65}{100}$$

Notice that 65 and 100 have a common factor of 5. When analyzing a ratio, always look for what the numerator and denominator have in common.

1. How did Parker determine the total number? Explain Parker's calculations.

Diego

Instead of scaling down and then scaling up, it is more efficient to scale up from the start. I can use the structure of this proportion to determine the scale factor.

Since $78 \div 65 = 1.2$, I can scale up in one step by multiplying by 1.2.

$$\frac{78}{?} \overset{\cdot 1.2}{=} \frac{65}{100}$$

$$100 \cdot 1.2 = 120$$

$$\frac{78}{120} = \frac{65}{100}$$

2. Use Diego's method to determine the unknown value.

$$\frac{45}{100} = \frac{126}{?}$$

3. Determine the whole in each situation. Explain your reasoning.

   a. You got a quiz back and your teacher wrote +16, and 80% at the top. How many points was the quiz worth?

   c. Jasmine made a 30% deposit on the purchase of a computer. She gave the clerk $168. What is the price of the computer?

ACTIVITY
3.4 Parts and Wholes in Problems

An accountant is reviewing a store's financial statements. But some of the information is unknown. All of the employee names, their bonus percent, and bonus amounts should be listed in the table.

Employee Name	Bonus Percent	Bonus Amount	Total Sales
Luna	18%	$540	
Nahimana	21%		$3657.14
Kayla	15%	$650	

1. Help the accountant by determining each employee's bonus amount or total sales. Complete the table and show your work.



2. Mason liked Isaiah's method for calculating percents of a number and thought it would work for calculating the whole. He said that when you can multiply to determine the part of a whole, maybe you can divide to determine the whole when you know only the part.

   He gave this example:

Percent	Whole	Part	Unknown Value
20% of	$1500 is	_____	0.2×1500
20% of	_____ is	$300	$300 \div 0.2$

   Is Mason correct? Does this always work? Explain your thinking.

3. Determine each value.

   a. 15 is 25% of what number?

   b. 15 is 30% of what number?

4. A department store recently had a big sale where the sales team marked the prices of items 25% off of the regular price. Now that the sale is over, Isaiah needs to mark each of the items back up to its original price. The table lists the items and their sale prices. Help Isaiah complete the table.

Item	Sale Price	Original Price
Shirt	$24.00	
Sweater	$59.95	

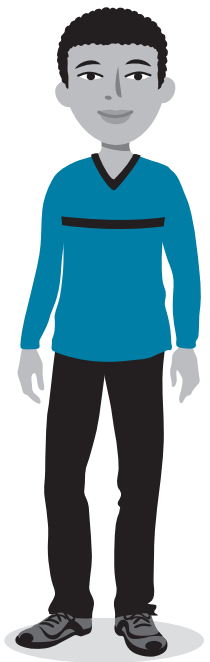
5. The department store realizes it isn't making enough money. The store manager decides to mark up prices by 20%.

   Complete the table with the new price for each item.

Item	Original Price	New Price
Pants	$29.00	
Shoes	$65.00	

Ask Yourself . . .
How can you use percents in everyday life?

25% OFF the regular price means 75% OF the regular price!



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PROBLEM SOLVING



6. The department store orders toasters from a company that produces three different models of toasters. The company has found that the percent of each shipment that is defective differs by model. Model A's defect rate is 2.5%, Model B's defect rate is 1.75%, and Model C's defect rate is 3.2%.

   On a particular shipment, the company forgets to mark the total number shipped of each model. You only know that you received 5 defective Model A toasters, 7 defective Model B toasters, and 16 defective Model C toasters. How many of each model were shipped?

ACTIVITY
3.5 Parts and Wholes in Geometry

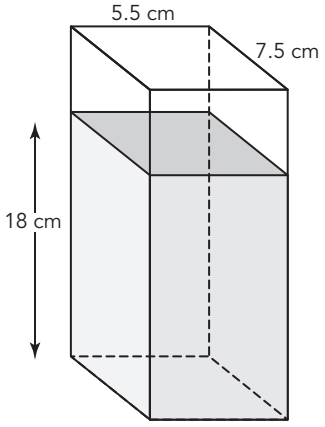


You can apply what you have learned about parts, wholes, percents, and ratio reasoning to solve percent problems in geometry too.

1. Adriana's new dog pen is a rectangular pen that measures 12 yards by 4 yards. She reduced the area of her old rectangular dog pen by 60% after adopting out 6 puppies. List some possible dimensions of Adriana's old dog pen. Explain your reasoning.

2. The tank shown is 75% full of water.

   a. What is the height of the tank? Explain how you solved the problem.

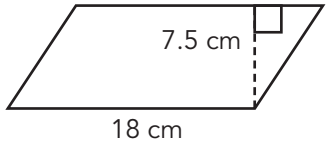


   b. Suppose the outside of the tank is covered with paper only up to the water level. What percent of the total surface area of the tank would be covered? Round to the nearest whole percent. Be sure to include the top of the tank in the total.

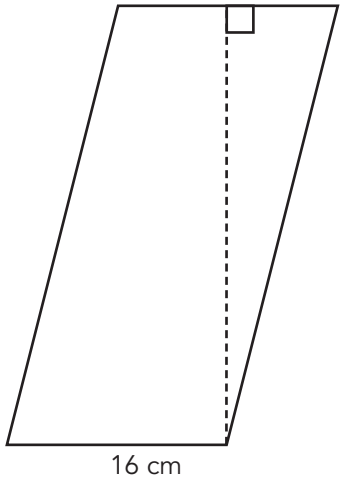
The ***surface area*** of a prism or pyramid is the total area of all its two-dimensional faces.

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3. The area of Parallelogram A is 25% of the area of Parallelogram B. What is the height of Parallelogram B? Show your work.



Parallelogram A



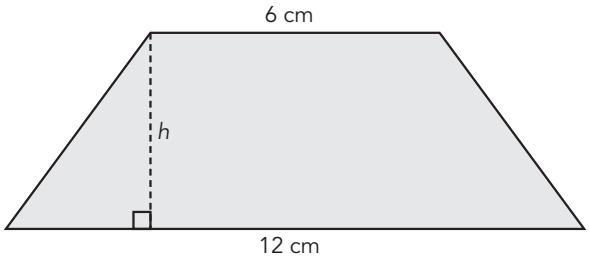
Parallelogram B

Remember . . .

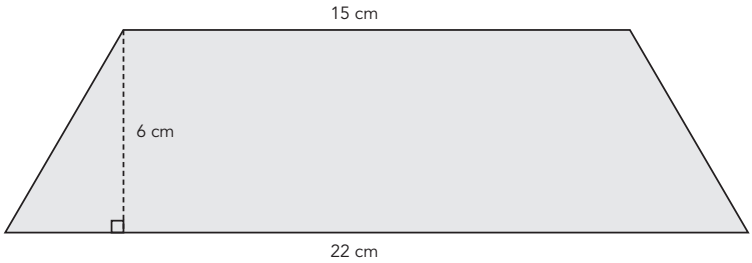
The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$.

4. Trapezoid A is 33% of the area of Trapezoid B. Determine the height of Trapezoid A.

Trapezoid A:



Trapezoid B:



Talk the Talk

Try and Try Again



For each question, demonstrate two different ways to determine the answer.

1. Angelina's goal is to score a 90% on the upcoming science test. When there are 40 questions on the test, how many does Angelina need to answer correctly?

2. Today, Avery made 40% of the burritos sold during the lunch rush at work. He made 28 burritos during this time, what is the total number of burritos made during the lunch rush?

Lesson 3 Assignment

Write

Compare different ways to determine the whole in a percent problem: using double number lines, writing a proportion, and using division.

Remember

Percent problems often have a part, a percent, and a whole. When you know the part and the percent, you can use a variety of strategies to determine the whole.

Practice

1. A manager at the department store keeps track of "points" for each employee. Employees earn points by being on time for work and for keeping the department neat. On a particular day, he gives "smile" points for each time an employee smiles at a customer. He recorded the smile points that each employee received, along with the total number of points for that employee. He had a problem with his computer, though, and some of the entries were deleted. Help the manager complete the table.

Employee Name	Smile Points	Percent of Total Points	Total Number of Points
Sebastian	15	5%	
Diego		8%	325
Logan	6	2%	
Mei	21	6%	
Sarah	45	12%	

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Lesson 3 Assignment

2. The Music Department of a department store sold 12 jazz CDs last month. Jazz sales during that month made up 2% of the Music Department's total amount of sales.

   a. Determine the number of CDs that the store sold during that month.

   b. Suppose that the store sells 14 jazz CDs during the next month and the percent of sales from jazz CDs is still 2%. What is the total number of CDs that the store will sell?

3. Calculate each value.

   a. 12 is 20% of what number?

   b. 28 is 35% of what number?

   c. 84 is 42% of what number?

   d. 32 is 80% of what number?

   e. 35% of 60 is what number?

   f. 25% of 132 is what number?

   g. 5% of 40 is what number?

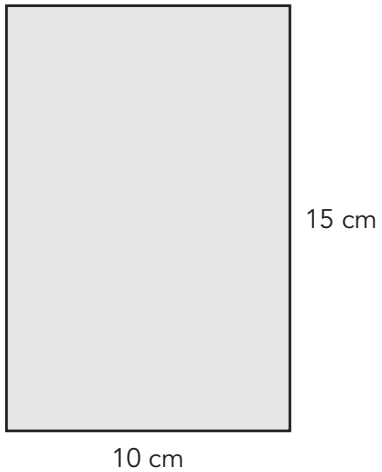
   h. 15% of 80 is what number?

4. The area of a rectangle A is 30% of the area of rectangle B. What is the height of rectangle A?

Rectangle A



Rectangle B



Lesson 3 Assignment

Prepare

Complete each common measurement conversion.

1. 1 foot = _____ inches
2. 1 kilogram = _____ grams
3. 1 meter = _____ centimeters
4. 1 gallon = _____ quarts
5. Which of the previous questions include units that are part of the U.S. customary system of measurement, and which include units that are part of the metric system?

Complete each equivalent fraction.

1. $\frac{6}{8} = \frac{\square}{12}$
2. $\frac{\square}{16} = \frac{15}{40}$
3. $\frac{6}{15} = \frac{\square}{10}$

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TOPIC 2 SELF-REFLECTION

Name: _____

Percents

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Percents* topic by:

TOPIC 2: *Percents*	Beginning of Topic	Middle of Topic	End of Topic
writing a percent as a ratio of a number to 100.	○	○	○
writing equivalent fractions, decimals and percents.	○	○	○
modeling percents as rates per 100 on a hundredths grid.	○	○	○
determining a percent of a number as a rate per 100.	○	○	○
ordering fractions, decimals and percents.	○	○	○
using benchmark percents to calculate common percents of quantities.	○	○	○
applying ratio reasoning to solve mathematical and real-world problems to determine the percent of a number or the whole.	○	○	○
solving real-world problems that involve finding the whole, given a part and a percent.	○	○	○

continued on the next page

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TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Percents* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?



TOPIC 2 SUMMARY

Percents Summary

NEW KEY TERM
- benchmark percent

LESSON 1 Percent, Fraction, and Decimal Equivalence

Percent can be used to represent a part-to-whole relationship with a whole of 100. The symbol "%" means "out of 100." Think of a percent as a fraction in which the denominator is 100.

You can use percents, fractions, and decimals interchangeably.

For example, you can write 15 out of 100 as the fraction $\frac{15}{100}$, or $\frac{3}{20}$. Written as a decimal, 15 out of 100 is 0.15. Because percent means "out of 100," 15 out of 100 is equivalent to 15%.

When the denominator is a factor of 100, scale up the fraction to write it as a percent.

$$\frac{4}{5} \xrightarrow{\cdot 20} \frac{80}{100}$$

$$\frac{4}{5} = \frac{80}{100} = 80\%$$

When the denominator is not a factor of 100, you can divide the numerator by the denominator to write the fraction as a decimal, which you can then write as a percent.

$\frac{3}{8} = 3 \div 8$

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -\ 24 \\ \hline 60 \\ -\ 56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$$\frac{3}{8} = 0.375 = 37.5\%$$

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Common Equivalent Fractions, Decimals, and Percents									
Fraction	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
Decimal	0.2	0.25	$0.\overline{3}$	0.4	0.5	0.6	$0.\overline{6}$	0.75	0.8
Percent	20%	25%	$33\frac{1}{3}\%$	40%	50%	60%	$66\frac{2}{3}\%$	75%	80%

LESSON 2

Using Estimation and Benchmark Percents

When ordering numbers expressed as fractions, decimals, and percents, you can first write the numbers in the same form before comparing.

For example, to order the numbers 0.88, 90%, and $\frac{17}{20}$ from least to greatest, you can write each number as a percent.

$0.88 = \frac{88}{100} = 88\%$ $\qquad$ $\frac{17}{20} = \frac{85}{100} = 85\%$

The numbers in order from least to greatest are $\frac{17}{20}$, 0.88, and 90%.

You can estimate percents when using a visual model.



For example, the shaded part appears to be about $\frac{1}{3}$ of the whole circle, and $\frac{1}{3} \approx 33\%$.

A **benchmark percent** is a percent that is commonly used, such as 1%, 5%, 10%, 25%, $33\frac{1}{3}\%$, 50%, and 100%. With fractions and decimals, benchmarks can be used to make estimations. With percents, however, you can use benchmarks to calculate any whole percent of a number. For example, determine each value if 400 is 100%. There is more than one way to use benchmark percents to determine the values.

You can determine any whole percent of a number by using 10%, 5%, and 1%.

For example, what is 28% of 500?

28% = 10% + 10% + 5% + 1% + 1% + 1%

10% of 500 is $500 \cdot \frac{1}{10}$, or 50.

5% of 500 is $50 \cdot \frac{1}{2}$, or 25.

1% of 500 is $25 \cdot \frac{1}{5}$, or 5.

50 + 50 + 25 + 5 + 5 + 5 = 140

28% of 500 is 140.

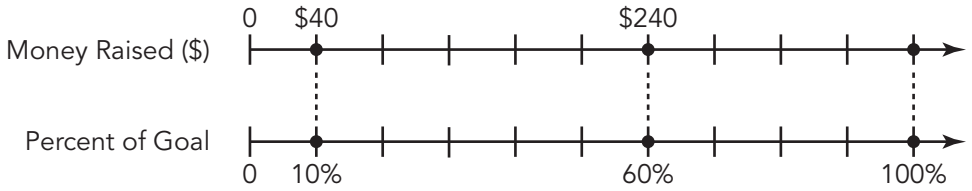
a. 50%	50% is half of 100%. $400 \cdot \frac{1}{2} = 200$
b. 25%	25% is half of 50%. $200 \cdot \frac{1}{2} = 100$
c. 10%	10% is one-fifth of 50%. $200 \cdot \frac{1}{5} = 40$
d. 5%	5% is half of 10%. $40 \cdot \frac{1}{2} = 20$
e. 1%	1% is one-fifth of 5%. $20 \cdot \frac{1}{5} = 4$

LESSON 3 Determining the Part and the Whole in Percent Problems

Percent problems often have a part, a percent, and a whole. When you know the part and the percent, you can use a variety of strategies to determine the whole.

One strategy is a double number line.

For example, Camilla's homeroom raised $240 for charity, which is 60% of its goal. Camilla uses a double number line to record the amount of money raised and the percent of the goal raised.



Camilla's homeroom has raised $240, which is 60% of the goal.

To determine the value that corresponds to 10%, Camilla divided the amount raised so far by 6: $240 ÷ 6 = $40.

Since 10% · 10 = 100%, she can multiply $40 by 10 to determine the homeroom's goal: $40 · 10 = $400.

You can also use proportions to determine the whole in percent problems.

For example, Parker is told that 65% of the students, or 78 students, prefer pizza for lunch according to a recent survey. He wants to know how many students were surveyed.

He wrote a proportion and determined that 120 students were surveyed.

$$\frac{\text{part}}{\text{whole}} \qquad \frac{78}{?} = \frac{65}{100}$$

$$\frac{65}{100} \overset{\div 5}{=} \frac{13}{20} \overset{\cdot 6}{=} \frac{78}{?}$$

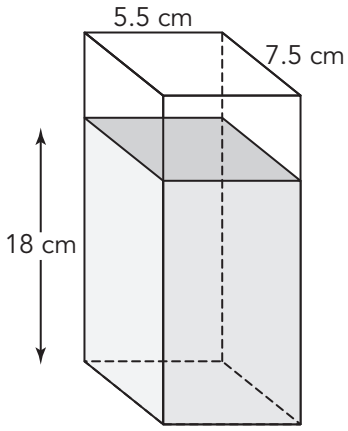
$$\frac{78}{120} = \frac{65}{100}$$

You can use these strategies to solve geometry problems as well.

For example, the tank shown is 75% full of water. What is the height of the tank?

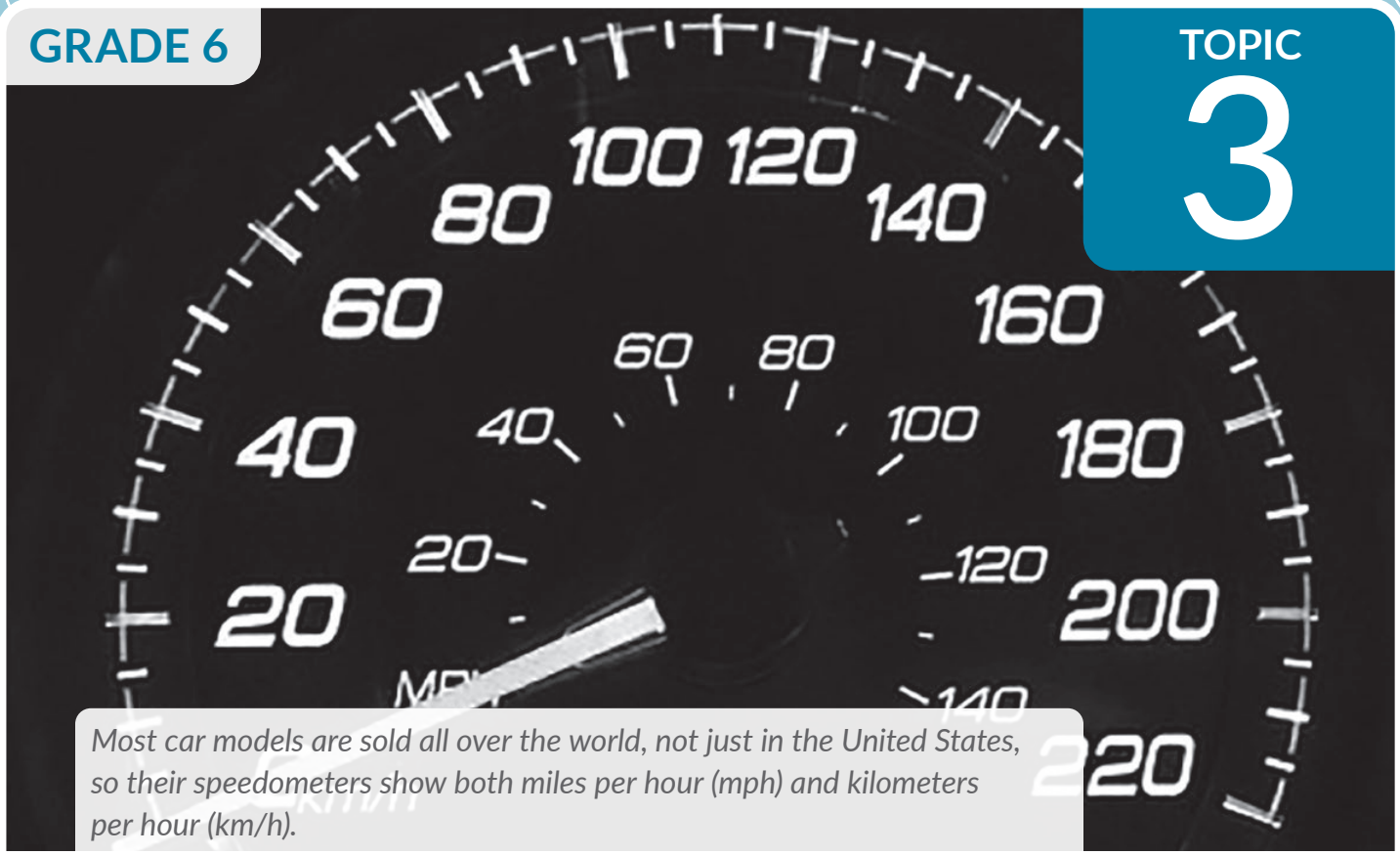
You can set up a proportion to solve for the height of the water tank. Since 18 cm represents the height of the tank when it is 75% full of water, you can determine the height of the tank in centimeters when it is 100% full of water.

The height of the tank is 24 centimeters.



GRADE 6

TOPIC 3



Most car models are sold all over the world, not just in the United States, so their speedometers show both miles per hour (mph) and kilometers per hour (km/h).

Unit Rates and Conversions

LESSON 1 Using Ratio Reasoning to Convert Units 393

LESSON 2 Introduction to Unit Rates 415

LESSON 3 Multiple Representations of Unit Rates. 433

1 Using Ratio Reasoning to Convert Units

OBJECTIVES

- Use proportional reasoning with double number lines to convert measurement units.
- Use ratio reasoning with ratio tables to convert measurement units.
- Use scaling up or scaling down to convert and transform measurement units appropriately.
- Use unit analysis to convert and transform measurement units appropriately.

NEW KEY TERM

- convert

In previous grades, you have worked with the U.S. customary system and the metric system of measurement. This year, you have also learned about ratios.

How can you use ratio reasoning to convert from one measurement unit to another in order to solve problems?

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Customary to Whom?

In the U.S., customary units are primarily used for business, personal, and social purposes. Sciences, including the medical field, use the metric system.

You've learned about the relationships between inches and feet, feet and yards, quarts and gallons, meters and millimeters—to name a few.

1. Name a U.S. customary system unit and a metric system unit that would be an appropriate size to measure each object or quantity.

Object/Quantity	U.S. Customary System	Metric System
Your height		
Length of your pencil		
Distance from your school to the beach		
Weight of your math book		
Amount of water in a bottle		
Amount of water in a swimming pool		

2. Circle the most appropriate measurement for each item.

a. The weight of a dog
- 15 pounds
- 18 ounces
- 1 ton
- 25 fluid ounces

b. The amount of gas in a car's tank
- 50 milliliters
- 2 kiloliters
- 55 liters
- 12 kiloliters

c. The height of your classroom
- 90 inches
- 1 mile
- 2 yards
- 12 feet

d. The height of a basketball hoop
- 3 meters
- 70 centimeters
- 500 millimeters
- 1 kilometer

ACTIVITY
1.1 Reasoning About Unit Conversions

You can use more than one measurement to describe the same length, weight, or capacity. For example, you may say that the length of a football field is 100 yards or 300 feet. You could also say that the length of the football field is about 90 meters. In each case, the lengths are the same—you just say them in different ways.

There are many situations in which you need to *convert* measurements to different units. To **convert** a measurement means to change it to an equivalent measurement in different units.

When you convert a measurement to a different unit, the size of the object does not change; only the units and the number of those units change.

1. Name a situation in which converting one measurement to another would be necessary or useful.

Use your knowledge of metric and U.S. customary measures to answer each question.

2. The numeric value of which measurement will be greater?

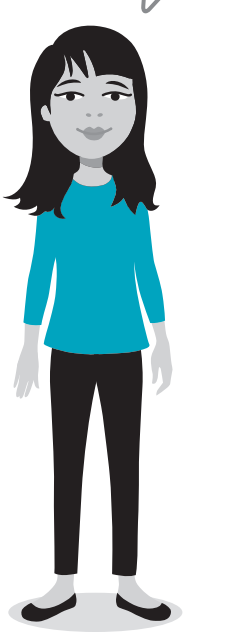
   a. The length of a table in inches or in feet

   b. The length of a table in meters or in centimeters

   c. The length of a table in inches or in yards

   d. The distance from school to your house in meters or in kilometers

   e. The weight of your math book in ounces or in pounds



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3. How did you decide which value would be greater in Question 2?

Because most conversions compare two quantities using multiplicative strategies, the conversions within systems that you already know can be written using ratio language. They can also be written symbolically in terms of equality.

Ratio Language	Symbolically
For every 12 inches, there is exactly 1 foot.	12 in. = 1 ft
For every yard, there are 36 inches.	1 yd = 36 in.
For every mile, there are 1760 yards.	1 mi = 1760 yds
For every meter, there are 1000 millimeters.	1 m = 1000 mm
For every 1 kilometer, there are 1000 meters.	1 km = 1000 m

When a conversion ratio is presented for use in converting between units of measure, it is often written as an equation: 12 in. = 1 ft. However, it can also be written as a ratio in fractional form: $\frac{12 \text{ in.}}{1 \text{ ft}}$.

4. Rewrite each common conversion using ratio language and as a ratio in fractional form.

Because conversions compare two quantities that are measured in different units, conversion ratios can also be called conversion rates.

a. 3 ft = 1 yd

b. 5280 ft = 1 mi

c. 1 g = $\frac{1}{1000}$kg

d. 4 qt = 1 gal

e. $1 \text{ m} = 100 \text{ cm}$

f. $\frac{1}{1000} \text{ m} = 1 \text{ mm}$

Because these measurement conversions are ratios, you can use ratio reasoning to convert between units. For example, you can determine the number of millimeters in a 10-kilometer race or the number of fluid ounces in 500 gallons of a solution.

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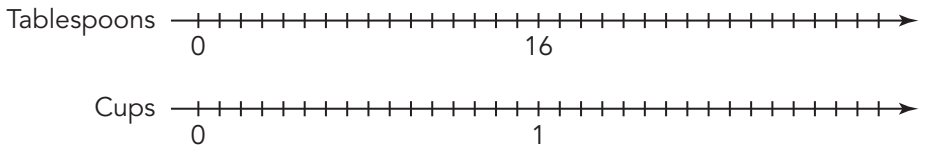
ACTIVITY
1.2 Using Double Number Lines to Convert Units

When you learned about ratios, you learned how to use proportional reasoning and double number lines to determine equivalent ratios. You can also use double number lines to convert from one unit to another.

You are baking cookies at your friend's house. After searching the cupboards and drawers, you cannot locate the measuring cups, but you can locate the tablespoon.

1. Use proportional reasoning and the double number line to determine how many tablespoons you need of each ingredient in the recipe.





a. 2 cups of sugar

b. $1\frac{3}{4}$ cups of flour

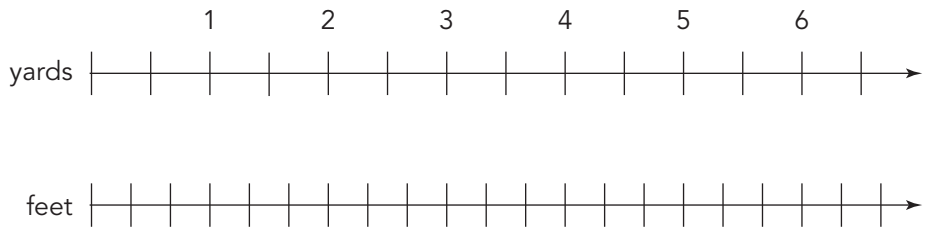
c. $\frac{1}{2}$ cup of raisins

2. Suppose you had found the cup but not the tablespoon. Use the double number line to determine how many cups you need when the recipe calls for 2 tablespoons of vanilla extract.

You want to redecorate your bedroom and need to measure the room for new carpeting, paint, and a border on the walls. You realize that you don't have a ruler, but you have a stick that you know has the length of 1 yard. You measure the room using the stick, but you need to know the dimensions in feet to purchase the materials. You record these measurements:

- The length of the room is 5 stick lengths.
- The width of the room is 4 stick lengths.
- The height of the room is 2.5 stick lengths.

3. Use a double number line to determine the measurement of each dimension in feet.



a. length

b. width

c. height

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ACTIVITY
1.3 Using Ratio Tables and Scaling to Convert Units

You can use ratio tables, like you did when determining equivalent ratios, as another strategy for converting units.

1. Complete the ratio table by converting between pounds and ounces.

Pounds	1	2		$1\frac{1}{4}$	$\frac{1}{2}$		
Ounces	16		4			6	40

2. What strategies did you use to determine the unknown values?

3. Complete the ratio table by converting between milliliters and liters.

Milliliters	1000	100		50	1		575
Liters	1		0.5			0.01	

4. What strategies did you use to determine the unknown values?

Ratio tables are helpful tools for converting within a given system of measurement. Scaling up or down is a similar strategy for determining equivalent ratios that you can more easily use to convert from one unit of measurement to another.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

WORKED EXAMPLE

You can use scaling up to determine how many ounces are in 2.5 pounds. Because you want to determine the number of ounces for a specific number of pounds, use the conversion rate 1 lb = 16 oz or $\frac{1 \text{ lb}}{16 \text{ oz}}$.

$$\frac{1 \text{ lb}}{16 \text{ oz}} = \frac{2.5 \text{ lb}}{? \text{ oz}} \longrightarrow \frac{1 \text{ lb}}{16 \text{ oz}} \overset{\cdot 2.5}{=} \frac{2.5 \text{ lb}}{40 \text{ oz}}$$

Use scaling up or down to answer each question.

5. The school cafeteria has eight very large cans of tomato sauce for making pizza. Each can contains 2 gallons of sauce. Is there more or less than 200 cups of sauce in these 8 cans?

6. Lucas, the quarterback for the Tigers Football team, can throw a football 40 meters. Mason, the quarterback for the Spartans Football team, can throw a football 0.06 kilometer. Who can throw farther? How do you know?



7. Luna says she is 1.5 yards tall. Nahimana is 5 feet tall. Luna says that she is taller, but Nahimana disagrees. Who is correct? Explain your reasoning.

8. Sebastian weighs 110 pounds, Gabriel weighs 98 pounds, Diego weighs 1481.5 ounces, and Logan weighs 1834.25 ounces. Place the dogs in order from the least weight to the greatest weight.

9. Kayla has a gold bracelet that weighs 24 grams. She wants to sell the bracelet, but she needs a minimum of 2000 milligrams of gold to sell it. Can Kayla sell her bracelet? Why or why not?

ACTIVITY 1.4

Using Unit Analysis to Convert Units

To use scaling up or down to convert one unit to another, you set up a proportion and use the conversion rate based on the given measurement that you are converting. In another strategy, *unit analysis*, you are multiplying by a form of 1 to rewrite the given measurement in a different unit. Sometimes, you may need to do this more than once to obtain the desired unit of measurement.

WORKED EXAMPLE

Determine the quantity in grams that is equivalent to 4.5 kilograms.

Scaling Up

$$\frac{1\text{ kg}}{1000\text{ g}} = \frac{4.5\text{ kg}}{?\text{ g}}$$

$\cdot 4.5$ (above)

$\cdot 4.5$ (below)

$$\frac{1\text{ kg}}{1000\text{ g}} = \frac{4.5\text{ kg}}{4500\text{ g}}$$

Unit Analysis

$$4.5\text{ kg}\left(\frac{1000\text{ g}}{1\text{ kg}}\right)$$

$$\frac{4.5\ \cancel{\text{kg}}}{1}\left(\frac{1000\text{ g}}{1\ \cancel{\text{kg}}}\right) = 4500\text{ g}$$

$$4.5\text{ kg} = 4500\text{ g}$$

In unit analysis, note how the units are carried through all calculations. Units are divided out in the same way that factors can be divided out.

$\cancel{\text{given unit}} \times \frac{\text{desired unit}}{\cancel{\text{given unit}}} = \text{desired unit}$

1. Analyze the Worked Example.

   a. Both strategies used a form of 1 to determine the equivalent number of grams in 4.5 kilograms. How is the form of 1 used in scaling up different from the form of 1 used in unit analysis?

   b. Why are the labels for kilograms crossed out in the unit analysis strategy?

Review the definition of **analyze** in the Academic Glossary.

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Ask Yourself . . .
Can others follow and understand your process and reasoning?

Avery and Carlos want to determine the number of miles in 31,680 feet using unit analysis.

Avery

$$31{,}680\ \text{ft}\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right) = 167{,}270{,}400\ \text{mi}$$

Carlos

$$31{,}680\ \text{ft}\ \frac{1\ \text{mi}}{5280\ \text{ft}} = 6\ \text{mi}$$

PROBLEM SOLVING



2. Explain why Avery's answer is not reasonable.

3. Explain what is different in how Avery and Carlos set up their multiplication problem. What is important about how the units are arranged in the conversion rates?

Use unit analysis to convert each unit of measurement. Check to make sure your answer is reasonable.

4. A giraffe is 18 feet tall. How tall is the giraffe in inches?

5. Many popular ice cream companies sell their product by both the gallon and the pint. When you buy 4 pints of ice cream, how many gallons have you purchased?

6. When the average adult hand is spread wide, the span from the tip of the thumb to the tip of the pinky is 22 centimeters. What is the measurement in millimeters?

A marathon is a long-distance foot race with an official distance of 42.195 kilometers that is usually run as a road race. Larger marathons can have tens of thousands of runners. Most of these marathon runners are not professional marathoners but run to raise funds for various charities.

7. Although a marathon is a popular distance for a race, there are many other distances in which runners can train to race. Complete the table shown by writing the unknown measurements.

Race	Kilometers	Meters
Short Distance	5	
Medium Distance	20	
Half Marathon		21,091
Ultramarathon	100	
Ironman Triathlon Swim		3864
Ironman Triathlon Bike		180,320

8. The length of the school playground is 32 yards 3 feet. What is the length of the playground in feet?

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Ask Yourself . . .
How can I convert each measurement to the desired units?

9. A giraffe is 4 meters 42 centimeters tall. How tall is the giraffe in millimeters?

10. A new community park stretches across 1 mile 880 yards. How many feet across is the park?

Conversion rates are also common in other contexts, like currency. During the 2016 Summer Olympics, the currency exchange rate between the U.S. dollar and the Brazilian real (pronounced "ray-all") was $1 US for every 3.17 BRL.

11. Adriana's family went to the Rio de Janeiro Olympics and she budgeted $500 to spend while she was gone.

    a. Write the conversion rate: _____ US = _____ BRL.

    b. Did Adriana budget more or less than 500 BRL? Explain.

    c. How many BRL could she spend in Rio de Janeiro?

    d. After Rio de Janeiro, Adriana's family traveled to Mexico, where 1 BRL was equal to 5.92 pesos. When Adriana had 295 BRL remaining, how many pesos did she have?

12. Angelina is preparing to re-carpet her room. She measured the room to have a length of 6 yards and a width of 8 yards. When she got to the carpet store, all of the measurements were in square feet.

    a. Determine how many square yards of carpet Angelina needs to buy to re-carpet her room.

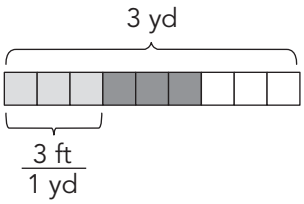
    b. Determine how many square feet of carpet Angelina needs to buy to re-carpet her room. How can you check your answer?

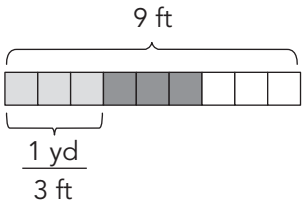


Talk the Talk

Larger or Smaller?

1. Compare the two conversions. How are they similar? How are they different?





2. When you convert a measurement with smaller units to a measurement with larger units, does the number of units increase or decrease?

3. When you convert a measurement with larger units to a measurement with smaller units, does the number of units increase or decrease?

4. What information is always needed to convert between measurement units?

For each conversion, explain which strategy you prefer to use and then convert the units.

5. 12 gal = _____ qt

6. 240 oz = _____ lb

7. 3 m 200 cm = _____ mm

8. 324 in. = _____ yd

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Lesson 1 Assignment

Write

Explain how to convert from one unit to another using ratio reasoning.

Remember

More than one unit can be used to describe the same length, weight, or capacity. To convert units means to change a measurement to an equivalent measurement in different units.

You can use models, ratio reasoning, and unit analysis to convert units using conversion rates.

Practice

Complete each customary measurement conversion.

1. There are 12 inches in 1 foot. How many feet are in 174 inches?

2. There are 3 feet in 1 yard. How many yards are in 12 feet?

Complete each metric measurement conversion.

3. There are 10 centimeters in 1 decimeter. How many decimeters are in 37 centimeters?

4. There are 100 meters in 1 hectometer. How many hectometers are in 201 meters?

Multiply by a unit rate to determine each measurement conversion using unit analysis.

5. How many pounds are in 480 ounces?

6. How many gallons are in 200 quarts?

Lesson 1 Assignment

7. How many ounces are in 1 ton?

8. How many yards are in 2 miles?

Use any strategy to convert between the specified units.

9. Mei will be traveling to Botswana, where the unit of currency is called the pula, which means "rain" in the local language. Suppose that $1 is equivalent to 7 pula.

   a. When Mei has $500 to spend in Botswana, how many pula will she have to spend?

   b. The safari lodge where she is staying in Chobe National Park costs 434 pula each night. What is the cost per night in dollars?

   c. When she goes to dinner at the safari lodge, the bill comes to 91 pula. How many dollars did Mei spend on dinner?

10. Isaiah is going to the hardware store for his Uncle Parker. He needs to buy 4 yards 1 foot of electrical wire and 14 quarts of liquid nails.

    a. The store only sells wire by the foot. How many feet does Isaiah need?

    b. The store only sells liquid nails by the gallon. How many gallons does Isaiah need?

11. Valentina is volunteering at a zoo and is helping weigh a penguin's egg. The egg weighs 0.15 kilogram.

    a. Is this more or less than the average weight of 145 grams? Explain.

    b. When Valentina expands the penguin area to be about 500 meters wider than it is now, how many more kilometers wide is the area?

Lesson 1 Assignment

12. There are 1000 millimeters in 1 meter and 10 millimeters in 1 centimeter. Convert 6 meters and 20 centimeters to millimeters.

Prepare

Determine each unknown quantity.

1. $\frac{18 \text{ arrows}}{3 \text{ bows}} = \frac{162 \text{ arrows}}{? \text{ bows}}$

2. $\frac{18 \text{ arrows}}{3 \text{ bows}} = \frac{? \text{ arrows}}{1 \text{ bow}}$

3. $\frac{8 \text{ shoes}}{80 \text{ socks}} = \frac{? \text{ shoes}}{1600 \text{ socks}}$

4. $\frac{8 \text{ shoes}}{80 \text{ socks}} = \frac{1 \text{ shoe}}{? \text{ socks}}$

2 Introduction to Unit Rates

OBJECTIVES

- Write unit rates.
- Use unit rates to solve problems involving unit pricing and better buys.
- Use unit rates and unit rate language to make comparisons.
- Use unit rates to solve problems involving constant speeds.
- Calculate unit rates.

NEW KEY TERM

- unit rate

Ratios and rates are useful in a variety of real-world situations. Most of your previous work with ratios and proportions involved writing equivalent ratios and rates, but ratios, specifically unit rates, can be used to answer many different types of questions.

How can you use unit rates in comparisons and to determine which deal is a better buy?

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Getting Started

Which One Would You Buy?

Emily and Parker go to the store to buy some laundry detergent for a neighbor. They see that the brand she wants comes in two different sizes:

26 fluid ounces for $9.75

20.5 fluid ounces for $7.25

1. Which size should Emily and Parker buy? Explain the reason for your decision.

ACTIVITY 2.1

Using Models to Estimate Unit Rates

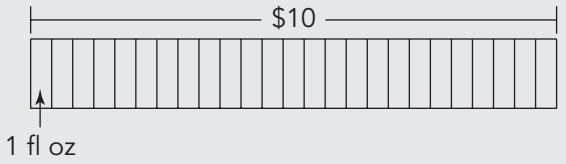
As you learned previously, a rate is a ratio in which the two quantities being compared are measured in different units.
A **unit rate** is a comparison of two measurements in which the numerator or denominator has a value of one unit.

