Wall Township Public Schools	Board Approval Date: 11/18/2025

Course Title: Precalculus Honors

Course Description: This course is designed for the mathematically talented students who are interested in pursuing mathematics at an advanced level, particularly Advanced Placement Calculus. The course focuses on trigonometry, function analysis, and an introduction to calculus designed to prepare students for success in future advanced mathematics courses. Graphing, problem solving, and analysis are stressed. Students are exposed to assessment strategies aligned to work expected in an AP Calculus course. Graphing calculator use is emphasized. The use of technology is infused to gather, analyze, and communicate mathematical information.

New Jersey Student Learning Standards - Mathematics

Vision for Mathematics Education in New Jersey

A New Jersey education in Mathematics builds quantitatively and analytically literate citizens prepared to meet the demands of college and career, and to engage productively in an information-driven society. All students will have access to a high-quality mathematics education that fosters a population that:

- leverages data in decision-making and as a lens for discussing, analyzing, and responding to practical questions.
- persists to make sense of and model problems arising in everyday life, society, and the workplace.
- thinks critically and strategically to assess quantitative relationships and to solutions to complex problems.
- employs precise reasoning and constructs viable arguments to deduce conclusions, recognize false statements and

assess peers' reasoning.
- interprets, evaluates and critiques the mathematics embedded in social, scientific and commercial systems, as well as the claims made in the private and public sectors.
- communicates precisely when conveying, representing, and justifying both qualitative and quantitative perspectives.

Standards for Mathematical Practice*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

**See end of document for additional information related to the mathematical practices.*

Inclusion of Climate Change Opportunities

( ) notes opportunities to integrate specific examples of climate change education provided by additional age-appropriate resources

The goal of inclusion of climate change education implementation is to foster generations of New Jersey students that can analyze, question, interpret, to think independently, and bring critical deduction to fulfill, and to lead in jobs created by burgeoning industries of the future green economy.

Course Sequence & Pacing	
Unit Title	**Estimated Number of Instructional Periods****
Unit 1: Unit 1- Introduction to Functional Analysis	24
Unit 2: Analysis of Polynomial Functions	13
Unit 3: Analysis of Rational Functions	12
Unit 4: Analysis of Implicitly Defined Functions	9
Unit 5: Analysis of Exponential and Logarithmic Functions	21
Unit 6: Introduction to Trigonometric Functions	18
Unit 7: Analysis of Trigonometric Functions	15
Unit 8: Parametric, Polar, and Vector Analysis	19
Unit 9: Limits and Continuity	15

**Instructional periods are scheduled for

Unit 1

Desired Results

Unit 1 Title: Introduction to Functional Analysis

Unit Summary: Creating a graph is not the same as interpreting the information displayed. Graphs and equations are alternative ways of displaying relationships between real life situations.

Learning Standards

NJSLS Math:

Interpreting Functions F-IF
A. Understand the concept of a function and use function notation
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain,then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y =f(x).

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

C. Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions F-BF
A. Build a function that models a relationship between two quantities
1. Write a function that describes a relationship between two quantities.★
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

B. Build new functions from existing functions
3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠1.

b. (+) Verify by composition that one function is the inverse of another.
c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Math Practices:
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

Interdisciplinary Connections

Career Readiness, Life Literacies, & Key Skills *(CLKS)***:**

With a growth mindset, failure is an important part of success.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

Innovative ideas or innovation can lead to career opportunities.

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities. Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

Computer Science & Design Thinking (*CS & DT***):**

Successful troubleshooting of complex problems involves multiple approaches including research, analysis, reflection, interaction with peers, and drawing on past experiences.

8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

Individuals select digital tools and design automated processes to collect, transform, generalize, simplify, and present large data sets in different ways to influence how other people interpret and understand the underlying information.

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena,including climate change.

Large data sets can be transformed, generalized, simplified, and presented in different ways to influence how individuals interpret and understand the underlying information.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

Technology innovation and improvement may be influenced by a variety of factors.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets	
Unit Essential Questions: How are functions and their graphs related? How can technology be used to investigate properties of functions and their graphs?	**Unit Enduring Understandings:** Graphs and equations are alternative ways for depicting and analyzing patterns of non-linear change. Mathematical models can be used to describe physical relationships; these are often non-linear.

