Georgia's K-12 Mathematics Standards: Learning Progressions

This document provides a visual progression of mathematics expectations within Georgia's K-12 Mathematics Standards across all grade levels for students, parents, and educators to make connections among key concepts as students move from grade level to grade level.

Georgia's K-12 Mathematics Standards
Mathematics Big Ideas and Learning Progressions, K-12

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*The Big Ideas extend to High School 4th course options beyond Advanced Algebra: Concepts and Connections. These Big Ideas can be found within each course standards document.

K-12 MATHEMATICS LEARNING PROGRESSION - GEORGIA

Key Concepts	ELEMENTARY SCHOOL (K-5)						MIDDLE SCHOOL (6-8)			HIGH SCHOOL (9-12)			
	K	1	2	3	4	5	6	7	8	Algebra: Concepts & Connections	Geometry: Concepts & Connections	Advanced Algebra: Concepts & Connections	Courses beyond Advanced Algebra
NUMERICAL REASONING													
Numbers	• Whole numbers to 100	• Whole numbers to 120 • Partition shapes into halves and quarters/fourths (fourths) with no shading	• Whole numbers to 1000 • Partition shapes into halves, thirds and quarters (fourths) with no shading	• Whole numbers to 10,000 • Unit fractions with denominators of 2, 3, 4, 6, and 8 • Represent fractions • Equivalence of simple fractions • Introduce shading to identify and compare fractional parts	• Whole numbers to 100,000 • Non-unit fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100 • Fractions with like denominators • Decimal fractions (tenths and hundredths)	• Multi-digit whole numbers • Fractions with unlike denominators • Fractions greater than 1 • Decimal fractions to thousandths	• Rational numbers as a concept ○ Integers ○ Fractions ○ Decimal numbers	• All rational numbers • Simple probability	• All rational numbers • Scientific notation • Numerical expressions with integer exponents • Approximate rational and irrational numbers (radicals) on a number line	• All rational numbers • Operations with radicals	• All numbers in The Real Number System	• All numbers in The Real Number System • Complex numbers	• Application of all numbers in the real number system • The Complex Number System (Precalculus and beyond)
Counting	• Counting forward to 100 • Counting backward from 20 • Counting objects to 20	• Counting forward and backward within 120 • Skip counting by 2s, 5s, and 10s • Counting objects to 120	• Counting forward and backward within 1000 • Skip counting by 2s, 5s, 10s, 25s, and 100s • Counting objects to 1000	• Counting unit fractions	• Counting non-unit fractions	• Counting decimal numbers	*Students should apply the foundational knowledge of counting to make sense of other mathematical ideas related to numbers in The Real Number System.*			*Students should apply the foundational knowledge of counting to make sense of other mathematical ideas related to numbers in The Real and Complex Number Systems.*			
