

Before This Module

Grade 2

In grade 2, students complete foundational work for multiplication and division. They form equal groups, write repeated addition sentences, arrange objects in rows and columns to form arrays up to 5 by 5, and discover how adding the number of objects in the rows or columns relates to repeated addition. At first, arrays are made with gaps between rows and columns and, later, are made with no gaps. Students build and manipulate arrays and use part-total language to express the composition and decomposition.

Grade 3 module 1 elevates the work of grade 2 through formal introduction of multiplication and division.

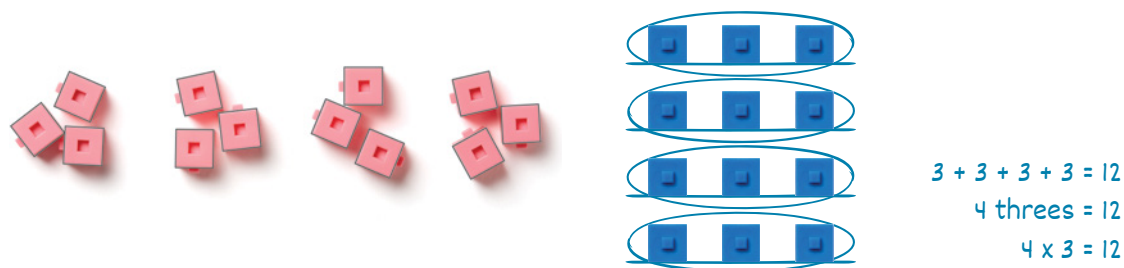
Overview

Multiplication and Division with Units of 2, 3, 4, 5, and 10

Topic A

Conceptual Understanding of Multiplication

Students connect their understanding of equal groups and repeated addition to multiplication. They identify the number of groups, the number in each group, and the total within equal-groups models and arrays. They write multiplication equations to represent the equal groups and arrays. Students interpret the meaning of the factors as the number of groups and the number in each group and solve word problems.



Topic B

Conceptual Understanding of Division

Students use equal-groups models and arrays to explore the two interpretations of division: measurement and partitive. They determine the total and either the number of groups or the number in each group based on the problem situation. They identify what is known and what is unknown, relate it to an unknown factor problem, and write a division equation. Students solve word problems involving division and make connections between multiplication and division.

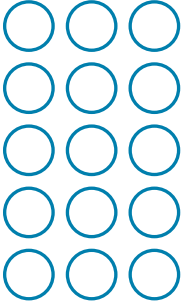


$$\square \times 3 = 15$$

$$\begin{array}{rcl} 15 & \div & 3 \\ \text{total} & & \text{number in} \\ & & \text{each group} \end{array} = \square$$

$$5 \times \square = 15$$

$$\begin{array}{rcl} 15 & \div & 5 \\ \text{total} & & \text{number of groups} \end{array} = \square$$



$$5 \times \square = 15$$

$$\begin{array}{rcl} 15 & \div & 5 \\ \text{total} & & \text{number of rows} \end{array} = \square$$

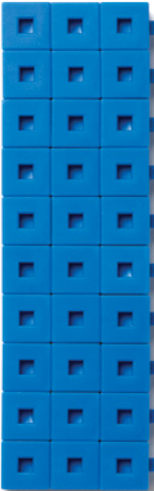
$$\square \times 3 = 15$$

$$\begin{array}{rcl} 15 & \div & 3 \\ \text{total} & & \text{number in each row} \end{array} = \square$$

Topic C

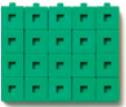
Properties of Multiplication

Students use the properties of multiplication to explore strategies they can use to multiply efficiently. They explore the commutative property of multiplication by skip-counting the rows and columns in arrays, which helps them build number sense while learning multiplication facts. Students use arrays and number bonds to model the distributive property when finding the products of unfamiliar facts.



3
6
9
12
15
18
21
24
27
30

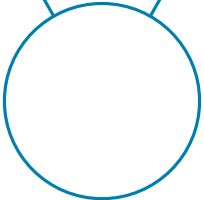
10, 20, 30

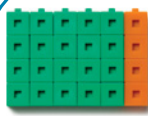


5 fours



1 four

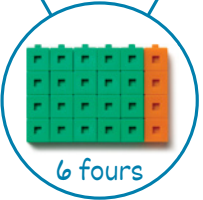


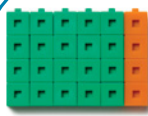


5 fours



1 four



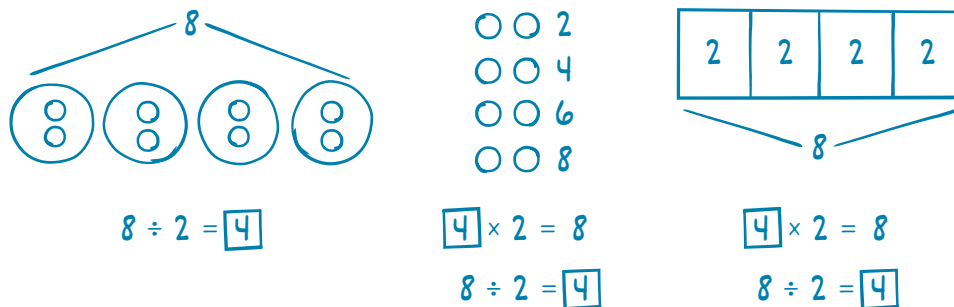


6 fours

Topic D

Two Interpretations of Division

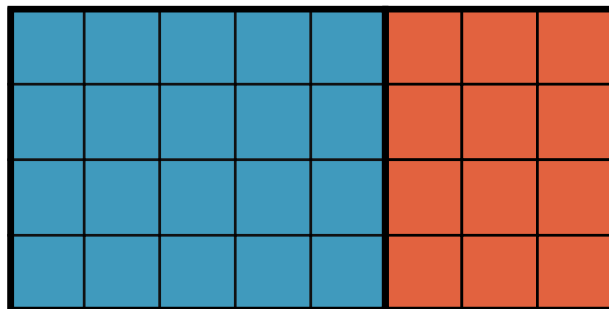
Students solidify their understanding of the relationship between multiplication and division and express division as both unknown factor problems and division equations. Students describe the quotient as either the number of groups or the size of each group and draw tape diagrams to represent the problems.



Topic E

Application of Multiplication and Division Concepts

Students apply the distributive property to complete multiplication and division problems and explore the foundations of the associative property of multiplication by breaking apart arrays into smaller arrays. Students solve two-step word problems using multiplication and division.



$$32 \div 4 = 5 + 3 = 8$$

$\wedge$
 20 12

After This Module

Grade 3 Module 3

In grade 3 module 3, students apply conceptual understanding and use the commutative, distributive, and associative properties to extend their learning of multiplication and division to units of 6, 7, 8, 9, 0, 1, and two-digit multiples of 10. Students solve one- and two-step word problems involving the four operations.

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Conceptual Understanding of Multiplication

Lesson 1 16
Organize, count, and represent a collection of objects.

Lesson 2 32
Interpret equal groups as multiplication.

Lesson 3 46
Relate multiplication to the array model.

Lesson 4 60
Interpret the meaning of factors as number of groups or number in each group.

Lesson 5 76
Represent and solve multiplication word problems by using drawings and equations.

Topic B 91

Conceptual Understanding of Division

Lesson 6 94
Explore measurement and partitive division by modeling concretely and drawing.

Lesson 7 110

Model measurement and partitive division by drawing equal groups.

Lesson 8 128

Model measurement and partitive division by drawing arrays.

Lesson 9 148

Represent and solve division word problems using drawings and equations.

Topic C 163

Properties of Multiplication

Lesson 10 166

Demonstrate the commutative property of multiplication using a unit of 2 and the array model.

Lesson 11 178

Demonstrate the commutative property of multiplication using a unit of 4 and the array model.

Lesson 12 190

Demonstrate the distributive property using a unit of 4.

Lesson 13 204

Demonstrate the commutative property of multiplication using a unit of 3 and the array model.

Lesson 14 216

Demonstrate the distributive property using units of 2, 3, 4, 5, and 10.

Topic D 233**Two Interpretations of Division****Lesson 15** 236

Model division as an unknown factor problem.

Lesson 16 254

Model the quotient as the number of groups using units of 2, 3, 4, 5, and 10.

Lesson 17 268

Model the quotient as the size of each group using units of 2, 3, 4, 5, and 10.

Lesson 18 284

Represent and solve measurement and partitive division word problems.

Topic E 301**Application of Multiplication and Division Concepts****Lesson 19** 304

Use the distributive property to break apart multiplication problems into known facts.

Lesson 20 318

Use the distributive property to break apart division problems into known facts.

Lesson 21 332

Compose and decompose arrays to create expressions with three factors.

Lesson 22 346

Represent and solve two-step word problems using the properties of multiplication.

Lesson 23 360

Represent and solve two-step word problems using drawings and equations.

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Why

Multiplication and Division with Units of 2, 3, 4, 5, and 10

Why are multiplication and division concepts taught in module 1 and module 3?

Understanding and applying multiplication and division concepts are part of the major work of grade 3. Beginning the year with multiplication and division concepts with units of 2, 3, 4, 5, and 10 maximizes time for students to develop conceptual understanding and build fluency. In module 2, students continue daily practice with multiplication and division through fluency activities. Students return to multiplication and division in module 3. They build upon the strong foundation established in module 1, now extending to more complex strategies with units of 6, 7, 8, 9, 0, and 1.

Multiplication concepts are the basis of much of the other work in grade 3 (e.g., area of plane figures, building fractions from unit fractions, and scaled bar and picture graphs). Beginning the year with multiplication allows for rich connections and enables multiplication to be the lens through which other concepts are explored.

How does the learning progress from module 1 to module 3?

Students' familiarity with skip-counting by twos, fives, and tens from earlier grade levels provides a natural starting point to establish the concepts of multiplication and division in module 1.

Early representations for multiplication and division in module 1 include equal groups and arrays, which are used to skip-count. Students learn the meaning of multiplication and division with familiar units of 5 and 10 and the smaller units of 2, 3, and 4, which allows for a smoother transition to more abstract representations, such as tape diagrams, before working with larger units.