What information can be used from an equation to sketch a function? How can a function be manipulated and how does it affect its graph?	Creating a graph is not the same as interpreting the information displayed. A variety of families of functions and methods can be used to model and solve real world situations.
Vocabulary: Parent functions, key characteristics, piecewise, transformation, zeros, domain, range, increasing, decreasing, relative extrema, odd function, even function, continuous, discontinuity, one-to-one, inverse function	
Students will be able to: Review graphs of constant, linear, absolute value, quadratic, and radical functions as shifts, reflections and/or stretches of their basic functions. Numerically understand shifts, reflections, and/or stretches of graphs of functions and graph piecewise defined functions. Compute values of the domain and range graphically and analytically. Interpret graphs of functions identifying intervals of increasing, decreasing, and constant function values as well as comparing the function to F(x) = 0. Compare and contrast functions and their graphs.	

Determine domain and range of a function graphically and analytically.

Perform operations on functions, including the composition of functions numerically, graphically and analytically.

Understand what it means for a function to be discontinuous and classify discontinuities as removable (point) or non-removable (jump or infinite).

Rewrite absolute value functions as piece-wise defined functions.

Classify functions as even, odd, or neither numerically, graphically and analytically.

Determine if a function is a one-to-one function or not.

If a function is a one-to-one function, find the inverse of a function numerically, graphically and analytically.

Learning Activities:

Formative: Other Evidence:Performance: Skill Demonstration
Teacher will incorporate various active learning strategies to assess student's comprehension of daily topics and then adjust teaching plans accordingly. Strategies to include: *communicators *exit passes *feedback conferences *Teacher created materials *gallery walks *quiz, quiz, trade *active answers

Assessment Evidence

Formative Assessments: Intro to Functional Analysis Part I Quiz (Formative) Intro to Functional Analysis Part II Quiz (Formative) Mid-Unit Test (Formative/Summative) Intro to Functional Analysis Part III Quiz (Formative)
Summative Assessments: Unit Test (Summative)
Common Benchmark Assessments: Cumulative Assessment
Alternative Assessments: Modified Unit Assessments, Modified Quizzes

Core Instructional & Supplemental Materials
Pearson Pre-Calculus Textbook
Practices quizzes
Teacher website
www.Desmos.com,
www.KhanAcademy.org,
www.njctl.org/courses/math/precalculus

Accommodations and Modifications:
Students with Disabilities, English Language Learners,
Students at Risk of Failure, Students with 504s, Gifted & Talented Students

Specific Strategies and Practices that Support Students with Disabilities:

Students receiving Special Education programming have specific goals and objectives, as well as accommodations and modifications outlined within their Individualized Education Plans (IEP) due to an identified disability and/or diagnosis. In addition to exposure to the general education curriculum, the instruction is differentiated based upon the student's needs. The IEP acts as a supplemental curriculum guide inclusive of instructional strategies that support each learner.

- Small group/One to one - Additional time - Review of directions - Student restates information - Space for movement or breaks - Extra visual and verbal cues and prompts - Preferential seating - Follow a routine/schedule - Rest breaks - Verbal and visual cues regarding directions and staying on task - Checklists - Immediate feedback Alternate or reduced work within Digital Platforms such as Math IXL, Delta Math, and Link It assignments

Specific Strategies and Practices that Support English Language Learners

English language learner (ELL) is a national-origin-minority student who is limited-English-proficient. This term is often preferred over limited-English-proficient (LEP) as it highlights accomplishments rather than deficits.

- Pre-teaching of vocabulary and concepts - Personal Glossary - Extended Time - Simplified/Verbal Instructions - Frequent Breaks - Visual learning, including graphic organizers - Use of cognates to increase comprehension - Teacher modeling - Pairing students with beginning English language skills with students who have more advanced English language skills - Scaffolding - Word walls - Think-pair-share - Cooperative learning groups Alternate or reduced work within Digital Platforms such as Math IXL, Delta Math, and Link It assignments

Students at Risk of Failure

- Home/School Connection - Alternative Assessments - Learning Contracts - Homogenous Grouping - Supplemental Instruction - Think-Pair-Share by Readiness - Homework Options - Differentiated Online Practice - Leveled Rubrics - Tiered Level of Supports - Tiered Activities - Small Group Instruction - Work Alone Choice - Group Work Choice Alternate or reduced work within Digital Platforms such as Math IXL, Delta Math, and Link It assignments - Opportunities

for make-up work or reduce number of assignments - Provide access to extra help when possible

Students with 504 Plans

Teachers are responsible for implementing designated services and strategies identified on a student's 504 Plan.
504 plans are formal plans that schools develop to give kids the support they need and covers any condition that limits daily activities in a major way. Students' accommodations are outlined in their individual 504 plan. -

Specific Strategies and Practices that Support Gifted & Talented Students

It is critical that all teachers are able to recognize a high-ability student who may need more depth and complexity in instruction.

