| Wall Township Public Schools | Board Approval Date: 8/19/2025 |
|---|---|

**Course Title: Algebra 2 Honors**

**Course Description:** This course differs from the non-honors course in the scope, pace and level of difficulty. This course is intended to build upon student work with linear, quadratic, and exponential functions. Students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Through the Mathematical Practice Standards students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

## New Jersey Student Learning Standards - Mathematics

**Vision for Mathematics Education in New Jersey**

A New Jersey education in Mathematics builds quantitatively and analytically literate citizens prepared to meet the demands of college and career, and to engage productively in an information-driven society. All students will have access to a high-quality mathematics education that fosters a population that:

- leverages data in decision-making and as a lens for discussing, analyzing, and responding to practical questions.
- persists to make sense of and model problems arising in everyday life, society, and the workplace.
- thinks critically and strategically to assess quantitative relationships and to solutions to complex problems.
- employs precise reasoning and constructs viable arguments to deduce conclusions, recognize false statements and assess peers' reasoning.
- interprets, evaluates and critiques the mathematics embedded in social, scientific and commercial systems, as well as the claims made in the private and public sectors.
- communicates precisely when conveying, representing, and justifying both qualitative and quantitative perspectives.

**Standards for Mathematical Practice***

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek

to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

**See end of document for additional information related to the mathematical practices.*

**Inclusion of Climate Change Opportunities**

( ) notes opportunities to integrate specific examples of climate change education provided by additional age-appropriate resources

The goal of inclusion of climate change education implementation is to foster generations of New Jersey students that can analyze, question, interpret, to think independently, and bring critical deduction to fulfill, and to lead in jobs created by burgeoning industries of the future green economy.

## Course Sequence & Pacing

| Unit Title | Estimated Number of Instructional Periods** |
|---|---|
| **Unit 1:** Relations and Functions | 14 periods |
| **Unit 2:** Linear Equations, Inequalities and Systems | 14 periods |
| **Unit 3:** Quadratic Functions | 14 periods |
| **Unit 4:** Polynomials and Polynomial Functions | 9 periods |
| **Unit 5:** Polynomial Equations | 10 periods |
| **Unit 6:** Inverses and Radical Functions | 11 periods |
| **Unit 7:** Exponential Functions | 9 periods |
| **Unit 8:** Logarithmic Functions | 10 periods |
| **Unit 9:** Rational Functions | 11 periods |
| **Unit 10:** Interpreting Data | 10 periods |
| **Unit 10:** Trigonometric Functions | 12 periods |
| **Benchmark Exams** | 6 |
| **Total:** | 130 periods |

**Instructional periods are scheduled for 56 minutes. Classes meet 3 times over 4 days.

# Instructional Notes

**The Standards for Mathematical Content**

The high school standards specify the mathematics that all students should study in order to be college and career ready. The high school standards are listed in conceptual categories and not course specific. It is important for instructors to pay close attention to the supplemental documents which align standards to courses and include clarifications on multi-course standards.

Conceptual Categories

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Limits_and_Clarifications_MultiCourseStandards Math.pdf

Students should spend the majority of their time on the major work of the grade (M). Supporting work (S) and, where appropriate, additional work (A) can engage students in the major work of the grade. See linked document for specifics.

Additional resources can be found at the end of this document.

(+) standards denote standards for the honors level of the course.

# Unit 1

## Desired Results

**Unit 1 Title:** Relations and Functions

**Unit Summary:** In this unit students will determine if functions are one-to-one and onto, determine linearity, intercepts, and symmetry of functions. They will also analyze and compare graphs, graph functions and inequalities in two variables and identify transformations of functions.

## Learning Standards

**NJSLS Math:**
**Functions**
**Interpreting Functions F.IF**
**B. Interpret functions that arise in applications in terms of the context.**
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
**C. Analyze functions using different representations.**
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
**b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
**c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

**Building Functions F.BF**
**B. Build new functions from existing functions.**

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**Algebra**
**Creating Equations A.CED ★**
**A. Create equations that describe numbers or relationships**
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

**Seeing Structure in Expressions A.SSE**
**A. Interpret the structure of expressions**
1. Interpret expressions that represent a quantity in terms of its context. ★

**b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of $P$ and a factor not depending on $P$.

**Math Practices:**
**Standards for Mathematical Practice**
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

## Interdisciplinary Connections

**Career Readiness, Life Literacies, & Key Skills** *(CLKS)*:
9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.
9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

**Computer Science & Design Thinking (*CS & DT*):**
8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.
8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

**Unit Essential Questions:**

How can analyzing a function help you understand the situation it models?

How can we determine if a relation is a function, and what does that tell us about the relationship between variables?

In what ways can key features of functions (such as intercepts, end behavior, and symmetry) be identified and interpreted from graphs and equations?

How do transformations affect the graphs and equations of functions?

Why is it important to analyze and compare different types of functions and their representations?

**Unit Enduring Understandings:**

Functions can be represented in multiple ways—verbally, algebraically, numerically, and graphically—and these representations help describe relationships between quantities.

Identifying features such as domain, range, intercepts, end behavior, and symmetry provides insight into the behavior and nature of functions.

Transformations such as translations, reflections, stretches, and compressions alter the graph of a function in predictable ways.

Analyzing and comparing functions allows us to make informed decisions and predictions in real-world and mathematical contexts.

Graphing inequalities and piecewise-defined functions expands our ability to model complex scenarios using mathematical tools.

| How can we use graphs to model and solve real-world problems involving functions and inequalities? | |
|---|---|

**Vocabulary:** Domain, Codomain, Range, Continuous Function, Linear Function, Linear Equation, Intercept, Line Symmetry, Point Symmetry, Even Functions, Odd Functions, Extrema, Maximum, Minimum, Relative Maximum, Relative Minimum, End Behavior, Linear Inequality, Boundary, Closed Half-Plane, Open Half-Plane, Constraint, Piecewise-Defined Function, Step Function, Greatest Integer Function, Absolute Value Function, Parent Function, Family of Graphs, Constant Function, Identify Function, Translation, Dilation, Reflection.

**Students will be able to:**

- Determine the continuity of functions and whether functions are one-to-one and/or onto
- Determine the linearity, intercepts and symmetry of functions
- Identify extrema and end behavior of functions
- Sketch graphs of functions and compare two functions represented in different ways
- Graph linear functions and inequalities in two variables
- Write and graph piecewise-defined, and absolute value functions
- Identify and use transformations of functions

**Learning Activities:** *Suggested or Potential Learning Activities*
Lesson Launch/Opener
Foldables
Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

Climate Change Example: Activity: Modeling Global Temperature Rise- Analyze real-world data to explore function types, graph behavior, and transformations in the context of climate change.

## Assessment Evidence

**Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for understanding

**Summative Assessments:** Cumulative Assessments

| |
|---|
| **Common Benchmark Assessments:** Common Unit Assessments, LinkIt! Benchmarks |
| **Alternative Assessments:** Reveal online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments |

## Core Instructional & Supplemental Materials

| |
|---|
| Reveal Algebra 2 McGraw Hill |
| ALEKS, DeltaMath, Khan Academy, Desmos, Graphing Calculators |
| LinkIt! Benchmarks, LinkIt! Test Bank |

## Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students

**Specific Strategies and Practices that Support Students with Disabilities:**
Students receiving Special Education programming have specific goals, objectives, accommodations, and modifications outlined in their Individualized Education Plans (IEPs) due to an identified disability and/or diagnosis. In addition to access to the general education curriculum, instruction is differentiated based on individual student needs. The IEP serves as a supplemental curriculum guide, inclusive of instructional strategies tailored to support each learner. *Examples of supports and accommodations include* small group or one-to-one instruction, additional time, review and clarification of directions, students restating instructions to demonstrate understanding, space for movement or scheduled breaks, extra visual and verbal cues and prompts, preferential seating, consistent routines and schedules, rest breaks, visual and verbal reminders for directions and task focus, checklists, immediate feedback, and alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