Place Value	• Compose and decompose numbers within 20 • Identify and write numerals to 20	• Compose and decompose 2-digit numbers	• Hundreds, tens and ones in 3-digit numbers	• Round numbers to 1000 to nearest 10 or 100 • Read & write multi-digit whole numbers to thousands	• Magnitude of place value • Multi-digit whole numbers to 100,000 • Round multi-digit whole numbers • Fractions with denominators of 10 or 100	• Magnitude of place value extended to decimal numbers • Powers of 10 to 10^3 • Read & write decimal numbers to thousandths place • Round decimal numbers to hundredths place	*Students should apply the foundational knowledge of place value to make sense of other mathematical ideas related to numbers in The Real Number System.*			*Students should apply the foundational knowledge of place value to make sense of other mathematical ideas related to numbers in The Real and Complex Number Systems.*			
Comparisons	• Comparing objects up to 10 • Comparing numbers of objects in a set from 1-10	• Comparing numbers to 100	• Comparing numbers to 1,000	• Comparing numbers to 10,000 • Unit fractions	• Multi-digit numbers • Fractions less than 1 • Decimal fractions to hundredths place	• Decimal fractions to thousandths place • Fractions greater than 1	• Integers • Unit rates • Ratios • Numerical data distributions • Measures of variation • Absolute value • Display and analyze categorical and quantitative (numerical) data	• Rational numbers • Probabilities • Random sampling	• Rational and irrational numbers (radicals) • Compare proportional relationships presented in different ways	• Rate of change (slope) • Intercept • Distributions of two or more data sets.		• Recognize the purpose of and differences among different types of studies. • Population distributions, sample data distributions, and sampling distributions	• Application of all numbers in the real number system • The Complex Number System (Precalculus and beyond)
Computational Fluency	• Fluency with addition and subtraction within 5	• Fluency with addition and subtraction within 10	• Fluency using mental math up to 20 • Fluency with strategies within 100	• Fluency with multiplication and division with single-digit numbers • Fluency with addition and subtraction within 1,000	• Fluency with addition and subtraction with multi-digit whole numbers	• Fluency with multiplication and division with multi-digit whole numbers	• All operations with whole numbers, fractions, and decimal numbers • Write & evaluate numerical expressions • Convert fractions with denominators of 2, 4, 5 and 10 to the decimal notation	• Operations with rational numbers • Rational numbers • Convert fractions to decimal numbers	• Operations with scientific notation • Scientific notation in real situations seen in everyday life • Expressions with integer exponents	• Operations with real numbers (rational and irrational) • Multiplication of irrational numbers		• Operations with all real numbers in the real number system. Specifically, operations with complex numbers.	• Application of all numbers in the real number system • The Complex Number System (Precalculus and beyond)

