Before This Module

Grade 3 Module 1

In grade 3 module 1, students build a conceptual understanding of multiplication as a number of equal groups (e.g., $4 \times 3 = 12$ can be interpreted as 4 groups of 3 is 12).

Grade 3 Module 2

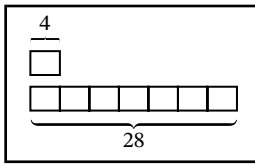
In grade 3 module 2, students compose and decompose metric measurement units and relate them to place value units up to 1 thousand. They use place value understanding and the vertical number line to round two- and three-digit numbers. Grade 3 students also add and subtract two- and three-digit numbers by using a variety of strategies, including the standard algorithm.

Overview

Place Value Concepts for Addition and Subtraction

Topic A

Multiplication as Multiplicative Comparison

	28 is 7 times as many as 4.	$28 = 7 \times 4$

Students identify, represent, and interpret multiplicative comparisons in patterns, tape diagrams, multiplication equations, measurements, and units of money. They describe the relationship between quantities as *times as much as* or use other language as applicable to a given context (e.g., *times as many as*, *times as long as*, and *times as heavy as*). Students use multiplication or division to find an unknown quantity in a comparison.

Topic B

Place Value and Comparison Within 1,000,000

56,348
50,000 + 6,000 + 300 + 40 + 8
fifty-six thousand, three hundred forty-eight
56 thousands 3 hundreds 4 tens 8 ones

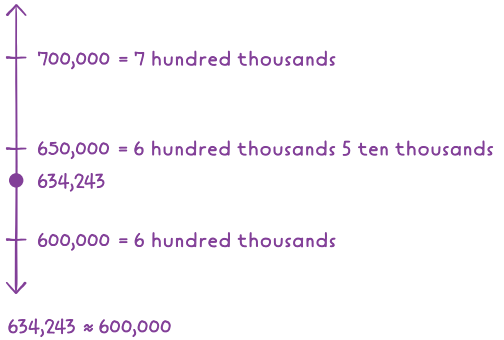
Students name the place value units of ten thousand, hundred thousand, and million. They recognize the multiplicative relationship between place value units—the value of a digit in one place is ten times as much as the value of the same digit in the place to its right. Students write and compare numbers with up to 6 digits in standard, expanded, word, and unit forms.

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Topic C

Rounding Multi-Digit Whole Numbers

Students name multi-digit numbers in unit form in different ways by using smaller units (e.g., 245,000 as 24 ten thousands 5 thousands or 245 thousands), and they find 1 more or 1 less of a given unit in preparation for rounding on a vertical number line. Students round four-digit, five-digit, and six-digit numbers to the nearest thousand, ten thousand, and hundred thousand. They determine an appropriate rounding strategy to make useful estimates for a given context.



Topic D

Multi-Digit Whole Number Addition and Subtraction

Students build fluency with addition and subtraction of numbers of up to 6 digits by using the standard algorithm. They add and subtract to solve two-step and multi-step word problems. The Read–Draw–Write process is used to help students make sense of the problem and find a solution path. Throughout the topic, students round to estimate the sum or difference and check the reasonableness of their answers.

800,000 - 500,000 = 300,000

$$\begin{array}{r} {\scriptstyle 7\ 10\ 2\ \ 13\ 11} \\ 803,415 \\ -\ 461,973 \\ \hline 341,442 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 461,973 \\ +\ 341,442 \\ \hline 803,415 \end{array}$$

Topic E

Metric Measurement Conversion Tables

Students use multiplicative comparisons to describe the relative sizes of metric units of length (kilometers, meters, centimeters), mass (kilograms, grams), and liquid volume (liters, milliliters). They express larger units in terms of smaller units and complete conversion tables. Students add and subtract mixed unit measurements.

1 kilometer is 1,000 times as long as 1 meter.

1 km = 1,000 × 1 m

1 kilometer = 1,000 meters

After This Module

Grade 5 Modules 1 and 4

In grade 5 modules 1 and 4, students extend the work of grade 4 by adding, subtracting, rounding, and comparing multi-digit numbers with digits to the thousandths place. Students recognize that the value of a digit in one place is $\frac{1}{10}$ of what it represents in the place to its left.

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Multiplication as Multiplicative Comparison

Lesson 1 16
Interpret multiplication as multiplicative comparison.

Lesson 2 40
Solve multiplicative comparison problems with unknowns in various positions.

Lesson 3 58
Describe relationships between measurements by using multiplicative comparison.

Lesson 4 76
Represent the composition of larger units of money by using multiplicative comparison.

Topic B 95

Place Value and Comparison Within 1,000,000

Lesson 5 98
Organize, count, and represent a collection of objects.

Lesson 6 140
Demonstrate that a digit represents 10 times the value of what it represents in the place to its right.

Lesson 7 160
Write numbers to 1,000,000 in unit form and expanded form by using place value structure.

Lesson 8 180
Write numbers to 1,000,000 in standard form and word form.

Lesson 9 200
Compare numbers within 1,000,000 by using >, =, and <.

Topic C 219

Rounding Multi-Digit Whole Numbers

Lesson 10 222
Name numbers by using place value understanding.

Lesson 11. 256
Find 1, 10, and 100 thousand more than and less than a given number.

Lesson 12 278
Round to the nearest thousand.

Lesson 13 294
Round to the nearest ten thousand and hundred thousand.

Lesson 14 310
Round multi-digit numbers to any place.

Lesson 15 324
Apply estimation to real-world situations by using rounding.

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Topic D 340
Multi-Digit Whole Number Addition and Subtraction

Lesson 16 346
Add by using the standard algorithm.

Lesson 17 364
Solve multi-step addition word problems by using the standard algorithm.

Lesson 18 380
Subtract by using the standard algorithm, decomposing larger units once.

Lesson 19 398
Subtract by using the standard algorithm, decomposing larger units up to 3 times.

Lesson 20 414
Subtract by using the standard algorithm, decomposing larger units multiple times.

Lesson 21 430
Solve two-step word problems by using addition and subtraction.

Lesson 22 448
Solve multi-step word problems by using addition and subtraction.

Topic E 465
Metric Measurement Conversion Tables

Lesson 23 468
Express metric measurements of length in terms of smaller units.

Lesson 24 488
Express metric measurements of mass and liquid volume in terms of smaller units.

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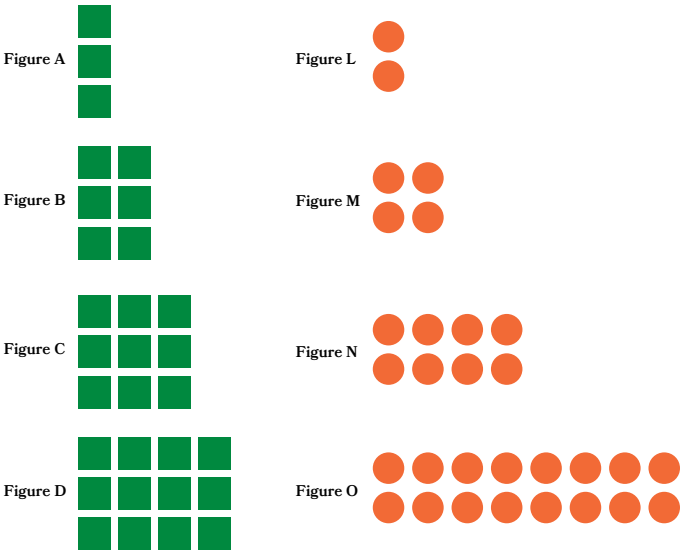
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Why

Place Value Concepts for Addition and Subtraction

Why does the place value module begin with a topic on multiplicative comparisons?

Beginning with multiplicative comparison enables students to build on their prior knowledge of multiplication from grade 3 and provides a foundation upon which students can explore the relationships between numbers and place value units. This placement also activates grade 3 knowledge of multiplication and division facts within 100 and provides students with opportunities to continue building fluency with the facts in preparation for multiplication and division in modules 2 and 3.

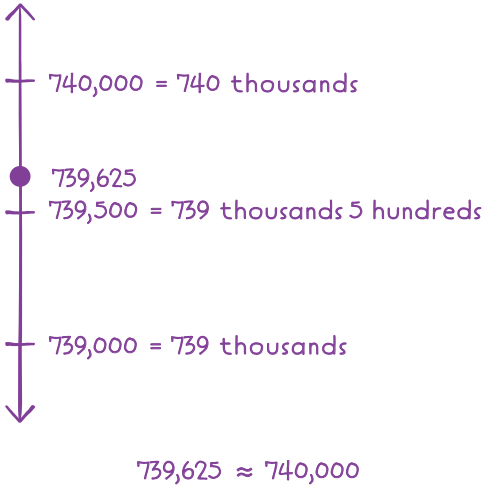


Students are familiar with additive comparison—relating numbers in terms of *how many more* or *how many less*. Multiplicative comparison—relating numbers as *times as many*—is a new way to compare numbers. Students use multiplicative comparison throughout the year to relate measurement units, whole numbers, and fractions. This important relationship between factors, where one factor tells how much larger the product is compared to the other factor, is foundational to ratios and proportional relationships in later grades. Taking time to develop this understanding across the grade 4 modules sets students up for success with interpreting multiplication as scaling in grade 5 and applying or finding a scale factor in scale drawings, dilations, and similar figures.

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Why is the vertical number line used for rounding numbers?

The vertical number line is used to help support conceptual understanding of rounding. In grade 3, students first see the vertical number line as an extension of reading a vertical measurement scale. Using the context of temperature, students identify the tens (i.e., benchmarks) between which a temperature falls, the halfway mark between the benchmark temperatures, and the benchmark temperature the actual temperature is closer to. Students then generalize to round numbers to the nearest ten and hundred.



In grade 4, students round numbers with up to 6 digits to any place. They continue to use the vertical number line as a supportive model. Labeling the benchmark numbers and halfway tick mark in both standard form and unit form helps emphasize the unit to which a number is being rounded. This way, the place values line up vertically, helping students see the relationship between the numbers.

The pictorial support of the vertical number line when rounding is eventually removed, but the conceptual understanding of place value remains as students round mentally. These experiences with the vertical number line prepare students for representing ratios with vertical double number lines and graphing pairs of values in the coordinate plane.

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Why are metric units of measurement addressed in this module? When are customary units of measurement addressed?

Work with metric units of length, mass, and liquid volume in topic E provides an opportunity for students to apply their place value understanding to a measurement context. Students convert metric units that have relationships involving hundreds (e.g., meters to centimeters) and thousands (e.g., kilograms to grams). They apply multi-digit addition and subtraction strategies, including the standard algorithm, to add and subtract mixed-unit measurements. Introducing metric units in module 1 also provides the opportunity to use the units in word problem contexts throughout the rest of the year.

Kilograms	Grams
1	1,000
2	2,000
5	5,000
12	12,000
583	583,000

Customary units are included within modules 2 and 3 because the relative sizes of customary units of measurement do not align with the place value unit structure. Customary units of length are addressed in module 2 when students work with two-digit multiplication, area, and perimeter. Additionally, units of time and customary units of weight and liquid volume are addressed in module 3 alongside multiplication and problem solving.