Seeing the relationship between 10 and 5—that is, 10 as 5 doubled—provides a foundation for students’ understanding of 4 as 2 doubled. In module 3, that understanding extends to 6 as 3 doubled and 8 as 4 doubled. The units of 6, 7, 8, and 9 are introduced in module 3 after students have developed some proficiency with smaller units and strategies based on the commutative and distributive properties. These strategies allow students to multiply and divide with larger units by creating easier problems using smaller and familiar units.

After students have developed some proficiency with the other single-digit factors, they can explore and understand the factors of 0 and 1 through patterns. Students learn why multiplication and division with 0 and 1 are unique and develop the meaning behind them to support understanding of the identity property in later grades.

Why do lessons focus on certain representations and tools for multiplication and division? Is it acceptable for students to use other representations and tools when they are not included in a lesson?

Most lessons include multiple representations and tools to facilitate access for all students. Allow students to demonstrate their understanding using representations and tools that make sense to them, even if those representations or tools are not a focus in the lesson.

Terminology

The following terms are critical to the work of grade 3 module 1. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

commutative property of multiplication

Changing the order of the factors in a multiplication expression does not change its product. For example, if $7 \times 3 = 21$ is known then $3 \times 7 = 21$ is also known because $7 \times 3 = 3 \times 7$. (Lesson 10)

division, divide, divided by, $\div$

To partition a total into groups of a specific size or into a specific number of equal groups. For example, 15 divided by 3 (written $15 \div 3$) can mean either of the following: 15 put into groups of 3 or 15 put into 3 equal groups. (Lessons 7 and 8)

factor

The numbers used in a multiplication expression. For example, in 3×4 , the numbers 3 and 4 are the factors. (Lesson 4)

multiplication, multiply, $\times$

The multiplication expression 3×5 means 3 groups of 5. To multiply two whole numbers is to find the total, or product, for a multiplication expression. (Lesson 2)

parentheses, ()

Symbols used for grouping and ordering parts of an expression. For example, in $3 \times (5 + 1)$, the $5 + 1$ is grouped between the parentheses. The parentheses make it clear that the 3 is being multiplied by $5 + 1$, not just by 5. (Lesson 12)

In grade 3, students gain the general understanding that operations inside parentheses are done before operations outside parentheses. Fluency with parentheses begins in grade 5 and is made formal in grade 6.

product

The total when one number is multiplied by another. For example, in $3 \times 4 = 12$, the number 12 is the product. (Lesson 3)

quotient

The number resulting from the division of two numbers. For example, in $28 \div 4 = 7$, the number 7 is the quotient. (Lesson 15)

rotate

To turn, used in reference to turning arrays 90 degrees. When an array is rotated, the rows become columns and the columns become rows. (Lesson 10)

The degree measure of an angle is introduced to students in grade 4.

size of group

The number in the group. For example, the multiplication equation $3 \times 4 = 12$ represents 3 groups of 4; so, the size of each group is 4. (Lesson 12)

Familiar

array

column

equal groups

equal shares

equal sharing

equation

estimate (noun)

estimate (verb)

expression

number in (or size of) each group

number of groups

repeated addition

row

skip-count

unit

unit form

unknown

Academic Verbs

Module 1 does not introduce any academic verbs from the grade 3 list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	100-bead demonstration rekenrek	60	Paper plates, small
4	Color tiles, plastic, set of 400	25	Pencils
25	Colored pencils, set of 8	25	Personal whiteboards
1	Computer with internet access	25	Personal whiteboard erasers
240	Crackers, round	1	Projection device
25	Dry-erase markers	25	Scissors
14	Envelopes	144	Sticky notes
1,300	Interlocking cubes, 1 cm	1	<i>Teach</i> book
24	<i>Learn</i> books	25	Wood rulers, inch and metric

Visit <http://eurmath.link/EurekaMaterials> to learn more.

Please see lesson 1 for a list of organizational tools (cups, rubber bands, graph paper, etc.) suggested for the counting collection.

Before This Module

Grade 2 Modules 1 and 2

In grade 2, students describe and apply place value concepts to two- and three-digit numbers. They count and bundle ones, tens, and hundreds up to 1,000. Students read and write numbers in standard, unit, and expanded forms and apply place value understanding to add and subtract two- and three-digit numbers by using a variety of strategies. Simplifying strategies consist of composing and decomposing tens and hundreds to make problems easier to compute mentally and developing various written methods to record student thinking.

Students also estimate and measure length by using a variety of tools and units in the customary and metric systems of measurement.

Grade 3 uses familiar place value concepts to expand student understanding of metric measurement of weight and liquid volume and to develop fluency in addition and subtraction within 1,000.

Overview

Place Value Concepts Through Metric Measurement

Topic A

Understanding Place Value Concepts Through Metric Measurement

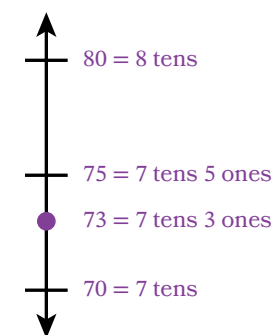
Students estimate and measure weight and liquid volume by using grams, kilograms, liters, and milliliters. Familiar concepts, such as composing and decomposing place value units, are applied to the relationship between the smaller unit (gram or milliliter) and the unit that represents 1,000 (kilogram or liter). The number line is applied as a familiar model for understanding how to read circular and vertical measurement scales. Students solve one-step word problems that have measurement contexts.



Topic B

Rounding to the Nearest Ten and Hundred

Reading temperatures on a thermometer provides an initial context for students to round to the nearest ten or hundred. To round a number, students determine the two consecutive tens or hundreds that the number is between and then determine which ten or hundred is closer by thinking about the number in relation to the halfway mark. The number line, when presented vertically, provides a new perspective on a familiar tool and is used to help students to round numbers. Students then apply their rounding skills to estimate sums and differences.



Topic C

Simplifying Strategies to Find Sums and Differences

Students apply their knowledge of the vertical number line to the scale on a scaled bar graph. They represent data in a scaled bar graph and solve addition and subtraction problems related to scaled bar graphs. They explore a variety of addition and subtraction strategies based on place value, the properties of operations, and the relationship between addition and subtraction. The strategies include adding and subtracting like units, making the next ten or hundred, taking from a ten or hundred, and using compensation. Emphasis is placed on thinking flexibly and building toward the use of mental math. Throughout the topic, students share and critique solution strategies for addition and subtraction problems and explain how their choice in strategy helps them to add or subtract efficiently.

$$17 + 98 = 15 + 100 = 115$$

$$15 + (2 + 98)$$

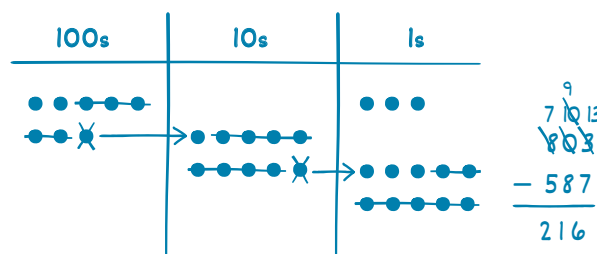
$$800 - 258 = 542$$

$$799 - 257 = 542$$

Topic D

Two- and Three-Digit Measurement Addition and Subtraction

Students use concrete and pictorial place value models alongside vertical form to represent and record the work of the standard algorithm for addition and the standard algorithm for subtraction. They compose and decompose units as needed and estimate to assess the reasonableness of their answers. Students apply their computation skills to select an appropriate strategy and solve one- and two-step word problems involving measurement contexts and units.



After This Module

Grade 4 Module 1

In grade 4, students apply their understanding of measurement units to convert weight, liquid volume, and length measurements from larger units to smaller units.

Students generalize place value and rounding concepts and relationships to larger, multi-digit numbers. They add and subtract multi-digit numbers by using the standard algorithms for addition and subtraction.

$$\begin{array}{r} 378,642 \\ + 527,169 \\ \hline 905,811 \end{array}$$

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Place Value Concepts Through Metric Measurement

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Topic A 14

Understanding Place Value Concepts Through Metric Measurement

Lesson 1 18

Connect the composition of 1 kilogram to the composition of 1 thousand.

Lesson 2 34

Estimate the weight of familiar objects and read scales when weighing objects.

Lesson 3 52

Use all four operations to solve one-step word problems involving weight.

Lesson 4 72

Connect decomposition of 1 liter to the decomposition of 1 thousand.

Lesson 5 90

Estimate and measure liquid volume using a vertical number line and connect composition of 1 liter to composition of 1 thousand.

Lesson 6 104

Use all four operations to solve one-step word problems involving liquid volume.

Lesson 7 120

Solve one-step word problems using metric units.

Topic B 135

Rounding to the Nearest Ten and Hundred

Lesson 8 138

Read temperatures on a thermometer using number line concepts.

Lesson 9 152

Round two-digit numbers to the nearest ten on the vertical number line.

Lesson 10 164

Round two- and three-digit numbers to the nearest ten on the vertical number line.

Lesson 11 178

Round to the nearest hundred on the vertical number line.

Lesson 12 192

Estimate sums and differences by rounding.

Topic C 206

Simplifying Strategies to Find Sums and Differences

Lesson 13 210

Collect and represent data in a scaled bar graph and solve related problems.

Lesson 14 226

Use place value understanding to add and subtract like units.

Lesson 15 242

Use the associative property to make the next ten to add.

Lesson 16	258
Use compensation to add.	

Lesson 17	272
Use place value understanding to subtract efficiently using take from a ten.	

Lesson 18	288
Use place value understanding to subtract efficiently using take from a hundred.	

Lesson 19	302
Use compensation to subtract.	

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Two- and Three-Digit Measurement Addition and Subtraction

Lesson 20	320
Add measurements using the standard algorithm to compose larger units once.	

Lesson 21	336
Add measurements using the standard algorithm to compose larger units twice.	

Lesson 22	352
Subtract measurements using the standard algorithm to decompose larger units once.	

Lesson 23	368
Subtract measurements using the standard algorithm to decompose larger units twice.	

Lesson 24	382
Subtract measurements using the standard algorithm to decompose larger units across two place values.	

Lesson 25	396
Solve two-step word problems.	

