

Before This Module

Kindergarten Module 3

In kindergarten, students compare the number of objects in a set using language such as *more than*, *fewer than*, and *the same as*. They compare numbers to 10 using language such as *greater than*, *less than*, and *equal to*.

Kindergarten Module 5

Kindergartners represent composition and decomposition situations using number bonds and number sentences. They solve *add to with result unknown* and *put together with total unknown* problem types.

Kindergarten Module 6

At the end of kindergarten, students decompose teen numbers as ten ones and some more ones and write the decomposition as a 10+ fact.

Overview

Counting, Comparison, and Addition

Topic A

Count and Compare with Data

Data contexts provide natural opportunities for counting. Students collect data by answering questions, sorting sets, and making observations. They create bar graphs, picture graphs, and tally charts to visually represent the data. As students count to find totals and visually compare quantities, they recognize that linear organizations are useful. Students compare what they see using language such as *more than*, *fewer than*, and *equal to*, and represent these statements numerically using the symbols $>$, $<$, and $=$. Number paths and tally marks provide students opportunities for counting on from 5.

		4			4
		6			6
		4			4

There are **14** bears.

There are **more** medium bears than small bears.
 $6 > 4$

There are **fewer** large bears than medium bears.
 $4 < 6$

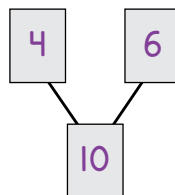
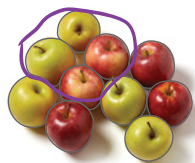
The number of small and large bears is **equal**.
 $4 = 4$

Topic B

Count On from a Visible Part

Students progress from finding totals by using the Level 1 strategy of counting all objects to using the Level 2 strategy of counting on from a known part. At first, objects are shown as two parts, such as the dots that appear on a pair of dice. Students choose a part they know, or can subitize, without counting one by one. They begin the count by naming the known part and then keep counting the objects in the second part to find the total: Fooouuur, 5, 6, 7, 8, 9, 10. Students advance to counting on from a part embedded within a total. For example, given a collection of apples, students represent two parts (4 apples and 6 apples) and the total (10 apples) by using number bonds and number sentences. They realize they can count on from either part and get the same total.

Fooouuur, 5, 6, 7, 8, 9, 10

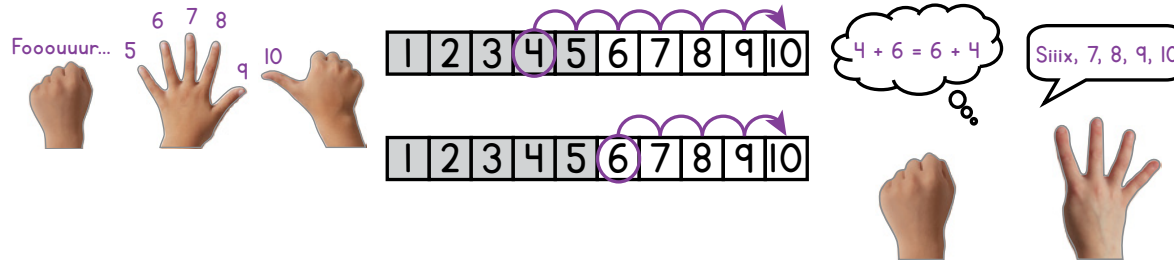


$$4 + 6 = 10$$

Topic C

Count On to Add

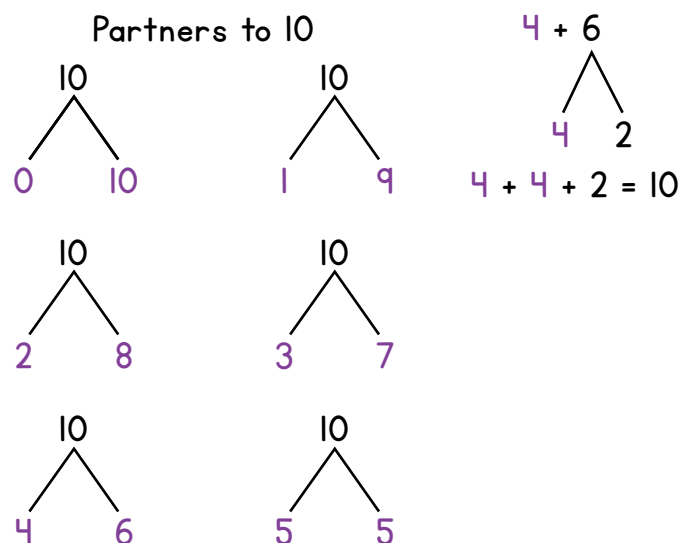
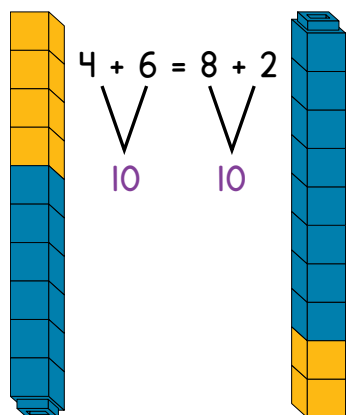
Now students count on to find totals for expressions (e.g., $4 + 6$) rather than for sets of countable objects. Because the parts are no longer presented as sets of objects that can be counted, students must hold the first addend in mind and count on the second addend by tracking with fingers. Students experiment with counting on from both numbers by using a number path to determine the efficiency of starting with the larger addend. Recognizing this efficiency and knowing that starting with either addend results in the same total, students begin to find totals by strategically counting on from the larger part. Students also look for patterns when adding 0 and 1.



Topic D

Make the Same Total in Varied Ways

This topic deepens understanding about the meaning of the equal sign, which earlier topics introduced through data and counting on. Students recognize that the expressions on both sides of the equal sign have the same total. In this topic, students reason about more complex number sentences to determine whether they are true or false. For example, $4 + 6 = 8 + 2$ is true because $4 + 6 = 10$ and $8 + 2 = 10$. This work leads to decomposing numbers and finding the partners for each (e.g., 10 is 1 and 9, 2 and 8, 3 and 7, 4 and 6, etc.). Their burgeoning number sense allows students to decompose addends to make equivalent, often easier, problems.



After This Module

Grade 1 Module 2

Graphs provide context for adding to find the total of all the data points.

Students use counting strategies from this module to find unknown addends and to subtract.

Grade 1 Module 3

With Level 1 and Level 2 strategies well established in the first modules, module 3 focuses on Level 3 strategies that involve making easier problems. To access Level 3 strategies such as make ten, students practice

- decomposing numbers 5 through 9,
- finding the partner that makes 10 for any number,
- developing fluency with $10 + n$ facts, and
- working with three addend expressions.

Grade 1 Module 4 and 5

Students will use number paths as a measuring tool. They will also use $>$, $<$, and $=$ symbols to compare measurements.

Students will use familiar $>$, $<$, and $=$ symbols to compare two-digit numbers by using place value concepts.

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Counting, Comparison, and Addition

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Why

Counting, Comparison, and Addition

What are the 3 counting levels?

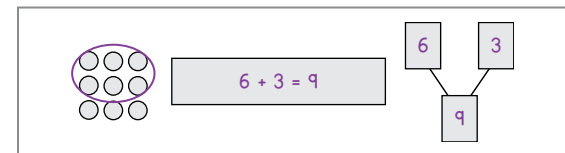
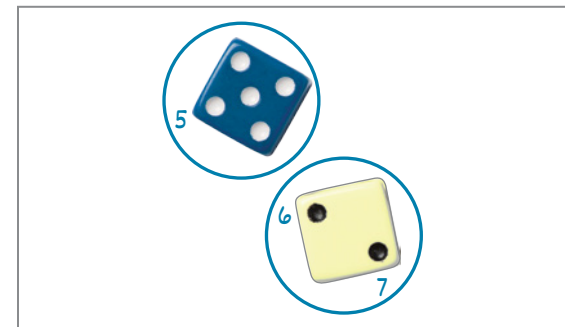
Students in K–2 advance through three strategy levels as they count, add, and subtract. All levels are valid strategies. However, each next level offers greater efficiency for problem solving.

- **Level 1, Direct Modeling by Counting All or Taking Away:** Students represent problems with groups of objects, fingers, or drawings. They model the action by composing or decomposing groups and then they count the result.
- **Level 2, Counting On:** Students count to solve, but they shorten the process of counting by starting with the number word of one part. They use different methods, such as fingers, to keep track of the count.
- **Level 3, Convert to an Easier Equivalent Problem:** Students work flexibly with numbers. They decompose and compose parts to create equivalent, easier problems.

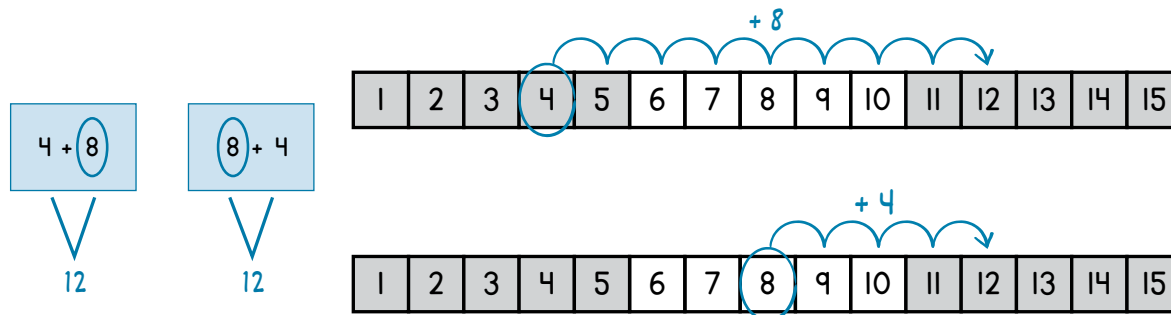
What stages do students move through as they develop skills with counting on?

Counting on is foundational to more efficient addition strategies, mastery of facts within 20, and finding an unknown part. It takes practice for students to trust that counting all and counting on strategies each produce the same total. Several complexities are involved:

- When presented with two parts composed of discrete objects, students intuitively count the objects to find the total. Rather than count all the objects starting at 1, they subitize one part and say how many (the quantity). Then they point to each object in the second part to count on. They understand that the last number stated is the total. They recognize that counting on is addition, recording the parts and total in number bonds and number sentences.



- When given one set of discrete objects, students will subitize an embedded part and count on to find the total. Students may point to the remaining objects as they count on, or they may begin to use their fingers to keep track. Students begin to realize that they can count on from either part and get the same result.
- When presented with an addition expression, students state the first addend (possibly by making a fist). Then they count on the second addend, tracking with fingers. They stop when the number of fingers is the same as the second addend. The last number said is the unknown total.
- Students first experience using one hand to count on, when the addend is 5 or less, and using two hands to count on when the addend is 6 through 9.
- Students will see that the sums are the same, or equal, when counting on from either addend. They use number paths to show that counting on from the larger addend is more efficient. Finally, they choose to count on from the larger addend by thinking of $8 + 4$ when presented with $4 + 8$.



Which word problem types, or addition and subtraction situations, are used in this module?

The table shows examples of addition and subtraction situations.¹ Darker shading in the table indicates the four kindergarten problem types. Students in grades 1 and 2 work with all problem types. Grade 2 students reach proficiency with the unshaded problem types.

Grade 1 students revisit the following problem types that were mastered in kindergarten. However, in grade 1, the problems may use numbers within 20 (not just within 10) and students solve them by using Level 2 and Level 3 strategies.

- **Add to with result unknown:** Both parts are given. An action joins the parts to form the total.

Hope has 7 rocks. She adds 3 more rocks. How many rocks does she have now?
(Lesson 13)

- **Put together/take apart with total unknown:** Both parts are given. No action joins or separates the parts. Instead, the parts are distinguished by an attribute such as type, color, size, or location.