-Homogenous Grouping - Small Group In-Depth Instruction - Differentiated Online Practice - Alternative Assessments - Pre-Assess to provide Differentiated Instruction to Accelerate Learning

-Enrichment/Extension Activities within Digital Platforms such as Math IXL, Delta Math, and Link It assignments

Unit 2

Desired Results

Unit 2 Title: Analysis of Polynomial Functions

Unit Summary: In this unit students will focus on polynomial functions of degree 3 or higher to identify end behavior and zeros of polynomial functions. Techniques will be presented for solving certain kinds of polynomial equations that will enhance students' abilities to manipulate algebraically the polynomial functions that model real world situations. Additionally, students will study methods for finding zeros of polynomial functions.

Learning Standards

NJSLS Math:

Arithmetic with Polynomials and Rational Expressions A -APR

A. Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

C. Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity (x2 + y2) 2 = (x2 – y2) 2 + (2xy) 2 can be used to generate Pythagorean triples

5. (+) Know and apply the Binomial Theorem for the expansion of (x + y) n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Interpreting Functions F-IF .
B. Interpret functions that arise in applications in terms of the context
★ 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
★ C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Math Practices:
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Innovative ideas or innovation can lead to career opportunities.

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities. Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

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Technology innovation and improvement may be influenced by a variety of factors.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**
How do the characteristics of graphs relate to their corresponding equations?	Functions can be written in many ways- some forms can be helpful under different circumstances.
How can algebra help us get information about a graph from an equation?	The vocabulary is just as important as the math.
How can the graph of a function be used to determine the essential characteristics of the function?	Functions families have common characteristics.
What is the best method for graphing polynomials?	We can use algebra to help graph functions.
How can technology assist in the understanding of graphing functions?	Technology is a resource for discoveries and investigations.
	There is a difference between using technology to find the answer compared to using technology to aid in

	supporting the answer.

Vocabulary: End behavior, polynomials, zeros, multiplicity, degree, turning points, concavity, factored form, standard form, intercepts, long division, synthetic division, remainder theorem, factor theorem, rational roots theorem, real and complex zeros, conjugate pairs.

Students will be able to:

Solve polynomial equations by factoring and make connections between the solutions and the graph of the function.

Solve polynomial inequalities.

Review polynomial synthetic division and apply the Factor and Remainder Theorems for polynomial functions.

Understand the graphical connection between the factors of a function and the zeros of the graph, including the multiplicity of those zeros.

Identify graphical properties of polynomial functions including relative extrema, intervals of increasing and decreasing, points of inflection, and intervals of concavity.

Sketch approximate graphs of polynomial functions using the zeros, the multiplicity of the zeros and end behavior.

Apply Descartes' Rule of Signs to determine possible combinations and types of zeros of polynomial functions and find equations of polynomial functions with integral coefficients given their roots.

Develop the Rational Root Theorem to identify all of the possible rational roots of a polynomial function.

Find all roots, real and imaginary, by applying the Rational Roots Theorem of polynomial functions.

Analyze polynomial functions numerically, graphically and analytically.

Learning Activities:

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Teacher will incorporate various active learning strategies to assess student's comprehension of daily topics and then adjust teaching plans accordingly. Strategies to include: *communicators *exit passes *feedback conferences *Teacher created materials *gallery walks *quiz, quiz, trade *active answers

Assessment Evidence

Formative Assessments:

Analysis of Polynomials Quiz Part I (Formative)
Analysis of Polynomials Quiz Part II (Formative)
Analysis of Polynomials Quiz Part III (Formative)

Summative Assessments:

Mid-Unit Test (Summative)

Unit Test (Summative)
Common Benchmark Assessments: Cumulative Assessment
Alternative Assessments: Modified Unit Assessments, Modified Quizzes

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Unit 3

Desired Results

Unit 3 Title: Analysis of Rational Functions

Unit Summary: The unit continues with the study of rational functions and their graphs. Students will find the domain, explore limits, identify vertical and horizontal asymptotes, and use transformations to graph a rational function. Finally, students will learn the basic techniques for solving polynomial inequalities and use those techniques to solve inequalities involving rational functions.