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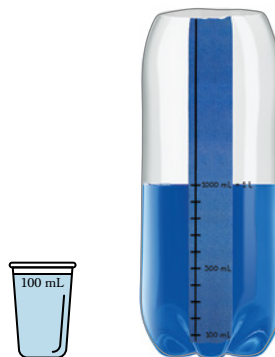
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Why

Place Value Concepts Through Metric Measurement

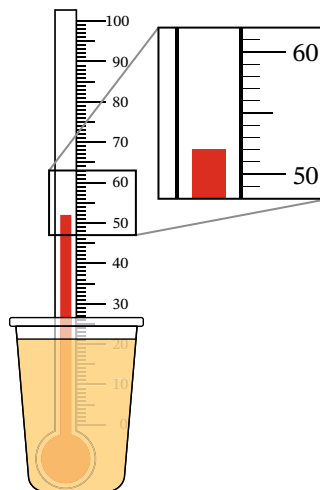
Why relate metric units to the place value system?

One of the advantages of the metric system of measurement is its base-ten structure. In grade 2, students connect the base-ten system and the metric system for measuring length by using centimeters and meters. Relating place value concepts to measurement provides a natural application to strengthen understanding and highlight connections. The composition and decomposition of 1 thousand as 10 hundreds, 100 tens, and 1,000 ones parallels the composition and decomposition of 1 kilogram as 10 hundred grams, 100 ten grams, and 1,000 grams and 1 liter as 10 hundred milliliters, 100 ten milliliters, and 1,000 milliliters. Tens and hundreds are used to show the progression from 1 to 1,000, but emphasis is placed on creating the new units of 1 liter and 1 kilogram from milliliters and grams.



Why are some measurement units, such as milliliters and degrees, included in lessons?

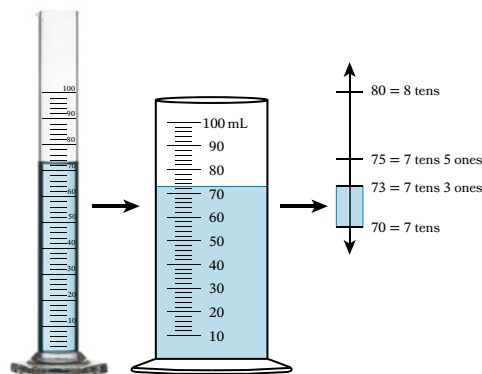
Selectively including milliliters and degrees provides opportunities to make rich connections that would not be possible without addressing those units. For example, by working with milliliters, students can see that the structure for measuring metric liquid volume is the same as the structure for measuring metric weight and that those structures are similar to the place value system. Similarly, students' understanding of the vertical number line for measurement and rounding is enhanced when the scale on the thermometer is connected to the vertical number line, thereby introducing a unit, degrees Fahrenheit, that is familiar outside their study of mathematics. Because they are likely to encounter the units



introduced in these lessons outside the classroom, familiarity with these units can help students better understand their experiences.

Why is the vertical number line the primary tool used for rounding?

The vertical number line is introduced in familiar contexts. Students interpret the scale on a beaker and the scale on a thermometer as vertical number lines when measuring liquid volume and temperature. This activity provides natural opportunities for discussions about benchmark numbers, more or less than halfway, and informally rounding amounts by using *about*. Students apply these experiences to rounding numbers to the nearest ten and hundred.



Why are so many addition and subtraction strategies and algorithms used?

Fluency means accurately, efficiently, and flexibly applying strategies and algorithms to solve problems. A strategy or algorithm may be efficient for solving one problem but time-consuming for another (e.g., the standard algorithm may not be the most efficient strategy for finding $500 - 499$).

Students analyze problems and select efficient solution strategies, many of which develop into mental math over time. They record their work in a way that makes sense to them.

Students are not expected to master all the strategies taught in module 2 topics C and D. Rather, they are expected to make informed decisions about which strategy to use on a problem-by-problem basis. For example, when the standard algorithms for addition and subtraction are taught, students are expected to evaluate each problem and select an efficient method, which may or may not be the standard algorithm, and recording notation that will help them find the correct answer. Throughout grade 3, addition and subtraction problems are intentionally presented horizontally. A vertical presentation implies use of the standard algorithm and vertical form notation. In contrast, a horizontal

$$98 + 56 = \underline{154}$$

$$56 \xrightarrow{+100} 156 \xrightarrow{-2} 154$$

presentation is more conducive to students thinking flexibly about number relationships to choose the most efficient strategy.

Why is telling and writing time found in a different module in *Eureka Math*²?

The grade 2 understanding of intervals on a number line is further developed here in module 2 by using vertical number lines and then again in module 5 with the study of fraction concepts by using horizontal number lines. Halves and quarters as they relate to units of time are more easily applied after extensive experience with fractional units on a number line. In module 6, students use a horizontal number line as a tool to solve problems involving intervals of time.

Terminology

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New

capacity

Capacity is the maximum amount a container can hold. It is measured by using units of liquid volume such as liters and milliliters. (Lesson 4)

Capacity is about the container. Liquid volume is about the liquid, regardless of what container it is in.

gram (g)

A gram is a unit for measuring weight. A centimeter cube weighs about 1 gram. 1,000 grams make 1 kilogram. (Lesson 1)

kilogram (kg)

A kilogram is a unit for measuring weight. A dictionary weighs about 1 kilogram. A kilogram is 1,000 grams. (Lesson 1)

liquid volume

Liquid volume is the amount of space a liquid takes up. It can be measured by using metric units such as liters and milliliters. (Lesson 4)

liter (L)

A liter is a unit for measuring liquid volume. An average-size water bottle holds about half a liter. A liter is 1,000 milliliters. (Lesson 4)

milliliter (mL)

A milliliter is a unit for measuring liquid volume. A small spoon can hold about 5 milliliters of liquid. 1,000 milliliters make 1 liter. (Lesson 4)

operation

Addition, subtraction, multiplication, and division are operations. In an expression involving two numbers, the symbol between the numbers represents the operation. For example, multiplication is the operation in the expression 4×5 . (Lesson 3)

round, rounding

To round is to find a benchmark number (e.g., a multiple of 10 or 100) that is close to a given number. For example, rounding 286 to the nearest ten would be 290, and rounding 286 to the nearest hundred would be 300. (Lesson 9)

scaled bar graph

A scaled bar graph is a bar graph where the tick marks count by a whole number greater than 1. For example, the tick marks on a scaled bar graph could count by fives (i.e., 0, 5, 10, 15, etc.). (Lesson 13)

standard algorithm (for addition and subtraction)

The standard algorithm is the process of adding (or subtracting) like units in a systematic way. Like units are added (or subtracted) one place value at a time. Units are regrouped during (or before) the process, as necessary. (Lesson 20)

≈

A symbol used when two expressions are about equal to each other. For example, 286 is about 290, so we can write $286 \approx 290$. (Lesson 10)

Familiar

about

addend

bar graph

benchmark

centimeters

comparative language: more than, less than, about the same

compose

decompose

divide

estimate (noun)

estimate (verb)

exchange, bundle, unbundle, rename

halfway

heavier, lighter

horizontal

interval

measure

mental math

meters

multiply

number line

place value units

plot

simplifying strategy

temperature

tick mark

vertical

weight

Academic Verbs

determine

examine

locate

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

9	2-liter containers	24	<i>Learn</i> books
1	Bottle of liquid food coloring, blue	1	Pad of chart paper
30	Clear plastic cups, about 150 mL (5 oz)	1	Pad of sticky notes
1	Container, about 1 liter	25	Pencils
1	Container, greater than 1 liter	9	Permanent markers
1	Container, less than 1 liter	25	Personal whiteboards
1	Demonstration thermometer	25	Personal whiteboard erasers
6	Digital compact scale with 5-cup bowl	9	Plastic pitchers, 1.5 L or larger
25	Dry-erase markers	6	Platform scales
14	Envelopes	1	Projection device
25	<i>Eureka Math</i> ® place value disks set, ones to thousands	1	Resealable bag, gallon size
1	<i>Eureka Math</i> ® tape measure, 150 cm	24	Resealable bags, sandwich size
1	<i>Eureka Math</i> ® whole number place value cards	1	Roll of painter's tape
1	Graduated cylinder, 100 mL	1	Syringe, 10 mL
1	Graduated cylinder, 1,000 mL	1	<i>Teach</i> book
11	Index cards	1	Teacher computer or device
2,443	Interlocking cubes, 1 cm		

Visit <http://eurmath.link/materials> to learn more.

Before This Module

Grade 3 Module 1

In grade 3 module 1, students relate repeated addition, equal groups, and arrays to multiplication and division. With a focus on units of 2, 3, 4, 5, and 10, students apply the commutative and distributive properties as strategies to multiply and divide and write expressions with three factors as a foundation of the associative property. Students use their understanding of multiplication and division concepts to reason about and solve one-step word problems. Throughout grade 3 module 2, fluency activities support students in developing proficiency with these factors.

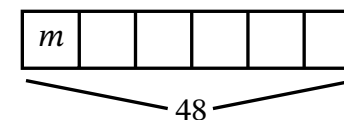
Overview

Multiplication and Division with Units of 0, 1, 6, 7, 8, and 9

Topic A

Multiplication and Division Concepts with an Emphasis on Units of 6 and 8

Students begin module 3 with a counting collection of pregrouped objects to apply their understanding of multiplication from module 1 and to provide formative data about their counting strategies. Throughout the topic, students apply multiplication and division concepts, representations, and strategies from module 1 to the units of 6 and 8. Students see sixes as 2 threes and eights as 2 fours. They use the commutative, distributive, and associative properties to help them multiply and divide. Students advance their representations to more abstract arrays and tape diagrams and begin to use a letter to represent an unknown quantity.