5 markers are in the box. 3 markers are out of the box. How many markers are there in all? (Lesson 7)

- **Put together/take apart with both addends unknown:** Only the total is given. Students take apart the total to find both parts. This situation is the most open ended because the parts can be any combination of numbers that make the total.

There are 5 dogs. What are all the ways they can be inside the house or out in the yard?
(Lesson 18)

Students are invited to solve word problems intuitively. Each lesson presents an accessible problem that can also be extended. Some students may directly model all components of the problem with manipulatives or by drawing. Others may use their fingers, a number path, or drawing to count on from one part. This variety is important because it presents an opportunity for students to discuss their reasoning.

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	<i>A</i> bunnies sat on the grass. <i>B</i> more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	<i>A</i> bunnies were sitting on the grass. Some more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies hopped over to the first <i>A</i> bunnies? $A + \square = C$	Some bunnies were sitting on the grass. <i>B</i> more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	<i>C</i> apples were on the table. I ate <i>B</i> apples. How many apples are on the table now? $C - B = \square$	<i>C</i> apples were on the table. I ate some apples. Then there were <i>A</i> apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate <i>B</i> apples. Then there were <i>A</i> apples. How many apples were on the table before? $\square - B = A$
Put Together /Take Apart	<i>A</i> red apples and <i>B</i> green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has <i>C</i> flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	<i>C</i> apples are on the table. <i>A</i> are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
Compare	Difference Unknown "How many more?" version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many more apples does Julie have than Lucy? $A + \square = C$ $C - A = \square$	Bigger Unknown "More" version suggests operation. Julie has <i>B</i> more apples than Lucy. Lucy has <i>A</i> apples. How many apples does Julie have? $A + B = \square$	Smaller Unknown "Fewer" version suggests operation. Lucy has <i>B</i> fewer apples than Julie. Julie has <i>C</i> apples. How many apples does Lucy have? $C - B = \square$ $\square - B = C$

Count On: Drawing	Count On: Number Path
Count On: Fingers	Count On: Number Facts $7 + 3 = 10$

¹ Common Core Standards Writing Team, *Progressions for the Common Core (draft), Grades K–5, Counting and Cardinality & Operations and Algebraic Thinking*, 9.

Teachers use students' thinking to advance the class toward the objective. They watch how students solve the problem, select work to share, and ask questions that engage the class in others' thinking. Observations about how students use counting on in these lessons may be useful for preparing to teach topics B and C. The Read-Draw-Write problem-solving routine begins in module 2.

Why is lesson 25 optional?

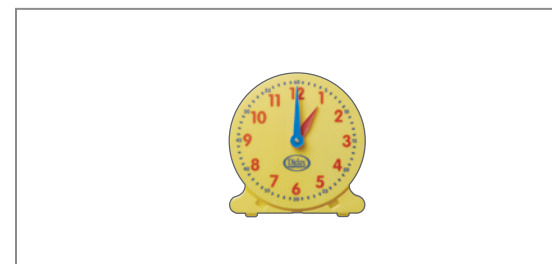
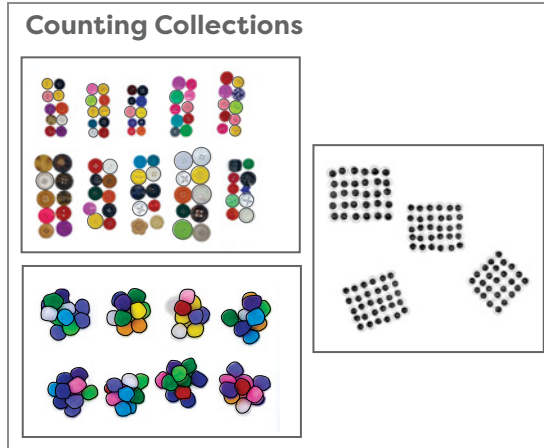
Students count a collection of objects in lesson 25. Counting collections lessons engage students in self-directed learning and provide opportunities for informal assessment. This lesson can be used in the module when the timing best meets the needs of the class. Note that counting collections lessons require preparation. Make sure to read the Lesson Preparation in advance.

Counting collections are best used as a frequent routine, as students benefit from opportunities to internalize the procedure, choose new collections, and try new counting strategies. They will be included in future lessons, however, consider doing them more often as time allows.

Why does this module include time?

Lesson 17 briefly introduces telling time to the hour. This initial exposure provides a starting place for ongoing informal practice before module 4, where telling time to the hour and to the half hour are directly addressed. Beginning with lesson 17, consider

- periodically pausing the class at the top of an hour to ask what time it is, and
- pointing out the time when events regularly happen on the hour, such as lunch at 12:00 or dismissal at 3:00.



Terminology

The following terms are critical to the work of grade 1 module 1. This resource groups terms into categories of New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it appears in the lesson. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

doubles

An addition number sentence or expression where both parts are the same number. Doubles can also be shown with objects. For example, $3 + 3 = 6$, $2 + 2$, 4 fingers on one hand and 4 on the other, or two dice showing 6 dots. (Lesson 9)

expression

An expression is like a number sentence, but there is no equal sign. (Lesson 14)

In grade 1, an expression is a number, or combination of sums and differences, that can be evaluated. For example, $10 + 2$ is an expression that evaluates to 12. However, $10 +$ is not an expression because it cannot be evaluated. In later grades, expressions will include multiplication and division. Some or all of the numbers in an expression may be replaced with symbols that represent unknowns.

graph

A way of organizing and representing, or showing, information so that we can ask and answer questions. (Lesson 2)

In module 1, students make graphs by aligning number paths, titling the graph, and labeling categories with pictures, letters, or words.

hour hand

The short hand on a clock, which points to the hour. (Lesson 17)

minute hand

The long hand on a clock, which points to the minutes. (Lesson 17)

o'clock

We say “o'clock” to say the time when the minute hand points to 12. For example, when the hour hand points to 1 and the minute hand points to 12, we say that the time is one o'clock. (Lesson 17)

unknown

A number that we need to figure out. (Lesson 12)

Familiar

compare

is greater than

is more than

number path

organize

partners

sort

tally mark

total

Academic Verbs

convince

represent

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	100-bead demonstration rekenrek	2	Number cubes, wooden, set of 12
1	20-bead rekenrek—wood	1	Pad of chart paper
1	Centimeter cubes, set of 500	2	Pads of sticky notes
1	Craft sticks, box of 200	24	Pencils
24	Crayon sets	24	Personal whiteboards
24	Cups, 6–8 ounce	24	Personal whiteboard erasers
1	Demonstration clock	1	Plastic pattern blocks set, 0.5 cm
2	Dot dice, set of 12	1	Projection device
24	Dry-erase markers	1	Small box
1	<i>Eureka Math</i> ^{2™} Addition expression cards, 13 decks	1	<i>Teach</i> book
25	<i>Eureka Math</i> ^{2™} Centimeter number paths	1	Teacher computer or device
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, basic student set of 12	1	Teddy bear counters, set of 96
25	<i>Eureka Math</i> ^{2™} Large number paths	1	Two-color counters, 200 pieces
24	<i>Learn</i> books	1	Unifix [®] Cubes, set of 1,000
1	Markers, set of 8		

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 25 for a list of organizational tools (cups, rubber bands, graph paper, etc.) suggested for the counting collection.

Before This Module

Kindergarten

Kindergarten students learn to represent *take from* story problems by using objects, drawings, and number sentences. They solve problems such as $5 - 3 = \underline{\quad}$ both concretely and pictorially.

Grade 1 Module 1

Counting on to find totals provides a foundation for counting on or counting back to find an unknown part.

Reasoning about the meaning of the equal sign and determining whether number sentences are true or false helps prepare students to write true number sentences and find unknowns in various equations.

Systematic study of partners to 6 through partners to 10 supports understanding subtraction equations as unknown addend problems.

While interpreting graphs, students compare categories by using *greater than*, *less than*, and *equal to* language.

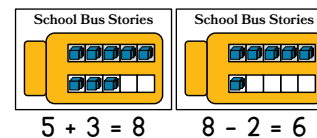
Overview

Addition and Subtraction Relationships

Topic A

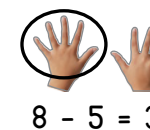
Reason About *Take From* Situations

To bridge from addition to the various types of subtraction situations, students solve *result unknown* stories about a school bus. They notice what happens when they add to and take from the total number of people on the bus.



As they solve sets of related *take from* problems that include subtract 0, subtract all, subtract 1 less than the total, and subtract 1, they notice patterns and formulate conjectures to solve similar problems efficiently.

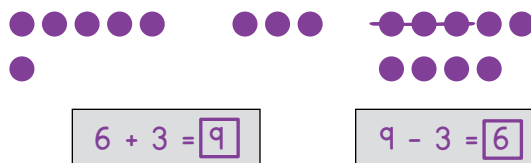
This topic also includes subtraction by using the familiar unit of five. Students subtract 5 by taking away 5 fingers all at once. With 5 as a point of reference, they take away 4 and 6 fingers all at once too. Taking away a part all at once is foundational to the Level 3 subtraction strategy known as take from ten that is introduced in module 3.



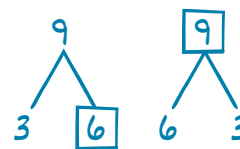
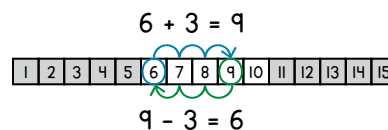
Topic B

Relate and Distinguish Addition and Subtraction

Students learn the Read-Draw-Write problem-solving process to make sense of, represent, and solve related *add to* and *take from* word problems. Students read and reread problems to draw models and solve. Then they represent their solutions by writing a number sentence.



In addition to drawing, students use number paths and number bonds to represent related addition and subtraction problems. The number path allows them to see subtraction as undoing addition by reversing the action. Number bonds help students discover that related addition and subtraction problems have the same parts and total, but what the unknown represents will change. See these examples:



- When you have a total and subtract a part, the unknown represents the other part.
- When you add parts together, the unknown represents the total.

Topic C

Find an Unknown Part in *Change Unknown* Problems

Students solve *add to with change unknown* and *take from with change unknown* word problems. They may

- use a labeled drawing to identify the total, the known part, and the unknown part;

Drawing	Fingers	Number Path

- count on from the known part to the total, noting how many they counted; or
- count back from the total to the known part, noting how many they counted.

When situations imply an *add to* action, students are likely to represent their thinking with addition. When a *take from* action is implied, students are likely to represent their thinking with subtraction.

They use Level 2 strategies when working with equations: counting on to find an unknown addend ($9 + \square = 14$) and counting back to find an unknown subtrahend ($14 - \square = 9$).

Topic D

Find an Unknown Part by Using Addition and Subtraction

Put together and *take apart* situations have no implied action, so students are equally likely to find the unknown part with either addition or subtraction. Students come to understand that the answer to a subtraction problem is an unknown part. To find the answer, they can count on, count back, or simply think about the parts they know that make the total. They determine that counting back is efficient when the known part is small compared to the total. To find the value of various unknowns in equations, they use known part-whole relationships and related facts.

$$10 - 6 = \boxed{4}$$

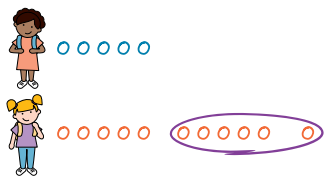
$$6 + \boxed{4} = 10$$



Topic E

Represent and Solve Comparison Problems

Students move toward work with comparison word problems first by making groups equal. They learn the strategies to figure out how many more are in one group than in another: take away the extra amount in one group or add the extra amount to one group. They record the action as a true addition or subtraction number sentence.



$$11 - \boxed{6} = 5$$



$$5 + \boxed{6} = 11$$

After This Module

Grade 1 Module 3

Students continue to solve *result unknown* and *part unknown* word problems and find unknown parts and totals in equations.

However, instead of using Level 2 counting strategies to solve, students use Level 3 strategies that involve forming easier yet equivalent problems.