Learning Standards

NJSLS Math:
Arithmetic with Polynomials and Rational Expressions A -APR
D. Rewrite rational expressions
6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Reasoning with Equations and Inequalities A -REI

A. Understand solving equations as a process of reasoning and explain the reasoning

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

D. Represent and solve equations and inequalities graphically
11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
Interpreting Functions F-IF

C. Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

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Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**
How do the characteristics of graphs relate to their corresponding equations? How can algebra help us get information about a graph from an equation? How can the graph of a function be used to determine the essential characteristics of the function? How are horizontal asymptotes connected to end behavior? How can technology assist in the understanding of graphing functions?	Functions can be written in many ways- some forms can be helpful under different circumstances. The vocabulary is just as important as the math. Functions families have common characteristics. We can use algebra to help graph functions. Technology is a resource for discoveries and investigations. There is a difference between using technology to find the answer compared to using technology to aid in supporting the answer.

Vocabulary: End behavior, zeros, multiplicity, factored form, intercepts, long division, synthetic division,real and complex zeros, vertical asymptotes, horizontal asymptotes, oblique asymptotes.

Students will be able to:

Solve rational equations and inequalities.

Discover graphical and analytical connections including restricted values, domain, and intercepts for rational functions.

Discover properties of rational functions analytically including point discontinuities and vertical asymptotes.

Determine equations of horizontal and slant asymptotes of rational functions.

Develop graphs of rational functions.

Understand the numerical behavior of rational functions at discontinuities.

Learning Activities:

Formative: Other Evidence:Performance: Skill Demonstration
Teacher will incorporate various active learning strategies to assess student's comprehension of daily topics and then adjust teaching plans accordingly. Strategies to include: *communicators *exit passes *feedback conferences *Teacher created materials *gallery walks *quiz, quiz, trade *active answers

Assessment Evidence

Formative Assessments:

Rational Functions Quiz (Formative)

Summative Assessments: Unit Test (Summative)
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Specific Strategies and Practices that Support English Language Learners
English language learner (ELL) is a national-origin-minority student who is limited-English-proficient. This term is often preferred over limited-English-proficient (LEP) as it highlights accomplishments rather than deficits.

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Students at Risk of Failure

- Home/School Connection - Alternative Assessments - Learning Contracts - Homogenous Grouping - Supplemental Instruction - Think-Pair-Share by Readiness - Homework Options - Differentiated Online Practice - Leveled Rubrics - Tiered Level of Supports - Tiered Activities - Small Group Instruction - Work Alone Choice - Group Work Choice Alternate or reduced work within Digital Platforms such as Math IXL, Delta Math, and Link It assignments - Opportunities for make-up work or reduce number of assignments - Provide access to extra help when possible

Students with 504 Plans
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Specific Strategies and Practices that Support Gifted & Talented Students

It is critical that all teachers are able to recognize a high-ability student who may need more depth and complexity in instruction.

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-Enrichment/Extension Activities within Digital Platforms such as Math IXL, Delta Math, and Link It assignments

Unit 4

Desired Results

Unit 4 Title: Analysis of Implicitly Defined Functions

Unit Summary:
Conic sections are symbolic and graphic representations that can be applied to real-life phenomena.

Learning Standards

NJSLS Math:
Interpreting Functions F - IF
C. Analyze functions using different representations
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Expressing Geometric Properties with Equations G-GPE
A. Translate between the geometric description and the equation for a conic section
1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

2. Derive the equation of a parabola given a focus and directrix.

3. (+)Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of

distances from the foci is constant.

Math Practices:
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

Interdisciplinary Connections

Career Readiness, Life Literacies, & Key Skills *(CLKS)*:

With a growth mindset, failure is an important part of success.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

Innovative ideas or innovation can lead to career opportunities.

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities. Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

Computer Science & Design Thinking ***(****CS & DT****)*:

Successful troubleshooting of complex problems involves multiple approaches including research, analysis, reflection, interaction with peers, and drawing on past experiences.

8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

Individuals select digital tools and design automated processes to collect, transform, generalize, simplify, and present large data sets in different ways to influence how other people interpret and understand the underlying information.

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena,including climate change.