Addition & Subtraction	• Single-digit numbers within 10	• Within 20 (using properties of operations) • Within 100 (using base ten understanding)	• Within 1,000 (using tools and strategies)	• Within 10,000	• Within 100,000 • Fractions with like denominators	• Fractions with unlike denominators • Decimal fractions to the hundredths place	*Students should apply the foundational knowledge of part-whole thinking with addition and subtraction to algebraic reasoning opportunities in the secondary years. The part-whole strategies for addition and subtraction acquired in elementary school are foundational for the intermediate algebra concepts taught in middle school.*			*Students should apply the foundational knowledge of part-whole thinking with addition and subtraction to advanced algebraic reasoning opportunities in the secondary years. The part-whole strategies for addition and subtraction acquired in elementary school serve as a foundation for the advanced algebra concepts taught in high school.*		
Multiplication & Division	*In the primary K-1 years, students are building foundational knowledge by acquiring a conceptual understanding of number and quantity. A major focus is the use of counting strategies. This foundational knowledge is essential and will be applied to the study of addition, subtraction, multiplication, and division in later years through part-whole reasoning.*		• Building arrays	• Within 100 • Multiply by multiples of 10	• Factors and multiples • Prime and composite numbers • Multiply by multi-digit whole numbers • Divide by 1-digit divisors	• Multiply multi-digit whole numbers • Multiply fractions and whole numbers • Divide unit fractions and whole numbers • Reason about multiplying by a fraction >, <, or = to 1	*Students should apply the foundational knowledge of part-whole thinking with multiplication and division to algebraic reasoning opportunities in the secondary years. The part-whole strategies for multiplication and division acquired in elementary school are foundational for the intermediate algebra concepts taught in middle school.*			*Students should apply the foundational knowledge of part-whole thinking with multiplication and division to algebraic reasoning opportunities in the secondary years. The part-whole strategies for multiplication and division acquired in elementary school serve as a foundation for the advanced algebra concepts taught in high school.*		
PATTERNING & ALGEBRAIC REASONING													
Patterns	• Repeating patterns with numbers and shapes • Explain the rationale for the pattern	• Growing and repeating patterns of 1s, 5s, and 10s • Repeated operations, shapes or numbers	• Numerical patterns involving addition and subtraction	• Numerical patterns related to multiplication • Make predictions based on patterns	• Generate number and shape patterns that follow a rule • Represent and describe patterns	• Generate two numerical patterns using a given rule • Identify relationships using a table	• Greatest common factor & least common multiple	• Constant of proportionality	• Integer exponents • Perfect squares and perfect cubes	• Arithmetic sequences • Geometric sequences		• Represent data with matrices. • Operations with matrices and scalars • Linear programming applications	• Identifying patterns and relationships related to all function types
Expressions	*In the early years, students are building foundational knowledge by acquiring a conceptual understanding of number and quantity. This foundational knowledge is essential and will be applied to the algebraic concepts explored in the secondary years. Students must develop a solid foundation in numeracy in K-5 in order to be prepared for the algebra involved in the study of expressions.*					**Numerical Reasoning** • Simple numerical expressions involving whole numbers with or without grouping symbols • Express fractions as division problems	• Write, analyze, and evaluate numerical and algebraic expressions • Identify, generate, and evaluate algebraic expressions • Identify like terms in an algebraic expression	• Add, subtract, factor & expand linear expressions • Rewrite expressions • Fluency with combining like terms in an algebraic expression • Linear expressions with rational coefficients	• Expressions with integer exponents • Linear expressions • Operations with algebraic expressions	• Exponential expressions • Quadratic expressions	• Expressions of varying degrees • Add, subtract, multiply single variable polynomial expressions. • Adding, subtracting and multiplying polynomial expressions. • Factoring and expanding polynomials	• Use exponential and logarithmic expressions to model real-life phenomena. • Radical expressions • Rational expressions • Polynomial expressions • Matrices • Linear programming applications	• Application of multiple types of expressions in real-world contexts • Finding equivalent expressions in advanced algebraic situation • Performing advanced manipulation of expressions
Variable Equations & Inequalities	*In the early years, students are building foundational knowledge by acquiring a conceptual understanding of number and quantity. This foundational knowledge is essential and will be applied to the algebraic concepts explored in the secondary years. Students must develop a solid foundation in numeracy in K-5 in order to be prepared for the algebra involved in the study of expressions, equations, and functions.*						• Write and solve one-step equations & inequalities	• Construct & solve multi-step algebraic equations and inequalities	• Analyze and solve linear equations and inequalities	• Exponential equations • Quadratic equations • Equations of parallel and perpendicular lines • Analyze and solve linear inequalities	**Geometric & Spatial Reasoning** • Equations involving geometric measurement • Write equations of circles in standard form.	• Exponential and logarithmic equations • Radical equations • Rational equations • Polynomial equations • Solve systems of quadratic and linear functions	• Application of all types of variable equations and inequalities • Performing advanced manipulation of equations and inequalities with multiple variables