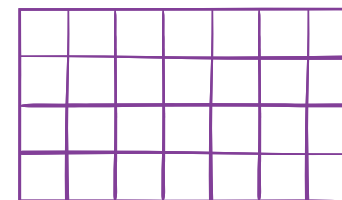


$$48 \div 6 = m$$
$$m = 8$$

Topic B

Multiplication and Division Concepts with an Emphasis on the Unit of 7

With a focus on 7 as a factor, students extend their work with the properties of operations and use different strategies to multiply efficiently. They create expressions with known facts to find products and quotients of unfamiliar facts by flexibly applying the distributive property and regrouping factors by using the associative property. Students apply their learning as they solve one-step word problems.



$$4 \times 7 = j$$
$$28 = j$$

There are 28
juice boxes.

$$4 \times 7 = 4 \times (5 + 2)$$
$$= (4 \times 5) + (4 \times 2)$$
$$= 20 + 8$$
$$= 28$$

Topic C

Analysis of Patterns Using Units of 9, 0, and 1

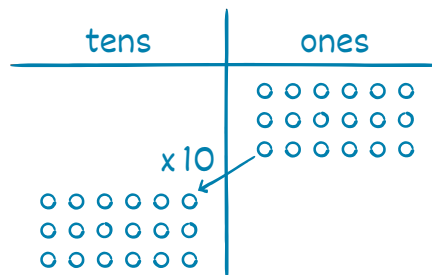
Students explore patterns to identify and apply strategies for multiplying and dividing with 9, 0, and 1. They look for and generalize patterns in the multiplication table and in input-output tables to solve problems. They synthesize their understanding of multiplication with one-digit factors to write their own one-step word problems and assess the reasonableness of solutions to two-step word problems.

+1 ten (9) -1	$2 \div 1 = 2$
+1 ten (18) -1	$3 \div 1 = 3$
+1 ten (27) -1	$4 \div 1 = 4$
+1 ten (36) -1	$5 \div 1 = 5$
+1 ten (45) -1	$6 \div 1 = 6$
+1 ten (54) -1	$7 \div 1 = 7$
+1 ten (63) -1	$8 \div 1 = 8$
+1 ten (72) -1	$9 \div 1 = 9$
+1 ten (81) -1	$30 \div 1 = 30$
+1 ten (90) -1	$500 \div 1 = 500$

Topic D

Multiplication with Multiples of 10 and Further Application of Concepts

To culminate the module, students use place value models, the properties of operations, and familiar facts to multiply multiples of 10 by one-digit numbers. They apply their learning to solve two-step word problems and share solution strategies. Students revisit the counting collection from topic A, this time applying their new understandings from module 3. Opportunities to extend learning are available with an optional lesson on multiplying and dividing by 11 and 12 and an optional lesson on solving word problems in a multi-part task.



After This Module

Grade 4 Module 2

In grade 4 module 2, students apply grade 3 strategies for multiplication and division to multiply and divide numbers of up to 4 digits by one-digit numbers. Students complete measurement conversions and solve multiplicative comparison and multi-step word problems. Students advance their prior experiences with the properties of operations by identifying factors, multiples, and prime numbers within 100.

Contents

Multiplication and Division with Units of 0, 1, 6, 7, 8, and 9

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Topic A	11
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Multiplication and Division Concepts with an Emphasis on Units of 6 and 8

Lesson 1	14
-----------------------	----

Organize, count, and represent a collection of objects.

Lesson 2	32
-----------------------	----

Count by units of 6 to multiply and divide by using arrays.

Lesson 3	50
-----------------------	----

Count by units of 8 to multiply and divide by using arrays.

Lesson 4	66
-----------------------	----

Decompose pictorial arrays to create expressions with three factors.

Lesson 5	82
-----------------------	----

Use the break apart and distribute strategy to multiply with units of 6 and 8.

Lesson 6	100
-----------------------	-----

Use the break apart and distribute strategy to divide with units of 6 and 8.

Topic B	115
----------------------	-----

Multiplication and Division Concepts with an Emphasis on the Unit of 7

Lesson 7	118
-----------------------	-----

Count by units of 7 to multiply and divide by using arrays and tape diagrams.

Lesson 8	132
-----------------------	-----

Use the break apart and distribute strategy to multiply with units of 7.

Lesson 9	150
-----------------------	-----

Model the associative property as a strategy to multiply.

Lesson 10	166
------------------------	-----

Use parentheses in expressions with different operations.

Lesson 11	178
------------------------	-----

Use the break apart and distribute strategy to divide with units of 7.

Lesson 12	190
------------------------	-----

Solve one-step word problems involving multiplication and division.

Topic C	205
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Analysis of Patterns Using Units of 9, 0, and 1

Lesson 13	210
------------------------	-----

Count by units of 9 to multiply.

Lesson 14	224
------------------------	-----

Apply strategies and identify patterns to multiply with units of 9.

Lesson 15	238
------------------------	-----

Reason about and explain patterns of multiplication and division with units of 1 and 0.

Lesson 16	252
------------------------	-----

Identify patterns by using the multiplication table.

Lesson 17	268
------------------------	-----

Identify and complete patterns with input-output tables.

Lesson 18 284

Create multiplication and division word problems.

Lesson 19 298

Solve two-step word problems by using the four operations and assess the reasonableness of solutions.

Topic D 313**Multiplication with Multiples of 10 and Further Application of Concepts****Lesson 20** 316

Multiply by multiples of 10 by using the place value chart.

Lesson 21 328

Multiply by multiples of 10 by using place value strategies and the associative property.

Lesson 22 340

Solve two-step word problems involving multiplication of single-digit factors and multiples of 10.

Lesson 23 356

Identify patterns and apply strategies to multiply with units of 11 and 12. (Optional)

Lesson 24 372

Organize, count, and represent a collection of objects.

Lesson 25 396

Apply multiplication and division concepts to complete a multi-part task. (Optional)

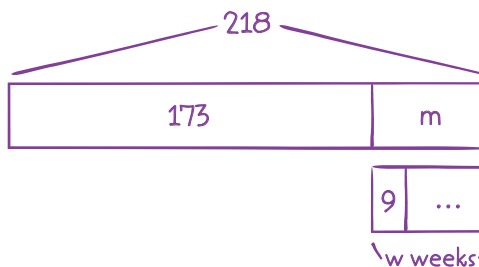
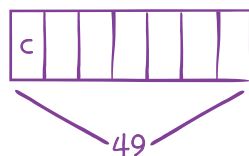
Resources**Standards** 414**Achievement Descriptors: Proficiency Indicators** 418**Terminology** 432**Math Past** 434**Materials** 438**Works Cited** 440**Credits** 442**Acknowledgments** 443

Why

Multiplication and Division with Units of 0, 1, 6, 7, 8, and 9

Why do tape diagrams representing multiplication and division look different in module 3 than they do in modules 1 and 2?

Students draw tape diagrams as part of the Read-Draw-Write process to help them make sense of what is known and unknown in a word problem and to determine a solution path. In modules 1 and 2,



students revisit and complete tape diagrams after they solve to confirm and check the reasonableness of their answers. In module 3, as students develop proficiency with multiplication and division facts, they learn to use tape diagrams more flexibly and sometimes skip the tape diagram check after solving the problem.

Why are lessons 23 and 25 considered optional?

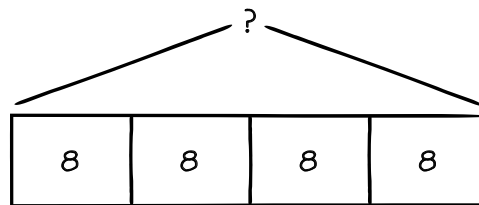
In lesson 23, students apply multiplication strategies and use patterns to skip-count by units of 11 and 12. This lesson was added to provide further application of strategies and in recognition that some state standards expand multiplication to include the factors of 11 and 12. Consider including the lesson if your state standards support it or if your students are ready to apply their strategies to larger factors.

$$\begin{array}{l} 12 \times 8 = 96 \\ \swarrow \searrow \\ 10 \quad 2 \\ 10 \times 8 = 80 \\ 2 \times 8 = 16 \\ 80 + 16 = 96 \end{array}$$

In lesson 25, students reason about, represent, and solve a multi-part word problem task. Students identify and use information that is given in a table, another problem, or a previous solution to make sense of and solve each part. Consider including the lesson to help students develop mathematical modeling skills and to help them determine what information is essential when using mathematics to solve complex, everyday problems. Multi-part task lessons are also included in modules 4 and 5.

Item	Number in Each Pack
Granola Bars	10
Yogurt	4
Fruit Snacks	12
Juice Boxes	8

Liz buys 4 packs of juice boxes.
How many juice boxes does she buy in all?



$$4 \times 8 = 32$$

Liz buys 32 juice boxes in all.

How do students continue to develop fluency with multiplication facts after module 3?

In module 4, students use their multiplication skills and strategies to find the areas of rectangles and rectangular arrays. In modules 4, 5, and 6, fluency activities reinforce multiplication and division concepts and skills through counting the math way, relating multiplication and division, and finding unknown factors. Sprints also provide practice with multiplication and division. A lesson at the end of module 6 includes activities that reinforce fluency. These activities can be implemented at any time after module 3 and can be repeated to aid in developing fact fluency.

$$15 \div 5$$

$$24 \div 8$$

$$18 \div 6$$

$$27 \div 9$$

Terminology

The following terms are critical to the work of grade 3 module 3. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

multiple

A multiple of a given number is the product of the given number and another whole number. For example, 56 is a multiple of 7 because $56 = 8 \times 7$.

The numbers you say when skip-counting by a given number are multiples of that number. For example, 0, 7, 14, 21, 28, 35, ... are multiples of 7. (Lesson 13)

Familiar

array

column

commutative property

distribute

division, divide, divided by, $\div$

equal groups

equation

estimate (noun)

estimate (verb)

even number

expression

factor

multiplication, multiply, $\times$

number bond

odd number

parentheses

product

quotient

row

skip-count

tape diagram

unit

unit form

unknown

value

Academic Verbs

apply

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	100-bead demonstration rekenrek	24	<i>Learn</i> books
12	Chart paper sheets	25	Pencils
25	Colored pencils, set of 8	25	Personal whiteboards
25	Dry-erase markers	25	Personal whiteboard erasers
24	Envelopes	1	Projection device
12	<i>Eureka Math</i> ^{2™} Numeral Cards, set of 12	25	Scissors
25	<i>Eureka Math</i> place value disks set, ones to thousands	150	Sticky notes
2	Highlighters	1	<i>Teach</i> book
780	Interlocking cubes, 1 cm	1	Teacher computer or device

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 1 for a list of organizational tools (cups, rubber bands, graph paper, etc.) suggested for the counting collections.

Before This Module

Grade 2 Modules 3 and 6

In grade 2 module 3, students recognize and draw two-dimensional shapes based on attributes such as the number of sides and angles and the presence of parallel sides or right angles. They use attributes to describe, with precise language, the differences between shapes.