One way to compare the values of items is to calculate the unit rate for each item.

Emily estimated unit rates for two detergents this way:

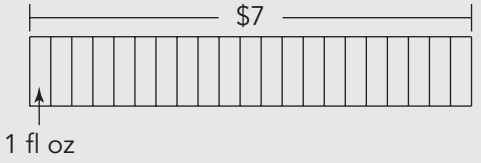
Emily

The larger bottle of detergent is about 25 fluid ounces for about $10.



So, each fluid ounce costs about $\frac{\$10}{25 \text{ fl oz}}$, which is $\frac{\$2}{5 \text{ fl oz}}$, or $\frac{\$0.40}{1 \text{ fl oz}}$.

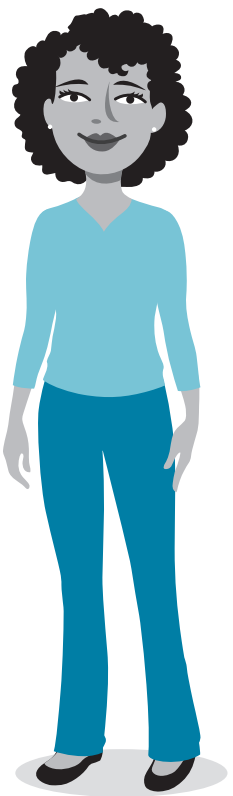
The smaller bottle of detergent is about 21 fluid ounces for about $7.



So, each fluid ounce costs about $\frac{\$7}{21 \text{ fl oz}}$, which is $\frac{\$1}{3 \text{ fl oz}}$, or about $\frac{\$0.33}{1 \text{ fl oz}}$.

That means that you pay less for each fluid ounce of the smaller bottle of detergent, so it is the better buy.

You may have encountered unit rates at the grocery store. Unit rates can help you determine which of two or more items is the best buy.

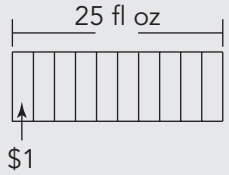


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Parker estimated the unit rates this way:

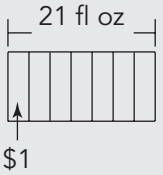
Parker

For the larger bottle of detergent, you spend about \$10 for about 25 fluid ounces.



So, for each dollar you spend on the larger bottle of detergent, you get about $\frac{25 \text{ fl oz}}{\$10}$, or $\frac{2.5 \text{ fl oz}}{\$1}$.

For the smaller bottle of detergent, you spend about \$7 for about 21 fluid ounces.



So, for each dollar you spend on the smaller bottle of detergent, you get about $\frac{21 \text{ fl oz}}{\$7}$, or $\frac{3 \text{ fl oz}}{\$1}$.

Because you get more detergent in the smaller bottle for each dollar you spend, the smaller bottle is the better buy.

1. Emily and Parker both chose the smaller bottle of detergent as the better buy, but for different reasons. Explain the differences in their reasoning.

2. Calculate the actual unit rate for each of the two sizes of detergent in two different ways.

ACTIVITY
2.2 Writing Unit Rates

Unit rates can be written with either quantity as the unit.

1. Each situation relates a quantity and a price. Calculate the two different unit rates associated with each situation: price per item and number of items per dollar. When necessary, round your answer to the nearest hundredth.

   a. A bottle of 250 vitamins costs $12.50.

   b. A pack of 40 AAA batteries costs $25.95.

   c. A package of 24 rolls of toilet paper costs $16.25.

   d. A box of 500 business cards costs $19.95.

2. Not all unit rates involve money. Write two different unit rates associated with each situation.

   a. The 5 goats eat 12 tomatillos.

   b. The exchange rate is 10 U.S. dollars for every 9 euros.

   c. The average stalactite grows 30 mm every 10 years.

   d. Sandy buys 500 coffee pods every year.

3. For each part of Question 2, identify which unit rates are useful in discussing the situation.

A stalactite is a formation that hangs from the ceilings of caves.

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The unit rate needed to solve a problem is often asked for in the question.

4. For each situation, identify the unit rate that would answer the question. Explain how you decided which unit rate to write.

   a. How many tomatillos did each goat eat?

   b. About how many euros is each U.S. dollar worth?

   c. How much does each stalactite grow in a month?

   d. How many coffee pods can Sandy use each week?

ACTIVITY 2.3

Using Unit Rates to Determine the Best Buy

PROBLEM SOLVING



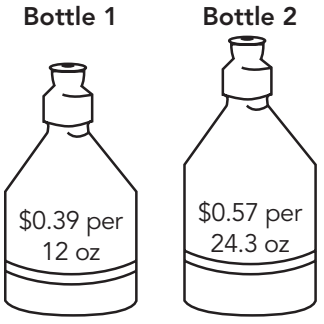
Movie theaters sell popcorn in notoriously large quantities. The smallest size popcorn usually contains at least 2 servings of popcorn. And, when you're eating all of that popcorn, you will also need some water!

1. Compare the prices for various sizes of popcorn sold at the local movie theater.

Mega Bag (32 oz)	$10.24
Giant Bag (24 oz)	$6.00
Medium Bag (16 oz)	$4.48
Kid's Bag (8 oz)	$2.40

   a. What is the unit rate price per ounce for each bag of popcorn?

   b. What size popcorn is the best buy? Explain your reasoning.



2. The movie theater sells bottles of water at various prices and in various sizes. Write the price of each bottle as a rate and then as a unit rate. Which bottle is the best buy? Explain how you know.



3. Use unit rates to determine which is the better buy. Explain your reasoning.

   a. 22 vitamins for $1.97 or 40 vitamins for $3.25

   b. 24.3 ounces for $8.76 or 32.6 ounces for $16.95

$1 for 2 tosses $2 for 5 tosses $5 for 10 tosses $10 for 25 tosses

4. On a recent trip to the state fair, you saw a sign for the price of the ring toss. Which "deal" should you take? Explain your reasoning.

ACTIVITY
2.4 Using Unit Rates to Make Comparisons

The local paper published these rates on gas mileage for a few new cars.

Avalar can travel 480 miles on 10 gallons of gas.
Sentar can travel 400 miles on 8 gallons of gas.
Comstar can travel 360 miles on 9 gallons of gas.

1. Change each rate to a unit rate so that it reports miles per one gallon of gas.

   a. Avalar

   b. Sentar

   c. Comstar

2. How did you calculate each unit rate?

3. How can unit rates help you to compare these cars?

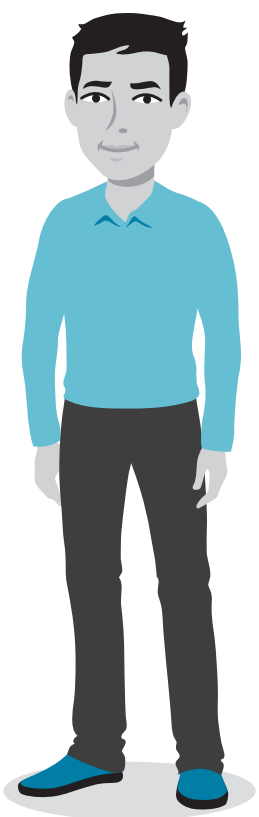
4. Guests at a dinner party are seated at three tables. Each table has large, round loaves of bread instead of individual rolls. Each person at the table shares the loaves equally.
   Table 1 has six guests and has two loaves of bread.
   Table 2 has eight guests and has three loaves of bread.
   Table 3 has 10 guests and has four loaves of bread.



a. Predict at which table the guests will get the largest serving of bread.

b. Determine how much bread each guest at each table will receive. Was your prediction accurate?

Make a prediction before you begin calculating and then use ratio reasoning to answer each question.



5. Nahimana can run 3 laps in 9 minutes. Kayla can run 2 laps in 7 minutes. Who is the faster runner?

6. Sebastian and Gabriel are making mini-cakes for the school bake sale. Sebastian makes 5 mini-cakes every 25 minutes. Gabriel makes 3 mini-cakes every 10 minutes. When they both continue to make mini-cakes at the same rate for the same amount of time, which boy will make more cakes?

7. On Monday, the school cafeteria sold 4 chocolate milks for every 10 white milks. On Tuesday, the cafeteria sold 1 chocolate milk for every 3 white milks. On which day did the cafeteria sell more chocolate milks per number of white milks sold?

8. A tour bus drove 120 miles in 2 hours, and a school bus drove 180 miles in 3 hours. Which bus drove faster?

ACTIVITY
2.5 Problem Solving with Unit Rates

Unit rates are helpful when solving problems about constant speeds.

1. In the spring, the gym teachers at the middle school sponsor a bike-a-thon to raise money for new sporting equipment. Students seek sponsors to pledge a dollar amount for each mile they ride.

   a. Diego can ride 12.5 miles per hour. At this rate, how far will he ride in 5 hours?

   b. Adriana can ride 14.75 miles per hour. At this rate, how far will she ride in 6 hours?

   c. When Angelina rides 56.25 miles in 5 hours, how far will she ride in 7 hours?

   d. Logan got a cramp in his leg after riding 27.5 miles in 2 hours and had to stop. Supposing he hadn't gotten the cramp and had continued to ride at the same rate, how far would he have ridden in 3 hours?

2. Sarah, Camilla, Mei, and Valentina are all training for the local marathon.

   a. Sarah can run 6.5 miles per hour. At this rate, how far will she run in the first 3 hours of the marathon?

   b. Camilla runs 13.5 miles in 2 hours. What is her rate?

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c. Mei is the slowest runner in the group. She can run 5.5 miles per hour. At this rate, how many miles will she run in the first 3 hours of the marathon?

d. Valentina wants to run the 26.2 miles of the marathon in 4.5 hours. At what rate will she have to run to reach this goal?

e. At a workout designed to increase speed, Sarah runs 800 meters in $2\frac{1}{2}$ minutes. Camilla runs 1600 meters in $4\frac{1}{2}$ minutes. Who ran the fastest in this workout?



3. Jasmine left her notebook on the bus and her friend Luna picked it up for her. On Saturday, they decide to meet to give Jasmine her notebook. They live 7.5 miles from each other and plan to walk and meet between their homes. Luna can walk 3 miles per hour, and Jasmine can walk 4.5 miles per hour. Jasmine makes the suggestion, "It will take the same amount of time when you stay put and I run 7.5 miles per hour." Is Jasmine's suggestion correct?

Unit rates are also useful when calculating the price of multiple items.

4. Complete each table.

   a. Carpet is sold by the square yard. Classroom carpet sells for $10.50 per square yard.

1 yd^2	40 yd^2	50 yd^2	100 yd^2
$10.50			

   b. Pink Lady apples are sold by the pound. One pound of Pink Lady apples costs $2.99.

1 lb	2 lb	5 lb	10 lb	20 lb
$2.99				

   c. Purchases in your county have a 7 percent sales tax added for every dollar of the purchase price.

$1	$5	$10	$20	$50	$100
$0.07					

5. How did you use a unit rate to complete each table in Question 4?

Talk the Talk

Shopping for Cereal

Lucas and Mason love cereal, but don't want to spend a lot of money. After scanning the aisle in the grocery store for the lowest prices, the boys make the following statements.

- Lucas says, "I found oat puffs for $0.14 per ounce. That's the cheapest cereal in the aisle!"
- Mason replies, "It's not cheaper than rice cereal! The unit price for that is 6.25 oz per dollar."

Who is correct? Explain your reasoning.

Lesson 2 Assignment

Write

Define the term *unit rate* in your own words.

Remember

Unit rates that involve money, like $1.25 per pound, or speed, like 60 miles per hour, are very common. But not all unit rates are about money or speed.

Practice

1. Write a unit rate for each situation.

   a. 254 words typed in 4 minutes

   b. 5 trays with 90 ice cubes

   c. 4 hot dogs eaten in 45 seconds

   d. 8 hours to drive 528 miles

2. Nahimana needs to buy apples to bake pies for the fair. She needs 13 pounds of apples. At one market, she finds apples selling for $1.89 a pound. At another market, she finds a 15-pound bag of apples for $26.99. Which market has the better deal?

3. Carlos needs to buy new contact lenses. His ophthalmologist sells 8-lens boxes in packs of 2 for $52 and 10-lens boxes in packs of 4 for $120. Which option is the better deal?

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Lesson 2 Assignment

4. Pet Store A claims in their advertisement that they have the best price in town for ChowChow dog food. They sell 20-pound bags for $16.95. Pet Store B also claims to have the best price in town for ChowChow dog food. They are selling 30-pound bags for $24.95. Which store has a valid claim?

5. During his last race, Isaiah biked 43 kilometers in 2 hours. Supposing that he maintains that same speed, how far will he travel in 3 hours?

6. Complete the table. Bananas are sold by the pound. One pound of bananas costs $0.55 per pound.

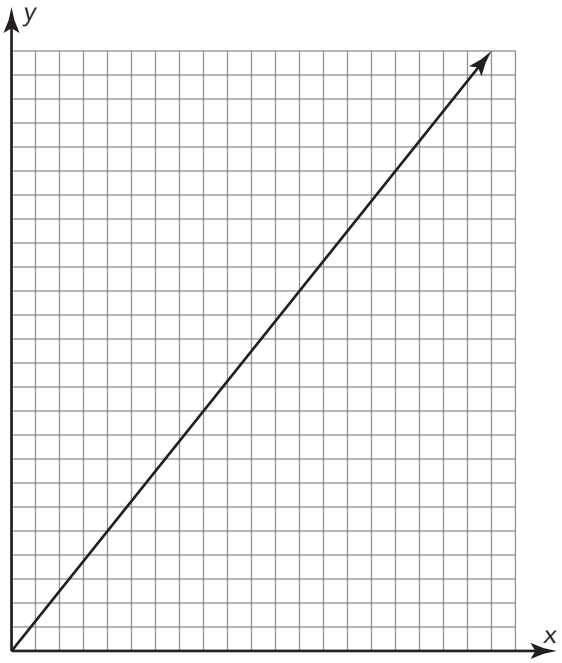
1 lb	2 lb	5 lb	10 lb
$0.55			

Lesson 2 Assignment

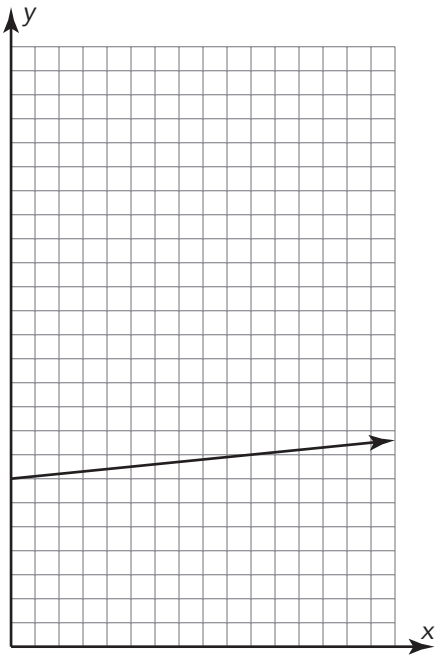
Prepare

For each graph, determine whether it represents equivalent ratios.
Explain your reasoning.

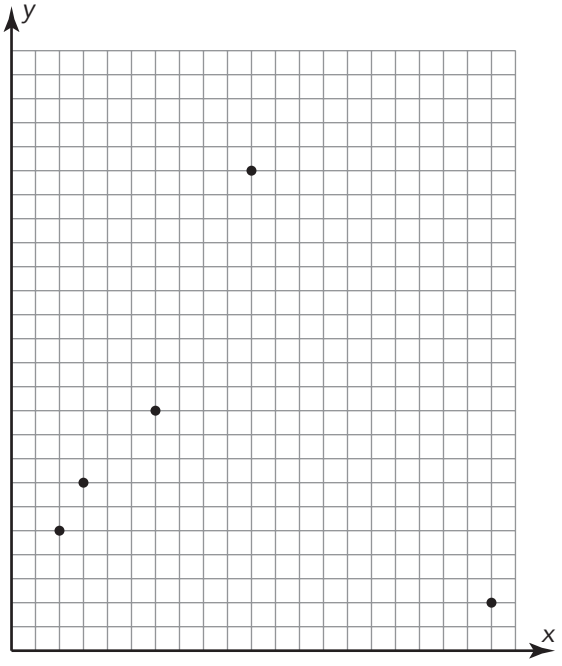
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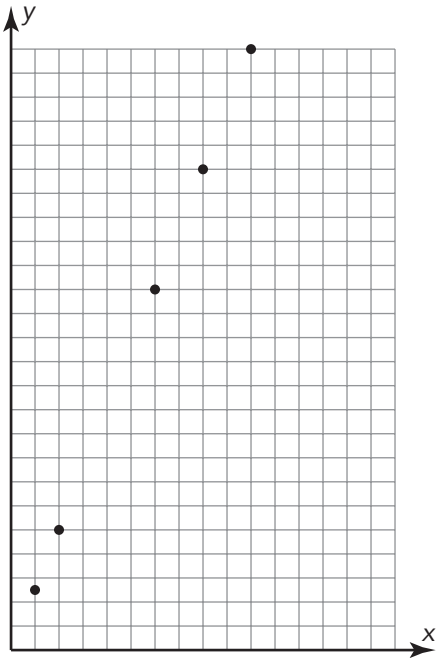
2.



3.



4.



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3 Multiple Representations of Unit Rates

OBJECTIVES

- Represent and identify unit rates using tables and graphs.
- Recognize that $(x, 1)$ and $(1, y)$ are both points on the graph of a unit rate.
- Graph unit rates in real-world situations involving unit pricing and constant speed.
- Compare unit rates based on their graphs.

You know about special ratios called rates and have used unit rates to convert measurements, determine the better buy, and solve problems about constant speeds.

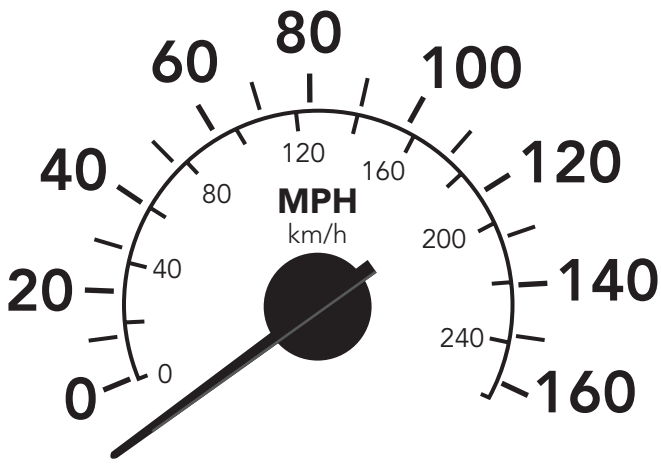
How can you use graphs of rates to solve other types of problems?

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Getting Started

The Need . . . for Speed

In cars, a speedometer shows the driver the rate at which the car is moving—its speed. Many speedometers are like double number lines arranged in a circular shape. The top number line on this speedometer shows the rate in miles per hour (mph), and the bottom number line shows the rate in kilometers per hour (km/h).



Use the speedometer to estimate.

1. At about what rate, in kilometers per hour, is the car moving when it is traveling at 60 miles per hour?

2. At about what rate, in miles per hour, is the car moving when it is traveling at 60 kilometers per hour?

3. About how long would it take to drive 90 kilometers at 55 miles per hour? Explain your reasoning.

ACTIVITY
3.1 Unit Rates in Tables and Graphs

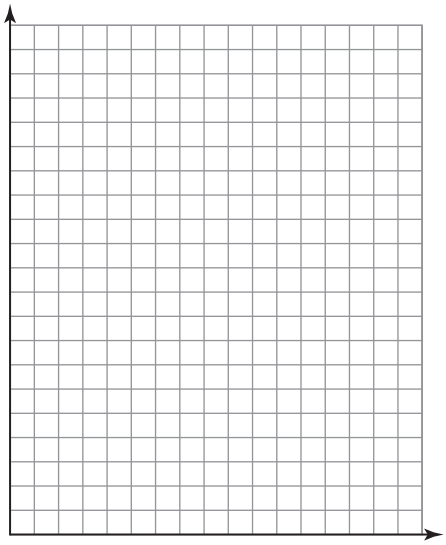
The 6th-grade chorus made and sold their own mixture of trail mix at basketball games to raise money for an upcoming trip. During the first basketball game, they sold 1-lb bags for $2.80. They got many requests to sell different-sized bags of their trail mix. The group decided to vary the size of the bags, but wanted to make sure that the cost-to-pounds rate stayed the same.

Trail Mix Weight (lb)	Cost ($)
0.25	
0.5	
0.75	
1	
1.25	
1.5	

1. Complete the table to display the cost for various quantities of trail mix. Create a graph from your table of values. Be sure to label the axes and name the graph.

In a unit rate, one or both of the values are 1.

2. Identify two points on the graph that represent unit rates. Write each unit rate in words and explain its meaning.



3. Explain how your graph displays equivalent rates.

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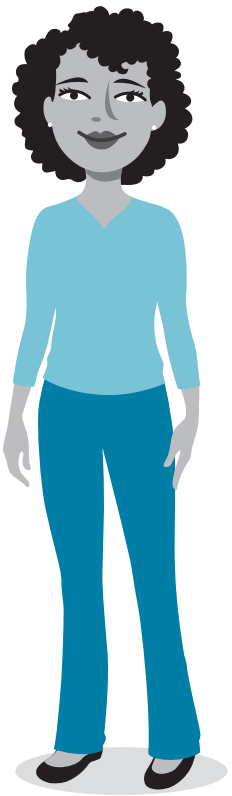
ACTIVITY
3.2 Unit Rates and Dimensions

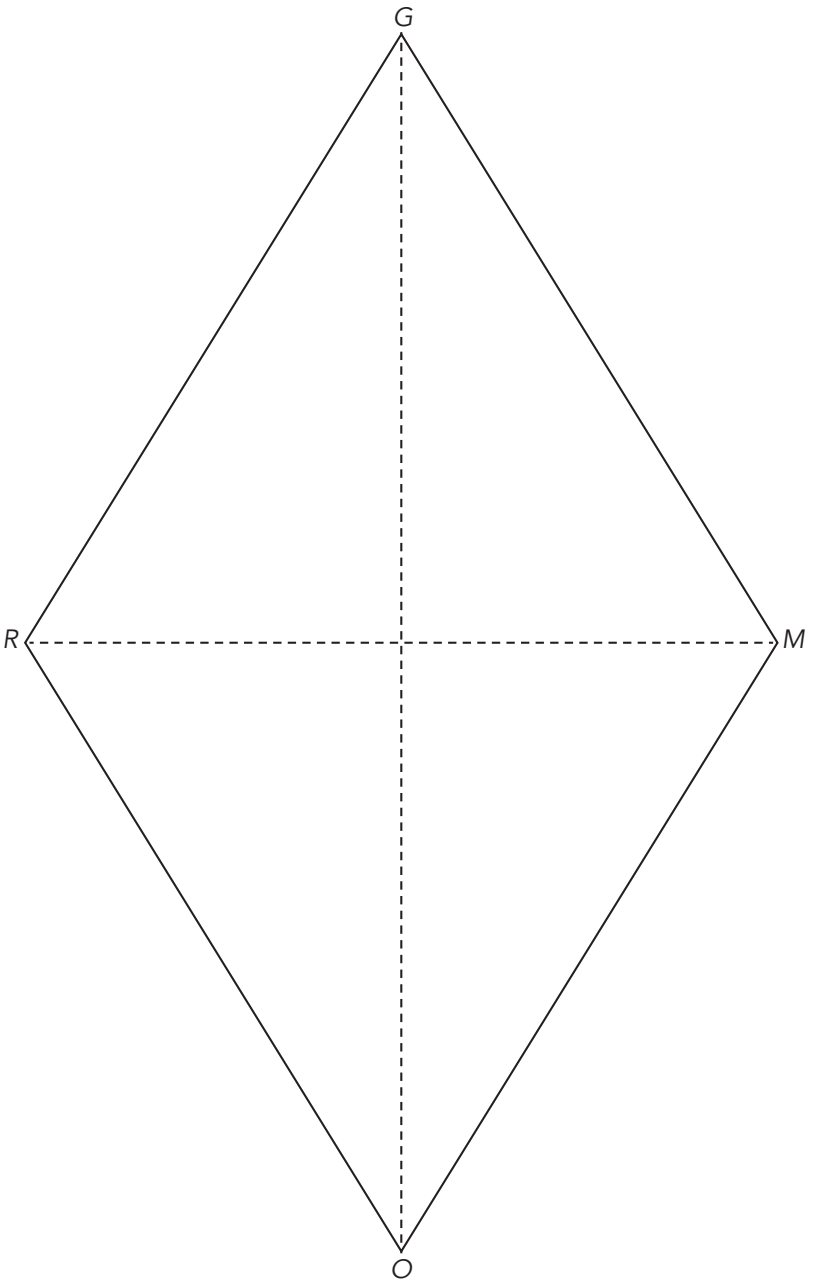
A diagonal is a line segment that connects opposite vertices of a polygon.

A rhombus is considered a Golden Rhombus when the diagonals are in a very specific ratio, known as φ or phi (pronounced "fi" or "fee"). A Golden Rhombus is shown and your task is to determine the ratio of the diagonals.

A Golden Rhombus

Look around your classroom to identify tools, besides a ruler, that you can use to measure the lengths of the diagonals.

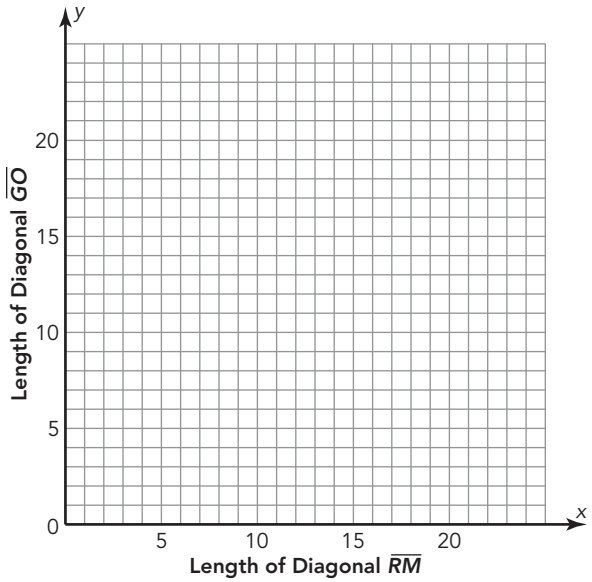




1. Use standard and non-standard tools to measure the lengths of the diagonals using 6 different units of measure and record them in the table. Be sure to include inches and centimeters as two of your units.

Unit of Measure	Length of Diagonal $\overline{RM}$	Length of Diagonal $\overline{GO}$

2. Graph the lengths of the diagonals on the coordinate plane.



3. Use a straightedge to connect the ratios plotted on the graph. Describe the pattern that the points appear to follow.

4. What does the pattern of ratios on the graph tell us about the ratios?

5. Write two unit rates that relate the length of diagonal $\overline{GO}$ and the length of diagonal $\overline{RM}$.

6. Describe where you can locate the unit rates on the graph.

Remember . . .
Another way to represent line segment GO is to write it as $\overline{GO}$.

7. Suppose you measure the Golden Rhombus in units called "ujeni". Use the unit rates to answer each question.

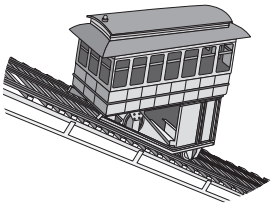
   a. When the length of diagonal $\overline{GO}$ is 15 ujeni, what is the length of diagonal $\overline{RM}$ in ujeni?

   b. When the length of diagonal $\overline{RM}$ is 15 ujeni, what is the length of diagonal $\overline{GO}$ in ujeni?

ACTIVITY
3.3 Graphing Rates

Opened in 1887, and designed to move coal workers from their homes atop Mt. Washington down to the coal factories along the river in Pittsburgh, Pennsylvania, the Duquesne Incline still serves as a mode of transportation for commuters who live in the area.

An incline is like a trolley that goes up and down a mountain.



Emily takes the incline to work each morning. The length of the incline is 800 feet, and it takes 90 seconds to ride from the top of Mt. Washington to the bottom.

1. Identify which of the following statements are true. Explain your reasoning for each.

   a. Emily travels approximately 178 feet every 20 seconds.

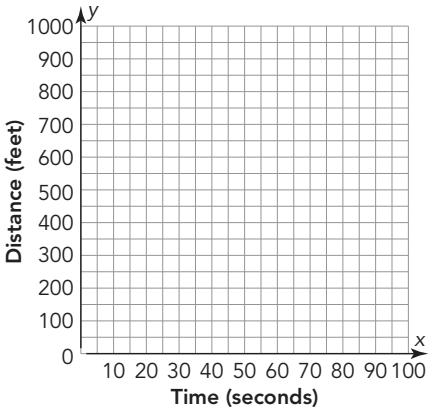
   b. She travels approximately 600 feet per minute.

   c. In 75 seconds, Emily travels approximately 750 feet.

   d. She travels approximately 44 feet every 5 seconds.

   e. She travels 8.9 feet per second.

2. Plot the correct rates from Question 1 on the coordinate plane. How can you use the graph to verify correct and incorrect statements from Question 1?

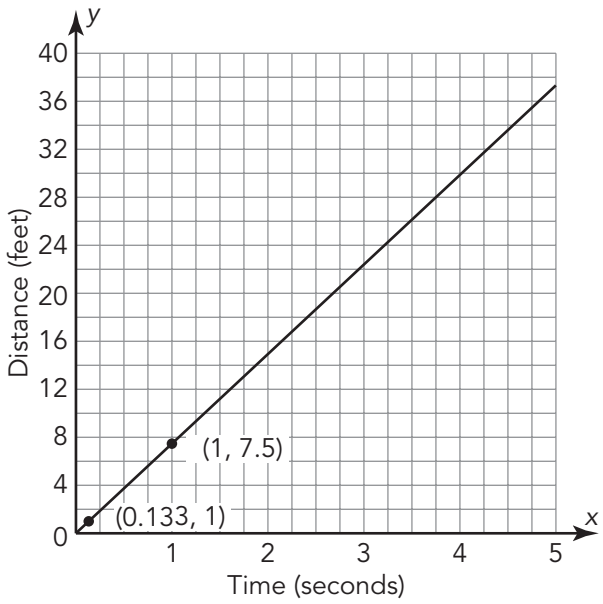


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Talk the Talk

Once Upon a Unit Rate

Write a story with unit rates that corresponds to this graph. Include 3 questions and their answers that someone could solve using the graph. Be prepared to share your story with the rest of the class.



Lesson 3 Assignment

Write

Explain what each described point would represent on a graph of equivalent rates.

- The point with an x-coordinate of 1
- The point with a y-coordinate of 1

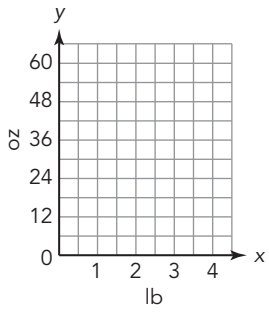
Remember

You can represent rates and unit rates in a variety of different ways—in tables, on graphs, and in stories and other situations.

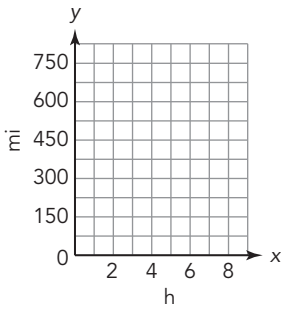
Practice

Graph the rates in each pair on a coordinate plane. Explain whether or not the rates are equivalent.

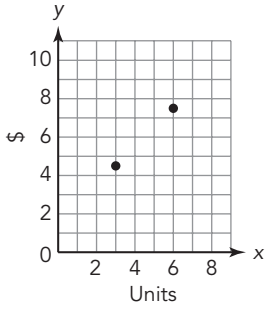
1. $\frac{48 \text{ oz}}{3 \text{ lb}}, \frac{64 \text{ oz}}{4 \text{ lb}}$



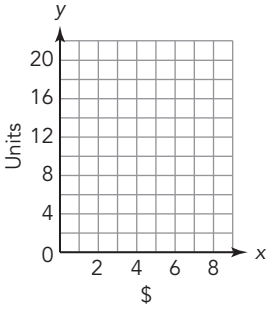
2. $\frac{150 \text{ mi}}{2.5 \text{ hr}}, \frac{525 \text{ mi}}{8.75 \text{ hr}}$



3. $\frac{\$4.50}{3}, \frac{\$7.50}{6}$

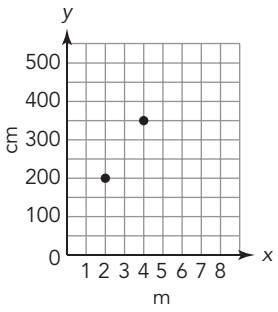


4. $\frac{10}{\$7}, \frac{12}{\$8.40}$

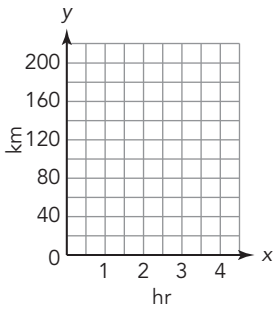


Lesson 3 Assignment

5. $\frac{200 \text{ cm}}{2 \text{ m}}, \frac{350 \text{ cm}}{4 \text{ m}}$



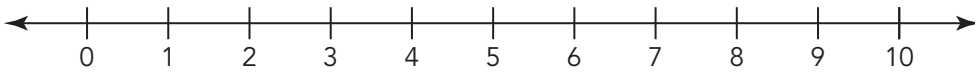
6. $\frac{90 \text{ km}}{1 \text{ hr}}, \frac{180 \text{ km}}{2 \text{ hr}}$



Prepare

Plot each number on the number line. Then, insert a $>$ or $<$ symbol to make each inequality statement true.

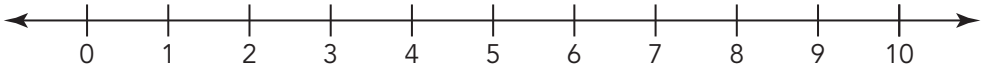
1. $\frac{3}{4}$ _____ $\frac{3}{2}$



2. 5.6 _____ 5.06



3. 7.65 _____ 6.75



TOPIC 3 SELF-REFLECTION

Name: _____

Unit Rates and Conversions

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Unit Rates and Conversions* topic by:

TOPIC 3: *Unit Rates and Conversions*	Beginning of Topic	Middle of Topic	End of Topic
applying ratio reasoning strategies (e.g., ratio tables, strip diagrams, double number lines) to convert measurement units in real-world and mathematical problems.	○	○	○
applying ratio reasoning to determine when to multiply or divide to convert measurement units in real-world and mathematical problems.	○	○	○
identifying and calculating a unit rate.	○	○	○
explaining the relationship between a ratio $a : b$ and a unit rate $\frac{a}{b}$ where $b = 1$.	○	○	○
applying ratio reasoning strategies and the concept of unit rate to solve real-world problems.	○	○	○
using multiple conversions to convert to a desired unit.	○	○	○

continued on the next page

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TOPIC 3 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Unit Rates and Conversions* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?



TOPIC 3 SUMMARY

Unit Rates and Conversions Summary

NEW KEY TERMS
- convert [convertir]
- unit rate

LESSON 1 Using Ratio Reasoning to Convert Units

There are many situations in which you need to convert measurements to different units. To **convert** a measurement means to change it to an equivalent measurement in different units. When you convert a measurement to a different unit, the size of the object does not change; only the units and the number of those units change.

You can write conversions using ratio and rate language. They are also written symbolically in terms of equality.

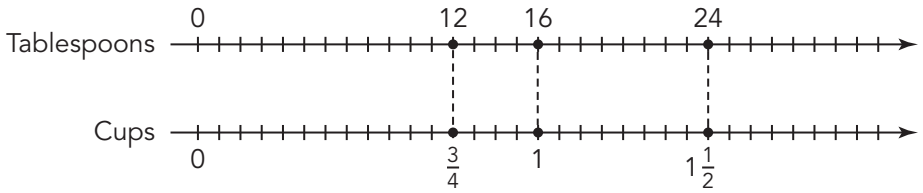
Ratio Language	Symbolically
For every 12 inches, there is exactly 1 foot.	12 in. = 1 ft
For every yard, there are 36 inches.	1 yd = 36 in.
For every mile, there are 1760 yards.	1 mi = 1760 yd
For every meter, there are 1000 millimeters.	1 m = 1000 mm
For every 1 kilometer, there are exactly 1000 meters.	1 km = 1000 m

A conversion ratio is also called a conversion rate because two quantities that are measured in different units are being compared. For example, you can write the rate of inches to feet in fractional form: $\frac{12 \text{ in.}}{1 \text{ ft}}$.

Because these measurement conversions are ratios, you can use ratio and rate reasoning to convert between units, such as double number lines.

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For example, the double number line shown represents the ratio of tablespoons to cups.



Using the double number line, you can determine that there are 12 tablespoons in $\frac{3}{4}$ cup or that there are $1\frac{1}{2}$ cups in 24 tablespoons.

Using a ratio table is another strategy for converting units. For example, this table represents the ratio of pounds to ounces.

Pounds	1	2	$\frac{1}{4}$	$1\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$2\frac{1}{2}$
Ounces	16	32	4	20	8	6	40

+ =

You can add values in different columns to determine new equivalent rates.

Scaling up or down is a similar strategy for determining equivalent ratios that can more easily be used to convert from one unit of measurement to another.

For example, you can use scaling up to determine how many ounces are in 2.5 pounds. Because you want to determine the number of ounces for a specific number of pounds, use the conversion rate 1 lb = 16 oz or $\frac{1 \text{ lb}}{16 \text{ oz}}$.

Scaling Up

$$\frac{1 \text{ lb}}{16 \text{ oz}} = \frac{2.5 \text{ lb}}{? \text{ oz}} \longrightarrow \frac{1 \text{ lb}}{16 \text{ oz}} \overset{\cdot 2.5}{=} \frac{2.5 \text{ lb}}{40 \text{ oz}}$$

You can also use unit analysis to determine the quantity in grams that is equivalent to 4.5 kilograms. In unit analysis, you multiply by a form of 1 to rewrite the given measurement in a different unit.

Unit Analysis

$$4.5 \text{ kg} \left(\frac{1000 \text{ g}}{1 \text{ kg}}\right)$$

$$\frac{4.5 \cancel{\text{kg}}}{1} \left(\frac{1000 \text{ g}}{1 \cancel{\text{kg}}}\right) = 4500 \text{ g}$$

$$\frac{1 \text{ kg}}{1000 \text{ g}} = \frac{4.5 \text{ kg}}{4500 \text{ g}}$$

$$4.5 \text{ kg} = 4500 \text{ g}$$

You can use multiple conversions to convert to a desired unit. A new community park stretches across 1 mile 880 yards. How many feet across is the park?

$$1 \text{ mile} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} = 5280 \text{ ft}$$

$$880 \text{ yards} \cdot \frac{3 \text{ ft}}{1 \text{ yard}} = 2640 \text{ ft}$$

$$5280 + 2640 = 7920 \text{ feet}$$

The park is 7290 feet across.

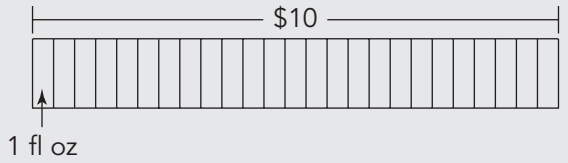
LESSON 2 Introduction to Unit Rates

A rate is a ratio in which the two quantities being compared are measured in different units. A **unit rate** is a comparison of two measurements in which the denominator has a value of one unit.

One way to compare the values of products is to calculate the unit rate for each item.

