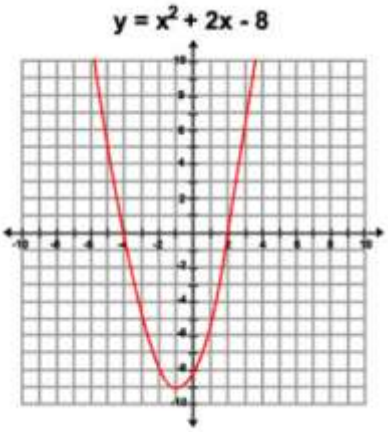
APEX LESSON 2.7 GRAPHS of QUADRATIC FUNCTIONS

Objectives:

- Determine the number of times that a given function crosses the *x*-axis.
- Find the vertex and *x*-intercepts of the graph of a quadratic function.
- Use the quadratic formula to find the vertex of a given function.
- Determine if a discriminant is positive, negative, or zero when given the graph of the function.
- Match a graph to its quadratic equation.

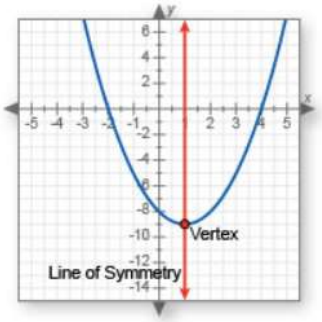
When **quadratic equations** are graphed, they form **parabolas**. The shape of the parabola and where it is on the coordinate plane can provide useful information about the function.

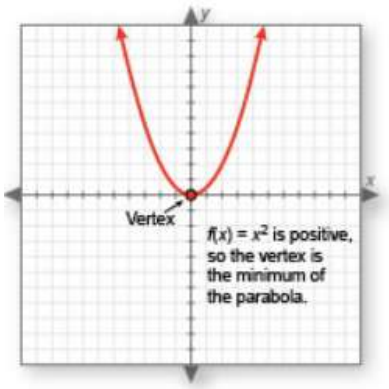
For instance, the **vertex of a parabola** is the point at which the direction of the **function** changes. The points where the parabola intersects the *x*-axis (also known as *solutions*, **roots**, or *zeros*) are the *x*-values of the function when *y* equals zero.

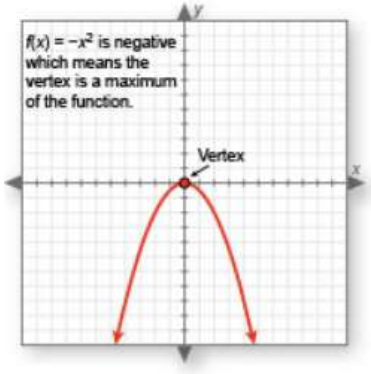


Features of Parabola

Every graph of a **quadratic function** is a **parabola**, and each parabola has a vertex at its center. Parabolas are symmetrical curves, meaning that each side of the curve is an exact reflection of the other side. The line of symmetry (also called the *axis of symmetry*) that divides the parabola into two mirror images runs directly through its vertex, as shown in the image at right.





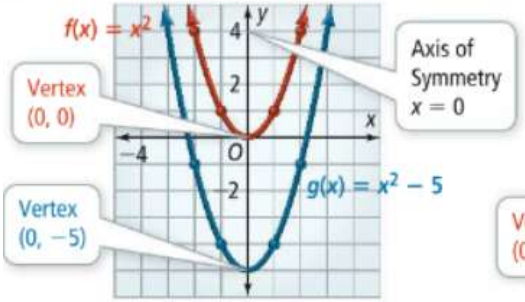


Examples

Problem 2 Graphing Translations of $f(x) = x^2$

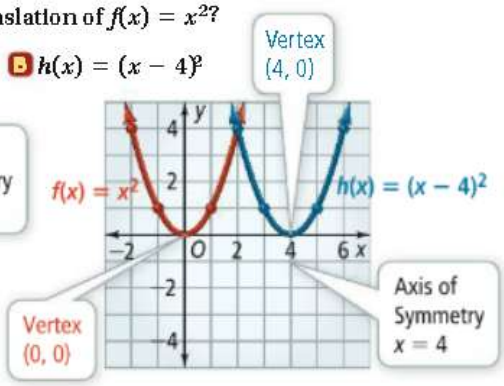
Graph each function. How is each graph a translation of $f(x) = x^2$?

A $g(x) = x^2 - 5$



Translate the graph of f down 5 units to get the graph of $g(x) = x^2 - 5$.

