

Before This Module

Kindergartners do not need prior experiences with *Eureka Math* or any preschool curriculum to be successful at the beginning of this module.

However, many modules from prekindergarten directly support the work in this module. They prepare students to successfully count groups of 0 to 10 objects and find or write the matching numeral. They also set the foundation for using one-to-one correspondence as a way to determine whether there are enough.

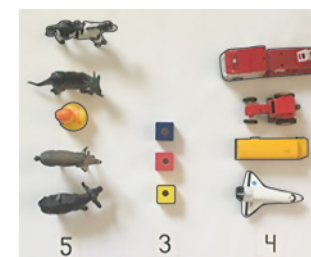
Overview

Counting and Cardinality

Topic A

Classify to Make Categories and Count

Classifying objects into groups by attribute creates a natural context for counting and working with numbers. Students are introduced to four core ideas about counting, which are collectively referred to as the number core. (See the *Why* section.)



Topic B

Answer *How Many* Questions with Up to 5 Objects

Students integrate the elements of the number core while considering different ways and reasons to count. They learn three strategies to help them count accurately regardless of configuration: touch and count, move and count, and mark and count.



Topic C

Write Numerals and Create Sets of Up to 5 Objects

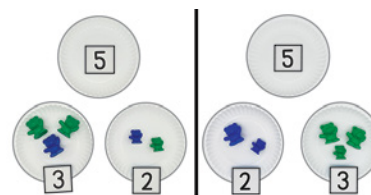
Students begin to write numerals and experience more complex counting situations, like counting sounds and words that can be heard but not seen. They count out objects to match a given number. They also count to find out whether they have enough of something.



Topic D

Decompose Numbers

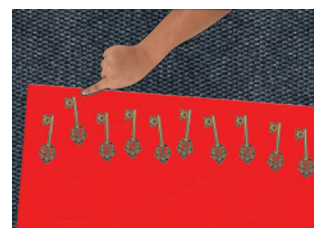
Students explore number relationships. This begins with the familiar context of sorting and then moves to story problems. Students begin to use number sentences like 5 is 2 and 3. These early experiences with problem solving set the stage for work with addition and subtraction concepts in modules 4 and 5.



Topic E

Answer *How Many* Questions with Up to 10 Objects

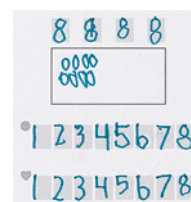
Topic E builds on the work of topic B as students apply number core concepts to sets of 6 to 10 objects. As the size of the group gets larger, tracking the count becomes more challenging. Students consider how to choose an accurate counting strategy.



Topic F

Write Numerals and Create Sets of Up to 10 Objects

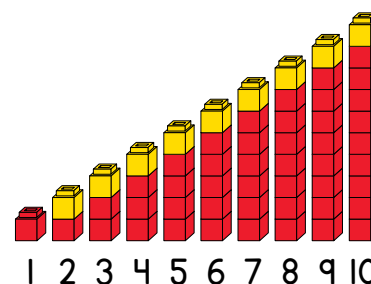
Topic F provides space to practice counting while students learn to write 6, 7, 8, 9, and 10. Students count things that happen over time, like a person's jumps or the runs in a baseball game, and see that writing numbers is a good way to record and remember that information.



Topic G

Analyze the Count Sequence

Students analyze the count sequence focusing on the ideas that each successive number is 1 more when counting forward and 1 less when counting backward. The number stairs model clearly illustrates these patterns.



After This Module

Kindergarten Module 2

Students apply their ability to isolate and sort by attributes, including number, as they analyze and compare shapes in module 2. They count sides and corners as they name, compare, model, and compose shapes.

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Lesson 21 236

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Lesson 22 246

Count sets in scattered configurations and match to a numeral.

Lesson 23 284

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Topic F 293**Write Numerals and Create Sets of Up to 10 Objects****Lesson 24** 296

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Topic G 339**Analyze the Count Sequence****Lesson 29** 342

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Lesson 30 352

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Organize, count, and represent a collection of objects.