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Terminology

The following terms are critical to the work of grade 4 module 1. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

billion
A billion is a unit composed of 10 hundred millions. It also names a place in the place value system. *A billion is a thousand millions.* (Lesson 8)

convert
To convert is to express a measurement in one unit in terms of a different unit (i.e., a renaming of a unit). For example, 2 meters can be converted to 200 centimeters. (Lesson 23)

ten thousand
Ten thousand is a unit composed of 10 thousands. It also names a place in the place value system. (Lesson 5)

hundred thousand
Hundred thousand is a unit composed of 10 ten thousands. It also names a place in the place value system. (Lesson 5)

kilometer
A kilometer is a unit for measuring distance or length. A kilometer is 1,000 meters. *A kilometer is about 2 and 1 half laps around an athletic track.* (Lesson 23)

million
A million is a unit composed of 10 hundred thousands. It also names a place in the place value system. *A million is a thousand thousands.* (Lesson 5)

mixed units
Numbers that are named by using more than one type of unit have mixed units (e.g., 1 meter 75 centimeters). (Lesson 23)

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Familiar

bundle

centimeter

exchange

expanded form

gram

kilogram

liter

mass

meter

milliliter

regroup

rename

round

standard algorithm

standard form

unbundle

unit form

word form

≈

Academic Verbs

express

justify

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Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

24	Colored pencils
25	Dry-erase markers
6	Envelopes
24	*Learn* books
6	Markers
9	Meter sticks
25	Pencils
25	Personal whiteboards
25	Personal whiteboard erasers
25	*Eureka Math*® place value disks set, ones to millions
1	Projection device
25	Rulers
3	Sticky notes, pad
1	*Teach* book
1	Teacher computer or device

Visit http://eurmath.link/materials to learn more.

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Before This Module

Grade 3 Modules 1 and 3

In grade 3, students develop conceptual understanding of multiplication and division and become fluent with multiplication and division facts within 100. They multiply multiples of 10 by one-digit numbers, recognizing that they can use familiar facts and only the unit changes. Models and methods used in grade 3 include drawing equal groups and arrays, using the break apart and distribute strategy, writing equations in unit form and standard form, and applying the commutative and associative properties.

Grade 3 Module 4

In grade 3, students find the areas and perimeters of rectangles. They tile rectangles and count the number of square units to determine the areas, eventually recognizing that the area can be found by multiplying the number of square units in each row by the number of square units in each column. Students find the perimeter by adding and by using multiplication, focusing on the relationships between the side lengths in rectangles and other polygons.

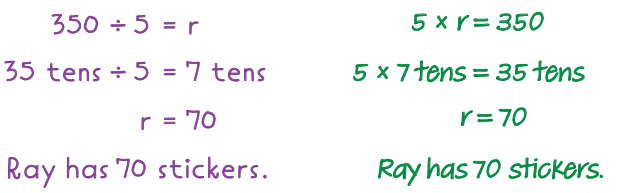
Overview

Place Value Concepts for Multiplication and Division

Topic A

Compose and Decompose Units of Ten

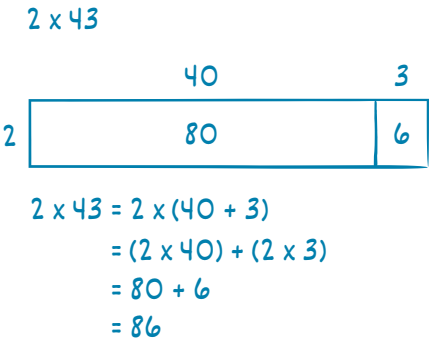
Students begin multi-digit multiplication and division by multiplying and dividing multiples of 10 by one-digit numbers. They develop conceptual understanding by representing the multiplication and division with place value disks or on a place value chart and by naming multiples of 10 in unit form. Students multiply and divide by using equations. They apply the associative property of multiplication, which allows them to make use of familiar facts to multiply (e.g., $5 \times 60 = 5 \times 6 \times 10 = (5 \times 6) \times 10$). To divide, students rename two-digit and three-digit multiples of 10 as tens to make use of familiar division facts, and they relate division to an unknown factor problem. Students multiply and divide to find the area or unknown side length of a rectangle with the newly formalized area formula $A = l \times w$.



Topic B

Multiplication of Tens and Ones by One-Digit Numbers

Students multiply two-digit numbers by one-digit numbers by using the distributive property. They decompose the two-digit numbers into tens and ones and then multiply each part by the one-digit number. Students use a place value chart, an area model, and equations to represent the multiplication. Equations are written in both unit form and standard form to record the multiplication represented in the models. When multiplying by using equations, they



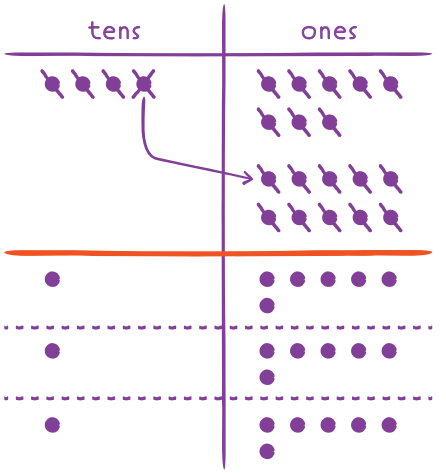
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write or think about numbers in unit form and recognize that, although the units change, the multiplication facts are familiar. Students apply their learning to solve one-step word problems. An optional lesson at the end of the topic provides an opportunity for students to use simplifying strategies, such as compensation and decomposition, to multiply.

Topic C

Division of Tens and Ones by One-Digit Numbers

Students divide two-digit and three-digit numbers by one-digit numbers by using the break apart and distribute strategy. They decompose the two-digit or three-digit total into tens and ones and then divide each part by the one-digit number. Students use a place value chart, an area model, and equations to represent the division. Equations are written in both unit form and standard form to record the division represented in the models. When dividing by using equations, students write or think about the numbers in unit form. Students recognize that, although the units change, the division facts are familiar. They apply their learning to solve one-step word problems. Three-digit division in this module is limited to expressions such as $126 \div 3$ where the number of hundreds is less than the divisor.



$48 \div 3 = (3 \text{ tens} + 18 \text{ ones}) \div 3$
$= (3 \text{ tens} \div 3) + (18 \text{ ones} \div 3)$
$= (30 \div 3) + (18 \div 3)$
$= 10 + 6$
$= 16$

Topic D

Problem Solving with Measurement

Students express larger customary units of length (i.e., yards and feet) in terms of smaller units (i.e., feet and inches) by using tape diagrams, number lines, and conversion tables. They focus on the relationships between the units and use repeated addition, skip-counting, and multiplication to complete the conversions. Students also convert mixed units to a smaller unit (e.g., yards and feet to feet) and add and subtract with mixed units by using number lines, number bonds, the arrow way, and equations. The measurement units provide a context for investigating the perimeter of rectangles. The relationships between the side lengths and perimeter of a rectangle are formalized. Students apply their understanding of customary units of length and the formulas for area and perimeter to solve problems including those that have both additive and multiplicative comparisons.

The area of the rectangle is 44 square feet. The perimeter is 30 feet. What are the length and width of the rectangle?



$2 \times (l + w) = 30$
$30 \div 2 = 15$
$15 = 11 + 4$
$11 \times 4 = 44$

The length of the rectangle is 11 feet.
The width of the rectangle is 4 feet.

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After This Module

Grade 4 Module 3

In module 3, students extend the work of module 2. They apply place value understanding and their understanding of the properties of operations to multiply and divide larger numbers. Students multiply and divide numbers of up to four digits by one-digit numbers and multiply two-digit numbers by two-digit numbers. They develop methods for recording products, quotients, partial products, and partial quotients vertically.

Topic E

Factors and Multiples

$72 = 8 \times 9$
$= (2 \times 4) \times 9$
$= 2 \times (4 \times 9)$
$= 2 \times 36$

Students use what they know about multiplication and division to help them identify factors, multiples, prime numbers, and composite numbers within 100. They list factor pairs by using arrays, division, and the associative property. Number bonds and equations help students to organize their thinking. They skip-count to find multiples and recognize relationships between factors and multiples—a number is a multiple of each of its factors and factors can be used to find other factors. Students explore properties of prime and composite numbers within 100. They apply their understanding of factors and multiples to determine whether a number is divisible by another number and to find an unknown term in shape or number patterns.

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Place Value Concepts for Multiplication and Division

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Compose and Decompose Units of Ten

Lesson 1 18
Multiply multiples of 10 by one-digit numbers by using the associative property of multiplication.

Lesson 2 32
Divide two- and three-digit multiples of 10 by one-digit numbers.

Lesson 3 46
Investigate and use a formula for the area of a rectangle.

Topic B 63

Multiplication of Tens and Ones by One-Digit Numbers

Lesson 4 68
Multiply by using familiar strategies.

Lesson 5 84
Multiply by using place value strategies and the distributive property.

Lesson 6 102
Multiply with regrouping by using place value strategies and the distributive property.

Lesson 7 120
Multiply by using an area model and the distributive property.

Lesson 8 140
Multiply by applying the distributive property and write equations.

Lesson 9 158
Solve multiplication word problems.

Lesson 10 178
Multiply by applying simplifying strategies. (Optional)

Topic C 197

Division of Tens and Ones by One-Digit Numbers

Lesson 11 200
Divide by using familiar strategies.

Lesson 12 214
Divide two-digit numbers by one-digit numbers by using an area model.

Lesson 13 230
Divide three-digit numbers by one-digit numbers by using an area model.

Lesson 14 248
Divide two-digit numbers by one-digit numbers by using place value strategies.

Lesson 15 270
Divide three-digit numbers by one-digit numbers by using place value strategies.

Lesson 16 286
Divide by using the break apart and distribute strategy.

Topic D 307

Problem Solving with Measurement

Lesson 17 310
Express measurements of length in terms of smaller units.

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Lesson 18 328
Investigate and use formulas for the perimeter of a rectangle.

Lesson 19 344
Apply area and perimeter formulas to solve problems.

Lesson 20 362
Solve word problems involving additive and multiplicative comparisons.

Topic E 377
Factors and Multiples

Lesson 21 380
Find factor pairs for numbers up to 100 and use factors to identify numbers as prime or composite.

Lesson 22 396
Use division and the associative property of multiplication to find factors.

Lesson 23 414
Determine whether a whole number is a multiple of another number.

Lesson 24 432
Recognize that a number is a multiple of each of its factors.

Lesson 25 450
Explore properties of prime and composite numbers up to 100 by using multiples.

Lesson 26 466
Use relationships within a pattern to find an unknown term in the sequence.

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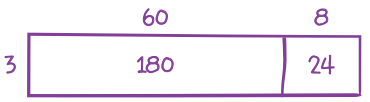
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Why

Place Value Concepts for Multiplication and Division

Why are multiplication and division included together within modules 2 and 3 rather than having one module dedicated to multiplication and one module dedicated to division?