Ratios & Rates	*In the early years, students are building foundational knowledge by acquiring a conceptual understanding of fractions and decimals. This knowledge will be applied to the concept of ratios and rates in middle school.*						**Numerical Reasoning with ratios and rates:** • Concept of ratio and rate • Equivalent ratios, percents, unit rates • Convert within measurement systems	• Compute unit rates associated with ratios of fractions • Determine unit rates	• Interpret unit rate as the slope of a graph	• Convert units and rates given a conversion factor	• Side ratios of similar triangles • Trigonometric ratios	• Average rate of change of quadratic, exponential, logarithmic, and radical functions • Trigonometric ratios	• Apply the concept of ratio and rate reasoning to solve contextual, real-life, mathematical problems
Proportional Relationships	*In the early years, students are building foundational knowledge by acquiring a conceptual understanding of fractions and decimals. This knowledge will be applied to the concept of proportional relationships later.*						*In Grade 6, students should develop a foundation for understanding proportions through the development of ratio and rate reasoning, as well as part-whole computational strategies related to fractions, decimals, and percents.*	• Use proportional relationships • Solve multi-step ratio and percent problems • Scale drawings of geometric figures • Use similar triangles to explain slope	*In Grade 8, students should extend their understanding of proportions to derive the equation* $y = mx + b$.	• Apply the concept of proportionality to functions and their graphs	• Apply the concept of proportionality to functions and their graphs	• Apply the concept of proportionality to functions and their graphs	• Apply the concept of proportionality to functions and their graphs
Graphing	• Categorical data displayed using objects and pictures	• Categorical data collected using tables and tally marks • Categorical data interpreted from pictographs and bar graphs	• Categorical data with no more than four categories • Data organized using tables and tally marks • Categorical data interpreted from bar graphs and pictographs • Add and subtract to create and obtain information from data organized in bar graphs, pictographs, tables, and tally charts	• Categorical data with scales > 1 • Collect, organize, and analyze numerical data with dot plots • Measure objects to collect data to represent on a dot plot	• Determine the difference between how categorical and numerical data are represented. • Data represented using bar graphs pictographs and dot plots • Measure objects to collect data to represent on a dot plot	• Plot ordered pairs in first quadrant • Numerical data on dot plots and categorical data on bar graphs • Display and analyze data distributions on dot plots • Find mean as a measure of center and balance point for the data set	• Plot order pairs in all four quadrants • Show rational numbers on a number line • Draw polygons on a coordinate grid • Find the length of a side of a polygon graphed on the coordinate plane (same x- or y- coordinate)	• Proportional relationships	• Linear functions • Comparing linear and non-linear functions • Systems of linear equations (including parallel and perpendicular) • Linear inequalities • Analyze data distributions	• Linear functions with function notation • Exponential functions • Quadratic functions • Systems of linear inequalities	• Equations of circles in standard form	• Exponential and logarithmic functions • Extension of quadratic functions • Radical functions • Polynomial functions • Rational functions	• Apply the concept of graphing to create mathematical models to solve real-life problems and explain real-life phenomena