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Why

Counting and Cardinality

What is the number core? How is it tied to counting?

In this module, children have sustained interaction with four core ideas for describing the number of objects in a group. These ideas are collectively referred to as the number core.

- **The number word list**—Students say numbers in the appropriate count sequence (1, 2, 3, ...).
- **One-to-one correspondence**—When counting, students pair one object with one number word, being careful not to count any objects twice or skip any objects.
- **Cardinality**—Students say a number to tell how many are in a group. They may be able to tell how many by subitizing, counting, or matching to group of known quantity. When counting, students recognize that the last number said represents the number of objects in the group.
- **Written numerals**—Students read and write the symbols used to represent numbers. They also connect the written numeral with the number of objects in a set.

Students must integrate all aspects of the number core to count and use numbers fluently. The majority of kindergarten activities should involve three or more elements of the number core in conjunction. The number core components are not learned in isolation.

The number core plays a foundational role in work with number relations, operations, and place value understanding and is thus a critical start to the kindergarten year.



Students use one-to-one correspondence when they say one number word for each spider they touch.



Students show how many are in a set (cardinality) using written numerals.

Why isn't there a lesson for each number from 1 to 10?

Module 1 focuses on strategies rather than specific numbers. When students learn strategies for counting accurately, they are able to apply them to different numbers arranged in different configurations.

For instance, when counting in a circular configuration, the challenging part is identifying the starting and stopping points so that every object is counted once and only once. Once a student has learned a strategy for marking the start, that strategy can be useful in successfully counting any number of objects in a circular configuration. The same strategy for marking the start can also be used to count the sides and corners of shapes.

As early as possible, we want students to approach problem solving with a strategy that allows them to be accurate and efficient. When a lesson focuses on a single number, students go into the counting task anticipating the total. Counting tasks in which students do not already know the total but genuinely want to know the total open them to the process of choosing a strategy to solve a problem (MP1). With good questioning, students can begin to evaluate whether their chosen strategy is effective.

A focus on strategies rather than specific numbers means that the size of a set can be varied to challenge individual students. This approach creates accessibility and engagement for kindergartners with a range of counting experiences.

What is counting the math way? How does it affect later learning?

Fingers are familiar manipulatives that students have with them all day, every day, and if used intentionally, fingers can support conceptual understanding and problem solving throughout elementary school. Around the world, people use their fingers to represent numbers in many different ways. Depending on their cultural experiences, young children may begin counting on a thumb, pointer finger, pinkie, or even a section of a finger. Students learn to count the math way in module 1, starting with 1 on the left pinkie finger and continuing to 10 on the right pinkie finger.



The mathematical advantage of counting the math way is that students count from left to right without interruption, just as they do on the number path, and eventually, the number line. Students see and feel the quantity increase as they count forward. The steady increase in distance from the starting point is a physical and visual model for understanding the magnitude of a number.

Counting the math way has advantages for understanding number relationships that will be important in kindergarten and beyond.

- **Unitizing.** When students unitize, they make use of five as a whole instead of thinking of it as 5 individual pieces. The structure of our hands makes it easier for students to unitize 5 when they think about 6 as 5 and 1, 7 as 5 and 2, and so on. Unitizing is a key step to using Level 2 counting on strategies.
- **Partners to 10.** Fluency with partners to 10 is critical to Level 3 problem-solving strategies. When counting the math way, students can easily see that the raised fingers and the lowered fingers are partners to 10. Other ways of finger counting do not always keep the fingers representing each part next to one another.
- **Embedded numbers.** The ability to decompose a number into embedded parts is foundational to many Level 3 strategies. Showing parts within a total is easy when both parts are 5 or less. When modeling $2 + 3$, a student can show one part on each hand. It is more challenging to see the parts when modeling $6 + 2$. As fine motor skills develop, students can wiggle the fingers representing one part while holding the other part steady. Counting the math way creates consistency with how the embedded number looks inside the total.