$204 \div 3 = (180 + 24) \div 3$
$= (180 \div 3) + (24 \div 3)$
$= 60 + 8$
$= 68$

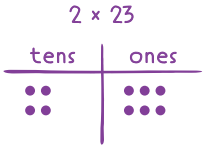
$3 \times 68 = 3 \times (60 + 8)$
$= (3 \times 60) + (3 \times 8)$
$= 180 + 24$
$= 204$

Modules 2 and 3 in grade 4 are structured similarly to the introduction of multiplication and division in modules 1 and 3 in grade 3—the modules are distinguishable by number choice rather than by operation, and they highlight the relationship between multiplication and division. Module 2 focuses on multiplying two-digit numbers by one-digit numbers by naming the two-digit numbers as tens and ones, and on dividing two-digit and three-digit numbers by one-digit numbers by naming the two-digit and three-digit numbers as tens and ones. Module 3 builds fluency with multiplication and division and extends the work up to four-digit numbers—thousands, hundreds, tens, and ones. In both modules, students can use their understanding of multiplication to find quotients by thinking of division as an unknown factor problem. Similar models, strategies, and recording methods provide coherence with each operation to support students' conceptual understanding of multiplication and division.

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Why are multiple models used for multiplication and division?

Multiple models are used for multiplication and division to support student understanding. Students use place value disks, drawings on a place value chart, area models, and equations written in unit form and standard form.



2 × 23 = 2 × (2 tens + 3 ones)
= (2 × 2 tens) + (2 × 3 ones)
= (4 tens) + (6 ones)
= 40 + 6
= 46

The use of place value disks, drawings on a place value chart, and equations written in unit form support students in isolating the individual place value units in the total and in each equal group. When students identify a number of units, they can use familiar multiplication and division facts to find the number of each unit in the product or quotient. This builds a foundation for students to understand the value of each digit as they record multiplication and division vertically in module 3.

Representing multiplication and division with area models begins in grade 3 and continues through grade 5 and beyond. The area model is used for both multiplication and division and demonstrates the relationship between the two operations, whether used with whole numbers, fractions, or algebraically. The area model is also useful for keeping track of how a number, either the total or a factor, is decomposed and for ensuring that each part of that number is multiplied or divided. As the numbers get larger, the area model can be more efficient to draw than other models and can be a helpful scaffold in the movement from pictorial representations to abstract recordings.

The common thread between the models and methods is the distributive property. When multiplying, students see a multi-digit factor broken apart and see that each part is multiplied to find partial products. When dividing, students see how the total is decomposed and how each part is divided to find partial quotients.



138 ÷ 6 = (120 + 18) ÷ 6
= (120 ÷ 6) + (18 ÷ 6)
= 20 + 3
= 23

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Why is the *distributive property* used in reference to multiplication and the *break apart and distribute strategy* used in reference to division?

In grade 3, students use the break apart and distribute strategy with multiplication and division. In grade 4, the distributive property is formally named, and students recognize that the distributive property relates multiplication and addition. For example, $7 \times 23 = 7 \times (20 + 3) = (7 \times 20) + (7 \times 3)$.

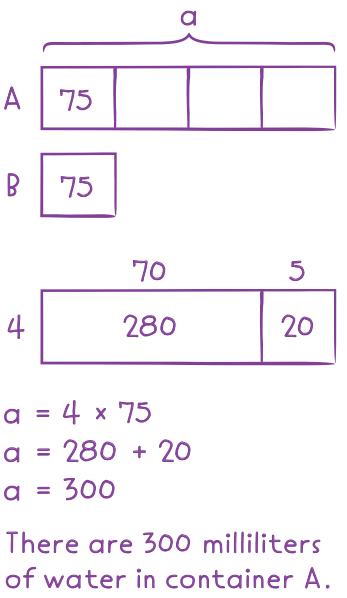
The distributive property is more formally the distributive property of multiplication over addition. Thus, the distributive property does not apply to division. With division, the break apart and distribute strategy continues to be referenced and describes how students approach a division problem. They break apart the total into smaller parts and then divide each part by the divisor. For example, $72 \div 8 = (40 \div 8) + (32 \div 8)$.

In grade 5, students learn that dividing by n is equivalent to multiplying by $\frac{1}{n}$, so the distributive property can be applied. For example, $72 \div 8 = 72 \times \frac{1}{8} = (40 + 32) \times \frac{1}{8} = \left(40 \times \frac{1}{8}\right) + \left(32 \times \frac{1}{8}\right) = (40 \div 8) + (32 \div 8)$.

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Are a tape diagram and an area model the same thing? Why might both be drawn when problem solving?

Tape diagrams are used to help students make sense of word problems and show the relationship between what is known and unknown in a word problem. Students use tape diagrams to find solution paths and then use another means to find their answers. For example, they might see from a tape diagram that they need to multiply 75 by 4 to solve a problem. The tape diagram does not help them to multiply 4 and 75, but an area model can. An area model is used to show the relationship between the factors and product or the relationship between the total, divisor, and quotient. With multiplication, students use the area model to show one factor broken into smaller parts, usually by place value, that are more manageable to multiply. With division, students use the area model to show the total broken into smaller parts that are more manageable to divide.



Why is lesson 10 considered optional?

Lesson 10 provides students with an opportunity to use simplifying strategies, like compensation and decomposition, to multiply in ways other than decomposing by place value. Such strategies can sometimes be used to find efficient ways to multiply. Students reason about the numbers, think flexibly, and make use of structure to find a product. For example, to find 3×79, they see that it may be more efficient to multiply 3 and 80 first and then subtract 3 than it is to multiply 7 tens 9 ones by 3. To find 6×73, students may decompose 73 in a way other than by place value to make it easier for them to multiply. For example, they might find 6×50, 6×20, and 6×3 and then add the partial products. This lesson is considered optional because students are not expected to reach mastery with these simplifying strategies. Consider including this lesson to help students further develop their number sense and mathematical reasoning skills and to connect simplifying strategies for addition to simplifying strategies for multiplication.

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Terminology

The following terms are critical to the work of grade 4 module 2. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

associative property of multiplication
The associative property of multiplication means that grouping the factors in a three-factor multiplication expression in different ways does not change the product. For example, $7 \times (6 \times 10) = (7 \times 6) \times 10$. (Lesson 1)

composite number
A whole number greater than 1 is a composite number if it has more than two factors. For example, 9 and 15 are composite numbers. *A whole number greater than* 1 *that is not a composite number is a prime number.* (Lesson 21)

distributive property
The distributive property means that decomposing one factor in a two-factor multiplication expression and multiplying each part by the other factor does not change the product. For example, $2 \times (30 + 4) = (2 \times 30) + (2 \times 4)$. *The full name of the property is the distributive property of multiplication over addition. Referring to it as the distributive property is sufficient at grade 4.* (Lesson 5)

divisible
A number is divisible by a second number if the second number is a factor of the given number. For example, 30 is divisible by 3 since $30 = 3 \times 10$. *The phrase divided evenly is sometimes used as well (e.g.,* 30 *can be divided evenly by* 3*).* (Lesson 22)

divisor
In a division expression, the number you are dividing by is the divisor. For example, in the expression $18 \div 3$, the divisor is 3. (Lesson 14)

formula
A formula is a rule or equation that is used to determine something. For example, a formula for the area of a rectangle with side lengths l and w is $A = l \times w$ and a formula for the perimeter of the rectangle is $P = 2 \times (l + w)$. (Lesson 3)

partial product
When decomposing one factor in a two-factor multiplication expression and multiplying each part by the other factor, the resulting products are called partial products. For example, $2 \times 34 = 2 \times (30 + 4) = (2 \times 30) + (2 \times 4) = 60 + 8$, and the expressions 2×30 and 2×4 or 60 and 8 are partial products. (Lesson 5)

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partial quotient
In a division expression, when a total is decomposed into parts and each of those parts is divided by the divisor, the resulting quotients are called partial quotients. For example, $72 \div 4 = (60 \div 4) + (12 \div 4) = 15 + 3$, and the expressions $60 \div 4$ and $12 \div 4$ or 15 and 3 are partial quotients. (Lesson 16)

prime number
A whole number greater than 1 is a prime number if its only factors are 1 and itself. For example, 7 and 11 are prime numbers. *A whole number greater than 1 that is not a prime number is a composite number.* (Lesson 21)

term (in a pattern)
A number or shape pattern is an ordered list of items (numbers or objects or images) and any one item in the list is called a term. For example, in the number pattern $1, 2, 4, 8, 16, \ldots$, the fourth term in the list is 8. (Lesson 26)

Familiar

area

area model

commutative property of multiplication

factor

foot

inch

length

multiple

perimeter

product

quotient

total

width

yard

Academic Verb

claim

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Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	Chart paper, tablet
25	Colored pencils (red and blue)
25	Crayons (red, green, and blue)
25	Dry-erase markers
24	*Learn* books
1	Meter stick
27	Paper strips, 1" × 12"
25	Pencils
25	Personal whiteboards
25	Personal whiteboard erasers
25	*Eureka Math*$^{2™}$ place value disks set, ones to millions
1	Projection device
25	Rulers
1	*Teach* book
1	Teacher computer or device

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Before This Module

Grade 4 Module 2

In module 2, students multiply two-digit numbers by one-digit numbers by using the distributive property and divide two- and three-digit numbers by one-digit numbers by using the break apart and distribute strategy. They rename the larger factor or total as tens and ones and then multiply or divide each part, as applicable. They use a place value chart, area model, and equations to represent the multiplication and division. Students express single customary units and mixed customary units of length in terms of smaller units. They show the conversions by using tape diagrams, number lines, and conversion tables and add or subtract to find the sum or difference of measurements. Students also identify factors and multiples of numbers within 100.

Overview

Multiplication and Division of Multi-Digit Numbers

Topic A

Multiplication and Division of Multiples of Tens, Hundreds, and Thousands

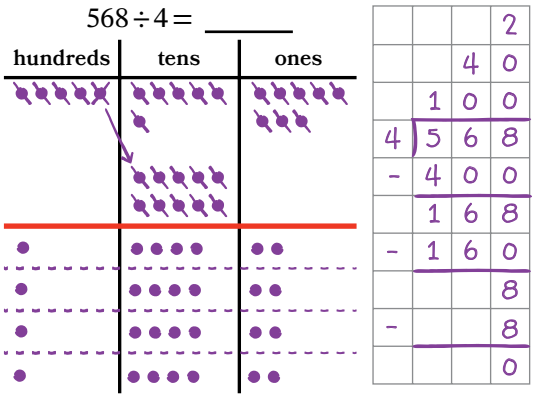
Students multiply and divide multiples of 10, 100, and 1000 by focusing on place value units. They use place value disks and write equations in unit form to help them recognize that they can use familiar multiplication and division facts to find products and quotients. Application of the associative property helps students to rewrite two-factor multiplication expressions as three-factor expressions so, again, they can multiply by using familiar facts. To help prepare for multiplication of two-digit numbers by two-digit numbers, an area model is used to show that multiplying a multiple of 10 by a multiple of 10 results in a number with the unit of hundreds.

$$\begin{aligned} 2100 \div 3 &= 21 \text{ hundreds} \div 3 \\ &= 7 \text{ hundreds} \\ &= 700 \end{aligned}$$

Topic B

Division of Thousands, Hundreds, Tens, and Ones

Students divide numbers of up to four digits by one-digit numbers. They draw an area model, represent the divisor as one side length, and compose the unknown side length by building up to the total. Students also represent the division on a place value chart. They decompose the totals into place value units, divide each unit, and record long division in vertical form alongside the place value chart to reinforce conceptual understanding. They recognize that, although the value of the unit is different, the process of dividing each unit remains the same.