Grade 1 Module 4

Students solve the following comparison situations involving length: *difference unknown* (how much longer and shorter), *longer length unknown*, and *shorter length unknown*.

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Lesson 2 34

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Lesson 3 48

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Lesson 4 64

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Relate and Distinguish Addition and Subtraction

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Lesson 6 96

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Topic C 128

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Why

Addition and Subtraction Relationships

How does the story of subtraction unfold in this module?

Module 2 presents three interpretations of subtraction. In-depth experience is necessary with all three interpretations to avoid the misconception that subtraction always means *take away*.

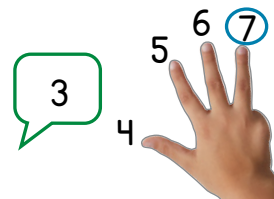
- Take from: Move or remove objects from a set, resulting in a smaller set
- Part-whole: Find an unknown part when given a part and the total
- Comparison: Compare two different sets and identify the difference between the two

Subtraction can be more challenging than addition because the given (or known) part is embedded within the total. Students represent the total and then isolate the known part to find the unknown part.

Why is counting on or counting back to find an unknown part more difficult than counting on to find a total?

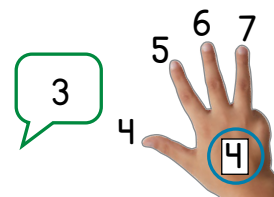
When students count on to find an unknown total, they

- start with the known part,
- count on the amount of the second part, and
- land on the total.



When students count on to find an unknown part in problems such as $3 + \square = 7$, $7 - \square = 3$ and $7 - 3 = \square$, they have an additional step. They

- start with the known part,
- count on until they get to the total, and
- identify how many they counted on.



Similarly, when students count back to solve a problem such as $7 - 3 = \square$, they land on the answer. However, when they count back to find an unknown part in a problem such as $7 - \square = 3$, they must also identify how many they counted back.

Which word problem types, or addition and subtraction situations, are used in this module?

The table shows examples of addition and subtraction situations.¹ Darker shading in the table indicates the four kindergarten problem types. Students in grades 1 and 2 work with all problem types. Grade 2 students reach proficiency with the unshaded problem types.

In this module, grade 1 students revisit the following problem types that were mastered in kindergarten. However, in grade 1, the problems may use numbers within 20 (not just within 10) and students solve them by using Level 2 and Level 3 strategies.

- **Add to with result unknown:** Both parts are given. An action joins the parts to form the total. (Lessons 1, 5, 6)
- **Take from with result unknown:** The total and one part are given. An action takes away one part from the total. (Lessons 1, 5, 6)
- **Put together/take apart with total unknown:** Both parts are given. No action joins or separates the parts. Instead, the parts are distinguished by an attribute such as type, color, size, or location. (Lesson 14)

The following four problem types are introduced in this module and mastered in grade 1:

- **Add to with change unknown:** One part and the total are given. An action joins the known part and the unknown part to form the total. (Lessons 9 and 13)
- **Take from with change unknown:** The total and one part are given. An action takes away the unknown part from the total to form the known part. (Lessons 11 and 13)
- **Put together/take apart with addend unknown:** The total and one part are given. No action joins or separates the parts. (Lesson 14)

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
Put Together /Take Apart	Total Unknown A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Both Addends Unknown ¹ Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	Addend Unknown ² C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
Compare	Difference Unknown "How many more?" version. Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? $A + \square = C$ $C - A = \square$	Bigger Unknown "More" version suggests operation. Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? $A + B = \square$ "Fewer" version suggests wrong operation. Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have?	Smaller Unknown "Fewer" version suggests operation. Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$ "More" version suggests wrong operation. Julie has B more apples than Lucy. Lucy has C apples. How many apples does Lucy have?

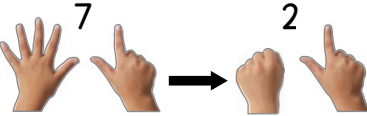
¹ Common Core Standards Writing Team, *Progressions for the Common core (draft), Grades K-5, Counting and Cardinality & Operations and Algebraic Thinking*, 9.

- **Compare with difference unknown:** Two given quantities are compared to find how many more or how many fewer. (Lessons 21 and 22)

What representations are used to solve these word problem types and how do they help students develop their understanding of subtraction?

- **Take from with result unknown** situations in topic A help students differentiate subtraction from addition by implying a take away action. They lend themselves to using fingers as tools to solve.

7 people are on a bus.
5 people get off the bus.
How many people are on the bus now?



$7 - 5 = \square$

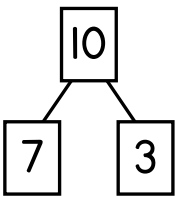
- Solving related **add to with result unknown** and **take from with result unknown** situations in topic B help students relate addition to subtraction. Recognizing the relationship between operations is critical to understanding subtraction as an unknown addend problem. Making drawings, including number bonds, to represent situations helps students see part-whole relationships and interpret solutions as either a part or a total.

7 people are on a bus.
3 people get on the bus.
How many people are on the bus now?

00000 000

10 people are on a bus.
3 people get off the bus.
How many people are on the bus now?

00000 00000



$7 + 3 = \square$

$10 - 3 = \square$

- **Add to with change unknown** and **take from with change unknown** situations in topic C provide contexts that extend students' ability to count on to find a total to the new application of counting on or counting back to find an unknown part. The **add to** or **take from** action implied by the context supports students in finding the unknown, or the value of the change, either by counting on or by counting back (using labeled drawings, fingers, or number paths).

3 people are on the bus.
Some more people get on the bus.
Now there are 10 people on the bus.
How many people got on the bus?

10 people are on the bus.
Some people get off the bus.
Now there are 3 people on the bus.
How many people got off the bus?

$3 + \square = 10$

$10 - \square = 3$

- **Put together/take apart with addend unknown** situations invite students to choose an operation to find the unknown part, since no action is implied by the context. They can solve a subtraction problem by counting on, which is often more reliable than counting back. They can also consider the related addition problem to answer the question *What goes with this part to make the total?* The number bond shows part-whole relationships and is the primary model used to help students relate addition and subtraction. Knowing how the numbers in a bond relate to each other helps students write related facts.

There are 7 bugs.
3 bugs are green.
The rest are brown.
How many bugs are brown?

$3 + \square = 7$

$7 - \square = 3$

$7 - 3 = \square$

- **Compare with difference unknown** situations are represented so that the difference between the size of the two quantities is evident. Students must identify the parts that are the same in both quantities and interpret the difference as either the extra part in one group or the part needed to make the two groups equal. Because of one common misconception, when students read the word *more* in comparison problems, some are inclined to add to find the total rather than compare to find the difference.

7 people are on the yellow bus.
3 people are on the blue bus.
How many more people are on the yellow bus than the blue bus?

Y	0	0	0	0	0	0	0
B	0	0	0	x	x	x	x
Y	0	0	0	x	x	x	x
B	0	0	0				

$$3 + \square = 7$$

$$7 - \square = 3$$

How does this module advance students' module 1 work with equality?

In module 1, students use the equal sign to do the following:

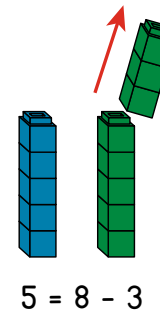
- Join parts to make a total ($5 + 3 = 8$)
- Match a total with its parts ($8 = 5 + 3$)
- Represent the equality of two expressions ($5 + 3 = 6 + 2$)

In these instances, students primarily use computational thinking: *When I find the total on either side of the equal sign, are the totals the same amount?*

In module 2, students extend their understanding of equality and use the equal sign to do the following:

- Determine an unknown part in equations ($8 - 5 = \square$, $5 + \square = 8$, $8 - \square = 5$)
- Find how many more one group has than another ($5 + \underline{3} = 8$ or $8 - \underline{3} = 5$)

In these instances, students focus on relational thinking: *What makes one group more than another group? How can I make the groups equal?* In doing so, they represent their thinking by using various equations, including $5 = 8 - 3$ or even $5 + 1 = 8 - 2$.



Terminology

The following terms are critical to the work of grade 1 module 2. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

equation

An equation is like a number sentence. It is different because we can write it to show that a number is unknown. (Lesson 10)

A number sentence with an equal sign can always be called an equation. An equation cannot be called a number sentence when it has a symbol for an unknown, such as a box, a blank,

a letter, or a question mark. Throughout grade 1 we use number sentence whenever it is applicable so that language used with students is consistent. Equation is used to draw attention to when students are finding an unknown, signified with a gray box. Students are not expected to distinguish between these terms in grade 1.

Familiar

compare

double

efficient

equal

expression

false

graph

minus

part

related

represent

subtract

symbol

take away

total

true

unknown

Academic Verb

combine

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	Chart paper, pad	24	<i>Learn</i> books
1	Computer with internet access	7	Markers
240	Craft sticks	25	Pencils
24	Crayons, set of 3 colors	132	Pennies
25	Dry-erase markers	24	Personal whiteboards
1	<i>Eureka Math</i> ^{2™} Addition expression cards, 13 decks	24	Personal whiteboard erasers
24	<i>Eureka Math</i> ^{2™} Centimeter number paths	1	Projection device
2	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, basic student set of 12	2	Sticky notes (pad)
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, demonstration set	1	<i>Teach</i> book
2	Farm animal counters, set of 72	1	Unifix [®] Cubes, set of 1,000

Visit <http://eurmath.link/materials> to learn more.

Before This Module

Kindergarten

Kindergarten students know teen numbers ($10 + n$) as ten ones and some ones.

Grade 1 Module 1

Students worked with $10 + n$ facts and developed fluency with all compositions for totals 5–10. Students use these skills to make ten and take from ten.

Grade 1 Module 2

Students began to use a Level 3 strategy for making a problem easier: using the relationship between addition and subtraction to subtract.

For example, students thought about a related addition fact to solve a subtraction problem.

To solve $10 - 6 = ?$ students think:

I know $6 + 4 = 10$, so $10 - 6 = 4$.

Overview

Properties of Operations to Make Easier Problems

Topic A

Make Easier Problems with Three Addends

Students work with three-addend problems in which two addends are partners to 10. They learn to group addends to make ten so that the problem becomes the same as finding the total for a $10 + n$ fact. This makes the problem easier for students who have command of $10 + n$ facts. Although students do not name the commutative and associative properties, they intuitively apply them as they think of addends in any order and group two of them first.

$$1 + 5 + 9 = 15$$

$$5 + 9 + 1 = 15$$

$$10 + 5 = 15$$

Topic B

Make Easier Problems to Add

Students work with two-addend problems in which the addends do not make ten. They apply the make ten strategy by

- thinking of one addend's partner to 10,
- decomposing the other addend to make ten, and
- adding 10 to the remaining part.

Lessons introduce complexity by presenting problems in which the second addend is closer to ten or where the two addends have a similar value, such as $6 + 7$. These problems require students to reason about which addend they should use to make ten. Students need time and practice for the make ten strategy to become efficient; at first, it may not be easier than counting on.

$$9 + 6 = 15$$



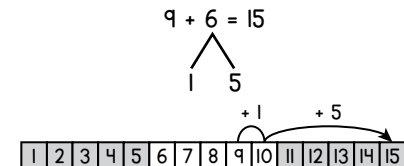
$$9 + 1 + 5$$

$$10 + 5 = 15$$

Topic C

Make Easier Addition Problems with a Linear Model

Students make ten to add and represent their thinking on a number path. They represent and compare two- and three-addend equations on the number path. Instead of showing an addend as a series of single hops, students show it with one larger hop. This is the first time that a single hop encompasses more than one space on the number path.