Large data sets can be transformed, generalized, simplified, and presented in different ways to influence how individuals interpret and understand the underlying information.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

Technology innovation and improvement may be influenced by a variety of factors.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**
How are multiple representations of conic sections related and used to model real world situations? How can the variable of time be incorporated into the coordinate plane?	The characteristics of conic sections and their representations are useful in solving real-world problems.

Vocabulary: conic section, parabola, circle, ellipse, hyperbola, focus, directrix, major and minor axes, vertices,

Students will be able to:

Derive and apply the distance formula to solve problems in the coordinate plane.

Analyze the symmetry of graphs of implicitly defined equations.

Analyze implicitly defined equations whose graphs are circles and semi-circles.

Analyze implicitly defined equations whose graphs are ellipses.

Analyze implicitly defined equations whose graphs are hyperbolas.

Learning Activities:

Formative: Other Evidence:Performance: Skill Demonstration
Teacher will incorporate various active learning strategies to assess student's comprehension of daily topics and then adjust teaching plans accordingly. Strategies to include: *communicators *exit passes *feedback conferences *Teacher created materials *gallery walks *quiz, quiz, trade *active answers

Assessment Evidence

Formative Assessments:

Analysis of Implicit Functions Quiz (Formative)

Summative Assessments:

Unit Test (Summative)

Common Benchmark Assessments: Cumulative Assessment
Alternative Assessments: Modified Unit Assessments, Modified Quizzes

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Teacher website
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www.KhanAcademy.org,
www.njctl.org/courses/math/precalculus

Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students
Specific Strategies and Practices that Support Students with Disabilities: Students receiving Special Education programming have specific goals and objectives, as well as accommodations and modifications outlined within their Individualized Education Plans (IEP) due to an identified disability and/or diagnosis. In

addition to exposure to the general education curriculum, the instruction is differentiated based upon the student's needs. The IEP acts as a supplemental curriculum guide inclusive of instructional strategies that support each learner.

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Unit 5

Desired Results

Unit 5 Title: Analysis of Exponential and Logarithmic Functions

Unit Summary: In this unit of exponential and logarithmic functions, students will begin with the characteristics of exponential functions and their graphs, as well as use formulas for compound interest. They will continue with the study logarithmic functions as the inverse function of exponential functions. At that point, students will learn the properties of logarithms to expand and condense logarithmic expressions, and then expand that understanding to exponential and logarithmic equations. Finally, students will apply their skills to modeling data with real world examples of exponential growth and decay

Learning Standards

NJSLS Math:
Reasoning with Equations and Inequalities A -REI
D. Represent and solve equations and inequalities graphically
11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Seeing Structure in Expressions A-SSE
B. Write expressions in equivalent forms to solve problems
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12) 12t ≈1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.★

Building Functions F-BF
A. Build a function that models a relationship between two quantities 1. Write a function that describes a relationship between two quantities.★
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

B. Build new functions from existing functions
5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
Interpreting Functions F-IF

C. Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.

Linear and Exponential Models★ F-LE
A. Construct and compare linear and exponential models and solve problems
4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to abct= d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Math Practices:
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Learning Targets	
Unit Essential Questions:	**Unit Enduring Understandings:**

How can an exponential function represent a real-world scenario? How can the properties of logarithms be used to solve equations? Why does simplifying or expanding a logarithmic expression help us solve problems?	Functions families have common characteristics. We can use algebra to help graph functions. Knowing how to read an equation is essential for graphing the function. Mathematics applies to the sciences.
Vocabulary: Exponential function, transformations, e, logarithms, domain, expand, condense, growth, decay.	
Students will be able to: Use the properties of exponents to solve exponential equations by rewriting both sides of the equation as a power of the same base. Investigate graphs and equations of exponential functions. Classify exponential functions as growth or decay functions. Find equations of basic exponential functions, with and without technology, given function values. Graph exponential functions, identifying horizontal asymptotes and applying shifts and/or reflections to basic functions.	

Analyze exponential functions numerically, graphically, and analytically.

Define a logarithm to be the inverse of an exponential and simplify simple logarithms using basic properties.

Simplify complex logarithmic expressions applying all of the properties of logarithms.

Solve exponential and logarithmic equations using the inverse properties of logarithms and exponentials.

Solve application problems involving exponential and logarithmic equations.

Develop graphs of logarithmic functions, identifying vertical asymptotes, domain, range, and end behavior graphically, analytically and numerically.