FUNCTIONAL & GRAPHICAL REASONING

Function Families	*In the early years, students are building foundational knowledge by acquiring a conceptual understanding of patterns and algebra through the Patterning and Algebraic Reasoning big idea. This knowledge will be applied to the concept of proportional relationships and then functional relationships in later grades and courses.*	*In the early middle school years, students are building foundational knowledge by acquiring a conceptual understanding of ratios and proportions. This foundational knowledge will be applied to the concept of functions in later grades and courses.*	• Linear functions • Line of best fit	• Linear functions with function notation • Parent graphs of function families • Exponential functions • Quadratic functions	• Function notation to represent transformations	• Exponential and logarithmic functions • Extension of quadratic functions • Radical functions • Polynomial functions • Fundamental Theorem of Algebra • End behavior of polynomial functions • Rational functions • Factor polynomials	• Construct and interpret a variety of function types. • Make sense of functions in the context of solving real-life problems • Create and interpret mathematical models of a variety of function types

GEOMETRIC & SPATIAL REASONING

Shapes and Properties	• Identify, sort, classify, analyze, and compare 2D & 3D based on attributes using informal language • Positional words	• Identify, sort, and classify 2D & 3D shapes based on specific attributes using formal language and geometric properties • Compose 2D shapes & 3D shapes	• Describe, compare and sort 2-D and 3-D shapes given a set of attributes • Identify lines of symmetry in everyday objects	• Quadrilaterals • Parallel & perpendicular line segments, points, lines, line segments, & right angles and presence or absence of these in quadrilaterals • Lines of symmetry with quadrilaterals	• Points, lines, line segments, rays, angles, and parallel & perpendicular line segments • Classify, compare, & contrast polygons based on presence or absence of parallel or perpendicular line segments, angles of a specified size or side lengths.	• Classify polygons based on geometric properties • Relationships between categories and subcategories of shapes	• Apply geometric and spatial reasoning involving shapes and properties to solve a variety of problems	• Measure angles using non-standard and standard tools • Write & solve equations using supplementary, complementary, vertical, and adjacent angles	• Introduction to Pythagorean Theorem and the converse	• Apply the distance formula, midpoint formula, and slope of line segments to solve real-world problems. • Apply slope criteria for parallel and perpendicular lines	• Develop and use precise definitions to prove theorems and solve geometric problems • Prove slope criteria for parallel and perpendicular lines • Transform polygons using rotations, reflections, dilations, and translations. • Congruence and trans-formations • Triangle congruence • Use congruence to prove relationships in geometric figures • Similarity and dilations • Similar triangles • Use similarity to prove relationships in geometric figures • Formal proofs & theorems about triangles • Trigonometric ratios (Sin, Cos, & Tan)	• Unit circle • Trigonometric equations	• Apply geometric and spatial reasoning in the context of solving a variety of complex problems
Geometric Measurement	*In the early years, students should build a solid foundation in numeracy in order to apply that foundational knowledge and the part-whole strategies acquired to relevant contexts, such as geometric measurement. K-2 students should build a foundation in measurement reasoning in order to later apply that knowledge to geometric contexts to solve problems.*			• Area of rectangles • Perimeter of rectangles	• Area and perimeter of composite rectangles • Angle measurement	• Volume of right rectangular prisms	• Area of triangles, quadrilaterals, and polygons • Surface area • Volume of right rectangular prisms with fractional edge lengths	• Relationship between parts of a circle • Area & circumference of a circle • Area and surface area of figures decomposed into triangles, quadrilaterals & circles • Volume of cubes, right prisms & cylinders	• Pythagorean Theorem to determine distance between two points • Volume of cones, cylinders, and spheres	• Use distance formula, midpoint formula, and slope to calculate perimeter and area of triangles and quadrilaterals.	• Volumes of prisms, cones, cylinders, pyramids, and spheres • Approximate volumes of irregular objects • Approximate density of irregular objects		

MEASUREMENT REASONING

Measurement	• Measurable attributes of length, height, width and weight • Classify and sort up to 10 objects by attributes • Display and interpret categorical data with up to 10 data points on graphs	• Measure length in non-standard units • Compare, describe and order up to 3 objects using length in non-standard units • Display and interpret categorical data (with up to 3 categories)	• Measure length to nearest whole unit • Use tools such as constructed rulers and standard rulers • Choose units (in, ft, yd) appropriately • Display and interpret categorical data (with up to 4 categories)	• Measure liquid volume, length and mass in customary units • Use rulers to measure lengths in halves and fourths of an inch • Analyze numerical and categorical data with whole number values	• Measure liquid volume, distance, and weight using the metric measurement system • Use rulers to measure lengths to nearest $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ of an inch • Analyze data using dot plots (with values to the nearest $\frac{1}{8}$ of a unit)	• Measure length and mass in metric units • Convert between units of measurement • Create dot plots with measurements in fractions	*In high school, students should be able to apply measurement reasoning in the context of solving a variety of complex problems. Students should be able to attend to precision and use their understanding of measurement to create and interpret mathematical models and solve complex, real-life problems.*