 1	 6
 2	 7
 3	 8
 4	 9
 5	 10

This is a module about numbers to 10. Why do you suggest counting collections with as many as 25 objects?

Counting collections are rich tasks that allow students to explore numbers and engage with the mathematical practices. They are also authentic assessment opportunities for teachers.

The first counting collection, which occurs in lesson 6, is a diagnostic assessment as much as it is an opportunity to introduce counting strategies. Young children with regular access to high-quality math experiences may walk into kindergarten with the ability to organize, count, and represent sets of 50 or more objects. Providing larger counting collections gives those students a chance to share the full range of their abilities so teachers can differentiate instruction to meet their needs.

We suggest beginning with collections of between 5 and 25 objects for the following reasons:

- Research indicates that students may count farther when counting objects than they do when asked to rote count.¹
- When students cross into the twenties, we see whether they can use the repeated pattern of number names in English (e.g., saying twenty-one instead of twenty-eleven).
- Young students are good at choosing a collection that matches up with their counting abilities when the collections have fewer than 25 objects. With larger collections, students can't perceive the difference between collection sizes with enough accuracy to stay within a comfortable range.
- Experiencing success in early kindergarten activities is important to building students' self-confidence and the sense that they are mathematicians.

The number of objects in a collection should increase in subsequent lessons (lessons 19 and 33) if any student shows mastery through 25. Try 40 or 50 objects and slowly increase the number until you find the spot where the student begins to need help with the number word list, or with organizing to maintain one-to-one correspondence.



¹ Carpenter, Thomas P., *Young Children's Mathematics*, p. 26

Terminology

The following terms are critical to the work of grade K module 1. This resource groups terms into categories called New and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

count

When we want to find out how many of something there are, we count. (Lesson 3)

enough

When there is enough of something, we don't need any more. For example, each dog gets 1 treat, each person gets 1 pair of shoes, or there is 1 fork for each plate. (Lesson 7)

line

Objects organized straight across (or up and down) are in a line. (Lesson 8)

In kindergarten, students only work with lines as an organizational concept. In later grades, they will be asked to consider lines

geometrically, as a straight path with no thickness that extends in both directions without end.

number

How many things there are. We write numbers to show how many things there are. (Lesson 5)

In kindergarten, number is commonly used to refer to a number (the concept of 5), its numeral (5), and the English word for that number (five). Kindergarten students are not asked to distinguish between these concepts.

number sentence

A number sentence tells how we put together or take apart a number, like *5 is 3 and 2* and *3 take away 1 is 2*. (Lesson 15 and Lesson 18)

In module 4, we will also use the term number sentence to describe an equation that uses symbols like +, −, and =. For example, $5 = 2 + 3$ and $3 - 1 = 2$ are number sentences.

sort

Put a set of objects into different groups. We might organize by color, size, or something else. (Lesson 2)

strategy

This is a plan to solve a problem. We can use a strategy to make sure we count correctly. (Lesson 6)

In module 1, students learn several counting strategies, such as touch and count and move and count. Later, students will learn strategies for activities other than counting, such as adding and subtracting.

Academic Verbs

explain

notice

record

solve

Materials

The following list of materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

3	Carrots	25	Pencils
1	Chart paper, tablet	25	Personal whiteboards
25	Counting collections	25	Personal whiteboard erasers
25	Crayon sets	1	Projection device
25	Cups	1	Puppet or stuffed animal
25	Dry-erase markers	1	Set of number gloves, left and right
1	<i>Eureka Math</i> ^{2™} 5-Group™ cards, demonstration set	48	Small resealable plastic bags
25	<i>Eureka Math</i> ^{2™} Bingo boards	25	Sorting bags
24	<i>Eureka Math</i> ^{2™} Hide Zero® cards, basic student set 12	1	<i>Teach</i> book
1	<i>Eureka Math</i> ^{2™} Hide Zero® cards, demonstration set	1	Teacher computer or device
12	<i>Eureka Math</i> ^{2™} Match cards, set of 12	3	Teddy bear counters, set of 96
12	<i>Eureka Math</i> ^{2™} Numeral Cards	275	Two-color beans, red and white
24	<i>Learn</i> books	1	Unifix® Cubes, set of 1,000
1	Pad of sticky notes	1	White paper, ream
96	Paper plates		

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 6 for a list of organizational tools (cups, rubber bands, graph paper, etc.) suggested for counting collections.

