APEX LESSON 3.1 POLYNOMIAL FUNCTIONS November 16-20, 2020

Objectives:

- Categorize expressions as monomial, binomial, trinomial, polynomial, or non-polynomial.
- Write polynomials in standard form and identify their coefficients, variables, and powers.
- Determine the degree and formal name of a given polynomial.
- Explain the similarities between integers and polynomials as closed sets under addition, subtraction, and multiplication.
- Apply multiple methods for multiplying and collecting like terms of higher-order polynomials.

What exactly is a polynomial?

Is it a number? *Yes.* Is it a variable? *Yes.* Is it an expression? *Yes.*

Polynomials are a single term or sum/difference of terms. They have variables raised to whole-number exponents ≥ zero. The variable cannot be under a radical, in the exponent, or in the denominator of a fraction.

Four examples of polynomials:

$$-2x+7$$

$$x^2+3x$$

$$4x^2-x+6$$

$$2x^3-5x^2+x+3$$

Non-Examples of Polynomials

Remember... these are NOT polynomials!

$\frac{a^2}{b^3}$ Fractions, Division

Square Roots $\sqrt{x^2+2x+2}$

9^{2x} Variables as the exponent

Negatives as the exponent $8x^{-3}$

Describing Polynomials

Classify by degree (highest exp. when in standard form)			Classify by type (number of terms)		
Degree	Name	Example	Terms	Name	Example
0	Constant	5	1	Monomial	$2x^3$
1	Linear	$21x+17$	2	Binomial	x^3+7x
2	Quadratic	$4x^2-6x+2$	3	Trinomial	$7x^2-3x+9$
3	Cubic	$10x^3-42$	4	4-term polynomial	$2x^4+3x^3+21x+4$
4	4th degree	$5x^4+7x^3+5x^2+5x+5$			
5	5th degree	$-x^5+1$	5	5-term polynomial	$x^4+x^3+x^2+x+1$

A **polynomial expression** is an expression that can be written in the following form:

$$a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$$

Every polynomial is made up of groups of letters and numbers joined by multiplication and separated by plus or minus signs. Each group is called a **term**.

a is a coefficient, a constant by which a variable is multiplied.

$$ax^n$$

n is the power of the term and is always a whole number.

x is a variable, an unknown quantity that can stand for any number.

POLYNOMIALS in Standard Form

Standard form: put terms in order of decreasing exponents

Standard form	Classify by Degree	# of terms Classify by Type	Leading Coefficient
A] $4x^2 - 2(2x^2 - 3x + 2)$ $\cancel{4x^2} - \cancel{4x^2} + 6x - 4$ $6x - 4$	linear	binomial	6
B] $2(x^3 - 5x + 8) - 5(3 - 2x) - 1$ $2x^3 - \cancel{10x} + \cancel{16} - \cancel{15} + \cancel{10x} - \cancel{1}$ $2x^3$	cubic	monomial	2
C] $\frac{1}{2}(2x - 3)^2$ $\frac{1}{2}(2x-3)(2x-3)$ $\frac{1}{2}(4x^2 - 12x + 9)$ $2x^2 - 6x + \frac{9}{2}$	quadratic	trinomial	2

Adding Polynomials

Add: $(x^2 + 3x + 1) + (4x^2 + 5)$

Step 1: **Underline like terms:**

$(x^2 + 3x + 1) + (4x^2 + 5)$

Notice: '3x' doesn't have a like term.

Step 2: **Add the coefficients of *like terms*, do not change the powers of the variables:**

$(x^2 + 4x^2) + 3x + (1 + 5)$

$\boxed{5x^2 + 3x + 6}$

To Subtract Polynomials:

① Set up the subtraction.

$(x^2 + 5) - (2x^2 - 1)$

② Distribute the negative.

$(x^2 + 5) - (2x^2 - 1)$

$= x^2 + 5 - 2x^2 - (-1)$

$= x^2 + 5 - 2x^2 + 1$

③ Combine like terms. (underlined)

$x^2 + 5 - 2x^2 + 1$

$= \boxed{-x^2 + 6}$

Multiplication of Polynomials

Example 1 Multiplying with a monomial

Use a multiplication strategy to expand the expression.
When multiplying, add the exponents. Combine like terms.

Simplify the expression. Write the answer in standard form.

A] $6x^3(5x^2-3x+2)$

$30x^5-18x^4+12x^3$

B] $4x(3x^2+2)(x^2-5)$

$(12x^3+8x)(x^2-5)$

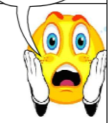
$12x^5-60x^3$
$+8x^3-40x$

$12x^5-52x^3-40x$

Example 2 Multiplying with a binomial

Rewrite any powers of a binomial first.
Use repeated distribution to expand the expression.

Do NOT distribute the exponent!



Simplify the expression. Write the answer in standard form.

A] $(x+5)(2x^2-x-3)$

$2x^3-x^2-3x$
$+10x^2-5x-15$

$2x^3+9x^2-8x-15$

B] $(2x^2-1)(x-5)^2$

$(2x^2-1)(x-5)(x-5)$

x^2-5x
$-5x+25$

$(2x^2-1)(x^2-10x+25)$

$2x^4-20x^3+50x^2$
$-x^2+10x-25$

$2x^4-20x^3+49x^2+10x-25$

Synthetic Division

Synthetic Division can only be used if the divisor is a linear factor.

1. Write down the coefficients of the dividend (insert dummy terms if necessary).
2. Change the sign of the constant in the divisor.
3. Bring down the first coefficient of the dividend.
4. Multiply, add, repeat.
5. The answer is the sequence of coefficients of the new polynomial but one degree less than the original polynomial.
6. The last term is the remainder, put that over the divisor.

Example:

Divide $2x^3+6x^2+29$ by $x+4$

-4	2	6	0	29
		-8	8	-32
	2	-2	8	-3

Coefficients of quotient — Remainder

$$(2x^3+6x^2+29)\div(x+4)=2x^2-2x+8-\frac{3}{x+4}$$

Exercises:

- Student will answer and solve 3.1.1
  Study: Polynomial Basics (Study Guide)

Synthetic Division

3	1	−5	−2	24
		3	-6	-24
	1	-2	-8	0

Answer: x^2-2x-8

Long Division

$$\begin{array}{r}
x^2-2x-8 \\
x-3\,\overline{)\,x^3-5x^2-2x+24} \\
\underline{x^3-3x^2} \\
-2x^2-2x+24 \\
\underline{-2x^2+6x} \\
-8x+24 \\
\underline{-8x+24} \\
0
\end{array}$$