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Topic C

Multiplication of up to Four-Digit Numbers by One-Digit Numbers

Students apply the distributive property to multiply numbers of up to four digits by one-digit numbers. They break apart the larger factor by place value and multiply the number of each unit by the one-digit factor. They represent the multiplication by using place value charts, area models, and vertical form. Students record partial products in vertical form by recording each partial product separately and by recording them together on one line.

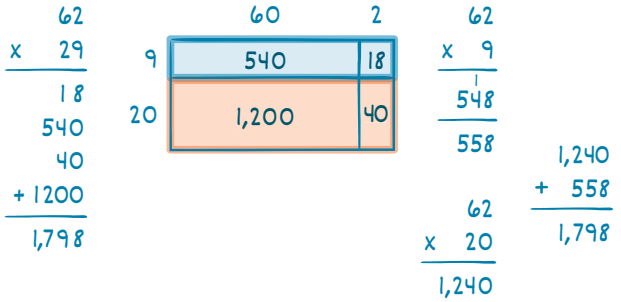
	4,	8	7	9
×				2
			1	8
		1	4	0
	1	6	0	0
+	8	0	0	0
	9,	7	5	8

	4,	8	7	9
×				2
	$^{1}8$	$^{1}6$	$^{1}4$	8
	9,	7	5	8

Topic D

Multiplication of Two-Digit Numbers by Two-Digit Numbers

Students apply the associative and distributive properties to multiply a two-digit number by a multiple of 10 and then progress to multiplying two-digit numbers by two-digit numbers. Area models are used to represent the multiplication and to help students recognize how each factor is broken apart and multiplied.



Students see that each part of one factor is multiplied by each part of the other factor. They record four partial products in the area model and in vertical form alongside the area model and then transition to recording two partial products in the same way. Students add the partial products to find the product.

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After This Module

Grade 5 Modules 1 and 2

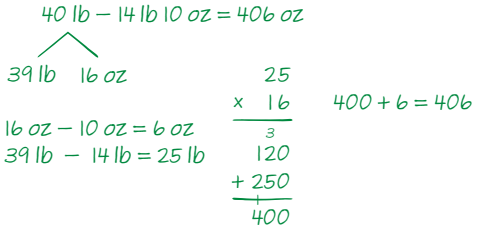
In grade 5 module 1, students multiply multi-digit whole numbers and develop fluency with the standard algorithm for multiplication. Students also divide with two-digit divisors and continue building conceptual understanding of multi-digit whole-number division. They find whole-number quotients and remainders. In module 2, students transition from finding whole-number quotients and remainders to fractional quotients.

Grade 5 Modules 1, 3, and 4

In modules 1, 3, and 4 of grade 5, students use multiplicative relationships to convert metric and customary units involving whole numbers, fractions, and decimals. In addition to expressing larger measurement units in terms of smaller units, they express smaller measurement units in terms of larger units.

Topic E

Problem Solving with Measurement



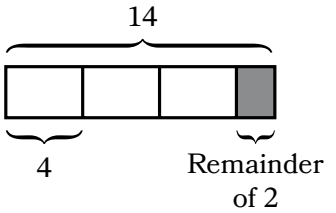
Students use multiplicative relationships to convert units of time and customary units of weight and liquid volume to smaller units. They use conversion tables and number lines to express larger measurement units in terms of smaller units and recognize that the smaller units are all multiples of the same number. Students notice relationships in the conversion tables and use the tables to convert other amounts. Throughout the topic, students add and subtract mixed units by using different methods including the method of expressing larger units in terms of smaller units before adding or subtracting and the method of adding or subtracting like units, renaming as necessary.

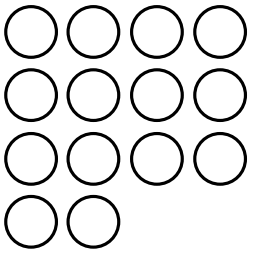
Topic F

Remainders, Estimating, and Problem Solving

Students divide with numbers that result in whole-number quotients and remainders. They recognize the remainder as the amount remaining after finding a whole-number quotient, and they solve word problems that require interpretation of the whole-number quotient and remainder. Students estimate quotients by finding a multiple of the divisor that is close to the total and then dividing. They reason about the relationship between their estimate and the actual quotient and apply their thinking to assess the reasonableness of their answers to division word problems. Students use the four operations to solve multi-step word problems. They draw tape diagrams that represent the known and unknown information in the problem to help them find a solution path. After solving, students assess the reasonableness of their answers.

$14 \div 3$





$14 = (3 \times 4) + 2$

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Divide multiples of 100 and 1000.

Lesson 2 32
Multiply by multiples of 100 and 1000.

Lesson 3 46
Multiply a two-digit multiple of 10 by a two-digit multiple of 10.

Topic B 63

Division of Thousands, Hundreds, Tens, and Ones

Lesson 4 66
Apply place value strategies to divide hundreds, tens, and ones.

Lesson 5 86
Apply place value strategies to divide thousands, hundreds, tens, and ones.

Lesson 6 102
Connect pictorial representations of division to long division.

Lesson 7 122
Represent division by using partial quotients.

Lesson 8 144
Choose and apply a method to divide multi-digit numbers.

Topic C 157

Multiplication of up to Four-Digit Numbers by One-Digit Numbers

Lesson 9 160
Apply place value strategies to multiply three-digit numbers by one-digit numbers.

Lesson 10 178
Apply place value strategies to multiply four-digit numbers by one-digit numbers.

Lesson 11. 194
Represent multiplication by using partial products.

Lesson 12 214
Multiply by using various recording methods in vertical form.

Topic D 232

Multiplication of Two-Digit Numbers by Two-Digit Numbers

Lesson 13 236
Multiply two-digit numbers by two-digit multiples of 10.

Lesson 14 254
Apply place value strategies to multiply two-digit numbers by two-digit numbers.

Lesson 15 274
Multiply with four partial products.

Lesson 16 290
Multiply with two partial products.

Lesson 17 308
Apply the distributive property to multiply.

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Topic E 324

Problem Solving with Measurement

Lesson 18 326
Express units of time in terms of smaller units.

Lesson 19 342
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Lesson 20 360
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Remainders, Estimating, and Problem Solving

Lesson 21 382
Find whole-number quotients and remainders.

Lesson 22 398
Represent, estimate, and solve division word problems.

Lesson 23 418
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Lesson 24 438
Solve multi-step word problems and assess the reasonableness of solutions.

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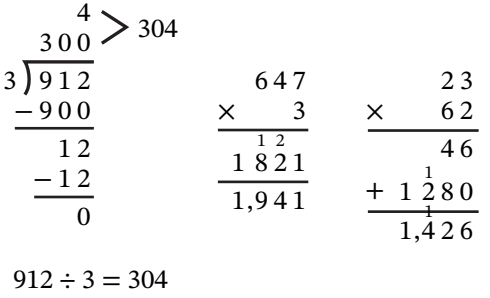
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Why

Multiplication and Division of Multi-Digit Numbers

Why does work with division come before multiplication in module 3?



The work of module 3 builds on the work of module 2. In module 2, students multiply and divide with tens and ones. Division follows multiplication and students relate the methods used for division with those they use for multiplication (e.g., distributive property, area model). Module 2 ends with a focus on factors and multiples. To continue the progression of learning from module 2 and provide an opportunity for application of the work with factors and multiples, division precedes multiplication in module 3. Students relate the methods used to divide with tens and ones to division of three- and four-digit numbers. Although the work focuses on division, students use their fluency with multiplication facts to help them divide. They then transition back to multiplication, first multiplying three- and four-digit numbers by one-digit numbers and then multiplying two-digit numbers by two-digit numbers.

Why do remainders come at the end of the module and not earlier in students' work with division?

Division resulting in whole-number quotients and remainders presents an added complexity to division, both conceptually and computationally. To provide ample time to refine students' skills and conceptual understanding of both division and multiplication, remainders are introduced last in the module. Students find whole-number quotients and remainders and then immediately have opportunities to apply their new learning to situations and word problems in which the whole-number quotients and remainders are interpreted.

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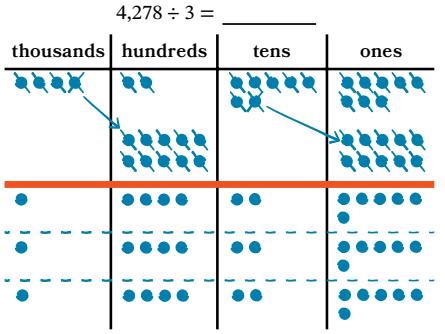
What is the intent of using vertical form for multiplication and division?

In grade 4, students multiply and divide by using strategies based on place value and the properties of operations. This module uses the representations of place value charts, area models, and vertical form because they are based on place value understanding. The intent of vertical form is to provide a written method to record the process of multiplication and division by using partial products and partial quotients. Vertical form can be more efficient than representing a problem with an area model or a place value chart. Because students mature in their understanding of multi-digit multiplication and division at different times, expect and accept a variety of representations. Fluency with the multiplication and division algorithm is not expected until grade 5 and grade 6, respectively.

$$\begin{array}{r} 708 \\ \times \quad 5 \\ \hline 40 \\ +3\,500 \\ \hline 3{,}540 \end{array} \qquad \begin{array}{r} 3{,}568 \\ \times \quad 4 \\ \hline {\scriptstyle 2\ 2\ 3} \\ 12\,042 \\ \hline 14{,}272 \end{array}$$

Why is vertical form introduced alongside the place value chart for multiplication and division?

Similar to what students experience with addition and subtraction, vertical form is introduced alongside the place value chart for multiplication and division to support conceptual understanding and the transition from a pictorial representation to a written representation. Each action represented in the place value chart (e.g., renaming units, adding or subtracting like units, distributing units, finding the total quantity of each unit) has a direct connection to a recording within vertical form. As students become proficient with recording in vertical form, they internalize the process and no longer require drawing on the place value chart to find the unknown or explain their work.



				6
			2	0
		4	0	0
	1,	0	0	0
3	4,	2	7	8
-	3	0	0	0
	1	2	7	8
-	1	2	0	0
			7	8
-			6	0
			1	8
-			1	8
				0

Additionally, students not yet fluent with multiplication and division facts may find the place value chart helpful in keeping track of their calculations within vertical form.