**Specific Strategies and Practices that Support English Language Learners**
An English Language Learner (ELL) is a national-origin minority student who is limited-English proficient. The term "ELL" is preferred over "limited-English proficient" as it emphasizes potential and progress rather than deficits. *Instructional strategies to support ELLs include* pre-teaching vocabulary and concepts, use of personal glossaries, extended time, simplified and verbal instructions, frequent breaks, visual aids including graphic organizers, use of cognates to support comprehension, teacher modeling, peer pairing (pairing beginning ELLs with more advanced ELLs), scaffolding techniques, word walls, think-pair-share activities, cooperative learning groups, and alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

**Students at Risk of Failure**

Students at risk of failure benefit from targeted supports that address both academic and social-emotional needs, providing multiple access points to learning and increasing the likelihood of success. *Support strategies include* strengthening the home/school connection, alternative assessments, learning contracts, homogeneous grouping, supplemental instruction, think-pair-share by readiness, homework options, differentiated online practice, leveled rubrics, tiered levels of support and activities, small group instruction, choice to work alone or in groups, alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt, opportunities to make up work or reduce the number of assignments, and access to extra help when available.

**Students with 504 Plans**
Teachers are responsible for implementing the accommodations and strategies outlined in a student's 504 Plan. A 504 Plan is a formal document designed to provide support for students with physical or mental impairments that significantly limit one or more major life activities. Accommodations are individualized and legally binding, and support must be consistently provided as outlined in the plan.

**Specific Strategies and Practices that Support Gifted & Talented Students**
It is essential that teachers can identify high-ability students who may require more depth and complexity in instruction. These learners benefit from opportunities to accelerate and enrich their learning. *Strategies to support gifted and talented students include* homogeneous grouping, small group in-depth instruction, differentiated online practice, alternative assessments, pre-assessment to guide instruction and accelerate learning, and enrichment and extension activities within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

# Unit 2

## Desired Results

**Unit 2 Title:** Linear Equations, Inequalities, and Systems

**Unit Summary:** In this unit, students will deepen their understanding of linear equations, inequalities, and systems as tools

for modeling and solving real-world problems. They will explore multiple representations of linear relationships, including standard, slope-intercept, and point-slope forms, and apply algebraic and graphical methods to solve equations and inequalities—both with and without absolute value. Students will also investigate systems of equations and inequalities in two and three variables, using strategies such as graphing, substitution, and elimination to determine solutions. Emphasis is placed on interpreting results, choosing appropriate methods, and justifying mathematical reasoning in context.

## Learning Standards

**NJSLS Math:**
**Algebra**
**Creating Equations A.CED**
**A. Create equations that describe numbers or relationships.**
1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

**Reasoning with Equations and Inequalities A.REI**
**D. Represent and solve equations and inequalities graphically.**
11. Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

**Functions**
**Interpreting Functions F.IF**
**B. Interpret functions that arise in applications in terms of the context.**
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

**Math Practices:**

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
4. Model with mathematics
5. Use appropriate tools strategically
7. Look for and make use of structure

## Interdisciplinary Connections

**Career Readiness, Life Literacies, & Key Skills** *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

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9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

**Computer Science & Design Thinking (***CS & DT***):**

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8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

| **Unit Essential Questions:** | **Unit Enduring Understandings:** |
| --- | --- |
| How are equations, inequalities, and systems of equations or inequalities best used to model real-world situations? | Equations and inequalities represent relationships and can be solved using a variety of algebraic and graphical strategies. |

| | |
|---|---|
| In what ways do different forms and methods for solving systems of equations and inequalities help us analyze relationships and make decisions? | Linear equations can be represented in multiple forms, each offering unique insights into their characteristics and applications.<br><br>Systems of equations and inequalities model multiple constraints and relationships; they can be solved and interpreted using graphical and algebraic methods. |

**Vocabulary:** absolute value, bounded, consistent, dependent, elimination, empty set, equation, extraneous solution, feasible region, inconsistent, inequality, independent, linear programming, optimization, ordered triple, root, solution, substitution, system of equations, system of inequalities, unbounded, zero

**Students will be able to:**

- Solve linear equations and inequalities in one variable
- Solve equations and inequalities involving absolute value algebraically
- Write linear equations in standard, slope-intercept, and point-slope form
- Solve systems of equations by graphing
- Solve systems of equations by using substitution or elimination
- Solve systems of inequalities in two variables
- Solve systems of equations in three variables
- Solve equations and inequalities involving absolute value by graphing

**Learning Activities:** *Suggested or Potential Learning Activities*
Lesson Launch/Opener
Foldables
Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m, where c(m) is the number of molecules of carbon dioxide.

## Assessment Evidence

**Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for understanding

**Summative Assessments:** Cumulative Assessments

**Common Benchmark Assessments:** Common Unit Assessments, LinkIt! Benchmarks

**Alternative Assessments:** Reveal online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments

## Core Instructional & Supplemental Materials

Reveal Algebra 2 McGraw Hill

ALEKS, DeltaMath, Khan Academy, Desmos, Graphing Calculators

LinkIt! Benchmarks, LinkIt! Test Bank

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# Unit 3

## Desired Results

### **Unit 3 Title:** Quadratic Functions

**Unit Summary:**
In this unit, students explore the structure and applications of quadratic functions and inequalities. They will learn to graph quadratic functions and inequalities, identifying key features such as vertex, axis of symmetry, direction of opening, and intercepts. Students will solve quadratic equations and inequalities using multiple methods, including factoring, completing the square, using the quadratic formula, and graphing. Emphasis will be placed on selecting the most efficient method for a given problem. Additionally, students will apply their understanding of quadratic functions to model and solve real-world problems, interpreting the meaning of solutions in context. By the end of the unit, students will be able to fluently analyze and manipulate quadratic expressions, equations, and graphs to

make informed mathematical decisions.

## Learning Standards

**NJSLS Math:**

**Number and Quantity**
**The Real Number System N.RN**
**A. Extend the properties of exponents to rational exponents**

3. Simplify radicals, including algebraic radicals (e.g. $\sqrt[3]{54} = 3\sqrt[3]{2}$ , simplify $\sqrt{32x^2}$ ).

**The Complex Number System N.CN**
**A. Perform arithmetic operations with complex numbers.**
1. Know there is a complex number $i$ such that $i^2 = –1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.
2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
**C. Use complex numbers in polynomial identities and equations.**
7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers.

**Algebra**
**Arithmetic with Polynomials and Rational Expressions A.APR**
**C. Use Polynomial identities to solve problems**
4. (+) Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

**Creating Equations ★ A.CED**
**A. Create equations that describe numbers or relationships.**
1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

**Reasoning with Equations and Inequalities A.REI**
**A.** Understand solving equations as a process of reasoning and explain the reasoning
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**B. Solve equations and inequalities in one variable**

4. Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x-p)^2=q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2=49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a\pm bi$ for real numbers $a$ and $b$.

**C. Solve systems of equations**

6. Solve systems of linear equations algebraically (include using the elimination method) and graphically, focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y=-3x$ and the circle $x^2+y^2=3$.

**D. Represent and solve equations and inequalities graphically.**

11. Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

**Seeing Structure in Expressions A.SSE.**

**A. Interpret the structure of expressions**

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4-y^4$ as $\left(x^2\right)^2-\left(y^2\right)^2$, thus recognizing it as a difference of squares that can be factored as $\left(x^2-y^2\right)\left(x^2+y^2\right)$.