In grade 2 module 6, students compose and decompose rectangular arrays with up to 5 rows and 5 columns of squares, both concretely and pictorially. They describe the arrays in unit form and write repeated addition equations to find the total.

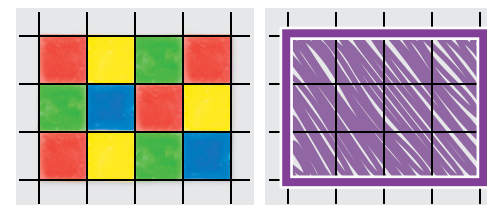
Overview

Multiplication and Area

Topic A

Foundations for Understanding Area

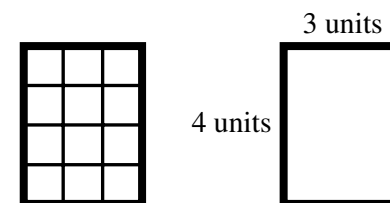
Students expand their understanding of polygons to recognize area as an attribute. They tile polygons with square units, and they count each square or skip-count to find the area of each polygon. As the topic progresses, students move to representing area pictorially on grid paper. They measure the side lengths of rectangles with a ruler and relate each measure to the number of tiles that compose that side length.



Topic B

Concepts of Area Measurement

Students formalize the use of multiplication to determine the area of a rectangle. They complete grids to show the rows and columns of squares within a rectangular array and connect the number of rows and columns to the side lengths. Students transition to using area models and relate the side lengths to area by writing multiplication equations.



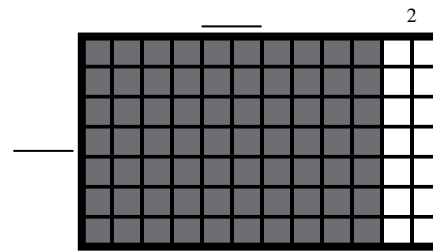
$$4 \times 3 = 12 \text{ or } 3 \times 4 = 12$$

Area: 12 square units

Topic C

Applying Properties of Operations to Area

Students apply multiplication strategies from modules 1 and 3 to find the areas of rectangles. They compose (and decompose) larger rectangles from (and into) smaller rectangles. They use the commutative, associative, and distributive properties to multiply side lengths and to find all possible side lengths of rectangles with a given area.



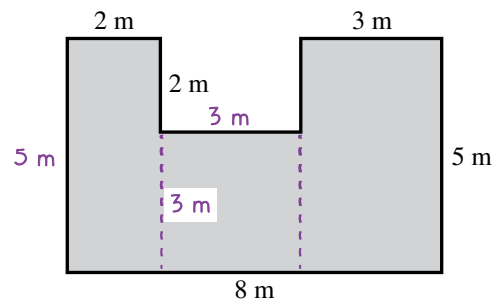
$$\begin{aligned} 7 \times 12 &= 7 \times (10 + 2) \\ &= (7 \times 10) + (7 \times 2) \\ &= 70 + 14 \end{aligned}$$

Area: 84 square units

Topic D

Applications of Area

Students apply area concepts and strategies to a variety of mathematical and real-world problems. They compose and decompose rectilinear shapes to find area, utilizing the attributes of rectangles to find unknown side lengths when necessary. They consider reasonableness related to area in real-world situations and use line plots to analyze measurement data based on area.



$$10 + 9 + 15 = 34$$

Area: 34 square meters

After This Module

Grade 4 Module 2

In grade 4 module 2, students formalize multiplication as a formula for finding the area of a rectangle when given its side lengths. They apply the formula to solve mathematical and real-world problems. Area models are used to represent multiplication and division problems and partial products and quotients.

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Multiplication and Area

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Foundations for Understanding Area

Lesson 1	14
Explore attributes of squares, rectangles, and trapezoids.	

Lesson 2	32
Recognize area as an attribute of polygons.	

Lesson 3	46
Tile polygons to find their areas.	

Lesson 4	60
Compose rectangles to compare areas.	

Lesson 5	76
Relate side lengths to the number of tiles on a side.	

Topic B	93
----------------------	----

Concepts of Area Measurement

Lesson 6	96
Tile rectangles with squares to make arrays and relate the side lengths to area.	

Lesson 7	110
Draw rows and columns to complete a rectangular array and determine its area.	

Lesson 8	124
Determine the area of a rectangle by using side lengths.	

Lesson 9	142
Multiply side lengths to find the area of a rectangle.	

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Applying Properties of Operations to Area

Lesson 10	160
Compose large rectangles and reason about their areas.	

Lesson 11	174
Decompose to find the total area of a rectangle.	

Lesson 12	192
Find all possible side lengths of rectangles with a given area.	

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Applications of Area

Lesson 13	210
Apply area understanding to real-world situations.	

Lesson 14	224
Reason to find the area of composite shapes by using grids.	

Lesson 15	242
Reason to find the area of composite shapes by using rectangles.	

Lesson 16	258
Solve historical math problems involving area.	

Lesson 17 274
Apply area concepts to a real-world context.

Lesson 18 288
Find the area of shapes and represent area data on a line plot.

Lesson 19 308
Apply area concepts to complete a multi-part task.

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Why

Multiplication and Area

Why does topic A limit the types of polygons and geometric concepts taught?

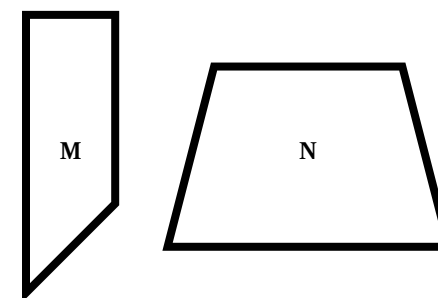
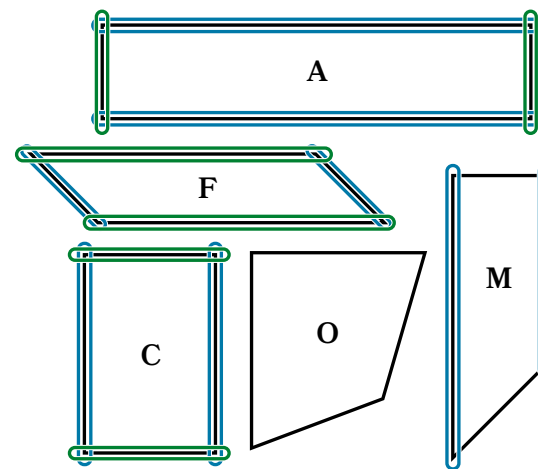
In module 4, students are asked to apply prior geometric understanding to find the areas of rectangles. The first few lessons of topic A engage them in recalling the attributes of squares, rectangles, and trapezoids while creating an opportunity to introduce them to the attribute of area. Students tile squares, rectangles, and trapezoids with unit squares to find their areas. These polygons are included within the lessons to enhance students' understanding of partitioning and equal shares for measuring area with the familiar fractional unit of halves. Additional two-dimensional figures and their attributes are included in module 6.

Why is a trapezoid defined as a quadrilateral with at least 1 pair of parallel sides instead of a quadrilateral with exactly 1 pair of parallel sides?

The term *trapezoid* can have two different meanings.

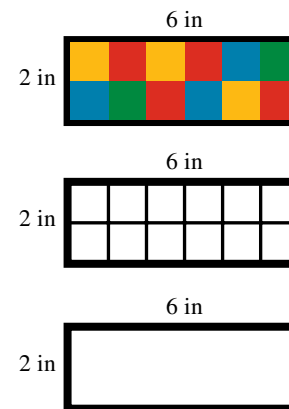
- Exclusive definition: A trapezoid is a quadrilateral with exactly 1 pair of parallel sides.
- Inclusive definition: A trapezoid is a quadrilateral with at least 1 pair of parallel sides.

Both definitions are legitimate, and at grade 3 there is not a significant advantage to either. The inclusive definition is chosen primarily for the conveniences it allows at later grades and for consistency with most geometry textbooks for college-bound students. One nice consequence of using the inclusive definition is that it means trapezoids and parallelograms have a similar relationship to the relationship between rectangles and squares: In each case, the latter is always also the former.



How is the relationship between multiplication and area formalized?

In topics A and B, students experience a progression from concrete to pictorial to abstract as they gain a deep conceptual understanding of area. They transition from concretely tiling to pictorially tiling by making rows and columns, and then they transition to using area models with only side lengths labeled. Students use repeated reasoning and notice area's relationship to the rows and columns in rectangles and the side lengths of rectangles. Strategies for finding area progress from counting all to skip-counting by rows or columns to multiplying side lengths. Armed with conceptual understanding, students move to procedural thinking and articulate that the product of the side lengths is equal to the area of the rectangle. They make progress in the module by using multiplication as an efficient strategy to find area. Specifically naming and applying the area formula occurs in grade 4.



What is the focus of lesson 19, the multi-part task lesson?

Module 4 lesson 19 is the second of three grade 3 multi-part task lessons. Each task lesson provides an opportunity for students to apply their learning and focuses on major work of the grade. The task lessons include interrelated problems that require students to gather information from a graphic or text other than that found within the problems themselves. The problems in lesson 19 require students to apply what they know about area and the side lengths of rectangles to determine the relevant information. Students apply their solutions to answer the questions posed by the problems.

Terminology

The following terms are critical to the work of grade 3 module 4. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

area

The area of a shape is the amount of flat space it takes up. (Lesson 2)

area model

An area model is a rectangle that represents flat space, or area. (Lesson 8)

An area model does not show a grid inside the rectangle. An array does.

length

A rectangle's length is either of its two different side lengths (often the longer one). The other side length is called the width. (Lesson 9)

side length

Side length is the length of a side in a polygon. (Lesson 4)

square centimeter

Square centimeters are a unit for measuring area.