Learning Activities:

Formative: Other Evidence:Performance: Skill Demonstration
Teacher will incorporate various active learning strategies to assess student's comprehension of daily topics and then adjust teaching plans accordingly. Strategies to include: *communicators *exit passes *feedback conferences *Teacher created materials *gallery walks *quiz, quiz, trade *active answers

Assessment Evidence

Formative Assessments:

Analysis of Exponential Functions Quiz (Formative)
Analysis of Logarithms Quiz (Formative)

Summative Assessments: Mid-Unit Test (Summative) *If necessary* Unit Test (Summative)
Common Benchmark Assessments: Cumulative Assessment
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Unit 6

Desired Results

Unit 6 Title: Introduction to Trigonometric Functions

Unit Summary: Trigonometry involves patterns that can be applied throughout all aspects of trigonometry. These patterns can be applied to the future learning of mathematics.

Learning Standards

NJSLS Math:
Trigonometric Functions F-TF
A. Extend the domain of trigonometric functions using the unit circle
1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for πx, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Circles G-C

B. Find arc lengths and areas of sectors of circles
5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector

Similarity, Right Triangles, and Trigonometry G-SRT
C. Define trigonometric ratios and solve problems involving right triangles
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
D. Apply trigonometry to general triangles

9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces)

Math Practices:
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

Interdisciplinary Connections

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Computer Science & Design Thinking ***(****CS & DT****)*****:**

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Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**
When is it appropriate to use an estimated answer rather than an exact answer? Do patterns exist in trigonometry to better evaluate expressions? As a point travels around a circle, how do the angles formed relate to one another?	One representation may sometimes be more helpful than another; and, used together, multiple representations give a fuller understanding of a problem. Precalculus connects discrete pieces of mathematics from your past learning.

When is one method of measuring an angle more appropriate than another method?	Trigonometry allows for indirect measurement of triangles when given limited information. Patterns exist in trigonometry and can be used to simplify evaluative processes.
Vocabulary: right triangle trigonometry, oblique triangle, degrees, radians, co-terminal angles, reference angles, unit circle, law of sines, law of cosines	
Students will be able to: Understand and apply the three basic trigonometric ratios to solve right triangles and application problems. Apply the Law of Sines and Law of Cosines to solve and find areas of oblique triangles and application problems. Find co-terminal angle(s), reference angle, and convert to radians. Apply the six basic trigonometric ratios to find values for angles on the unit circle. Determine the value of the six trigonometric functions of a given angle not on the unit circle: (1) knowing a point through which the terminal side of the angle passes, or (2) given characteristics and constraints of the angle. Evaluate trigonometric functions of angles and solve simple trigonometric equations without the use of a	

calculator.

Learning Activities:
Formative: Other Evidence:Performance: Skill Demonstration
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Assessment Evidence

Formative Assessments:
Intro to Trig Part I (Formative)
Intro to Trig Part II (Formative)

Summative Assessments:
Unit Test (Summative)

Common Benchmark Assessments: Cumulative Assessment

Alternative Assessments: Modified Unit Assessments, Modified Quizzes

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Unit 7

Desired Results

Unit 7 Title: Analysis of Trigonometric Functions

Unit Summary: A variety of methods exist for solving trigonometric equations and proving identities which transfer to advanced algebraic concepts.

Learning Standards

NJSLS Math:

Creating Equations★ A -CED

A. Create equations that describe numbers or relationships

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

with Equations and Inequalities A -REI

B. Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable

Trigonometric Functions F-TF

C. Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$,or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

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Technology innovation and improvement may be influenced by a variety of factors.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

Learning Targets	
Unit Essential Questions:	**Unit Enduring Understandings:**
What does it mean when we solve a trigonometric equation?	The unit circle is a gateway between geometry and algebra.
How do we represent multiple solutions on a given domain?	Context is critical when using estimation.