**Functions**

**Interpreting Functions F.IF**

**B. Interpret functions that arise in applications in terms of the context.**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

**C. Analyze functions using different representations**

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

**Math Practices:**

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
4. Model with mathematics
5. Use appropriate tools strategically
7. Look for and make use of structure

## Interdisciplinary Connections

**Career Readiness, Life Literacies, & Key Skills** *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

**Computer Science & Design Thinking (***CS & DT***):**

8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

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8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

| **Unit Essential Questions:** | **Unit Enduring Understandings:** |
| --- | --- |
| What are important characteristics of a quadratic function?<br><br>What real world situations can be modeled by quadratic functions and equations?<br><br>When is it most appropriate to use the square root property or completing the square to simplify or solve a quadratic expression?<br><br>How does the quadratic formula and discriminant help us determine the nature and number of solutions to a quadratic equation?<br><br>What are effective strategies for graphing and solving quadratic inequalities?<br><br>How can we solve systems that include both linear and quadratic equations, and what do their solutions represent? | Quadratic functions have predictable graph shapes and features—such as vertex, axis of symmetry, and intercepts—that help us understand and solve real-world and mathematical problems.<br><br>Complex numbers, including pure imaginary numbers, expand our number system, allowing us to solve all quadratic equations, even when no real solutions exist.<br><br>Factoring is a foundational algebraic method for solving certain types of quadratic equations efficiently.<br><br>The square root property and completing the square are powerful tools for simplifying and solving quadratic expressions, especially when equations are not easily factorable.<br><br>The quadratic formula is a universal method for solving any quadratic equation, and the discriminant within it provides insight into the number and type of solutions.<br><br>Quadratic inequalities can be analyzed and solved using graphing and algebraic methods, helping to identify ranges of possible values in a variety of contexts. |

| | |
|---|---|
| | Systems of linear and quadratic equations can be solved using substitution or graphing, and their solutions represent points of intersection with real-world significance. |

**Vocabulary:** average rate of change, axis of symmetry, completing the square, complex conjugates, complex number, difference of squares, discriminant, factored form, imaginary unit i, maximum, minimum, perfect square trinomials, projectile motion problems, pure imaginary number, quadratic equation, quadratic function, quadratic inequality, quadratic relations, rate of change, rationalizing the denominator, standard form of a quadratic equation, vertex, and vertex form.

**Students will be able to:**

- Students will graph quadratic equations.
- Students will simplify radicals with rational, irrational, and imaginary numbers.
- Students solve quadratic quotations by graphing.
- Students perform operations with pure imaginary and complex numbers.
- Students solve quadratic equations by factoring.
- Students simplify quadratic expressions by using the square root property and completing the square.
- Students use the quadratic formula and discriminant to solve quadratic equations and determine the number of real roots.
- Students graph and solve quadratic inequalities.
- Students solve systems of linear and quadratic equations.
- Factor and Solve polynomials over the set of complex numbers.

**Learning Activities:** *Suggested or Potential Learning Activities*
Lesson Launch/Opener
Foldables
Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

**Climate Change Example:** Students may create equations and/or inequalities to represent the economic impact of climate change.

**Climate Change Example:** Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m, where c(m) is the number of molecules of carbon dioxide.

**Climate Change Example:** Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).

## Assessment Evidence

**Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for understanding

**Summative Assessments:** Cumulative Assessments

**Common Benchmark Assessments:** Common Unit Assessments, LinkIt! Benchmarks

**Alternative Assessments:** Reveal online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments

## Core Instructional & Supplemental Materials

Reveal Algebra 2 McGraw Hill

ALEKS, DeltaMath, Khan Academy, Desmos, Graphing Calculators

LinkIt! Benchmarks, LinkIt! Test Bank

## Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students

**Specific Strategies and Practices that Support Students with Disabilities:**

Students receiving Special Education programming have specific goals, objectives, accommodations, and modifications outlined in their Individualized Education Plans (IEPs) due to an identified disability and/or diagnosis. In addition to access to the general education curriculum, instruction is differentiated based on individual student needs. The IEP serves as a supplemental curriculum guide, inclusive of instructional strategies tailored to support each learner. *Examples of supports and accommodations include* small group or one-to-one instruction, additional time, review and clarification of directions, students restating instructions to demonstrate understanding, space for movement or scheduled breaks, extra visual and verbal cues and prompts, preferential seating, consistent routines and schedules, rest breaks, visual and verbal reminders for directions and task focus, checklists, immediate feedback, and alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

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Students at risk of failure benefit from targeted supports that address both academic and social-emotional needs, providing multiple access points to learning and increasing the likelihood of success. *Support strategies include* strengthening the home/school connection, alternative assessments, learning contracts, homogeneous grouping, supplemental instruction, think-pair-share by readiness, homework options, differentiated online practice, leveled rubrics, tiered levels of support and activities, small group instruction, choice to work alone or in groups, alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt, opportunities to make up work or reduce the number of assignments, and access to extra help when available.

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**Specific Strategies and Practices that Support Gifted & Talented Students**

It is essential that teachers can identify high-ability students who may require more depth and complexity in instruction. These learners benefit from opportunities to accelerate and enrich their learning. *Strategies to support gifted and talented students include* homogeneous grouping, small group in-depth instruction, differentiated online practice, alternative assessments, pre-assessment to guide instruction and accelerate learning, and enrichment and extension activities within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

# Unit 4

## Desired Results

**Unit 4 Title:** Polynomials and Polynomial Functions

**Unit Summary:** In this unit students develop a deeper understanding of polynomial functions by examining their structure, behavior, and graphical representations. They learn to classify polynomials based on degree and number of terms, and explore how the degree and leading coefficient affect the graph's end behavior. Students analyze key features of polynomial graphs, including turning points, intercepts, and symmetry. They practice operations with polynomials—addition, subtraction, multiplication—and use both long and synthetic division to divide polynomials. The module also introduces the binomial theorem and its application to expand powers of binomials. By the end of the unit, students will be able to model and solve real-world problems using polynomial expressions and functions.

## Learning Standards

**NJSLS Math:**

**Algebra**

**Arithmetic with Polynomials and Rational Expressions A.APR**

**A. Perform arithmetic operations on polynomials.**

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**C. Use polynomial identities to solve problems.**

5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle.

**D. Rewrite rational expressions.**

6. Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using inspection, long division, or, for the more complicated examples, a computer algebra system.

**Seeing Structure in Expressions A.SSE.1**

**A. Interpret the structure of expressions**

1. Interpret expressions that represent a quantity in terms of its context. ★

**b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of $P$ and a factor not depending on $P$.

**Creating Equations A.CED ★**

**A. Create equations that describe numbers or relationships**

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## Functions

### Interpreting Functions F.IF

**B. Interpret functions that arise in applications in terms of the context.**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

**C. Analyze functions using different representations.**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

**c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

**Math Practices:**

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics

5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

## Interdisciplinary Connections

**Career Readiness, Life Literacies, & Key Skills** *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

**Computer Science & Design Thinking** ***(****CS & DT****)*:

8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

| **Unit Essential Questions:** | **Unit Enduring Understandings:** |
|---|---|
| What do the key features of a polynomial function reveal about its graph and behavior?<br><br>How are polynomials similar to and different from other types of functions we've studied? | Polynomial functions exhibit predictable patterns and behavior that can be analyzed through their graphs, including end behavior, turning points, and intercepts.<br><br>Polynomials are closed under addition, subtraction, and multiplication, which means these operations result in another |

| | |
|---|---|
| How can operations on polynomials (add, subtract, multiply, divide) be used to solve real-world problems or model relationships?<br><br>What strategies can we use to simplify polynomial expressions and make them easier to analyze or solve?<br><br>Why are binomials important in mathematics, and how can we expand them efficiently? | polynomial — a property similar to integers.<br><br>Dividing polynomials (using long or synthetic division) helps simplify expressions and identify factors, which is crucial for solving polynomial equations.<br><br>(+) Binomial expansion demonstrates the power of patterns and can be systematically understood using tools like Pascal's Triangle or the Binomial Theorem.<br><br>Understanding the structure and behavior of polynomial functions helps model and solve complex, real-world problems in science, economics, and engineering. |

**Vocabulary:** Power Function, Monomial Function, Polynomial, Standard Form of a Polynomial, Degree of a Polynomial, Polynomial Function, Binomial, Trinomial, Synthetic Division, Pascal's Triangle

**Students will be able to:**

- Analyze polynomial functions by examining key features and graphing
- Analyze the graphs of polynomial functions by identifying key features
- Add, Subtract, Multiply Polynomials
- Divide polynomials by using long and synthetic division
- Expand Powers of binomials

**Learning Activities:** *Suggested or Potential Learning Activities*
Lesson Launch/Opener
Foldables
Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

Climate Change Example: Modeling Arctic Sea Ice Loss with Polynomials- Students will analyze and model Arctic sea ice extent over time using polynomial functions, interpret key features of the function, and make future predictions based on the model.