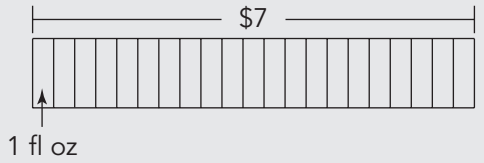
For example, a brand of laundry detergent comes in two different sizes: 26 fluid ounces for $9.75 or 20.5 fluid ounces for $7.25.

The larger bottle of detergent is about 25 fluid ounces for about $10.



So, each fluid ounce costs about $\frac{\$10}{25 \text{ fl oz}}$, which is $\frac{\$2}{5 \text{ fl oz}}$, or $\frac{\$0.40}{1 \text{ fl oz}}$.

The smaller bottle of detergent is about 21 fluid ounces for about $7.



So, each fluid ounce costs about $\frac{\$7}{21 \text{ fl oz}}$, which is $\frac{\$1}{3 \text{ fl oz}}$, or about $\frac{\$0.33}{1 \text{ fl oz}}$.

That means that you pay less for each fluid ounce of the smaller bottle of detergent, so it is the better buy.

A unit rate is written with either quantity as the unit. In the example above, the unit rate was determined as the price per fluid ounce. Another way to write the unit rate is the number of fluid ounces per dollar. For the larger bottle of detergent, you get about $\frac{2.5 \text{ fl oz}}{\$1}$, and for the smaller bottle of detergent, you get about $\frac{3 \text{ fl oz}}{\$1}$.

Unit rates are helpful when solving problems about constant speeds.

For example, suppose Jasmine can ride 50 miles in 4 hours. At this rate, how far will she ride in 7 hours?

$$\frac{50 \text{ miles}}{4 \text{ hours}} = \frac{12.5 \text{ miles}}{1 \text{ hour}} \qquad \frac{12.5 \text{ miles}}{1 \text{ hour}} = \frac{87.5 \text{ miles}}{7 \text{ hours}}$$

Scale down to determine the unit rate. Then, scale up to determine the equivalent rate needed to solve the problem.

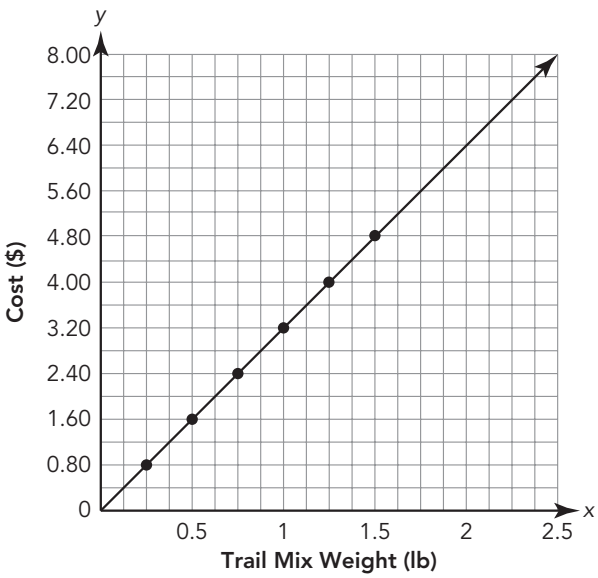
LESSON 3 Multiple Representations of Ratios

You can represent rates and unit rates in a variety of different ways—in tables, on graphs, and in stories and other situations.

For example, the 6th-grade chorus is selling bags of trail mix in various sizes to raise money for an upcoming trip. The group wants the ratio of cost-to-pounds to stay the same no matter the size of the bag. They decide to sell 1-pound bags for $3.20.

The table shown displays the cost for various quantities of trail mix. These ratios are plotted on the graph and connected with a line.

Trail Mix Weight (lb)	Cost ($)
0.25	0.80
0.5	1.60
0.75	2.40
1	3.20
1.25	4.00
1.5	4.80



The graph displays equivalent rates because each ordered pair that falls on the line is a multiple of (x, y) and is equivalent to the ratio $\frac{y}{x}$. You can use the graph to determine that the unit rate *cost* : *weight* is $3.20 per pound and that the unit rate *weight* : *cost* is about 0.3 pound per dollar.

GRADE 6

MODULE 3

Moving Beyond Positive Quantities

TOPIC 1 Signed Numbers and the Four Quadrants 451

TOPIC 2 Operating with Integers 541

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GRADE 6

TOPIC 1



If you think of the surface of the ocean as 0, then a diver is in the negative numbers until he comes back up.

Signed Numbers and the Four Quadrants

LESSON 1 Introduction to Negative Numbers 453

LESSON 2 Absolute Value 473

LESSON 3 Rational Number System 493

LESSON 4 Extending the Coordinate Plane 509

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1 Introduction to Negative Numbers

OBJECTIVES

- Use positive and negative numbers to describe quantities having opposite directions.
- Explain the meaning of 0 in contexts represented by positive and negative numbers.
- Identify and represent a number and its opposite on a number line.
- Represent, interpret, and order positive and negative rational numbers using number lines and inequality statements.

NEW KEY TERMS

- negative numbers
- infinity

You have used numbers equal to or greater than 0 to represent real-world situations.

How can you use numbers less than 0 to describe real-world situations?

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Getting Started

Number Line Geography

1. What do you know about a number line?



2. Label the number line and be sure to include 0. Then, plot and label a single point of your choice on the number line.

   a. Draw a ray, or an arrow, beginning at your point to represent the numbers greater than or equal to the value at your point.



   b. Draw a ray, or an arrow, beginning at your point to represent the numbers less than or equal to value at your point.



   c. At the ends of a number line, there are arrows going in both directions. What do these arrows indicate?

   d. What do you think is on the number line to the left of 0?

ACTIVITY
1.1 Investigating Time on a Number Line

Let's use a number line to represent time.

Your teacher will assign students to participate in the activity. Be sure to record what happens on the number line.

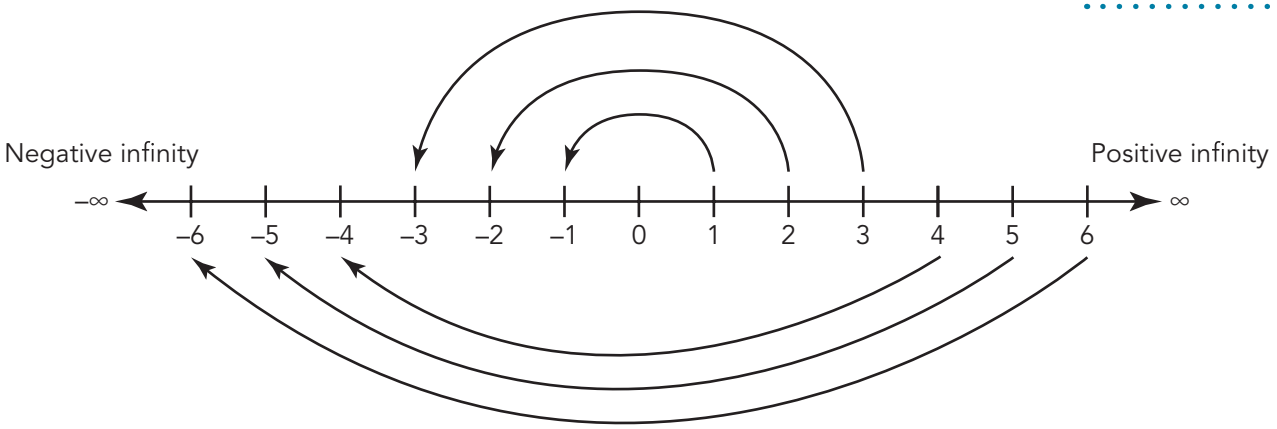
1. For each student, plot and label the point where the student stands on the number line. Also, identify what time the point represents.



Student *A*: Stand at 0 to represent the time right now.
Student *B*: Stand at the point that represents 3 hours from now.
Student *C*: Stand at the point that represents 3 hours ago.
Student *D*: Stand at the point that represents 5 hours from now.
Student *E*: Stand at the point that represents 2 hours ago.
Student *F*: Stand at the point that represents 7 hours ago.

A number line can be created by reflecting the positive numbers across zero. The values to the left of zero on the number line are called **negative numbers** and are labeled with a negative sign. The positive values extend to positive infinity, and the negative numbers extend to negative infinity. **Infinity**, represented by the symbol ∞, means a quantity with no end or bound. The number line goes on forever in both directions!

A *negative number* is written with a negative sign. You can write a positive number with a positive sign or without any sign. For example, positive 5 can be written as +5 or 5.



2. Describe the change in the values of the numbers as you move to the right on the number line.

3. Describe the change in the values of the numbers as you move to the left on the number line.

Consider your class time number line.

4. Describe the locations of the points that represent time in the future.

5. Describe the locations of the points that represent time in the past.

6. How would you label your number line differently from one created by a class that starts at a different time?

7. What observations can you make about where a given number of hours before or after time 0 is plotted? What do you notice about its distance from 0? For example, what do you notice about 3 hours before and 3 hours after now? Or 6 hours before and 6 hours after now?

ACTIVITY
1.2 Representing Opposites on a Number Line

Let's think more about both sides of 0 on a number line.

Your teacher will model a number line.

1. Create and label a number line according to the model.

2. Plot and label the location where each student stands on the number line. In the table, identify the value represented by the location where the student is standing.

   Student *A*: Stand at 0.
   Student *B*: Stand at 4.5.
   Student *C*: Stand at the opposite of 4.5.
   Student *D*: Stand at -6.
   Student *E*: Stand at the opposite of -6.
   Student *F*: Stand at a location between 2 and 3.
   Student *G*: Stand at the location that is the opposite of Student *F*.

Student	Value
A	
B	
C	
D	
E	
F	
G	

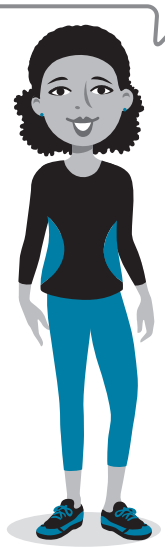
3. Describe the number line relationship of the students who were opposites of each other.

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Opposite numbers are reflections of each other across 0 on the number line.

- The opposite of a positive number is a corresponding negative number.
- The opposite of a negative number is a corresponding positive number.

There is only one number that is its own opposite.



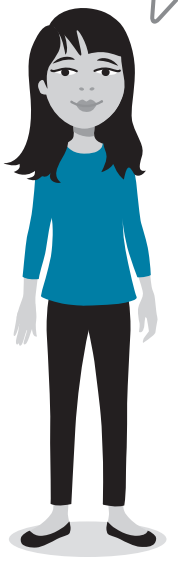
Attaching a negative sign to a number means reflecting that number across 0 on the number line.

4. Use symbols to represent the opposite of 4.5 and the value it represents.
   $-(4.5) =$ _____

5. Use symbols to represent the opposite of -6 and the value it represents.
   $-(-6) =$ _____

6. What do you notice about the distance from 0 of corresponding opposite numbers?

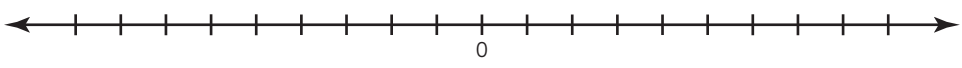
Don't forget to label the number line!



7. What is the opposite of 0?

8. Name the opposite of each number. Then, plot each number and its opposite on the number line.

$1\frac{1}{2}$	-5	-9.9
25%	$-\frac{3}{4}$	$\frac{13}{4}$



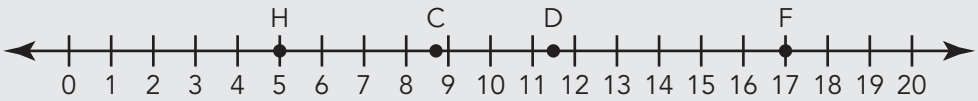
ACTIVITY
1.3 Representing Money on a Number Line

Daniela and her friends are trying to decide whether they can go to the movies. Each ticket costs $9.00. After checking their wallets, each friend comments on how much money they have.

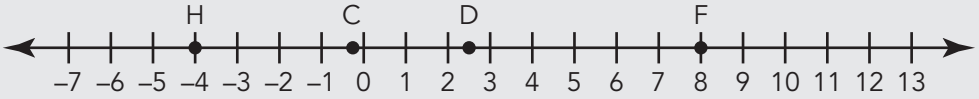
- Daniela: I have $2.50 more than the movie costs.
- Hannah: Oh, I don't have enough money. I'm $4.00 short.
- Fernando: Not only can I buy a ticket, but I have just enough money to buy the $8.00 snack combo!
- Catalina: When I find one more quarter, I can go.

Michael and Olivia created different number lines to represent the scenario.

Michael



Olivia



Analyze each representation of the scenario.

1. What does each point represent on Michael's number line?

2. What does each point represent on Olivia's number line?

3. Michael and Olivia are thinking about 0 differently. Explain what 0 represents on each number line.

4. Suppose the four friends decide to go to a matinee instead, where the ticket price is $7.50.

   a. How would Michael's number line change?



   b. How would Olivia's number line change?

ACTIVITY
1.4 Temperature Connection

Number lines can also be vertical, like a thermometer or a measure of elevation.

1. Discuss and write a sentence to describe the meaning of each statement.

   a. The weather forecaster predicts the temperature will be below zero.

   b. A submarine travels at 3000 feet below sea level.

   c. Badwater Basin in Death Valley, California, is 86 meters below sea level.

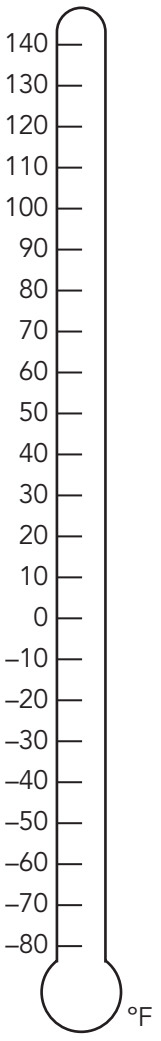
2. Mark each temperature on the thermometer shown.

   a. According to the National Park Service, the highest temperature on record in the United States is 134°F. It occurred in 1913 in Death Valley, California.

   b. According to The Weather Channel, the lowest temperature on record is −80°F. It occurred at Prospect Creek Camp, Alaska.

   c. According to The Weather Channel, the lowest temperature recorded in the contiguous 48 states is −70°F. It occurred in Montana.

   d. According to Current Results Weather and Science Facts, the highest winter average temperature in the United States is 78°F, which occurs in Honolulu, Hawaii.

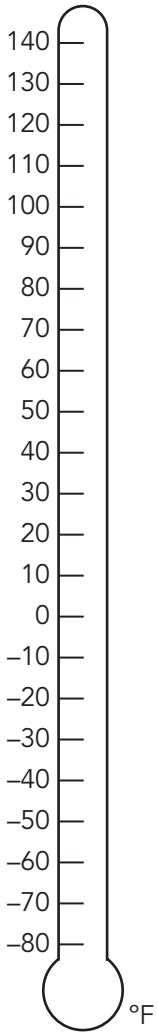


3. Which is colder, the lowest temperature recorded in Alaska or the lowest temperature recorded in Montana? How do you know?



4. Koda and Alexander were comparing 25 degrees to −27 degrees.
   - Koda wrote $25 < -27$ and justified her comparison by stating that the farther a number is from zero, the greater the number.
   - Alexander wrote $25 > -27$ and justified his comparison by stating that the greater temperature will be above the second temperature on a thermometer.

   Who is correct? Explain your choice.



5. Plot each set of temperatures on the thermometer. Then, insert a $>$ or $<$ symbol to make each number sentence true.

   a. −26°F _____ −31°F

   b. −6°F _____ −17°F

   c. −9°F _____ 8°F

6. Use the thermometer to order the temperatures from least to greatest.

   25°F −33°F 0°F 105°F −40°F −5°F 67°F

ACTIVITY 1.5

Comparing and Ordering Rational Numbers

1. Madison, Ana, Jacob, Maria, and Amir each spent time during the month running. Order the time the individuals spent running during the month from least to greatest.

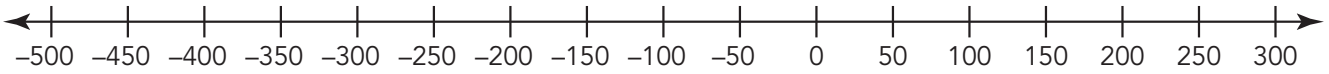
   Portion of the Month:

   Madison: $\frac{8}{25}$
   Ana: 23.6%
   Jacob: 0.175
   Maria: $\frac{3}{8}$
   Amir: 33.4%

Ask Yourself . . .
How can you use rational numbers in everyday life?

2. There was a craft show at the local school. Each table reported the amount of money in dollars and cents made at the table after accounting for the money spent in supplies. Represent the amount made at each table on the number line.

   Table A: −28.78
   Table B: 300.25
   Table C: 0.28
   Table D: −360.20
   Table E: 285.99
   Table F: −158.92
   Table G: 201.35



3. Which table lost the most money?

4. Which table made the most money?

5. Which table almost broke even?

6. For each pair of tables, write an inequality statement to compare the amount of money made at the tables. Interpret the statements in context.

   a. Table D and Table F

   b. Table A and Table C

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7. Ms. Sanchez asked Gabriela and Javier to order the following set of numbers.
$-0.35, \frac{2}{3}, 42\%, \frac{5}{4}, -2.35$

Gabriela

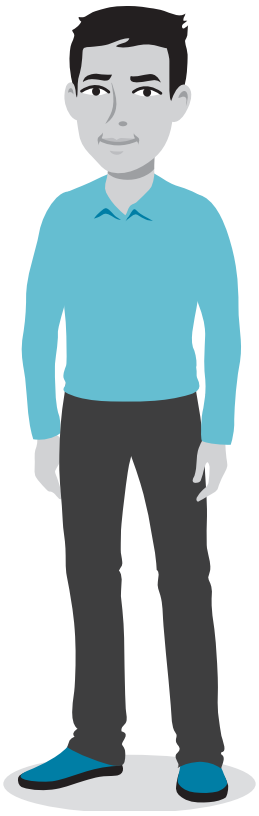
42% is the greatest value because 42 is greater than any of the other numbers.

Javier

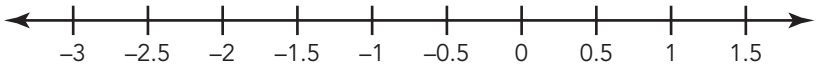
$-2.35 > -0.35$ because -2.35 is farther from 0 than -0.35.

a. Are Gabriela and Javier correct? Explain your reasoning.

How do you know which rational number is greater by looking at the number line?



b. Plot and label the values on the number line.

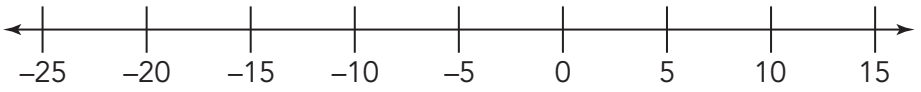


You can compare different types of numbers by plotting the numbers on a number line.

8. Use the number line to answer each question.

a. Plot each value on the number line.

$-6\frac{2}{3}$ -20 15% 10.5 $-17\frac{1}{2}$ -7.98 12 -3 -13



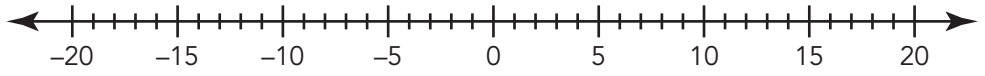
b. Which of the numbers has the least value? How do you know?

c. Which of the numbers has the greatest value? How do you know?

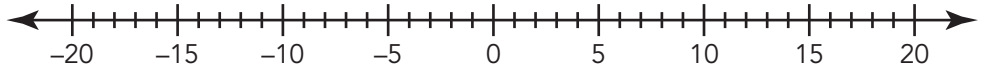
d. Order the numbers from least to greatest.

9. Plot and label each rational number on the number line. Then, insert a $>$, $<$, or $=$ symbol to make each number sentence true.

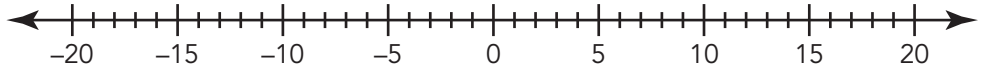
a. -10.25 _____ $-15\frac{2}{3}$



b. -17 _____ -17



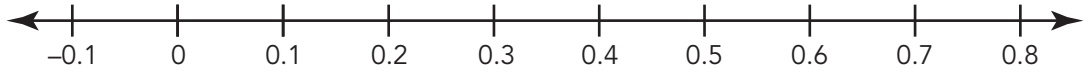
c. $5\frac{2}{3}$ _____ -8.28



d. 32% _____ 0.35



e. 45% _____ $\frac{2}{3}$



Talk the Talk

Putting It All Together

1. What does 0 mean on a number line?

2. What does *opposite* mean in terms of a number line?

3. Compare the types of numbers. Use what you know about number lines to explain your reasoning.

   a. Which is greater—a negative or a positive rational number?

   b. Which is greater—zero or any positive rational number?

   c. Which is greater—zero or any negative rational number?

   d. How do you decide which of two numbers is greater when both numbers are positive?

   e. How do you decide which of two numbers is greater when both numbers are negative?

4. Your sixth-grade cousin goes to school in a different state. His math class has not yet started comparing positive and negative numbers. Write him an email explaining how to compare any two numbers. Be sure to include 1 or 2 examples and enough details that he will be able to explain it to his class.

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Lesson 1 Assignment

Write

Write a sentence to explain the relationship between *opposites* and *negative numbers*.

Remember

The rational number line is used to represent positive numbers, negative numbers, and zero. The values to the left of zero on the number line are reflections of the values on the right across 0.

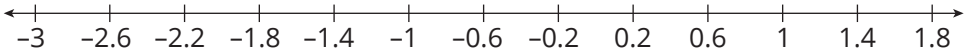
Practice

1. Plot each number and its opposite on a number line.

   a. -1 b. 0.1 c. $1\frac{3}{4}$ d. -1.9 e. 0.009

2. Plot the numbers on a number line and order from least to greatest.

   0.125 $1\frac{1}{5}$ $-\frac{4}{9}$ $\frac{4}{11}$ $-\frac{3}{2}$ -2.75



3. The Ravine Flyer II is a steel and wood roller coaster that takes advantage of the terrain in Erie, PA, to make the ride more exciting. Although the coaster is only 80 feet high, it follows the line of a cliff in order to drop to -35 feet (0 represents the height of the cliff).

   a. Plot the highest and lowest points of the roller coaster on a vertical number line.

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Lesson 1 Assignment

b. Explain why a vertical number line better represents the problem context better than a horizontal number line.

c. How many total feet does the roller coaster drop?

4. The Monster is a roller coaster that uses a design similar to the Ravine Flyer II. The Monster reaches a height of 120 feet, but then drops to −25 feet. Order the highest and lowest points of the two roller coasters from least to greatest.

5. An amusement park wants to design a coaster that rises 60 feet above ground and then drops the same distance below ground through a tunnel. Represent the underground depth with a number, and explain its relationship with the above ground height.

6. All the students in a class were reading the same book. Order the amount of the book these five students read last week from least to greatest.

   Portion of the Book:
   Student A: $\frac{14}{24}$
   Student B: 25.3%
   Student C: 0.425
   Student D: $\frac{5}{8}$
   Student E: 47.2%

7. Five musicians each spent time practicing during the week. Order the time the musicians spent practicing during the week from greatest to least.

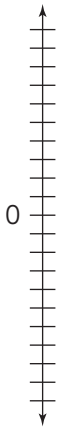
   Portion of the Week:
   Piano player: $\frac{32}{70}$
   Trumpet player: 63.4%
   Guitarist: 0.489
   Singer: $\frac{7}{18}$
   Drummer: 32.5%

Lesson 1 Assignment

Prepare

Plot and label each set of numbers on the number line and describe the relationship between the numbers.

1. 5 and -5
2. $2\frac{3}{4}$ and $-2\frac{3}{4}$
3. 8.634 and -8.634



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2 Absolute Value

OBJECTIVES

- Explain the meaning of the absolute value of a rational number as its distance from 0 on a number line.
- Interpret the meaning of absolute value as the magnitude for a positive or negative quantity in a real-world context.
- Evaluate the absolute value of a quantity.
- Compare and order numbers expressed as absolute value and distinguish absolute value comparisons from statements about order.

NEW KEY TERM

- absolute value

Numbers can be described by their distance from 0 on the number line.

How can you use numbers less than 0 to describe real-world situations?

Getting Started

Going the Distance



1. Plot a point at −7 on the number line.

2. Describe the distance from −7 to 0.

3. Plot as many other points as possible on the number line that are the same distance from 0 as −7.

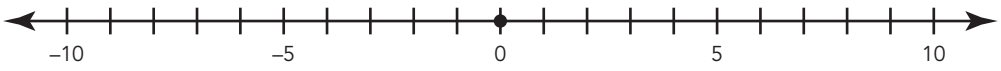
4. How many numbers did you plot? Why do you think this is true?

ACTIVITY
2.1 Absolute Value as Magnitude

Let's revisit the number line from the *Human Number Line* lesson.

Your teacher will assign students to participate in the activity.
Be sure to record what happens on the number line.

- Student A: Stand on 0 and hold one end of the string provided by your teacher.
- Student B: Hold the other end of the string and stand on the number line as far as possible from Student A. Are there other places on the number line that you could stand and be as far from Student A as possible?
- Repeat this activity with two more pieces of string of different lengths and two additional students, Students C and D. Student A will hold the 0 end of each string.



1. Compare the locations where each student stood.

   a. What do you notice about the distances each time the students moved?

   b. What do you notice about the approximate values for the numbers where each stood?

The magnitude, or **absolute value,** of a number is its distance from zero on a number line. The symbol for absolute value is | |. The expression $|n|$ is read as "the absolute value of a number n."

Because distance cannot be negative, the *absolute value* of a number is always positive or 0.

2. Plot 5 on the number line.

a. How far is 5 from 0?

b. $|5| =$ _____

3. Plot -7.2 on the number line.

a. How far is -7.2 from 0?

b. $|-7.2| =$ _____

4. Explain what each statement means. Name any other values that have the same absolute value, when possible.

a. $|-5|$

b. $|1\frac{5}{6}|$

c. $|0.75|$

d. $|-1.36|$

Use your investigation and a number line to answer each question.

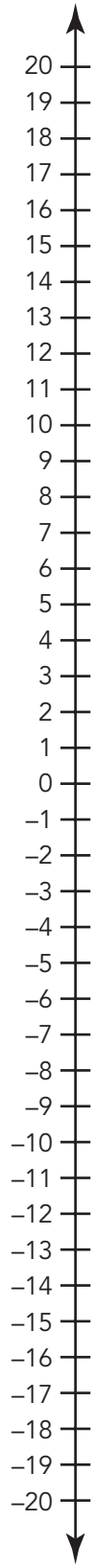
5. Can two different numbers have the same absolute value? Provide examples when possible.

6. What do you notice about the absolute value of

   a. any positive number?

   b. any negative number?

   c. zero?



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ACTIVITY 2.2

Interpreting Absolute Value Statements

Absolute values are used in real-world applications when you are interested in only the number and not in the sign of the number. When you look at temperature changes, you could say the temperature "fell by," "decreased by," or "increased by" an absolute value.

1. Complete the table with an appropriate situation, absolute value statement, and/or number. For the last row, assign the correct units to the number based on your situation.

Situation	Absolute Value Statement	Numeric Example (with units)
The temperature went from 55°F to 5°F.	The temperature fell by 50°F.	−50°F
The bank account balance went from $2500 to $2250.		
The bank account balance went from $495 to $615.		$120
	The water level increased by 4.9 feet.	
During the hike, the elevation went from 1125 feet to 1750 feet.		
		−10 _____

You also use absolute value statements to describe how numbers compare with other numbers. You often use these statements without thinking about "less than" or "greater than." Rather, you use words like "debt," "lost," "colder," "depth," "above," "hotter," or "below."

2. Complete the table with an appropriate situation, absolute value statement, and/or example. For the last row, assign the correct units to the numeric example based on your situation.

Situation	Absolute Value Statement	Numeric Example (with units)
A water level less than $2\frac{1}{2}$ miles below sea level	More than $2\frac{1}{2}$ miles below sea level	-3 miles
An account balance less than $-\$30$	A debt greater than \$30	
A weight less than -7.5 pounds of previous weight	Lost more than 7.5 pounds	
A dive to a height less than -350 feet		
	Colder than 10 degrees below 0	
	A depth greater than 15 m	
	A golf tournament stroke total more than 7 strokes below par	
		-100 _____



PROBLEM SOLVING



ACTIVITY 2.3

Using Absolute Value to Solve Real-World Problems

1. In many buildings, particularly outside of the United States, the ground floor of a building is labeled as *G* or *Lobby*. The first floor of the building is one floor above the ground floor. The building pictured has a lobby, 10 floors of offices, and 4 floors of garage below the lobby.





Ask Yourself . . .
How else can you represent this information?

a. Huyen has an office on the 9th floor and parks on the 3rd floor below the ground floor. William and Abby are determining how many floors Huyen must go up from her car to reach her office.

William represents the 9th floor as 9 and the 3rd floor below ground as -3. Therefore, since $9 - 3 = 6$, Huyen traveled 6 floors to get from her car to her office.

Abby says that the ground floor to the 9th floor is 9 floors, and from the ground floor to the 3rd garage level is 3 floors. Huyen traveled $|9| + |-3| = 9 + 3 = 12$ floors.

Who is correct? Explain your reasoning.

Write a numeric expression using absolute values that would represent each situation. Then, calculate the answer.

b. Luis parks his car on the 2nd floor below ground and works on the 7th floor. How many floors must he go up from his car to reach his office?

c. Olivia is working on the 8th floor. At lunch, she goes to her car on the 4th floor below ground and then back up to the lobby. How many total floors does Olivia travel?

d. When Malik goes from his office on the 10th floor to a meeting on the 5th floor, how many floors does he travel and in which direction?

2. The table shows the Top Notch company's bank balances. The table represents the first 10 weeks of operation. Amounts within parentheses represent overdrafts.

Week	1	2	3	4	5	6	7	8	9	10
Balance	$159.25	($201.35)	$231.57	($456.45)	($156)	($12.05)	$281.34	$175	$192.34	$213

a. Use estimation to determine the gains/losses between consecutive weeks.

Ask Yourself . . .
When might it be more efficient to use a numerical expression instead of counting to determine a difference?

b. Between which two weeks did Top Notch have the largest gain in money? What was the actual gain?

c. Between which two weeks did Top Notch have the largest loss in money? What was the actual loss?

d. What was the difference between the company's lowest balance and its highest balance?

e. Order the estimated gains and losses that you determined in part (a) from least to greatest. Use a negative sign to indicate losses.

f. Order the estimated gains and losses that you determined in part (a) from least to greatest according to their absolute values. What does the absolute value mean in the context of this problem?

g. Why are the orders different in parts (e) and (f)?

3. As part of a long-term science experiment, scientists connected two rulers end-to-end so that the zeros were at the same place. They measured the water level in a pond by placing the connected rulers in the pond so that the water level aligned at zero. They measured the water level each week for 10 weeks.

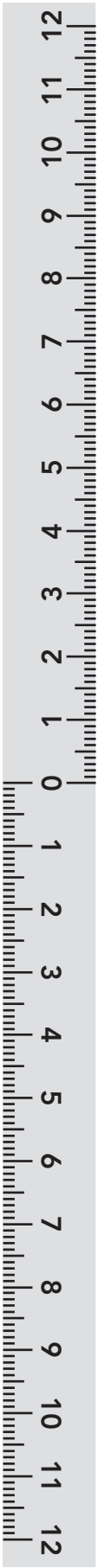
Week	1	2	3	4	5	6	7	8	9	10
Water level (inches)	$2\frac{3}{4}$	$-2\frac{1}{8}$	$1\frac{7}{8}$	$-\frac{3}{4}$	$\frac{3}{4}$	$1\frac{1}{8}$	$2\frac{7}{8}$	$1\frac{1}{4}$	-2	$2\frac{3}{16}$

a. What do the positive numbers represent? What do the negative numbers represent?

b. Between which two weeks did the water level change the most? What was the change?

c. Between which two weeks did the water level change the least? What was the change?

d. How much did the water level change between Weeks 4 and 5? What was the change?



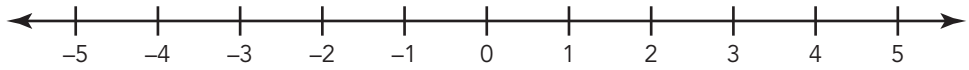
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Talk the Talk

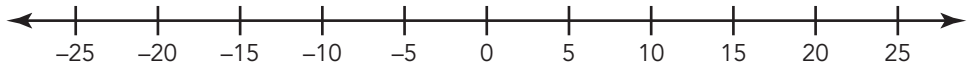
You Absolutely MUST Compare These!

Insert a $>$, $<$, or $=$ symbol to make each statement true.
Justify each answer in terms of the definition of absolute value and number lines.

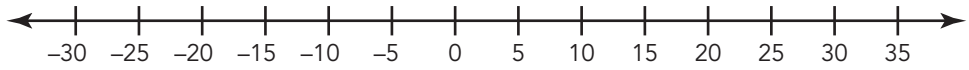
1. $|-4.67| \_\_ |3|$



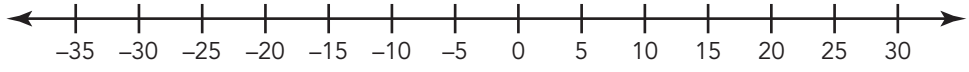
2. $|-15| \_\_ |15|$



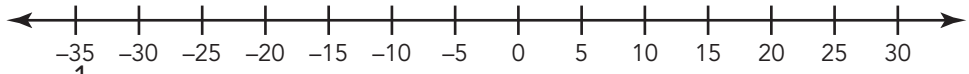
3. $|25\frac{9}{10}| \_\_ |-33\frac{2}{3}|$



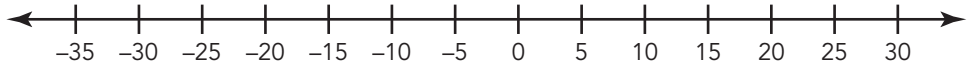
4. $|13.45| \_\_ |-27|$



5. $|-15.34| \_\_ |-1\frac{11}{12}|$



6. $|-19\frac{1}{2}| \_\_ |5.5|$



Lesson 2 Assignment

Write

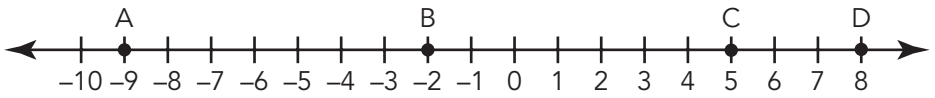
Explain the relationship between a number, its opposite, and its absolute value.

Remember

The *absolute value* of a rational number is its distance from zero on a number line. A numeric expression using absolute values can be used to calculate the distance between positive and negative numbers.

Practice

1. Use the number line to answer each of the following questions:



a. Where is *A* located?

b. What is the opposite of *A*?

c. How far is *A* from 0?

d. What is $|A|$?

e. Where is *B* located?

f. What is the opposite of *B*?

Lesson 2 Assignment

g. How far is B from 0?

h. What is $|B|$?

i. Where is C located?

j. What is the opposite of C?

k. How far is C from 0?

l. What is $|C|$?

m. Where is D located?

n. What is the opposite of D?

o. How far is D from 0?

p. What is $|D|$?

Lesson 2 Assignment

2. Angel is a wrestler for his high school wrestling team in the winter. Angel needs to stay around 140 pounds in the off-season. He charted his weight over the summer by listing the differences his weight was from 140 pounds. He used negative numbers when his weight was under 140 pounds and positive numbers when his weight was above 140 pounds.

Week	1	2	3	4	5	6	7	8	9	10	11	12
Weight Difference	+4.5	+2.1	−1.5	−0.5	−2.5	+1.5	−3.75	−2.8	0	+1.3	−1.5	−5

a. Was the amount his weight varied from 140 pounds in week 4 more or less than the amount it varied from 140 pounds in week 8? Insert a $>$, $<$, or $=$ symbol to make the statement true. Explain your answer.

$$|-0.5| \bigcirc |-2.8|$$

b. Was the amount his weight varied from 140 pounds in week 6 more or less than the amount it varied from 140 pounds in week 11? Insert a $>$, $<$, or $=$ symbol to make the statement true. Explain your answer.

$$|+1.5| \bigcirc |-1.5|$$

Lesson 2 Assignment

c. Use absolute values to determine the difference in Angel's weight from week 7 to week 10.

d. Use absolute values to determine the difference in Angel's weight from week 8 to week 12.

3. The table shown tracks Angel's weight changes that he reports to his coach for the first 4 weeks of school. Complete the table to explain the changes.

Situation	Absolute Value Statement	Rational Number
His weight went from 140 to 135 pounds.	His weight fell by 5 pounds.	
His weight went from 135 pounds to 141 pounds.		6 lbs
His weight went from 141 pounds to 140.5 pounds.		
His weight went from 140.5 pounds to 139 pounds.		

Lesson 2 Assignment

4. Weather experts collect many types of data to study and analyze, including extreme temperature changes. The interior West of North America experiences great temperature changes due to Chinook Winds. The table shows extreme temperature rises in three cities.

Place	Granville, ND	Fort Assiniboine, MT	Spearfish, SD
Date	Feb. 21, 1918	Jan. 19, 1892	Jan. 22, 1943
Time Period	12 hours	15 minutes	2 minutes
Temperature Change	From $-33°F$ to $50°F$	From $-5°F$ to $37°F$	From $-4°F$ to $45°F$

For each city, write an absolute value equation and use it to determine how much the temperature rose.

a. Granville, ND

b. Fort Assiniboine, MT

c. Spearfish, SD

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Lesson 2 Assignment

5. Michael measured the rainfall and evaporation using a rain gauge in his backyard for 8 days. Michael marked his rain gauge with values from -6 inches to $+6$ inches and filled the gauge with water to the zero mark. For each question, write an expression using absolute value and then calculate the answer.

Days	1	2	3	4	5	6	7	8
Gauge Reading	0.5	-1.3	3.7	4.2	2.1	-0.9	-2.4	5.6

a. On how many days out of the eight did it rain?

b. Between which two consecutive readings did it rain the most? How many inches were recorded?

c. Between which two consecutive readings was evaporation the greatest? How many inches of water evaporated?

Lesson 2 Assignment

d. Calculate the gain or loss of water in the rain gauge between days 1 and 2. Express the change in the water level in the gauge as a positive or negative number.

e. Calculate the gain or loss of water in the rain gauge between days 2 and 3. Express the change in the water level in the gauge as a positive or negative number.

Prepare

Represent each decimal or precent as a fraction in lowest terms.

1. 0.3

2. 2.8

3. $\frac{3}{4}\%$

4. 212%

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3 Rational Number System

OBJECTIVES

- Classify numbers according to their number systems.
- Apply and extend an understanding of whole numbers and integers to the system of rational numbers.
- Understand ordering of rational numbers.

NEW KEY TERMS

- integers
- ellipsis
- rational numbers
- Density Property

You use many different types of numbers in math class and in the world, including positive and negative whole numbers, fractions, and decimals.

How can you organize and classify different types of numbers?

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Getting Started

Sort It Out!

Cut out the cards found at the end of the lesson. Then, analyze and sort the numbers into different groups. You may group them in any way you feel is appropriate, but you must sort the numbers into more than one group.

1. For each of your groups,
   - create a title that fits the numbers in that group.
   - list the numbers included.
   - write a rationale for why you grouped those particular numbers.