1 square centimeter is defined as the area of a square with side lengths of 1 centimeter. (Lesson 4)

square inch

Square inches are a unit for measuring area. 1 square inch is defined as the area of a square with side lengths of 1 inch. (Lesson 4)

square units

Square units are a unit for measuring area. 1 square unit of area is defined as the area of a square with side lengths of 1 unit. (Lesson 2)

unit square

A unit square is a square with side lengths of 1 unit. (Lesson 7)

width

A rectangle's width is either of its two different side lengths (often the shorter one). The other side length is called the length. (Lesson 9)

Familiar

array

attribute

break apart and distribute

line plot

parallel

polygon

quadrilateral

rectangle

right angle

square

trapezoid

Academic Verbs

observe

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

750	Centimeter tiles, square	8	Markers, set of 8
8	Chart paper, sheets	1	Meter stick, wood
2	Color tiles, plastic, set of 400	25	Pencils
11	Colored paper, sheets	25	Personal whiteboards
24	Colored pencils, set of 8	25	Personal whiteboard erasers
25	Dry-erase markers	1	Projection device
74	Envelopes	25	Rulers, inch and metric
12	<i>Eureka Math</i> ® Numeral Cards, set of 12	25	Scissors
25	Highlighters, set of 2 colors	140	Sticky notes
25	Index cards	1	<i>Teach</i> book
600	Interlocking cubes, 1 cm	1	Teacher computer or device
24	<i>Learn</i> books		

Visit <http://eurmath.link/materials> to learn more.

Before This Module

Grade 2 Module 3

In grade 2, students identify fractions geometrically as parts of shapes. They partition rectangles and circles into two, three, and four equal parts or shares and name the equal parts as halves, thirds, or fourths. Students compose equal parts to make a whole. They describe the parts as a “half of,” “third of,” or “fourth of” the whole shape and identify the number of parts required to make the whole. Students recognize that parts can be equal in size even when they are shaped differently.

Grade 3 module 5 elevates the work of grade 2 through the formal introduction of fractions as numbers.

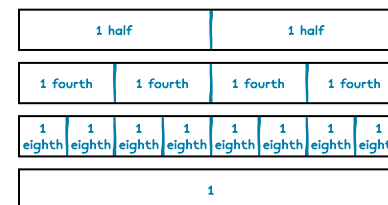
Overview

Fractions as Numbers

Topic A

Partition a Whole into Equal Parts

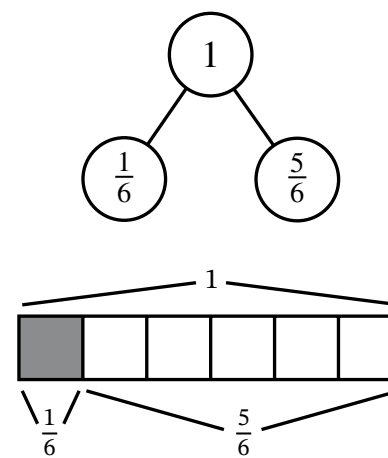
Students begin to formalize their understanding of fractions as numbers by transitioning from recognizing fractional parts of geometric shapes to partitioning and recognizing fractional parts of concrete objects and pictorial models. They begin to understand fractions as numbers by naming the fractional parts of the whole in unit form and describing the relationship between the number of fractional parts and the size of each part. Students define 1 of a fractional unit as a unit fraction, identify the number of unit fractions needed to make 1 whole, and name unit fractions in fraction form.



Topic B

Unit Fractions and Their Relationship to the Whole

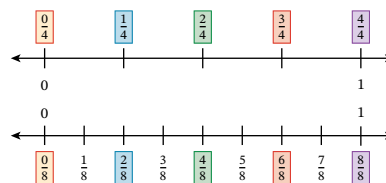
Students iterate unit fractions to create non-unit fractions concretely, pictorially, and numerically in unit form and fraction form. They compose and decompose wholes with unit and non-unit fractions by using number bonds and tape diagrams. After establishing that the fractions refer to the same whole, students compare unit fractions and fractions with the same numerator concretely and pictorially by reasoning about the size of the parts.



Topic C

Fractions on the Number Line

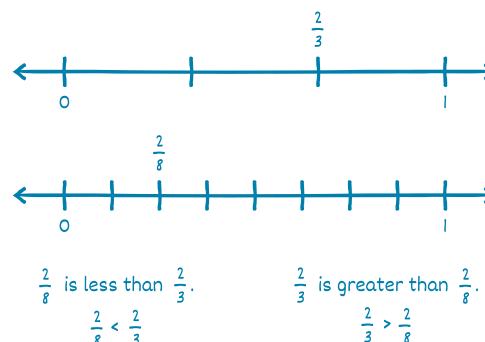
Students continue to formalize their understanding of fractions as numbers by representing fractions from 0 to 1 on a number line. They partition number lines with fraction tiles, and they compose and decompose 1 whole by using strategies similar to those used with number bonds and tape diagrams in topic B. Students relate a fraction's value to its size, its location on a number line, and its distance from 0 to find pairs of equivalent fractions. They generalize their understanding of fractions on a number line to build a ruler that they use to measure lengths within fractions of an inch and to create line plots representing fractional length data.



Topic D

Comparing Fractions

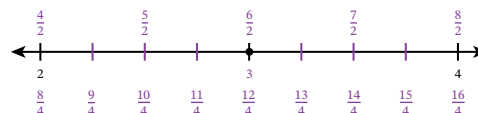
Students extend a number line to include fractions greater than 1 by drawing and partitioning number lines that represent specified intervals. They count and skip-count by fractional units to name fractions greater than 1 and to recognize fractions that are equivalent to whole numbers. They generalize their understanding of location and distance on a number line to compare fractions.



Topic E

Equivalent Fractions

With a progression of strategies similar to those in earlier topics, students use one or two number lines to find pairs of equivalent fractions greater than 1. They identify fractions equivalent to whole numbers and express whole numbers as fractions with a denominator of 1 (e.g., $\frac{4}{1}$). The module culminates with two lessons in which students apply what they have learned about fractions to new problem situations: creating a ruler without other measurement tools and completing a multi-part task.



After This Module

Grade 4 Module 4

In grade 4 module 4, students build on their understanding of fraction composition, decomposition, equivalence, and comparison to include additional fractional units and mixed numbers. Students extend their thinking to more abstract reasoning about fraction relationships including using benchmark fractions to compare the relative sizes of fractions and mixed numbers and using multiplication and division to generate equivalent fractions. Students apply what they know about whole number operations to add and subtract fractions with like units, including mixed numbers, and they multiply fractions by a whole number.

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Partition a Whole into Equal Parts

Lesson 1	16
-----------------------	----

Partition a whole into equal parts and name the fractional unit.

Lesson 2	32
-----------------------	----

Partition different wholes into fractional units concretely.

Lesson 3	50
-----------------------	----

Partition a whole into fractional units by folding fraction strips.

Lesson 4	64
-----------------------	----

Partition a whole into fractional units pictorially and identify the unit fraction.

Lesson 5	80
-----------------------	----

Partition a whole into fractional units and write fractions in fraction form.

Topic B	95
----------------------	----

Unit Fractions and Their Relationship to the Whole

Lesson 6	98
-----------------------	----

Build non-unit fractions less than 1 from unit fractions concretely.

Lesson 7	112
-----------------------	-----

Identify and represent a whole as two parts: a unit fraction and a non-unit fraction.

Lesson 8	128
-----------------------	-----

Identify and represent a whole as two non-unit fractions.

Lesson 9	142
-----------------------	-----

Compare unit fractions by reasoning about their size concretely.

Lesson 10	156
------------------------	-----

Compare non-unit fractions less than 1 with the same numerator by using tape diagrams.

Topic C	171
----------------------	-----

Fractions on the Number Line

Lesson 11	174
------------------------	-----

Locate fractions from 0 to 1 on a number line by using fraction tiles.

Lesson 12	190
------------------------	-----

Represent fractions from 0 to 1 on a number line.

Lesson 13	208
------------------------	-----

Identify equivalent fractions from 0 to 1 with tape diagrams and on number lines.

Lesson 14	226
------------------------	-----

Recognize that equivalent fractions share the same location on a number line.

Lesson 15	240
------------------------	-----

Identify fractions on a ruler as numbers on a number line.

Lesson 16	254
------------------------	-----

Measure lengths and record data on a line plot.

Topic D 269**Comparing Fractions****Lesson 17** 272

Represent fractions greater than 1 on a number line and identify fractions equivalent to whole numbers.

Lesson 18 288

Compare fractions with like units by using a number line.

Lesson 19 304

Compare fractions with unlike units but the same numerator by using number lines.

Lesson 20 328

Compare fractions with related units by using a number line.

Lesson 21 344

Compare various fractions by representing them on number lines.

Topic E 359**Equivalent Fractions****Lesson 22** 362

Identify fractions equivalent to whole numbers by using number lines.

Lesson 23 374

Reason to find fractions equivalent to whole numbers by using patterns and number lines.

Lesson 24 386

Generate equivalent fractions greater than 1 by using a number line.

Lesson 25 402

Express whole numbers as fractions with a denominator of 1.

Lesson 26 418

Create a ruler with 1-inch, half-inch, and quarter-inch intervals.

Lesson 27 432

Apply fraction concepts to complete a multi-part task. (Optional)

Resources**Standards** 450**Achievement Descriptors: Proficiency Indicators** 452**Terminology** 462**Math Past** 464**Materials** 466**Works Cited** 468**Credits** 470**Acknowledgments** 471

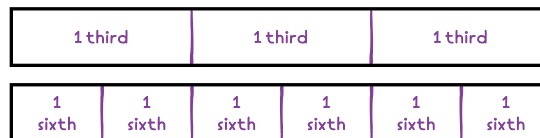
Why

Fractions as Numbers

How does the curriculum support students in partitioning fraction models with a sufficient degree of precision?

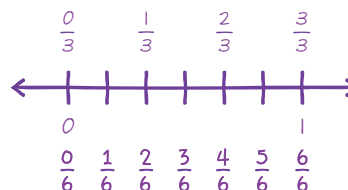
Students are expected to estimate, rather than measure, the placement of lines and tick marks when partitioning fraction models. The relationships between fractional units (e.g., halves, fourths, and eighths; thirds and sixths; and fifths and tenths) is established and reinforced throughout the module.