Daily Tool Kits

In module 1, students and teachers work with hands-on materials to explore the math concepts introduced in each lesson. The list of materials below includes the most frequently used items in module 1. Consider creating a tool kit for each student to minimize materials preparation for each lesson. Having student and teacher tool kits on hand each day allows for smooth transitions and drastically decreases lesson prep time.

For best results, begin module 1 with 5 Unifix[®] Cubes, 5 two-colored beans, Hide Zero[®] cards 0–5, a pencil, and a crayon. Increase the number of cubes, beans, and Hide Zero[®] cards to 10 before beginning topic E.

Student Daily Tool Kit

- Unifix[®] Cubes (10)
- Two-color beans in resealable bag (10)
- *Eureka Math*^{2™} Hide Zero[®] cards, student set (0–10)
- Pencil
- Crayon



Teacher Daily Tool Kit

- Set of number gloves, left and right
- Two-color beans in resealable bag (10)
- Unifix[®] Cubes (10)
- *Eureka Math*^{2™} Hide Zero[®] cards, demonstration set (0–10)



Self-Created Materials

Number Gloves

To create number gloves, you will need a new pair of cleaning gloves and a permanent marker. Lay out the gloves with the palms facing up. Start with the right-hand glove, writing numbers on the fingertips beginning with 1 on the pinkie finger. Continue across the fingers, ending with 5 on the thumb. Continue with the left-hand glove, beginning with 6 on the thumb and ending with 10 on the pinkie finger. Keep the gloves in your teacher tool kit for multiple uses in module 1.



Two-Color Beans

To create two-color beans, you will need lima beans and a can of red spray paint. Lay the beans on newspaper or a protected surface. Spray one side of the beans and allow them to dry. The other side will remain the original color of the bean. Keep the beans in a cool, dry place in your teacher tool kit for multiple uses in module 1.



Sorting Bags

In topic A, students sort objects in a variety of ways and explore attributes such as number, size, shape, and color. They use sorting bags, or bags of strategically selected objects, that can be classified by attribute. Use sorting bags to provide extra practice in stations, small group instruction, or at other times throughout module 1.

Use the chart to create sorting bags that vary by attribute and complexity. Create one or more sorting bags for each attribute.

You do not need to create a class set for every bag type because students choose from the assortment of bags each day.

Students are not limited to one way of sorting items in a bag, even if the bag is designed with a specific attribute in mind. For example, students may sort a bag of buttons by color, shape, or size. They may also sort into unique categories based on their personal experience. Giving students the autonomy to sort as they choose allows them to explore categories and attributes independently.

Difficulty	Attributes					
	Color	Size	Name or Type	Shape	Location	Use or Activity
Complex	Different objects with the same pattern	Different objects with contrasting dimensions (e.g., long, thin items; short, wide items; long, wide items; short, thin items)	Similar objects with different attributes (e.g., insects and spiders)	Different objects, 2-D and 3-D shapes	Similar objects, 3 types of transportation (e.g., students who ride the bus, walk, or bike)	Objects with multiple uses, 3 uses (e.g., use to play, use to write, use to build)
Moderate	Different objects, 3 colors	Different objects, 3 sizes (small, medium, and large)	Different objects, 3 types (e.g., fly, cat, and fish)	Different objects, 3 shapes	Different objects, 3 locations	Different objects, 3 uses
Easy	Same objects, 2 colors	Same objects, 2 sizes	Different objects, 2 obvious types (e.g., fly and cat)	Same objects, 2 shapes (e.g., round button and square button)	Different objects, 2 obvious locations (e.g., apple in lunch box, car in toy box)	Different objects, 2 obvious uses (e.g., use to eat, use to write)

