

Before This Module

Grade 4 Module 1

Students read, write, compare, and round multi-digit whole numbers to millions in standard, expanded, word, and unit forms. They describe the relationship between a digit in one place and the digit in the next larger place by using the multiplicative comparison 10 *times as much as*.

Grade 4 Modules 2 and 3

Students multiply and divide whole numbers of up to four digits by one-digit numbers (including expressing quotients with whole-number remainders) and multiply 2 two-digit numbers. They use methods based on the place value chart, area models, the commutative and associative properties of multiplication, and the distributive property.

Overview

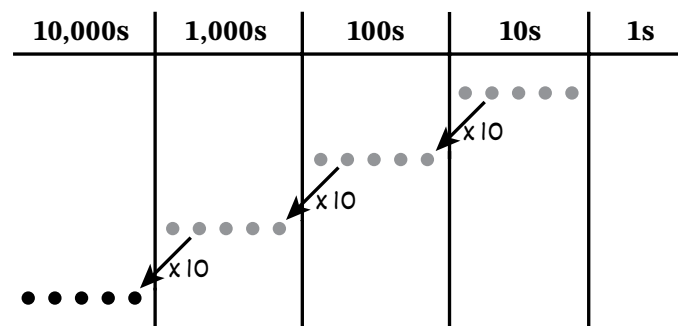
Place Value Concepts for Multiplication and Division with Whole Numbers

Topic A

Place Value Understanding for Whole Numbers

Students use multiplicative comparison statements to explain that a digit in one place represents 10 times as much as what it represents in the place to the right. Students notice how digits of a number shift when they multiply or divide by a power of 10 and express a power of 10 in exponential form. Then students find products and quotients by using powers of 10 and convert metric measurements from larger to smaller units.

1, 731, 225
1,000,000 is 1,000 times
as much as 1,000.



$$\begin{aligned} 50 \times 1,000 &= \underline{50 \times 10 \times 10 \times 10} \\ &= \underline{50 \times 10^3} \\ &= \underline{50,000} \end{aligned}$$

Topic B

Multiplication of Whole Numbers

Students build fluency with multiplying multi-digit numbers by using the standard algorithm. They use place value understanding to visualize the decomposition of factors while they multiply a single digit at a time by another single digit in the standard algorithm.

	400	20	7
2	800	40	14
50	20,000	1,000	350

$$\begin{array}{r}
 427 \\
 \times 52 \\
 \hline
 854 \\
 + 21350 \\
 \hline
 22,204
 \end{array}$$

Topic C

Division of Whole Numbers

Students use methods based on place value to find quotients of whole numbers with up to four-digit dividends and two-digit divisors. They estimate quotients, then use tape diagrams, area models, and vertical form to record quotients and remainders.

926 ÷ 23

Area Model

	10	10	10	10	
23	230	230	230	230	6

Quotient: 40
Remainder: 6

926 ÷ 23

Area Model

	30	10
23	690	230

$$\begin{array}{r}
 690 \\
 + 230 \\
 \hline
 920 \\
 926 - 920 = 6
 \end{array}$$

Quotient: 40
Remainder: 6

926 ÷ 23

Vertical Form

10	
30	Estimates:
23 $\overline{) 926}$	→ 900 ÷ 30 = 30
-690	
236	→ 230 ÷ 23 = 10
-230	
6	

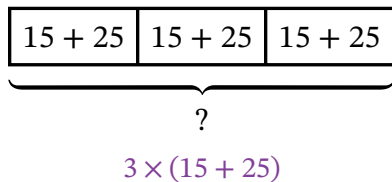
Quotient: 40
Remainder: 6

Topic D

Multi-Step Problems with Whole Numbers

Students move between written, pictorial, and numeric representations of mathematical statements. They use tape diagrams to determine when parentheses are needed in expressions and evaluate expressions containing grouping symbols.

3 times the sum of 15 and 25



$$(26 - 8) \div 2$$

There are 26 people at the park. 8 people go home. The rest of the people make 2 equal groups to play a game. How many people are in each group?

$$(26 - 8) \div 2 = 18 \div 2 = 9$$

There are 9 people in each group.

After This Module

Grade 5 Module 4

Students use place value knowledge and *times as much as* language to learn about decimal numbers. Students see how the strategies they use for whole-number operations extend to operations with decimal numbers. They convert metric measurements from smaller units to larger units.

Grade 6 Modules 2 and 4

In module 2, students learn to divide whole numbers with any number of digits by using the standard algorithm. In module 4, students build upon grade 5 knowledge by writing and evaluating numerical expressions with terms that have whole-number bases and exponents.

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Place Value Understanding for Whole Numbers

Lesson 1 18

Relate adjacent place value units by using place value understanding.

Lesson 2 58

Multiply and divide by 10, 100, and 1,000 and identify patterns in the products and quotients.

Lesson 3 80

Use exponents to multiply and divide by powers of 10.

Lesson 4 102

Estimate products and quotients by using powers of 10 and their multiples.

Lesson 5 120

Convert measurements and describe relationships between metric units.

Lesson 6 136

Solve multi-step word problems by using metric measurement conversion.

Topic B 154

Multiplication of Whole Numbers

Lesson 7 158

Multiply by using familiar methods.

Lesson 8 174

Multiply two- and three-digit numbers by two-digit numbers by using the distributive property.

Lesson 9 194

Multiply two- and three-digit numbers by two-digit numbers by using the standard algorithm.

Lesson 10 214

Multiply three- and four-digit numbers by three-digit numbers by using the standard algorithm.

Lesson 11 232

Multiply two multi-digit numbers by using the standard algorithm.

Topic C 246

Division of Whole Numbers

Lesson 12 250

Divide two- and three-digit numbers by multiples of 10.

Lesson 13 270

Divide two-digit numbers by two-digit numbers in problems that result in one-digit quotients.

Lesson 14 288

Divide three-digit numbers by two-digit numbers in problems that result in one-digit quotients.

Lesson 15 306

Divide three-digit numbers by two-digit numbers in problems that result in two-digit quotients.

Lesson 16 326

Divide four-digit numbers by two-digit numbers.

Topic D 346

Multi-Step Problems with Whole Numbers

Lesson 17 350
Write, interpret, and compare numerical expressions.

Lesson 18 368
Create and solve real-world problems for given numerical expressions.

Lesson 19 382
Solve multi-step word problems involving multiplication and division.

Lesson 20 400
Solve multi-step word problems involving the four operations.

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Why

Place Value Concepts for Multiplication and Division with Whole Numbers

Why does multiplication and division of whole numbers come first?

After much consideration of our students' learning, teachers' input, and a review of the research around how students learn and how mathematical concepts progress, we decided it makes the most sense to put place value concepts and operations with whole numbers first. Why?

1. The major emphasis of grade 5 standards involves understanding the place value system, performing operations with multi-digit whole numbers, and applying and extending knowledge of whole-number operations to fractions and decimals. Beginning the year with a focus on place value and whole-number operations sets up students for success as they move into operations with fractions in modules 2 and 3, then with decimals in module 4.
2. Beginning the year with learning how to multiply multi-digit numbers provides an opportunity for students to develop fluency with using the standard algorithm throughout the year, as required by the standards.
3. Multiplying and dividing multi-digits numbers gives rise to developing estimation skills and to introducing powers of 10 in a meaningful way. Powers of 10 are not just the numbers on a place value chart, rather they are powerful tools for making estimates of products and quotients and for checking the reasonableness of answers.

$$537 \times 3,296 \approx \underline{500} \times \underline{3,000} = \underline{1,500,000}$$

				3,	2	9	6
			×		5	3	7
			2	~~3~~	~~0~~	~~7~~	2
			9	~~8~~	~~8~~	8	0
+	1	~~6~~	~~4~~	~~8~~	0	0	0
	1,	7	6	9,	9	5	2

When do students learn about decimals? Why?

Grade 5 module 4 addresses work with decimals and parallels the content of module 1. Students begin by relating adjacent place value units and use comparison language, such as 1 tenth is 10 times as much as 1 hundredth, just as they did with whole numbers. It makes sense mathematically to position decimals in module 4 after an in-depth study of whole numbers in module 1 and then fractions in modules 2 and 3. This move also makes sense pedagogically because students can use $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ to describe relationships between numbers on the place value chart and to perform operations on decimals.

tens	ones	tenths	hundredths	thousandths
3	7	5	9	4

- a. The digit 3 is in the tens place. It has a value of 30.

I notice students only convert from larger metric units to smaller metric units in this module. Why?

Metric conversions are limited to moving from larger units (such as kilometers) to smaller units (such as meters) in module 1 because conversions that move from smaller to larger units are best performed by using fractions or decimals. Students learn to multiply fractions in module 3 and they learn to multiply decimals in module 4. The remaining part of the metric conversion standard is fully met in module 4 as an application of decimals.