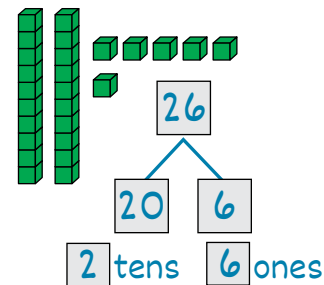


Topic D

Reason about Ten as a Unit to Add or Subtract

In topic D, students learn to recognize ten as a unit. By building, drawing, and visualizing quantities, they see that all two-digit numbers are composed of tens and ones. Grouping quantities into units of ten and some ones helps students recognize the equivalence of 1 ten and 10 ones. They rename the groups of ten in terms of units by referring to each as a *ten*, rather than 10 ones.

Students apply this work to addition and subtraction with teen numbers. They decompose the teen number into 1 ten and some ones and use the structure of ten as a tool for adding or subtracting.



$ \begin{array}{c} 15 + 3 = 18 \\ \swarrow \quad \searrow \\ 10 \quad 5 \\ \quad \quad 5 + 3 = 8 \\ \quad \quad 10 + 8 = 18 \end{array} $	$ \begin{array}{c} 15 - 3 = 12 \\ \swarrow \quad \searrow \\ 10 \quad 5 \\ \quad \quad 5 - 3 = 2 \\ \quad \quad 10 + 2 = 12 \end{array} $
---	---

Topic E

Make Easier Problems to Subtract

Students practice several Level 3 subtraction strategies: take from ten, counting on by using ten, and counting back by using ten. Each of these strategies involves decomposing the total or the known part and manipulating the quantities by using the structure of ten. As they learn these strategies and explore the relationship between them, students analyze problems and ask themselves: “Should I take from the ones, ten, or ones and ten?” They have opportunities to self-select their strategy and the tools to represent their thinking based on the answer to that question.

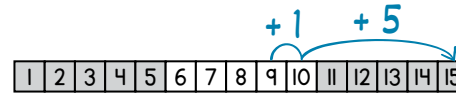
To solve a problem such as $15 - 9$ by using the take from ten strategy, students draw the teen total as a ten and some ones (15 is 10 and 5). They take the subtrahend from ten (10 minus 9) and add the parts that remain (1 and 5 is 6).

Students also relate take from ten to the familiar strategy of counting on. They use the number path to make or get to a ten and then add or hop the remaining part. Additionally, students break apart the subtrahend and count back to get to a ten.

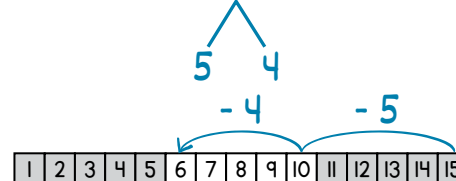
These strategies for making subtraction problems easier may not seem easier at first. Students need time to become efficient, but with steady practice, these become mental math strategies by the end of the year.

$$15 - 9 = 6$$

$$\begin{array}{r} 10 \quad 5 \\ 10 - 9 = 1 \\ 1 + 5 = 6 \end{array}$$

$$15 - 9 = 6$$


$$1 + 5 = 6$$

$$15 - 9 = 6$$


After This Module

Grade 1 Module 4

Students continue to explore tens and ones as they measure lengths.

Grade 1 Module 5

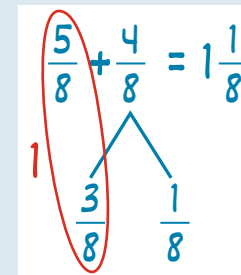
Students advance their place value understanding by working with larger groups of tens and ones.

They apply Level 3 strategies from module 3 to solve addition and subtraction problems with larger numbers.

Grades 2–5

Students continue to use Level 3 strategies to add and subtract whole numbers and fractions. They

- make a hundred or make one (a whole)

$$\frac{5}{8} + \frac{4}{8} = 1\frac{1}{8}$$


- and take from 1 or 100.

$$282 - 98 = 184$$

$$\begin{array}{r} 182 \quad 100 \\ 100 - 98 = 2 \\ 182 + 2 = 184 \end{array}$$

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Make ten when an addend is 5.

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Make ten when the first addend is 9.

Lesson 7 102

Make ten when the first addend is 8 or 9.

Lesson 8 116

Make ten when the second addend is 8 or 9.

Lesson 9 130

Make ten with either addend.

Lesson 10 142

Make ten when there are three addends.

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Make Easier Addition Problems with a Linear Model

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Count on to make the next ten within 100.

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Why

Properties of Operations to Make Easier Problems

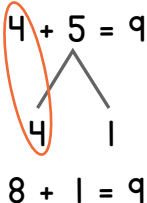
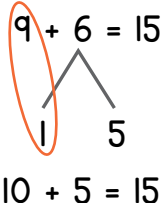
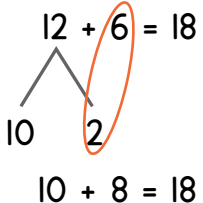
What are Level 3 strategies for addition and subtraction?

Module 3 marks a critical moment in grade 1 when students transition away from finding totals by counting all (Level 1 strategy) or counting on (Level 2 strategy) to finding totals by making an equivalent yet easier problem (Level 3 strategy). The following tables show different ways to make addition or subtraction problems easier. These strategies often rely on knowledge of the properties of operations, such as the commutative and associative properties of addition, and leverage ten as a benchmark number. Level 3 strategies take time and practice before they are truly easier for students than Level 2 strategies.

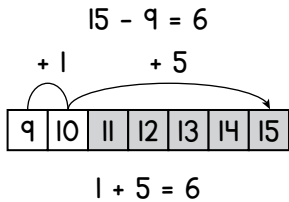
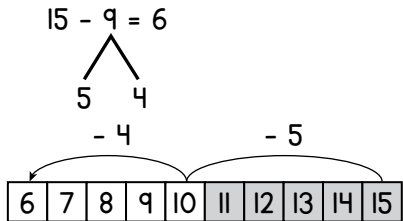
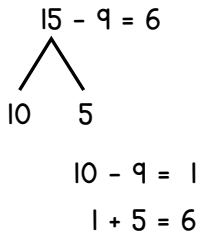
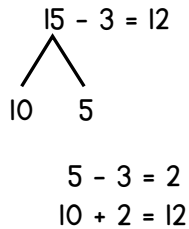
Students learn the following Level 3 strategies, though they may not master them all. After learning them, students may self-select strategies that are most efficient for them personally or that are most appropriate for the problem. It is important for students to internalize the concepts embedded within these strategies to improve their number sense. Ultimately, students who have a command of these concepts use numbers flexibly to solve problems efficiently.

These strategies extend to the ranges of numbers that students work with in later grades. In grades 1–3, students apply these strategies for making an easier problem to larger units, such as hundreds and thousands. In grades 4–6, they apply these strategies to smaller units, such as decimals and fractions.

Level 3 addition strategies

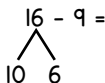
Decompose an addend to make a known fact.	Decompose an addend to make ten.	Decompose an addend into a ten and ones; add the ones.
 $4 + 5 = 9$ $8 + 1 = 9$	 $9 + 6 = 15$ $10 + 5 = 15$	 $12 + 6 = 18$ $10 + 8 = 18$

Level 3 subtraction strategies

Count on using ten (use the relationship between addition and subtraction).	Decompose the subtrahend (known part) to get to ten.	Decompose the minuend (total) and take from ten.	Decompose the minuend (total) and take from the ones.
$15 - 9 = 6$  $1 + 5 = 6$	$15 - 9 = 6$  $- 4$ $- 5$	$15 - 9 = 6$  $10 - 9 = 1$ $1 + 5 = 6$	$15 - 3 = 12$  $5 - 3 = 2$ $10 + 2 = 12$

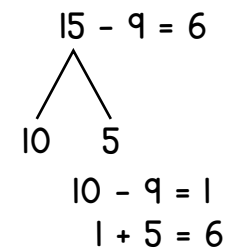
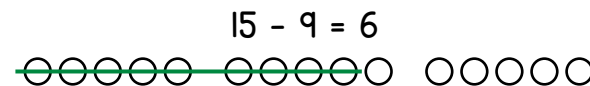
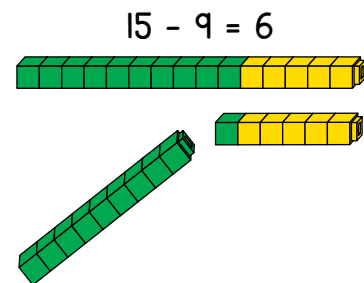
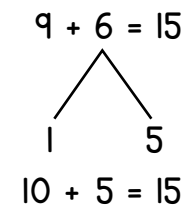
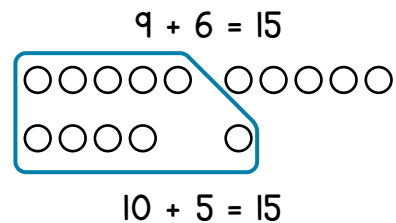
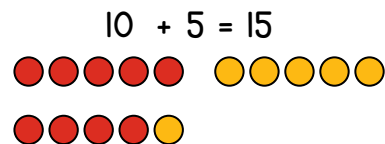
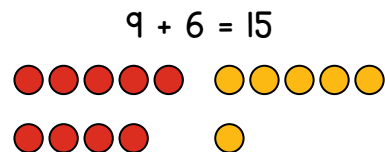
What skills do students need to be successful with Level 3 strategies?

To use Level 3 strategies successfully, students must think about numbers flexibly. This often involves decomposing and composing. The tables list the essential skills that students learn in kindergarten and early grade 1 and show how they apply the skills as they use Level 3 strategies.

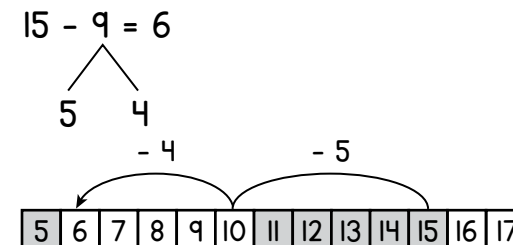
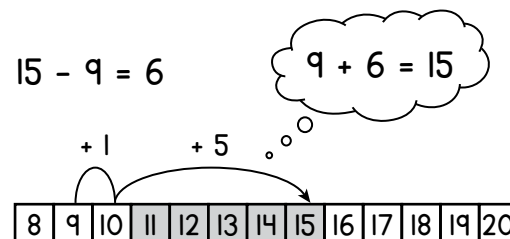
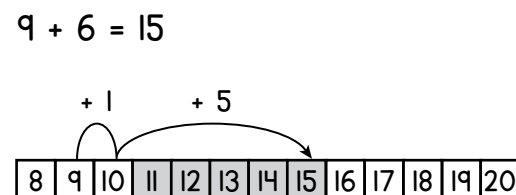
Make Ten with 9	Essential Skills	Take 9 from Ten	Essential Skills
Think of 9's partner to 10.  Break the second addend into 1 and the remaining part.  Add 9 and 1 to make 10.  Add the remaining part to 10. $10 + 6 = 16$	Know the addend's partner to 10. Know all decomposition for amounts less than 10. Know all compositions for 10. Know all teen numbers as a $10 + n$ fact.	Decompose the teen minuend into ten and some ones.  Take the subtrahend, 9, from ten. $10 - 9 = 1$ Add the remaining parts. $1 + 6 = 7$	Know all teen numbers as a $10 + n$ fact. Know all decompositions for 10. Know all compositions for numbers to 10.

What tools do students use to apply Level 3 strategies?

Students represent how they make ten or take from ten in a variety of ways. Students may use concrete tools, such as cubes or fingers; pictorial tools, such as 5-groups; or abstract tools, such as number bonds and number sentences. Students move fluidly between concrete, pictorial, and abstract tools as they progress to mentally solving problems within 20.



The number path is a linear model used to represent strategies such as counting on or counting back by using ten.