2. Compare your sort with your classmates' sorts. Create a list of the different ways your class grouped the numbers.

ACTIVITY
3.1 Analyzing Number Sorts

1. Daniela grouped these numbers together. Why do you think she put these numbers in the same group?

   $$0,\ -452,\ 9,\ 24,\ |-3|,\ -3,\ -(-9),\ |-452|$$

2. Javier had a group similar to Daniela's but he did not include -452 and -3. Why do you think Javier omitted these numbers from his group?

3. Destiny said that she created two groups: Group 1 contains all the numbers that someone could write as fractions, and Group 2 contains all the numbers that someone could not write as fractions. Analyze Destiny's sorting idea.

   a. Which numbers do you think Destiny placed in Group 2?



   b. Gabriela is not sure about Destiny's sort. She thinks that she could write all of the numbers fractions. Is Gabriela correct? Explain why or why not.

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You have used different sets of numbers, including the set of natural, or counting, numbers and the set of whole numbers.

4. Identify the numbers from the sort that are in each set.

   a. Natural numbers

   b. Whole numbers

Notice the three periods before and after specific numbers in the set. These three periods are called an **ellipsis,** and they are used to represent infinity in a number set.



Throughout this topic, you have been learning about the set of *integers*. **Integers** are the set of whole numbers with their opposites. The integers can be represented by the set {..., −3, −2, −1, 0, 1, 2, 3, ...}.

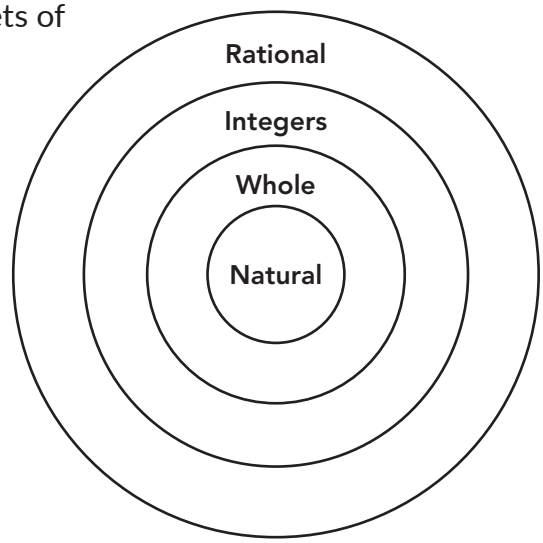
5. Identify the numbers from the sort that are included in the set of integers.

You have also worked with *rational numbers* throughout this year. **Rational numbers** are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

6. Identify the numbers from the sort that are included in the set of rational numbers.

ACTIVITY
3.2 Classifying Numbers

There are many ways you can classify numbers. As you saw in the previous activity, many of the classifications are subsets of other classifications. The diagram shows the different sets of numbers you have encountered in your mathematical experiences.



Natural numbers are a subset of whole numbers.

Whole numbers are a subset of integers.

Integers are a subset of rational numbers.

Pin the number on the bullseye! Your teacher will direct students to pin (or tape) a number card to its correct location in the diagram of the rational number set.

1. For each value, check all of the number sets to which it belongs.

Number	Natural Number	Whole Number	Integer	Rational Number
3				
3.222				
0				
−4.5				
$-\frac{3}{5}$				
54				
−5				
$\frac{23}{3}$				
0.667				
−1,364,698				

2. Complete the table with the unknown examples and descriptions.

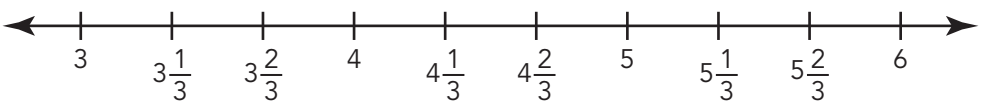
	Natural Numbers	Whole Numbers	Integers	Rational Numbers
Examples	1, 2, 3, ...		..., −3, −2, −1, 0, 1, 2, 3, ...	
Description	Counting numbers	Natural numbers and 0		

ACTIVITY
3.3 Density

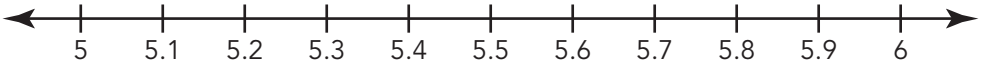
The **density property** states that between any two rational numbers there is another rational number. The property is not true for natural numbers, whole numbers, or integers. For example, there is no integer between 25 and 26. There is no whole number or natural number between 12 and 13.

1. Plot the given rational numbers. Then plot and label a rational number between each pair of rational numbers.

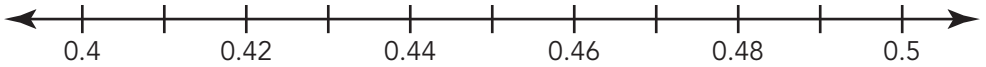
   a. $4\frac{1}{3}$ and $4\frac{2}{3}$



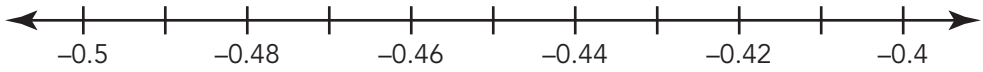
   b. 5.5 and 5.6



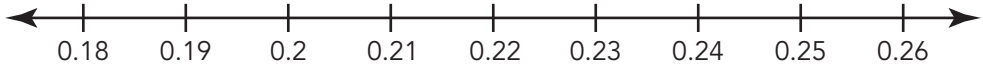
   c. 0.45 and 0.46



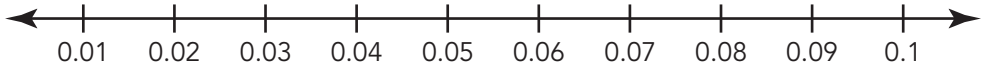
d. -0.45 and -0.46



e. 23% and 0.24



f. 5.5% and 0.065



Complete each rational number line with a partner.

2. Create a number line from 0 to 1. Your goal is to plot and label a rational number closer to 1 than your partner.



Partner 1: Plot a rational number, A, between 0 and 1 that is close to 1.

Partner 2: Plot a rational number, B, between A and 1.

Repeat at least two more times.

3. Create a number line from -1 to 0. Your goal is to plot and label a rational number closer to 0 than your partner.



Partner 1: Plot a rational number, A, between -1 and 0 that is close to 0.

Partner 2: Plot a rational number, B, between A and 0.

Repeat at least two more times.

4. Create a number line from -6 to -5. Your goal is to plot and label a rational number closer to -5 than your partner.



Partner 1: Plot a rational number, A, between -6 and -5 that is close to -5.

Partner 2: Plot a rational number, B, between A and -5.

Repeat at least two more times.

Talk the Talk

Do They Always Belong?

Determine whether each statement is true or false. Justify your answer using definitions and/or examples.

1. All whole numbers are rational numbers. True False

2. All rational numbers are whole numbers. True False

3. All rational numbers are integers. True False

4. All integers are rational numbers. True False

5. All whole numbers are integers. True False

6. All integers are whole numbers. True False

0	-5.78	$2\frac{15}{16}$	$\frac{3}{4}\%$	-452
$\frac{1}{2}$	24	9	$\frac{6}{7}$	$-\frac{6}{7}$
-0.5	0.5	$-\frac{1}{2}$	2.5%	5.78
-3	$\lvert -3 \rvert$	$-\frac{2}{3}$	$\frac{1}{1000}$	0.001
-6.41	$\lvert 6.41 \rvert$	$-(-9)$	$\lvert -452 \rvert$	-0.3
225%	$6\frac{1}{4}$	25%	0.25%	$\left\lvert \frac{215}{16} \right\rvert$

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Why is this page blank?

So you can cut out the numbers on the other side

Lesson 3 Assignment

Write

Define each term in your own words.

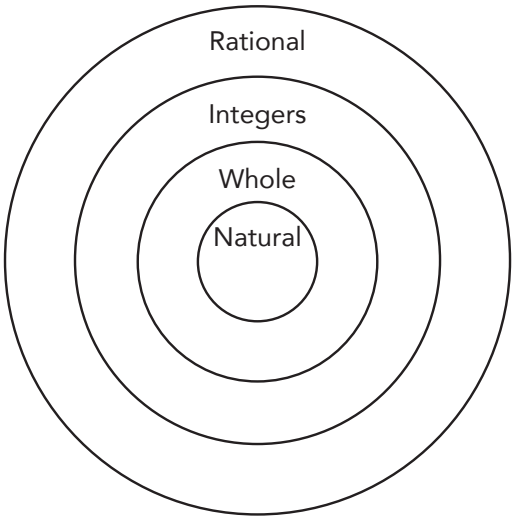
1. The set of rational numbers
2. The Density Property

Remember

Rational numbers include all numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and b is not zero.

Practice

1. Write each of the numbers in the correct section of the Venn diagram.
   $-2, 0, 6, \frac{1}{2}, -0.25, 11, -6, 1, 14.2, 23$



2. Write all the sets of numbers to which each value belongs.

   a. The tundra covers about $\frac{1}{5}$ of Earth's surface.

   b. The average annual temperature is $-18°$ Fahrenheit.

   c. As of 2002, there are 48 varieties of land mammals found in the tundra region.

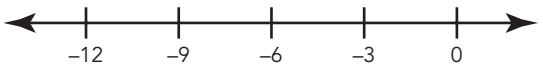
   d. The permafrost is a layer of frozen soil that is located below Earth's surface at -1496 m.

   e. During the summer months, the low temperature averages about 37.4° F.

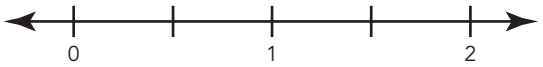
Lesson 3 Assignment

3. Hannah collects data about some animals. Determine a rational number between each pair of rational numbers. Plot all three numbers on a number line.

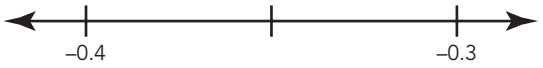
   a. A mole's runway is between 3 and 12 inches under the ground.



   b. The musky rat kangaroo weighs between $\frac{3}{4}$ and $\frac{3}{2}$ pounds.



   c. The percent of change of the Alaskan polar bear population in the past year was between -0.33 and -0.32.



4. All the students in a class were reading the same book. Order the amount of the book these five students read last week from least to greatest.
   Portion of the Book:
   Student A: $\frac{14}{24}$
   Student B: 25.3%
   Student C: 0.425
   Student D: $\frac{5}{8}$
   Student E: 47.2%

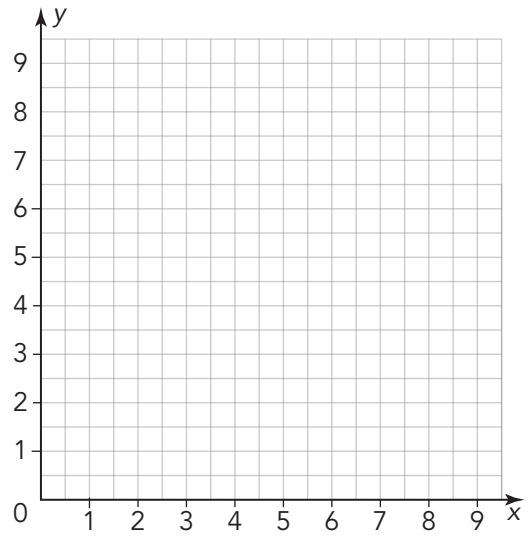
5. Five musicians each spent time practicing during the week. Order the time the musicians spent practicing during the week from greatest to least.
   Portion of the Week:
   Piano player: $\frac{32}{70}$
   Trumpet player: 63.4%
   Guitarist: 0.489
   Singer: $\frac{7}{18}$
   Drummer: 32.5%

Lesson 3 Assignment

Prepare

Plot each point.

A (3, 5), B (0, $\frac{9}{2}$), C (6, 1.5), D (8.5, 0), E (0, 0)



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4 Extending the Coordinate Plane

OBJECTIVES

- Identify the four quadrants of the coordinate plane and the characteristics of points located in each.
- Locate and plot ordered pairs of positive and negative rational numbers on the coordinate plane.
- Use absolute value to determine distances on the coordinate plane.
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.

NEW KEY TERM

- quadrants

You can locate and plot ordered pairs of positive numbers on a coordinate plane.

How can you extend the plane to include ordered pairs of any rational number?

Getting Started

All About Extending

Consider the coordinate plane that you have used to graph points where both the *x*- and *y*-coordinates were zero or positive numbers.

1. Based on what you have learned about number lines:

   a. What do you know about the number line that makes up the *x*-axis? Extend that number line and label it appropriately.

   b. What do you know about the number line that makes up the *y*-axis? Extend that number line and label it appropriately.

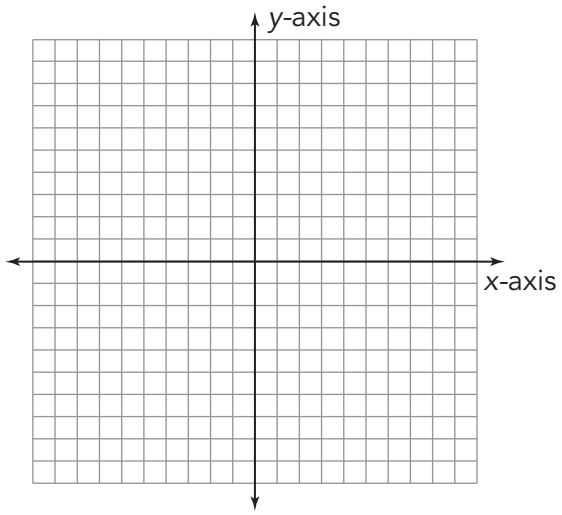
2. The point where the *x*-axis and *y*-axis intersect is known as the *origin*. Label the point of intersection with its coordinates.

By extending the number lines that form the axes, you have created the entire coordinate plane.

3. How many regions are created when you extend the coordinate plane to all rational numbers?

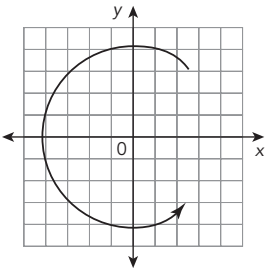
The regions on the coordinate plane are called **quadrants**. They are numbered with Roman numerals from one to four (I, II, III, IV), starting in the upper right-hand quadrant and moving counterclockwise.

4. Label each of the quadrants on your coordinate plane.



The coordinate plane is often called the *Cartesian* coordinate plane, named for René Descartes.

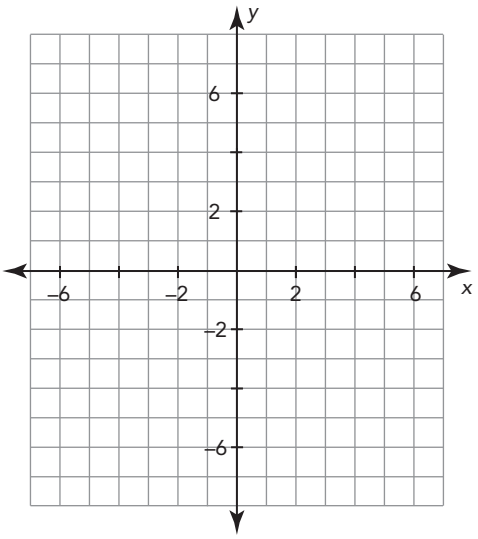
Counterclockwise refers to the opposite direction to the way in which the hands of a clock move.



ACTIVITY
4.1 Human Coordinate Plane

Your teacher is going to direct students to stand at certain locations on the human coordinate plane.

1. For each student, plot and label the point where the student is standing on the coordinate plane. Then, record the coordinates of that point in the table.

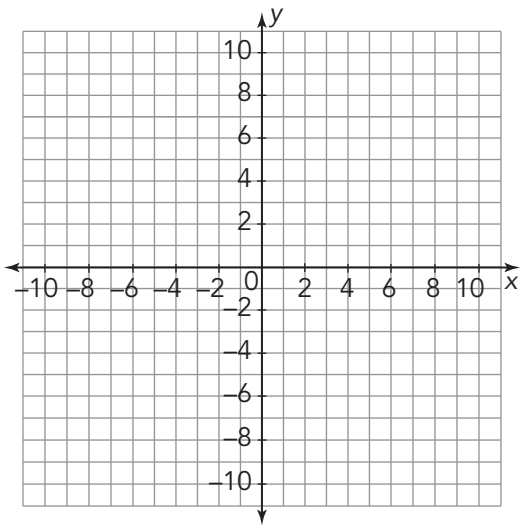


Student	Location	Student	Location
A		H	
B		I	
C		J	
D		K	
E		L	
F		M	
G		N	

2. Where did each student always start? How did each student know which direction to go first?

3. What do you notice about the coordinates of the points that are in the same quadrant of the coordinate plane?

Your teacher is going to select students to plot ordered pairs that meet specific conditions. The students will select locations that satisfy those conditions.





4. For each student, plot and label the point where the student is standing on the coordinate plane. Then, record the coordinates of that point in the table.

Student	Condition	Location
A	Anywhere	
B	Negative x-coordinate	
C	Negative y-coordinate	
D	On an axis	
E	In QII	
F	In QIII	

5. Compare the ordered pairs you have plotted and identified in this activity. What is similar about the points you graphed in each region or axis of the graph?

a. QI:

b. QII:

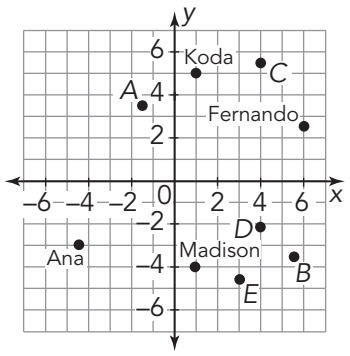
c. QIII:

d. QIV:

e. x-axis:

f. y-axis:

6. A teacher has all the students who were absent for this activity come by after school to recreate the experience. Some students recorded their names, while other students noted their location with a letter.



a. What are Fernando's, Ana's and Koda's coordinates?

b. Alexander had the coordinate $\left(-\frac{3}{2}, 3.5\right)$. Which point represents Alexander?

c. Which student(s) and/or point(s) are in Quadrant IV?

d. Which student is the same distance from the *x*-axis as point *A*?

e. Jacob had the coordinates $\left(4, 5\frac{1}{2}\right)$. Which point represents Jacob?

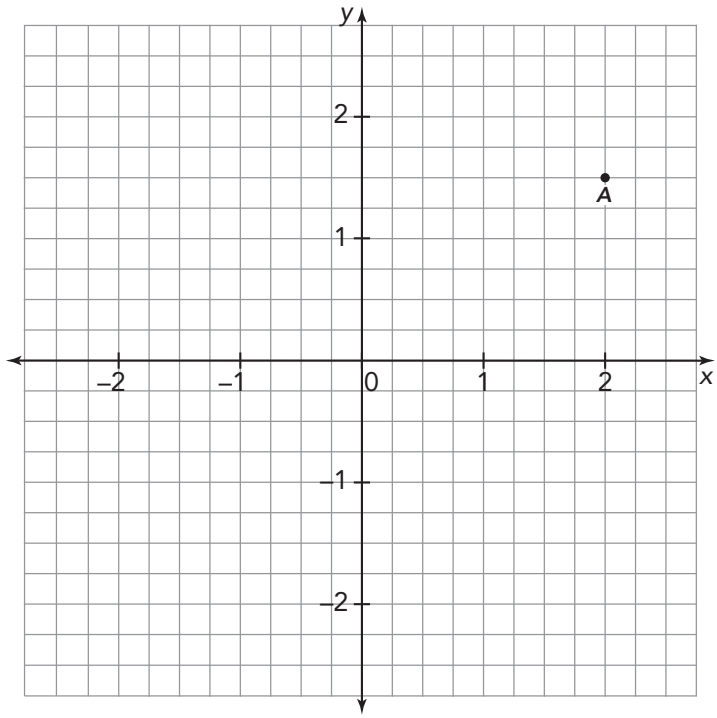
f. The teacher is 5 units to the left of Fernando. What are the coordinates of the teacher?

g. How far is the teacher from Madison?

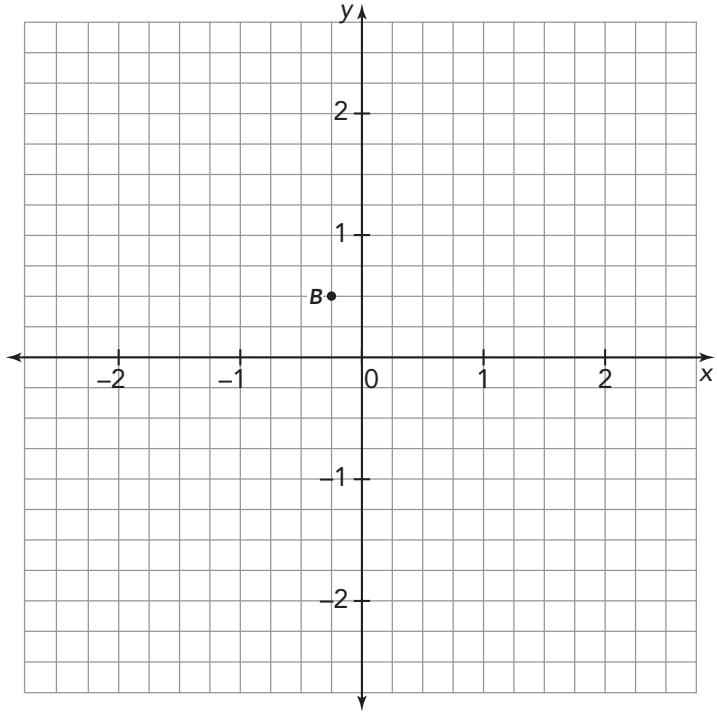
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ACTIVITY
4.2 Horizontal and Vertical Distance on the Coordinate Plane

1. The graph shows the point *A*. Plot the point *C* in Quadrant IV that is 3 units from point *A*.

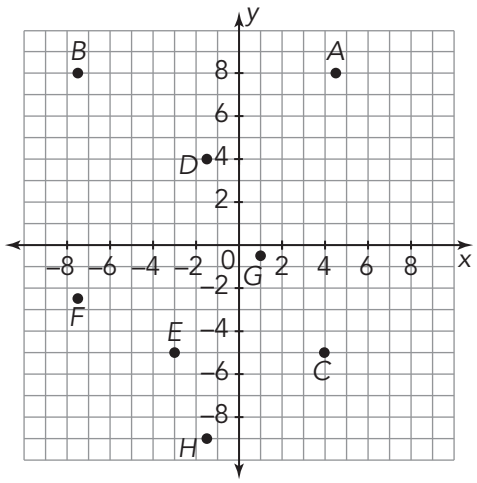


2. The graph shows the point *B*. Plot the point *D* in Quadrant I that is $\frac{3}{4}$ unit from point *B*.



You have used absolute value to determine differences between values. In the same way, you can use absolute value to determine distances on the coordinate plane.

3. Consider points *A* and *B*.



a. Use the coordinate plane to determine the distance from point *A* to point *B*.

b. Describe how the coordinates of points *A* and *B* are similar.

c. Write a numeric expression using the absolute value of the *x*-coordinates of the points and calculate the distance.

4. Consider points *B* and *F*.

a. Use the coordinate plane to determine the distance from point *B* to point *F*.

b. Describe how the coordinates of points *B* and *F* are similar.

c. Write a numeric expression using the absolute value of the *y*-coordinates of the points and calculate the distance.

5. Write a numeric expression using absolute value and calculate the distance from:

   a. $(-\frac{5}{4}, -4.5)$ to $(3.75, -4.5)$.

   b. point *B* to $(-7.5, 3.25)$.

6. Which point is $4\frac{1}{2}$ units away from $\left(-\frac{3}{2}, -\frac{1}{2}\right)$?

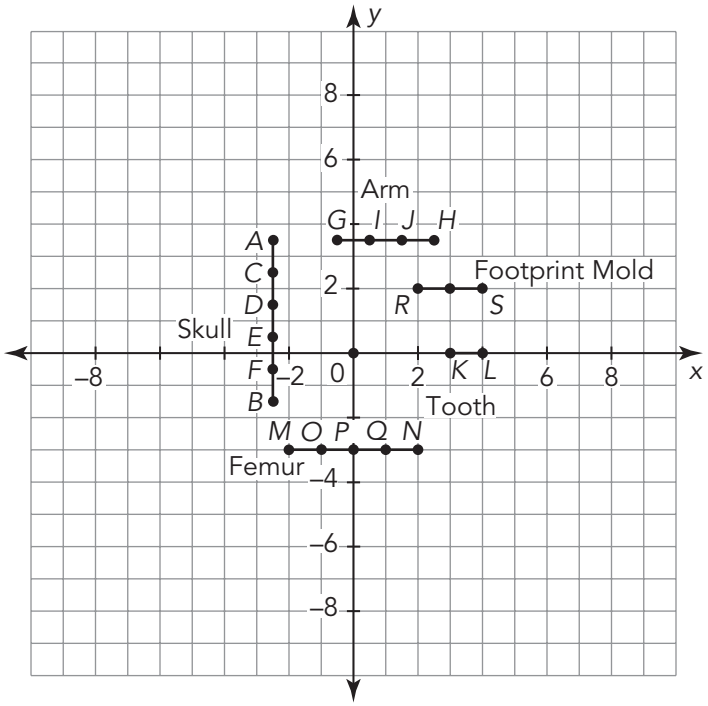
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ACTIVITY
4.3 T-Rex Dig

In the T-Rex Dig game, players place the "bones" of their dinosaur horizontally or vertically on a coordinate grid. Players then take turns guessing the location of each other's dino bones using coordinates. Once a player has located all of the other player's dino bones, the game is over.

Let's look at a sample game board and questions that might be asked to uncover all the dino bones.

1. Use the game board to answer questions about the T-Rex fossils. (Each grid line is 1 foot long.)



a. How long is the T-Rex's skull? Write an absolute value equation to justify your answer.

Ask Yourself . . .
How can you organize and record your mathematical ideas?

b. How many coordinates must be guessed to completely "uncover" the skull?

c. How long is the T-Rex's arm? Write an absolute value equation to justify your answer.

d. What is the greatest number of quadrants crossed by any one fossil?

e. Are any fossils on an axis? When any fossils are on an axis, identify the axis, the fossil, and the coordinates of the fossil(s).

2. Your turn! Use the graph paper provided at the end of the lesson. Use the bottom grid to plot and label your 5 fossils. You may want to label some of the coordinates to help you as you play the game. Use the top grid to record the coordinates you ask of your partner.

As you play the game, ask your opponent mathematical questions. For example, you can ask:

- Is the femur symmetric over an axis?
- How many of your fossils are vertical?
- Are any of the fossils on an axis? (But you can't ask which axis!)
- Do any of the fossils share an ending x-coordinate with another fossil?

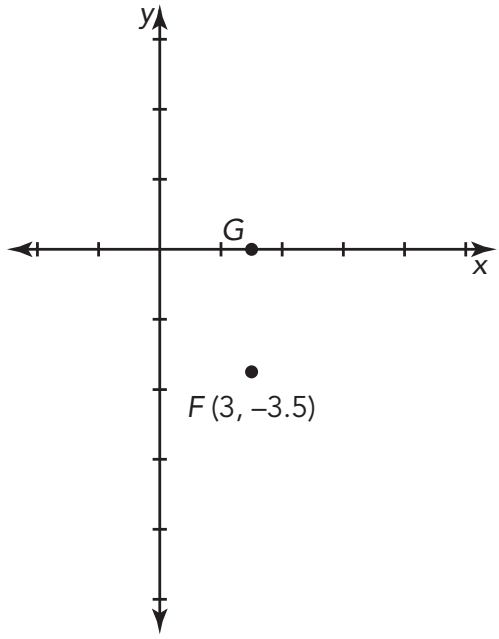
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Talk the Talk

Determining Coordinates

Use the graph and information provided to answer each question.

- The graph shows the locations of point *F* and point *G*.
- Point *G* is on the *x*-axis and has the same *x*-coordinate as point *F*.
- Point *H* is located at $(-4, a)$.
- The distance from point *F* to point *G* is half the distance from point *F* to point *H*.

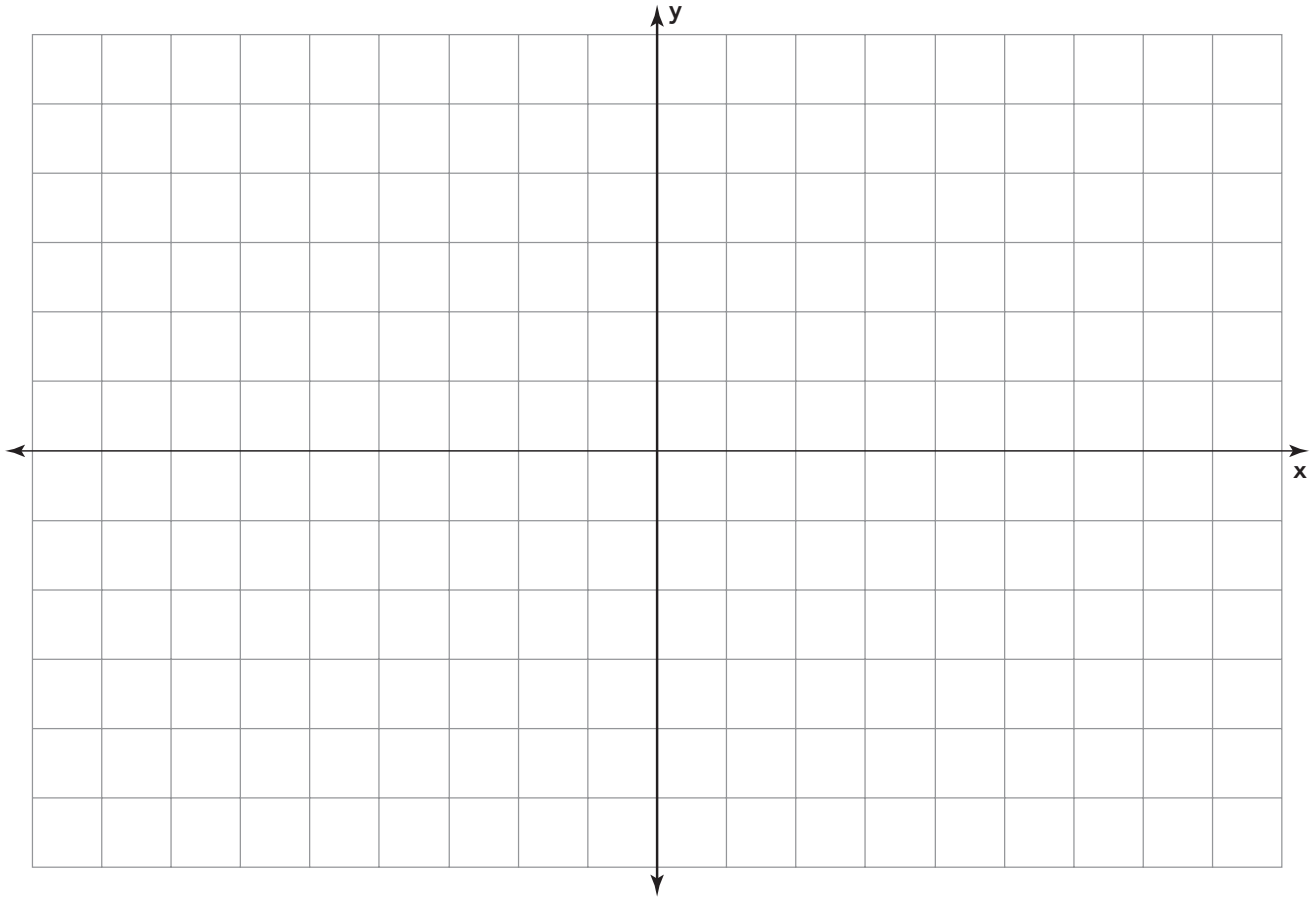


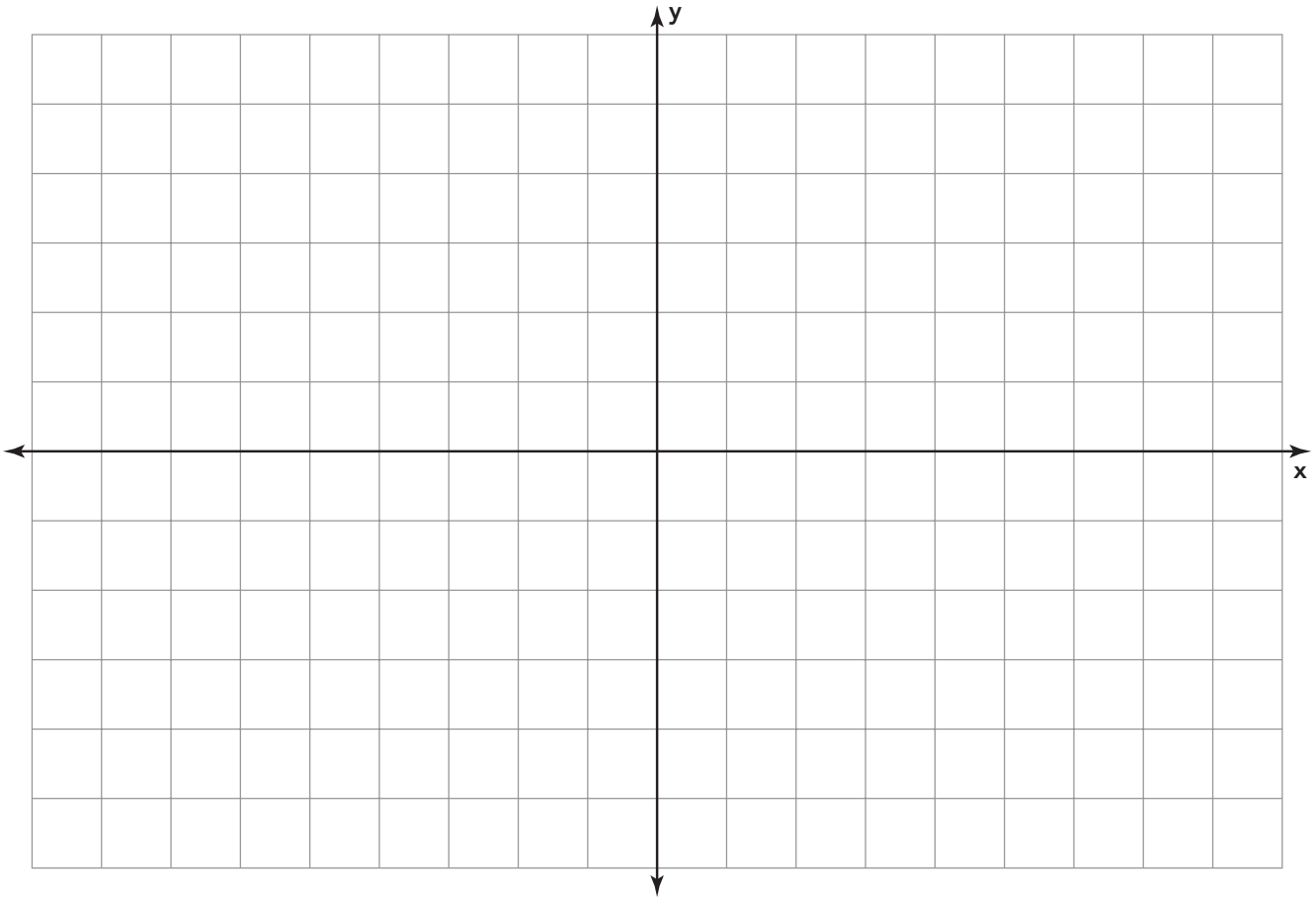
1. What is the value of a? Explain how you determined this coordinate.

2. Plot point J so that the distance from point F to point J is the same as the distance from point F to point H. Explain how you decided where to plot point J.

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T-Rex Dig Game Board





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So you can tear out the game board on the other side

Lesson 4 Assignment

Write

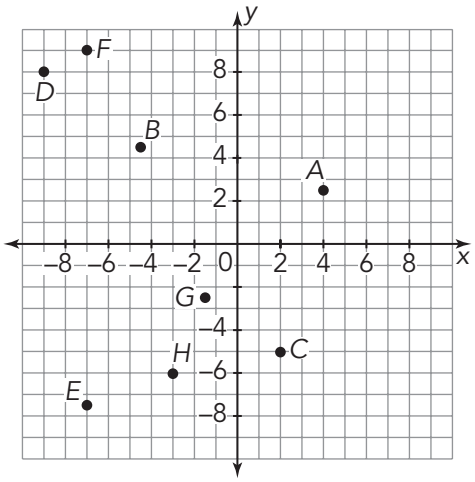
Use the terms *axis*, *quadrant*, and *coordinates* to explain how ordered pairs that differ only by sign are related to each other.

Remember

The Cartesian coordinate plane is formed by two perpendicular number lines that intersect at the zeros, or the origin. The intersecting number lines divide the plane into four regions, called *quadrants*.

Practice

1. Identify the ordered pair associated with each point graphed on the coordinate plane.



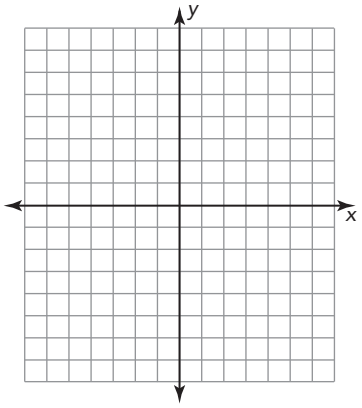
2. Plot and label the locations of points *P* through *Z* on a coordinate plane. Draw line segments from point to point, beginning and ending at point *P*. Describe the resulting figure.

P $\left(0, \frac{25}{5}\right)$ *Q* $(1, 3)$ *R* $(4, 3)$

S $(1.5, 1)$ *T* $\left(3.7, -\frac{30}{10}\right)$ *V* $\left(0, -\frac{2}{3}\right)$

W $\left(-3.7, -\frac{30}{10}\right)$ *X* $(1.5, 1)$ *Y* $(-4, 3)$

Z $(-1, 3)$



Lesson 4 Assignment

3. Plot the ordered pair (a, b) in Quadrant I of a coordinate plane and the ordered pair (c, d) in Quadrant III. Plot and label each additional ordered pair. Explain how you knew where to plot each point.

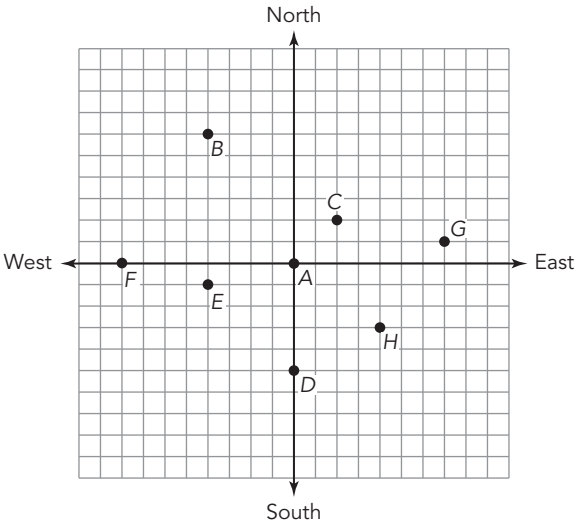
   a. $(-a, b)$ b. $(a, -b)$ c. $(-a, -b)$

   d. $(-c, d)$ e. $(c, -d)$ f. $(-c, -d)$

4. The coordinate plane shown represents a map of Amir's neighborhood. Each square represents one city block. Amir's house is located at point A, which is the origin. The other points represent the following locations.

   B – Bank
   C – Paul's friend Javier's house
   D – Grocery Store
   E – Post Office
   F – Middle School
   G – Playground
   H – Orthodontics Office

   Explain how Amir can get to the given destination from his house when he first walks east or west (right or left) and then walks north or south (up or down). Then, determine the coordinates of the destination point and the quadrant in which the point is located.