Module 1	Module 4
32 m = _____ cm	25 mm = _____ cm
32 m = 32×1 m	25 mm = 25×1 mm
= 32×100 cm	= 25×0.1 cm
= 3,200 cm	= 2.5 cm

Terminology

The following terms are critical to the work of grade 5 module 1. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

centigram

A centigram is a metric unit for measuring weight. A gram is 100 centigrams. A large ant weighs about 1 cg. (Lesson 5)

centiliter

A centiliter is a metric unit for measuring capacity or liquid volume. A liter is 100 centiliters. (Lesson 5)

dividend

In a division expression, the number being divided by another number is the dividend. For example, in the expression $18 \div 3$, the dividend is 18. (Lesson 12)

exponent

An exponent represents how many times the same number is used as a factor. (Lesson 3)

exponential form

A number with an exponent is in exponential form. (Lesson 3)

kiloliter

A kiloliter is a metric unit for measuring capacity or liquid volume. A kiloliter is 1,000 liters. (Lesson 5)

milligram

A milligram is a metric unit for measuring weight. A gram is 1,000 milligrams. A small feather weighs about 1 mg. (Lesson 5)

millimeter

A millimeter is a metric unit for measuring length. A meter is 1,000 millimeters. (Lesson 5)

power of 10

A number that can be written as a product of 10s, or as a 10 with an exponent, is a power of 10. 100 is a power of 10. (Lesson 3)

Familiar

centimeter

convert

distributive property

divisor

express

factor

kilogram

partial products

partial quotients

quotient

remainder

standard algorithm

Academic Verbs

analyze

consider

evaluate

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

3	Blank paper, sheets	25	Pencils
5	Chart paper, sheets	25	Personal whiteboards
25	Dry-erase markers	25	Personal whiteboard erasers
8	Envelopes	1	Projection device
24	<i>Learn</i> books	1	<i>Teach</i> book
9	Meter sticks, wood	1	Teacher computer or device

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 1 for a list of organizational tools (cups, rubber bands, graph paper, etc.) suggested for the counting collection.

Before This Module

Grade 4 Module 4

In grade 4 module 4, students build conceptual understanding of fractions by partitioning and shading area models. They learn that fractions can be decomposed into parts in the same manner as whole numbers. Students use area models to make equivalent fractions and show the renaming numerically. By applying an understanding of equivalent fractions, students learn to compare fractions by creating a common numerator or a common denominator.

Overview

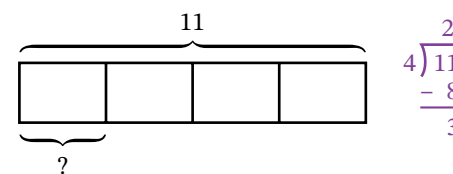
Addition and Subtraction with Fractions

Topic A

Fractions and Division

Students use equal sharing to understand a fraction as the result of dividing the numerator by the denominator. They use models and vertical form to divide and represent remainders as fractions. Then students solve word problems involving division.

Mr. Evans pours 11 liters of water equally into 4 containers. How many liters of water are in 1 container?



$$11 \div 4 = 2\frac{3}{4}$$

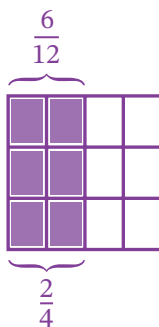
There are $2\frac{3}{4}$ liters of water in 1 container.

Topic B

Addition and Subtraction of Fractions by Making Like Units

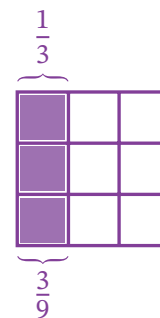
Students use models and equations to make like units before they add and subtract fractions. They analyze expressions to determine whether the units in fractions are related or unrelated, which indicates whether one or more fractions must be renamed to make like units.

$$\frac{3}{4} + \frac{6}{12} = \frac{3}{4} + \frac{2}{4} = \frac{5}{4}$$



$$\frac{6}{12} = \frac{6 \div 3}{12 \div 3} = \frac{2}{4}$$

$$\frac{7}{9} - \frac{1}{3} = \frac{7}{9} - \frac{3}{9} = \frac{4}{9}$$

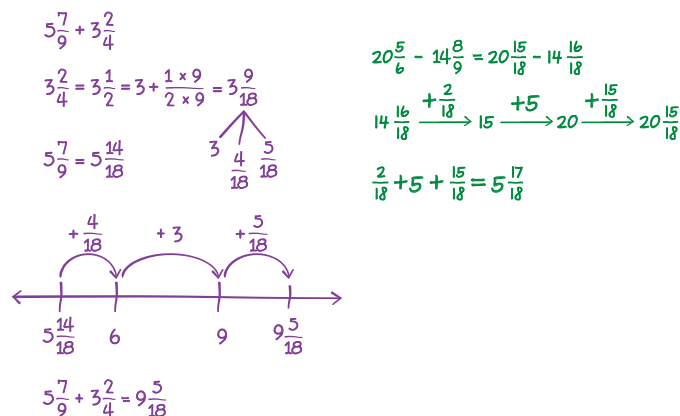


$$\frac{1}{3} = \frac{1 \times 3}{3 \times 3} = \frac{3}{9}$$

Topic C

Addition and Subtraction of Fractions, Whole Numbers, and Mixed Numbers

Students apply knowledge of addition and subtraction of whole numbers to help them add and subtract mixed numbers. They use familiar models such as a number bond, a number line, and the arrow way to show their thinking.

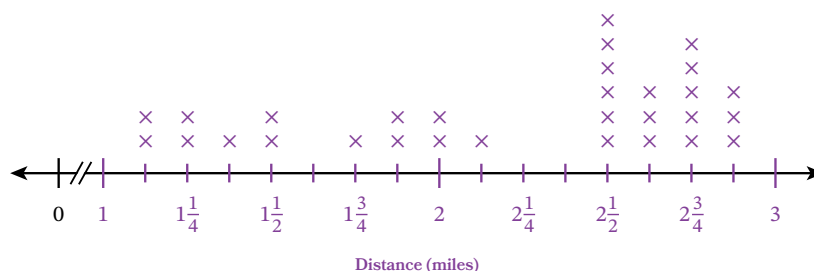


Topic D

Problem Solving and Line Plots with Fractional Measurements

Students create line plots from a given set of data. They consider all data points to decide how much of the number line should be shown and how to label the number line to present the data so it can be easily read, analyzed, and understood. Students find the sum of measurements in the data set and use models to equally redistribute the sum among all data points.

Distances Walked by Miss Song



After This Module

Grade 5 Module 3

The study of fractions continues in grade 5 module 3 when students learn to multiply with fractions and divide with whole numbers and unit fractions. Students apply what they know about addition of fractions to make sense of multiplication of fractions and to check their answers.

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Fractions and Division

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Interpret a fraction as division.

Lesson 2	34
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Interpret a fraction as division by writing remainders as fractions.

Lesson 3	50
-----------------------	----

Represent fractions as division by using models.

Lesson 4	68
-----------------------	----

Solve word problems involving division and fractions.

Topic B	85
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Addition and Subtraction of Fractions by Making Like Units

Lesson 5	88
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Add and subtract fractions with related units by using pictorial models.

Lesson 6	108
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Add and subtract fractions with related units by using area models to rename fractions.

Lesson 7	126
-----------------------	-----

Add and subtract fractions with related units by finding equivalent fractions numerically.

Lesson 8	142
-----------------------	-----

Add and subtract fractions with unrelated units by finding equivalent fractions pictorially.

Lesson 9	162
-----------------------	-----

Add and subtract fractions with unrelated units by finding equivalent fractions numerically.

Topic C	179
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Addition and Subtraction of Fractions, Whole Numbers, and Mixed Numbers

Lesson 10	182
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Add whole numbers and mixed numbers and add mixed numbers with related units.

Lesson 11	202
------------------------	-----

Add mixed numbers with unrelated units.

Lesson 12	220
------------------------	-----

Subtract whole numbers from mixed numbers and mixed numbers from whole numbers.

Lesson 13	238
------------------------	-----

Subtract mixed numbers from mixed numbers with related units.

Lesson 14	258
------------------------	-----

Subtract mixed numbers from mixed numbers with unrelated units.

Topic D 275**Problem Solving and Line Plots with Fractional Measurements****Lesson 15** 278

Represent data on a line plot.

Lesson 16 294

Solve problems by using data from a line plot.

Lesson 17 312

Solve problems by equally redistributing a total amount.

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Why

Addition and Subtraction with Fractions

When should students put fractions in simplest form? Why?

Referring to the simplest form of anything can be tricky because what is simple to one person may not appear simple to another. Further, in some cases what is known as simplest form can be more difficult to interpret if a context is involved. For example, $\frac{54}{60}$ is a simpler form than $\frac{9}{10}$ if a problem asks how many hours someone works on a project.

Students are not expected to rename fractions in one form or another unless the context of a problem requires students to do so. For example, a number such as $\frac{30}{20}$ is an acceptable answer for a sum or difference. However, if $\frac{30}{20}$ is the result when a student is asked a question about how many more kilometers one person ran than another, students are expected to express their answer as a mixed number, such as $1\frac{10}{20}$. A mixed number is often easier to interpret than a fraction greater than 1 in context-based problems.

Why are students sometimes asked to express their answer in the largest unit instead of being asked to reduce the fraction or write their answer in lowest terms?

Much of the traditional language involved with fractions, such as *reduce* and *lowest terms*, may either cause confusion or be misunderstood. Let's first consider the term *reduce*.

Reduce means to make something smaller in amount or size. It is inaccurate to say "reduce a fraction" because it is not the fraction that is reduced; rather the numerator and denominator are proportionally reduced to make a fraction with the same value. To avoid the misconception that reducing a fraction makes a fraction with a lesser value, students are not given the direction to reduce fractions.

Now consider the phrase *lowest terms*. The mathematical use of *term* is not introduced to students until grade 6, which is when students can make sense of a term as a single number, a variable, or a product of numbers and variables in an expression. In grade 4, students use *term*, but only when referring to any one item in an ordered list of items in

a pattern. Because *lowest terms* is a phrase students cannot yet make sense of mathematically, students are not asked to write fractions in lowest terms.

Instead of being directed to reduce or write fractions in lowest terms, students are asked to express their answer by using the largest unit. This is a more accurate description because when a fraction is “reduced,” it is actually written with a larger fractional unit. For example, the fraction $\frac{30}{100}$ is 30 hundredths in unit form. 10 hundredths has the same value as 1 tenth, so 30 hundredths has the same value as 3 tenths, and $\frac{30}{100}$ is equivalent to $\frac{3}{10}$. Tenths are a larger unit than hundredths, so the fraction $\frac{30}{100}$ written in the largest possible unit is the fraction $\frac{3}{10}$. Students enter grade 5 with a solid understanding of the term *unit*.