Terminology

The following terms are critical to the work of grade 1 module 3. This resource groups terms into categories called New and Familiar. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

addend

Addends are the parts in an addition equation or expression. (Lesson 1)

ten

A ten is a unit that is equal to 10 ones. For example, 2 tens is equal to 20. (Lesson 15)

unit

Units tell us what we are counting. Some examples of units are ones, tens, blocks, hours, and minutes. (Lesson 16)

Familiar

compare

double

efficient

equal

equation

expression

graph

minus

number path

part

partners

related

represent

subtract

take away

total

true

unknown

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	10-sided dice, set of 24	24	<i>Learn</i> books
1	20-bead demonstration rekenrek	12	Paper clips
2	Centimeter cubes, set of 500	37	Pencils
1	Chart paper, pad	24	Personal whiteboards
1	Computer with internet access	24	Personal whiteboard erasers
1	Craft sticks, 6 color, package of 1,000	3	Plastic pattern blocks set, 0.5 cm
3	Cups	1	Projection device
2	Dot dice, set of 12	1	Sticky notes, pad
24	Dry-erase markers	1	<i>Teach</i> book
2	<i>Eureka Math</i> ² ® Hide Zero [®] cards, basic student set of 12	1	Teddy bear counters, set of 96
1	<i>Eureka Math</i> ² ® Hide Zero [®] cards, demonstration set	2	Two-color counters, 200 pieces
2	Farm animal counters, set of 72	1	Unifix [®] Cubes, set of 1,000

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 15 for a list of organizational tools (cups, number paths, etc.) suggested for the counting collection.

Before This Module

Kindergarten

Students identified height and length as measurable attributes. They directly compared the length of two objects by aligning the endpoints, and used terms such as taller, longer, and shorter to describe the relationship between them. They informally explored the relationship of number and unit size to length.

Grade 1 Module 2

Students drew pictures to solve *comparison with difference unknown* word problems. They answered the question *How many more?* rather than *How much longer or shorter?*

Grade 1 Module 3

Students prepared for measuring lengths with 10-centimeter sticks and cubes by representing concrete and pictorial quantities as tens and ones using unit form, number bonds, and number sentences.

Overview

Comparison and Composition of Length Measurements

Topic A

Direct and Indirect Length Comparison

The first set of lessons gives students the opportunity to learn and understand aspects of measurement without focusing on numbers and units. At first students directly compare the length of two objects by aligning the endpoints. Then they use direct comparison to order, or seriate, three to six objects. Ordering a set of objects by length requires repeated direct comparisons. Students come to understand that each object is longer than those that come before it and shorter than those that come after it when ordering from shortest to tallest.

Next, students compare objects via indirect measurement, or what is known as transitivity of measurement. They compare the lengths of two objects that cannot be directly compared by comparing them both to a third object.

The monkey is longer than the zookeeper's finger.

The frog is shorter than the zookeeper's finger.

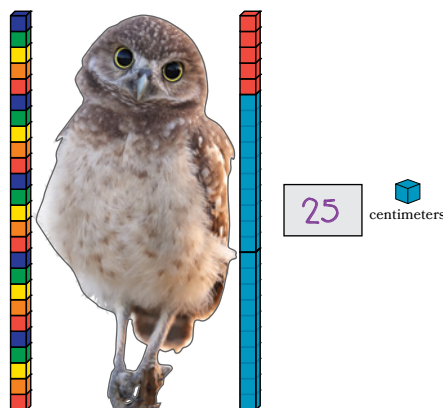
So, the monkey is longer than the frog.



Topic B

Length Measurement and Comparison

Students measure the height or length of objects by placing centimeter cubes and/or 10-centimeter sticks(s) alongside an object, end-to-end, without gaps or overlaps. They count the number of times the length unit, or centimeter cube, needs to be iterated and express the length and the number of length units needed. Students show how they measured the object by making drawings and labeling them. They consider ways to represent total lengths in terms of tens and ones. They also compare the numerical lengths and use a number sentence with the symbols $>$, $=$, or $<$ to record the relationship.



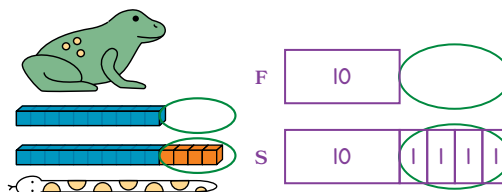
Topic C

Comparison Word Problems with Measurement

Topic C leverages length situations to work toward mastery of comparison word problems. The table shows examples of addition and subtraction situations¹. Darker shading in the table indicates the four kindergarten problem types. Students in grades 1 and 2 work with all problem types. Grade 2 students reach proficiency with the unshaded problem types. This module revisits a word problem type and introduces new problem types. See the following examples below.

- **Compare with difference unknown:** Two given lengths are compared to find how much longer or how much shorter one is than the other. (Lesson 10, 11)

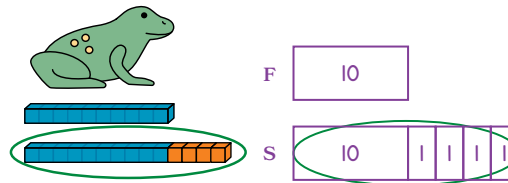
The frog is 10 centimeters long. The snake is 14 centimeters long. How much longer is the snake than the frog? How much shorter is the frog than the snake?



¹ Common Core Standards Writing Team, *Progressions for the Common Core (draft)*, Grades K–5, Counting and Cardinality & Operations and Algebraic Thinking, 9.

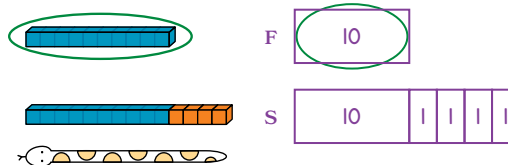
- **Compare with bigger quantity (or longer length) unknown:** The smaller length and the difference between the lengths are given to find the longer length. (Lesson 12)

The frog is 10 centimeters long. The snake is 4 centimeters longer than the frog. How long is the snake?



- **Compare with smaller quantity (or shorter length) unknown:** The longer length and the difference between the lengths are given to find the shorter length. (Lesson 13)

The snake is 14 centimeters long. The frog is 4 centimeters shorter than the snake. How long is the frog?



To solve *compare with difference unknown* problems, students measure pairs of objects and compare the actual lengths or drawings of the lengths. They see the difference, or how much longer (or shorter) one is than the other, in two primary ways:

- They identify the extra cubes in the longer length (the ones that do not match up).
- They think about the missing cubes in the shorter length that are needed to make them equal.

To solve *compare with longer or shorter unknown* problems, students first measure the object and represent the known length. Then to find the unknown length, they represent the known length again and add or take away the difference.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
Put Together / Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
Compare	Difference Unknown "How many more?" version. Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? $A + \square = C$ $C - A = \square$	Bigger Unknown "More" version suggests operation. Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? $A + B = \square$ "Fewer" version suggests wrong operation. Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Julie have? $A + B = \square$	Smaller Unknown "Fewer" version suggests operation. Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$ "More" version suggests wrong operation. Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

After This Module

Grade 1 Module 5

Students formally study the place value of two-digit numbers. They understand that 10 ones compose 1 ten and that the two digits represent amounts of tens and ones.

Grade 1 Module 6, part 2

Non-standard units, such as paper clips, are used to measure lengths. Students solve comparison word problems using tape diagrams.

Grade 2 Module 1, part 1

Students estimate and measure using both customary and metric units. They iterate units to create a 10-centimeter ruler and a meter stick. In doing so, they also connect metric measurement to place value units.

Measurement comparison word problems continue, albeit with larger lengths.

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Comparison and Composition of Length Measurements

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Direct and Indirect Length Comparison

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Compare and order objects by length.

Lesson 2	30
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Reason to order and compare heights.

Lesson 3	44
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Compare the lengths of two objects indirectly by using a third object.

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Length Measurement and Comparison

Lesson 4	60
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Measure accurately with centimeter cubes.

Lesson 5	70
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Measure and compare lengths.

Lesson 6	84
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Measure and order lengths.

Lesson 7	98
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Use 10-centimeter sticks and centimeter cubes to measure.

Lesson 8	110
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Draw to represent a length measurement.

Lesson 9	122
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Represent a total length as units of tens and ones.

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Comparison Word Problems with Measurement

Lesson 10	138
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Compare to find how much longer.

Lesson 11	152
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Compare to find how much shorter.

Lesson 12	166
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Find the unknown longer length.

Lesson 13	180
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Find the unknown shorter length.

Lesson 14 (Optional)	192
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Measure to find patterns.

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Why

Comparison and Composition of Length Measurements

Why do we use centimeter cubes to measure in grade 1?

In this module, students measure using a standard metric length unit: centimeters. They use both centimeter cubes and 10-centimeter sticks. This provides meaningful and concrete experiences for students to count by using tens and ones. This provides a foundation that will be further developed in the next module.

Young students may have difficulty measuring with a ruler because of how it differs from determining the number of objects in a group, such as a basket of apples. When counting to determine the number of objects in a set, each object has a clear beginning and ending. Quantity is a *discrete* attribute. In contrast, the length of an object extends from one endpoint of the object to the other without any breaks. It is a *continuous* attribute. To determine the length of an object, we partition the length into equal parts called length units. The number of length units that make up the length depends on the size of the unit chosen, and because length is continuous, it can always be subdivided into smaller and smaller length units. Thus, determining length by using a ruler, which is continuous in nature, causes two main issues:

- To count how many centimeters long an object is, you need to count the space between two hashmarks, recognizing that each hashmark represents the end of one centimeter and the start of the next one.
- Depending on the length unit used, a larger number doesn't necessarily imply a longer object. For example, it takes 2 paper clips or 10 centimeter cubes to measure the same crayon. This differs from what students are used to where a set with a greater number of objects has more.

Grade 1 helps students bridge this gap by having students measure by using centimeter cubes. Because length is determined by counting the (discrete) group of cubes that were used to measure, this tool allows students to treat length as if it were discrete. They will learn to work with rulers and number lines as models of continuous attributes in grade 2.

Students will also measure with non-standard length units, such as paper clips, in module 6 part 2. Emphasizing non-standard units too early may defeat the purpose it is intended to achieve by interfering with students' development of basic measurement concepts (such as no gaps, no overlaps), which are required to understand the need for standard units. At this time, students will also discover the importance of using standard, same-sized length units.

Terminology

The following terms are critical to the work of grade 1 module 4. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

length

Length is the space between the endpoints of an object; when we measure up and down, we sometimes call it height. (Lesson 4)

Technically, height is also a measurement of length. Colloquially, we distinguish between length and height depending on whether we are measuring “side to side” or “up and down.”

measure

To measure is to use a tool to find out how long or tall something is. (Lesson 4)

In grade 1, students only measure length. However, other attributes, such as weight or volume, can also be measured with different tools.

Familiar

compare

efficient

equal

false

fewer

greater

less

minus

number path

number sentence

one(s)

order

part

represent

subtract

take away

ten(s)

total

true

unit

unknown

Academic Verbs

Module 4 does not introduce any academic verbs from the grade 1 list.

Materials

The following list of materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

12	10-inch paper plates	24	<i>Learn</i> books
1	10-sided dice, set of 24	13	Markers
1	100-bead demonstration rekenrek	25	Paper, sheets
3	Base 10 rods, plastic, set of 50 (also called 10-centimeter sticks in the lessons)	24	Pencils
2	Centimeter cubes, set of 500	24	Personal whiteboards
1	Chart paper, pad	24	Personal whiteboard erasers
1	Computer with internet access	3	Plastic pattern blocks sets, 0.5 cm
1	Craft sticks, 6 color, package of 1,000	1	Projection device
13	Crayons	12	Resealable plastic bags
2	Dot dice, set of 12	24	Scissors
24	Dry-erase markers	1	Spool of string
1	<i>Eureka Math</i> ^{2™} Owl Posters, set of 7	13	Standard small paper clips, 3 cm size
13	Glue sticks	1	Sticky notes, pad
		1	<i>Teach</i> book

Visit <http://eurmath.link/materials> to learn more.

Before This Module

Kindergarten Module 6

Students begin to develop place value understanding when they come to see that teen numbers are composed of 10 ones and some more ones. They do not formalize the notion of “a ten” as a unit. Students also count to 100 by tens and by ones.