Lesson 4 Assignment

a. Bank

b. Orthodontics Office

c. Javier's house

d. Playground

e. Post Office

5. Identify the ordered pairs associated with *B* and *E*. Describe how the ordered pairs are similar.

6. Write a numeric expression using the absolute value of the *y*-coordinates of the points to calculate the distance between *B* and *E*.

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Lesson 4 Assignment

7. How can a numeric expression using absolute values help you calculate the distance from one point to another on the coordinate plane when the points are on the same vertical or horizontal line?

Prepare

Sketch a number line and plot each value.

1. -2
2. 0
3. 1
4. $\frac{1}{2}$
5. 3

TOPIC 1 SELF-REFLECTION

Name: _____

Signed Numbers and the Four Quadrants

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents I **am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Signed Numbers and the Four Quadrants* topic by:

TOPIC 1: *Signed Numbers and the Four Quadrants*	Beginning of Topic	Middle of Topic	End of Topic
identifying a rational number and its opposite.	○	○	○
using rational numbers to represent quantities in real-world situations.	○	○	○
identifying and positioning a rational number on a vertical or horizontal number line.	○	○	○
identifying and recognizing numbers with opposite signs as numbers on opposite sides of 0 on a number line.	○	○	○
classifying whole numbers, integers, and rational numbers using a visual representation to describe relationships between sets of numbers.	○	○	○
ordering rational numbers on a number line.	○	○	○
ordering a set of rational numbers arising from real-world contexts.	○	○	○

continued on the next page

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TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Signed Numbers and the Four Quadrants*	Beginning of Topic	Middle of Topic	End of Topic
defining the absolute value of a rational number as its distance from 0 on a number line.	○	○	○
interpreting absolute value as magnitude for a positive or negative quantity in a real-world situation.	○	○	○
identifying the absolute values of rational numbers.	○	○	○
identifying in which quadrant a point is located based on the signs of the numbers in its ordered pair.	○	○	○
plotting points in all four quadrants of the coordinate plane using ordered pairs of rational numbers.	○	○	○
calculating the distances between two points with the same first coordinate or the same second coordinate using absolute value.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Signed Numbers and the Four Quadrants* topic.

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?



TOPIC 1 SUMMARY

Signed Numbers and the Four Quadrants Summary

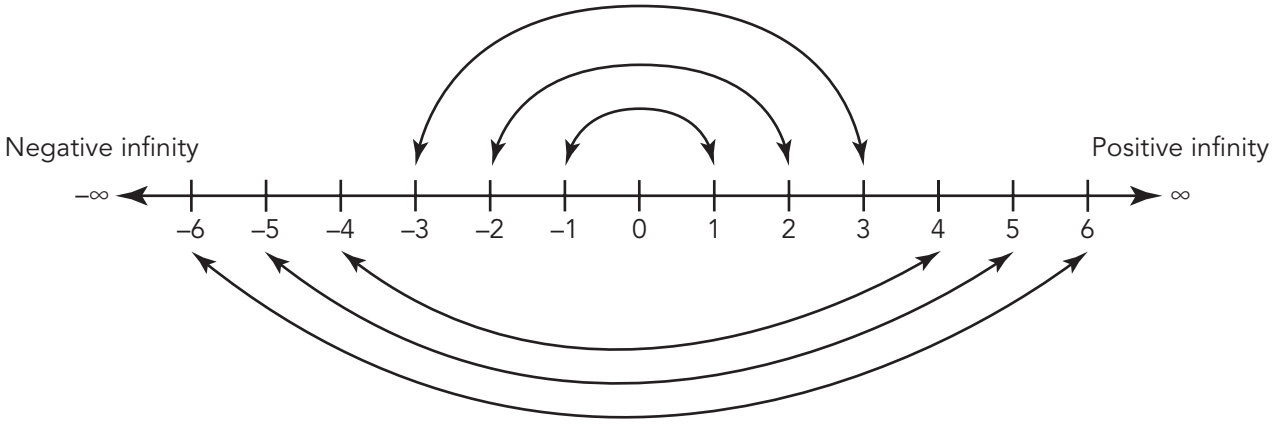
NEW KEY TERMS

- negative numbers [números negativos]
- infinity [infinito]
- absolute value [valor absoluto]
- integers [enteros]
- ellipsis
- rational numbers [números racionales]
- Density Property [Propiedad de Densidad]
- quadrants [cuadrantes]

LESSON 1 Introduction to Negative Numbers

You can create a number line by reflecting the positive numbers across zero. The values to the left of zero on the number line are called **negative numbers** and are labeled with a negative sign. You can write a positive number with a positive sign or without any sign. For example, positive 5 can be written as +5 or 5.

The positive values extend to positive infinity, and the negative numbers extend to negative infinity. **Infinity**, represented by the symbol ∞, means a quantity with no end or bound.

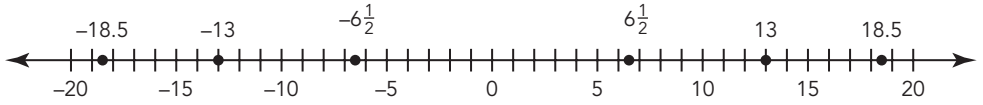


Opposite numbers are reflections of each other across 0 on the number line.

- The opposite of a positive number is a corresponding negative number.
- The opposite of a negative number is a corresponding positive number.

Attaching a negative sign to a number means reflecting that number across zero on the number line. The number 0 is the only number that doesn't have an opposite.

For example, the numbers $6\frac{1}{2}$, -13, -18.5, and their opposites are plotted on the number line.



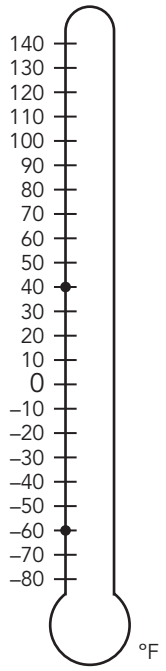
Number lines can also be vertical, like a thermometer or a measure of elevation.

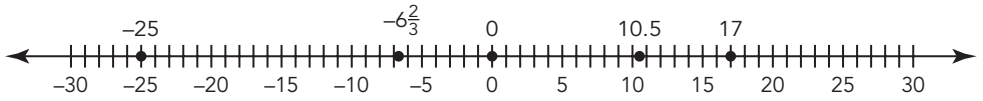
You can use a thermometer to plot temperatures and to compare and order temperatures. In vertical number lines like this one, the greater the value, the higher up on the number line.

For example, compare 40 degrees to -60 degrees. By plotting each temperature on the thermometer, you can see that 40 degrees is above -60 degrees. Therefore $40 > -60$.

You can compare different types of numbers by plotting the numbers on a number line.

For example, the numbers, $-6\frac{2}{3}$, 10.5, -25, 17, and 0 have been plotted on the number line. You can use the number line to order the values from least to greatest.





From the number line you can determine that -25 has the least value because it is the farthest to the left; 17 has the greatest value because it is the farthest to the right. The numbers ordered from least to greatest are -25, $-6\frac{2}{3}$, 0, 10.5, and 17.

LESSON 2

Absolute Value

The magnitude, or **absolute value**, of a number is its distance from zero on a number line. The symbol for absolute value is | |. The expression $|n|$ is read as "the absolute value of a number n." Because distance cannot be negative, the absolute value of a number is always positive or 0.

$|9| = 9$, because 9 is 9 units from 0 on a number line.

$|-3.8| = 3.8$, because -3.8 is 3.8 units from 0 on a number line.

Absolute values are used in real-world applications when you are interested in only the number and not in the sign of the number. You also use absolute value statements to describe how numbers compare with other numbers.

Situation	Absolute Value Statement	Numeric Example
The temperature went from 55°F to 5°F.	The temperature fell by 50°F.	−50°F
The bank account balance went from $550 to $795.	The balance increased by $245.	$245
A water level went from 10.3 feet to 6.7 feet.	A water level fell by 3.6 feet.	−3.6 feet
A water level less than $-2\frac{1}{2}$ miles below sea level	More than $2\frac{1}{2}$ miles below sea level	−3 miles
A temperature less than −5° F	Colder than 5°F below 0	−8°F
An account balance less than −$100	A debt greater than $100	−$110

Absolute value equations calculate the distance between positive and negative numbers to solve real-world problems.

For example, the Top Notch company's bank balances are shown. The table shown represents the first 6 weeks of operation. Overdrafts, which are a negative balance, are represented by amounts within parentheses. What was the gain or loss between Weeks 2 and 3?

Week	1	2	3	4	5	6
Balance	$159.25	($201.35)	$231.57	($456.45)	($156)	($12.05)

At the end of week 2, the company had a negative balance of $201.35 and at the end of week 3 it had a positive balance of $231.57. The company had a gain between these two weeks because it went from a lesser balance to a greater balance. The gain is equal to the sum of the absolute values of the two balances.

$$|-\$201.35| + |\$231.57| = \$201.35 + \$231.57 = \$432.92$$

LESSON 3

Rational Number System

Integers are the set of whole numbers and their opposites. The integers can be represented by the set { ... , −3, −2, −1, 0, 1, 2, 3, ... }. The three periods before and after the numbers in the set are called an **ellipsis**, and they are used to represent infinity in a number set.

Rational numbers are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and b does not equal 0.

There are many ways you can classify numbers. Many of the classifications are subsets of other classifications. The diagram shows the different sets of numbers you have encountered in your mathematical experiences.

Natural numbers are a subset of whole numbers.

Whole numbers are a subset of integers.

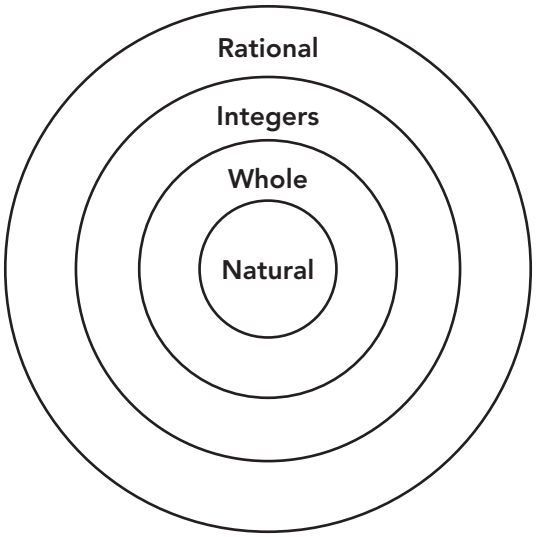
Integers are a subset of rational numbers.

The number 2 is a rational number, an integer, a whole number, and a natural number.

The number 0 is a rational number, an integer, and a whole number.

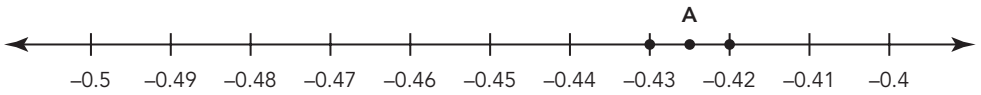
The number -11 is a rational number and an integer.

The numbers 12.5 and $-\frac{3}{4}$ are both rational numbers.



The **density property** states that between any two rational numbers there is another rational number.

For example, consider the rational numbers -0.42 and -0.43 and the number line shown. The number represented by point A is another rational number that falls between -0.42 and -0.43 such that $-0.43 < A < -0.42$. Point A could represent the value -0.425.



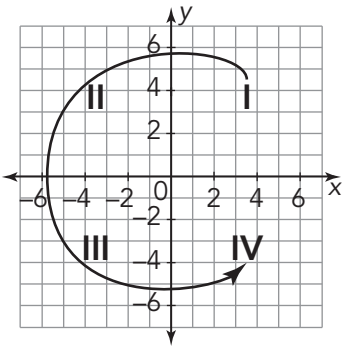
The property is not true for natural numbers, whole numbers, or integers. For example, there is no integer between -25 and -26. There is no whole number or natural number between 12 and 13.

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LESSON 4

Extending the Coordinate Plane

The Cartesian coordinate plane is formed by two perpendicular number lines that intersect at the zeros, or the origin. The intersecting number lines divide the plane into four regions, called **quadrants**.



The quadrants are numbered with Roman numerals from one to four (I, II, III, IV) starting in the upper right-hand quadrant and moving counterclockwise.

To plot an ordered pair on the coordinate plane, begin at the origin (0, 0), and first move the distance along the *x*-axis given by the *x*-value of the ordered pair. Move right for a positive value and move left for a negative value. Then, move the distance along the *y*-axis given by the *y*-value of the ordered pair. Move up for a positive value and move down for a negative value.

The table shows the signs of the coordinates for ordered pairs in each quadrant.

Quadrant II	Quadrant I
(−, +)	(+, +)
Quadrant III	**Quadrant IV**
(−, −)	(+, −)

For example, the following points are plotted on the coordinate plane:

$A\ (-4, \frac{1}{2})$

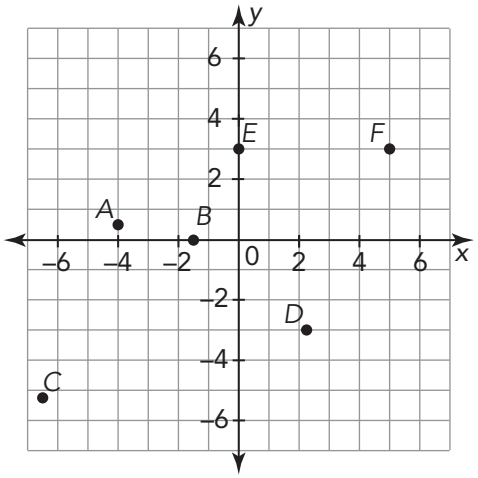
$B\ (-\frac{3}{2}, 0)$

$C\ (-6.5, -5.25)$

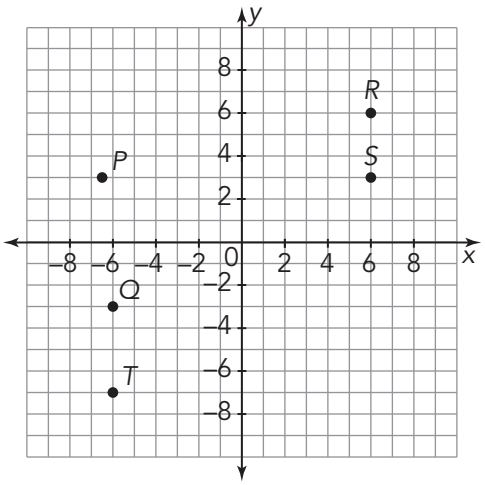
$D\ (2\frac{1}{4}, -3)$

$E\ (0, 3)$

$F\ (5, 3)$



You can use absolute value to determine distances on the coordinate plane.



For example, the distance from point P to point Q is $|3| + |-3| = 3 + 3 = 6$ units.

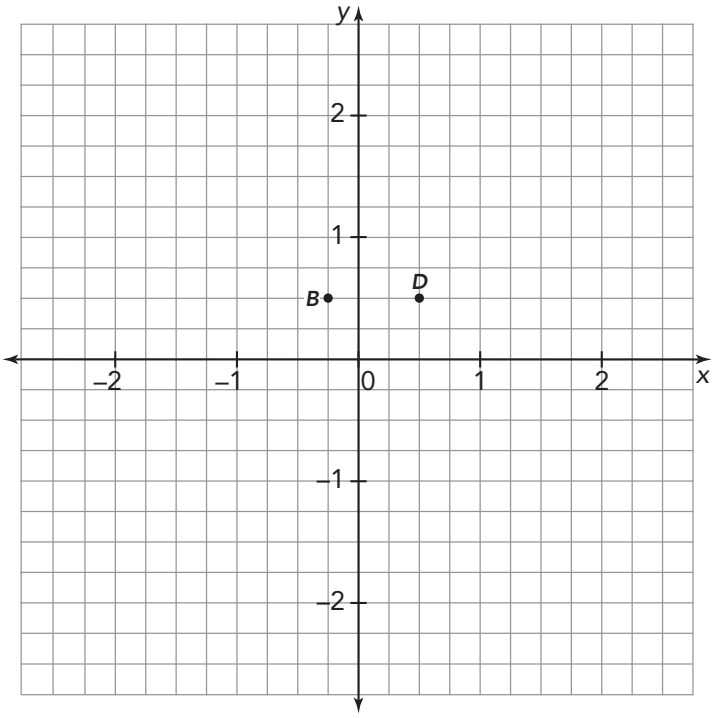
The distance from point P to point S is $|-6.5| + |6| = 6.5 + 6 = 12.5$ units.

The distance from point R to point S is $|6| - |3| = 6 - 3 = 3$ units.

The distance from point Q to point T is $|-7| - |-3| = 7 - 3 = 4$ units.

You can plot a point on the coordinate plane that is a certain distance from a given point.

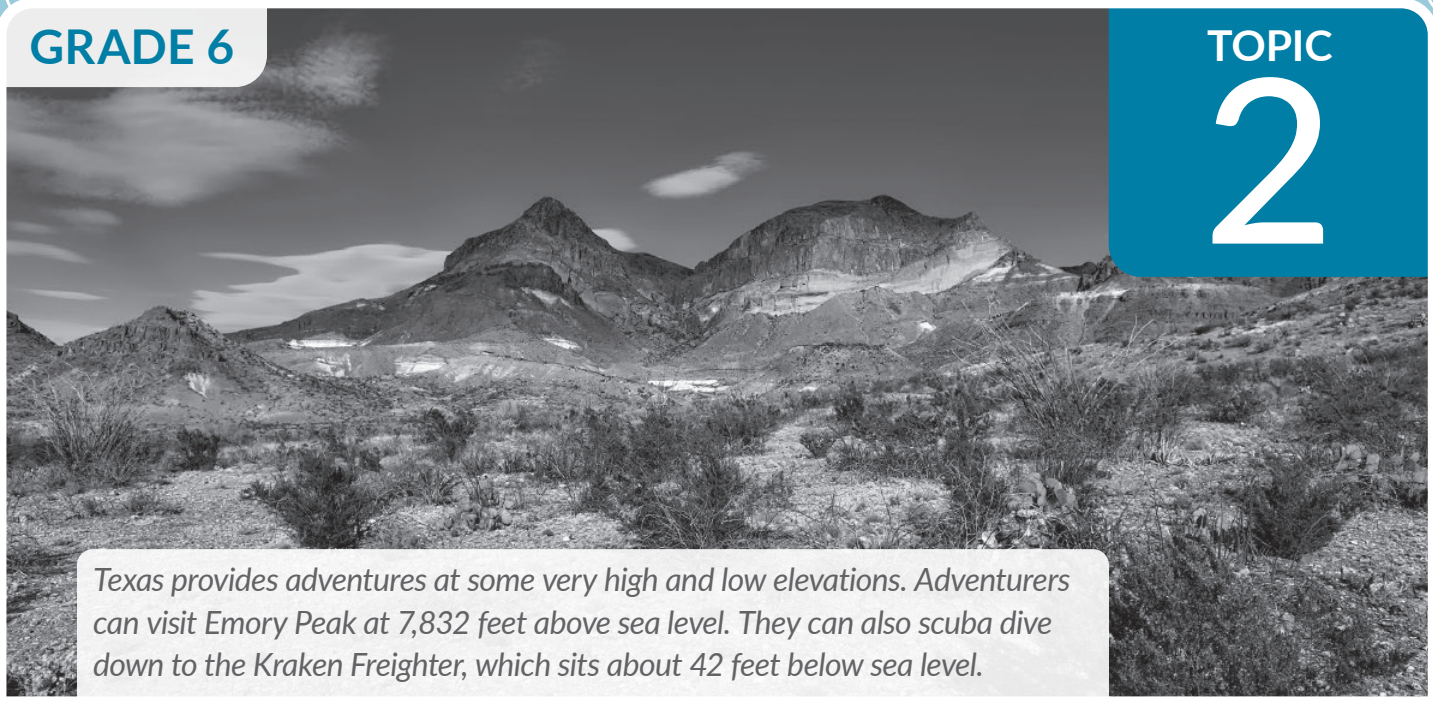
The graph shows the point B. Plot the point D in Quadrant I that is $\frac{3}{4}$ units from point B.



The coordinate of point D are $(\frac{1}{2}, \frac{1}{2})$.

GRADE 6

TOPIC 2



Texas provides adventures at some very high and low elevations. Adventurers can visit Emory Peak at 7,832 feet above sea level. They can also scuba dive down to the Kraken Freighter, which sits about 42 feet below sea level.

Operating with Integers

LESSON 1 Using Models to Understand Integer Addition **543**

LESSON 2 Adding Integers, Part I **557**

LESSON 3 Adding Integers, Part II **573**

LESSON 4 Subtracting Integers **599**

LESSON 5 Multiplying and Dividing Integers **633**

1 Using Models to Understand Integer Addition

OBJECTIVES

- Represent numbers as positive and negative integers.
- Use a number line diagram to represent the sum of positive and negative integers.

You have learned about negative numbers and can plot locations on a number line.

Does addition and subtraction work the same with negative numbers as with positive numbers?

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Getting Started

Hut! Hut! Hike!

The playing field and footballs are located at the end of the lesson. You also need two number cubes, one red and one black.

You and a partner are going to play **Math Football**. You will take turns rolling two number cubes to determine how many yards you can advance the football toward your end zone.

Player 1 will be the Home Team and Player 2 will be the Visiting Team. In the first half, the Home Team will move toward the Home end zone, and the Visiting Team will move toward the Visiting end zone.

Rules

Players both start at the zero yard line and take turns. On your turn, roll two number cubes, one red and one black. The number on each cube represents a number of yards. Move your football to the left the number of yards shown on the red cube. Move your football to the right the number of yards shown on the black cube. Start each of your next turns from the ending position of your previous turn.

Scoring

When players reach their end zone, they score 6 points. When players reach their opponent's end zone, they lose 2 points. An end zone begins on either the +10 or −10 yard line.

Example:

	Player	Starting Position	Results of the Number Cubes Roll	Ending Position
First Turn	Home Team	0	Red 3 and Black 5	+2
	Visiting Team	0	Red 5 and Black 6	+1
Second Turn	Home Team	+2	Red 1 and Black 6	+7
	Visiting Team	+1	Red 6 and Black 2	−3

1. Read through the table. After two turns, which player is closer to their end zone?

ACTIVITY
1.1

Signed Numbers as Values with Directions

Let's play Math Football. Begin by selecting the home or visiting team. Your teacher will set the length of time for each half. You will play two halves. Make sure to switch ends at half-time with the Home Team moving toward the Visiting end zone, and the Visiting Team moving toward the Home end zone. Good luck!

Ask Yourself: What tools or strategies can you use to solve this problem?

1. Once your game is finished, answer each question.

   a. When you are trying to get to the Home end zone, which number cube do you want to show the greater value? Explain your reasoning.

   b. When you are trying to get to the Visiting end zone, which number cube do you want to show the greater value? Explain your reasoning.

   c. Did you ever find yourself back at the same position you ended on your previous turn? Describe the values on the cubes that would cause this to happen.

   d. Describe the roll that causes you to move your football the greatest distance either left or right.

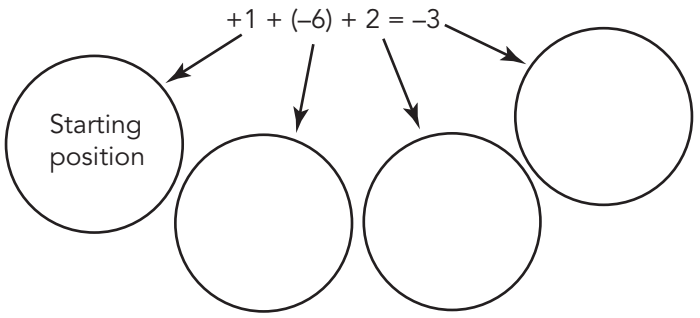
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ACTIVITY
1.2 Writing Equations with Signed Numbers

You can write equations to describe the results of number cube rolls. Think of the result of rolling the red number cube as a negative number and the result of rolling the black number cube as a positive number.

	Player	Starting Position	Results of the Number Cubes Roll	Ending Position	Equation
First Turn	Home Team	0	Red 3 and Black 5	+2	$0 + (-3) + 5 = +2$
	Visiting Team	0	Red 5 and Black 6	+1	$0 + (-5) + 6 = +1$
Second Turn	Home Team	+2	Red 1 and Black 6	+7	$+2 + (-1) + 6 = +7$
	Visiting Team	+1	Red 6 and Black 2	−3	$+1 + (-6) + 2 = -3$

1. Describe each part of the equation for the second turn of the Visiting Team player.



Play Math Football again. But this time, work with your partner to get to the Home end zone together in the first half and the Visiting end zone in the second half. Write equations to record your moves.

2. Think about the number cube rolls you made in the game.

   a. What kind of rolls move you closer to the Home end zone?

   b. What kind of rolls move you closer to the Visiting end zone?

3. Write an equation for each situation. Use the game board for help.

   a. The Home Team player starts at the zero yard line and rolls a red 6 and a black 2. What is the ending position?

      Equation: _____

   b. The Visiting Team player starts at the zero yard line and rolls a red 5 and a black 4. What is the ending position?

      Equation: _____

   c. The Home Team player starts at the 5 yard line and rolls a red 2 and a black 2. What is the ending position?

      Equation: _____

   d. The Visiting Team player starts at the -5 yard line and rolls a red 4 and a black 6. What is the ending position?

      Equation: _____



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e. Suppose the Home Team player is at the +8 yard line. Complete the table and write two equations that will put the player into the Home end zone.

Starting Position	Roll of the Red Number Cube	Roll of the Black Number Cube	Equation
+8			
+8			

f. Suppose the Visiting Team player is at the −8 yard line. Complete the table and write two equations that will put the player into the Visiting end zone.

Starting Position	Roll of the Red Number Cube	Roll of the Black Number Cube	Equation
−8			
−8			

Talk the Talk

Mission: Possible, and Impossible

Consider the moves you made in the Math Football game.

1. In which direction would you move when you roll:

   a. A greater number on the black cube than on the red cube?

   b. A greater number on the red cube than on the black cube?

   c. Two black cubes?

   d. A black cube and a red cube?

   e. Two red cubes?

2. Is it possible to decrease in value when rolling two black cubes? Explain your reasoning.

3. Is it possible to increase in value when rolling two red cubes? Explain your reasoning.

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Home Team
Visiting Team
10
9
8
7
6
5
4
3
2
1
0
–1
–2
–3
–4
–5
–6
–7
–8
–9
–10
Black
Red

Why is this page blank?

So you can cut out the Game Field on the other side.

Lesson 1 Assignment

Write

In your own words, explain how to decide whether the sum of two numbers is less than, equal to, or greater than 0.

Remember

Combining positive and negative moves together on a line results in a move to the left, a move to the right, or staying in the same position, depending on the size of the positive and negative values.

Practice

1. Determine the ending position by adding and subtracting the indicated steps from each starting position.

Starting Position	Steps Backward	Steps Forward	Ending Position
+3	4	5	
+7	6	2	
+5	2	4	
0	5	8	
−4	3	7	
+1	7	9	
−6	1	5	
−2	5	6	
+8	3	1	
−9	2	4	

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Lesson 1 Assignment

2. Write an equation to represent the movement indicated by the starting point, steps backward, and steps forward.

Starting Position	Steps Backward	Steps Forward	Equation
+2	4	7	
−7	3	5	
+6	9	4	
+4	6	1	
−5	2	9	
0	5	3	
−3	1	4	
−8	2	6	
0	8	2	
+9	7	8	

Lesson 1 Assignment

Prepare

A large hotel has a ground floor (street level) and 26 floors of guest rooms above street level, which can be modeled by positive integers. There are 5 floors of parking below street level, which can be modeled by negative integers. In this hotel, street level is represented by zero.

Write an integer addition expression that models the hotel elevator's motion in each case.

1. The elevator starts at street level, goes up 7 floors, and then goes down 3 floors.

2. The elevator starts at street level, goes up 10 floors, and then goes down 12 floors.

3. The elevator starts at street level, goes down 4 floors, and then goes up 11 floors.

4. The elevator starts at street level, goes down 2 floors, goes up 5 floors, and finally goes down 3 floors.

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2 Adding Integers, Part I

OBJECTIVES

- Model the addition of integers on a number line.
- Develop a rule for adding integers.
- Identify $p + q$ as the number located a distance of $|q|$ from p.

You have been adding and subtracting positive numbers most of your life. In previous courses, you learned how to add numbers using a number line.

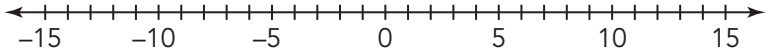
How can a number line be a helpful tool when adding positive and negative numbers?

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Getting Started

Getting on Line

Use the number line and determine the number described by each. Explain your reasoning.



1. The number that is 7 more than −9

2. The number that is 2 more than −6

3. The number that is 10 more than −8

4. The number that is 10 less than 6

5. The number that is 5 less than −4

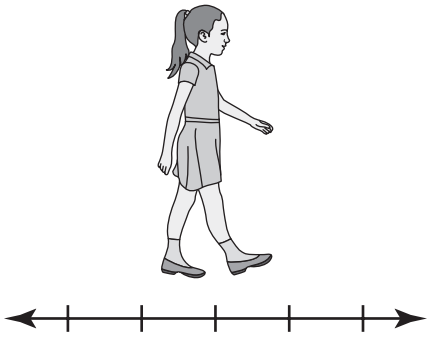
6. The number that is 2 less than −4

ACTIVITY
2.1 Walking the Number Line

Walking a number line can help you to add positive and negative numbers.

Walk the number line for an addition sentence:

- Start at zero and walk to the value of the first term of the expression.
- To indicate addition, turn to face up the number line, toward the greater positive numbers.
- Walk forward when adding a positive number or walk backward when adding a negative number.



Your teacher will select a classmate to walk the line for each of the given problems. Help your classmate by preparing the directions that are needed.

1. Complete the table.

	Where You Start	Direction You Face	Walk Backward or Forward	Final Location
$1 + 3$				
$0 + (-4)$				
$-3 + 5$				
$-1 + (-4)$				

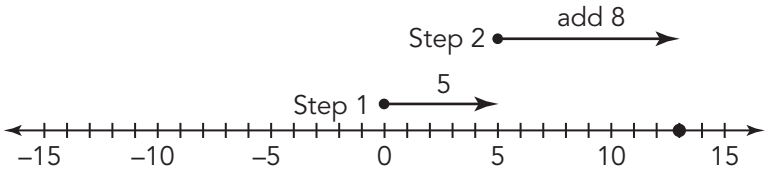
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This Worked Example represents the movement created by walking the number line.

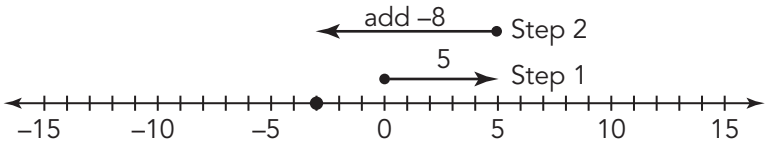
WORKED EXAMPLE

A number line can be used to model integer addition.
When adding a positive integer, move to the right on a number line.
When adding a negative integer, move to the left on a number line.

Example 1: The number line shows how to determine 5 + 8.



Example 2: The number line shows how to determine 5 + (−8).



In an addition sentence, the terms being added together are called addends.

Compare the first steps in each example.

2. What distance does the first addend in each example show?

Remember that the *absolute value* of a number is its distance from 0.

Ask Yourself . . .
What observations can you make?

3. Describe the graphical representation of the first addend. Where does it start and in which direction does it move? How does this movement represent walking the line?

4. What is the absolute value of the first addend in each example?

Compare the second steps in each example.

5. What distance does the second addend in each example show?

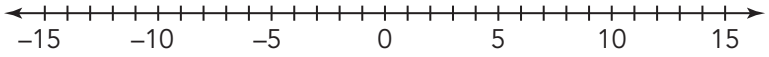
6. Why did the arrows for the second addend both start at the endpoints of the first addend but then continue in opposite directions? Explain your reasoning.

7. What is the absolute value of the second term in each example?

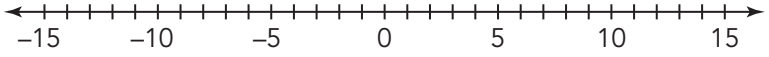
8. Use the number line to determine each sum. Show your work.

Think about walking the line as you model these sums on the number line.

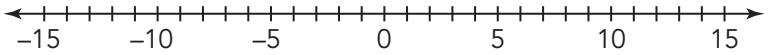
a. $-3 + 7 =$ _____



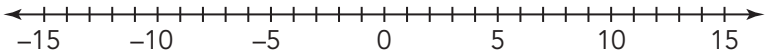
b. $3 + (-7) =$ _____



c. $-3 + (-7) =$ _____



d. $3 + 7 =$ _____



Notice that the first addend in each expression in parts (a) through (d) was either 3 or -3.

9. What do you notice about the distances shown by these addends on the number lines?

10. What is the absolute value of each addend?

Notice that the second addend in each expression was either 7 or -7.

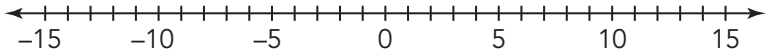
11. What do you notice about the distances shown by these addends on the number lines?

12. What is the absolute value of each addend?

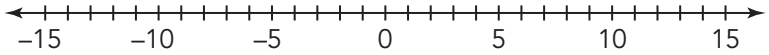
ACTIVITY
2.2 Adding on Number Lines

Now that you have the feel for how to move on the number line when adding negative numbers, it is time to practice with more examples. Use the number line to determine each sum. Show your work.

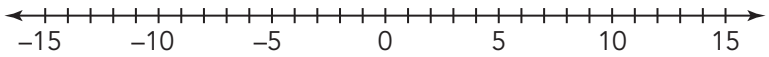
1. $-9 + 5 =$ _____



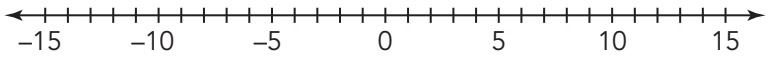
2. $9 + (-5) =$ _____



3. $-9 + (-5) =$ _____



4. $9 + 5 =$ _____



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Notice that the first addends in each expression in Questions 1 through 4 was either 9 or -9.

5. What do you notice about the distances shown by these addends on the number lines?

6. What is the absolute value of each addend?



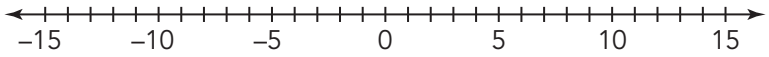
Notice that the second addend in each expression was either 5 or -5.

7. What do you notice about the distances shown by these addends on the number lines?

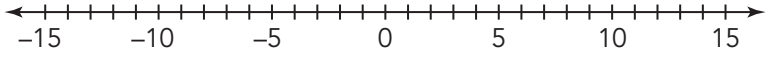
8. What is the absolute value of each addend?

Use the number line to determine each sum. Show your work.

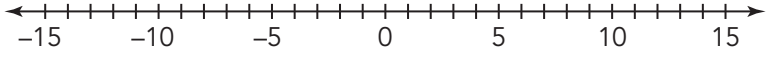
9. $-8 + 2 =$ _____



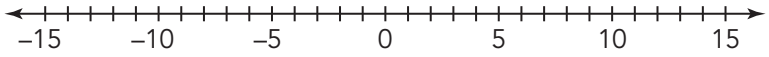
10. $8 + (-2) =$ _____



11. $-8 + (-2) =$ _____

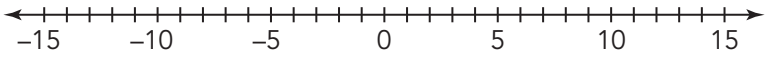


12. $8 + 2 =$ _____

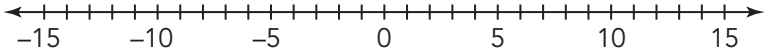


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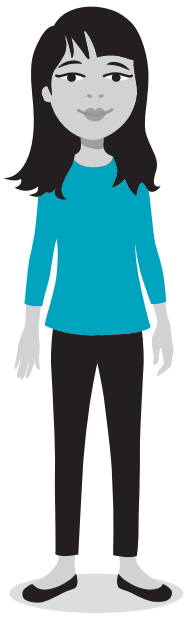
13. $-4 + 11 =$ _____



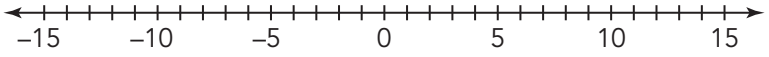
14. $4 + (-11) =$ _____



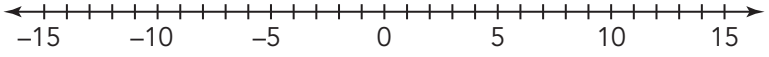
Can you see why the absolute value is important when adding signed numbers?



15. $-4 + (-11) =$ _____



16. $4 + 11 =$ _____



Talk the Talk

Patterns on the Line

Demonstrate what you have learned about adding two numbers using a number line.

1. Describe the patterns from the *Adding on Number Lines* activity, when you:

   a. Add two positive numbers.

   b. Add two negative numbers.

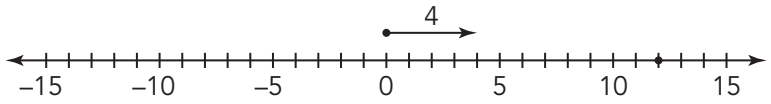
   c. Add a negative and a positive number.

2. Do you think these patterns will hold true for all numbers, even fractions and decimals? Explain your reasoning.

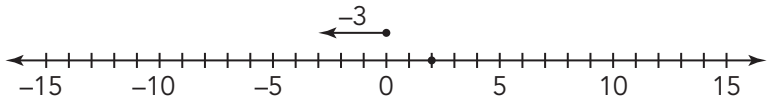
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3. Complete each number line model and equation.

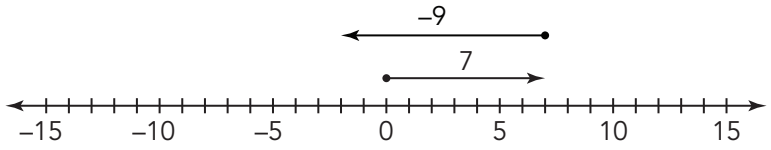
a. $4 + \_\_\_\_\_ = 12$



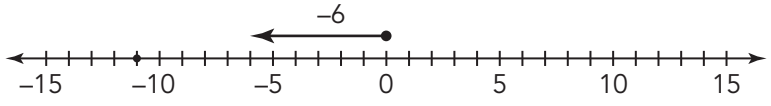
b. $-3 + \_\_\_\_\_ = 2$



c. $7 + \_\_\_\_\_ = -2$



d. $-6 + \_\_\_\_\_ = -11$



Lesson 2 Assignment

Write

Explain how walking the line is the same as representing addition and subtraction on the number line.

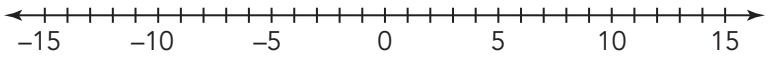
Remember

When adding a positive integer on a number line, move to the right on the number line. When adding a negative integer, move to the left on the number line.

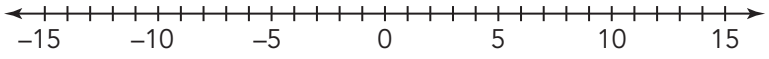
Practice

Use the number line to determine each sum. Show your work.