When students are finding equivalent fractions, why do they write the factors in the opposite order in grade 5 than in grade 4?

In grade 3, students interpret the factors in the expression 3×4 as 3 groups of 4. Therefore, in grade 4, students find equivalent fractions by writing an expression such as $\frac{3 \times 4}{3 \times 5}$.

This is interpreted as tripling the number of units selected and tripling the total number of units, especially when using an area model to make an equivalent fraction. By grade 5, students develop a deeper understanding of multiplication and begin to move away from pictorial models and equal-groups thinking. They are comfortable enough working with the multiplication expressions that it is more natural to write the factors of the original fraction first (e.g., 4 and 5) followed by the factors they use to rename the fraction (e.g., 3 and 3), rather than vice versa. In grade 5, students continue to use the area model to find equivalent fractions but move toward renaming fractions numerically. For these reasons, equivalent fractions are presented with an expression such as $\frac{4 \times 3}{5 \times 3}$, which honors the knowledge students have gained over the years and allows them to attend to the needs of the problem instead of focusing on ordering factors in a particular way.

Terminology

The following terms are critical to the work of grade 5 module 2. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

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New

minuend

The minuend is the starting value, or the total in a part-total relationship. (Lesson 12)

subtrahend

The subtrahend is the number being subtracted from the starting value, or the part in a part-total relationship. (Lesson 12)

Familiar

common denominator

denominator

equivalent fractions

fraction form

mixed number

numerator

Academic Verb

conclude

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

15	Chart paper, sheets	1	Projection device
25	Dry-erase markers	1	Roll of tape
24	<i>Learn</i> books	24	Scissors
125	Markers	216	Sticky notes
101	Paper, blank sheets	1	<i>Teach</i> book
25	Pencils	1	Teacher computer or device
25	Personal whiteboards	32	Unifix® Cubes
25	Personal whiteboard erasers		

Visit <http://eurmath.link/materials> to learn more.

Before This Module

Grade 4 Module 4

In grade 4 module 4, students decompose a non-unit fraction as a sum of unit fractions and then write the sum as a product of a whole number times a unit fraction. They solve word problems involving multiplication of a fraction by a whole number and express answers as fractions and mixed numbers.

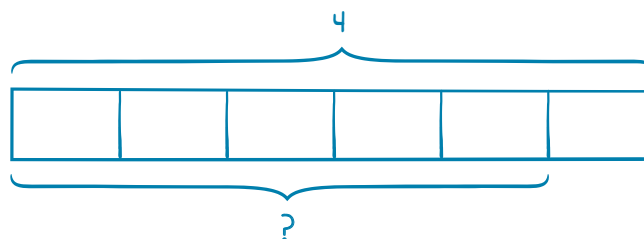
Overview

Multiplication and Division with Fractions

Topic A

Multiplication of a Whole Number by a Fraction

Students extend their understanding of fractions from parts of a whole (e.g., 1 third of a shape) to parts of a set or a number (e.g., 1 third of a group of 12 items). They find fractions of a set and then transition to finding a fraction of a whole number. Students learn that finding a fraction of a whole number means they are finding the product of a fraction and a whole number. They apply this learning to converting customary measurement units.



$$6 \text{ units} = 4$$

$$1 \text{ unit} = \frac{4}{6}$$

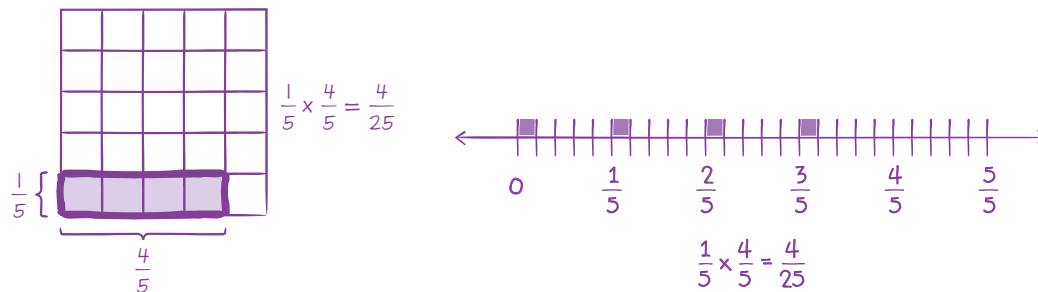
$$5 \text{ units} = 5 \times \frac{4}{6} = \frac{20}{6}$$

$\frac{5}{6}$ of 4 is 5 parts when 4 is partitioned into 6 equal parts.

Topic B

Multiplication of Fractions

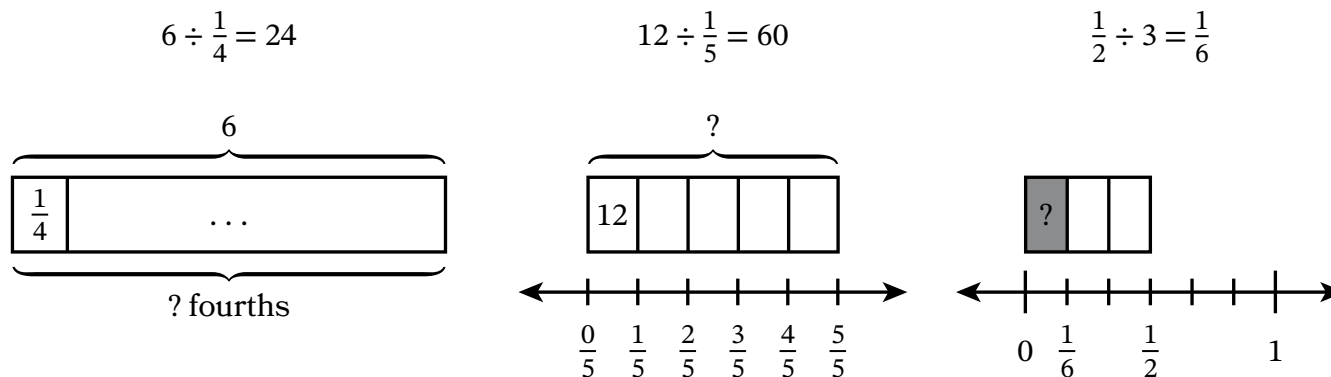
Students use area models and number lines to multiply fractions by unit fractions and then fractions by fractions. Throughout the topic, they reason about the value of products by considering whether the factors involved are greater than 1 or less than 1.



Topic C

Division with a Unit Fraction and a Whole Number

Students use tape diagrams and number lines to divide a whole number by a unit fraction and to divide a unit fraction by a whole number. They solve word problems that involve multiplication of fractions and division of whole numbers and fractions, and students explain the relationship between multiplication and division.



After This Module

Grade 5 Module 5

In module 5, students learn to find the area of rectangles with side lengths that are fractions. The context of area is also used to show how to multiply mixed numbers.

Grade 6 Module 2

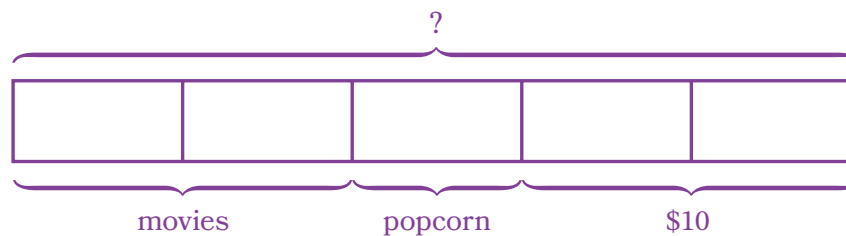
In grade 6, students expand their understanding of division involving fractions to division of fractions in general.

Topic D

Multi-Step Problems with Fractions

Students apply their previous learning about all operations with fractions to compare and evaluate expressions that contain grouping symbols. They create and solve word problems involving fractions, and they write equations with parentheses for word problems that require multiple steps to solve.

Toby spends $\frac{2}{5}$ of his money on movie tickets. He spends $\frac{1}{3}$ of his remaining money on popcorn. He has \$10 left. How much money did Toby have to begin with?



$$\left(\frac{1}{2} \times 10\right) \times 5 = 25$$

Toby had \$25 to begin with.

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Multiplication of a Whole Number by a Fraction

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-----------------	----

Find fractions of a set with arrays.

Lesson 2	36
-----------------	----

Interpret fractions as division to find fractions of a set with tape diagrams and number lines.

Lesson 3	54
-----------------	----

Multiply a whole number by a fraction less than 1.

Lesson 4	72
-----------------	----

Multiply a whole number by a fraction.

Lesson 5	90
-----------------	----

Convert larger customary measurement units to smaller measurement units.

Lesson 6	110
-----------------	-----

Convert smaller customary measurement units to larger measurement units.

Topic B	129
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Multiplication of Fractions

Lesson 7	132
-----------------	-----

Multiply fractions less than 1 by unit fractions pictorially.

Lesson 8	152
-----------------	-----

Multiply fractions less than 1 pictorially.

Lesson 9	172
-----------------	-----

Multiply fractions by unit fractions by making simpler problems.

Lesson 10	192
------------------	-----

Multiply fractions greater than 1 by fractions.

Lesson 11	210
------------------	-----

Multiply fractions.

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Division with a Unit Fraction and a Whole Number

Lesson 12	230
------------------	-----

Divide a nonzero whole number by a unit fraction to find the number of groups.

Lesson 13	252
------------------	-----

Divide a nonzero whole number by a unit fraction to find the size of the group.