Students first partition 1 whole into fractional units by using a fraction strip that they fold into equal parts. From this experience, students begin to recognize the relationships between fractional units. For example, folding halves



in half makes fourths, and folding fourths in half makes eighths. In later lessons, they extend this thinking to partitioning a number line. Partitioning strategies such as making fourths by making halves of halves are explicitly taught, and problems are intentionally sequenced to use the connections between related units (i.e., halves, fourths, and eighths; thirds and sixths; and fifths and tenths). Students are supported in reasoning about how many tick marks they should make between whole numbers before partitioning them. Units that are often more difficult to estimate, such as fifths, sevenths, ninths, and tenths, are used sparingly.

To support them in partitioning fraction models, students are generally provided with a drawing of the model to partition. The drawings are large enough that a slightly imprecise partition should be sufficiently close for students to see and be able to describe the relationship between the quantities. Most of the models that students are asked to partition are rectangular or linear (e.g., number lines). Nonrectangular models, including circles, are generally pre-partitioned for students.



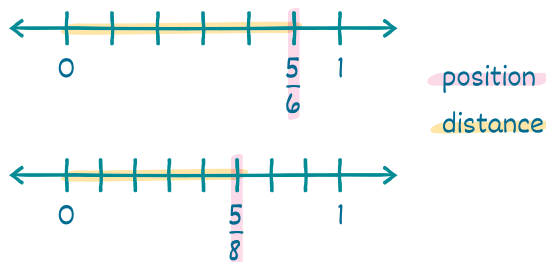
Why aren't the terms *numerator* and *denominator* introduced in this module?

To support students in understanding a fraction as a single number, the terms *numerator* and *denominator* are not used in this module. Naming the fractional unit and the particular number of that unit as the *denominator* and *numerator* before students have a solid understanding that a fraction represents a single quantity can perpetuate a misconception that a fraction is made up of two numbers: one number “over” another number. To support student understanding of a fraction as representing a single quantity, fractions should be read aloud in unit form (e.g., 1 half) or by describing the number the fraction represents (e.g., 1 out of 2 equal parts) rather than as, for example, “1 over 2.” The terms *numerator* and *denominator* are introduced in grade 4.

Shape	Number of Shaded Units	Total Number of Units	Shaded Fraction in Unit Form	Shaded Fraction in Fraction Form
	1	2	1 half	$\frac{1}{2}$

Why is distance from zero used to compare fractions, even though in later grades that strategy won't work for negative numbers?

Students conceptually understand a fraction on a number line in two ways: as a position or location on the number line, and as a distance from zero. So, it is natural for them to use the same two ways of thinking when comparing fractions: the fraction located to the right is greater, and the fraction further from zero is greater. The first way of thinking will continue to work once negative numbers are introduced in grade 6. The second way of thinking will need to be amended because, for example, -5 is further from zero than -3 , but -5 is less than -3 . When comparing two negative numbers, the second way of thinking is the exact opposite: the number further from zero is less, not greater. This opposite relationship is consistent with the way students will learn about negative numbers themselves—as the (additive) opposites of positive numbers. The introduction of negative numbers in grade 6 will challenge students' understanding of the number system in general, which is what makes the use of this comparison strategy acceptable for grade 3.



Why is lesson 27 considered optional?

Lesson 27 is an extension of the other lessons in the module and provides an additional opportunity for students to apply their learning. In it, students reason about, represent, and solve a multi-part word problem task. They identify and use multiple pieces of information that are given in tables, in other problems, or in previous solutions, to make sense of and solve each part. Consider including the lesson to help students develop mathematical-modeling skills and to help them determine what information is essential when using mathematics to solve complex everyday problems.

Robin's Pizza

Total number of equal slices	8
Number of slices with peppers	1
Number of slices with olives	2
Number of slices with mushrooms	5
Number of people equally sharing the pizza	4

David's Pizza

Total number of equal slices	6
Number of slices with peppers	3
Number of slices with olives	2
Number of slices with mushrooms	1
Number of people equally sharing the pizza	3

Terminology

The following terms are critical to the work of grade 3 module 5. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

eighths

Eighths are the fractional unit made by partitioning a whole into 8 equal parts. (Lesson 2)

equivalent

Two fractions (or a fraction and a whole number) are equivalent if they name the same number. (Lesson 12)

$\frac{2}{4}$ and $\frac{3}{6}$ are equivalent fractions because they both name the number $\frac{1}{2}$. (Lesson 13)

fifths

Fifths are the fractional unit made by partitioning a whole into 5 equal parts. (Lesson 2)

fraction form

Fraction form is a way to write fractions where the numerator represents the number of units and the denominator represents the fractional unit. For example, the fraction 3 fourths (in unit form) can be written in fraction form as $\frac{3}{4}$. (Lesson 5)

fraction greater than 1

A fraction greater than 1 is a fraction that represents a number greater than 1 or an amount larger than the whole. For example, $\frac{3}{3}$ is equal to 1, so $\frac{4}{3}$, $\frac{5}{3}$, $\frac{6}{3}$, etc. are fractions greater than 1. (Lesson 17)

fractional unit

Fractional units are the units created by partitioning a whole into a number of equal parts. Fractional units name something that can be counted, composed, and decomposed. For example, thirds are a fractional unit, just like inches are a measurement unit and tens are a place value unit. (Lesson 1)

ninths

Ninths are the fractional unit made by partitioning a whole into 9 equal parts. (Lesson 2)

sixths

Sixths are the fractional unit made by partitioning a whole into 6 equal parts. (Lesson 1)

tenths

Tenths are the fractional unit made by partitioning a whole into 10 equal parts. (Lesson 2)

unit fraction

A unit fraction is exactly one of a specific fractional unit. For example, 1 half, 1 third, 1 fourth, and 1 fifth (or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$) are unit fractions. (Lesson 3)

Familiar

>, =, < symbols

equal parts

equal shares

fourths

fraction

half, halves

half of, one third of, one fourth of

line plot

number line

partition

plot

thirds

tick mark

unit form

whole

whole number

Academic Verbs

identify

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	Analog clock	2	Paper circle, 9" diameter
20	Blank paper, sheets	84	Paper rectangles, 2" × 4"
8	Chart paper, sheets	2	Paper rectangles, 9" × 12"
8	Clay, 2-ounce pieces	350	Paper strips, 1" × 6", white
55	Clear plastic cups, about 150 mL (5 oz)	350	Paper strips, 1" × 6", gray
25	Colored pencils, set of 8	25	Pencils
8	Construction paper, brown, 2" × 6" rectangles	25	Personal whiteboards
8	Construction paper, yellow, 1" × 12" strips	25	Personal whiteboard erasers
4	Containers, such as small buckets	1	Projection device
25	Crayons	25	Ruler, inch and metric
25	Dry-erase markers	25	Scissors
73	Envelopes	1	Sentence strips, 75 pcs
1	<i>Eureka Math</i> ® Numeral Cards, set of 12 decks	1	Spool of string
25	Glue sticks	75	Straws, 25 each of 3 colors
25	Highlighters, set of 2 colors	3	Sticky notes, pads
35	Index cards	1	<i>Teach</i> book
24	<i>Learn</i> books	1	Teacher computer or device
25	Markers, set of 8	3	Wax craft sticks, 3-foot sticks
1	Masking tape, roll		

Visit <http://eurmath.link/materials> to learn more.

Before This Module

Grade 2 Module 3

In grade 2 module 3, students tell and write time to the nearest five minutes, use *a.m.* and *p.m.*, and describe quarter hours by using *quarter past* and *quarter to*.

Grade 3 Modules 2, 4, and 5

In grade 3 module 2, students create scaled bar graphs to represent categorical data.

In module 4, students identify attributes of quadrilaterals, including right angles and parallel sides. Students also name different types of quadrilaterals by using attributes. They define and recognize *area* as an attribute of polygons and determine the areas of rectangles by using side lengths. Students represent area data on line plots.

In module 5, students partition wholes into fractional parts on the number line. They use rulers to measure to the nearest quarter inch and plot fractional length data on line plots.

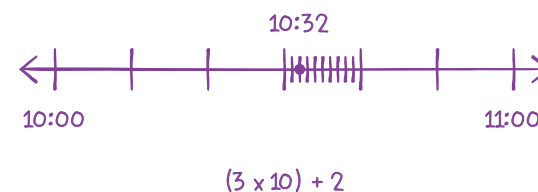
Overview

Geometry, Measurement, and Data

Topic A

Tell Time and Solve Time Interval Problems

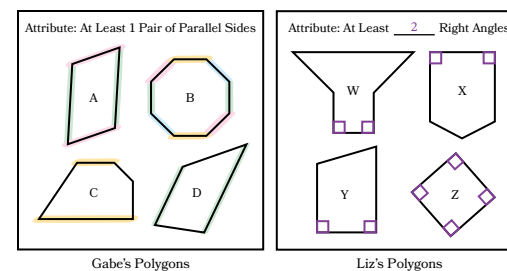
Students use a number line to represent the scale of a clock. They count by fives or tens and then by ones to tell time to the nearest minute. Students apply familiar strategies to solve time interval word problems where the unknown is the finish time, the start time, or the elapsed time. They plot time data in fractions of hours on line plots, and they answer questions about the data. In an optional lesson, students count coins and create money word problems.



Topic B

Attributes of Two-Dimensional Figures

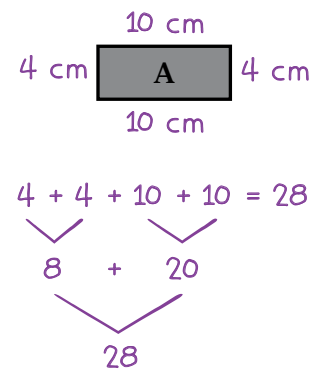
Students describe, define, and sort quadrilaterals by using attributes such as pairs of parallel sides, sides that have the same length, and right angles. They determine attributes that are important for defining a shape. Students also describe and sort polygons and recognize that polygons with the same name can look different based on their other attributes. They use that understanding to draw polygons to match a list of attributes. Students compose polygons to make other polygons. They relate the attributes of the composed polygons to the attributes of the individual polygons.



Topic C

Problem Solving with Perimeter

Students define perimeter and understand it as an attribute of two-dimensional shapes. To find the perimeter of a polygon, they measure the side lengths with a ruler and find the sum. To find the perimeter of a shape with curves, they use string and a ruler. Students use known attributes to find unknown side lengths and perimeters of polygons, including regular polygons. They reason about the relationship between area and perimeter and determine the perimeters of rectangles with the same area and the areas of rectangles with the same perimeter. The topic culminates when students solve real-world problems involving perimeter and unknown measurements.