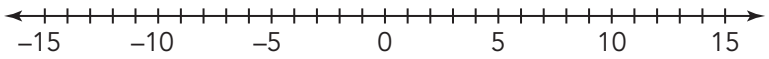
1. $-6 + 4$



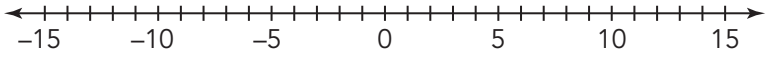
2. $-9 + (-2)$



3. $13 + (-12)$

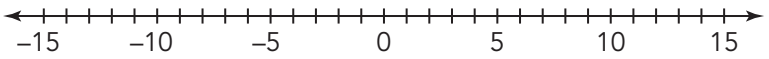


4. $7 + (-14)$

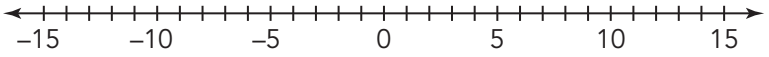


Lesson 2 Assignment

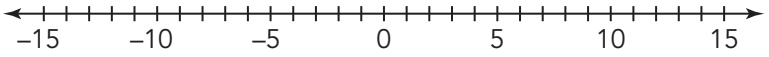
5. $7 + (-1)$



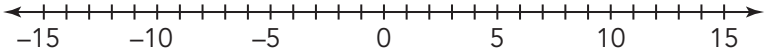
6. $3 + (-13)$



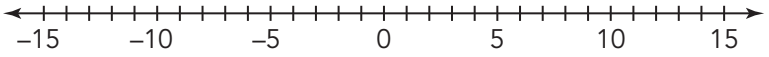
7. $8 + (-8)$



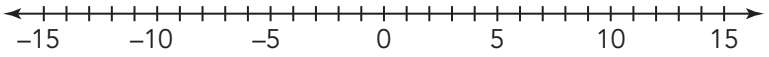
8. $-2 + 8$



9. $-13 + 3$



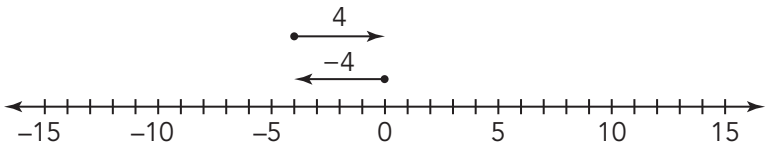
10. $0 + (-12)$



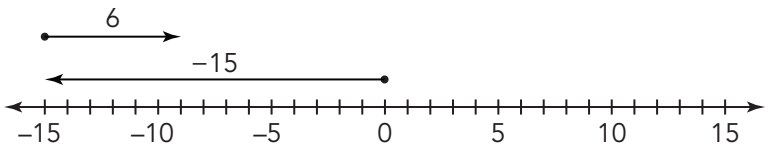
Lesson 2 Assignment

Complete each number line model and equation.

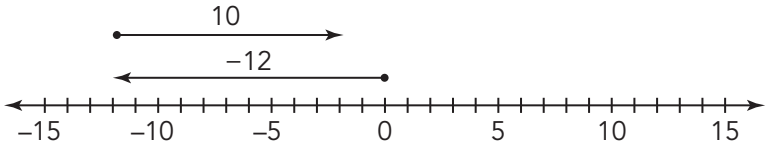
11. $-4 + \_\_\_\_\_ = 0$



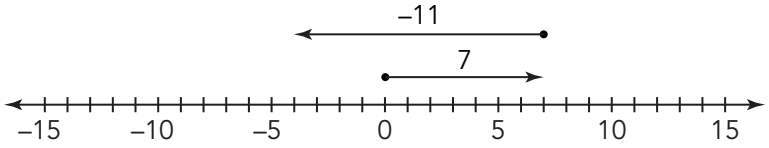
12. $-15 + \_\_\_\_\_ = -9$



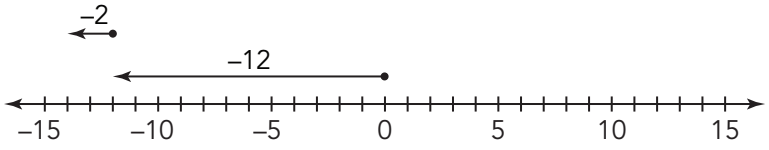
13. $\_\_\_\_\_ + 10 = -2$



14. $\_\_\_\_\_ + (-11) = -4$



15. $-12 + \_\_\_\_\_ = -14$



Lesson 2 Assignment

Prepare

Use a number line to determine each sum. Then, write a sentence to describe the movement you used on the number line to compute the sum of the two integers.

1. $-2 + 1$

2. $-5 + 8$

3. $-2 + (-3)$

4. $4 + (-6)$

3 Adding Integers, Part II

OBJECTIVES

- Describe situations in which opposite quantities combine to make 0.
- Model the addition of integers using two-color counters.
- Develop a rule for adding integers.
- Apply previous understandings of addition and subtraction to add integers.

NEW KEY TERMS

- additive inverse property
- additive inverses

You know how to use a number line to model adding positive and negative numbers.

Do the patterns you noticed from the number line model apply to other models for adding positive and negative numbers?

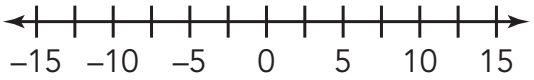
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Getting Started

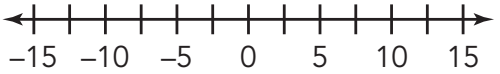
Creating Zero

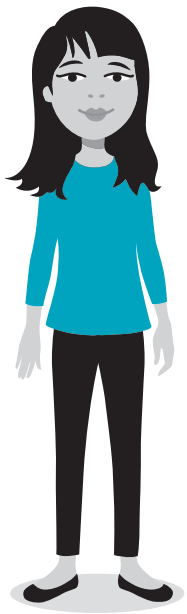
Use a number line to illustrate how the sum of two numbers can be zero.

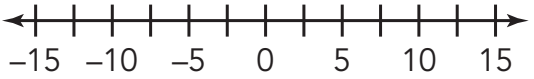
1. Write 3 examples of number sentences that sum to zero and draw the number line models to support your solutions.



How can you end at zero when you start at zero?







2. What pattern do you notice?

3. Describe a real-world situation in which two numbers would sum to zero. Write the number sentence that could be used to represent the situation.

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ACTIVITY
3.1 Additive Inverses

The **additive inverse property** states that the sum of a number and its opposite is zero. Two numbers with the sum of zero are called **additive inverses**.

Addition of integers can also be modeled using two-color counters that represent positive (+) charges and negative (–) charges. One color, usually red, represents the negative number, or negative charge. The other color, usually yellow, represents the positive number, or positive charge. In this book, gray shading will represent the negative number, and no shading will represent the positive number.

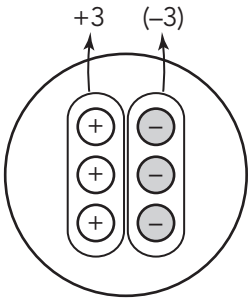
⊖ = –1 ⊕ = +1

Can you create two-color-counter models of the sums you wrote in the *Creating Zero* activity?



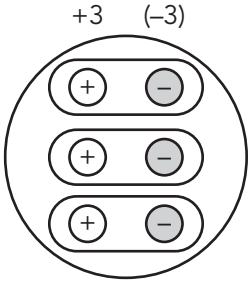
WORKED EXAMPLE

You can model the expression 3 + (–3) in different ways using two-color counters:



3 + (–3) = 0

Three positive charges and three negative charges have no charge.



3 + (–3) = 0

Each positive charge is paired with a negative charge.

Each pair of positive and negative charges has no charge.

1. What is the value of each ⊕ and ⊖ pair in the second model?

2. Describe how you can change the numbers of ⊕ and ⊖ counters in the model, but leave the sum unchanged.

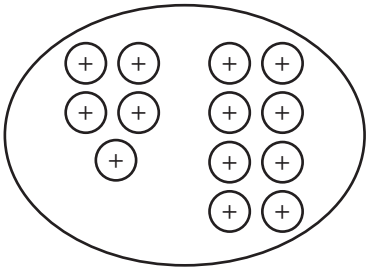
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ACTIVITY
3.2 Adding Integers with Two-Color Counters

Let's consider two examples where integers are added using two-color counters.

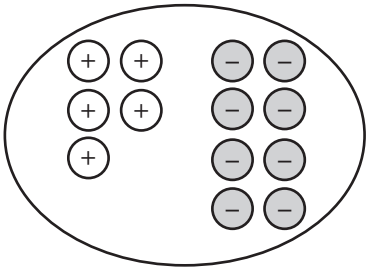
WORKED EXAMPLE

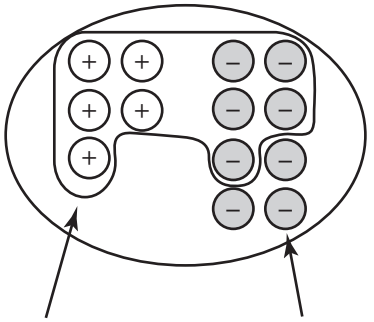
Example 1: $5 + 8$



There are 13 positive counters in the model. The sum is 13.

Example 2: $5 + (-8)$





There are five (+)(−) pairs. The value of those pairs is 0.

There are 3 (−), or negative counters, remaining.

There are 3 negative counters remaining. The sum of $5 + (-8)$ is -3.

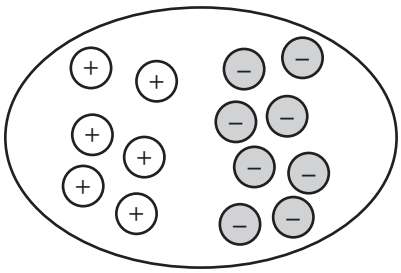
1. Create another model to represent a sum of −3. Write the appropriate number sentence.

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

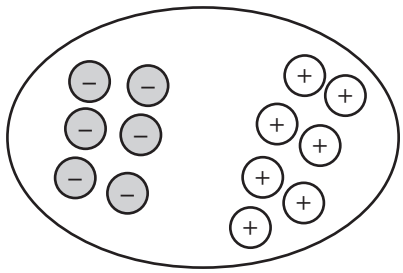
2. Share your model with your classmates. How are they the same? How are they different?

3. Write a number sentence to represent each model.

a.



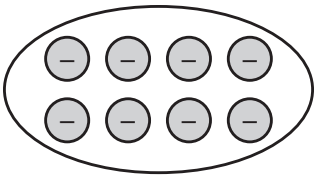
b.



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4. Does the order in which you wrote the integers in your number sentence matter? How do you know?

The students were then asked to write a number sentence for the given model.



Fernando

$-8 + 0 = -8$

Alexander

$-1 + (-7) = -8$

5. Analyze the number sentences written by Fernando and Alexander.

   a. Explain why both number sentences are correct.

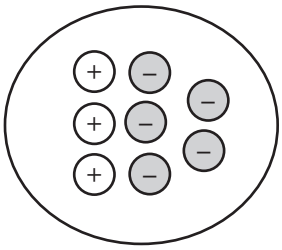
   b. Write an additional number sentence that could describe the model.

6. Write the number sentence in Question 5 a second way.

Fernando and Alexander used two-color counters to represent the number sentence $3 + (-5)$.



7. The students placed the same counters on their desks, but they reported different sums. Fernando reported the sum as 8, and Alexander said the sum was -2. Use the model to explain who is correct. What was the error made by the incorrect student?



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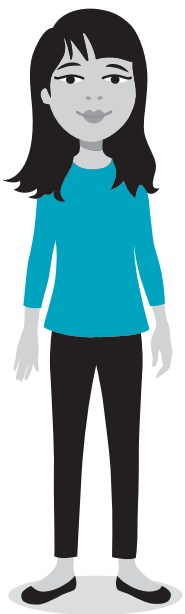
8. Draw a model for each and then complete the number sentence.

a. $-9 + (-4) =$ _____

b. $-4 + (-5) + 4 =$ _____

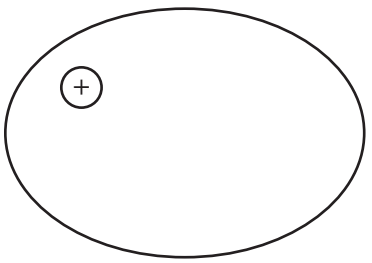
c. $3 + 6 + (-4) =$ _____

How does this process change when you have 3 addends instead of 2?

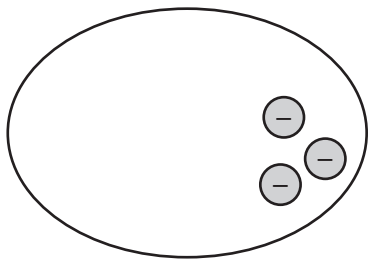


9. Complete the model to determine the unknown integer.

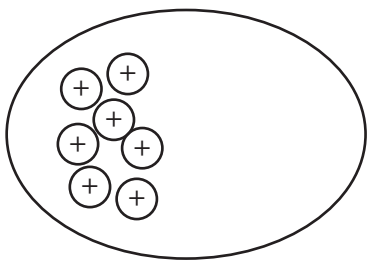
a. $1 +$ _____ $= -4$



b. $-2 + (-1) +$ _____ $= 7$



c. $4 + 3 +$ _____ $= -1$



10. Describe the set of integers that makes each sentence true.

    a. What integers when added to -7 give a sum greater than 0?

    b. What integers when added to -7 give a sum less than 0?

    c. What integers when added to -7 give a sum of 0?

You have now used two models to represent adding integers.

11. For each problem, draw both models to represent the number sentences and determine the sums.

    a. $(-3) + (-7)$

Think about how the absolute values of the addends compare with each other.

b. $8 + (-6) + (-7)$

c. $(-6) + 4 + 9$

12. Explain the similarities and differences of the models in helping you determine the sum of two integers.

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ACTIVITY
3.3 Rules for Adding Integers

Visual models provide concrete representations of new ideas, like adding signed numbers. But, you probably do not want to draw visual models when you have large numbers, lots of addends, fractions, or decimals.

Look back over the activities in this lesson and write rules for adding integers.

1. When adding two integers, what will the sign of the sum be when:

   a. Both integers are positive?

   b. Both integers are negative?

   c. One integer is positive and one integer is negative?



2. Write a rule that states how to determine the sum of any two integers that have the same sign.

3. Write a rule that states how to determine the sum of any two integers that have opposite signs.

Cut out the expressions provided at the end of the lesson.

4. Without computing the sums, sort the expressions into two piles: those that have a positive sum and those that have a negative sum.

   a. How can you decide which expressions have a positive sum and which have a negative sum?

   b. Tape or glue the expressions in the space provided.

      Positive Sums Negative Sums

   c. Use your rules to determine the sum of each expression.

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5. Determine each unknown addend.

a. _____ + (−25) = 34

b. _____ + 26 = 12

c. 8 + _____ = −24

d. −12 + _____ = −24

6. Determine the sums.

a. −5 + (−12) + 8

b. −42 + (−53) + (−28)

c. −26 + 8 + (−35)

d. 7 + (−41) + (−56)

e. −3 + (−22) + 10

f. −9 + (−12) + (−5)



Talk the Talk

Summarizing Sums

1. Use the graphic organizer provided to represent additive inverses. Write an example, using both a number sentence and a real-world situation. Then, represent your number sentence in words, using a number line model, and using a two-color-counter model.

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EXAMPLE: NUMBER SENTENCE AND REAL-WORLD SITUATION

IN WORDS

ADDITIVE INVERSES AND ZERO

NUMBER-LINE MODEL

TWO-COLOR-COUNTER MODEL

2. Write a number sentence that meets the given conditions. When it is not possible to create the number sentence, explain why not.

   a. Two positive addends with a positive sum.

   b. Two positive addends with a negative sum.

   c. Two negative addends with a positive sum.

   d. Two negative addends with a negative sum.

   e. A positive addend and a negative addend with a positive sum.

   f. A positive addend and a negative addend with a negative sum.

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Rules for Adding Integers Cutouts

$-32 + 24 + (-26)$	$-35 + (-15)$
$21 + (-41) + (-15)$	$-20 + (-28) + 60$
$55 + (-37) + 98$	$67 + 119$

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Why is this page blank?

So you can cut out the rules on the other side.

Lesson 3 Assignment

Write

Define the term *additive inverse property* in your own words.

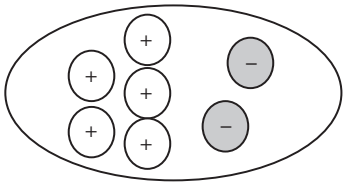
Remember

When two integers have the same sign and are added together, the sign of the sum is the sign of both integers. When two integers have opposite signs and are added together, the absolute values of the integers are subtracted, and the sign of the sum is the sign of the integer with the greater absolute value.

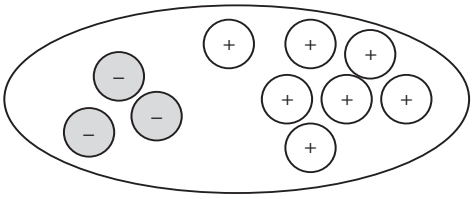
Practice

1. Write a number sentence for each two-color-counter model. Then, determine the sum.

a.



b.



2. Use two-color counters or a number line to draw a model for each number sentence. Then, determine the sum.

a. $3 + (-6)$

b. $-7 + (-4)$

c. $2 + 5$

d. $10 + (-8)$

e. $4 + (-7) + 15$

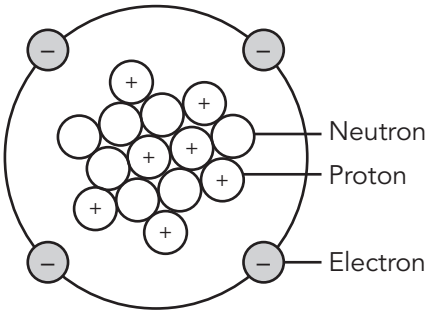
f. $-12 + 20 + (-6)$

g. $-75 + (-16) + 20$

h. $-6 + (-41) + (-13)$

Lesson 3 Assignment

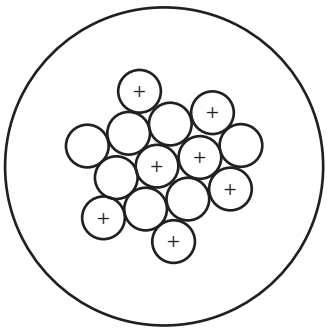
3. An atom is made up of protons, neutrons, and electrons. The protons carry a positive (+) charge and make up the nucleus of an atom with the neutrons. Neutrons ◯ do not carry a charge. The electrons carry a negative (−) charge and circle the nucleus. Atoms have no positive or negative charge. This means that they must have the same number of protons and electrons. A partial model of a nitrogen atom is shown.



Nitrogen Atom

   a. How many electrons should be drawn on the model of a nitrogen atom so that it has the same number of protons and electrons? How do you know?

   b. Complete the model of the nitrogen atom by drawing in the electrons.



   c. Write a number sentence to represent the sum of the number of protons and electrons in a nitrogen atom.

Lesson 3 Assignment

d. Use a number line to show the sum of the number of protons and electrons in the nitrogen atom.

Determine each sum.

4. $45 + (-27)$

5. $32 + (-98)$

6. $-153 + 74$

7. $-63 + (-41)$

8. $527 + (-289)$

9. $-32 + 98$

10. $-47 + (-95)$

11. $-51 + 134$

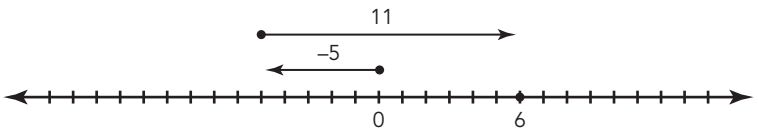
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Lesson 3 Assignment

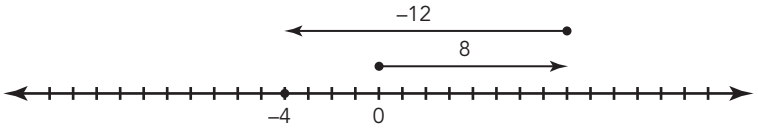
Prepare

For each number line model, write the number sentence that the model describes and draw a two-color-counter model to represent the number sentence.

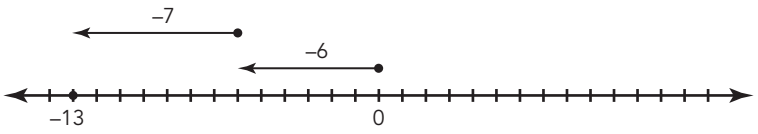
1.



2.



3.



4 Subtracting Integers

OBJECTIVES

- Model subtraction of integers on a number line.
- Model subtraction of integers using two-color counters.
- Develop a rule for subtracting integers.
- Apply previous understandings of addition and subtraction to subtract integers.

NEW KEY TERM

- zero pair

You have added integers using number lines and two-color-counter models.

How can you use these models to subtract integers?

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Getting Started

Take It Away

Each situation described has two different conclusions. Describe how you might model each on a number line.

1. You owe your friend $10.

   a. You borrow an additional $5.



   b. Your friend takes away $5 of that debt.

2. The temperature is −7°.

   a. Overnight it gets 12° colder.

   b. During the day, it gets 12° warmer.

3. You have charged $65 on a credit card.

   a. You return an item purchased with that card that costs $24.

   b. You purchase an additional item with that card that costs $24.

4. You dug a hole in the ground that is 20 inches deep. Your dog sees the pile of dirt and thinks it's a game.

   a. He knocks 6 inches of dirt back into the hole.

   b. He digs the hole another 4 inches deeper.

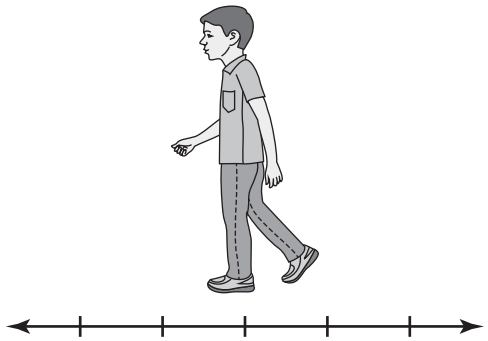
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ACTIVITY
4.1 Subtracting Integers on a Number Line

Think about how you moved on the number line when you were learning to add positive and negative numbers in the previous lesson. Let's walk the line to generate rules for subtracting integers.

Walk the number line for a subtraction sentence:

- Start at zero and walk to the value of the first term of the expression.
- To indicate subtraction, turn to face down the number line, towards the lesser negative numbers.
- Walk forward when subtracting a positive number, or walk backward when subtracting a negative number.



Your teacher will select a classmate to walk the line for each of the given problems. Help your classmate by preparing the directions that are needed.

1. Complete the table.

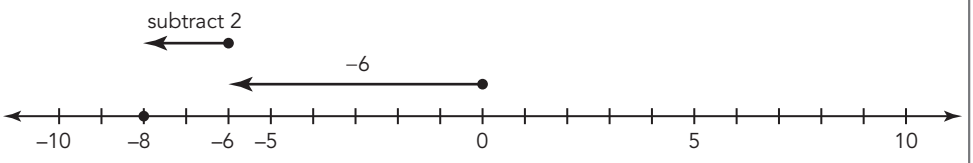
	Where You Start	Direction You Face on the Number Line	Walk Backward or Forward	Final Location
$1 - 3$				
$0 - (-4)$				
$-3 - 5$				
$-1 - (-4)$				

Jacob thought about how he could take what he learned from walking the line and create a number line model on paper. He said, "Subtraction means to move in the opposite direction."

Analyze Jacob's examples.

WORKED EXAMPLE

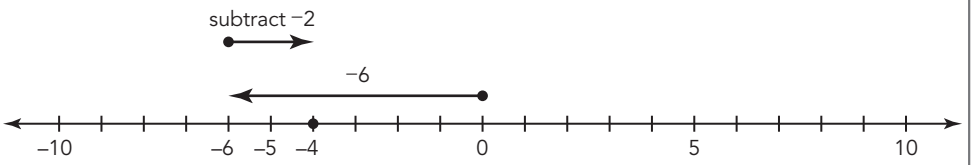
Example 1: $-6 - (+2)$



First, I moved to -6. Then, I went in the opposite direction of adding $(+2)$ because I am subtracting $(+2)$. So, I went two units to the left and ended up at -8.

$-6 - (+2) = -8$

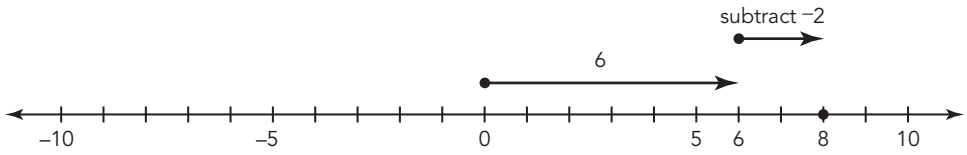
Example 2: $-6 - (-2)$



In this problem, I started by moving to -6. Because I am subtracting (-2), I went in the opposite direction of adding -2. So, I moved right two units and ended up at -4.

$-6 - (-2) = -4$

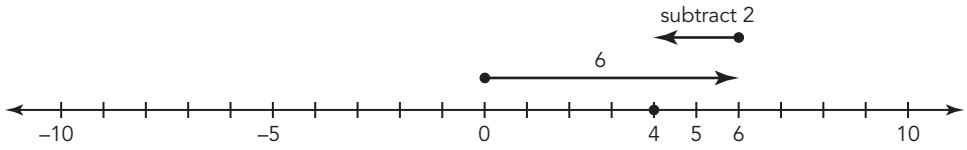
Example 3: $6 - (-2)$



2. $6 - (-2) =$ _____

Explain the movement Jacob modeled on the number line to determine the answer.

Example 4: $6 - (+2)$

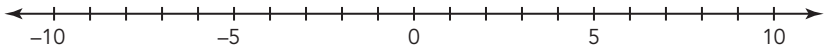


3. $6 - (+2) =$ _____

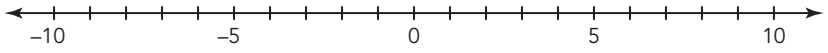
Explain the movement Jacob modeled on the number line to determine the answer.

4. Use the number line to complete each number sentence.

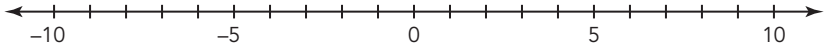
a. $-4 - (-3) =$ _____



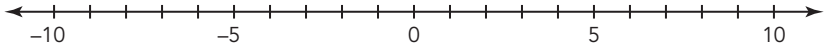
b. $-4 - (-4) =$ _____



c. $-4 - 3 =$ _____



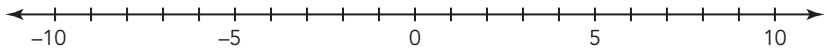
d. $-4 - 4 =$ _____



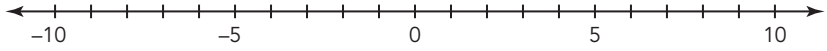


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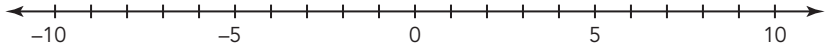
e. $4 - (-3) =$ _____



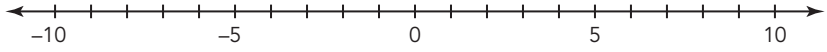
f. $4 - 4 =$ _____



g. $4 - 3 =$ _____



h. $4 - (-4) =$ _____



5. What patterns did you notice when subtracting the integers in Question 4? Describe an addition problem that is similar to each subtraction problem.

   a. Subtracting two negative integers

   b. Subtracting two positive integers

   c. Subtracting a positive integer from a negative integer

   d. Subtracting a negative integer from a positive integer

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ACTIVITY 4.2

Subtracting Integers with Two-Color Counters

Subtraction can mean to "take away" objects from a set. *Subtraction* can also mean a comparison of two numbers, or the "difference between them."

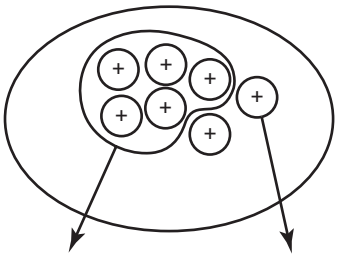
The number line model and the two-color-counter model used in the addition of integers can also be used to investigate the subtraction of integers.

WORKED EXAMPLE

Using just positive or just negative counters, you can show subtraction using the "take away" model.

Example 1: $7 - 5$

First, start with seven positive counters.



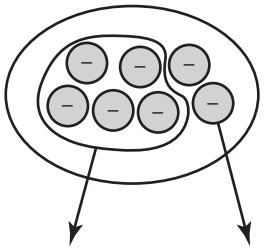
Then, take away five positive counters.

Two positive counters remain.

$$7 - 5 = 2$$

Example 2: $-7 - (-5)$

First, start with seven negative counters.



Then, take away five negative counters.

Two negative counters remain.

$$-7 - (-5) = -2$$

1. How are Worked Examples 1 and 2 similar? How are they different?

To subtract integers using both positive and negative counters, you will need to use *zero pairs*.

$$\oplus + \ominus = 0$$

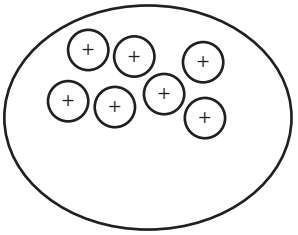
Recall that the value of a $\ominus$ and $\oplus$ pair is zero. So, together they form a **zero pair**. You can add as many pairs as you need and not change the value.

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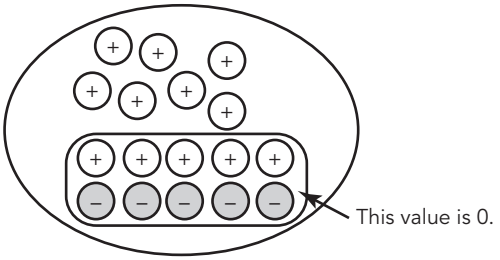
WORKED EXAMPLE

Example 3: $7 - (-5)$

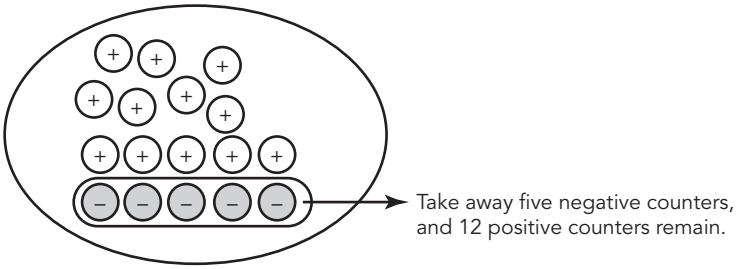
Start with seven positive counters.



The expression says to subtract five negative counters, but there are no negative counters in the first model. Insert five negative counters into the model. So that you don't change the value, you must also insert five positive counters.



Now, you can subtract, or take away, the five negative counters.

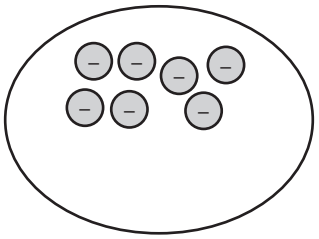


$$7 - (-5) = 12$$

2. Why is the second model equivalent to the original model?

Example 4: $-7 - 5$

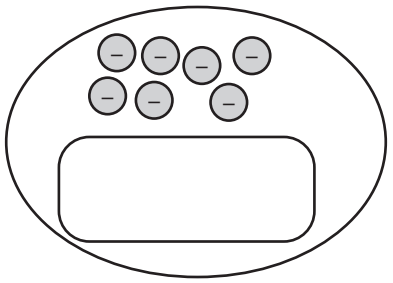
Start with seven negative counters.



3. The expression says to subtract five positive counters, but there are no positive counters in the model.

   a. How can you insert positive counters into the model and not change the value?

   b. Complete the model.



   c. Now, subtract, or take away, the five positive counters. Determine the difference.



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4. Draw a representation for each subtraction problem. Then, calculate the difference.

   a. $4 - (-5)$

   b. $-4 - (-5)$

c. $-4 - 5$

d. $4 - 5$

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5. How could you model $0 - (-7)$?

   a. Draw a sketch of your model. Then, determine the difference.

   b. In part (a), does it matter how many zero pairs you add? Explain your reasoning.

6. Does the order in which you subtract two numbers matter? Draw models and provide examples to explain your reasoning.

7. Are the rules you wrote at the end of the previous activity true for the two-color-counter models? What else did you learn about subtracting integers?

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ACTIVITY
4.3 Analyzing Integer Subtraction

You probably have noticed some patterns when subtracting signed numbers on the number line and with two-color counters. Let's explore these patterns to develop a rule.

$-8 - 5 = -13$
$-8 - 4 = -12$
$-8 - 3 = -11$
$-8 - 2 = -10$
$-8 - 1 = -9$
$-8 - 0 = -8$

1. Analyze the number sentences shown.

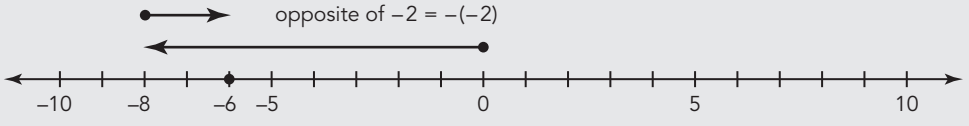
   a. What patterns do you see? What happens as the integer subtracted from -8 decreases?

   b. From your pattern, predict the answer to $-8 - (-1)$.

Consider the subtraction expression $-8 - (-2)$.

Jacob's Method

Start at -8. Since I'm subtracting, I go in the opposite direction of adding (-2), which means I go to the right 2 units. The answer is -6.



Amir's Method



I see another pattern. Since subtraction is the inverse of addition, you can think of subtraction as adding the opposite number.

$-8 - (-2)$ is the same as $-8 + (+2)$

$-8 + 2 = -6$

2. How is Amir's method similar to Jacob's method?

3. Use Amir's method to fill in each blank.

$10 - (-4) = 10 + (\_\_\_\_\_) = \_\_\_\_\_$

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4. Determine each difference.

a. $-9 - (-2) =$

b. $-3 - (-3) =$

c. $-7 - 5 =$

d. $24 - 8 =$

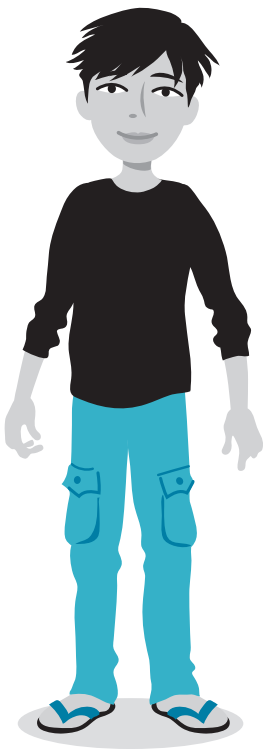
e. $-5 - (-3) - (-12) =$

f. $7 - (-23) - 30 =$

g. $-18 - (-10) - 4 =$

h. $10 - (-3) - (-6) =$

I can change any subtraction problem to addition when I add the opposite of the number that follows the subtraction sign.



5. Determine the unknown integer in each number sentence.

a. $3 + \_\_\_\_\_ = 7$

b. $2 + \_\_\_\_\_ = -7$

c. $\_\_\_\_\_ + -20 = -10$

d. $\_\_\_\_\_ - 5 = 40$

e. $\_\_\_\_\_ - (-5) = 40$

f. $\_\_\_\_\_ + 5 = 40$

g. $6 + \_\_\_\_\_ = 52$

h. $-6 + \_\_\_\_\_ = 52$

i. $-6 + \_\_\_\_\_ = -52$

ACTIVITY 4.4

Distance Between Rational Numbers

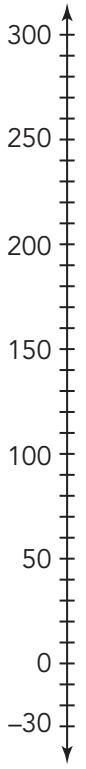
PROBLEM SOLVING



Amusement parks are constantly trying to increase the level of thrills on their rides. One way is to make the roller coasters drop faster and farther. A certain roller coaster begins by climbing a hill that is 277 feet above ground. Riders go from the top of that hill to the bottom, which is in a tunnel 14 feet under ground, in approximately 3 seconds!

Determine the vertical distance from the top of the roller coaster to the bottom of the tunnel.

1. Plot the height and depth of the first hill of the roller coaster on the number line.



Consider Koda's and Daniela's methods for determining the vertical distance.

Koda

Previously, I learned that you could add the absolute values of each number to calculate the distance.

$$|277| + |-14| = 277 + 14$$
$$= 291$$

The vertical distance is 291 feet.

Ask Yourself: What precise mathematical language do you need to communicate your mathematical reasoning?

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Daniela

I learned in previous courses that the difference between two numbers on a number line can be determined with subtraction. Because absolute value measures distance, I need the absolute value of the difference.

$$\begin{aligned} |277 - (-14)| &= |277 + (+14)| \\ &= |291| \\ &= 291 \end{aligned}$$

The vertical distance is 291 feet.

2. Describe how Koda and Daniela used absolute value differently to determine the vertical distance from the top of the roller coaster to the bottom of the tunnel.

3. Destiny says that order does not matter. Instead of calculating the distance from the top to the bottom, she says that you can calculate the vertical distance from the bottom to the top. Is Destiny correct? Determine whether Destiny is correct using both Koda's strategy and Daniela's strategy.

As demonstrated in Daniela's strategy, the distance between two numbers on the number line is the absolute value of their difference. Use Daniela's strategy to solve each problem.

4. The first recorded Olympic Games began in 776 BC. Called the *Ancient Olympics*, games were held every four years until being abolished by Roman Emperor Theodosius I in 393 AD.

   a. Represent the start and end years of the Ancient Olympics as integers.

   b. Determine the length of time between the start and end of the Ancient Olympics.

   c. Determine the length of time between the start of the Ancient Olympics and the Modern Olympics, which began in 1896.

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d. When you research the ancient calendar, you will learn that there actually was no Year 0. The calendar went from −1 BC to 1 AD. Adjust your answer from part (c) to account for this.

5. A drop tower ride drops the car with riders from a height of 125 feet. The car drops to a height of −30 feet. The car returns to a height of 18 feet where the riders get off. How far does the car travel from when the car drops until the riders get off?

Talk the Talk

Determining the Difference

Use what you have learned about adding and subtracting with integers to think about patterns in addition and subtraction.

1. Determine whether these subtraction sentences are always true, sometimes true, or never true. Give examples to explain your thinking.

   a. positive − positive = positive

   b. negative − positive = negative

   c. positive − negative = negative

   d. negative − negative = negative

2. When you subtract two negative integers, will the answer be greater than or less than the number you started with? Explain your thinking.

3. What happens when a positive number is subtracted from zero?

4. What happens when you subtract a negative number from zero?

5. Just by looking at the problem, how do you know whether the sum of two integers is positive, negative, or zero?

6. How are addition and subtraction of integers related?

7. Write a rule for subtracting positive and negative integers.

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Lesson 4 Assignment

Write

Define the term *zero pair* in your own words.

Remember

You can change any subtraction problem to an addition problem without changing the answer. Subtracting two integers is the same as adding the opposite of the subtrahend, the number you are subtracting.

Practice

1. Draw both a model using two-color counters and a model using a number line to represent each number sentence. Then, determine the difference.

   a. $-8 - (-5)$

   b. $-4 - 9$

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Lesson 4 Assignment

c. $2 - (-8)$

d. $3 - 12$

2. Determine each difference without using a number line.

a. $7 - (-13)$

b. $10 - (-1)$

c. $-16 - 3$

d. $-9 - 7$

Lesson 4 Assignment

e. $-1 - (-2)$

f. $-5 - (-5)$

g. $19 - (-19)$

h. $-8 - (-8)$

i. $40 - (-20)$

j. $-800 - (-300)$

k. $-16 - (-5) - 8$

l. $4 - 30 - (-12)$

m. $21 - 4 - (-6)$

n. $-32 - (-34) - (-8)$

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Lesson 4 Assignment

3. The highest temperature ever recorded on Earth was 136°F at Al Aziziyah, Libya, in Africa. The lowest temperature ever recorded on Earth was −129°F at Vostok Station in Antarctica. Determine the difference between the two temperatures.