Lesson 14	272
------------------	-----

Divide a unit fraction by a nonzero whole number.

Lesson 15	292
------------------	-----

Divide by whole numbers and unit fractions.

Lesson 16	312
------------------	-----

Reason about the size of quotients of whole numbers and unit fractions and quotients of unit fractions and whole numbers.

Lesson 17	326
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Solve word problems involving fractions with multiplication and division.

Topic D 343

Multi-Step Problems with Fractions

Lesson 18 346
Compare and evaluate expressions with parentheses.

Lesson 19 364
Create and solve one-step word problems involving fractions.

Lesson 20 384
Solve multi-step word problems involving fractions and write equations with parentheses.

Lesson 21 404
Solve multi-step word problems involving fractions.

Lesson 22 422
Evaluate expressions involving nested grouping symbols. (Optional)

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Why

Multiplication and Division with Fractions

How are the measurement and partitive interpretations for division important to this module?

When students begin their work with division in grade 3, they learn to interpret the problem based on the context. For $12 \div 3 = 4$, the quotient 4 can mean the size of each group (e.g., 4 marbles in 3 bags) or it can mean the number of groups (e.g., 4 bags each with 3 marbles). These interpretations are revisited in grade 5 module 1 when students divide multi-digit whole numbers. Knowing these interpretations is important for students because it helps them make sense of the numbers in problems and it supports students with reasoning about whether their answers are correct.

As students begin to divide with fractions in this module, understanding the meaning of the numbers in the division equation supports students with making sense of the problem and relating the division equation to multiplication. Because the quotient is always less than the dividend when they divide by whole numbers greater than 1, students often need support with division by fractions less than 1 because the quotient is greater than the dividend.

This module aims to anchor division with unit fractions in contexts. Therefore, students can reason about the size of the quotient and the dividend and interpret an equation by using questions such as, How many thirds are in 2? 2 is $\frac{1}{3}$ of what number? $\frac{1}{3}$ is 2 groups of what?

Please note that students are not required to categorize problems as measurement or partitive division, but rather interpret the meaning of the divisor and quotient within the problem.

Why do students learn to multiply and divide with fractions before they do so with decimals?

Students have conceptually worked with fractions for longer than they have worked with decimals. Starting as early as in kindergarten, students informally model fractions

as parts of a whole (shape) and identify equal parts. In grade 3, the concept of a fraction is formalized, and in grade 4, operations work with fractions begins.

It is not until grade 4 that the fractional units of tenths and hundredths are introduced as decimals. Because tenths and hundredths are both place value units and fractional units, students use what they know about fractions to support their conceptual development with decimals. For the same reason, students learn to multiply and divide with fractions before they do so with decimals.

Why is lesson 22 considered optional?

In grade 5, students are expected to use parentheses, brackets, and braces in numerical expressions and to be able to evaluate expressions

$$4,628 \times \left[\left((3 \div 5) - \frac{3}{5} \right) + \left(\frac{7}{13} - (14 \div 26) \right) \right] = 0$$

that contain these grouping symbols. Lesson 22 extends students' understanding of how to evaluate expressions with grouping symbols by presenting students with expressions that have nested grouping symbols. Students read an expression and identify expressions in grouping symbols as *a number*. For example, in the equation shown, they read the expression on the left side of the equal sign as 4,628 *times another number*. Then students read the expression within the brackets as *a number plus another number*. They continue to read the expression until they find an expression that can be evaluated (i.e., $3 \div 5$) because it is not part of *another number*. This natural extension to grade 5 expectations provides an opportunity for students to practice the skills they recently acquired with adding, subtracting, multiplying, and dividing with fractions and multi-digit whole numbers. Students learn that these seemingly complicated expressions are just different ways to represent numbers. Some of the more complicated expressions that students work with can be easily evaluated, which may help students embrace challenging problems in the future.

Terminology

The following terms are critical to the work of grade 5 module 3. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

Module 3 does not introduce any new terms.

Familiar

convert

cup

denominator

gallon

mixed number

numerator

ounce

pint

pound

product

quart

quotient

whole number

Academic Verb

demonstrate

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

36	Blue construction paper, $4\frac{1}{2}'' \times 2''$ strips	25	Personal whiteboards
1	Centimeter cubes, set of 1,000	25	Personal whiteboard erasers
25	Dry-erase markers	1	Projection device
24	Envelopes	36	Red construction paper, $4\frac{1}{2}'' \times 2''$ strips
24	<i>Learn</i> books	25	Scissors
2	Paper, blank sheets	1	<i>Teach</i> book
25	Pencils	1	Teacher computer or device

Visit <http://eurmath.link/materials> to learn more.

Before This Module

Grade 4 Module 5

In grade 4 module 5, students relate decimal fractions to decimal numbers. They write and compare decimal numbers to hundredths by using models and by renaming them as fractions. Students also add decimal fractions with denominators 10 and 100 by renaming them as fractions.

Grade 5 Module 1

Students apply place value understanding to multiply and divide whole numbers by powers of 10. They build fluency with the standard algorithm to multiply multi-digit whole numbers. Students also divide whole numbers by using tape diagrams, area models, and vertical form to record quotients and remainders.

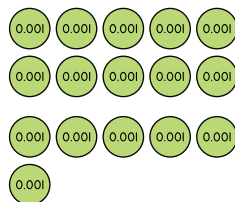
Overview

Place Value Concepts for Decimal Operations

Topic A

Understanding Decimal Numbers with Place Value and Fraction Thinking

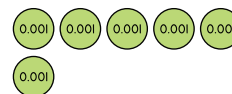
Students represent decimal numbers to thousandths by using a variety of concrete and pictorial models and name the numbers in different forms. Students describe relationships between adjacent decimal place value units as *10 times as much as* the next smaller unit and $\frac{1}{10}$ *as much as* the next larger unit. Students compare two decimal numbers to thousandths and round decimal numbers to any place value.



16 thousandths

$$\frac{16}{1,000} \quad 0.016$$

sixteen thousandths



1 hundredth 6 thousandths

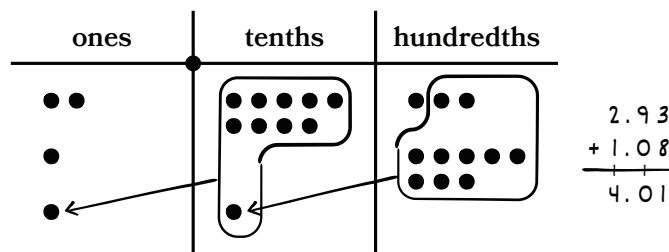
$$\frac{16}{1,000} \quad 0.016$$

sixteen thousandths

Topic B

Addition and Subtraction of Decimal Numbers

Students apply the methods they use to add and subtract whole numbers to add and subtract decimal numbers. By the end of the topic, they apply place value understanding and use concrete and pictorial models, recording the work in vertical form, to support the transition to the standard algorithm.



Topic C

Multiplication of Decimal Numbers

Students apply the methods they use to multiply whole numbers to multiply decimal numbers to hundredths. They rely on unit form and their understanding of multiplication as equal groups to make sense of products of decimal numbers and whole numbers. Then students transition to multiplying two decimal numbers by using fraction multiplication to determine the product and make sense of its units.

After This Module

Grade 6 Module 2

In grade 6 module 2, students fluently add, subtract, multiply, and divide decimals by using the standard algorithm for each operation.

Grade 6 Module 3

In grade 6 module 3, students define *rational numbers*. They plot, order, and compare rational numbers on a number line.

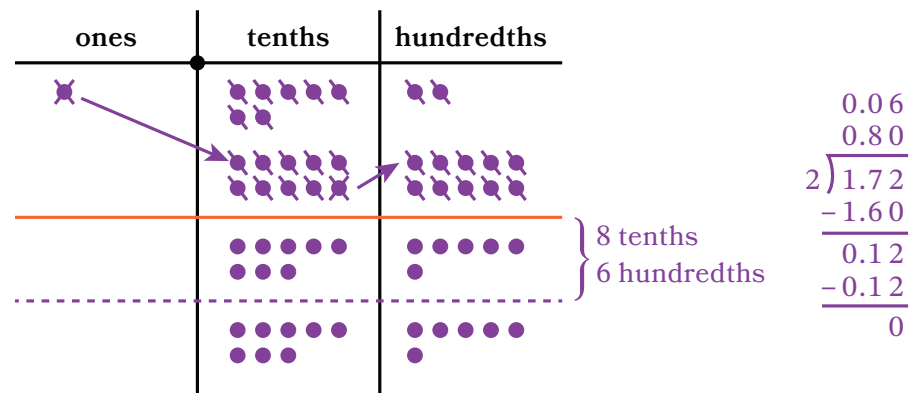
Topic D

Division of Decimal Numbers

Students apply the methods they use to divide whole numbers to divide decimal numbers to hundredths.

With a continued emphasis on unit thinking, students rename decimal numbers in unit

form, use whole-number division methods to divide, and then rename the quotient in decimal form. Students also connect dividing whole numbers by unit fractions to dividing numbers by 0.1 and 0.01.



Topic E

Applications of Decimals

Students apply their understanding of decimal place value, relationships between decimals and fractions, and computation with decimals, fractions, and whole numbers to convert measurements in both the metric and customary measurement systems. They use tape diagrams to interpret and evaluate numerical expressions, and they create word problems that can be represented by a given expression or tape diagram.

$$\begin{aligned}
 360.7 \text{ cm} &= \underline{\hspace{1cm}} \text{ m} \\
 360.7 \text{ cm} &= 360.7 \times 1 \text{ cm} \\
 &= 360.7 \times 0.01 \text{ m} \\
 &= 3.607 \text{ m}
 \end{aligned}$$

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Place Value Concepts for Decimal Operations

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Represent thousandths as a place value unit.	
Lesson 3	62
Represent decimal numbers to the thousandths place in different forms.	
Lesson 4	78
Relate the values of digits in a decimal number by using place value understanding.	
Lesson 5	92
Multiply and divide decimal numbers by powers of 10.	
Lesson 6	110
Compare decimal numbers to the thousandths place.	
Lesson 7	128
Round decimal numbers to the nearest one, tenth, or hundredth.	
Lesson 8	148
Round decimal numbers to any place value unit.	