Bags that can be sorted in more than one way are especially interesting for students. They can also be used for decomposition activities beginning in module 1 topic D and continuing through module 4. In the following examples, the number of objects

in each group and the number of possible groups may vary depending on the attribute used to sort. This keeps students engaged in counting in topic A, and in considering different ways to decompose a number in topic D and beyond.

Easy	• Large yellow bears, small yellow bears, large blue bears, small blue bears (size, color) • Yellow banana, yellow school bus, yellow lemon, yellow marker, red apple, red fire truck, red marker, green lettuce, green car (color, type, use) • Green apple, peas, watermelon, banana, yellow squash, corn, red apple, raspberry, cherry (color, shape, type, size)
Moderate	• Small white rocks, small black rocks, medium black rocks, large gray rocks (size, color, shape) • Small round brown buttons, large round brown buttons, medium pink round buttons, small pink heart-shaped buttons, large square blue buttons, small green round buttons (color, size, shape) • Bird, insect, airplane, boat, duck, pool float (transportation, air or water; type, alive or not; use, fly or float)
Complex	• Polka-dot shirt, polka dot pants, polka dot plate, striped jacket, striped cup, plaid socks, plaid napkin (color, use) • Fish, fox, dolphin, dog, donkey, clam, cow, coyote, camel (name, first sound; location, water or land) • Scissors, pencil, crayon, marker, eraser, toothbrush, comb, flip-flops, umbrella, bathing suit (use, location)

Counting Collections

Throughout the kindergarten year, students will organize, count, and record collections of objects. Each class needs a variety of counting collections made of everyday materials. At the beginning of the year, collections of 4–25 objects are appropriate for most students. By the end of the year, kindergartners may be ready to count collections of 100 or more objects.

Does this sound like a lot of materials management? Establishing routines and an organization system for counting collections makes the process much more manageable, and the student learning will be worth it.



Counting Collection Bins
4–10 objects
11–25 objects
26–45 objects
46–100 objects
101–150 objects

Creating and Organizing Collections

Counting collections can be assembled by using materials easily found in classrooms or homes. Buttons, beans, bottle caps, pom-poms, erasers, craft sticks, crayons ... almost anything can be part of a counting collection. Put each collection in a resealable bag or container.

We suggest sorting and storing collections in color-coded bins based on the number of objects in the collection. Place a corresponding colored sticker on each collection so students can easily return them to the proper bin. Begin the year with a written inventory of how many objects are in each bag to help gauge student counting accuracy, especially with larger collections. Consider having a designated container for items that were missed during cleanup.

Routine Materials

Develop a routine set of materials that each pair of students needs for counting collection lessons. One counting collection and two pieces of paper per student pair are necessary, but providing a work mat and two clipboards will help kindergarten students stay organized. A piece of 11" x 17" paper makes a good work mat. Counting collection lessons are a great opportunity for formative assessment, so we recommend that teachers have an observational checklist available.

Choice Materials and Organizing Tools

Students may choose to use organizing tools as they count and record. Cups, bowls, 10-frame cartons, number paths, 10-frames, and two-hand mats can support collection organization, counting, and recording. Teacher notes provide specific ideas for using organizing tools to support or challenge students. Display and make available these types of materials for students to use as they wish.

Before This Module

Kindergarten Module 1

In the previous module, students identified the attributes of an object, used attributes to sort objects, and discussed sorting strategies and rules. Students also learned strategies to count and keep track of their count.