B $h(x) = (x - 4)^2$



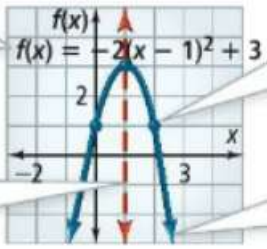
Translate the graph of f to the right 4 units to get the graph of $h(x) = (x - 4)^2$.

Problem 4 Using Vertex Form

A What is the graph of $f(x) = -2(x - 1)^2 + 3$?

Step 1 Identify the constants $a = -2, h = 1$, and $k = 3$. Because $a < 0$, the parabola opens downward.

Step 2 Plot the vertex $(h, k) = (1, 3)$ and draw the axis of symmetry $x = 1$.



Step 3 Plot two points. $f(2) = -2(2 - 1)^2 + 3 = 1$. Plot (2, 1) and the symmetric point (0, 1).

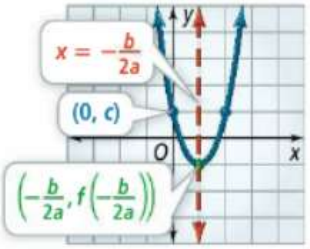
Step 4 Sketch the curve.

Take note

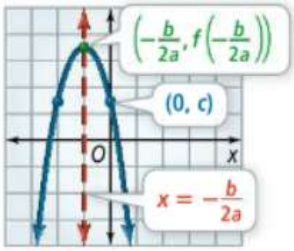
Properties Quadratic Function in Standard Form

- The graph of $f(x) = ax^2 + bx + c, a \neq 0$, is a parabola.
- If $a > 0$, the parabola opens upward. If $a < 0$, the parabola opens downward.
- The axis of symmetry is the line $x = \frac{b}{2a}$.
- The x-coordinate of the vertex is $\frac{b}{2a}$. The y-coordinate of the vertex is the y-value of the function for $x = -\frac{b}{2a}$, or $y = f\left(-\frac{b}{2a}\right)$.
- The y-intercept is $(0, c)$.

$y = ax^2 + bx + c, a > 0$



$y = ax^2 + bx + c, a < 0$



Problem 2 Graphing a Function of the Form $y = ax^2 + bx + c$

What is the graph of $y = x^2 + 2x + 3$?

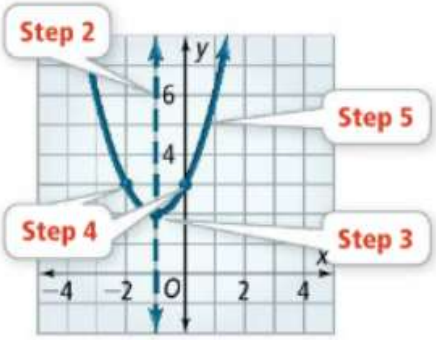
Step 1 Identify a, b, and c.
$a = 1, b = 2, c = 3$

Step 2 The axis of symmetry is $x = \frac{b}{2a}$.
$x = -\frac{2}{2(1)}$
Lightly sketch the line $x = -1$.

Step 3 The x-coordinate of the vertex is also $\frac{b}{2a}$, or -1.
The y-coordinate is
$y = (-1)^2 + 2(-1) + 3 = 2$.
Plot the vertex $(-1, 2)$.

Step 4 Since $c = 3$, the y-intercept is $(0, 3)$. The reflection of $(0, 3)$ across $x = -1$ is $(-2, 3)$. Plot both points.

Step 5 $a > 0$ confirms that the graph opens upward. Draw a smooth curve through the points you found in Steps 3 and 4.



Problem 3 Converting Standard Form to Vertex Form

What is the vertex form of $y = 2x^2 + 10x + 7$?

$y = 2x^2 + 10x + 7$ Identify a and b.

$x = -\frac{b}{2a}$ Find the x-coordinate of the vertex.

$= -\frac{10}{2(2)}$

$= -2.5$

$y = 2(-2.5)^2 + 10(-2.5) + 7$ Substitute $x = -2.5$ into the equation.

$= -5.5$

The vertex is $(-2.5, -5.5)$.

$y = a(x - h)^2 + k$ Write the vertex form.

$y = 2[x - (-2.5)]^2 + (-5.5)$ Substitute $a = 2$, $h = -2.5$, $k = -5.5$.

$y = 2(x + 2.5)^2 - 5.5$ Simplify.

The vertex form is $y = 2(x + 2.5)^2 - 5.5$.