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Terminology

The following terms are critical to the work of grade 4 module 3. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

cup
A cup is a unit for measuring liquid volume. A juice glass has a capacity of about 1 cup. Cup, pint, quart, gallon, ounce, and pound are all customary units. (Lesson 20)

gallon
A gallon is a unit for measuring liquid volume. A large plastic container of milk has a capacity of about 1 gallon. (Lesson 20)

long division
Long division is a process used to find the quotient of a division expression. The recording of the process is referred to as vertical form. (Lesson 6)

ounce
An ounce is a unit for measuring weight. Five quarters weigh about 1 ounce. (Lesson 19)

pint
A pint is a unit for measuring liquid volume. A pint is 2 cups. An average-size water bottle has a capacity of about 1 pint. (Lesson 20)

pound
A pound is a unit for measuring weight. A soccer ball weighs about 1 pound. (Lesson 19)

quart
A quart is a unit for measuring liquid volume. There are 4 quarts in a gallon. 1 quart is a little less than 1 liter. (Lesson 20)

remainder
The remainder is an amount remaining after finding a whole-number quotient in a division expression. For example, $17 \div 5$ has a whole-number quotient of 3 and a remainder of 2. (Lesson 21)

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Familiar

algorithm

area

area model

array

associative property of multiplication

commutative property of multiplication

compare

customary units

distribute

distributive property

divide, division

divisor

equation

expression

factor

hours, minutes, seconds

length

metric units

mixed units

multiple

multiply, multiplication

partial product

partial quotient

perimeter

place value

product

quotient

regroup, rename

rows, columns

total

width

yard

Academic Verbs

indicate

interpret

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Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	Chart paper, sheet
25	Dry-erase markers
24	*Learn* books
25	Pencils
25	Personal whiteboards
25	Personal whiteboard erasers
24	*Eureka Math*^{2} place value disks sets, ones to millions
1	Projection device
1	*Teach* book
1	Teacher computer or device
1	Timer

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Before This Module

Grade 3 Module 5

Students formalize their understanding of fractions as numbers in grade 3. Using concrete objects and pictorial models, students define 1 of a fractional unit as a unit fraction. They use unit form and fraction form to represent how unit fractions can be iterated to create non-unit fractions. Students represent fractions on a number line, which further helps them solidify their understanding of fractions as numbers. The distance and location on the number line are used to compare fractions with either the same numerator or the same denominator. Students recognize that when two fractions share the same location or are the same distance from zero on the number line, they are equivalent.

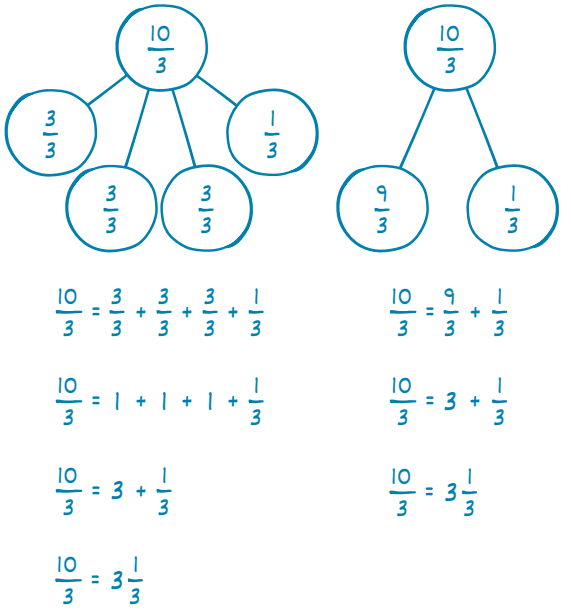
Overview

Foundations for Fraction Operations

Topic A

Fraction Decomposition and Equivalence

Students decompose fractions into a sum of unit fractions and into a sum of non-unit fractions. They use familiar models such as number bonds, tape diagrams, and number lines to represent fractions. They recognize that the area model may be a useful model to represent fractions. Students decompose fractions greater than 1 into a sum of a whole number and a fraction less than 1. This decomposition helps students rename fractions greater than 1 as equivalent mixed numbers. Students express mixed numbers as a sum of a whole number and a fraction less than 1. Then they rename the whole number as an equivalent fraction that they then add to the fraction. This helps students rename mixed numbers as equivalent fractions greater than 1.

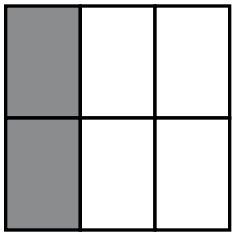


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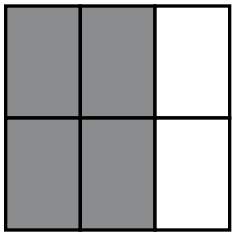
Topic B

Equivalent Fractions

Students generate equivalent fractions and equivalent mixed numbers. They decompose fractional units to find an equivalent fraction with smaller units and record their work with multiplication. They compose fractional units to find an equivalent fraction with larger units and record their work with division. Students use area models, as well as tape diagrams and number lines, to represent fractions and compose or decompose fractional units to generate equivalent fractions.



$\frac{1}{3} = \frac{2 \times 1}{2 \times 3} = \frac{2}{6}$



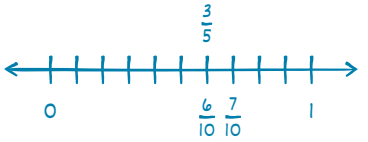
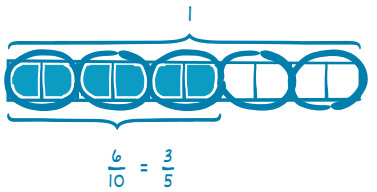
$\frac{2}{3} = \frac{2 \times 2}{2 \times 3} = \frac{4}{6}$

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Topic C

Compare Fractions

Students use various methods to compare fractions less than 1, fractions greater than 1, and mixed numbers. They consider the relationship between the numbers and use what they know about unit fractions to compare fractions to benchmark numbers such as 0, $\frac{1}{2}$, and 1. When the fractions have related numerators or denominators, students use what they know about generating equivalent fractions to rename one fraction to create a common numerator or common denominator. They rename both fractions as equivalent fractions to compare any two fractions. They use similar methods to compare fractions greater than 1 and mixed numbers.

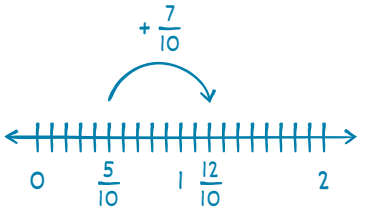
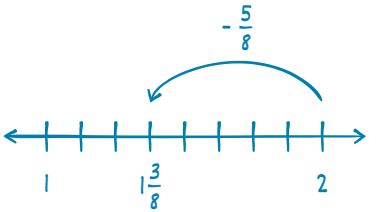
Use the Benchmarks 0, $\frac{1}{2}$, and 1	Create a Common Denominator	Create a Common Numerator
$\frac{1}{2} ___ \frac{5}{8}$ $\frac{1}{2} < \frac{5}{8}$ $\frac{1}{2}$ is equivalent to $\frac{4}{8}$, and $\frac{4}{8}$ is less than $\frac{5}{8}$. So $\frac{1}{2}$ is also less than $\frac{5}{8}$. $\frac{4}{5} ___ \frac{5}{6}$ $\frac{4}{5} < \frac{5}{6}$ Both fractions are only 1 fractional unit away from 1. $\frac{1}{6}$ is smaller than $\frac{1}{5}$, so $\frac{5}{6}$ is closer to 1 than $\frac{4}{5}$.	$\frac{7}{10} ___ \frac{3}{5}$ $\frac{7}{10} > \frac{3}{5}$  $\frac{3}{5} = \frac{2 \times 3}{2 \times 5} = \frac{6}{10}$. $\frac{7}{10} > \frac{6}{10}$ because both fractions have the same fractional unit and 7 is greater than 6. So $\frac{7}{10} > \frac{3}{5}$.	$\frac{6}{10} ___ \frac{3}{4}$ $\frac{6}{10} < \frac{3}{4}$  $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$. $\frac{3}{5} < \frac{3}{4}$ because fifths are smaller units than fourths and 3 of a smaller unit is less than 3 of a larger unit. So $\frac{6}{10} < \frac{3}{4}$.

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Topic D

Add and Subtract Fractions

Students estimate to assess reasonableness when solving word problems to establish an underlying theme for the topic. They add and subtract fractions with like units and subtract a fraction from a whole number. Using unit form and a number line helps students relate their previous work with adding and subtracting whole numbers to adding and subtracting fractions. They see similar part-total relationships and the importance of adding and subtracting like units. Students may apply their part-total understanding to think of a subtraction problem as an unknown addend problem. In an optional lesson, students add fractions with related units by generating equivalent fractions.

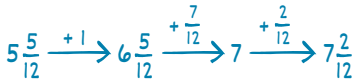
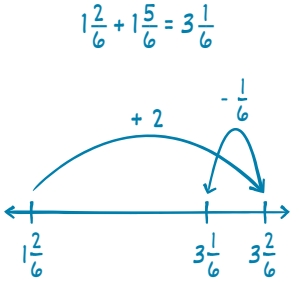
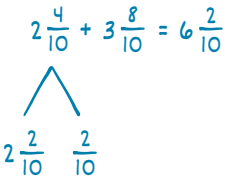
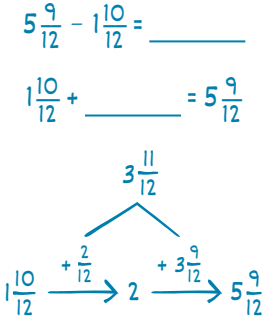
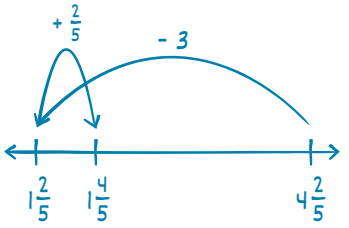
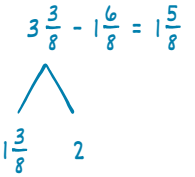
Add or Subtract by Using Unit Form or Fraction Form	Add or Subtract by Using a Number Line	Subtract by Renaming or Decomposing the Total
5 twelfths + 2 twelfths = 7 twelfths $\frac{5}{12} + \frac{2}{12} = \frac{7}{12}$	$\frac{5}{10} + \frac{7}{10} = \frac{12}{10}$ 	$1 - \frac{5}{6} = \frac{1}{6}$ $\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$
4 fifths − 3 fifths = 1 fifth $\frac{4}{5} - \frac{3}{5} = \frac{1}{5}$	$2 - \frac{5}{8} = 1\frac{3}{8}$ 	$2 - \frac{3}{10} = 1\frac{7}{10}$ 

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Topic E

Add and Subtract Mixed Numbers

Students add a fraction to a mixed number and add two mixed numbers. They also subtract a fraction from a mixed number and subtract two mixed numbers. Students apply previously learned strategies for adding and subtracting whole numbers to add and subtract mixed numbers. They use number bonds, the arrow way, and an open number line to represent and record the addition and subtraction. Understanding a mixed number as a sum of a whole number and a fraction helps students as they compose and decompose mixed numbers to add and subtract. Students create and interpret line plots, including solving addition and subtraction problems with fractional data.

Add or Subtract by Counting On by Using the Arrow Way	Add or Subtract by Using Compensation on an Open Number Line	Add by Decomposing a Part or Subtract by Decomposing the Total
		
		

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Topic F

Repeated Addition of Fractions as Multiplication

Students use what they know about multiplying whole numbers to multiply fractions and mixed numbers by whole numbers. They use unit form and the associative property to multiply a fraction by a whole number (e.g., $3 \times \frac{5}{6} = (3 \times 5)$ sixths). To multiply a mixed number by a whole number, students express the mixed number as a sum and then apply the distributive property.