4. The highest point in the United States is Denali, Alaska, at about 6773 yards above sea level. The lowest point in the United States is the Badwater Basin in Death Valley, California, at about 94 yards below sea level. Determine the number of yards between the lowest and highest points.

Lesson 4 Assignment

Prepare

Determine each sum.

1. $-2 + (-2)$

2. $-9 + (-4)$

3. $-3 + 8$

4. $5 + (-16)$

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5 Multiplying and Dividing Integers

OBJECTIVES

- Multiply integers using models.
- Develop rules for multiplying integers.
- Develop rules for dividing integers.

You know the reasoning and rules to add and subtract integers.

How do you multiply and divide integers?

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Getting Started

Addition or Multiplication?

Consider the addition problem $(-8) + (-8) + (-8) + (-8)$.

1. Rewrite the addition problem as a multiplication problem.

2. Is the product from Question 1 positive or negative? Explain your reasoning.

Consider the addition problem $(-1) + (-1) + (-1) + (-1) + (-1)$.

3. Rewrite the addition problem as a multiplication problem.

4. Is the product from Question 3 positive or negative? Explain your reasoning.

5. What relationship helped you answer Questions 2 and 4?

ACTIVITY
5.1 Modeling the Multiplication of Integers

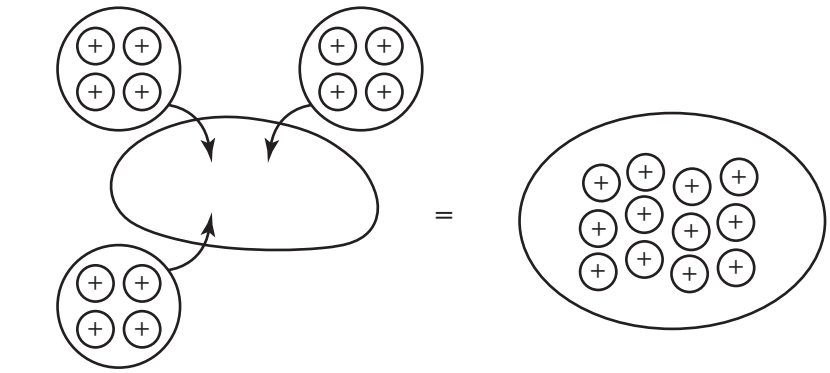
When thinking about multiplying integers, remember that multiplication can be represented as repeated addition.

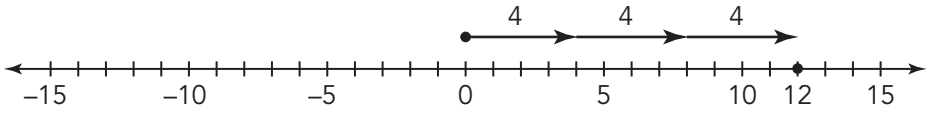
WORKED EXAMPLE

Consider the expression $3 \cdot 4$.

As repeated addition, it is represented as $4 + 4 + 4$.

You can think of $3 \cdot 4$ as three groups of 4.



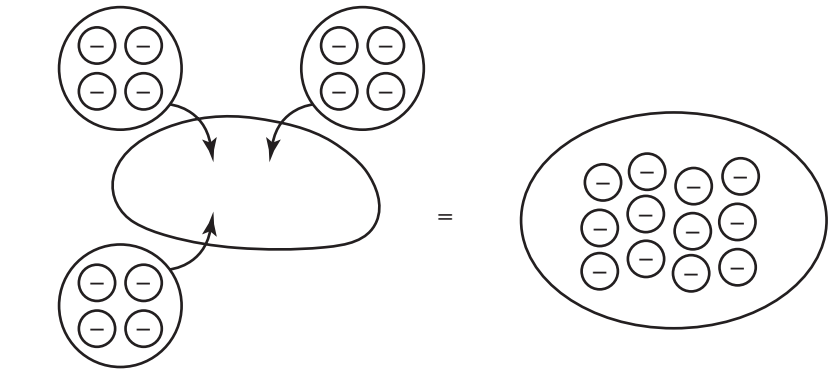


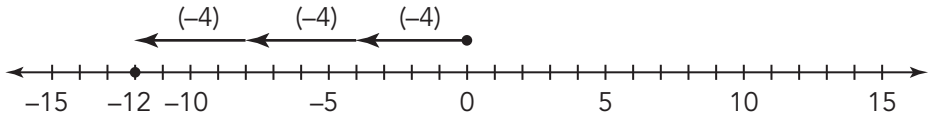
1. Explain how the number line in Worked Example 1 illustrates $3 \cdot 4$.

WORKED EXAMPLE

Consider the expression $3 \cdot (-4)$. As repeated addition, it is represented as $(-4) + (-4) + (-4)$.

You can think of $3 \cdot (-4)$ as three groups of (-4).

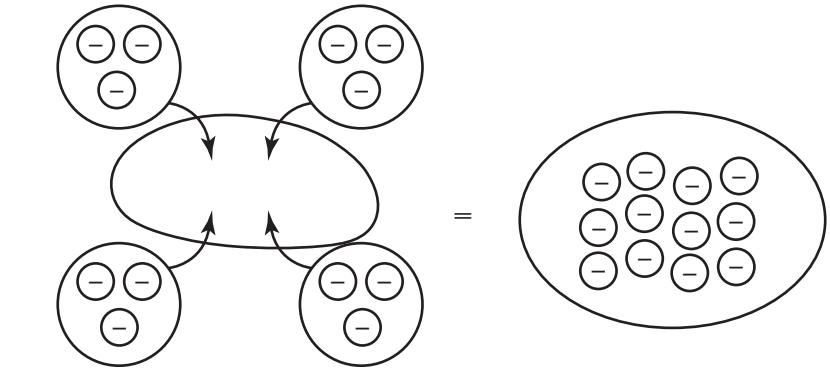


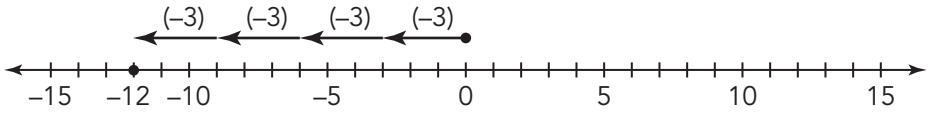


2. Explain how you can interpret each model in Worked Example 2 as three groups of *the opposite of 4*.

WORKED EXAMPLE

Consider the expression $4 \cdot (-3)$.





How is the number line model similar to models you have used to add positive numbers?



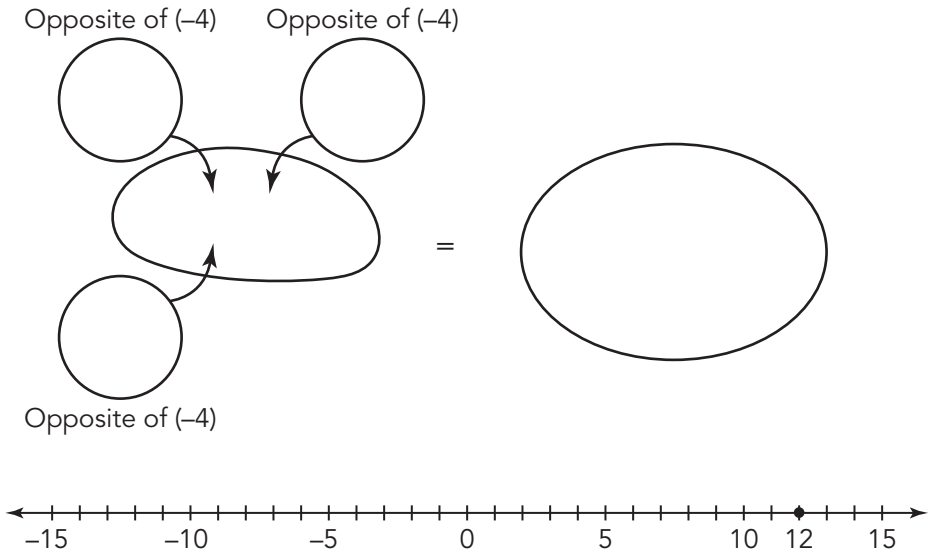
3. Interpret each model in Worked Example 3.

Think about how the 4 Worked Examples are alike and how they are different.

WORKED EXAMPLE

Consider the expression $(-3) \cdot (-4)$.

You know that $3 \cdot (-4)$ means "three groups of (-4)" and that -3 means "the opposite of 3." So, $(-3) \cdot (-4)$ means "the opposite of 3 groups of (-4)."



4. Complete the model by drawing in the appropriate counters and the appropriate rays in Worked Example 4.

   a. Determine the product and explain how your models illustrate this product.

   b. Another way to write this expression is $-((-4) + (-4) + (-4))$. Rewrite the expression as the opposite of a number: $-(\_\_\_\_\_)$.

   c. How does the expression you wrote in part (b) relate to the product? Explain why this makes sense.

5. Draw either a number line or two-color-counter model to determine each product. Describe the expression in words.

Use the examples when you need help.

a. $2 \cdot 3$

b. $2 \cdot (-3)$

c. $(-2) \cdot 3$

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d. $(-2) \cdot (-3)$

6. Complete the table.

Expression	Description	Addition Sentence	Product
$3 \cdot 5$	Three groups of 5	$5 + 5 + 5 = 15$	15
$(-3) \cdot 5$			
$3 \cdot (-5)$			
$(-3) \cdot (-5)$			

7. What do you notice about the products and their signs across the problems in this activity?

ACTIVITY
5.2 Signed Multiplication Facts

Analyze the sequence of products with 4.

1. What pattern do you notice in the products as the numbers multiplied by 4 decrease?

$4 \cdot 5 = 20$
$4 \cdot 4 = 16$
$4 \cdot 3 = 12$
$4 \cdot 2 = 8$
$4 \cdot 1 = 4$
$4 \cdot 0 = 0$



2. Continue the pattern to determine each product.

   a. $4 \cdot (-1) =$ _____

   b. $4 \cdot (-3) =$ _____

   c. $4 \cdot (-2) =$ _____

Analyze the sequence of products with -5.

3. Describe the pattern and then extend it by writing the next three number sentences.

$-5 \cdot 5 = -25$
$-5 \cdot 4 = -20$
$-5 \cdot 3 = -15$
$-5 \cdot 2 = -10$
$-5 \cdot 1 = -5$
$-5 \cdot 0 = 0$

4. How do these products change as the numbers multiplied by -5 decrease?

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Look back at the products you have determined in this lesson to answer each question.

5. Describe the sign of the product of two integers when:

   a. They are both positive.

   b. They are both negative.

   c. One is positive and one is negative.

   d. One is zero.

6. When you know that the product of two integers is negative, what can you say about the two integers? Give examples.

7. Describe a rule that will help you multiply any two integers.

8. Use your rule to evaluate each expression.

   a. $6 \cdot 5 =$ _____

   $6 \cdot (-5) =$ _____

   $-6 \cdot 5 =$ _____

   $-6 \cdot (-5) =$ _____

   b. $-3 \cdot 2 \cdot (-4) =$ _____

   $-3 \cdot (-2) \cdot (-4) =$ _____

   $3 \cdot (-2) \cdot 4 =$ _____

   $-3 \cdot (-2) \cdot 4 =$ _____

   $3 \cdot 2 \cdot (-4) =$ _____

   $-3 \cdot 2 \cdot 4 =$ _____



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Create some examples to test when you are not sure how to answer these questions.

9. Describe the sign of each product and how you know.

   a. The product of any even number of negative integers

   b. The product of any odd number of negative integers

How is determining the sign of the product different from when you add and subtract signed numbers?



10. What is the sign of the product of three positive integers and five negative integers? Explain your reasoning.

ACTIVITY
5.3 Signed Fact Families

When you studied division in previous courses you learned that multiplication and division were inverse operations. For every multiplication fact, you can write a corresponding division fact.

WORKED EXAMPLE

Consider the fact family for 4, 5, and 20.

$$4 \cdot 5 = 20$$

$$5 \cdot 4 = 20$$

$$20 \div 4 = 5$$

$$20 \div 5 = 4$$

Similarly, you can write fact families for integer multiplication and division.

Examples:

$$-7 \cdot 3 = -21 \qquad -8 \cdot (-4) = 32$$

$$3 \cdot (-7) = -21 \qquad -4 \cdot (-8) = 32$$

$$-21 \div (-7) = 3 \qquad 32 \div (-8) = -4$$

$$-21 \div 3 = -7 \qquad 32 \div (-4) = -8$$

1. What pattern(s) do you notice in each fact family?

Use fact families to help you determine each answer.

2. Fill in the unknown numbers to make each number sentence true.

   a. $56 \div (-8) =$ _____

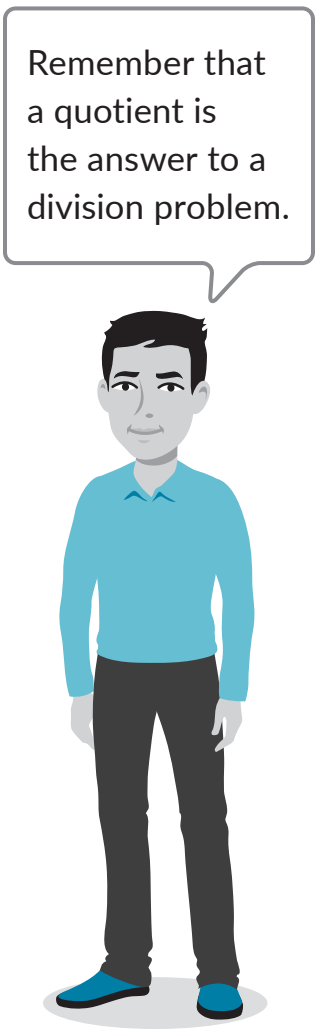
   b. $-63 \div$ _____ $= -7$

   c. _____ $\div (-8) = -4$

   d. _____ $\div (-8) = 0$

3. Describe the sign of the quotient of two integers when:

   a. Both integers are positive.



   b. One integer is positive and one integer is negative.

c. Both integers are negative.

d. The dividend is zero.

4. How do the answers to Question 3 compare to the answers to the same questions about the multiplication of two integers? Explain your reasoning.

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ACTIVITY
5.4 Putting It All Together

Fill in the unknown number to make each number sentence true.

1. a. $(-14)(-2)(3) =$ _____

   b. $-5 +$ _____ $+ 3 = 3$

   c. $-6 - (-3) + 9 - 2 =$ _____

   d. $(-3)(-2)(-10) \div (-6) =$ _____

   e. $(-4) + (-4) + (-4) + (-4) + (-4) + (-4) =$ _____ (-4)

2. After 14 holes on a putt-putt course, a player had a score of -2. The player scored a -1 on each of the last 4 holes. What was the player's final score?

3. Gabriela has \$42. She purchased 3 journals. Each journal cost \$6. She returned 8 packages of markers that cost \$4 each. How much money does Gabriela have now?

Talk the Talk

What's Your Sign?

Think about patterns in the signs of sums, differences, products, and quotients of integers.

1. Determine two different sets of single-digit integers that make each number sentence true.

   a. _____ = −42

   b. _____ = 56

   c. _____ = 63

   d. _____ = −48

2. Complete the table by writing the sign (+, −, or +/−) to describe each sum, difference, product, or quotient.

Description of Integers	Addition (Sum)	Subtraction (Difference)	Multiplication (Product)	Division (Quotient)
Two positive integers				
Two negative integers				
One positive and one negative integer				

3. Create a true multiplication or division number sentence that meets the given condition.

   a. Positive product

   b. Negative product

   c. Positive quotient

   d. Negative quotient

Lesson 5 Assignment

Write

Explain how you determine the sign of the product or quotient of three integers.

Remember

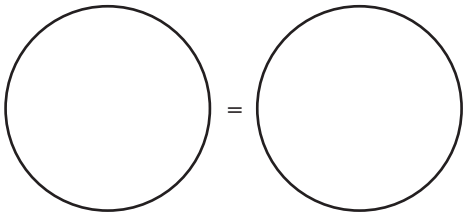
To multiply and divide integers, perform the usual multiplication and division algorithms and then apply the correct sign to the product or quotient.

Practice

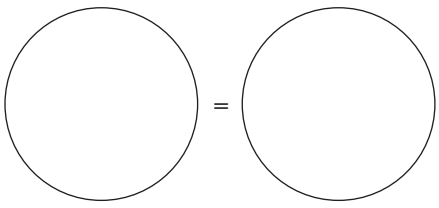
Draw a two-color-counter model to determine each product.

Describe the expression in words.

1. $6 \cdot (-3)$

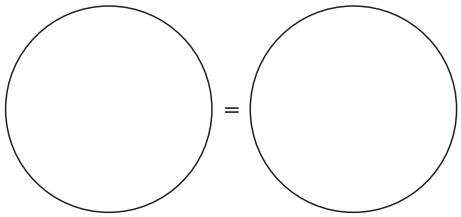


2. $-2 \cdot 5$



Lesson 5 Assignment

3. $-4 \cdot (-2)$



Complete a number line model to determine each product.

4. $-2 \cdot 7$

5. $-5 \cdot (-3)$

Lesson 5 Assignment

6. $3 \cdot (-3)$

Determine each product.

7. $5 \cdot 11$

   $5 \cdot (-11)$

   $-5 \cdot 11$

   $-5 \cdot (-11)$

8. $3 \cdot 4 \cdot 5$

   $-3 \cdot (-4) \cdot 5$

   $-3 \cdot 4 \cdot 5$

   $-3 \cdot (-4) \cdot (-5)$

   $3 \cdot 4 \cdot (-5)$

   $3 \cdot (-4) \cdot (-5)$

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Lesson 5 Assignment

Determine the integer that makes each number sentence true.

9. $\_\_\_\_\_ \cdot (-9) = -36$

10. $3 \cdot \_\_\_\_\_ = -24$

11. $14 \cdot \_\_\_\_\_ = 56$

12. $\_\_\_\_\_ \cdot (-6) = 30$

13. $9 = (-63) \div \_\_\_\_\_$

14. $-40 \div \_\_\_\_\_ = -8$

15. $16 \div \_\_\_\_\_ = -8$

16. $\_\_\_\_\_ \div (-6) = -4$

Lesson 5 Assignment

17. Fill in the unknown number to make each number sentence true.

    a. $(-24)(-3)(2) = \_\_\_\_\_$

    b. $-42 \div \_\_\_\_\_ = 6$

    c. $-2 + \_\_\_\_\_ + 8 = -3$

    d. $-8 - (-5) + 12 - 6 = \_\_\_\_\_$

    e. $(-5) + (-5) + (-5) + (-5) + = \_\_\_\_\_ \cdot (-5)$

18. On a very cold day, the temperature at noon was 8°F. By 2:00, the temperature had increased 2°F. Then the temperature changed by −3°F each hour for 4 hours. What was the temperature then?

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Lesson 5 Assignment

Prepare

Write each power of ten as a product of factors. Then, calculate the product.

1. 10^2 = _____ = _____

2. 10^5 = _____ = _____

3. 10^3 = _____ = _____

4. 10^4 = _____ = _____

5. 10^7 = _____ = _____

TOPIC 2 SELF-REFLECTION

Name: _____

Operating with Integers

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represent **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skil**l.

I can demonstrate an understanding of the standards in the *Operating with Integers* topic by:

TOPIC 2: *Operating with Integers*	Beginning of Topic	Middle of Topic	End of Topic
demonstrating addition and subtraction on a horizontal or vertical number line diagram.	○	○	○
describing situations where opposite quantities combine to make 0.	○	○	○
explaining how a number and its opposite have a sum of 0 and are additive inverses.	○	○	○
explaining why the sum of $p + q$ is located a distance of $\|q\|$ in the positive or negative direction from p on a number line.	○	○	○
representing integer operations with number lines and two-color counters.	○	○	○
explaining the rules of adding, subtracting, multiplying, and dividing signed numbers.	○	○	○
applying the distributive property to multiply integers and recognize that $(-1)(-1) = 1$.	○	○	○
using properties of operations to add, subtract, multiply, and divide integers.	○	○	○

continued on the next page

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TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Operating with Integers* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

Operating with Integers Summary

NEW KEY TERMS

- additive inverse property [propiedad del inverso aditivo]
- additive inverses [inversos aditivos]
- zero pair [par cero]

LESSON 1 Using Models to Understand Integer Addition

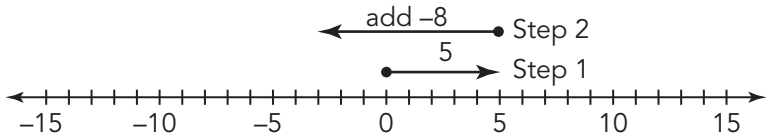
Combining positive and negative moves on a line results in moving to the left, moving to the right, or staying in the same position, depending on the size of the positive and negative values. For example, the table shows possible moves in the game of Math Football. In the game, the numbers on the red die are negative and the numbers on the black die are positive.

	Player	Starting Position	Results of the Number Cubes Roll	Ending Position
First Turn	Home Team	0	Red 3 and Black 5	+2
	Visiting Team	0	Red 5 and Black 6	+1
Second Turn	Home Team	+2	Red 1 and Black 6	+7
	Visiting Team	+1	Red 6 and Black 2	−3

LESSON 2 Adding Integers, Part I

You can use a number line to model integer addition. When adding a positive integer, move to the right on a number line. When adding a negative integer, move to the left on a number line.

For example, this number line model shows how to determine $5 + (-8)$.

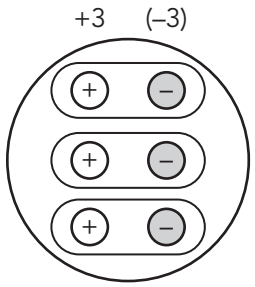


The *absolute value* of a number is its distance from 0. The absolute value of 5 is 5, and the absolute value of -8 is 8. The -8 has a greater absolute value, or distance from zero. Therefore, the sign of the sum is negative.

LESSON 3 Adding Integers, Part II

You can model the addition of integers using two-color counters that represent positive (+) charges and negative (–) charges.

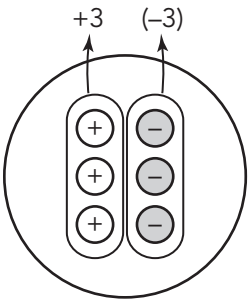
The expression $3 + (-3)$ can be modeled in different ways.



$3 + (-3) = 0$

Each positive charge is paired with a negative charge.

Each pair of positive and negative charges has no charge.



$3 + (-3) = 0$

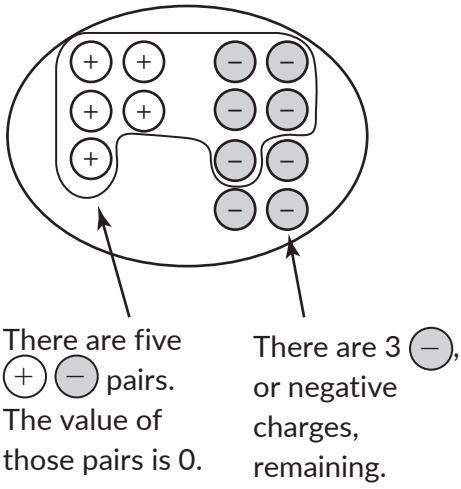
Three positive charges and three negative charges have no charge.

The **additive inverse property** states that the sum of a number and its opposite is zero. Two numbers with the sum of zero are called **additive inverses**. For example, $3 + (-3) = 0$. You can use additive inverses to model adding integers with different absolute values.

The expression $5 + (-8)$ is modeled here using two-color counters. There are 3 negative charges remaining. The sum of $5 + (-8)$ is -3.

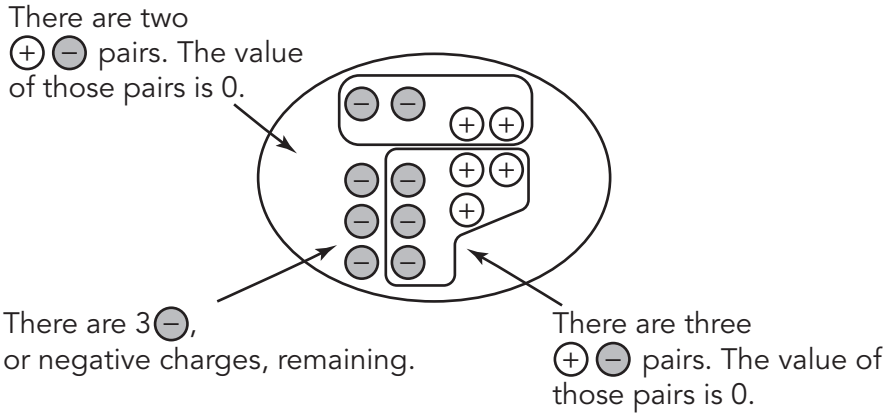
These are the rules for adding integers:

- When two integers have the same sign and are added together, the sign of the sum is the sign of both integers.
- When two integers have opposite signs and are added together, the absolute values of the integers are subtracted and the sign of the sum is the sign of the integer with the greater absolute value.



You can use similar strategies when adding 3 or more values.

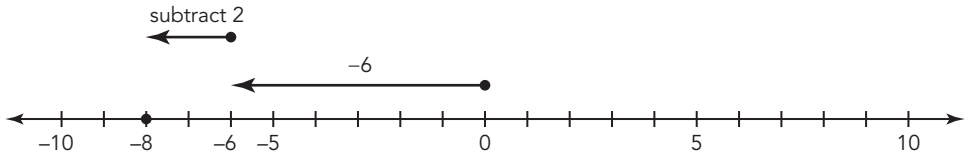
For example, the model shows how to determine $(-2) + (-6) + 5$.



LESSON 4 Subtracting Integers

You can use a number line to subtract integers. When subtracting an integer, you move in the opposite direction.

For example, the number line model shows how to determine $-6 - (+2)$.

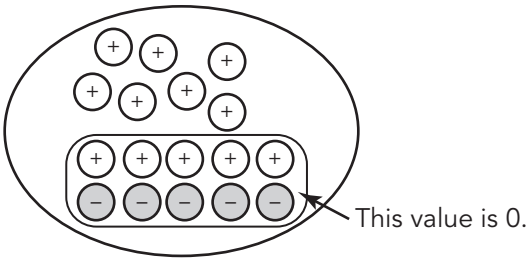


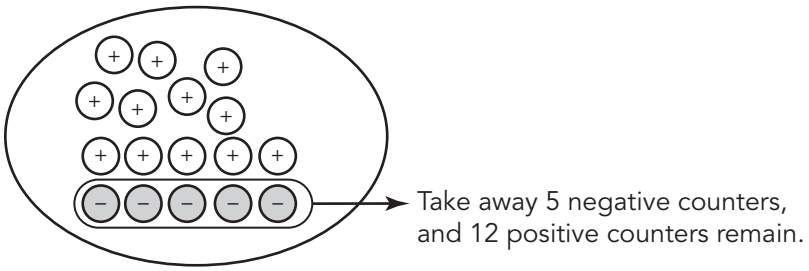
First, move to -6. Then, move in the opposite direction of adding $+2$.

You can use two-color counters to model the subtraction of integers. To subtract integers using both positive and negative counters, you will need to use *zero pairs*. The value of a negative charge and a positive charge is zero, so together they form a **zero pair**.

For example, the expression $7 - (-5)$ is modeled here using two-color counters.

Start with 7 positive counters. There are no negative counters to subtract, so insert 5 negative counters into the model. You must also insert 5 positive counters in order to not change the value.





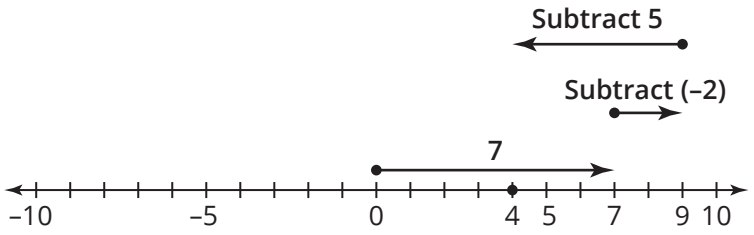
Now you can subtract, or take away, the 5 negative counters.

$7 - (-5) = 12$

You can change any subtraction problem to an addition problem without changing the answer. Subtracting two integers is the same as adding the opposite of the subtrahend, which is the number you are subtracting.

You can use similar strategies when subtracting 3 or more values.

For example, the number line model shows how to determine $7 - (-2) - 5$.

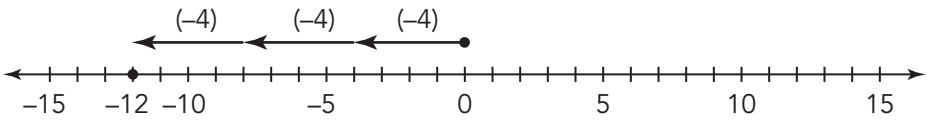


LESSON 5

Multiplying and Dividing Integers

When thinking about multiplying integers, remember that you can think about multiplication like repeated addition.

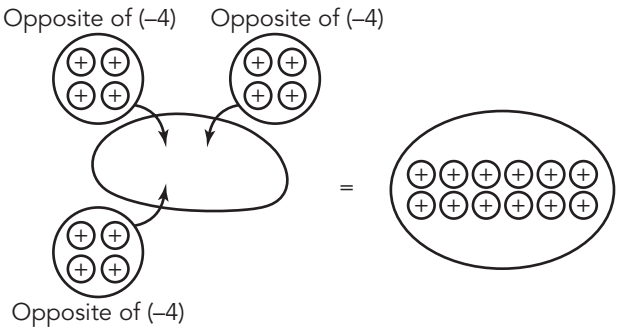
For example, consider the expression $3 \cdot (-4)$. As repeated addition, it is represented as $(-4) + (-4) + (-4)$.



You can think of $3 \cdot (-4)$ as three groups of -4.

Now, consider the expression $(-3) \cdot (-4)$.

You know that $3 \cdot (-4)$ means "three groups of -4" and that -3 means "the opposite of 3." So, $(-3) \cdot (-4)$ means "the opposite of 3 groups of -4."



To multiply and divide integers, perform the usual multiplication and division algorithms, and then apply the correct sign to the product or quotient. An odd number of negative signs in the expression gives a negative product or quotient. An even number of negative signs in the expression gives a positive product or quotient.

When you studied division in previous courses, you learned that multiplication and division were inverse operations. For every multiplication fact, you can write a corresponding division fact. Similarly, you can write fact families for integer multiplication and division.

For example, consider these two fact families.

$-7 \cdot 3 = -21$ $\quad$ $-8 \cdot (-4) = 32$

$3 \cdot (-7) = -21$ $\quad$ $-4 \cdot (-8) = 32$

$-21 \div (-7) = 3$ $\quad$ $32 \div (-8) = -4$

$-21 \div 3 = -7$ $\quad$ $32 \div (-4) = -8$

You can apply the order your understanding of operations to questions that involve multiple operations and integers. For example, consider the expression $(-6) \div (-2) + (-5)$

You can begin by dividing, $-6 \div (-2)$.

$-6 \div (-2) = 3$.

Finally, add 3 and -5.

$3 + (-5) = -2$

Math Glossary

A

absolute value

The absolute value, or magnitude, of a number is its distance from zero on a number line.

Example

The absolute value of -3 is the same as the absolute value of 3 because they are both a distance of 3 from zero on a number line.



$|-3| = |3|$

account balance

Account balance is the amount of money in an account at a given time.

addition property of equality

The addition property of equality states that if two values a and b are equal when you add the same value c to each, the sums are equal.

Examples

$12 = 12$ and $12 + 7 = 12 + 7$

If $a = b$, then $a + c = b + c$.

additive inverses

Two numbers with the sum of zero are called additive inverses.

Examples

$-19 + 19 = 0$ $\quad$ $a + -a = 0$

additive reasoning

Additive reasoning focuses on the use of addition and subtraction for comparisons.

Example

Vicki is 40 years old, and Ben is 10 years old. In 5 years, Vicki will be 45 and Ben will be 15. Vicki will always be 30 years older than Ben. This is additive reasoning.

algebraic expression

An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Examples

a $\quad$ $2a + b$ $\quad$ xy $\quad$ $\frac{4}{p}$ $\quad$ z^2

algorithm

An algorithm is a process or description of steps you can follow to complete a mathematical calculation.

Annual Percentage Yield (APY)

Annual Percentage Yield (APY) is a percentage that is paid to customers based on the account balance in an account for a year.

Example

Yvonne opens a checking account with an average monthly balance of $600. The account has a 2.5% APY.

(600)(0.025) = 15

Yvonne will earn $15 on this checking account over the course of the year.

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associate's degree

An associate's degree is a degree earned in a one- or two-year program in which students learn a specific skill or trade.

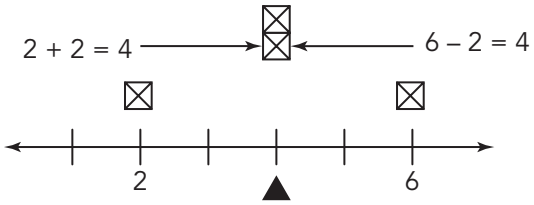
B

balance point

When you have all the points on a number line at the same value, the number line is balanced. The value where the number line is balanced is called the balance point.

Example

Consider the data set: 2, 6.

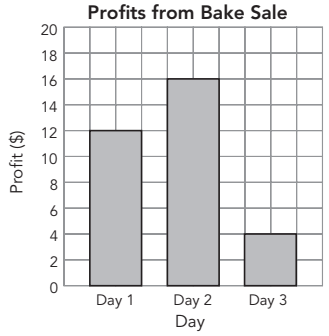


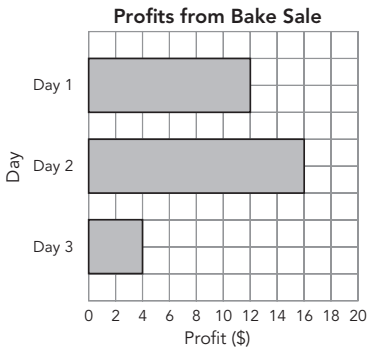
The balance point is 4.

bar graph

A bar graph displays categorical data using either horizontal or vertical bars on a graph. The height or length of each bar indicates the value for that category.

Examples



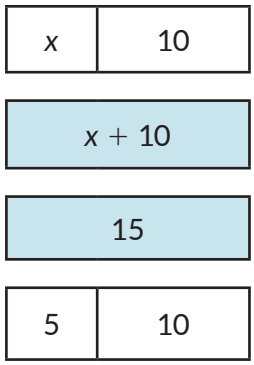


bar model

A bar model uses rectangular bars to represent known and unknown quantities.

Example

You can use a bar model to solve the equation $x + 10 = 15$.



The top bar can be split into two bars, x and 10. When this split happens in the bottom bar, with one bar containing 10, it shows that x is the same as 5, so $x = 5$.

base

The base of a power is the factor that is multiplied repeatedly in the power.

Examples

$2^3 = 2 \cdot 2 \cdot 2 = 8$ $8^0 = 1$

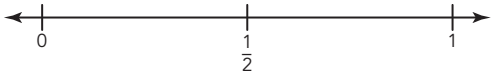
base base

benchmark fractions

Benchmark fractions are common fractions you can use to estimate the value of fractions.

Example

The numbers 0, $\frac{1}{2}$, and 1 are some benchmark fractions.



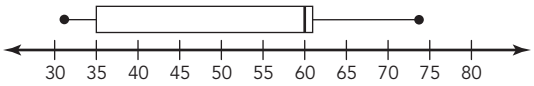
benchmark percents

A benchmark percent is a percent that is commonly used, such as 1%, 5%, 10%, 25%, $33\frac{1}{3}$%, 50%, and 100%.

box plot

A box-and-whisker plot, or just box plot, is a graph that displays the five-number summary of a data set: the median, the upper and lower quartiles (Q1 and Q3), and the minimum and maximum values.

Example



Data: 32, 35, 35, 53, 55, 60, 60, 61, 61, 74, 74

Minimum = 32

Q1 = 35

Median = 60

Q3 = 61

Maximum = 74

C

categorical data

Categorical data, or qualitative data, are data for which each piece of data fits into exactly one of several different groups or categories.

Examples

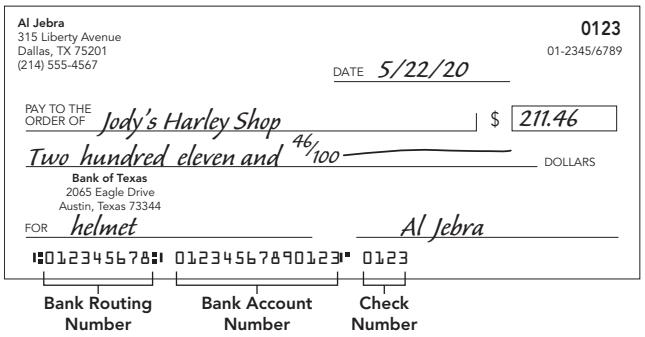
Animals: lions, tigers, bears, etc.

Colors: blue, green, red, etc.

check

A check is a written order to a bank to pay a specific amount of money to a person or company out of your checking account.

Example



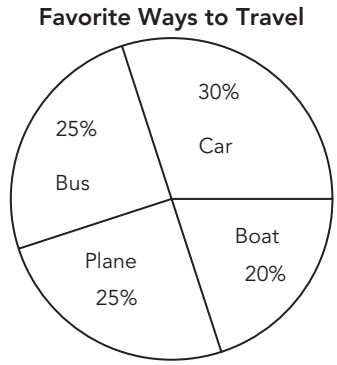
checking account

A checking account allows customers to safely store money in the bank and write checks.

circle graph

A circle graph, often called a *pie chart*, displays categorical data using sectors, or "wedges," of a circle.

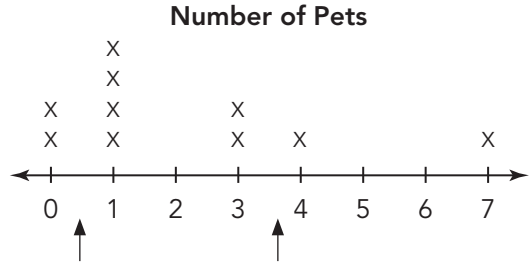
Example



clusters

Clusters are areas of the graph where data are grouped close together.

Example



There are clusters of data from 0 to 1 and from 3 to 4.

coefficient

A number that is multiplied by a variable in an algebraic expression is called a coefficient.

Examples

$14x$ $\frac{1}{3}(g)$ $w + 2.5$

coefficient

The coefficient is 1 even though it is not shown.

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common factor

A common factor is a number that is a factor of two or more numbers.

Example

Factors of 60: **1**, **2**, **3**, **4**, 5, **6**, 10, **12**, 15, 20, 30, 60

Factors of 24: 1, **2**, **3**, **4**, **6**, 8, **12**, 24

Common factors of 60 and 24: 1, 2, 3, 4, 6, and 12

commutative property of multiplication

The commutative property of multiplication states that for any numbers a and b, the product $a \cdot b$ is equal to the product $b \cdot a$.

Examples

$$\begin{array}{r} 29 \\ \cdot\ 3 \\ \hline 87 \end{array} = \begin{array}{r} 3 \\ \cdot\ 29 \\ \hline 27 \\ +\ 60 \\ \hline 87 \end{array} \qquad \underbrace{\frac{1}{5} \cdot \frac{2}{3}}_{\frac{2}{15}} = \underbrace{\frac{2}{3} \cdot \frac{1}{5}}_{\frac{2}{15}}$$

complex fraction

A complex fraction is a fraction that has a fraction in either the numerator, the denominator, or both the numerator and denominator.