## Assessment Evidence

**Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for Understanding

**Summative Assessments:** Cumulative Assessments

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# Unit 5

## Desired Results

**Unit 5 Title:** Polynomial Equations

**Unit Summary:** In this unit students build on their understanding of polynomial functions to solve polynomial equations. They explore multiple solution strategies, including graphing, factoring, and using the Remainder and Factor Theorems. Students also prove and apply polynomial identities and gain experience identifying zeros of polynomial functions. These skills enable them to construct accurate graphs of polynomial equations and interpret their features in context. The module emphasizes the connection between algebraic solutions and graphical representations, strengthening students' ability to analyze mathematical models. By the end, students can confidently evaluate, factor, and solve complex polynomial equations while understanding the significance of each solution in the broader function.

## Learning Standards

**NJSLS Math:**
**Functions**
**Interpreting Functions F.IF**
**B. Interpret functions that arise in applications in terms of the context.**
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. . Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
**C. Analyze functions using different representations.**
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
**c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

**Number and Quantity**
**Complex Number System N.CN**
**C. Use complex numbers in polynomial identities and equations.**
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## Algebra

**Arithmetic with Polynomials and Rational Expressions A.APR**

**B. Understand the relationship between zeros and factors of polynomials.**

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**C. Use polynomial identities to solve problems.**

4. Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

**Creating Equations A.CED**

**A. Create equations that describe numbers or relationships.**

5. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**Reasoning with Equations and Inequalities A.REI**

**D. Represent and solve equations and inequalities graphically.**

11. Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

## Math Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

8. Look for and express regularity in repeated reasoning

## Interdisciplinary Connections

**Career Readiness, Life Literacies, & Key Skills** *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

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8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

| **Unit Essential Questions:** | **Unit Enduring Understandings:** |
|---|---|
| How can we use graphs, factoring, and algebraic methods to solve polynomial equations? | Polynomial equations can be solved using multiple methods, including graphing, factoring, and algebraic techniques. |
| What do the zeros of a polynomial tell us about the function's behavior and its graph? | The zeros of a polynomial function represent x-values where the graph intersects the x-axis and provide insights into the function's structure. |
| Why are the Factor Theorem and Remainder Theorem useful tools in solving polynomial equations? | The Remainder Theorem and Factor Theorem allow us to evaluate and factor polynomials efficiently and are foundational to |

| | |
|---|---|
| How can polynomial identities be proven and used to simplify or rewrite expressions?<br><br>What strategies can we use to choose the most efficient method for solving a polynomial equation? | polynomial division.<br><br>Polynomial identities demonstrate mathematical structure and can be used to simplify complex expressions and solve problems more effectively.<br><br>Understanding and identifying patterns in polynomial behavior help in making predictions and constructing graphs. |

**Vocabulary:** Depressed Polynomial, Identity, Multiplicity, Polynomial Identity, Prime Polynomial, Quadratic Form, Synthetic Substitution

**Students will be able to:**

- Solve polynomial equations by graphing
- Solve polynomial equations by factoring and by writing them in quadratic form
- Prove polynomial identities and use them to describe numerical relationships
- Evaluate and factor functions by using the Remainder and Factor Theorems.
- Determine the numbers and types of roots and polynomial equations, find zeros, and use zeros to graph polynomial functions.
- Use the Fundamental Theorem of Algebra to determine the number and type of solutions for a polynomial function.

**Learning Activities:** *Suggested or Potential Learning Activities*

Lesson Launch/Opener
Foldables
Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

Climate Change Example: Modeling Carbon Emissions Using Polynomial Equations- Students will use real or simulated carbon emissions data to create a polynomial model, factor the polynomial equation, find its zeros, and interpret the results in the context of global carbon output and climate policy.

## Assessment Evidence

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|---|
| **Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for Understanding |
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| |
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# Unit 6

## Desired Results

**Unit 6 Title:** Inverse and Radical Functions

**Unit Summary:** In this unit, students will explore inverse and radical functions through algebraic operations and graphical analysis. They will learn to simplify expressions with radicals and rational exponents, determine and verify inverse functions, and graph square and cube root functions. Students will solve radical equations both algebraically and by graphing, developing a deeper understanding of function behavior. These skills will be applied to model and solve real-world problems.

## Learning Standards

**NJSLS Math:**
**Numbers and Quantity**
**The Real Number System N.RN**
**A. Extend the properties of exponents to rational exponents**
1. Explain how the definition of the meaning of ration exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of $5$ because we want $\left(5^{1/3}\right)^3 = 5^{(1/3)^3}$ to hold, so $\left(5^{1/3}\right)^3$ must equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**Algebra**
**Reasoning with Equations and Inequalities A.REI**
**A. Understand solving equations as a process of reasoning and explain the reasoning.**
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

**Seeing Structure in Expressions A.SSE.**
**A. Interpret the structure of expressions**

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $\left(x^2\right)^2 - \left(y^2\right)^2$, thus recognizing it as a difference of squares that can be factored as $\left(x^2 - y^2\right)\left(x^2 + y^2\right)$.

**Functions**
**Building Functions F.BF**
**A. Build a function that models a relationship between two quantities.**
1. Write a function that describes a relationship between two quantities. ★
b. Combine standard function types using arithmetic operations.
**B. Build new functions from existing functions.**
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions.
a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{(x+1)}{(x-1)}$ for $x \neq 1$.

**Interpreting Functions F.IF**
**B. Interpret functions that arise in applications in terms of the context.**
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
**C. Analyze functions using different representations.**
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions

**Math Practices:**
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics

5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

## Interdisciplinary Connections

**Career Readiness, Life Literacies, & Key Skills** *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

**Computer Science & Design Thinking** ***(****CS & DT****)*:

8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

**Unit Essential Questions:**

How can the inverse of a function be used to help interpret a real-world event or solve a problem?

What does it mean for two functions to be inverses, and how can this relationship be verified algebraically and graphically?

**Unit Enduring Understandings:**

Inverse functions "undo" each other and can be verified by composition and reflected symmetry across the line $y = x$.

Radical expressions and equations can be simplified and solved using properties of exponents and roots, as well as graphical representations.

| | |
|---|---|
| | Functions, including square and cube root functions, have distinct characteristics that influence their graphs and can be used to interpret and solve contextual problems. |

**Vocabulary:** composition of functions, conjugates, cube root function, index, inverse functions, inverse relations, like radical expressions, *n*th root, principal root, radical equation, radical function, radicand, rational exponent, square root function

**Students will be able to:**

- Simplify and graph operations on functions and determine compositions of functions
- Graph and verify inverse functions
- Students will rewrite radical expressions with rational exponents and vice versa.
- Simplify expressions involving radicals and rational exponents
- Graph and analyze square and cube root functions
- Simplify and perform operations with radical and rational exponent expressions
- Solve radical equations algebraically and by graphing

**Learning Activities:** *Suggested or Potential Learning Activities*
Lesson Launch/Opener
Foldables
Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

Climate Change Example: Students will analyze an exponential model of global $CO_2$ concentration and interpret the average rate of change over different time intervals to understand the implications of climate change.