Money	• Identify pennies, nickels and dimes and values	• Identify value of pennies, nickels, dimes and quarters	• Combination of coins • Problems involving dollars and all coins	• Using money to solve problems	• Using money as a tool or manipulative to solve problems	• Using money as a tool to solve problems involving decimals	*In middle and high school, students should be able to solve quantitative and algebraic problems that include money as a context. Students should be able to apply measurement reasoning in the context of solving a variety of complex problems. Students should be able to attend to precision and use their understanding of money to create and interpret mathematical models and solve complex, real-life problems. Additionally, students in the Advanced Financial Algebra course should be able to apply their concept of money to make complex financial decisions to prepare them for the future.*						
Time		• Tell & write time in hours and half hours • Measure elapsed time to the hour	• Time to the nearest five minutes • Distinguish between a.m. & p.m. • Elapsed time to hour or half hour	• Tell time to the nearest minute • Estimate relative time • Elapsed time to hour, half hour & quarter hour	• Intervals of time • Elapsed time to the nearest minute	• Solving problems involving time	*In middle and high school, students should be able to apply an understanding of time in the context of solving a variety of complex problems. Students should be able to attend to precision and use their understanding of time to create and interpret mathematical models and solve complex, real-life problems.*						
DATA & STATISTICAL REASONING													
Ask Statistical Questions; Investigate, Represent and Analyze Data	• Categorical data (up to 10 objects) • Comparing 2 categories	• Categorical data (up to 10 objects) • Comparing up to 3 categories • Represent data using tally marks and numerical values within 20	• Categorical data (up to 20 objects) • Display data on pictograph and bar graph	• Categorical and numerical data • Display data on pictographs, bar graphs and dot plots (line plots)	• Determining the best model and representation for categorical and numerical data • Measure to nearest $\frac{1}{8}$ of a unit to collect data • Display data distributions on dot plots (line plots) • Numerical data with addition & subtraction of fractions with like denominators	• Numerical data on dot plots and categorical data on bar graphs • Display and analyze data distributions on dot plots • Find mean as a measure of center and balance point for the data set	• Distinguish statistical and non-statistical questions • Describe distributions • Design simple experiments & collect data • Measures of center & variability	• Predict populations • Random sampling • Informal comparative inferences about two sample populations	• Scatter plots of bivariate data • Patterns of association of bivariate data • Straight lines to represent linear associations • Informally assess linear models and interpret slope and y-intercept	• Univariate and bivariate data • Compare and represent center (median and mean) and variability (interquartile range, standard deviation) or two or more distributions by and using technology. • Represent data on a scatter plot. • Calculate the line of best fit and interpret the correlation coefficient, *r*, of a linear fit using technology. • Distinguish between correlation and causation.	• Categorical data & two-way frequency tables • Compute and interpret expected values for real-life situations.	• Sample surveys, experiments, observational studies • Primary and secondary data • Ethics, privacy, potential bias, and confounding variables • Z-scores • Empirical rule • Descriptive and inferential statistics • Margin of error, confidence intervals • Normal distribution	• Create statistical investigative questions to solve real-life problems • Create, interpret, and make sense of mathematical displays of data used to solve the statistical investigative question • Determine the solution to the question posed through an analysis of data and through the use of statistical reasoning in a variety of contexts
PROBABILISTIC REASONING													
Probability	*In the early years, students should build a solid foundation in numeracy in order to apply that foundational knowledge and the part-whole strategies acquired to relevant contexts, such as probability. Students should be able to use quantitative and abstract reasoning to solve problems, attend to precision, and use their part-whole thinking strategies to compute efficiently and solve problems.*						*In Grade 6, students should build a solid foundation in numeracy to apply that foundational knowledge and the part-whole strategies acquired to relevant contexts, such as probability. Students should be able to use quantitative and abstract reasoning with ratios and proportions to solve problems, attend to precision, and use their part-whole thinking strategies to compute efficiently.*	• Represent probability • Approximate probability • Develop probability models (uniform & not uniform) • Find probabilities of simple events	*In Grade 8, students should be able to apply their basic knowledge of probabilistic reasoning to solve problems and make sense of the world around them.*	*In high school, students should be able to apply their knowledge of probabilistic reasoning to solve problems and make sense of the world around them.*	• Categorical data & two-way frequency tables • Interpret probabilities in context	*In the later years of high school, students should be able to apply a more complex knowledge of probabilistic reasoning to solve problems and make sense of the world around them.*	*In the later years of high school, students should be able to apply a more complex knowledge of probabilistic reasoning to solve problems and make sense of the world around them.*