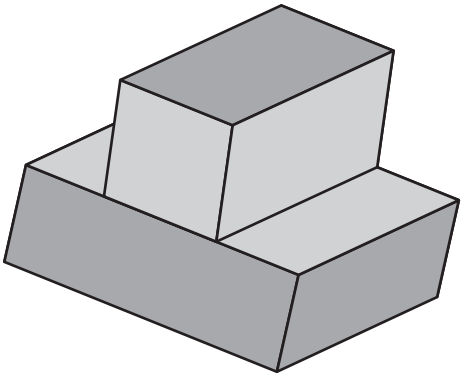
Examples

$\frac{\frac{3}{4}}{3}$, $\frac{7}{\frac{1}{2}}$ and $\frac{\frac{1}{4}}{\frac{2}{3}}$ are all complex fractions.

composite solid

A composite solid is made up of more than one geometric solid.

Example



continuous data

When quantitative data are a measurement of something and can have values that are between two counting numbers, the data are called continuous data.

convert

To convert a measurement means to change it to an equivalent measurement in different units.

Example

To convert 36 inches to feet, you can multiply:

$$36 \text{ in.} \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = \frac{36 \text{ ft}}{12}$$
$$= 3\text{ft}$$

credit card

A card that allows a person to borrow a certain amount of money and pay the borrowed money back at a later time is a credit card.

credit history

Credit history includes a record of all loans, credit card debt, late payments, etc.

credit report

A credit report is a detailed listing of an individual's credit history.

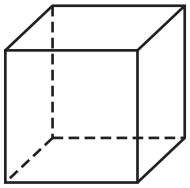
credit score

A credit score is a number used by lenders to rate how likely it is that a person will repay their debts.

cube

A cube is a polyhedron that has congruent squares as faces.

Example



D

data

Data are categories, numbers, or observations gathered in response to a statistical question.

Examples

Favorite foods of sixth graders
Heights of different animals at the zoo

debit

A debit occurs when money is taken out of the account automatically through the use of a debit card.

debit card

A debit card is a card that allows a bank's customer to pay for goods using money from their account.

density property

The density property states that between any two rational numbers, there is another rational number.

dependent quantity

The dependent quantity is the quantity that depends on another in a problem situation.

Example

Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

$\text{number of gallons} \cdot \frac{\text{miles}}{\text{gallon}} = \text{miles traveled}$

The dependent quantity is the total miles traveled. The number of miles traveled depends on the gallons of fuel.

dependent variable

The variable that represents the dependent quantity is called the dependent variable.

Example

Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

$\text{number of gallons} \cdot \frac{\text{miles}}{\text{gallon}} = \text{miles traveled}$

$$g \cdot m = t$$

The dependent quantity is the total miles traveled. Since t represents total miles traveled in the equation, t is the dependent variable.

deposit

A deposit is money put into the account. Deposits may come in the form of cash, checks, or transfers.

discrete data

Discrete data can only take on specific, individual values. Common examples of discrete data include the number of students in a class or shoe sizes.

distribution

The overall shape of a graph is called the distribution of data. A distribution is the way in which the data are spread out.

distributive property

The distributive property states that for any numbers a, b, and c, $a(b + c) = ab + ac$.

Example

$$\begin{aligned} 4(2 + 15) &= 4 \cdot 2 + 4 \cdot 15 \\ &= 8 + 60 \\ &= 68 \end{aligned}$$

division property of equality

The division property of equality states that when you divide equal values a and b by the same value c and $c \neq 0$, the quotients are equal.

Example

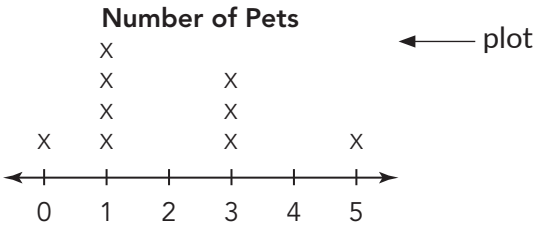
$12 = 12$ and $12 \div 7 = 12 \div 7$

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

dot plot

A dot plot (sometimes called a *line plot*) is a data display that shows discrete data on a number line with dots, Xs, or other symbols.

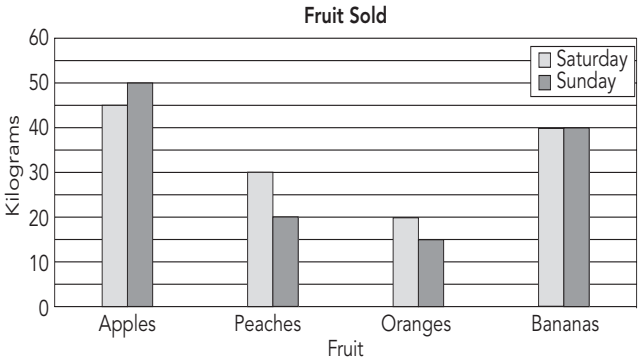
Example



double bar graph

A double bar graph is used when each category contains two different data sets. The bars may be vertical or horizontal.

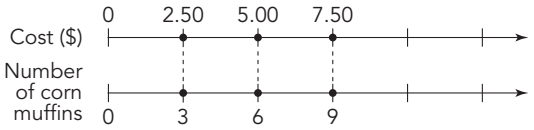
Example



double number line

A double number line is a model that is made up of two number lines used together to represent the ratio between two quantities.

Example

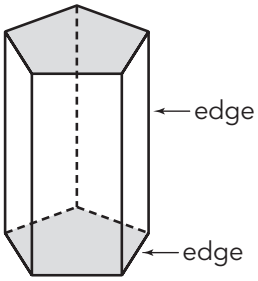


E

edge

An edge is the intersection of two faces of a three-dimensional figure.

Example



ellipsis

An ellipsis is a set of three periods used to represent infinity in a number set.

Example

$\{..., -2, -1, 0, 1, 2, ...\}$

ellipsis ellipsis

equation

An equation is a mathematical sentence that uses an equals sign to show that two quantities are the same as one another.

Examples

$$y = 2x + 4$$
$$6 = 3 + 3$$
$$2(8) = 26 - 10$$
$$\frac{1}{4} \cdot 4 = \frac{8}{4} - \frac{4}{4}$$

equivalent expressions

Two algebraic expressions are equivalent expressions if, when any values are substituted for variables, the results are equal.

Example

$$(x + 10) + (6x - 5) = 7x + 5$$
$$12 + 7 = 14 + 5$$
$$19 = 19$$

equivalent fractions

Fractions that represent the same part-to-whole relationship are equivalent fractions.

equivalent ratios

Equivalent ratios are ratios that represent the same part-to-part or part-to-whole relationship.

evaluate an algebraic expression

To evaluate an algebraic expression means to determine the value of the expression for a given value of each variable.

Example

Evaluate the expression $= \frac{(4x + (2^3 - y)}{p}$ for $x = 2.5$, $y = 8$, and $p = 2$.

- First replace the variables with numbers: $\frac{4(2.5) + (2^3 - 8)}{2}$
- Then calculate the value of the expression: $\frac{10 + 0}{2} = \frac{10}{2} = 5$

evaluate a numeric expression

To evaluate a numeric expression means to rewrite the expression as a single numeric value.

Example

$$19 - 4 \cdot 3$$
$$19 - 12$$
$$7$$

experiment (statistics)

An experiment is one method of collecting data in which a researcher imposes a condition and observes the results.

Example

A researcher conducts an experiment to investigate if 6th-graders perform better on an assessment if they read a textbook or watch a video about the material. The researcher randomly assigns half the students to read the text and half the students to watch the video. All students would be given the same assessment, and the scores of the students in the two groups would be compared.

exponent

The exponent of the power is the number of times the base is used as a factor.

Examples

$2^3 = 2 \cdot 2 \cdot 2$

exponent

$8^4 = 8 \cdot 8 \cdot 8 \cdot 8$

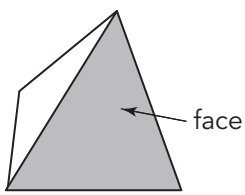
exponent

F

face

A face is one of the polygons that makes up a polyhedron.

Example



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frequency

A frequency is the number of times an item or number occurs in a data set.

Number Rolled	Tally	Frequency
2	𝍸 II	7

The number 2 was rolled 7 times, so its frequency was 7.

frequency table

A table used to organize data according to how many times a data value occurs is called a frequency table.

Example

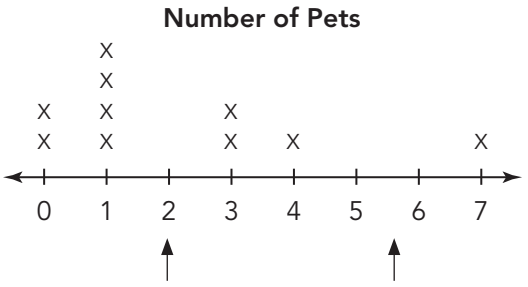
Number of Roller Coasters at Major Theme Parks	
Number of Roller Coasters	Frequency (f)
7–9	6
10–12	4
13–15	7
16–18	2

G

gaps

Gaps are areas of the graph where there are no data.

Example

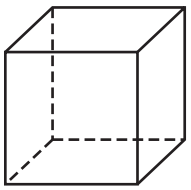


There are gaps between 1 and 3 and between 4 and 7.

geometric solid

A geometric solid is a bounded three-dimensional geometric figure.

Example



grant

A grant is money awarded to a person from the government or school to help pay tuition.

graph of an inequality

The graph of an inequality in one variable is the set of all points on a number line that make the inequality true.

Example



$x \leq 3$

greatest common factor (GCF)

The greatest common factor, or GCF, is the largest factor two or more numbers have in common.

Example

Factors of 16: **1**, **2**, **4**, 8, 16

Factors of 12: **1**, **2**, 3, **4**, 6, 12

Common factors: 1, 2, 4

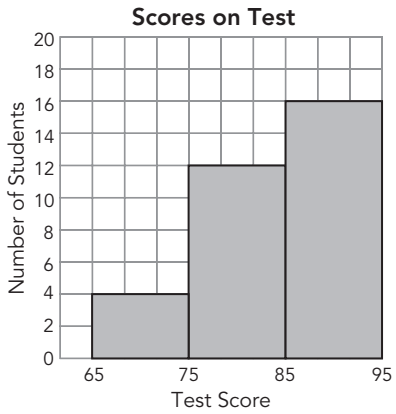
Greatest common factor: 4

H

histogram

A histogram is a graphical way to display quantitative or numerical data using vertical bars. The width of a bar represents an interval of data and is often referred to as a *bin*. The height of the bar indicates the frequency, or the number of data values included in any given bin.

Example



I

identity property of addition

The identity property of addition states that the sum of any number and 0 is the number.

Examples

$6 + 0 = 6$ $\quad \frac{3}{4} + 0 = \frac{3}{4}$

$5^2 + 0 = 5^2$ $\quad 0.125 + 0 = 0.125$

identity property of multiplication

The identity property of multiplication states that the product of any number and 1 is the number.

Examples

$6 \cdot 1 = 6$ $\quad \frac{3}{4} \cdot 1 = \frac{3}{4}$

$5^2 \cdot 1 = 5^2$ $\quad 0.125(1) = 0.125$

independent quantity

The independent quantity is the quantity the dependent quantity depends on.

Example

Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

$$\text{number of gallons} \cdot \frac{\text{miles}}{\text{gallon}} = \text{miles traveled}$$

The independent quantity is the number of gallons. The other quantity (miles traveled) is dependent upon this quantity.

independent variable

The variable that represents the independent quantity is called the independent variable.

Example

Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

$$\text{number of gallons} \cdot \frac{\text{miles}}{\text{gallon}} = \text{miles traveled}$$

$$g \cdot m = t$$

The independent quantity is the number of gallons. Since g represents the number of gallons in the equation, g is the independent variable.

inequality

An inequality is any mathematical sentence that has an inequality symbol.

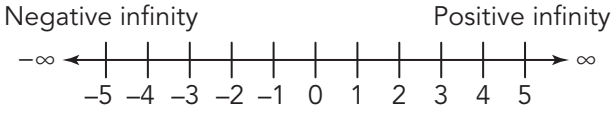
Examples

$8 > 2$ $\quad a \leq b$ $\quad 6.051 > 6.009$ $\quad 2x + 4 \geq 16$

infinity

Infinity, represented by the symbol ∞, means a quantity with no end or bound.

Example



integers

Integers are the set of whole numbers with their opposites.

Example

The set of integers can be represented as {... −3, −2, −1, 0, 1, 2, 3, ...}

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interest

Interest is an additional charge for borrowing money, usually a percentage of the amount borrowed.

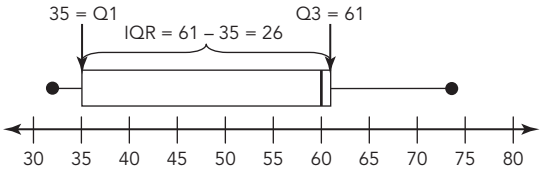
interest rate

The interest rate is the percentage of the balance on the credit card that is added to the cost of the purchase.

interquartile range (IQR)

The interquartile range, abbreviated IQR, is the difference between the third quartile, Q3, and the first quartile, Q1. The IQR indicates the range of the middle 50 percent of the data.

Example



inverse operations

Inverse operations are pairs of operations that reverse the effects of each other.

Examples

Addition and subtraction are inverse operations: $351 + 25 - 25 = 351$.

Multiplication and division are inverse operations: $351 \cdot 25 \div 25 = 351$.

K

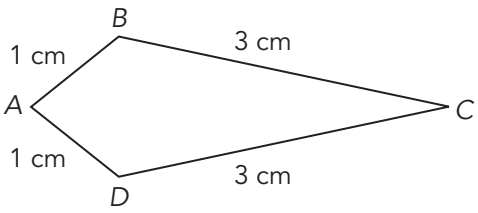
key

A key explains how each data set is represented by a color or a pattern in the graph.

kite

A kite is a quadrilateral with two pairs of consecutive congruent sides where opposite sides are not congruent.

Example



L

least common multiple (LCM)

The least common multiple, or LCM, is the smallest multiple (other than zero) that two or more numbers have in common.

Example

Multiples of 60: 60, **120**, 180, **240**, 300, 360, 420, 480 . . .

Multiples of 24: 24, 48, 72, 96, **120**, 144, 168, 192, 216, **240** . . .

Some common multiples of 60 and 24: 120, 240 . . .

Least common multiple of 60 and 24: 120

like terms

In an algebraic expression, like terms are two or more terms that have the same variable raised to the same power.

Examples

like terms

$4x + 3p + x + 2 = 5x + 3p + 2$

like terms

$24a^2 + 2a - 9a^2 = 13a^2 + 2a$

no like terms

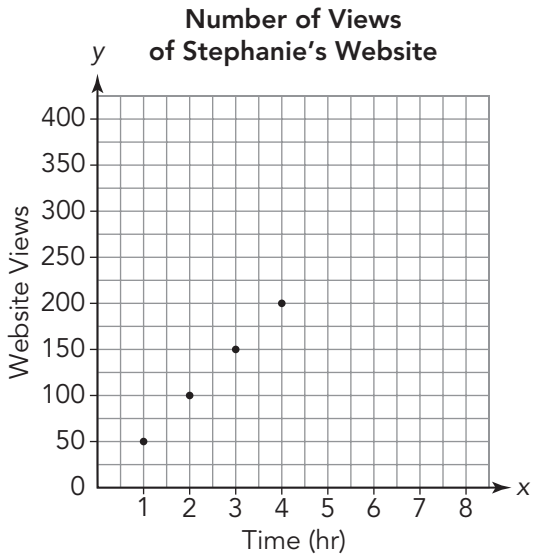
$m + m^2 - x + x^3$

linear relationship

When a set of points graphed on a coordinate plane forms a straight line, a linear relationship exists.

Example

The points graphed show a linear relationship.



literal equation

A literal equation is an equation in which the variables represent specific measures.

Examples

$A = lw$ $A = \frac{1}{2}bh$ $d = rt$

M

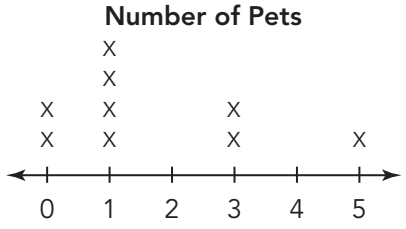
master's degree

A master's degree is a one- or two-year degree beyond an undergraduate degree.

mean

The mean is the arithmetic average of the numbers in a data set.

Example



$$\text{Mean} = \frac{0+0+1+1+1+1+3+3+5}{9}$$
$$= \frac{15}{9} = 1\frac{2}{3} \text{ pets}$$

measure of center

A measure of center tells you how the data values are clustered, or where the "center" of a graph of the data is located.

Examples

Mean, median, and mode are each a measure of center for data.

measure of variation

A measure of variation describes the spread of data values.

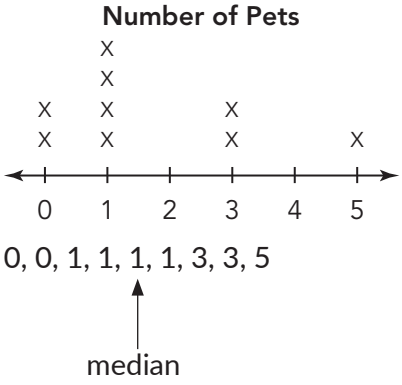
Example

Range is a measure of variation for data.

median

The median is the middle number in a data set when the values are placed in order from least to greatest or greatest to least.

Example



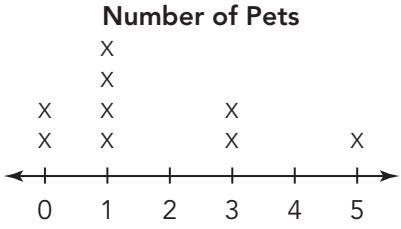
0, 0, 1, 1, 1, 1, 3, 3, 5

median

mode

The mode is the value or values that occur most frequently in a data set.

Example



0, 0, 1, 1, 1, 1, 3, 3, 5

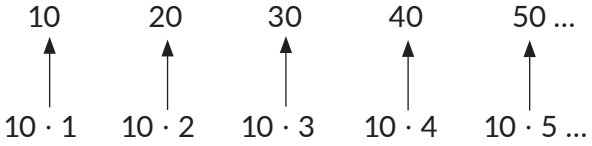
The mode of the data is 1.

multiple

A multiple is the product of a given whole number and another whole number.

Example

Multiples of 10:



multiplication property of equality

The multiplication property of equality states that if two values a and b are equal, when you multiply each by the same value c, the products are equal.

Example

$12 = 12$ and $12(7) = 12(7)$

If $a = b$, then $ac = bc$.

multiplicative inverse

The multiplicative inverse of a number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers. The product of any nonzero number and its multiplicative inverse is 1.

Examples

The multiplicative inverse of $\frac{3}{7}$ is $\frac{7}{3}: \frac{3}{7} \cdot \frac{7}{3} = \frac{21}{21} = 1$

The multiplicative inverse of 5 is $\frac{1}{5}: \frac{5}{1} \cdot \frac{1}{5} = \frac{5}{5} = 1$

multiplicative reasoning

Multiplicative reasoning focuses on the use of multiplication and division.

Example

Vicki is 40 years old, and Ben is 10 years old. Vicki is 4 times as old as Ben. In 5 years, Vicki will be 3 times as old as Ben.

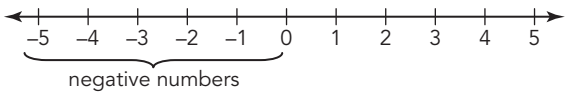
This is multiplicative reasoning.

N

negative numbers

The values to the left of zero on a number line are called negative numbers.

Example



numeric expression

A numeric expression is a mathematical phrase that contains numbers and operations.

Example

$5 \cdot 4 - 9$

O

observational study

An observational study is one method of collecting data in which a researcher collects data by observing the variable of interest.

Example

A researcher is interested in whether or not more men or women prefer a certain store. The researcher observes the number of men and women who visit the store over a number of hours and compares the values of the two groups.

one-step equation

A one-step equation is an equation that can be solved using only one operation.

order of operations

The order of operations is a set of rules that ensures the same result every time an expression is evaluated.

Example

$44 + (6 - 5) - 2 \cdot 75 \div 5^1$ Parentheses

$44 + 1 - 2 \cdot 75 \div 5^1$ Exponents

$44 + 1 - 2 \cdot 75 \div 5$ Multiplication and Division (from left to right)

$44 + 1 - 150 \div 5$

$44 + 1 - 30$ Addition and Subtraction (from left to right)

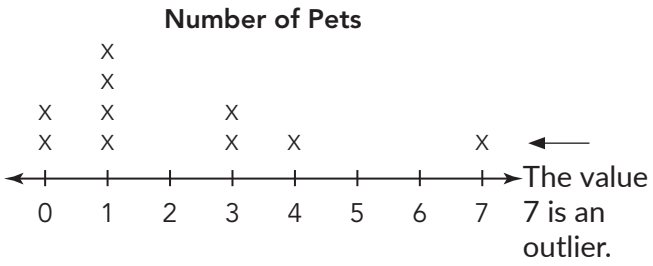
$45 - 30$

15

outliers

Outliers are data values that lie a large distance from the other data in a graph. Outliers usually accompany gaps in data.

Example



overdraft

An overdraft occurs when money is withdrawn from a bank account and the balance goes below zero. Most banks will charge a fee every time this happens.

P

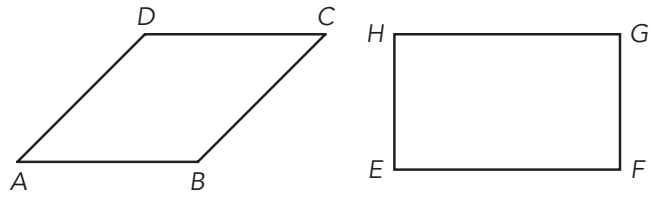
parallelogram

A parallelogram is a four-sided figure with two pairs of parallel sides and opposite sides that are equal in length.

Examples

In parallelogram *ABCD*, opposite sides *AB* and *CD* are parallel and equal in length; opposite sides *AD* and *BC* are parallel and equal in length.

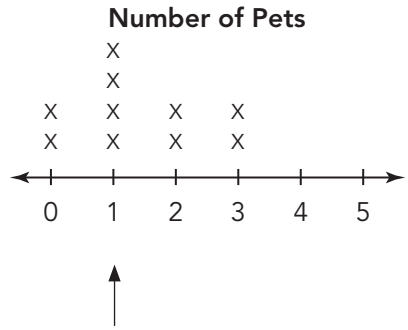
In parallelogram *EFGH*, opposite sides *EF* and *GH* are parallel and equal in length; opposite sides *FG* and *EH* are parallel and equal in length.



peaks

Peaks are values on a graph that contain more data points than the values on either side of it.

Example



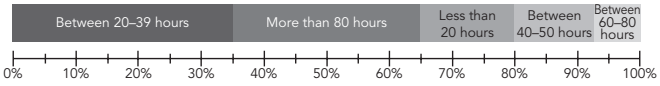
percent

A percent is a part-to-whole ratio where the whole is equal to 100. Percent is another name for hundredths. The percent symbol "%" means "per 100," or "out of 100."

percent bar graph

A percent bar graph shows a single bar that represents 100%, with sections that represent the frequency of each category. You can display a percent bar graph horizontally or vertically.

Example



perfect cube

A perfect cube is the cube of an integer.

Example

64 is a perfect cube: $4 \cdot 4 \cdot 4 = 64$

perfect square

A perfect square is the square of an integer.

Examples

9 is a perfect square: $3 \cdot 3 = 9$

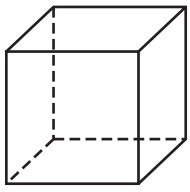
25 is a perfect square: $5 \cdot 5 = 25$

polyhedron

A polyhedron is a three-dimensional solid figure that has polygons as faces.

Example

A cube is a polyhedron. It has six square faces.



population

A population is an entire set of items from which data are collected.

Example

If you wanted to determine the average height of the students at your school, the number of students at the school would be the population.

post-secondary education

Post-secondary education refers to education beyond high school.

power

To represent repeated multiplication, use a power. A power has two elements: the base and the exponent.

Example

base → 6^2 ← exponent

power

private school

Colleges and universities are either public or private. A private school does not receive funding from the state. They raise funds from tuition and donations.

properties of inequalities

The properties of inequalities allow you to solve inequalities involving any numbers.

Examples

- addition property of inequalities
  If $a < b$, then $a + c < b + c$.
  If $a > b$, then $a + c > b + c$.
- subtraction property of inequalities
  If $a < b$, then $a - c < b - c$.
  If $a > b$, then $a - c > b - c$.
- multiplication property of inequalities
  If $a < b$, then $a \cdot c < b \cdot c$, for $c > 0$.
  If $a > b$, then $a \cdot c > b \cdot c$, for $c > 0$.
  If $a < b$, then $a \cdot c > b \cdot c$, for $c < 0$.
  If $a > b$, then $a \cdot c < b \cdot c$, for $c < 0$.
- division property of inequalities
  If $a < b$, then $\frac{a}{c} < \frac{b}{c}$, for $c > 0$.
  If $a > b$, then $\frac{a}{c} > \frac{b}{c}$, for $c > 0$.
  If $a < b$, then $\frac{a}{c} > \frac{b}{c}$, for $c < 0$.
  If $a > b$, then $\frac{a}{c} < \frac{b}{c}$, for $c < 0$.

proportion

A proportion is an equation that states that two ratios are equal.

Example

$\frac{1}{2} = \frac{4.5}{9}$

public school

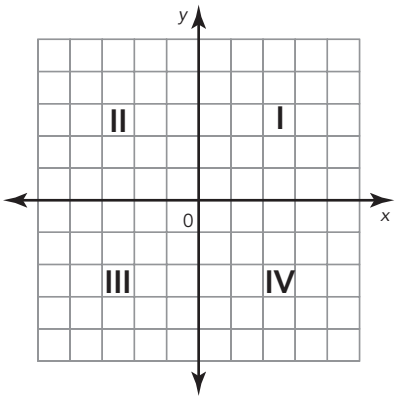
Colleges and universities are either public or private. A public school receives funding from the state government to offset the costs of running the university. The tuition to public universities is generally much less than the tuition to private schools.

Q

quadrants

The *x*- and *y*-axes divide the coordinate plane into four regions called quadrants. These quadrants are numbered with Roman numerals from one (I) to four (IV), starting in the upper right-hand quadrant and moving counterclockwise.

Example



quantitative data

Quantitative data, or numerical data, are data for which each piece of data can be placed on a numerical scale and compared.

Examples

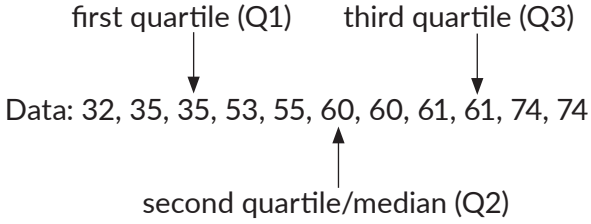
The zoo has 4 lions, 3 tigers, and 6 bears.

In 2006, Los Angeles had a population of about 3,849,378. In the same year, Atlanta had a population of about 429,500.

quartiles

Quartiles are a set of values that describe variation in a data set. When data in a set are arranged in order, quartiles are the numbers that split data into quarters (or fourths).

Example

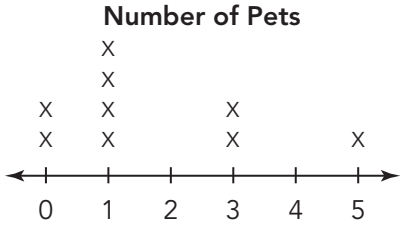


R

range

The range is the difference between the maximum and minimum values of a data set.

Example



0, 0, 1, 1, 1, 1, 3, 3, 5

$5 - 0 = 5$

The range of the data is 5.

rate

A rate is a comparison by division between two quantities that have different units.

Example

The speed of 60 miles in two hours is a rate:

$\frac{60 \text{ mi}}{2 \text{ h}} = \frac{30 \text{ mi}}{1 \text{ h}}$

ratio

A ratio is a multiplicative comparison between two quantities that may contain the same or different units.

Examples



The ratio of stars to circles is $\frac{3}{2}$, or 3:2, or 3 to 2.

The ratio of circles to stars is $\frac{2}{3}$, or 2:3, or 2 to 3.

rational numbers

Rational numbers are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples

-4, $\frac{1}{2}$, $\frac{2}{3}$, 0.67, and $\frac{22}{7}$ are examples of rational numbers.

reciprocal

The reciprocal of a number is also known as the multiplicative inverse of the number. (See *multiplicative inverse*.)

Examples

The reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$: $\frac{3}{7} \cdot \frac{7}{3} = \frac{21}{21} = 1$

The reciprocal of 5 is $\frac{1}{5}$: $\frac{5}{1} \cdot \frac{1}{5} = \frac{5}{5} = 1$

reflexive property of equality

The reflexive property of equality says that when both sides of an equation look exactly the same, their values are equal.

Examples

$7 = 7$

$a = a$

relative frequency table

A relative frequency table displays the ratio or percent of occurrences within a category to the total of the category.

Example

Test Scores for Mr. Watson's Math Test		
Test Score	Frequency	Relative Frequency
61–70	4	$\frac{4}{28} \approx 0.143 = 14.3\%$
71–80	7	$\frac{7}{28} = 0.25 = 25\%$
81–90	9	$\frac{9}{28} \approx 0.321 = 32.1\%$
91–100	8	$\frac{8}{28} \approx 0.286 = 28.6\%$

relatively prime

Two numbers that do not have any common factors other than 1 are called relatively prime.

Example

Positive whole number pairs that have a difference of 1 (4 and 5, 10 and 11, 15 and 16) are always relatively prime.

repeating decimal

A repeating decimal is a decimal in which a digit, or a group of digits, repeat(s) infinitely. Repeating decimals are rational numbers.

Examples

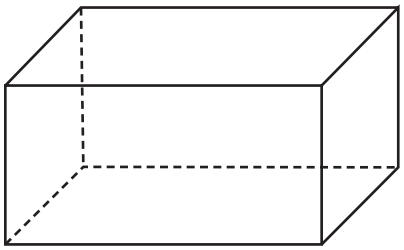
$\frac{1}{9} = 0.111\ldots$ $\frac{7}{12} = 0.58333\ldots$

$\frac{22}{7} = 3.142857142857\ldots$

right rectangular prism

A right rectangular prism is a polyhedron with three pairs of congruent and parallel rectangular faces.

Example



S

sample

A sample is a selection from a population.

Example

If you wanted to determine the average height of the students in your school, you could choose a certain number of students and measure their heights. The heights of the students in this group would be your sample.

scale factor

The scale factor is the factor that you multiply or divide both parts of a ratio by to scale up or scale down to an equivalent ratio.

scaling down

Scaling down means to divide both parts of the ratio by the same factor greater than 1, or multiply both parts of the ratio by the same factor less than 1.

Example

$$\frac{3}{6} = \frac{1}{2} \quad (\div 3 \text{ above and below})$$

scaling up

Scaling up means to multiply both parts of a ratio by the same factor greater than 1.

Example

$$\frac{1}{2} = \frac{3}{6} \quad (\cdot 3 \text{ above and below})$$

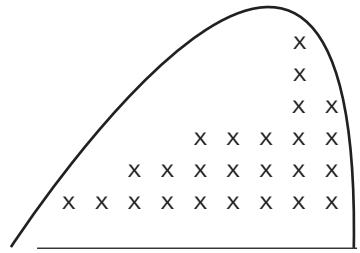
scholarship

A scholarship is money awarded to a person from the college or private institution to help a student pay for school. Scholarships may be academic, athletic, based on financial need, or awarded from private organizations for a variety of reasons.

skewed left distribution

In a skewed left distribution of data, the peak of the data is to the right side of the graph. There are only a few data points to the left side of the graph.

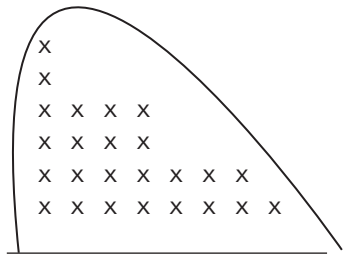
Example



skewed right distribution

In a skewed right distribution of data, the peak of the data is to the left side of the graph. There are only a few data points to the right side of the graph.

Example



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solution

A solution to an equation is any value for a variable that makes the equation true.

Example

The solution to the equation $2x + 4 = 8$ is $x = 2$.

solution set of the inequality

The set of all points that make an inequality true is the solution set of the inequality.

Examples

$x < 7$

The solution set for $x \geq 7$ is all the numbers greater than or equal to 7.

solve an inequality

To solve an inequality means to determine the values of the variable that make the inequality true.

Example

$2x + 4 \geq 16$

$2x \geq 12$

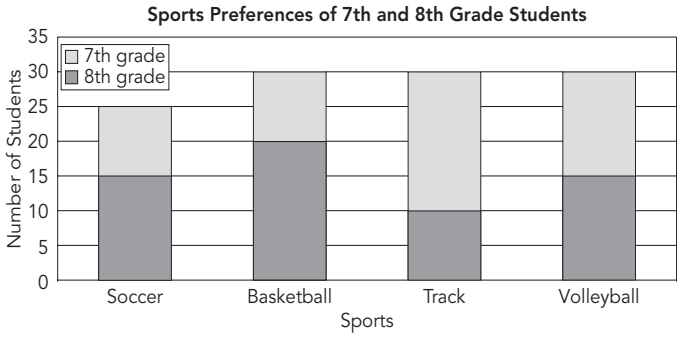
$x \geq 6$

Any value for x that is greater than or equal to 6 will make the inequality true.

stacked bar graph

A stacked bar graph is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole.

Example



statement

A statement is a summary of the account balance. A statement includes every transaction that occurred throughout the month including deposits, withdrawals, debits, and transfers.

statistical process

The statistical process has four components:

- Formulating a statistical question
- Collecting appropriate data
- Analyzing the data graphically and numerically
- Interpreting the results of the analysis

statistical question

A statistical question is a question that anticipates an answer based on data that vary.

Example

"What sport is the most popular in your school?" is a statistical question because it anticipates that the answers will vary since not everyone at your school is likely to have the same favorite sport.

"How many students are in Chess Club?" is NOT a statistical question because there is only one answer to the question.

stem-and-leaf plot

A stem-and-leaf plot is a graphical method used to represent ordered numerical data. Once the data are ordered, the stem and leaves are determined. Typically, the stem is all the digits in a number except the rightmost digit, which is the leaf.

Example

Books Read in Mr. Brown's Class

0	3, 6
1	0, 1, 5
2	
3	9, 9
4	0, 0, 0

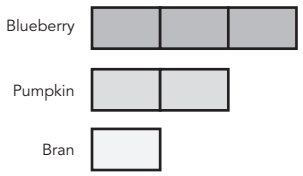
Key: 1 | 0 = 10

strip diagram

A strip diagram illustrates number relationships by using rectangles to represent ratio parts.

Example

A bakery sells packs of muffins in the ratio of 3 blueberry muffins : 2 pumpkin muffins : 1 bran muffin. The strip diagram represents the ratio of each type of muffin.



student loan

A student loan is money borrowed in order to pay for college or trade school.

subtraction property of equality

The subtraction property of equality states that when you subtract the same value c from equal values a and b, the differences are equal.

Examples

$12 = 12$ and $12 - 7 = 12 - 7$

If $a = b$, then $a - c = b - c$.

survey

A survey is one method of collecting data in which people are asked one or more questions.

Example

A restaurant may ask its customers to complete a survey with the following question:

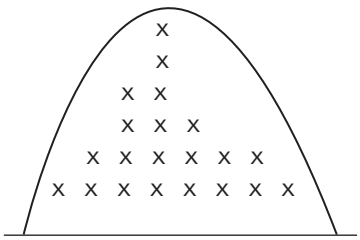
On a scale of 1–10, with 1 meaning "poor" and 10 meaning "excellent," how would you rate the food you ate?

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

symmetric distribution

In a symmetric distribution of data, the left and right halves of the graph are mirror images of each other. The peak is in the middle because there are many data values in the center.

Example



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symmetric property of equality

The symmetric property of equality states that if $a = b$, then $b = a$.

Example

$x = 3$ is the same as $3 = x$.

T

term

A term of an algebraic expression is a number, variable, or product of numbers and variables.

Example

The expression has four terms.

$3y + 5xy + \frac{1}{2}x + 6$

terms

terminating decimal

A terminating decimal has a finite number of digits, meaning that after a finite number of decimal places, all following decimal places have a value of 0. Terminating decimals are rational numbers.

Examples

$\frac{9}{10} = 0.9$ $\frac{15}{8} = 1.875$ $\frac{193}{16} = 12.0625$

transfer

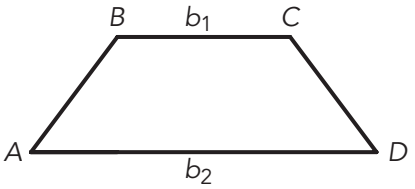
A transfer occurs when money is moved from one account to another.

trapezoid

A trapezoid is a quadrilateral with two bases that are parallel to each other, often labeled b_1 and b_2.

Example

Quadrilateral *ABCD* is a trapezoid. Side *BC* is parallel to side *AD*.



Triangle Inequality theorem

The Triangle Inequality theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Triangle Sum theorem

The Triangle Sum theorem states that the sum of the measures of the interior angles of a triangle is 180°.

tuition

Tuition is the fee paid in order to receive instruction at a school.

U

undergraduate degree

An undergraduate degree is a four-year degree from a college or university.

unit cube

A unit cube is a cube in which its sides are all 1 unit long.

unit fraction

A unit fraction is a fraction that has a numerator of 1 and a denominator that is a positive integer.

unit rate

A unit rate is a comparison of two different measurements in which the numerator or denominator has a value of one unit.

Example

The speed 60 miles in 2 hours can be written as a unit rate:

$\frac{60 \text{ mi}}{2 \text{ h}} = \frac{30 \text{ mi}}{1 \text{ h}}$.

The unit rate is 30 miles per hour.

unit rate of change

The unit rate of change describes the amount the dependent variable changes for every unit the independent variable changes.

V

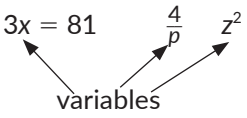
variability

In statistics, variability means that the value of the attribute being studied can change from one person or thing to another.

variable

A variable is a letter or symbol that is used to represent a number.

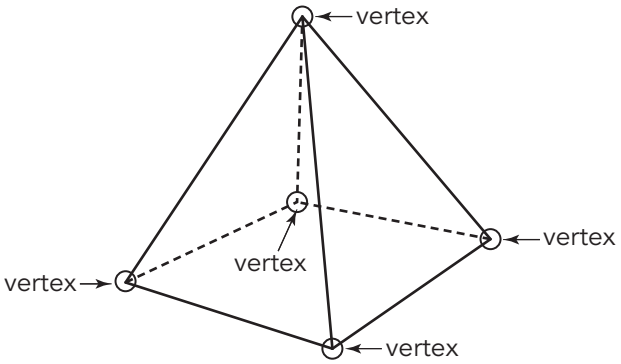
Examples



vertex

A vertex of a polyhedron is a point at which three or more of its edges meet.

Example



vocational school

A vocational school is a school in which students are taught the skills needed to perform a particular job.

volume

Volume is the amount of space occupied by an object. Volume is measured in cubic units.

W

withdrawal

A withdrawal occurs when money is taken out of an account, usually from a check, online payment, or Automated Teller Machine (ATM).

work-study program

A work-study program is a plan set up by the college that allows a student to work while in school to earn money.

Z

zero pair

A positive counter and a negative counter together make a zero pair, since the total value of the pair is zero.

Example

zero property of multiplication

The zero property of multiplication states that the product of any number and 0 is 0.

Examples

$6 \cdot 0 = 0$

$\frac{3}{4} \cdot 0 = 0$

$5^2 \cdot 0 = 0$

$0.125(0)\ 0 = 0$

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