## Assessment Evidence

**Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for understanding

**Summative Assessments:** Cumulative Assessments

**Common Benchmark Assessments:** Common Unit Assessments, LinkIt! Benchmarks

**Alternative Assessments:** Reveal online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments

## Core Instructional & Supplemental Materials

Reveal Algebra 2 McGraw Hill

ALEKS, DeltaMath, Khan Academy, Desmos, Graphing Calculators

LinkIt! Benchmarks, LinkIt! Test Bank

## Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students

**Specific Strategies and Practices that Support Students with Disabilities:**
Students receiving Special Education programming have specific goals, objectives, accommodations, and modifications outlined in their Individualized Education Plans (IEPs) due to an identified disability and/or diagnosis. In addition to access to the general education curriculum, instruction is differentiated based on individual student needs. The IEP serves as a supplemental curriculum guide, inclusive of instructional strategies tailored to support each learner. *Examples of supports and accommodations include* small group or one-to-one instruction, additional time, review and clarification of directions, students restating instructions to demonstrate understanding, space for movement or scheduled breaks, extra visual and verbal cues and prompts, preferential seating, consistent routines and schedules, rest breaks, visual and verbal reminders for directions and task focus, checklists, immediate feedback, and alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

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An English Language Learner (ELL) is a national-origin minority student who is limited-English proficient. The term "ELL" is preferred over "limited-English proficient" as it emphasizes potential and progress rather than deficits. *Instructional strategies to support ELLs include* pre-teaching vocabulary and concepts, use of personal glossaries, extended time, simplified and verbal instructions, frequent breaks, visual aids including graphic organizers, use of cognates to support comprehension, teacher modeling, peer pairing (pairing beginning ELLs with more advanced ELLs), scaffolding techniques, word walls, think-pair-share activities, cooperative learning groups, and alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

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Students at risk of failure benefit from targeted supports that address both academic and social-emotional needs, providing multiple access points to learning and increasing the likelihood of success. *Support strategies include* strengthening the home/school connection, alternative assessments, learning contracts, homogeneous grouping, supplemental instruction, think-pair-share by readiness, homework options, differentiated online practice, leveled rubrics, tiered levels of support and activities, small group instruction, choice to work alone or in groups, alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt, opportunities to make up work or reduce the number of assignments, and access to extra help when available.

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**Specific Strategies and Practices that Support Gifted & Talented Students**

It is essential that teachers can identify high-ability students who may require more depth and complexity in instruction. These learners benefit from opportunities to accelerate and enrich their learning. *Strategies to support gifted and talented students include* homogeneous grouping, small group in-depth instruction, differentiated online practice, alternative assessments, pre-assessment to guide instruction and accelerate learning, and enrichment and extension activities within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

# Unit 7

## Desired Results

### Unit 7 Title: Exponential Functions

**Unit Summary:**
In this unit, students explore exponential relationships and their applications in mathematical and real-world contexts. They will graph exponential growth and decay functions, identifying key characteristics such as asymptotes, intercepts, and rates of change. Students will solve exponential equations and inequalities using algebraic methods and graphing techniques, gaining fluency in switching between representations. The unit also includes analysis of expressions and functions involving the natural base *e*, helping students understand continuous growth and decay models. Additionally, students will work with geometric sequences—generating terms, writing formulas, and finding sums—to deepen their understanding of exponential patterns. By the end of the unit, students will be able to use technology to analyze data, recognize exponential behavior, and select the most appropriate function type to model real-world situations effectively.

## Learning Standards

**NJSLS Math:**
**Algebra**
**Creating Equations A.CED ★**
**A. Create equations that describe numbers or relationships**
1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

**Reasoning with Equations and Inequalities A.REI**
**D. Represent and solve equations and inequalities graphically.**
11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

**Seeing Structure in Expressions A.SSE.**
**A. Interpret the structure of expressions**

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $\left(x^2\right)^2 - \left(y^2\right)^2$, thus recognizing it as a difference of squares that can be factored as $\left(x^2 - y^2\right)\left(x^2 + y^2\right)$.

**B.Write expressions in equivalent forms to solve problems.**
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
c. Use the properties of exponents to transform expressions for exponential functions. For example, the expression $1.15^t$ can be rewritten as $\left(1.15^{1/12}\right)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
4. (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.★

**Functions**
**Building Functions F.BF**
B. Build new functions from existing functions.
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**Interpreting Functions F.IF**
**A. Understand the concept of a function and use function notation**
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.
**B. Interpret functions that arise in applications in terms of the context.**
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship . Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
**C. Analyze functions using different representations.**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=(1.02)^t$, $y=(.97)^t$, $y=(1.01)^{12t}$, $y=(1.01)^{\frac{t}{10}}$, and classify them as representing exponential growth or decay.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

**Linear and Exponential Models F.LE**

**A.** Construct and compare linear and exponential models and solve problems

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

**B.** Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

**Math Practices:**

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

## Interdisciplinary Connections

**Career Readiness, Life Literacies, & Key Skills** *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.
9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

**Computer Science & Design Thinking (*CS & DT*):**
8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.
8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

**Unit Essential Questions:**

How are real world situations involving quantities that grow or decline rapidly model mathematically?

How can we recognize and graph exponential growth and decay functions, and what do their key features tell us about real-world situations?

In what ways can exponential equations and inequalities be solved using algebraic and graphical methods?

What is the significance of the natural base *e*, and how is it used to model continuous growth and decay?

**Unit Enduring Understandings:**

Exponential growth and decay functions model many real-world phenomena, such as population growth and radioactive decay, and can be effectively analyzed through their graphs and key characteristics.

Exponential equations and inequalities can be solved using algebraic strategies and graphing techniques, allowing for flexible approaches depending on the context of the problem.

The natural base *e* represents continuous growth or decay and is essential in modeling real-world processes like compound interest and natural population changes.

Geometric sequences are closely connected to exponential functions, and understanding how to generate and sum their terms

| | |
|---|---|
| How do geometric sequences relate to exponential functions, and how can we use formulas to generate their terms and find their sums? | helps in analyzing patterns and solving problems involving repeated multiplication. |

**Vocabulary:** Asymptote, coefficient of determination, common ratio, compound interest, decay factor, e, explicit formula, exponential decay, exponential equation, exponential function, exponential growth, exponential inequality, finite sequence, geometric means, geometric sequence, geometric series, growth factor, infinite sequence, recursive formula, regression function, sequence, series, sigma notation, and term of a sequence.

**Students will be able to:**

- Students graph expert initial growth and Decay functions.
- Students solve exponential equations and inequalities algebraically and by graphing.
- Students analyze expressions and functions involving the natural base e.
- Students write explicit and recursive formulas for a given geometric sequence or series
- Students will calculate the sum of a finite or infinite geometric series.
- Students will model data and interpret the expression for a given situation in its context.
- Students will Construct and compare linear and exponential models and solve problems

**Learning Activities:** *Suggested or Potential Learning Activities*
Lesson Launch/Opener
Foldables
Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

**Climate Change Example:** Students may create equations and/or inequalities to represent the economic impact of climate change.

**Climate Change Example:** Students may relate the domain of a function c(m) representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for c(m).

**Climate Change Example:** Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).

## Assessment Evidence

**Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for understanding

**Summative Assessments:** Cumulative Assessments

**Common Benchmark Assessments:** Common Unit Assessments, LinkIt! Benchmarks

**Alternative Assessments:** Reveal online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments

## Core Instructional & Supplemental Materials

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## Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students

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# Unit 8

## Desired Results

**Unit 8 Title:** Logarithmic Functions

**Unit Summary:** In this unit, students will explore the relationship between exponential and logarithmic functions, recognizing logarithms as the inverse of exponentials. They will learn to write, evaluate, and graph logarithmic functions, as well as simplify and solve both logarithmic and exponential equations using common and natural logarithms. Students will apply properties of logarithms to manipulate expressions and solve problems efficiently. These skills will be used to model real-world scenarios, including exponential growth and decay.

## Learning Standards

**NJSLS Math:**
**Algebra**
**Creating Equations A.CED**
**A. Create equations that describe numbers or relationships.**
1. Create equations and inequalities in one variable and use them to solve problems.