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Addition and Subtraction of Decimal Numbers

Lesson 9	166
Add decimal numbers by using different methods.	
Lesson 10	180
Add decimal numbers by using place value understanding.	
Lesson 11	198
Subtract decimal numbers by using different methods.	
Lesson 12	214
Subtract decimal numbers by using place value understanding.	
Lesson 13	230
Solve word problems involving addition and subtraction of decimal numbers and fractions.	

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Multiplication of Decimal Numbers

Lesson 14	250
Multiply decimal numbers to hundredths by one-digit whole numbers by using different models.	
Lesson 15	268
Multiply decimal numbers to hundredths by one-digit whole numbers and multiples of 10, 100, or 1,000 by using different written methods.	
Lesson 16	282
Multiply decimal numbers to hundredths by two-digit whole numbers by using area models and vertical form.	

Lesson 17	300
Multiply decimal numbers to hundredths by two-digit whole numbers by using different methods.	

Lesson 18	316
Relate decimal-number multiplication to fraction multiplication.	

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Multiply a decimal number by a decimal number.	

Topic D 351

Division of Decimal Numbers

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Divide decimal numbers to hundredths by one-digit whole numbers and multiples of 10, 100, or 1,000 by using unit form and place value understanding.	

Lesson 21	374
Divide decimal numbers to hundredths by one-digit whole numbers and multiples of 10, 100, or 1,000 by using place value understanding and vertical form.	

Lesson 22	396
Divide decimal numbers to hundredths by two-digit whole numbers.	

Lesson 23	412
Relate division by 0.1 and 0.01 to division by a unit fraction.	

Lesson 24	428
Divide decimal numbers by decimal numbers, resulting in whole-number quotients.	

Lesson 25	442
Divide decimal numbers by decimal numbers, resulting in decimal-number quotients.	

Topic E 457

Applications of Decimals

Lesson 26	460
Solve a real-world problem involving metric measurements. (Optional)	

Lesson 27	480
Convert metric measurements involving decimals.	

Lesson 28	496
Convert customary measurements involving decimals.	

Lesson 29	512
Interpret, evaluate, and compare numerical expressions involving decimals.	

Lesson 30	530
Create and solve real-world problems for given numerical expressions involving decimals.	

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Why

Place Value Concepts for Decimal Operations

Why are multiple models used for decimal-number operations?

Multiple models are used for decimal-number operations to support depth of conceptual understanding, to connect operations with decimal numbers to students' work with whole numbers, and to promote choice, flexibility, and efficiency in problem solving. Through the use of familiar models and strategies, students come to recognize that the same place value relationships that they have relied on for addition, subtraction, multiplication, and division of whole numbers throughout grades 3–5 also support operations with decimal numbers.

Students use concrete and pictorial models such as place value disks, drawings on a place value chart, area models, and number lines, and they rely on a variety of representations, such as expressing operations in unit form and recording calculations in vertical form to organize and show their thinking. For example, students physically bundle and unbundle place value disks, or use place value drawings, and connect their work with the disks to what they see occurring as they record the operations in vertical form. As with whole numbers, area models help students keep track of how a number is decomposed and ensure that each part of that number is multiplied or divided. When students rename a sum, difference, product, or quotient in unit form, they can apply familiar arithmetic skills with whole numbers and then use the units to make sense of the placement of the decimal point.

	3 ones	2 tenths	4 hundredths
8	24 ones 24	16 tenths 1.6	32 hundredths 0.32

$$24 + 1.6 + 0.32 = 25.92$$

Why does the term *decimal number* in topics A–D change to *decimal* in topic E?

This module builds on learning from grade 4 module 5. Grade 4 uses *decimal number* exclusively to help avoid confusion with related terms, such as confusing *decimal number* and *decimal point*, and to highlight the idea that decimals are, in fact, numbers, simply written in a new form. In topics A–D, *decimal number* is used to both support a seamless transition from grade 4 and to notably contrast with *whole number*, particularly because a large emphasis is on place value understanding and on whole-number methods for the multiplication and division work. Grade 6 module 2 uses *decimal* exclusively. Mathematically, either *decimal number* or *decimal* is valid, so this module serves as an intentional transition point. Because the emphasis in topic E shifts to applying the operations, rather than focusing on the operations themselves, there is less need to notably distinguish between decimal numbers and whole numbers.

How is student understanding of decimal numbers enhanced by having previously studied whole numbers and fractions?

Student understanding of decimal numbers enhances after an in-depth study of whole numbers in module 1. Students are able to apply the same methods they use to multiply and divide whole numbers to multiply and divide decimal numbers. They also use their understanding of multiplying and dividing whole numbers by powers of 10 to describe relationships between adjacent decimal place value units. Students use their knowledge of fractions from modules 2 and 3 to use $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ to describe relationships between numbers on the place value chart and to relate fraction operations to decimal-number operations.

63.177
 ↓
 0.07
 ↓
 0.007

0.07 is 10 times as much as 0.007.

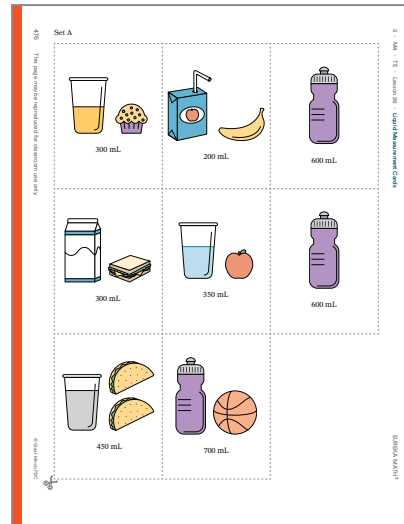
$$0.07 = 10 \times 0.007$$

0.007 is $\frac{1}{10}$ as much as 0.07.

$$0.007 = \frac{1}{10} \times 0.07$$

Why is lesson 26 considered optional?

Lesson 26 engages students in a hands-on, high-level task and provides an opportunity for students to apply a variety of measurement conversion and whole-number and decimal computation experiences to a real-world problem. Students work with decimal amounts of measurement units and intuitively convert measurements from larger units to smaller units before using formal methods, including equations, in lesson 27. The mathematics in this lesson stands alone; students can be successful with the mathematics in the remainder of the topic if this lesson is omitted.



Terminology

The following terms are critical to the work of grade 5 module 4. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

inequality

An inequality is a statement that compares two expressions by using $>$ or $<$. (Lesson 29)

thousandths

Thousandths are a place value unit. 1 one can be decomposed into 1,000 thousandths. (Lesson 1)

Familiar

associative property of multiplication

bundle, exchange, rename

commutative property of multiplication

compare

distributive property

expanded form

exponent

hundredths

long division

power of 10

round

tenths

Academic Verbs

Module 4 does not introduce any academic verbs from the grade 5 list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

2	Chart paper, sheets	25	Pencils
1	Dot dice, set of 12	25	Personal whiteboards
25	Dry-erase markers	25	Personal whiteboard erasers
12	<i>Eureka Math</i> ^{2™} deci-disks set, thousandths to ones	1	Projection device
12	<i>Eureka Math</i> ^{2™} place value disks set, ones to millions	25	Rulers, plastic
24	<i>Learn</i> books	13	Scissors
2	Markers	1	<i>Teach</i> book
8	Paper, sheets	1	Teacher computer or device

Visit <http://eurmath.link/materials> to learn more.

Before This Module

Grade 3 Module 2

Students measure liquid volume in metric units and relate metric units and place value relationships.

Grade 3 Module 4 Grade 4 Module 6

Students identify attributes of a variety of shapes, including the number and length of sides, the number and type of angles, and the presence of pairs of parallel sides. They also draw lines of symmetry in two-dimensional figures. In grade 3, students recognize area as an attribute of two-dimensional shapes and find the area of rectangles with whole-number side lengths by tiling and, eventually, by multiplying side lengths.

Grade 5 Module 3

Students use an area model and number lines to multiply fractions by fractions.

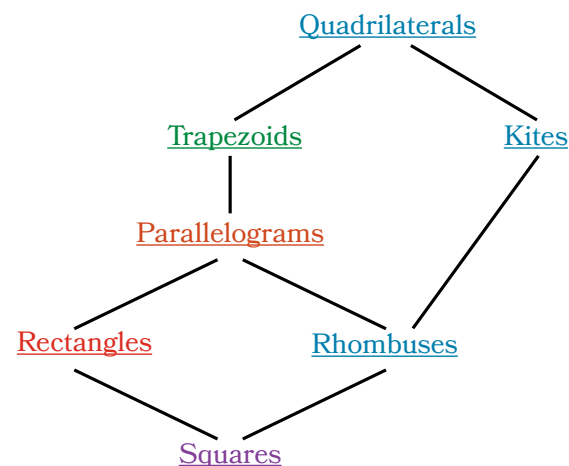
Overview

Addition and Multiplication with Area and Volume

Topic A

Drawing, Analysis, and Classification of Two-Dimensional Figures

Students construct, analyze, and classify trapezoids, kites, parallelograms, rectangles, rhombuses, and squares. They identify properties of quadrilaterals that involve pairs of parallel sides, angle measures, side lengths, diagonals, and lines of symmetry and use these properties to create a hierarchy of quadrilaterals. They use the hierarchy to determine the most specific name of any quadrilateral and all names for the quadrilateral.