$3 \times \frac{5}{6} = (3 \times 5)$ sixths

$= \frac{3 \times 5}{6}$

$= \frac{15}{6}$

After This Module

Grade 4 Module 5

Students extend their understanding of tenths and hundredths as fractional units to recognizing tenths and hundredths as place value units. They write tenths and hundredths in fraction form and in decimal form. Students apply their understanding of comparing and adding fractions to compare and add tenths and hundredths as decimal numbers.

Grade 5 Module 2

Students add and subtract fractions and mixed numbers with related or unlike units. They use benchmark numbers to estimate mentally and assess the reasonableness of answers. Students apply the same methods to generate equivalent fractions and make like units before they add or subtract by using familiar methods.

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Topic A 19

Fraction Decomposition and Equivalence

Lesson 1 22
Decompose whole numbers into a sum of unit fractions.

Lesson 2 40
Decompose fractions into a sum of unit fractions.

Lesson 3 56
Decompose fractions into a sum of fractions.

Lesson 4 72
Represent fractions by using various fraction models.

Lesson 5 88
Rename fractions greater than 1 as mixed numbers.

Lesson 6 106
Rename mixed numbers as fractions greater than 1.

Topic B 123

Equivalent Fractions

Lesson 7 126
Rename fractions as a sum of equivalent smaller unit fractions.

Lesson 8 148
Generate equivalent fractions with smaller units for unit fractions.

Lesson 9 166
Generate equivalent fractions with smaller units for non-unit fractions.

Lesson 10 182
Generate equivalent fractions with larger units.

Lesson 11 200
Represent equivalent fractions by using tape diagrams, number lines, and multiplication or division.

Lesson 12 220
Generate equivalent fractions for fractions greater than 1 and generate equivalent mixed numbers.

Topic C 237

Compare Fractions

Lesson 13 240
Compare fractions by using the benchmarks 0, $\frac{1}{2}$, and 1.

Lesson 14 254
Compare fractions with related denominators.

Lesson 15 270
Compare fractions with related numerators.

Lesson 16 288
Generate a common numerator or denominator to compare fractions.

Lesson 17 302
Apply fraction comparison strategies to compare fractions greater than 1.

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Topic D 315

Add and Subtract Fractions

Lesson 18 318
Estimate sums and differences of fractions by using benchmarks.

Lesson 19 334
Add and subtract fractions with like units.

Lesson 20 350
Subtract a fraction from a whole number.

Lesson 21 368
Solve addition and subtraction word problems and estimate the reasonableness of the answers.

Lesson 22 382
Add two fractions with related units. (Optional)

Topic E 398

Add and Subtract Mixed Numbers

Lesson 23 404
Add a fraction to a mixed number.

Lesson 24 420
Add a mixed number to a mixed number.

Lesson 25 432
Subtract a fraction from a mixed number, part 1.

Lesson 26 448
Subtract a fraction from a mixed number, part 2.

Lesson 27 462
Subtract a mixed number from a mixed number.

Lesson 28 478
Represent and solve word problems with mixed numbers by using drawings and equations.

Lesson 29 494
Solve problems by using data from a line plot.

Lesson 30 512
Represent data on a line plot.

Topic F 549

Repeated Addition of Fractions as Multiplication

Lesson 31 552
Decompose non-unit fractions into a product of a whole number and a unit fraction.

Lesson 32 566
Multiply a fraction by a whole number by using the associative property.

Lesson 33 582
Solve word problems involving multiplication of a fraction by a whole number.

Lesson 34 600
Multiply a mixed number by a whole number by using the distributive property.

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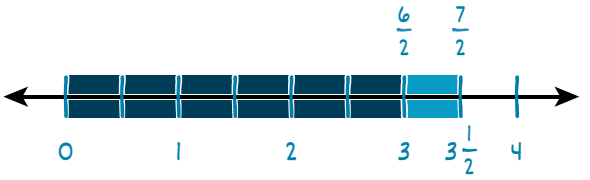
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Why

Foundations for Fraction Operations

Why do fractions greater than 1 and mixed numbers appear throughout the module?

Incorporating fractions less than 1, fractions greater than 1, and mixed numbers together throughout the module solidifies for students the idea that fractions are numbers. Numbers can be decomposed, renamed flexibly, and compared. Operations, such as addition, subtraction, and multiplication, can be performed with numbers. When students perform this work seamlessly with fractions less than 1, fractions greater than 1, and mixed numbers, they can connect their understandings and build on their previous knowledge of one category of numbers and apply it to the next category.



Rename a Fraction Greater Than 1 as a Mixed Number

$\frac{7}{2} = \frac{6}{2} + \frac{1}{2}$

$\frac{7}{2} = 3 + \frac{1}{2}$

$\frac{7}{2} = 3\frac{1}{2}$

Rename a Mixed Number as a Fraction Greater Than 1

$3\frac{1}{2} = \frac{6}{2} + \frac{1}{2}$

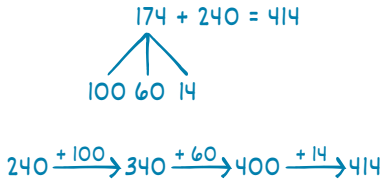
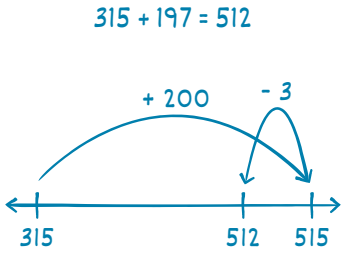
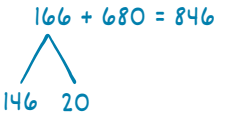
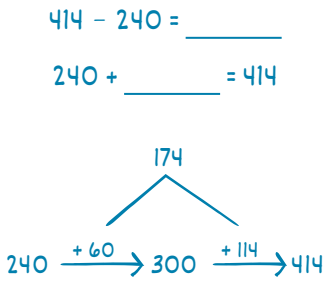
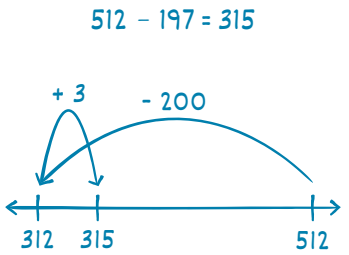
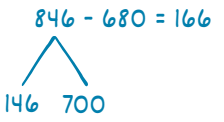
$3\frac{1}{2} = \frac{7}{2}$

Why are so many addition and subtraction strategies used?

The strategies used for adding and subtracting fractions and mixed numbers in topics D and E reflect the strategies students use in grades 1, 2, and 3 to add and subtract whole numbers. These strategies reinforce the idea of fractions as numbers—we can perform operations with fractions similar to the way we perform operations with whole numbers. Because fractions are numbers, they can be composed and decomposed. Students apply the part–total relationship found in addition and subtraction problems to compose and decompose the units of the parts and total.

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Fluency means being accurate and efficient and flexibly applying strategies to solve problems. A strategy may be efficient for solving one problem but time consuming for another. Students analyze problems and select efficient strategies, many of which develop into mental math over time. They select a model to record their work in a way that makes sense to them. Students are not expected to master all the strategies and models taught in module 4 topics D and E. Rather, they are expected to make informed decisions about which strategy to use on a problem-by-problem basis.

Add or Subtract by Counting On by Using the Arrow Way	Add or Subtract by Using Compensation on an Open Number Line	Add by Decomposing a Part or Subtract by Decomposing the Total
		
		

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Why is lesson 22 optional?

In grade 4, students are expected to add and subtract fractions with like units. Lesson 22 invites students to use what they know about generating equivalent fractions and to add two fractions with related units. Later in grade 4, students add tenths and hundredths. Lesson 22 introduces the idea of renaming one fraction before adding to create like units and prepares students to add tenths and hundredths in module 5.

$$\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$$

$$\frac{2}{3} = \frac{2 \times 2}{2 \times 3} = \frac{4}{6}$$

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Terminology

The following terms are critical to the work of grade 4 module 4. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

common denominator
Two or more fractions that have the same fractional unit have a common (or the same) denominator. *The phrase* find a common denominator *is sometimes used to refer to renaming two or more fractions with the same denominator.* (Lesson 14)

common numerator
Two or more fractions that have the same number of fractional units have a common (or the same) numerator. (Lesson 15)

denominator
The denominator of a fraction identifies a fractional unit and indicates how many of the fractional unit makes 1 whole. For example, the 4 in $\frac{3}{4}$ identifies the fractional unit is fourths. (Lesson 8)

mixed number
A mixed number is a whole number plus a fraction less than 1; the addition symbol is usually not written. For example, the mixed number $2\frac{3}{4}$ is the same amount as $2 + \frac{3}{4}$. (Lesson 5)

numerator
The numerator of a fraction identifies the number of fractional units. For example, the 3 in $\frac{3}{4}$ indicates 3 of the fractional unit $\frac{1}{4}$. (Lesson 8)

Familiar

$>$, $=$, $<$ symbols

associative property of addition

associative property of multiplication

data

distributive property

equivalent fraction

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fraction form

fraction greater than 1

fractional unit

line plots

multiple

partition

plot

unit form

unit fraction

Academic Verbs

Module 4 does not introduce any academic verbs from the grade 4 list.

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Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	Chart paper, tablet
25	Colored pencils, set of 2 colors
25	Dry-erase markers
25	Glue sticks
33	Index cards
24	*Learn* books
25	Paper strips, $2'' \times 11''$
50	Paper strips, $2\frac{1}{8}'' \times 5\frac{1}{2}''$
25	Pencils
25	Personal whiteboards
25	Personal whiteboard erasers
1	Projection device
25	Rulers
25	Scissors
1	String, 8 ft
1	*Teach* book
1	Teacher computer or device

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Before This Module

Grade 4 Module 4

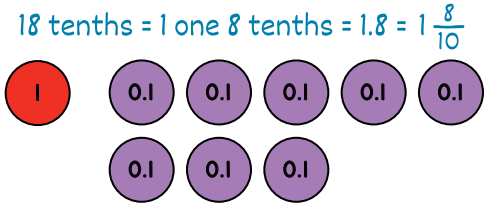
In module 4, students decompose whole numbers and fractions as sums of smaller fractions with the same unit. They also rename fractions greater than 1 as mixed numbers. Students compare fractions with different fractional units by using multiplication and division to generate fractions with like units. They add fractions with like units by using strategies similar to those used with whole number addition.