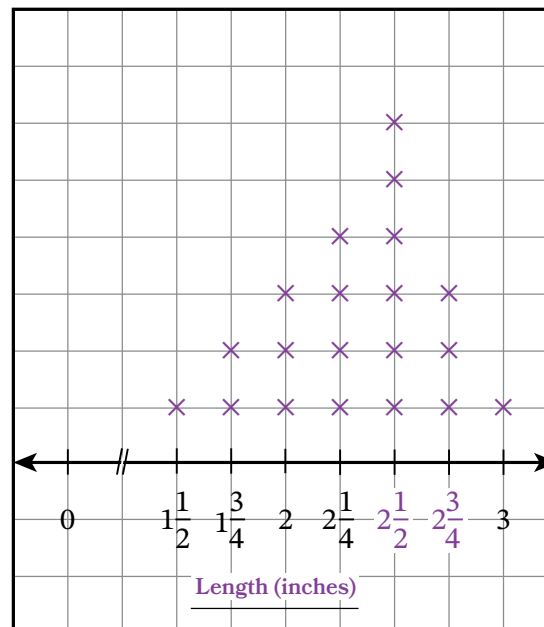
Topic D

Collecting and Displaying Data

Students collect measurement data involving fractional lengths by measuring the perimeter of circles to the nearest quarter inch. They represent this and other fractional data on line plots. Students also represent categorical data by using scaled picture graphs and scaled bar graphs.

Students identify place value patterns to name units up to 1 million by organizing, counting, and representing a collection with a total value greater than 1,000. In an optional lesson, students name and count numbers greater than 1,000. They end the topic by completing several activities to show and self-assess their fluency with multiplication and division within 100 and addition and subtraction within 1,000.

Title: Mrs. Smith's Caterpillars



After This Module

Grade 4 Modules 1, 2, and 4

In grade 4 module 1, students formally extend their understanding of the place value system to 1 million. They name the units of ten thousand, hundred thousand, and million and describe the relationship between each place value unit and the next largest unit by using *10 times as much as* language. They compare, round, and add and subtract with numbers of up to six digits.

In grade 4 module 2, students formalize the strategies used to find area and perimeter of rectangles as formulas. They use the formulas for area and perimeter of rectangles to solve a variety of problems.

In grade 4 module 4, work with line plots is extended to include eighths of a unit. Students use the information in line plots to solve problems involving addition and subtraction of fractions.

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Geometry, Measurement, and Data

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Topic A	11
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Tell Time and Solve Time Interval Problems

Lesson 1	16
-----------------------	----

Relate skip-counting by fives on the clock to telling time on the number line.

Lesson 2	32
-----------------------	----

Count by fives and ones on the number line as a strategy for telling time to the nearest minute on the clock.

Lesson 3	50
-----------------------	----

Solve time word problems where the end time is unknown.

Lesson 4	66
-----------------------	----

Solve time word problems where the start time is unknown.

Lesson 5	84
-----------------------	----

Solve time word problems where the change in time is unknown.

Lesson 6	104
-----------------------	-----

Solve time word problems and use time data to create a line plot.

Lesson 7	118
-----------------------	-----

Count coins and create money word problems. (Optional)

Topic B	137
----------------------	-----

Attributes of Two-Dimensional Figures

Lesson 8	140
-----------------------	-----

Compare and classify quadrilaterals.

Lesson 9	156
-----------------------	-----

Compare and classify other polygons.

Lesson 10	176
------------------------	-----

Draw polygons with specified attributes.

Lesson 11	194
------------------------	-----

Reason about composing polygons by using tetrominoes.

Lesson 12	212
------------------------	-----

Reason about composing polygons by using tangrams.

Topic C	227
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Problem Solving with Perimeter

Lesson 13	230
------------------------	-----

Decompose quadrilaterals to understand perimeter as the boundary of a shape.

Lesson 14	246
------------------------	-----

Measure side lengths in whole-number units to determine the perimeters of polygons.

Lesson 15	262
------------------------	-----

Recognize perimeter as an attribute of shapes and solve problems with unknown measurements.

Lesson 16	278
------------------------	-----

Solve problems to determine the perimeters of rectangles with the same area.

Lesson 17	294
------------------------	-----

Solve problems to determine the areas of rectangles with the same perimeter.

Lesson 18 308

Solve real-world problems involving perimeter and unknown measurements by using all four operations.

Topic D 323**Collecting and Displaying Data****Lesson 19** 328

Measure the perimeter of various circles to the nearest quarter inch by using string.

Lesson 20 344

Record measurement data in a line plot.

Lesson 21 358

Create and analyze a line plot for measurement data to the nearest half unit and quarter unit.

Lesson 22 370

Generate categorical data and represent it by using a scaled picture graph.

Lesson 23 388

Solve problems by creating scaled picture graphs and scaled bar graphs.

Lesson 24 412

Organize, count, and represent a collection of objects.

Lesson 25 446

Name and count numbers greater than 1,000. (Optional)

Lesson 26 466

Fluently multiply and divide within 100 and add and subtract within 1,000.

Resources**Standards** 478**Achievement Descriptors: Proficiency Indicators** 480**Terminology** 490**Math Past** 492**Materials** 494**Works Cited** 496**Credits** 498**Acknowledgments** 499

Why

Geometry, Measurement, and Data

How are the topics of this module related?

Each topic in this module involves problem solving and applications of fractions. Throughout the module, students solve problems involving time, perimeter and area, and data represented in line plots, scaled bar graphs, and scaled picture graphs. Students apply and deepen their understanding of fractions throughout the module by working with fractional data, such as length measurements (including perimeters), and by using partitioned number lines in the context of time interval problems, rulers, and the scales of line plots and scaled bar graphs.

Why is lesson 7 considered optional?

Lesson 7 extends the learning of the other topic A lessons by providing an opportunity for students to apply the strategies they used to solve time interval problems to a popular context—money.



Coins

$$50¢ \xrightarrow{+30¢} 80¢ \xrightarrow{+5¢} 85¢ \xrightarrow{+2¢} 87¢$$

Carla bought a pineapple for 129 cents.
She was left with 87 cents.

How much money did she start with?

$$87¢ \xrightarrow{+100¢} 187¢ \xrightarrow{+20¢} 207¢ \xrightarrow{+3¢} 210¢ \xrightarrow{+6¢} 216¢$$

She started with 216¢.

This lesson is included to serve as a bridge between the money word problem work in grade 2 and the money word problem work in grades 4 and 5. The money word problems in lesson 7 follow a similar structure to the time interval word problems, with problems where the ending amount is unknown, the starting amount is unknown, or the change in amount is unknown. Consider including the lesson to help students develop their ability to reason abstractly and quantitatively by contextualizing and decontextualizing mathematical situations.

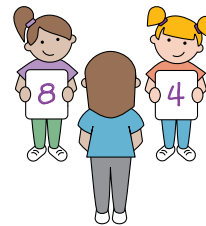
Why do lessons 24 and 25 introduce place value units to 1 million?

Lessons 24 and 25 intentionally extend student thinking around place value units in preparation for the major work of grade 4. By organizing, counting, and representing a collection with a total value greater than 1,000 in lesson 24, students study patterns to identify and broaden their understanding of the place value system. Students make sense of relationships and patterns in the place value system as they repeatedly bundle ten smaller units to compose one of the next larger unit, only to realize that much larger units are needed. Lesson 25 is an optional lesson where students repeatedly count and bundle bills to one million. This lesson builds the foundation for students seeing that the value of a digit is 10 times as much as the value of the same digit in the place to its right, a key concept for expanding whole-number relationships to decimals. Consider including lesson 25 to provide students with experience with numbers larger than 1,000 before grade 4.



How are the activities in lesson 26 intended to be used?

The activities in lesson 26 are intended to help students self-assess and enhance their progress toward the grade 3 fluency expectations for addition and subtraction within 1,000 and for multiplication and division within 100. Individually or collectively, the activities can be implemented any time after the completion of module 3 and repeated as desired. Consider using this lesson at the end of the year to support students in self-reflection and to provide experience with the activities so students can continue to use them outside of class.



Terminology

The following terms are critical to the work of grade 3 module 6. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

diagonal

A diagonal in a polygon is a line segment connecting two corners that are not next to each other. (Lesson 8)

measurement data

When the same type of measurement is taken for a group of objects, the group of measurements is called measurement data. One example of measurement data is a list of the heights of everyone in a classroom. (Lesson 21)

In grade 6, students start using the term numerical data to mean the same thing as measurement data. They also explore the differences between measurement data or numerical data and categorical data.

perimeter

The perimeter of a shape is the length of its boundary. (Lesson 13)

Informally and in grade 3, the term perimeter can also be used to mean the boundary itself (not just the length of the boundary). In grade 4, students more precisely define what they mean by shape, and they also more precisely use perimeter to mean the length of the boundary (not the boundary itself).

regular polygon

A regular polygon is a polygon with all sides the same length and all angles the same size. (Lesson 9)

In grade 4, students learn that angles are the same size if they have the same measure.

scaled picture graph

A scaled picture graph is a picture graph where each picture, or symbol, represents more than 1 object or observation. (Lesson 22)

Familiar

analog clock

area

attribute

data

fraction

frequent

hexagon

key

line plot

octagon

parallel

parallelogram

pentagon

plot

polygon

quadrilateral

quarter past, quarter to

rectangle

rhombus

right angle

scale

scaled bar graph

square

survey

tangram

trapezoid

Academic Verbs

Module 6 does not introduce any academic verbs from the grade 3 list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	Analog clock	25	Pencils
13	Chart paper, sheets	25	Personal whiteboards
1	Clear tape, roll	25	Personal whiteboard erasers
25	Colored pencils, sets of 8	1	Projection device
25	Dry-erase markers	25	Rulers
12	Envelopes	25	Scissors
12	Glue sticks	25	Sticky notes
25	Highlighters, sets of 6 colors	1	String, 124 ft
160	Index cards	1	<i>Teach</i> book
24	<i>Learn</i> books	1	Teacher computer or device
25	Markers	24	Wax craft sticks, 12-inch sticks

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 24 for a list of organizational tools (cups, rubber bands, graph paper, etc.) suggested for the counting collection.