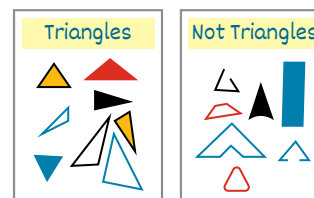
Overview

Two- and Three-Dimensional Shapes

Topic A

Analyze and Name Two-Dimensional Shapes

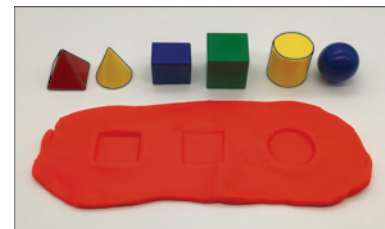
Most kindergarten students come to school with some knowledge about shapes. Those ideas may be based on visual examples or real-world experience: “That is a rectangle because it looks like a door.” They may be able to name some shapes based on the general look or feel of the shape. The goal of this topic is to turn their attention to the specific qualities that are used to classify shapes, such as number of sides and corners, known as defining attributes. This process helps students build a conceptual understanding of two-dimensional, or flat, shapes.



Topic B

Analyze and Name Three-Dimensional Shapes

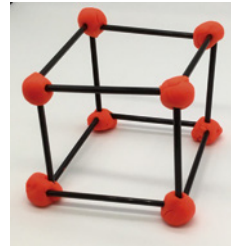
Students continue to focus on defining attributes and extend the list to include features of three-dimensional, or solid, shapes: faces and edges. Through sorting, they discover that some attributes are common to both flat and solid shapes, such as corners. Their spatial thinking evolves as they consider how geometric attributes affect the way a solid shape can be moved or the type of imprint it leaves.



Topic C

Construct Shapes

Prior experience with attributes paves the way for building, drawing, and composing shapes. To manipulate materials and successfully construct a shape, students draw from the mental images built in the first two topics. Experiences constructing and composing shapes provide students opportunities to observe how shapes look from multiple perspectives, how parts fit together to make a whole, and how shapes relate to one another.



After This Module

Kindergarten Module 3

The ability to focus on a specific set of attributes sets the stage for comparison. The measurable attributes of length, weight, quantity, volume, and area are the focus as students use and fine-tune the spatial reasoning skills developed in module 2. They communicate their observations about objects with structured comparison statements by using precise mathematical language. The module culminates with a return to counting and cardinality when students compare sets and numbers.

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Analyze and Name Two-Dimensional Shapes

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Find and describe attributes of flat shapes.	

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Classify shapes as triangles or nontriangles.	

Lesson 3	48
Classify shapes as circles, hexagons, or neither.	

Lesson 4	60
Classify shapes as rectangles or nonrectangles, with square rectangles as a special case.	

Lesson 5	72
Communicate the position of flat shapes by using position words.	

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Analyze and Name Three-Dimensional Shapes

Lesson 6	84
Distinguish between flat and solid shapes.	

Lesson 7	94
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Lesson 8	110
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Construct Shapes

Lesson 10	138
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Lesson 11	148
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Lesson 12	158
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Lesson 13	170
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Lesson 14	180
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Why

Two- and Three-Dimensional Shapes

Why do we need so many examples of each flat shape for this module?

Young children base their understanding of shapes on examples they can see, touch, and manipulate. To fully master the kindergarten geometry standards, students need access to a variety of two- and three-dimensional shapes.

Typical children's books and media show a very limited set of shapes—generally symmetric, regular shapes. Young children often base their early mental images, or prototypes, on this small subset of shapes.

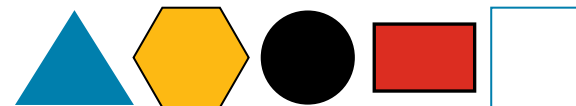
However, concepts of shape do not build like number concepts. Starting with a small set of prototype shapes and gradually introducing new examples of those shapes over time leads to limited and sometimes inflexible concepts of shape that have to be unlearned. Reliance on typical exemplar shapes can lead students to misclassify shapes because they do not look how students expect. For example, a kindergarten student might say, “That’s not a triangle. It’s too pointy.”