**Reasoning with Equations and Inequalities A.REI**
**D. Represent and solve equations and inequalities graphically.**
11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

**Seeing Structure in Expressions A.SSE.**
**A. Interpret the structure of expressions**

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $\left(x^2\right)^2 - \left(y^2\right)^2$, thus recognizing it as a difference of squares that can be factored as $\left(x^2 - y^2\right)\left(x^2 + y^2\right)$.

**Functions**
**Interpreting Functions F.IF**
**C. Analyze functions using different representations.**
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**Linear and Exponential Models F.LE**
A. Construct and compare linear and exponential models and solve problems.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

**Math Practices:**
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
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## Interdisciplinary Connections

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8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

| **Unit Essential Questions:** | **Unit Enduring Understandings:** |
|---|---|
| How are logarithms defined and used to model situations in the real world?<br><br>How can properties of logarithms be used to simplify expressions and solve exponential and logarithmic equations?<br><br>How can we interpret and communicate solutions to exponential and logarithmic equations in real-world contexts? | Logarithms are the inverse of exponential functions and can be used to solve equations where the variable is in the exponent.<br><br>The properties of logarithms—such as the product, quotient, and power rules—allow for the simplification of complex expressions and solving equations.<br><br>Exponential growth and decay can be modeled using exponential functions and solved using logarithms in both base 10 (common) and base *e* (natural) contexts. |

**Vocabulary:** common logarithm, logarithm, logarithmic equation, logarithmic function, natural base exponential function, natural logarithm

**Students will be able to:**

- Write and evaluate logarithms and graph logarithmic functions
- Simplify logarithmic expressions and solve logarithmic equations
- Simplify logarithmic expressions and solve exponential equations by using common logarithms
- Simplify logarithmic expressions and solve exponential equations by using natural logarithms
- Write exponential growth and decay equations and solve them using logarithms

**Learning Activities:** *Suggested or Potential Learning Activities*
Lesson Launch/Opener
Foldables

Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

Climate Change Example: Students will write and solve a logarithmic equation in one variable using the pH formula to understand how ocean acidification works.

## Assessment Evidence

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**Summative Assessments:** Cumulative Assessments

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# Unit 9

## Desired Results

### Unit 9 Title: Rational Functions

**Unit Summary:**
In this unit, students develop a deep understanding of rational expressions and functions. They will learn to simplify, multiply, divide, add, and subtract rational expressions, using factoring and common denominators to perform operations accurately. Students will graph and analyze reciprocal and general rational functions, identifying key features such as asymptotes, intercepts, and end behavior. The unit also explores different types of variation—direct, inverse, joint, and combined—equipping students with strategies to recognize and solve variation equations in real-world and mathematical contexts. By the end of the unit, students will be able to apply algebraic reasoning and graphical analysis to understand complex relationships and model situations involving ratios and rates of change.

## Learning Standards

**NJSLS Math:**
**Algebra**
**Arithmetic with Polynomials and Rational Expressions A.APR**
**D. Rewrite Rational Expressions**
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

**Creating Equations ★**
**A.CED.1**
A. Create equations that describe numbers or relationships.
1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

**Reasoning with Equations and Inequalities A.REI**
**A. Understand solving equations as a process of reasoning and explain the reasoning.**
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

**D. Represent and solve equations and inequalities graphically.**
11. Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

**Functions**
**Building Functions F.BF**
**B. Build new functions from existing functions.**
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

**Interpreting Functions F.IF**
**B. Interpret functions that arise in applications in terms of the context.**
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
**C. Analyze functions using different representations.**
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

**Math Practices:**
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

## Interdisciplinary Connections

**Career Readiness, Life Literacies, & Key Skills** *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.
9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

**Computer Science & Design Thinking (*CS & DT*):**
8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.
8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

| **Unit Essential Questions:** | **Unit Enduring Understandings:** |
|---|---|
| How are the rules for operations with rational numbers applied to operations with rational expressions and equations?<br><br>How can we simplify, multiply, and divide rational expressions accurately and efficiently?<br><br>What strategies are effective for adding and subtracting rational expressions with like and unlike denominators?<br><br>How do the graphs of reciprocal and other rational functions reflect their algebraic structure, including asymptotes and | Rational expressions can be simplified and manipulated using the same properties and operations as numerical fractions, including factoring, finding common denominators, and reducing to lowest terms.<br><br>Multiplying, dividing, adding, and subtracting rational expressions requires a solid understanding of polynomial operations and factoring techniques.<br><br>The graphs of reciprocal and other rational functions reveal important features, such as vertical and horizontal asymptotes, holes, and end behavior, which reflect the algebraic form of the |

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| discontinuities?<br><br>What key features help us analyze and compare rational functions?<br><br>In what ways do rational expressions and functions model relationships between quantities in real-life situations? | function.<br><br>Rational functions model real-world relationships involving rates, ratios, and variation, and their graphs help visualize how changes in variables affect outcomes.<br><br>Understanding rational expressions and functions enhances students' ability to model and solve complex problems, both algebraically and graphically. |
| **Vocabulary:** complex fraction, excluded values, horizontal asymptote, hyperbola, oblique asymptotes, point discontinuity, rational equation, rational expression, rational function, rational inequality, reciprocal function, and vertical asymptote. | |
| **Students will be able to:**<br>• Students will multiply and divide rational expressions.<br>• Since add and subtract rational expressions.<br>• Students graph and analyze reciprocal functions.<br>• Students graph and analyze rational functions.<br>• Students solve rational equations and inequalities algebraically and by graphing.<br>• Students will interpret key characteristics from a reciprocal and rational function. | |
| **Learning Activities:** *Suggested or Potential Learning Activities*<br>Lesson Launch/Opener<br>Foldables<br>Interactive Presentation/ Online Practice<br>Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos<br><br>**Climate Change Example:** Students may create equations and/or inequalities to represent the economic impact of climate change. | |

## Assessment Evidence

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|---|
| **Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for understanding |
| **Summative Assessments:** Cumulative Assessments |
| **Common Benchmark Assessments:** Common Unit Assessments, LinkIt! Benchmarks |
| **Alternative Assessments:** Reveal online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments |

## Core Instructional & Supplemental Materials

| |
|---|
| Reveal Algebra 2 McGraw Hill |
| ALEKS, DeltaMath, Khan Academy, Desmos, Graphing Calculators |
| LinkIt! Benchmarks, LinkIt! Test Bank |

## Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students

**Specific Strategies and Practices that Support Students with Disabilities:**
Students receiving Special Education programming have specific goals, objectives, accommodations, and modifications outlined in their Individualized Education Plans (IEPs) due to an identified disability and/or diagnosis. In addition to access to the general education curriculum, instruction is differentiated based on individual student needs. The IEP serves as a supplemental curriculum guide, inclusive of instructional strategies tailored to support each learner. *Examples of supports and accommodations include* small group or one-to-one instruction, additional time, review and clarification of directions, students restating instructions to demonstrate understanding, space for movement or scheduled breaks, extra visual and verbal cues and prompts, preferential seating, consistent routines and schedules, rest breaks, visual and verbal reminders for directions and task focus, checklists, immediate feedback, and alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

**Specific Strategies and Practices that Support English Language Learners**
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**Students at Risk of Failure**
Students at risk of failure benefit from targeted supports that address both academic and social-emotional needs, providing multiple access points to learning and increasing the likelihood of success. *Support strategies include* strengthening the home/school connection, alternative assessments, learning contracts, homogeneous grouping, supplemental instruction, think-pair-share by readiness, homework options, differentiated online practice, leveled rubrics, tiered levels of support and activities, small group instruction, choice to work alone or in groups, alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt, opportunities to make up work or reduce the number of assignments, and access to extra help when available.