Topic B

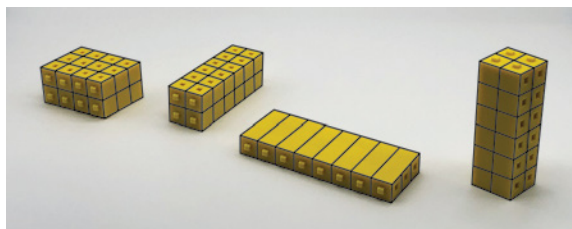
Areas of Rectangular Figures with Fraction Side Lengths

Students find areas of rectangles with fraction side lengths, first by tiling with tiles that are squares with unit-fraction side lengths and then by tiling with rectangles with fraction side lengths. Through reasoning, students determine that the area of any rectangle, including one with fraction side lengths, can be found by multiplying the rectangle's length by its width. Students use an area model to multiply mixed numbers, and then they solve real-world and mathematical problems involving multiplication of mixed numbers.

Topic C

Volume Concepts

In their first formal study of volume, students count the number of unit cubes that pack right rectangular prisms. Then they build right rectangular prisms with improvised units to find volume by using something other than unit cubes. They compose and decompose right rectangular prisms into layers in different ways, finding the volume of each layer and multiplying the number of layers by the volume of each layer to find the volume of the right rectangular prism. Students explore conceptual ideas about volume and capacity by differentiating between packing with cubes and filling with a liquid.



Topic D

Volume and the Operations of Multiplication and Addition

Students synthesize the work of topic C by determining that the volume of any right rectangular prism is calculated either by multiplying the area of the base by the height, $V = B \times h$, or by multiplying the three dimensions of the prism, $V = l \times w \times h$. They use these two formulas to find volumes and unknown dimensions of right rectangular prisms in both mathematical and real-world problems. Students find the volume of a figure composed of right rectangular prisms by decomposing the figure into right rectangular prisms, finding the volume of each prism, and adding the volumes together.

After This Module

Grade 5 Module 6

Students graph quadrilaterals in the coordinate plane and use the structure of the coordinate plane to name each quadrilateral and identify its properties.

Grade 6 Module 3

Students draw polygons in all four quadrants of the coordinate plane and use the coordinates of the vertices to solve real-world and mathematical problems, such as finding area and perimeter.

Grade 6 Module 5

Students find the volume of right rectangular prisms with fraction edge lengths by packing them with cubes with unit fraction edge lengths and by applying the formulas for volume.

Contents

Addition and Multiplication with Area and Volume

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Drawing, Analysis, and Classification of Two-Dimensional Figures

Lesson 1	18
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Analyze hierarchies and identify properties of quadrilaterals.

Lesson 2	36
-----------------------	----

Classify trapezoids based on their properties.

Lesson 3	56
-----------------------	----

Classify parallelograms based on their properties.

Lesson 4	76
-----------------------	----

Classify rectangles and rhombuses based on their properties.

Lesson 5	98
-----------------------	----

Classify kites and squares based on their properties.

Lesson 6	116
-----------------------	-----

Identify quadrilaterals from given properties.

Lesson 7	136
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Classify quadrilaterals in a hierarchy based on properties.

Topic B	152
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Areas of Rectangular Figures with Fraction Side Lengths

Lesson 8	158
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Find areas of square tiles with fraction side lengths by relating the tile to a unit square.

Lesson 9	174
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Organize, count, and represent a collection of square tiles.

Lesson 10	196
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Find the area of a rectangle with fraction side lengths by relating the rectangle to a unit square.

Lesson 11	218
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Find areas of rectangles with fraction side lengths by using multiplication.

Lesson 12	238
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Multiply mixed numbers.

Lesson 13	258
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Solve mathematical problems involving areas of composite figures with mixed-number side lengths.

Lesson 14	278
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Solve real-world problems involving areas of composite figures with mixed-number side lengths.

Lesson 15	294
------------------------	-----

Solve multi-step word problems involving multiplication of mixed numbers.

Topic C	309
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Volume Concepts

Lesson 16	312
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Identify attributes and properties of right rectangular prisms.

Lesson 17	334
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Find the volume of right rectangular prisms by packing with unit cubes and counting.

Lesson 18 354

Find the volume of right rectangular prisms by packing with improvised units.

Lesson 19 374

Compose and decompose right rectangular prisms to find their volume by using layers.

Lesson 20 394

Interpret volume as filling.

Lesson 21 412

Relate volumes of solids and liquid volume.

Topic D 426**Volume and the Operations of Multiplication and Addition****Lesson 22** 432

Find the volumes of right rectangular prisms by using the area of the base.

Lesson 23 452

Find the volumes of right rectangular prisms by multiplying the edge lengths.

Lesson 24 470

Solve word problems involving volumes of right rectangular prisms.

Lesson 25 490

Find the volumes of solid figures composed of right rectangular prisms.

Lesson 26 506

Solve word problems involving perimeter, area, and volume.

Lesson 27 524

Apply concepts and formulas of volume to design a sculpture by using right rectangular prisms, part 1.

Lesson 28 542

Apply concepts and formulas of volume to design a sculpture by using right rectangular prisms, part 2.

Resources**Standards** 554**Achievement Descriptors: Proficiency Indicators** 556**Terminology** 570**Math Past** 572**Materials** 576**Works Cited** 577**Credits** 579**Acknowledgments** 580

Why

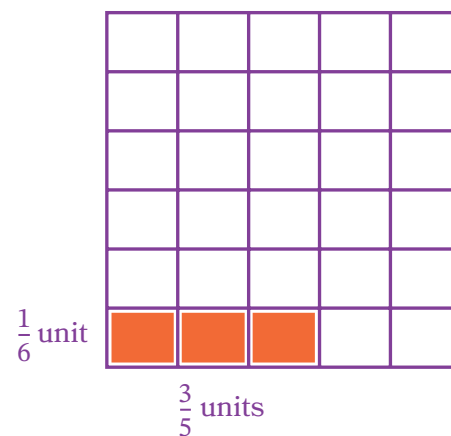
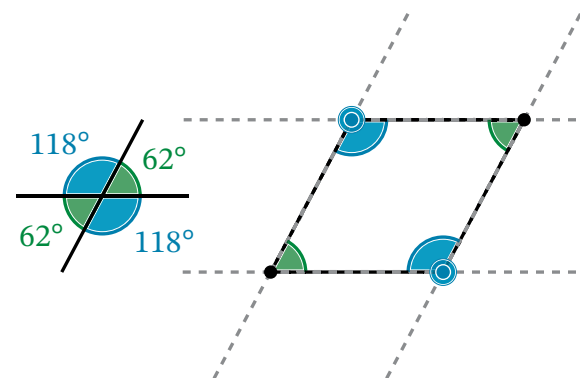
Addition and Multiplication with Area and Volume

Topic A includes significantly more digital interactives than any other topic in grade 5. Why?

The use of digital interactives provides pictorial representation to support the concrete-pictorial-abstract framework and deepen student understanding of quadrilaterals. Each study of a new quadrilateral in topic A introduces students to a concrete representation, such as constructing the quadrilateral by using paper, pencils, right-angle tools, and rulers. A digital interactive provides a pictorial representation that shows various figures that look different than, but are called the same name as, the quadrilateral students constructed. Digital interactives are an ideal way to make comparisons among two-dimensional figures because the measures of angles, sides, and diagonals visually shift as the figure is manipulated. Then students engage with abstract representation in the hierarchy by generalizing properties of the quadrilateral and recording those properties in the correct categories. The repeated sequence for each quadrilateral provides a structure for comparing and manipulating quadrilaterals, inviting students to generalize about the properties in each new digital interactive, which means that topic A has more digital interactives than do other grade 5 topics.

Topic A is about properties of two-dimensional figures, and topics C and D are about volume of three-dimensional figures. Why is topic B, area of rectangles with fraction side lengths, included in this module?

This module is a study of the attributes and properties of two- and three-dimensional figures. Topic A begins the module with a study of properties of two-dimensional figures including sides, angles, diagonals, and symmetry. This study of properties and attributes connects to the focus of topic B, area of rectangles with fraction side lengths, because area is an attribute of two-dimensional figures. Topics C and D focus on volume, an attribute of three-dimensional figures.

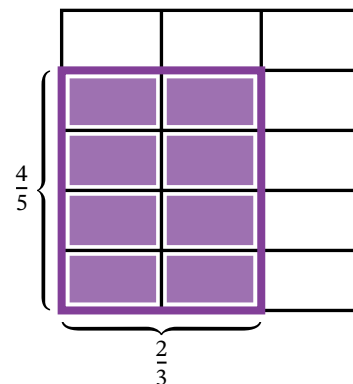


Why do students learn to multiply fractions again in module 5 when they already learned it in module 3?

A subtle distinction must be made between using an area model to multiply fractions and finding the area of a rectangle. In module 3, students use an area model to find the product of fractions before learning that for whole numbers a , b , c , and d , $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$, given that b and d are nonzero, but students are not asked to find the area of a rectangle with fraction side lengths until module 5 topic B. Even then, they cannot immediately apply the familiar formula $A = l \times w$ because they have only learned that the formula applies to whole numbers l and w . Thus, students must develop the conceptual understanding of what it means to find the area of a rectangle with fraction side lengths. To do this, topic B has students replicate their work with whole-number side lengths from grades 3 and 4: they tile a region with fraction unit squares and count the tiles to see that the area of the region is equal to the number of tiles it takes to cover the region. Students can then conclude that the area formula can be applied even when l and w are fractions.