Grade 1 Module 3

Students rename groups of ten ones as units of ten. They come to see that all two-digit numbers are composed of tens and ones.

Grade 1 Module 4

Students use 10-centimeter sticks (tens) and centimeter cubes (ones) to measure lengths. They state total lengths in terms of tens and ones.

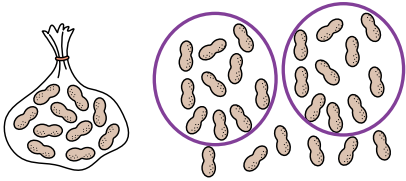
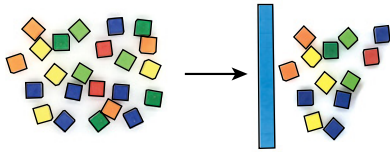
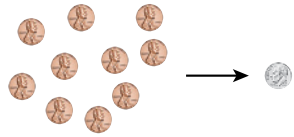
Overview

Place Value Concepts to Compare, Add, and Subtract

Topic A

Grouping Units in Tens and Ones

Topic A builds on work with tens and ones from modules 3 and 4. Lessons develop the idea that smaller units, such as ones, compose larger units, such as tens. The following place value models, which increase in complexity, help students to internalize the equivalence of 10 ones and 1 ten and to understand that two-digit numbers represent amounts of tens and ones.

Groupable	Pre-grouped	Nonproportional
Physically group 10 ones to compose a new unit of ten. The size of the new unit is proportionally larger than the base unit.  An illustration showing a bag of 10 brown beans on the left. To its right, two purple circles each contain 5 beans, with 5 more beans scattered below them, representing the process of grouping 10 individual ones into a new unit of ten.	Tens are composed by trading 10 ones for a new item that represents 1 ten. The new item is proportionally larger than the base unit.  An illustration showing a collection of 10 small, multi-colored cubes on the left. An arrow points to the right, where a single tall blue rectangular bar (representing a ten) is shown next to the remaining 5 cubes.	10 ones are traded for a new item that represents 1 ten. However, the new item is non-proportional. Visually, it is not 10 times larger than the base unit.  An illustration showing 10 small copper coins on the left. An arrow points to the right, where a single silver coin (representing a ten) is shown. The silver coin is visually much larger than the copper coins, illustrating a non-proportional trade.

Students represent two-digit totals in different ways. For example, a total of 21 may be shown as 21 ones, 1 ten 11 ones, or 2 tens 1 one. Students see that the digits used to write a numeral, such as 2 and 1 in 21, show how many tens and ones there are when the number is expressed in its “most composed” form.



Topic B

Use Place Value to Compare

Students compare two-digit numbers by using the place value structure of tens and ones rather than by using size or length. They represent comparisons with number sentences that include the symbols $>$, $=$, or $<$ and explain why the number sentences are true. Comparing numbers such as 39 and 93 helps students focus on the value of each digit. They see that if two numbers have different digits in the tens place, then they can use those digits to compare them efficiently.

tens	ones	
9	3	
tens	ones	
3	9	

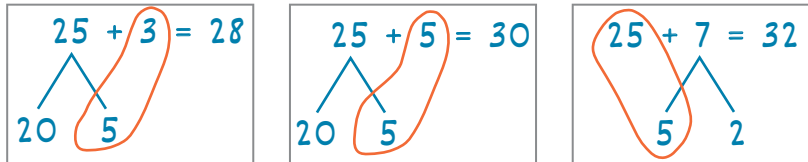
Topic C

Addition of One-Digit and Two-Digit Numbers

Students add a one-digit number to a two-digit number by using place value to make easier problems. They solve problems with the help of a variety of tools such as cubes, drawings, number bonds, and number paths, and they explain their thinking.

Students solve problems where the ones do not compose a new ten (e.g., $25 + 3 = 28$) by decomposing the two-digit addend into tens and ones, combining the ones with the

one-digit addend, and then adding the tens. They also find the total when the ones do compose a ten (e.g., $25 + 5 = 30$). Students decompose an addend to make the next ten, and then add the tens. Sometimes the ones compose a ten and some ones (e.g., $25 + 7 = 32$).



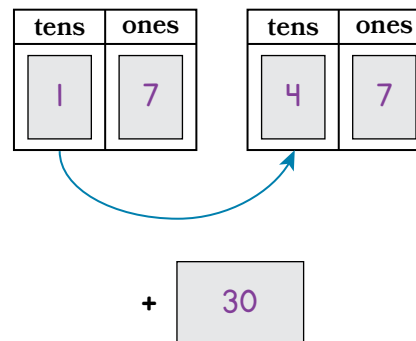
Topic D

Addition and Subtraction of Tens

Students learn three skills to complete grade 1 subtraction work and to prepare them for adding 2 two-digit numbers in topic E:

- Add tens to a multiple of ten.
- Add tens to any two-digit number.
- Subtract tens from a multiple of ten.

To add and subtract tens, students use the Level 2 strategies of counting on and counting back with tens. They advance to using the Level 3 strategy of representing an equation in unit form to find an easier one-digit fact they know. For example, rewriting $20 + 40$ as 2 tens + 4 tens helps students see that they can use $2 + 4$ to solve the problem. Similarly, rewriting $60 - 30$ as 6 tens - 3 tens helps students see that they can use $6 - 3$ to solve this problem. Then students add tens to a two-digit number by adding the tens first and recognizing that the ones-digit stays the same.



Topic E

Addition of Two-Digit Numbers

Students use place value understanding to make easier two-digit addition problems. They practice three strategies that involve decomposing one or both addends and composing the resulting parts. Students self-select strategies and tools for solving the problems based on the numbers in the problem and their preference. Students record their reasoning using a written method and they explain their thinking.

Add Like Units	Add Tens First	Add Ones First (May Make the Next Ten)
Decompose both addends into tens and ones, combine tens with tens and ones with ones. Then combine tens and ones. $\begin{array}{c} 15 + 15 = 30 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 10 \quad 5 \quad 10 \quad 5 \\ 20 + 10 = 30 \end{array}$	Decompose an addend into tens and ones, combine the tens with the other addend, and then add the ones. $\begin{array}{c} 15 + 15 = 30 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 10 \quad 5 \quad 10 \quad 5 \\ 25 + 5 = 30 \end{array}$	Decompose an addend to combine some (or all) of the ones with the other addend, in many cases to make the next ten. $\begin{array}{c} 15 + 15 = 30 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 10 \quad 5 \quad 5 \quad 10 \\ 20 + 10 = 30 \end{array}$

After This Module

Grade 1 Module 6 Part 2

Students count beyond 100 to 120 and begin to understand that 10 tens compose 100.

Students apply the same addition strategies they used in this module, but with addends where more than 1 or 2 new tens can be composed.

Grade 2 Module 1

Students extend their place value knowledge with larger numbers. Specifically, they learn that 10 tens make 1 hundred. They represent three-digit numbers in different ways by using place value understanding.

Grade 2 Module 2

Students continue to use place value to make easier problems when adding within 1,000. Strategies include add like units, count on with benchmark numbers, and make a ten or a hundred within 200. Students also extend place value thinking to make easier subtraction problems within 1,000.

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Place Value Concepts to Compare, Add, and Subtract

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Grouping Units in Tens and Ones

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Tell time to the hour and half hour by using digital and analog clocks.

Lesson 2 32
Count a collection and record the total in units of tens and ones.

Lesson 3 42
Recognize the place value of digits in a two-digit number.

Lesson 4 56
Represent a number in multiple ways by trading 10 ones for a ten.

Lesson 5 70
Reason about equivalent representations of a number.

Lesson 6 84
Add 10 or take 10 from a two-digit number.

Topic B 98

Use Place Value to Compare

Lesson 7 102
Use place value reasoning to compare two quantities.

Lesson 8 116
Use place value reasoning to write and compare 2 two-digit numbers.

Lesson 9 128
Compare two quantities and make them equal.

Topic C 140

Addition of One-Digit and Two-Digit Numbers

Lesson 10 144
Add the ones first.

Lesson 11 156
Add the ones to make the next ten.

Lesson 12 170
Decompose an addend to make the next ten.

Lesson 13 182
Reason about related problems that make the next ten.

Lesson 14 196
Determine which equations make the next ten.

Topic D 210

Addition and Subtraction of Tens

Lesson 15 214
Count on and back by tens to add and subtract.

Lesson 16 226
Use related single-digit facts to add and subtract multiples of ten.

Lesson 17 240
Use tens to find an unknown part.

Lesson 18 252
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Lesson 19 264
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Why

Place Value Concepts to Compare, Add, and Subtract

Why are there so many strategies and tools for adding large numbers?

One concept unifies the many ways to add two-digit numbers: make an equivalent yet easier problem. Break the addends into parts and combine the parts in ways that make sense. Exposing students to more than one way to make an easier problem increases number sense, encourages flexibility, and provides choice.

Presenting different strategies acknowledges that learners think about numbers in different ways. Also, the addends may lend themselves to certain strategies. For example, to solve $29 + 21$ efficiently, simply add the 1 from 21 to 29 to make 30. Then add 30 and 20. However, to solve $32 + 26$, it may be easier to think of 26 in terms of tens and ones, solve $32 + 20$, and add 6 more to make 58. Students who develop a toolbox of strategies will begin to select them intentionally.

Students also select different tools or models, such as manipulatives, drawings, number paths, number bonds, and number sentences. How they solve a problem is less important than an accurate solution, the written record of their thinking, and their explanation of their reasoning. Sharing such representations and engaging in discussion to analyze various ways to solve are critical features of instruction that deepen understanding of place value, operations, and quantity.

What skills do students need to add two-digit numbers by the end of the year?

Students need several skills to add 2 two-digit numbers. As they work through module 5 topic E and module 6 part 2, they continue to develop these embedded skills through practice. However, if students are still working toward proficiency with some of the first skills in the following chart, then they may benefit from directly modeling 2-digit addition problems using concrete or pictorial tools.

Add One-Digit Numbers	Break Up a Number into Tens and Ones	Identify the Next Ten	Know Partners to 10	Add a Two-Digit and a One-Digit Number	Add Tens to a Two-Digit Number
$8 + 5 = \boxed{13}$		$12 + \boxed{8} = 20$	$3 + 7 = \boxed{10}$	$14 + 9 = \boxed{23}$	$27 + 10 = \boxed{37}$

How do students deepen and advance their understanding of the equal sign?

Students evaluate number sentences such as $90 - 40 = 20 + 30$. They determine whether the number sentence is true or false by calculating the value of the expressions on either side of the equal sign. This provides practice with adding and subtracting tens while giving students opportunities to engage with uncommon and complex equation types.

Students also evaluate number sentences with equivalent expressions that represent different ways to make an easier problem. For example, the expression on either side of the equals sign in $10 + 10 + 2 + 6 = 12 + 10 + 6$ shows one way to break up the addends in $12 + 16$. Students find the expressions have the same value and conclude that the number sentence is true. This helps confirm that there are several valid ways to solve a problem.

Why is telling time included in a module on place value?

Module 5 centers on the idea that smaller units such as ones compose larger unit such as tens. Time also involves composing units. For example, minutes make up hours and half hours, and hours make up days. Work with units in various contexts (e.g., time, measurement, and shapes) develops reasoning that helps students understand numerical units such as ones, tens, and hundreds. Students revisit telling time to the half hour in module 6.