What does it mean to prove an identity? How can we use identities to simplify the process involved in solving an equation?	Equivalent expressions can be written in a variety of formats. Algebra and proof can be used to verify trigonometric identities.
Vocabulary: Trig functions, sum and difference identity, double angle identities	
Students will be able to: Use the nine basic trigonometric identities to rewrite expressions in terms of a single trigonometric ratio. Use the nine basic trigonometric ratios to prove that given equations are identities. Apply the trigonometric sum and difference identities. Apply the double angle trigonometric identities. Solve trigonometric equations, with and without technology.	
Learning Activities: Formative: Other Evidence:Performance: Skill Demonstration Teacher will incorporate various active learning strategies to assess student's comprehension of daily topics and then adjust teaching plans accordingly. Strategies to include: *communicators *exit passes *feedback conferences *Teacher created materials *gallery walks *quiz, quiz, trade *active answers	

Assessment Evidence
Formative Assessments: Analysis of Trigonometric Functions Part I (Formative) Analysis of Trigonometric Functions Part II (Formative) Analysis of Trigonometric Functions Part III (Formative)
Summative Assessments: Unit Test (Summative)
Common Benchmark Assessments: Cumulative Assessment
Alternative Assessments: Modified Unit Assessments, Modified Quizzes

Core Instructional & Supplemental Materials
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Practices quizzes
Teacher website
www.Desmos.com,
www.KhanAcademy.org,
www.njctl.org/courses/math/precalculus

Accommodations and Modifications:
Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students

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Students at Risk of Failure

- Home/School Connection - Alternative Assessments - Learning Contracts - Homogenous Grouping - Supplemental Instruction - Think-Pair-Share by Readiness - Homework Options - Differentiated Online Practice - Leveled Rubrics - Tiered Level of Supports - Tiered Activities - Small Group Instruction - Work Alone Choice - Group Work Choice

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-Enrichment/Extension Activities within Digital Platforms such as Math IXL, Delta Math, and Link It assignments

Unit 8

Desired Results

Unit 8 Title: Parametric, Polar, and Vector analysis

Unit Summary: Polar Coordinates and Vectors are alternative methods of describing rectangular coordinates.

Learning Standards

NJSLS Math:
Building Functions F-BF
A. Build a function that models a relationship between two quantities
1. Write a function that describes a relationship between two quantities.★
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

B. Build new functions from existing functions

The Complex Number System N -CN
B. Represent complex numbers and their operations on the complex plane.
4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

Vector and Matrix Quantities N -VM

A. Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).

2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

B. Perform operations on vectors.

4. (+) Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. (+) Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy).

b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠0, the direction of cv is either along v (for c > 0) or against v (for c < 0).

C. Perform operations on matrices and use matrices in applications.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

Math Practices:
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

Interdisciplinary Connections

Career Readiness, Life Literacies, & Key Skills *(CLKS)*:

With a growth mindset, failure is an important part of success.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
Innovative ideas or innovation can lead to career opportunities.

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities. Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving.

Computer Science & Design Thinking *(CS & DT)*:

Successful troubleshooting of complex problems involves multiple approaches including research, analysis, reflection, interaction with peers, and drawing on past experiences.

8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

Individuals select digital tools and design automated processes to collect, transform, generalize, simplify, and present large data sets in different ways to influence how other people interpret and understand the underlying information.

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena,including climate change.

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Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**
How are polar and rectangular graphs related? What are the advantages and disadvantages to using Cartesian and polar coordinate systems for graphing and solving problems?	There exists a definite and distinct bridge between the fields of trigonometry and physics. Multiple ways of representing real world phenomena can be used to solve problems.
Vocabulary: Parametric equation, Polar, pole, rose-curve, cardiod, limacon, vector, magnitude, resultant, dot-product, cross-product	
Students will be able to: Evaluate sets of parametric equations for given values of the parameter. Convert parametric equations to rectangular form eliminating the parameter. Create a parametric equation set from a rectangular equations. Sketch/graph parametric equations. Solve problems using parametric equations. Plot polar coordinates.	

Convert between polar and rectangular equation forms.

Sketch/graph basic polar equations.

Identify basic polar equations such as rose, cardioid, limacons, and lemniscates.

Write vectors in component form.

Describe a vector using magnitude and direction.

Add/Subtract vectors

Resolve a vector into its components.

Compute dot-product and cross-product of vectors. (Optional)

Learning Activities:
Formative: Other Evidence:Performance: Skill Demonstration
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Assessment Evidence

Formative Assessments: Parametric Quiz (Formative) Polar Quiz (Formative) Vector Quiz (Formative)
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Unit 9

Desired Results

Unit 9 Title: Limits and Continuity

Unit Summary: Students explore limits and continuity by analyzing functions numerically, graphically, and algebraically to understand their behavior near specific points. They evaluate limits using various methods, apply the Intermediate Value Theorem, and use infinite limits to identify asymptotes and determine continuity.