Overview

Place Value Concepts for Decimal Fractions

Topic A

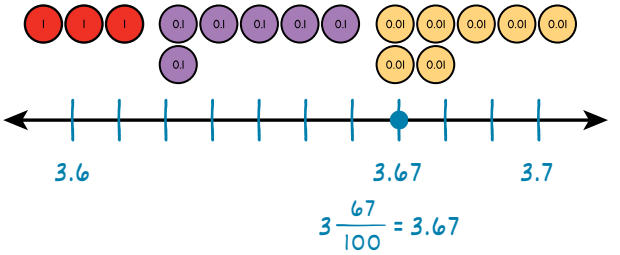
Exploration of Tenths



Within the familiar context of money, students use decimal points to record amounts of money as decimal numbers for the first time. Students see that numbers can be represented in different ways. They use tape diagrams, number lines, and area models to represent the fractional unit of tenths. They write tenths in unit form and fraction form and then see that decimal form is another way to write the numbers. Students decompose 1 one into 10 tenths and compose 10 tenths into 1 one by using familiar representations, including place value disks, and recognize tenths as both a fractional unit and a place value unit. They record mixed numbers of ones and tenths in unit form, fraction form, and decimal form.

Topic B

Tenths and Hundredths



Students decompose tenths into hundredths by using tape diagrams, number lines, and area models. They recognize hundredths as a fractional unit and write hundredths in fraction form and decimal form. Students see that the decomposition of 1 tenth as 10 hundredths and the composition of 10 hundredths as 1 tenth follows the same pattern as other place value units, and they recognize that hundredths is also a place value unit. Students use their knowledge of equivalent fractions and place value units to understand $\frac{10}{100} = \frac{1}{10}$ and,

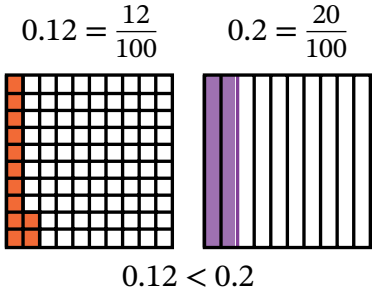
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for example, 0.21 is the same amount as 21 hundredths or 2 tenths 1 hundredth. Students record mixed numbers of ones, tenths, and hundredths in fraction form, decimal form, and unit form and express mixed numbers with units of tenths and hundredths in expanded form.

Topic C

Comparison of Decimal Numbers

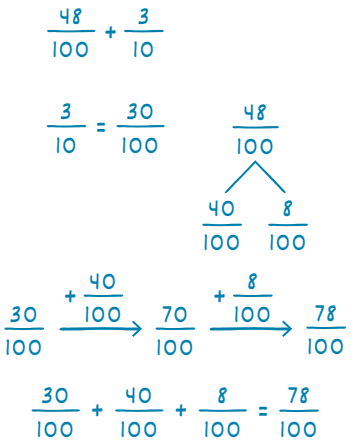
Students compare decimal numbers by applying their prior understanding of whole number and fraction comparison and by using strategies of their choice. They justify their comparisons and see how different strategies can be used. Students then use an area model, number line, and place value disks to represent decimal numbers. Students express decimal numbers in decimal form, fraction form, and unit form. They compare numbers by using different strategies such as making like units, comparing the value of each digit starting with the largest unit, and using mental math strategies. Students apply their knowledge to compare mixed numbers and to order decimal numbers.



Topic D

Addition of Tenths and Hundredths

Students extend their understanding of fraction equivalence and fraction addition with like units to add fractions and mixed numbers with the unlike units of tenths and hundredths. They rename tenths as hundredths to create like units and then use familiar strategies to add. For example, they might combine like units or break apart an addend to make an easier problem. Students also solve word problems that require the addition of metric measurements or amounts of money expressed as decimal numbers. They express the decimal numbers in fraction form, add the numbers in fraction form, and then use decimal form within the solution statement.



After This Module

Grade 5 Module 4

In grade 5, students expand their understanding of place value units to include thousandths. They compare and round decimal numbers that include units to the thousandths, and they add, subtract, multiply, and divide with units to the hundredths. Students convert metric units of measurement and solve word problems that include decimal numbers.

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Place Value Concepts for Decimal Fractions

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Topic A 13

Exploration of Tenths

Lesson 1 16
Organize, count, and represent a collection of money.

Lesson 2 42
Decompose 1 one and express tenths in fraction form and decimal form.

Lesson 3 56
Represent tenths as a place value unit.

Lesson 4 72
Write mixed numbers in decimal form with tenths.

Topic B 91

Tenths and Hundredths

Lesson 5 94
Decompose 1 one and express hundredths in fraction form and decimal form.

Lesson 6 114
Represent hundredths as a place value unit.

Lesson 7 136
Write mixed numbers in decimal form with hundredths.

Lesson 8 154
Represent decimal numbers in expanded form.

Topic C 171

Comparison of Decimal Numbers

Lesson 9 174
Compare measurements expressed as decimal numbers.

Lesson 10 190
Use pictorial representations to compare decimal numbers.

Lesson 11 206
Compare and order decimal numbers.

Topic D 222

Addition of Tenths and Hundredths

Lesson 12 226
Apply fraction equivalence to add tenths and hundredths.

Lesson 13 242
Apply fraction equivalence to add mixed numbers with tenths and hundredths.

Lesson 14 256
Solve word problems with tenths and hundredths.

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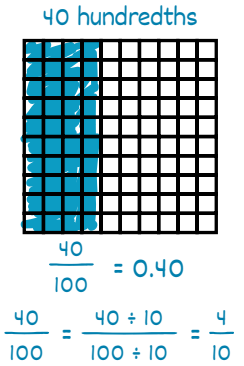
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Why

Place Value Concepts for Decimal Fractions

Why is the module titled *Place Value Concepts for Decimal Fractions* instead of *Place Value Concepts for Decimal Numbers*?

In grade 3 module 5 and grade 4 module 4, students build a conceptual understanding of fractions. This module explicitly broadens students' understanding of tenths and hundredths as place value units and emphasizes that tenths and hundredths are units smaller than ones that can be written in fraction form and decimal form. This understanding helps prepare students for operating with decimals in grades 5 and 6. The module title signals that conceptual understanding of decimal numbers is rooted in fractions.



Why is the term *decimal* omitted in grade 4?

The term *decimal* is sometimes used to reference decimal points and decimal numbers and can describe the form in which a number is written. Instead of using the term *decimal*, however, the grade 4 lessons consistently use the terms *decimal point, decimal number*, and *decimal form* to help students distinguish between the symbol, the number, and how the number is expressed. The term *decimal number* is also used to emphasize that decimal numbers are numbers and have similar characteristics as whole numbers. *Decimal form* describes one way a number can be written. Students also write numbers in unit form and fraction form. In later grades, once students have developed a foundational understanding of decimal numbers, the term *decimal* is used.

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Terminology

The following terms are critical to the work of grade 4 module 5. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

decimal form
A number written with a decimal point is written in decimal form. For example, the number 1 tenth, written in unit form, can be written in fraction form, $\frac{1}{10}$, or decimal form, 0.1 or .1. (Lesson 2)

decimal fraction
A fraction with a denominator of 10 or 100 is a decimal fraction. (Lesson 2)

Any fraction with a denominator that is a power of 10 *is a decimal fraction. In grade 4, students are limited to decimal fractions with denominators of* 10 *or* 100.

decimal number, decimal
A number written in decimal form is a decimal number. (Lesson 2)

decimal point
A decimal point is a symbol that looks like a period. It identifies which digit represents ones and which digit represents tenths. The digit to the left of the decimal point represents ones. The digit to the right of the decimal point represents tenths. (Lesson 1)

In some countries, a decimal comma is used to identify which digit represents ones and which digit represents tenths. For example, the decimal number 1 *and* 25 *hundredths is written as* $1{,}25$ *rather than* 1.25.

hundredths
Hundredths is a place value unit. 1 one can be decomposed into 100 hundredths. (Lesson 6)

tenths
Tenths is a place value unit. 1 one can be decomposed into 10 tenths. (Lesson 3)

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Familiar

denominator

equivalent fraction

expanded form

fraction form

fraction greater than 1

mixed number

numerator

sum

total

unit form

Academic Verbs

Module 5 does not introduce any academic verbs from the grade 4 list.

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Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

4	Blank paper, sheets
25	Dry-erase markers
25	*Eureka Math*$^{2™}$ deci-disks sets, thousandths to ones
1	*Eureka Math*$^{2™}$ decimal place value cards set
1	*Eureka Math*$^{2™}$ place value cards set, to millions
25	*Eureka Math*$^{2™}$ place value disks sets, ones to millions
9	Index cards
24	*Learn* books
1	Meter stick, wood
25	Pencils
25	Personal whiteboards
25	Personal whiteboard erasers
1	Projection device
25	Scissors
1	String, 8 ft
1	*Teach* book
1	Teacher computer or device

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Before This Module

Grade 3 Modules 4 and 6

In grade 3, students compare and classify polygons, including regular polygons, by attributes such as the number of sides, number of angles, right angles, pairs of parallel sides, and sides that have equal length. They draw polygons to match a list of attributes and recognize that some combinations of attributes are not possible. Describing, defining, and sorting quadrilaterals—squares, rectangles, rhombuses, parallelograms, and trapezoids—by their attributes is an emphasis.

Grade 3 students describe angles as right angles (i.e., square corners), angles larger than a right angle, or angles smaller than a right angle. They identify the size of the angles by matching the angles with the corner of an index card.

Grade 4 module 6 elevates the work of grade 3 through formal introduction of geometric figures, angle measurement, and further exploration of two-dimensional figures.

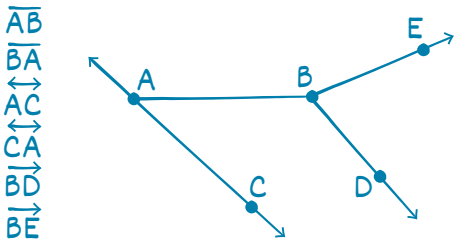
Overview

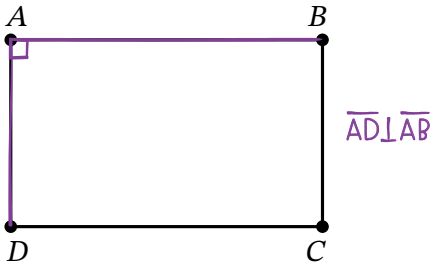
Angle Measurements and Plane Figures

Topic A

Lines and Angles

Students define, name, and draw points, lines, line segments, rays, angles, parallel lines, perpendicular lines, and intersecting lines. They identify different types of angles and describe them in relationship to each other (e.g., an obtuse angle is larger than a right angle and smaller than a straight angle). Students use their understanding of the words *parallel*, *perpendicular*, and *intersecting* to identify and name relationships between lines, line segments, rays, and sides in polygons.

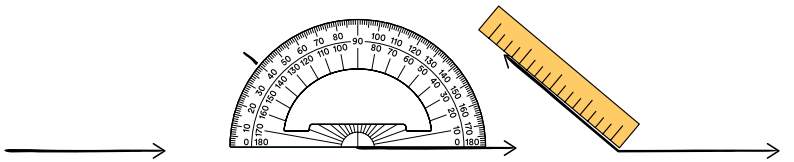




Topic B

Angle Measurement

Students apply fractional understanding to see an angle as a fractional turn through a circle that is measured in degrees—a $1°$ angle is $\frac{1}{360}$ of a turn through a circle. They describe turns in real-world situations, and they refine their definitions of angle types to include degree measures. Students use protractors to measure and draw angles with accuracy and use benchmark angles to estimate the measures of angles.