Terminology

The following terms are critical to the work of grade 1 module 5. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

digit

Numbers like 7 and 5 are called digits. When we write digits next to each other, we make another number. For example, we write the digits 7 and 5 next to each other to make 75. (Lesson 2)

compose

To compose means to be put together, or group. (Lesson 3)

place

A digit's place is its position in a number. Numbers with two digits have two places: the tens place and the ones place. (Lesson 3)

value

Value is how much something is worth. For example, in the number 53, the 5 is in the tens place, so it has a value of 50. (Lesson 3)

Familiar

addend

compare

efficient

equal

equation

expression

false

fewer

greater

less

minus

number sentence

one(s)

part

partners

represent

subtract

take away

ten(s)

total

true

unit

unknown

Academic Verbs

Module 5 does not introduce any academic verbs from the grade 1 list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	10-sided dice, set of 24	1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, student extension set of 12
1	100-bead demonstration rekenrek		
3	Base 10 rods, plastic, set of 50	1	<i>Eureka Math</i> ^{2™} Numeral Cards, set of 12 decks
1	Centimeter cubes, set of 500	24	<i>Learn</i> books
18	Chart paper sheets	25	Markers
1	Computer with internet access	24	Pencils
24	Counters	650	Pennies
12	Counting collections	24	Personal whiteboards
24	Crayons (1 red crayon, 1 green crayon, 1 blue crayon per student)	24	Personal whiteboard erasers
78	Dimes	1	Projection device
1	Dot dice, set of 12	12	Scissors
24	Dry-erase markers	1	<i>Teach</i> book
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, basic student set of 12	1	Unifix [®] Cubes, set of 1,000
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, demonstration set	12	Work mats

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 2 for a list of organizational tools (cups, plates, number paths, etc.) suggested for the counting collection.

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Before This Module

Grade K Module 2

Students identify, name, and describe squares, rectangles, and triangles by using defining attributes such as the number of closed, straight sides and the number of corners. They are exposed to trapezoids and rhombuses.

They also describe the faces of solid shapes and discuss functionality. However, triangular prisms are new to grade 1.

In kindergarten, students construct solid and flat shapes by using sticks, and they draw flat shapes by using a straightedge and dot paper.

Students compose two-dimensional and three-dimensional shapes and identify the smaller shapes used in the compositions.

Overview

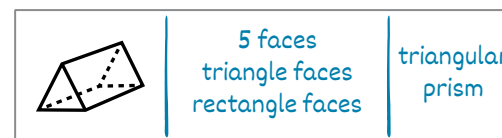
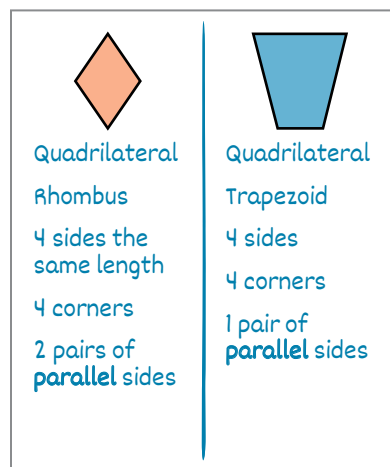
Part 1: Attributes of Shapes

Topic A

Attributes of Shapes

Students begin their study of geometry by describing and naming two-dimensional flat shapes by using defining attributes. At first, they count the number of sides on shapes to sort them into these categories: triangles, quadrilaterals, pentagons, and hexagons. They see that two shapes with the same number of sides that look different can still be given the same name. Students draw, name, and describe shapes (triangles, rectangles, squares, rhombuses, trapezoids, and hexagons) by using defining attributes such as square corners, parallel sides, and side lengths.

Students describe and name three-dimensional solid shapes, including cubes, cones, cylinders, rectangular prisms, triangular prisms, and pyramids. They look at the faces they see in unfolded nets to describe the solid shapes the nets will become. They also notice how a solid shape's attributes (its faces, edges, and corners) impact its functionality (like whether it rolls or stacks).



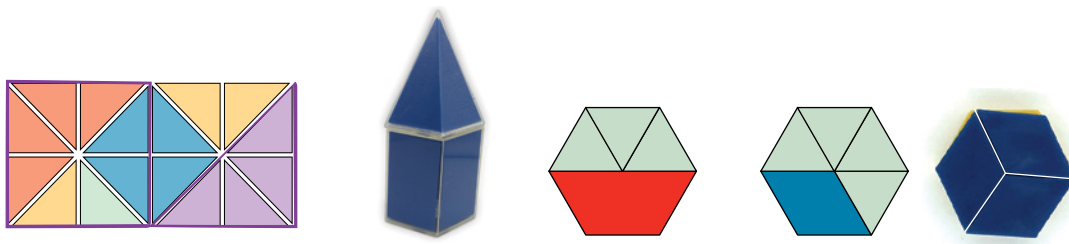
Topic B

Composition of Shapes

Students decompose and compose flat and solid composite shapes in increasingly complex ways:

- They identify shapes within a composite shape.
- They name composite shapes using defining attributes.
- They create composite shapes by combining shapes.

Geometric composition is an important concept because it deepens understanding of part-whole relationships in other areas, such as composing 10 ones to make 1 ten, decomposing 8 into 2 and 6, partitioning a whole into halves, or recognizing that a clock is partitioned into hours and minutes. After students compose a shape in a variety of ways, they compare the number of shapes they used. They realize that the smaller the shapes they use to make a composed shape, the more shapes they need to make the composition.



Topic C

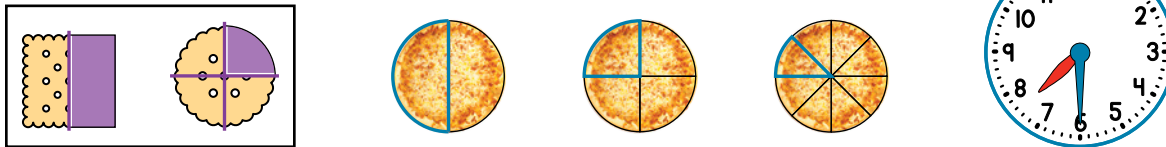
Halves and Fourths

In topic C, students continue to compose and decompose shapes and relate their work to basic fractions. They partition shapes in a variety of ways and determine if the parts are equal shares of the whole. They conclude that equal parts (or shares) are the same shape and size.

Then students partition circles and rectangles into 2 and 4 equal parts and name the shares as halves, fourths, and quarters. They identify 1 share as 1 half or 1 fourth of the whole and verbalize how many equal shares compose the whole (2 shares or 4 shares).

Students also determine that the more shares you partition a shape into, the smaller the shares get.

Finally, students connect their understanding of halves to telling time. They reason about the phrase *half past* and relate it to a half-circle. They recognize that half past can also be stated as a time, such as 2:30, because the minute hand has gone halfway around the clock. In optional lesson 15, students also analyze the movement and location of the hour hand, noting that it will be halfway between the 2 and the 3 at 2:30.



After This Module

Grade 2 Module 3

Students begin using the number of angles in a shape as a defining attribute of flat shapes. They use the number of faces, edges, and vertices as defining attributes of solid shapes.

Fraction work expands from working with halves and fourths to include thirds. Students refine their understanding of equal shares as they see that equal shares are the same size, but not always the same shape.

Grade 2 students relate fractions to telling time by using the language *quarter past* and *quarter to*.

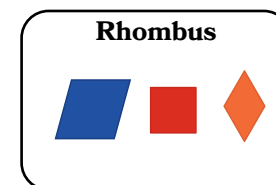
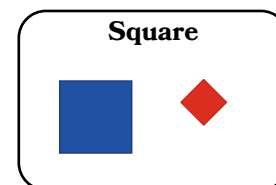
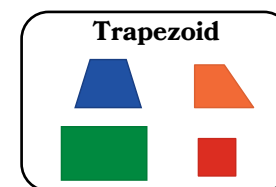
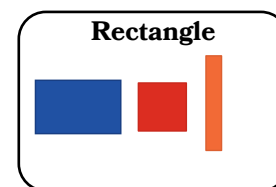
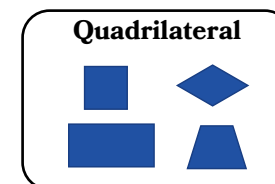
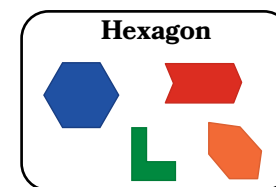
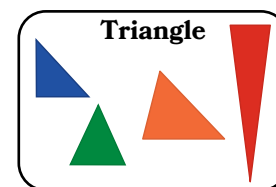
Why

Part 1: Attributes of Shapes

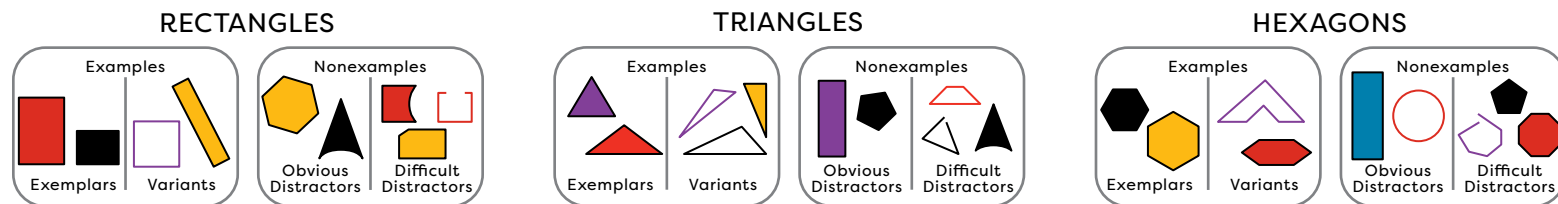
How do attributes of shapes help students name and describe them?

Grade 1 students expand their knowledge of defining attributes, or the mathematical characteristics of a shape, to describe flat shapes with increasing precision. They use attributes, such as the number of straight sides and whether the shape has equal-length sides, parallel sides, or square corners, to sort a variety of shapes into different categories. They find that the fewer attributes a shape category has, the more shapes that fit into that category. In contrast, the more attributes a category has, the fewer shapes that fit into that category. Students see that the same shape can have more than one name or fit into more than one category, depending on the attributes they are considering. This concept connects to students' experience of naming and representing numbers in various ways.

- **Triangle**—A triangle is any closed shape with 3 straight sides. In later grades, students use attributes, such as angles, to name specific triangles. Grade 1 students see a variety of triangles and may notice specific attributes, but the name *triangle* stays consistent.
- **Hexagon**—A hexagon is any closed shape with 6 straight sides. Students identify a variety of hexagons in grade 1, not just regular hexagons with 6 equal side lengths and 3 pairs of parallel sides.
- **Quadrilateral**—A quadrilateral is any closed shape with 4 straight sides. The following shapes can be called quadrilaterals: rectangle, square, trapezoid, and rhombus. Grade 1 students do not work with parallelograms or kites.
- **Trapezoid**—A trapezoid is any closed shape with 4 straight sides and at least 2 parallel sides. A square and a rectangle may be called a trapezoid.
- **Rectangle**—A rectangle is any closed shape with 4 straight sides and 4 right angles. A square can also be called a rectangle.
- **Rhombus**—A rhombus is any closed shape with 4 straight sides of equal length. A square can also be called a rhombus.
- **Square**—A square has 4 right angles and 4 straight sides of equal length. No other shapes can be called a square.



Students describe and name both exemplar and variant shapes. Exemplars are the shapes students typically visualize. They are often symmetrical with a horizontal base. Variants are less frequently encountered in children's books and media. To name these shapes, students must pay close attention to their defining attributes. When asked to sort shapes or to identify a specific shape within a set, students are shown some nonexamples, or distractors. Obvious distractors do not look much like the exemplars and are easier for students to categorize as nonexamples on sight. Difficult distractors look more like the exemplars, but they do not have all the defining attributes of the target shape. They require students to attend to defining attributes with more precision.



Terminology

The following terms are critical to the work of grade 1 module 6 part 1. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

composed shape

A composed shape is a shape that is made up of other shapes. For example, a rhombus made with two green triangle blocks is a composed shape. (Lesson 6)

These are usually known as composite shapes. Since students have already been introduced to the word compose, they are called composed shapes in this lesson for ease of language.

fourth/fourths/fourth of

A fourth is one of 4 equal parts.

A shape that is partitioned into 4 equal parts is partitioned into fourths.

One of 4 equal parts of a shape is 1 fourth of that shape. (Lesson 11)

half/halves/half of

A half is one of 2 equal parts.