Learning Standards

NJSLS Math:
This unit begins concepts in Calculus that are not included in the NJSLS for Mathematics

Mathematical Practice 1- Implementing Mathematical Processes
○ 1.A Identify the question to be answered or problem to be solved (not assessed)
○ 1.B Identify key and relevant information to answer a question or solve a problem (not assessed)
○ 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
○ 1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.
○ 1.E Apply appropriate mathematical rules or procedures, with and without technology.
○ 1.F Explain how an approximated value relates to the actual value.
● Mathematical Practice 2- Connecting Representations
○ 2.A Identify common underlying structures in problems involving different contextual situations.

○ 2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.
○ 2.C Identify a re-expression of mathematical information presented in a given representation.
○ 2.D Identify how mathematical characteristics or properties of functions are related in different representations.
○ 2.E Describe the relationships among different representations of functions and their derivatives.
● Mathematical Practice 3- Justification
○ 3.A Apply technology to develop claims and conjectures (not assessed).

○ 3.B Identify an appropriate mathematical definition, theorem, or test to apply
○ 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
○ 3.D Apply an appropriate mathematical definition, theorem, or test.
○ 3.E Provide reasons or rationales for solutions and conclusions.
○ 3.F Explain the meaning of mathematical solutions in context.
○ 3.G Confirm that solutions are accurate and appropriate.
● Mathematical Practice 4- Communication and Notation
○ 4.A Use precise mathematical language.
○ 4.B Use appropriate units of measure
○ 4.C Use appropriate mathematical symbols and notation (e.g., Represent a derivative using f '(x), y', and dy/dx).
○ 4.D Use appropriate graphing techniques.
○ 4.E Apply appropriate rounding procedures.

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Learning Targets

Unit Essential Questions:	**Unit Enduring Understandings:**
How does the math that you previously studied relate to the math that you are going to be studying? How do limits guarantee the continuity of a function? When do limits fail to exist? How are limits connected to the existence of vertical and horizontal asymptotes? What is the difference between calculating a limit and evaluating a function at a point?	The concept of a limit can be used to understand the behavior of functions. Continuity is a key property of functions that is defined using limits.

What are connections among these presentations of functions; graphically, numerically, analytically or verbally?	
Vocabulary: Domain, range, independent, dependent variable, graph, function, absolute value, increasing, decreasing, linear, quadratic, polynomial, coefficients, degree, cubic , power, root,, reciprocal , rational, algebraic, trigonometric, exponential, logarithmic , translations, composite , limit, right-hand limit, left-hand limit, vertical asymptote, continuous at a point, discontinuity, removable discontinuity, jump discontinuity, horizontal asymptote, infinite limits, limits at infinity, intermediate value theorem.	
Students will be able to: Numerically, graphically, and algebraically classify and analyze functions. Evaluate, if it exists, the value of a limit from a graphical approach, including one-sided limits. Evaluate limits analytically, including direct substitution, cancellation and rationalization, applying the properties of limits. Evaluate limits of exponential functions analytically. Evaluate limits of trigonometric functions analytically. Graphically and analytically, apply the three part definition of continuity to determine if a function is continuous at a point.	

Understand and apply the intermediate value theorem.

Distinguish between infinite limits and limits at infinity, and use them to identify asymptotes.

Learning Activities:
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Assessment Evidence

Formative Assessments:
Quiz #1: Objectives #1-3
Non-calculator quiz consisted of sample AP questions. The quiz will have 7 multiple choice questions and free response question.
Quiz #2: Objectives #4-7
Non-calculator quiz consisted of sample AP questions. The quiz will have 7 multiple choice questions and 1 free response question.

Summative Assessments:
Summative
Test #1: Unit #1 – Limits and Continuity

Test with calculator and non-calculator sections. Each section is comprised of 7 multiple choice questions and 1 free response question
Common Benchmark Assessments: Cumulative Assessment
Alternative Assessments: Modified Unit Assessments, Modified Quizzes

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Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. Reason abstractly and quantitatively Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
3. Construct viable arguments and critique the reasoning of others Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of

their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to

visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered$7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$ older students can see the 14 as $2 \times 7$and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check

whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)\left(x^2 + x + 1\right)$ and $(x - 1)\left(x^3 + x^2 + x + 1\right)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.