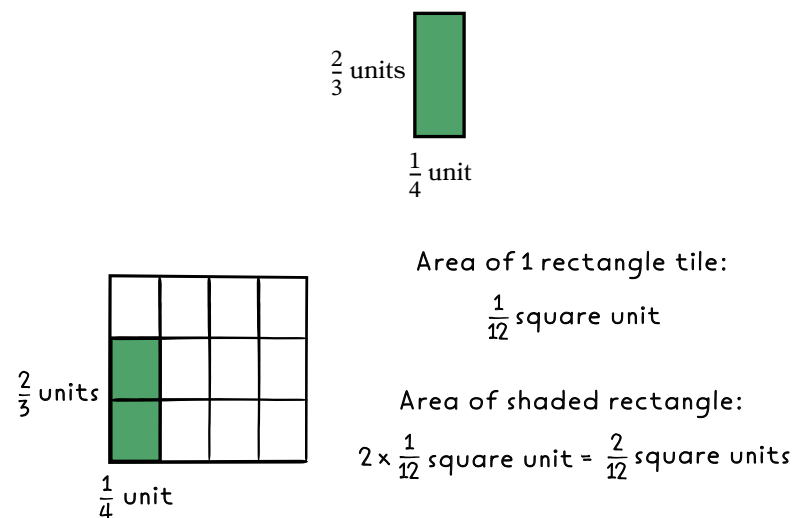
Module 3 Topic B

What is $\frac{4}{5} \times \frac{2}{3}$?



Module 5 Topic B

What is the area of the rectangle shown?



Terminology

The following terms are critical to the work of grade 5 module 5. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

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Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

base

The base of a prism is one of the faces of the prism, often thought of as the surface on which the prism rests. (Lesson 16)

composite figure

A composite figure is a geometric figure composed of two or more smaller figures. (Lesson 13)

cubic centimeter

Cubic centimeters are a unit for measuring volume. 1 cubic centimeter of volume is defined as the volume of a cube with side lengths of 1 centimeter. (Lesson 17)

cubic inch

Cubic inches are a unit for measuring volume. 1 cubic inch of volume is defined as the volume of a cube with side lengths of 1 inch. (Lesson 17)

cubic unit

Cubic units are a unit for measuring volume. 1 cubic unit of volume is defined as the volume of a cube with side lengths of 1 unit. (Lesson 17)

kite

A kite is a quadrilateral with at least two pairs of adjacent sides that are the same length. (Lesson 5)

midpoint

The midpoint of a line segment is the point on the line segment that divides it into two line segments of equal length. (Lesson 3)

plane

A plane is a flat, two-dimensional surface that extends forever. (Lesson 1)

property

A property is a characteristic that is true of every item in a category. (Lesson 1)

right rectangular prism

A right rectangular prism is a solid where all the faces are rectangles. (Lesson 16)

unit cube

A unit cube is a cube with edges that each measure 1 unit. (Lesson 17)

volume

The volume of a solid is the amount of three-dimensional space it takes up. (Lesson 17)

Familiar

acute angle

acute triangle

attribute

capacity

cube

diagonal

edge

equilateral triangle

face

figure

formula

height

intersect

isosceles triangle

line of symmetry

liquid volume

obtuse angle

obtuse triangle

parallel

parallelogram

perpendicular

quadrilateral

rectangle

rhombus

right angle

right triangle

square

square unit

straight angle

supplementary angles

three-dimensional figure

trapezoid

two-dimensional figure

unit square

vertex

Academic Verbs

Module 5 does not introduce any academic verbs from the grade 5 list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

25	4" protractor (180°)	24	<i>Learn</i> books
148	Blank paper, sheets	1	Patty paper, 4" x 4", 500 sheets
1	Centimeter cubes, set of 1,000	25	Pencils
1	Clear tape, roll	25	Personal whiteboards
25	Dry-erase markers	25	Personal whiteboard erasers
104	Envelopes	1	Projection device
12	<i>Eureka Math</i> ^{2™} Rectangular Prisms set	25	Rulers, plastic
24	Glue sticks	25	Scissors
24	Highlighters	72	Sticky notes
135	Index cards	1	<i>Teach</i> book
960	Interlocking cubes, 1 cm	1	Teacher computer or device

Please see lesson 9 for a list of organizational tools (cups, whiteboards, bags, etc.) suggested for the counting collection.

Before This Module

Grade 5 Module 4

Throughout prior grades, students use the number line as a tool for counting, comparing, and operating with whole numbers. Earlier in grade 5, students compare and round decimal numbers in a similar manner to how they compare and round whole numbers, by using the structure of number lines and plotting points to solve problems.

Grade 4 Module 2

In grade 4, students apply their understanding of factors and multiples to find an unknown term in shape or number patterns. They recognize that they can use what they know about the earlier terms in a sequence to find a later term without having to list all the terms.

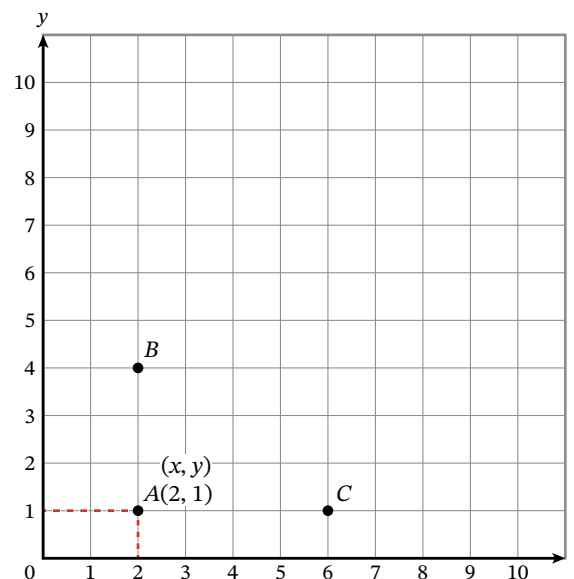
Overview

Foundations to Geometry in the Coordinate Plane

Topic A

Coordinate Systems

Students build on their understanding of number lines to construct a coordinate system composed of intersecting horizontal and vertical number lines. Students plot points and identify ordered pairs for points. They describe the location of a point in the coordinate plane as a horizontal distance from the y -axis and a vertical distance from the x -axis. Students conclude the topic by using a map on a coordinate plane to identify locations and describe distances and directions between those locations.



Topic B

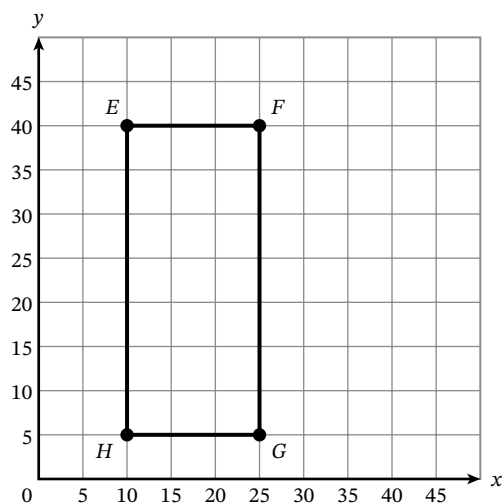
Patterns in the Coordinate Plane

Students extend their understanding of the coordinate plane by identifying properties of horizontal and vertical lines in the coordinate plane. Students then work with two number patterns simultaneously, generating terms when given rules and starting numbers, using the patterns to create ordered pairs, and plotting the points that represent the ordered pairs. Students transition to using tables and graphs to examine relationships between corresponding terms in two number patterns. They identify, describe, and compare addition, subtraction, multiplication, and division number relationships in the coordinate plane. Topic B ends with an optional lesson in which students identify and describe mixed-operation number patterns.

Topic C

Solve Mathematical Problems in the Coordinate Plane

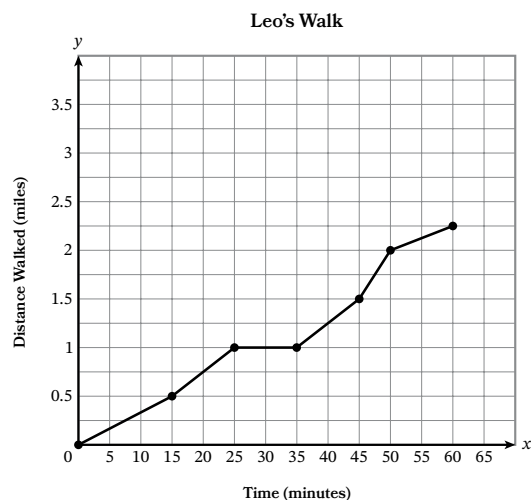
Students begin topic C by examining lines in the coordinate plane. They develop the understanding that lines have an infinite number of points. They realize that one point has many lines through it, but any two points can have only one line that passes through them both. Then students work with geometric figures in the coordinate plane. They classify angles, identify parallel and perpendicular line segments, and use those observations to classify quadrilaterals graphed in the coordinate plane. Students identify lines of symmetry and look for patterns in the coordinates of symmetric points. At the end of the topic, students solve problems by drawing rectangles in the coordinate plane and determining their vertices, perimeters, and areas.



Topic D

Solve Real-World Problems with the Coordinate Plane

In topic D, students recognize that the coordinate plane is a useful tool for representing data, modeling relationships, and solving real-world problems. They come to understand that graphs can tell stories. Students interpret the meaning of points and line segments in a line graph that represents real-world data. In both an optional lesson and a real-world problem-solving task, students revisit relationships between two number patterns. Students solve problems by using a graph to identify and describe the number patterns in the x - and y -coordinates.



After This Module

Grade 6 Module 1

Students model ratio relationships by plotting pairs of values in the first quadrant of the coordinate plane. They interpret the coordinates of points in a graph and use graphs to solve problems involving ratios and rates.