A shape that is partitioned into 2 equal parts is partitioned into halves.

One of 2 equal parts of a shape is 1 half of that shape. (Lesson 11)

half past

The phrase *half past* is used to tell the time when the minute hand is halfway around the clock. Half past two is the same time as 2:30. (Lesson 14)

parallel

Two sides that are across from each other and never touch, even when we imagine them going far out past the shape, are parallel. (Lesson 2)

partition

When we cut or break something into parts, we partition it. For example, we can partition a shape into halves. (Lesson 10)

Note that partitioning a shape does not mean the shape is in equal parts. A shape can be partitioned into equal parts or unequal parts.

quarter/quarter of

Quarter is another word for fourth. *Quarter of* and *fourth of* mean the same thing. (Lesson 12)

rhombus

A rhombus is a closed shape with 4 straight sides that are all the same length. (Lesson 2)

square corner

If a square fits perfectly in a shape's corner, then that corner is called a square corner. (Lesson 2)

trapezoid

A trapezoid is a closed shape with 4 straight sides. At least 2 of the sides are parallel. (Lesson 2)

In grade 3, students learn that a trapezoid has at least 1 pair of parallel sides, but it can also have 2 pairs of parallel sides. For example, students learn to recognize that a parallelogram is a kind of trapezoid. Grade 1 students are not expected to recognize this, and the examples of trapezoids they are given all have exactly 1 pair of parallel sides.

Familiar

circle

compose

cone

cube

cylinder

equal

flat shape

hexagon

hour hand

length

minute hand

part

pyramid

rectangle

rectangular prism

solid shape

sphere

square

triangle

whole

Academic Verb

sketch

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	100-bead demonstration rekenrek	24	Glue
2	Balls	24	<i>Learn</i> books
1	Centimeter cubes, set of 500	24	Pencils
1	Chart paper, tablet	24	Personal whiteboards
1	Computer with internet access	24	Personal whiteboard erasers
1	Craft sticks, 6 color, package of 1,000	3	Plastic pattern blocks sets, 0.5 cm
24	Crayons	1	Projection device
1	Demonstration clock	24	Scissors
24	Dry-erase markers	207	Sticky notes
12	<i>Eureka Math</i> ^{2™} Numeral Cards decks	1	<i>Teach</i> book
2	Geometric solids with nets, set of 12		

Visit <http://eurmath.link/materials> to learn more.

Before This Module

Grade K Module 6

Students count by ones and tens to tell how many within 100. They also write numbers from 0 to 20.

Grade 1 Modules 2, 3, 4

Students solve word problems by using the Read-Draw-Write process. They primarily use 5-group drawings. Their number sentences reflect how they solved the problem by using the drawing.

Grade 1 Module 5

Students use base-ten place value structure to read, write, and represent two-digit numbers. Students learn to think of 10 ones as 1 ten.

Students use Level 3 strategies (make easier problems) to add tens, one-digit and two-digit numbers, and 2 two-digit numbers within 50.

Overview

Part 2: Advancing Place Value, Addition, and Subtraction

Topic D

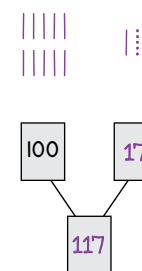
Count and Represent Numbers Beyond 100

Students work with three-digit numbers from 100 to 120 in the following ways: counting, reading numerals, writing totals, and representing numbers.

Students count 100 or more objects by making groups of tens and ones. They represent the same total by using different combinations of tens and ones and reason about why it is possible to compose numbers in different ways.

To write the number that represents the total, students relate the way they say the number to its written form. For example, *one hundred seventeen* is written as 117. Students recognize that the representation showing a group of 100, a group of 10, and 7 ones matches the numeral. They also read and write numbers greater than 100 by using patterns when numbers are presented in horizontal and vertical sequences.

91	101	111
92	102	112
93	103	113
94	104	114
95	105	115
96	106	116
97	107	117
98	108	118
99	109	119
100	110	120



Topic E

Deepening Problem Solving

In topic E, students represent and solve all of the K–2 problem types. They make sense of word problems by drawing and labeling a tape diagram. Drawing and analyzing the resulting tape diagram clarifies the relationships in the problem. This helps students identify the meaning of the unknown and write an addition or subtraction equation to solve.

Topic E also includes nonroutine problems with many different solution paths.

Topic F

Extending Addition to 100

Students add 2 two-digit numbers that have sums within 100. They use models such as ten-sticks and cubes, drawings, and number bonds. They make easier problems by decomposing one or both addends and combining the remaining parts. Students recognize that when composing the parts, it is sometimes necessary to compose a ten. They also recognize that when they have 10 tens, the total is 100. Students show how they made an easier problem by using a written method and through explaining their work.

Making Easier Two-Digit Addition Problems		
$36 + 49$ $30 + 40 = 70$ $6 + 9 = 15$ $70 + 15 = 85$	$36 + 49$ $36 + 40 = 76$ $76 + 9 = 85$	$36 + 49$ $49 + 1 = 50$ $50 + 35 = 85$
• Add like units (tens with tens and ones with ones). • Add tens first. • Make the next ten.		

After This Module

Grade 2 Module 1 Part 2

Students use a variety of tools to count by ones, tens, and hundreds within 1,000.

Students use place value to make use of benchmark numbers, such as 20 or 200, to add, subtract, and compare.

Grade 2 Module 2

Students use Level 3 strategies to add numbers up to three digits within 200 and relate them to a written recording.

Students use the Read–Draw–Write process and tape diagrams to solve two-step word problems.

Why

Part 2: Advancing Place Value, Addition, and Subtraction

Which new word problem types, or addition and subtraction situations, are used in this module?

The table shows examples of addition and subtraction situations.¹ Darker shading in the table indicates the four kindergarten problem types. Students in grades 1 and 2 work with all problem types. Grade 2 students reach proficiency with the unshaded problem types. Students work with all the problem types in this module. Listed below are the problem types students are introduced to for the first time in grade 1.

- **Add to with start unknown:** The part that represents the *add to* action and the total are given. The unknown is the starting part.

Kit has some dollars. She gets 7 more dollars for helping her mom. Now she has 15 dollars. How many dollars did Kit have to start? (Lesson 22)

- **Take from with start unknown:** The part that represents the *take from* action and the part remaining are given. The unknown is the total at the start.

Baz has some dollars. He spends 10 dollars. He still has 2 dollars left. How many dollars did Baz have to start? (Lesson 22)

- **Compare with longer length unknown (*shorter* suggests wrong operation):** The shorter length and the difference are known, but the longer length is unknown. The word *shorter* incorrectly suggests subtracting the difference from the smaller length.

Imani's shoe is 11 paper clips long. Imani's shoe is 6 paper clips shorter than Kioko's shoe. How many paper clips long is Kioko's shoe? (Lesson 24)

- **Compare with shorter length unknown (*longer* suggests wrong operation):** The longer length and the difference are known, but the shorter length is unknown. The word *longer* incorrectly suggests adding the difference to the longer quantity.

Kioko's book is 13 paper clips long. Kioko's book is 4 paper clips longer than Imani's book. How many paper clips long is Imani's book? (Lesson 24)

Table 2: Addition and subtraction situations by grade level.			
	Result Unknown		Start Unknown
	Change Unknown	Change Unknown	Change Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
Put Together /Take Apart	Total Unknown	Both Addends Unknown ¹	Addend Unknown ²
	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $\square + \square = C$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
Compare	Difference Unknown	Bigger Unknown	Smaller Unknown
	"How many more?" version. Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? $A + \square = C$ $C - A = \square$	"More" version suggests operation. Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? $A + B = \square$	"Fewer" version suggests operation. Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

¹ These word problem types come from the document *Grades K-5, Counting and Cardinality & Operations and Algebraic Thinking*, one of the *Progressions for the Common Core State Standards in Mathematics*. An explanation and example of some types are included here. See the table for examples. Darker shading indicates the four kindergarten problem subtypes. Grade 1 and grade 2 students work with all subtypes and variants. Students master the types that are not shaded in grade 2.

How are the grade 1 problem types represented by using tape diagrams?

Result Unknown		Change Unknown		Start Unknown	
Add to	Val gets 8 points. Then she gets 7 points. How many points does she have? $8 + 7 = ?$				
		Jade has 12 tickets. She gets some more tickets. Now she has 20 tickets. How many tickets did she get? $12 + ? = 20$ or $20 - 12 = ?$		Kit has some dollars. She gets 7 more dollars. Now she has 15 dollars. How many dollars did she start with? $? + 7 = 15$ or $15 - 7 = ?$	
Take From	Baz has 20 points. He uses 19 points. How many points does he have left? $19 + ? = 20$ or $20 - 19 = ?$				
		Zan has 15 points. He uses some points. Now he has 8 points left. How many points did he use? $? + 8 = 15$ or $15 - 8 = ?$		Baz has some dollars. He spends 10 dollars. He has 2 dollars left. How many dollars did he have at first? $? - 10 = 2$ or $10 + 2 = ?$	
Total Unknown		Addends Unknown		Addend Unknown	
Put Together/ Take Apart	Max has 12 tickets. Kat has 8 tickets. How many tickets do they have in all? $12 + 8 = ?$				
		Val hit 3 balloons. She got 19 points. What points were on the balloons? $19 = ? + ? + ?$		Tam has 9 points. Baz has some points. They have 15 points in all. How many points does Baz have? $9 + ? = 15$ or $15 - 9 = ?$	
Difference Unknown		Bigger/Longer Unknown		Smaller/Shorter Unknown	
Compare	Tam plays for 12 minutes. Max plays for 4 minutes. How many more minutes does Tam play than Max plays? $4 + ? = 12$ or $12 - ? = 4$	Jon plays for 8 minutes. Deb plays for 2 more minutes than Jon plays. How long does Deb play? $8 + 2 = ?$		Kit plays for 10 minutes. Ben plays for 4 fewer minutes than Kit plays. How long does Ben play? $10 - 4 = ?$ or $? + 4 = 10$	
	Math takes 12 minutes. Art takes 4 minutes. How many fewer minutes does art take than math takes? $4 + ? = 12$ or $12 - ? = 4$	(shorter suggests wrong operation) Val's hand is 6 cubes long. Val's hand is 2 cubes shorter than Mom's hand. How long is Mom's hand? $6 + 2 = ?$		(longer suggests wrong operation) Val's hand is 6 cubes long. Val's hand is 2 cubes longer than her friend's hand. How long is her friend's hand? $6 - 2 = ?$	

Terminology

The following terms are critical to the work of grade 1 module 6 part 2. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

Module 6 part 2 does not introduce any new terminology.

Familiar

addend	one(s)
compare	part
efficient	partners
equal	represent
equation	subtract
expression	take away
false	ten(s)
fewer	total
greater	true
less	unit
minus	unknown

Academic Verbs

Module 6 part 2 does not introduce any academic verbs from the grade 1 list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	100-bead demonstration rekenrek	12	Index cards
1	10-sided dice, set of 24	24	<i>Learn</i> books
3	Base 10 rods, plastic, sets of 50	13	Markers
2	Centimeter cubes, sets of 500	1	Paper bag or opaque box
1	Chart paper, tablet	3	Paper clips, boxes of 100
1	Computer with internet access	24	Pencils
1	Craft sticks, 6 color, package of 1,000	24	Personal whiteboards
120	Dimes	24	Personal whiteboard erasers
1	Dot dice, set of 12	1	Projection device
24	Dry-erase markers	1	String, spool
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, demonstration set	1	<i>Teach</i> book
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, basic student set of 12	1	Teddy bear counters, set of 96
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, student extension set of 12	1	Two-color counters, set of 200 pieces
12	<i>Eureka Math</i> ^{2™} Numeral Cards decks	1	Unifix [®] Cubes, set of 1,000

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 16 for a list of organizational tools (cups, plates, number paths, etc.) suggested for the counting collections.