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**Specific Strategies and Practices that Support Gifted & Talented Students**
It is essential that teachers can identify high-ability students who may require more depth and complexity in instruction. These learners benefit from opportunities to accelerate and enrich their learning. *Strategies to support gifted and talented students include* homogeneous grouping, small group in-depth instruction, differentiated online practice, alternative assessments, pre-assessment to guide instruction and accelerate learning, and enrichment and extension activities within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

# Unit 10

## Desired Results

### Unit 10 Title: Interpreting Data

**Unit Summary:**
In this unit, students will use quantitative reasoning and statistical analysis to investigate patterns and relationships in real-world data. They will define appropriate quantities for descriptive modeling, choose meaningful statistical measures (mean, median, standard deviation, interquartile range), and interpret the implications of their findings. Students will represent data visually using dot plots, histograms, box plots, and scatter plots to compare multiple datasets. They will fit and evaluate mathematical models (linear and exponential) to represent relationships between variables, analyze residuals, and assess the reliability and limitations of predictions. Through authentic, real-world contexts, students will develop the ability to use mathematics to analyze trends, make inferences, and communicate findings effectively.

## Learning Standards

**NJSLS Math:**

**Quantities ★ N.Q**

**A. Reason quantitatively and use units to solve problems.**

2. Define appropriate quantities for the purpose of descriptive modeling. *This standard is integrated with S.ID.A in Algebra 2 and requires the student to create a quantity of interest in the situation being described. For example, in a situation involving more than two different data sets, the student might decide that the interquartile range (IQR), as a measure of spread, is the key variable in a situation and work with the IQR.*

**Interpreting Categorical and Quantitative Data S.ID**

**A. Summarize, represent, and interpret data on a single count or measurement variable**

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). *Includes more than two data sets. Tasks have a real-world context.*
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *Includes more than two data sets. Tasks have a real-world context.*
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). *Includes more than two data sets. Tasks have a real-world context.*

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

**B. Summarize, represent, and interpret data on two categorical and quantitative variables**

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

6. Represent data on two quantitative variables on a scatter plot and describe how the variables are related.

a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. *Exponential functions include those with domains not in the integers. Tasks have a real-world context.*

b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. *Exponential functions include those with domains not in the integers. Tasks have a real-world context.*

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**Math Practices:**

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

## Interdisciplinary Connections

**Career Readiness, Life Literacies, & Key Skills** *(CLKS)*:

9.4.12.CI.1 Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task.

9.4.12.TL.3 Analyze the effectiveness of the process and quality of collaborative environments.

**Computer Science & Design Thinking (*CS & DT*):**

8.1.12.CS.4: Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

| **Unit Essential Questions:** | **Unit Enduring Understandings:** |
|---|---|
| How can we define and select the most meaningful quantities to analyze a real-world situation?<br><br>How can we summarize and compare different datasets to identify meaningful patterns and differences?<br><br>How can we use functions and models to represent the relationship between two variables?<br><br>How can statistical analysis and mathematical modeling guide decision-making? | Selecting the right quantitative measures—such as the rate of change or the interquartile range—allows us to focus on the most relevant aspects of a problem. This choice depends on the context and the purpose of the analysis.<br><br>Measures of center (mean, median) and spread (IQR, standard deviation) help us compare variables across different situations or time periods, revealing patterns and variability that raw data alone may not show.<br><br>Scatter plots and fitted models (linear, exponential) can describe relationships between variables. Assessing model fit with residuals helps determine whether the model accurately reflects real-world trends.<br><br>Quantitative models and statistical analysis can forecast possible outcomes, evaluate proposed solutions, and inform decisions—but they must be interpreted with awareness of their assumptions and limitations. |

**Vocabulary:** Mean, Median, Mode, Interquartile Range (IQR), Standard Deviation, Outlier, Dot Plot, Histogram, Box Plot, Scatter Plot, Correlation, Residual, Line of Best Fit, Linear Model, Exponential Model, Quantitative Variable, Categorical Variable, Frequency Table, Relative Frequency, Slope, Y-Intercept, Rate of Change, Data Distribution, Normal Distribution, Prediction, Model Fit

**Students will be able to:**

- Define appropriate quantities for descriptive modeling based on a given real-world scenario, selecting measures such as mean, median, interquartile range, or standard deviation to address a specific question.
- Represent data visually using dot plots, histograms, and box plots for one or more datasets, ensuring accuracy and clarity.
- Summarize and compare datasets by calculating and interpreting measures of center (mean, median) and spread (IQR, standard deviation), including comparisons across more than two datasets.
- Interpret differences in shape, center, and spread of multiple datasets in context, including consideration of outliers and their effects.
- Fit and use mathematical models (linear and exponential) to represent relationships between two quantitative variables, using technology where appropriate.
- Assess the fit of a function by analyzing residual plots and determining whether a model appropriately represents the data.
- Summarize and interpret categorical data using two-way frequency tables, calculating and explaining joint, marginal, and conditional relative frequencies.
- Fit a normal distribution to a data set when appropriate, and use it to estimate probabilities and population percentages.
- Communicate findings from statistical analyses and modeling using precise mathematical vocabulary and clear reasoning.

**Learning Activities:** *Suggested or Potential Learning Activities*
Lesson Launch/Opener
Foldables
Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

**Climate Change Example:**

Climate Change Example: Students may represent geoscience data, with plots on the real number line, as they analyze results from global climate models.

Climate Change Example: Students may represent geoscience data on two quantitative variables on a scatter plot and describe how the variables are related in order to analyze the data and the results from global climate models.

Climate Change Example: Students may use linear or exponential functions fitted to geoscience data to solve problems and analyze the results from global climate models to make an evidence-based forecast of the current rate of global climate change.

## Assessment Evidence

**Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for understanding

**Summative Assessments:** Cumulative Assessments

**Common Benchmark Assessments:** Common Unit Assessments, LinkIt! Benchmarks

**Alternative Assessments:** Reveal online practice, Modified Unit Assessments, Modified Projects, Other Modified Assessments

## Core Instructional & Supplemental Materials

Reveal Algebra 2 McGraw Hill

ALEKS, DeltaMath, Khan Academy, Desmos, Graphing Calculators

LinkIt! Benchmarks, LinkIt! Test Bank

## Accommodations and Modifications: Students with Disabilities, English Language Learners, Students at Risk of Failure, Students with 504s, Gifted & Talented Students

**Specific Strategies and Practices that Support Students with Disabilities:**
Students receiving Special Education programming have specific goals, objectives, accommodations, and modifications outlined in their Individualized Education Plans (IEPs) due to an identified disability and/or diagnosis. In addition to access to the general education curriculum, instruction is differentiated based on individual student needs. The IEP serves as a supplemental curriculum guide, inclusive of instructional strategies tailored to support each learner. *Examples of supports and accommodations include* small group or one-to-one instruction, additional time, review and clarification of directions, students restating instructions to demonstrate understanding, space for movement or scheduled breaks, extra visual and verbal cues and prompts, preferential seating, consistent routines and schedules, rest breaks, visual and verbal reminders for directions and task focus, checklists, immediate feedback, and alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

**Specific Strategies and Practices that Support English Language Learners**
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**Students at Risk of Failure**
Students at risk of failure benefit from targeted supports that address both academic and social-emotional needs, providing multiple access points to learning and increasing the likelihood of success. *Support strategies include* strengthening the home/school connection, alternative assessments, learning contracts, homogeneous grouping, supplemental instruction, think-pair-share by readiness, homework options, differentiated online practice, leveled rubrics, tiered levels of support and activities, small group instruction, choice to work alone or in groups, alternate or reduced assignments within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt, opportunities to make up work or reduce the number of assignments, and access to extra help when available.

**Students with 504 Plans**
Teachers are responsible for implementing the accommodations and strategies outlined in a student's 504 Plan. A 504 Plan is a formal document designed to provide support for students with physical or mental impairments that significantly limit one or more major life activities. Accommodations are individualized and legally binding, and support must be consistently provided as outlined in the plan.