Grade 6 Modules 3 and 5

Students extend the axes of the coordinate plane to include negative rational numbers. They identify and plot points in all four quadrants of the coordinate plane and find the distance between points with the same first or second coordinate. Students draw polygons in the coordinate plane and use the structure of the coordinate plane to solve real-world and mathematical problems.

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Identify mixed-operation relationships between corresponding terms in number patterns. (Optional)

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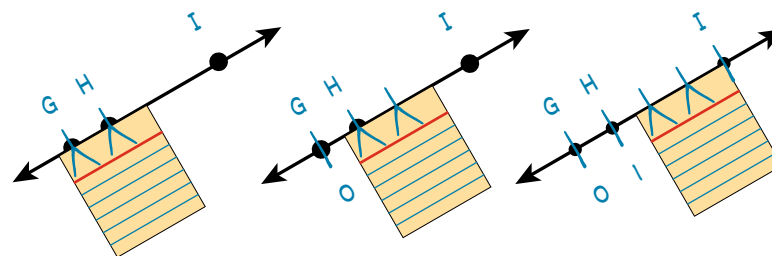
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Why

Foundations to Geometry in the Coordinate Plane

Why does the coordinate plane module start with plotting coordinates on number lines, including number lines of different orientations?

Beginning with plotting coordinates on number lines activates prior knowledge of plotting and determining locations on a number line as distances from 0. Students understand a number line as a coordinate system that can be used to describe the location of a point when they choose the location of 0 and an interval length. Students then build on this knowledge to construct a coordinate plane when they realize that a single number line is not sufficient to describe the location of points that are not collinear.



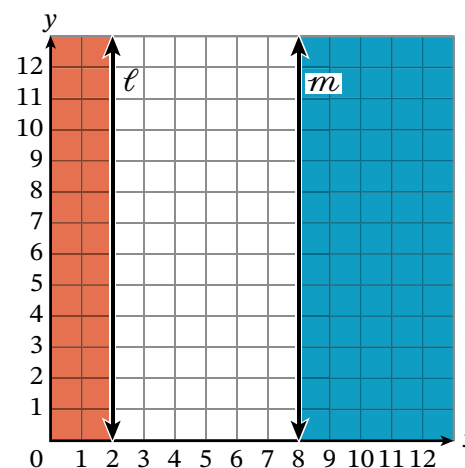
Why do students work with number relationships and mathematical problems before working with real-world problems in the coordinate plane?

Because students are learning what the coordinate plane is, how to use it, and why it is useful, topics A, B, and C introduce students to a geometric interpretation of movements in the first quadrant. To build foundational knowledge, students use the structure of the intersecting axes of the coordinate plane to describe and compare locations, movement within the plane, and distance from the origin and between points, and to solve problems with polygons. This work helps students develop a deeper knowledge of the structure of the coordinate plane.

The real-world context activities in topic D assume fluency with and deep understanding of the coordinate plane, including graphing and locating points, to solve problems.

Line m is a vertical line that intersects the x -axis at the point $(8, 0)$. Draw and label line m on the graph from part (a).

Use blue to shade the region of the plane where points are more than 8 units from the y -axis.

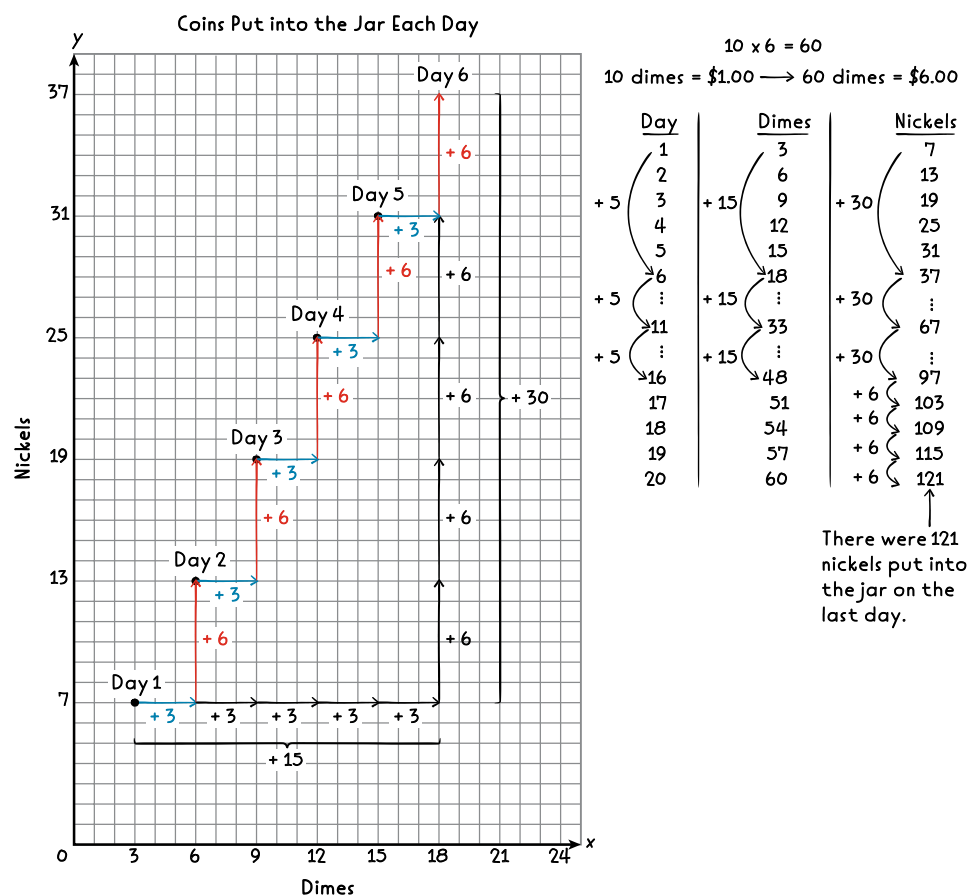


Why are lessons 10 and 19 considered optional?

The activities in lessons 10 and 19 are intended to extend students' work with single-operation number relationships. In lesson 10, students work with more than one operation in a number relationship to gain further experience with identifying and describing relationships between two number patterns. In lesson 19, students are exposed to two number patterns with points that do not lie on a line when graphed, but they can reason about the visual patterns to find additional ordered pairs. This work previews work with expressions and equations in later grades by providing further exposure to more complicated number relationships. Although some students may be able to recognize and apply multiple-operation relationships between the x - and y -coordinates, the expectation for students at grade 5 is limited to identifying single-operation number relationships, such as "The x -coordinate is 4 more than the y -coordinate."

What is the purpose of the problem-solving task in lesson 20?

The task in lesson 20 is meant to prepare students for the open-ended problem-solving tasks and the mathematical modeling cycle that they will encounter in later grades. In lesson 20, students apply their knowledge of number patterns, relationships between coordinates, and graphing to a complex, real-world problem, and they also choose their own model or strategy for solving the problem. The first lesson in grade 6 module 1 is a similar problem-solving task involving measurement conversions that extends students' thinking from the problem in lesson 20, and, together with lesson 20, lays a foundation on which students discover ratio relationships. A major theme of the mathematics that students learn in grades 6–8 is the relationships between two variables and how to model those relationships with equations, tables, graphs, and descriptions. Thus, lesson 20 foreshadows the learning and work that students will do with proportional relationships, linear relationships, and functions in the future.



Terminology

The following terms are critical to the work of grade 5 module 6. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules or in previous grade levels.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

axes

Axes are the pair of perpendicular number lines that create a coordinate system. Axes refers to both the x -axis and the y -axis together. (Lesson 2)

coordinate

A coordinate is a number used to identify the location of a point. (Lesson 1)

coordinate plane

A coordinate plane is a plane with a coordinate system used to describe locations of points in the plane. A coordinate plane has perpendicular axes that intersect at 0. (Lesson 2)

coordinate system

A coordinate system is a system that uses one or more numbers, or coordinates, to identify the location of a point. (Lesson 1)

ordered pair

An ordered pair is a pair of two coordinates that describe the location of a point in the coordinate plane. An ordered pair is written (x, y) , where x represents the point's x -coordinate and y represents the point's y -coordinate. (Lesson 2)

origin

The origin is the point where the x - and y -axes intersect. The origin is located at the point $(0, 0)$. (Lesson 2)

x -axis

The x -axis is the horizontal number line in a coordinate system. (Lesson 2)

x -coordinate

The x -coordinate is the value that describes a point's horizontal distance from the y -axis. (Lesson 2)

y -axis

The y -axis is the vertical number line in a coordinate system. (Lesson 2)

y -coordinate

The y -coordinate is the value that describes a point's vertical distance from the x -axis. (Lesson 2)

Familiar

acute angle

angle

area

endpoint

figure

graph

horizontal line

interval

length

line

line of symmetry

line segment

midpoint

number line

obtuse

operation

parallel

parallelogram

pattern

perimeter

perpendicular

plane

point

polygon

quadrilateral

ray

rectangle

scale

square

symmetry

term

tick mark

trapezoid

vertex

vertical line

Academic Verbs

Module 6 does not introduce any academic verbs from the grade 5 list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

96	Colored pencils	1	Projection device
25	Dry-erase markers	24	Sticky notes, 1.5" × 2"
50	Index cards	269	Sticky notes, 3" × 3"
24	<i>Learn</i> books	25	Straightedges
12	Markers	1	<i>Teach</i> book
11	Paper, sheets	1	Teacher computer or device
25	Pencils	1	Timer
25	Personal whiteboards	24	4" protractors (180°)
25	Personal whiteboard erasers		

Visit <http://eurmath.link/materials> to learn more.