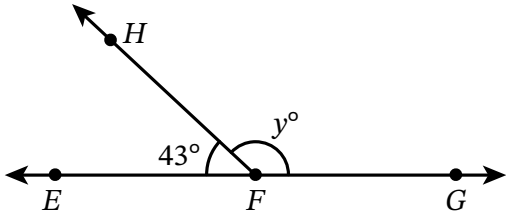


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Topic C

Determine Unknown Angle Measures

Students recognize and apply the additive nature of angle measure to find the unknown measures of angles within figures without using a protractor. They use what is known and the part–total relationship to determine an unknown angle measure when right angles, straight angles, and angles of known measures are decomposed. Students extend the strategy to find the measures of multiple unknown angle measures around a point.

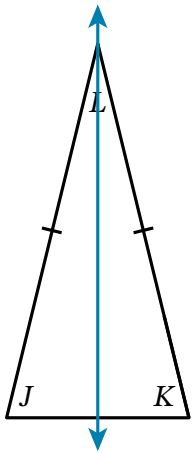


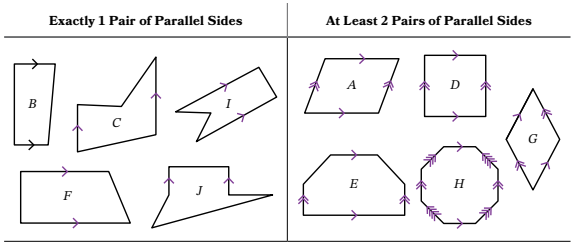
43 + y = 180
y = 137
The measure of ∠HFG is 137°.

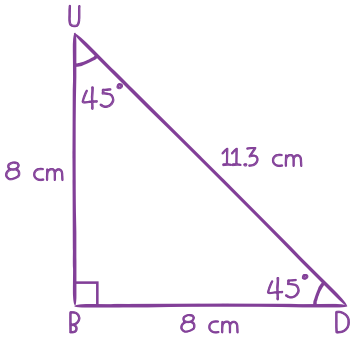
Topic D

Two-Dimensional Figures and Symmetry

Students recognize, identify, and draw lines of symmetry. They identify attributes of polygons including side length and the presence or absence of pairs of parallel sides, pairs of perpendicular sides, and angle types to sort and classify them. Special emphasis is placed on triangles. Students classify triangles based on side lengths and angle measures and draw triangles based on given attributes.







After This Module

Grade 5 Module 5

In grade 5, students advance to more sophisticated classifications of polygons based on sides and angles. They recognize categories and subcategories of polygons, for example, noting that a square is a special type of rhombus.

Grade 7 Modules 3 and 4

In grade 7 module 3, students use angle relationships to write and solve more complex equations to find the unknown measures of angles.

In grade 7 module 4, students consider side and angle relationships as they construct triangles.

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Lesson 2 34
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Lesson 3 54
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Lesson 4 68
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Lesson 5 88
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Two-Dimensional Figures and Symmetry

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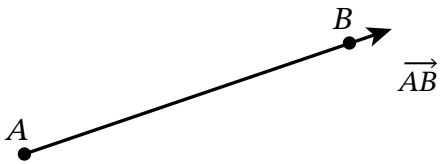
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Why

Angle Measurements and Plane Figures

Why is there an emphasis on detailed definitions, drawings, and notation?



Topic A presents students with many new geometric terms, most of which name or describe geometric figures. Along with learning the definition of each term, students identify examples of the figures, draw the figures, and use geometric notation to name them. Each task includes an emphasis on detail that is intended to help strengthen student understanding of each term by connecting the verbal definition with pictures and notations. For example, as they draw a ray, students relate the definition of a ray to the meaning of endpoint, point, and arrowhead. Then students use notation, which visually coincides with the definition, to name the ray, repeating the process with the definitions of lines, line segments, points, and angles.

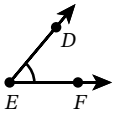
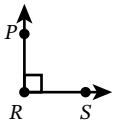
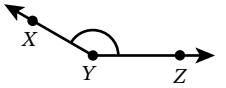
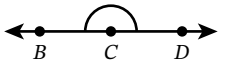
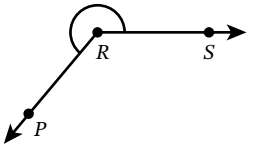
Students draw parallel and perpendicular line segments by using a right-angle tool and straightedge and use a protractor to draw angles of a given measure with accuracy. Students attend to detail as they consider how their tools can help them accurately draw figures that align with their definitions, which also deepens their understanding of the definitions.

When students continue their study of geometry, they reason about the complex relationships between sides, angles, intersecting lines, and more. Developing detail in drawings and notation can help students clearly communicate ideas and lays the groundwork for longer, more detailed geometric proofs. Similarly, in this module, students notate figures with symbols to identify equal side lengths in triangles and pairs of parallel sides. These details will be necessary later as students work with relationships between sides and angles in triangles and identify categories and subcategories of quadrilaterals.

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Why are students introduced to so much terminology?

The intention of introducing the terms in this module is to build a strong core foundation of geometry. The purposeful inclusion of formal terminology helps to make other definitions clearer or to prepare students for future learning. Following are some examples that demonstrate this intention.

Angle Type	Angle Measure	Example
acute angle	Between 0° and 90°	
right angle	90°	
obtuse angle	Between 90° and 180°	
straight angle	180°	
reflex angle	Between 180° and 360°	

To support their understanding of acute, right, and obtuse angle types, students are introduced to straight and reflex angles. By including straight and reflex angles, this module allows students to define angle types in relation to each other and to use those relationships when solving problems. Student understanding of $360°$ as a full turn through a circle is crucial to their understanding of degrees as units used to measure angles. With acute, right, obtuse, straight, and reflex angles in their tool kit, students can name any angle whose measure is within $360°$.

The formal relationships of complementary and supplementary angles is important in grade 7 when students write equations to find multiple unknown angle measures. However, the additive nature of angle measure is a key concept in grade 4, and therefore, this module helps students gain familiarity with these angles and their relationships. Students decompose right and straight angles into smaller angles and see that the total measure of the smaller angles is $90°$ and $180°$, respectively. The terms *complementary* and *supplementary* give students the precise vocabulary to name the relationships they discover.

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Classifying two-dimensional figures based on sides and angles is part of the work of grade 4. Students apply their understanding of angle types, parallelism, and perpendicularity to sort and classify polygons. As they sort triangles based on side lengths and angle measures, students classify triangles by name—acute, obtuse, right, equilateral, isosceles, and scalene. Using these names formalizes the process of classifying triangles and allows students to practice applying the terminology they have acquired.

Why is this module placed last in the year?

The deliberate sequence of the grade 4 modules allows students to build on their prior grade-level understanding, develop new learning, create foundational scaffolds and supports for future learning, and attend to the major work of the grade.

Fractional understanding is an integral part of the definition of *degree*, the unit used to measure angles. Placing geometric work last allows students time to develop fluency with grade 4 fraction and decimal concepts. They can then apply that knowledge to add and subtract new units of measure, *degrees*, and measure side lengths of polygons to the fraction of a whole-number length measurement.

Studying geometry last in the sequence also allows students to connect their algebraic work from the year to their familiarity with two-dimensional shapes. Their mathematical understanding matures as they make these connections and expand their geometric vocabulary.

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Terminology

The following terms are critical to the work of grade 4 module 6. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

acute angle
An acute angle is an angle that is smaller than a right angle; its measure is less than $90°$. (Lesson 2, Lesson 8)

acute triangle
An acute triangle is a triangle that contains all acute angles. (Lesson 18)

adjacent angles
Two angles with the same vertex that share a ray and do not overlap are called adjacent angles. (Lesson 14)

angle
An angle is a figure formed by two rays with the same endpoint. (Lesson 1)

complementary angles
Two angles are complementary angles if the sum of the two angle measures is $90°$. (Lesson 14)

degree
A degree is the unit used to measure angles. An angle that turns through $\frac{1}{360}$ of a circle is a one-degree ($1°$) angle. (Lesson 8)

equilateral triangle
An equilateral triangle is a triangle with all sides of equal length. (Lesson 18)

figure
A figure is a set of points in a flat space. *Sometimes the term* figure *refers to certain common shapes like triangles, rectangles, trapezoids, etc.* (Lesson 1)

intersect
Two or more lines (or line segments or rays) intersect if they meet or cross. They intersect at a point. (Lesson 4)

isosceles triangle
An isosceles triangle is a triangle with at least two sides of equal length. *Note that an equilateral triangle is also an isosceles triangle.* (Lesson 18)

line
A line is a straight path, with no thickness, that extends forever in both directions. (Lesson 1)

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line of symmetry
A line of symmetry is a line across a figure such that when the figure is folded along the line, 2 matching parts are created. (Lesson 17)

line segment
A line segment is two points and the set of points on the line between them. (Lesson 1)

obtuse angle
An obtuse angle is an angle that is larger than a right angle and smaller than a straight angle; its measure is greater than $90°$ and less than $180°$. (Lesson 2, Lesson 8)

obtuse triangle
An obtuse triangle is a triangle that contains an obtuse angle. (Lesson 18)

parallel
Two lines in a flat space are parallel if they never intersect. Line segments or rays are parallel if the lines containing them are parallel. (Lesson 5)

perpendicular
Two lines in a flat space are perpendicular if the angles formed at the point where they intersect are right angles. Line segments or rays are perpendicular if the lines that contain them are perpendicular. (Lesson 4)

point
A point indicates a precise location in a flat space and has no size. The points identifying a line segment are called endpoints. (Lesson 1)

ray
A ray is a point and all the points on the line in one direction away from the point. The point is called the endpoint of the ray. (Lesson 1)

reflex angle
A reflex angle is an angle that is larger than a straight angle and smaller than a full rotation, or whole turn; its measure is greater than $180°$ and less than $360°$. (Lesson 7)

right angle
A right angle measures $90°$. (Lesson 8)

right triangle
A right triangle is a triangle that contains a right angle. (Lesson 18)

scalene triangle
A scalene triangle is a triangle with no sides of equal length. (Lesson 18)

straight angle
A straight angle is formed by two rays, with a common endpoint, that together make a line. (Lesson 2)

supplementary angles
Two angles are supplementary angles if the sum of the two angle measures is $180°$. (Lesson 14)

vertex
A vertex is a point where two lines or segments or rays intersect. *The common endpoint of rays making an angle is referred to as the* vertex *of the angle.* Vertex *is also used to refer to any point of intersection of two sides of a polygon.* (Lesson 2)

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Familiar

benchmark

decompose

sum

Academic Verbs

construct

classify

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Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

25	4″ protractors (180°)
100	Blank paper, sheets
1	Cardstock circles, 4″ diameter, red (set of 25)
1	Cardstock circles, 4″ diameter, white (set of 25)
25	Colored pencils
25	Dry-erase markers
24	*Learn* books
25	Markers
25	Pencils
25	Personal whiteboards
25	Personal whiteboard erasers
1	Projection device
25	Rulers
25	Scissors
24	Sticky notes
1	*Teach* book
1	Teacher computer or device
36	Toothpicks

Visit http://eurmath.link/materials to learn more.

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