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It is essential that teachers can identify high-ability students who may require more depth and complexity in instruction. These learners benefit from opportunities to accelerate and enrich their learning. *Strategies to support gifted and talented students include* homogeneous grouping, small group in-depth instruction, differentiated online practice, alternative assessments, pre-assessment to guide instruction and accelerate learning, and enrichment and extension activities within digital platforms such as ALEKS, Reveal, Delta Math, and LinkIt.

# Unit 11

## Desired Results

### Unit 11 Title: Trigonometric Functions

**Unit Summary:** Students explore the foundational concepts of trigonometric functions. They begin by learning to measure angles in both degrees and radians and then extend their understanding to general angles using the unit circle. Students evaluate all six trigonometric functions and sine and cosine functions, analyze their periodic behavior, and study their transformations. By the end of the module, students will be able to identify and interpret key features of trigonometric graphs—such as amplitude, period, phase shift, and vertical shift. This unit builds essential skills for modeling cyclical phenomena in both mathematical and real-world contexts.

## Learning Standards

**NJSLS Math:**

**Algebra**

**Creating Equations A.CED ★**

**A. Create equations that describe numbers or relationships**

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**Functions**

**Interpreting Functions F.IF**

**B. Interpret functions that arise in applications in terms of the context.**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★

**C. Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

**e.** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**Building Functions**

**Building Functions F.BF**

B. Build new functions from existing functions.
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**Trigonometric Functions F.TF**

**A. Extend the domain of trigonometric functions using the unit circle.**

1. (+) Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. (+) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**B. Model periodic phenomena with trigonometric functions.**

5. (+)Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

**Math Practices:**

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
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## Interdisciplinary Connections

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**Computer Science & Design Thinking** ***(****CS & DT****)*:

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8.2.12.NT.1: Explain how different groups can contribute to the overall design of a product.

## Learning Targets

| **Unit Essential Questions:** | **Unit Enduring Understandings:** |
|---|---|
| How are angles measured in both degrees and radians, and how are they related? | Trigonometric functions extend beyond right triangles to general angles using the unit circle. |
| What is the unit circle, and how does it help define trigonometric functions for all real numbers? | The unit circle enables the definition of trigonometric functions for all real numbers, supporting the concept of periodicity. |
| How can trigonometric functions be used to model periodic behavior? | Graphs of sine, cosine, tangent, and their reciprocals can model periodic phenomena, showing consistent patterns in nature and science. |
| In what ways can the graphs of trigonometric functions be transformed and interpreted? | Key characteristics such as amplitude, period, midline, and phase shift allow students to interpret and transform trigonometric graphs. |
| How do inverse trigonometric functions allow us to solve for unknown angles? | Inverse trigonometric functions are used to determine unknown angles from given function values, making them essential in real-world applications like navigation, physics, and engineering. |

**Vocabulary:** Central Angle of a Circle, Coterminal Angles, Initial Side, Radian, Standard Position, Terminal Side, Quadrantal Angle, Reference Angle, Trigonometric Function, Trigonometric Ratio, Trigonometry, Circular Function, Cycle, Period, Periodic Function, Unit Circle, Oscillation, Midline, Amplitude, Frequency, Sinusoidal Function, Phase Shift, Vertical Shift.

**Students will be able to:**

- Draw angles in standard position and convert between degree and radian measure of angles.
- Find the values of trigonometric functions by using general angles and reference angles.
- Find trigonometric values by using the unit circle and the properties of periodic functions.
- Graph sine and cosine curves.
- Graph translations of trigonometric functions.

**Learning Activities:** *Suggested or Potential Learning Activities*
Lesson Launch/Opener
Foldables
Interactive Presentation/ Online Practice
Small Group Practice, Small Group Intervention, Small Group Activities, Differentiated Instruction (Reteach), Enrichment Practice (Challenge), ALEKS, Reveal online, Communicators, DeltaMath, Desmos

Climate Change Example: Modeling Earth's Seasonal Temperature Patterns with Trigonometric Functions- Students will use real climate data to create a sinusoidal model that represents average monthly temperatures of a specific location. They will interpret the amplitude, period, midline, and phase shift in the context of climate trends.

## Assessment Evidence

**Formative Assessments:** Quizzes, Warm-Ups, Exit Passes, Communicators, Classwork, Reveal Check for Understanding

**Summative Assessments:** Cumulative Assessments

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Reveal Algebra 2 McGraw Hill

| |
|---|
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| Standards for Mathematical Practice |
| --- |
| **1. Make sense of problems and persevere in solving them**<br>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. |
| **2. Reason abstractly and quantitatively**<br>Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. |
| **3. Construct viable arguments and critique the reasoning of others**<br>Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the |

arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

**4. Model with mathematics**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5. Use appropriate tools strategically**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**6. Attend to precision**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7. Look for and make use of structure**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three

and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered$7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$ older students can see the 14 as $2 \times 7$and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

**8. Look for and express regularity in repeated reasoning**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$.

Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)\left(x^2 + x + 1\right)$ and $(x - 1)\left(x^3 + x^2 + x + 1\right)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

# Algebra 2 Course Content Framework and Content Emphases

Students should spend the majority of their time on the major work of the grade (**M**). Supporting work (**S**) and, where appropriate, additional work (**A**) can engage students in the major work of the grade

## Number and Quantity Conceptual Category

### The Real Number System (N.RN)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Extend the properties of exponents to rational exponents | 1, 2, 3 | M |

### Quantities (N.Q)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Reason quantitatively and use units to solve problems | 2* | S |

### The Complex Number System (N.CN)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Perform arithmetic operations with complex numbers | 1, 2 | A |
| B. Use complex numbers in polynomial identities and equations | 7 | A |

## Algebra Conceptual Category

## Creating Equations (A.CED)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Create equations that describe numbers or relationships | 1* | S |

## Seeing Structure in Expressions (A.SSE)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Interpret the structure of expressions | 2 | M |
| B. Write expressions in equivalent forms to solve problems | 3c* | M |

## Arithmetic with Polynomials and Rational Expressions (A.APR)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Perform arithmetic operations on polynomials | 1 | M |
| B. Understand the relationship between zeros and factors of polynomials | 2, 3 | M |
| D. Rewrite rational expressions | 6 | S |

## Reasoning with Equations and Inequalities (A.REI)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Understand solving equations as a process of reasoning and explain the reasoning | 1*, 2 | M |
| B. Solve equations and inequalities in one variable | 4b* | S |

| | | |
|---|---|---|
| C. Solve systems of equations | 6*, 7 | A |
| D. Represent and solve equations and inequalities graphically | 11* | M |

# Functions Conceptual Category

## Interpreting Functions (F.IF)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Understand the concept of function and use function notation | 3* | S |
| B. Interpret functions that arise in applications in terms of the context | 4*, 6* | M |
| C. Analyze functions using different representations | 7b*, 7c, 7e*, 8b, 9* | S |

## Building Functions (F.BF)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Build a function that models a relationship between two quantities | 1a*, 1b, 2 | M |
| B. Build new functions from existing functions | 3*, 4a | A |

## Linear and Exponential Models (F.LE)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Construct and compare linear and exponential models and solve problems | 2*, 4 | S |
| B. Interpret expressions for functions in terms of the situation they model | 5* | A |

# Statistics and Probability Conceptual Category

## Interpreting categorical and quantitative data (S.ID)

| Cluster Letter and Heading | Standard Number | Content Emphasis |
|---|---|---|
| A. Summarize, represent, and interpret data on a single count or measurement variable | 1*, 2*, 3*, 4 | S |
| B. Summarize, represent, and interpret data on two categorical and quantitative variables | 5, 6a-b* | S |

Limits and Clarifications for Multi-Course Standards

Mathematics Content Guide for the New Jersey Graduation Proficiency Assessment (NJGPA)

NJ Digital Item Library

High School Mathematics Course Frameworks and Content Emphases - Identifies the standards that are Major, Supporting, and Additional in a core Algebra 2 course