When students work with a variety of examples and nonexamples, they are able to move from a visual level of understanding shapes toward the descriptive level. At the descriptive level, students begin to name a shape based on a set of defining attributes, such as the number of sides and corners, rather than how the shape looks.

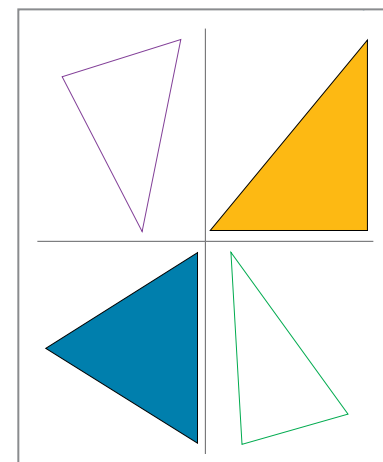
What are exemplars, variants, and distractors?

Kindergarten students sort two-dimensional shapes into two groups, examples and nonexamples. Students name these groups based on the target shape. Examples are called *triangles* and nonexamples are called *not triangles*.

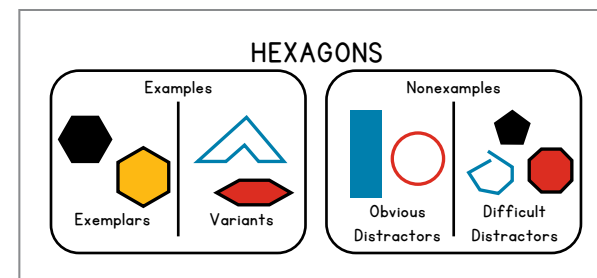
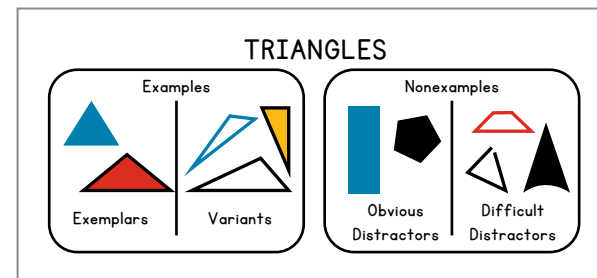
Within the example group, there are exemplars and variants. Exemplars are the shapes that young children typically visualize. They are often symmetrical with a horizontal base. Variants are less frequently encountered in typical children's books and media. To name these shapes, students must pay close attention to defining attributes. Expect to find many variants in the topic A lessons.



Typical Exemplar Shapes



Triangles Removable



The nonexample group can be broken into two parts, obvious distractors and difficult distractors. Obvious distractors do not look much like the exemplars and are easier for children to categorize as nonexamples on sight. They rarely require further investigation. Difficult distractors look more like exemplars, but they do not have all of the defining attributes of the target shape. Difficult distractors require students to attend to defining attributes with more precision. They will be part of all the two-dimensional shape sorts in topic A.

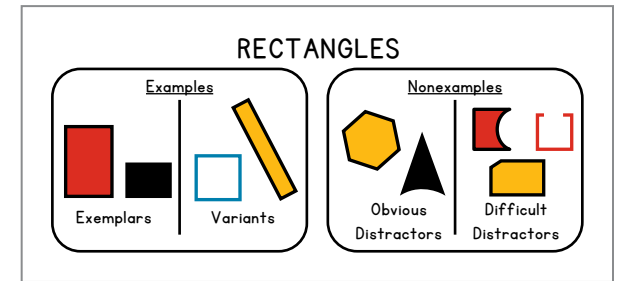
It is not necessary for students to distinguish between exemplars and variants or obvious and difficult distractors. Use the charts on the right to better understand the intentional selection of two-dimensional shapes for each lesson.

How precise do kindergarten students need to be when identifying, building, and drawing shapes?

It is common for students to enter kindergarten at a visual and syncretic level of geometric thinking, which means that their understanding of shape is largely driven by appearance and relationships that are not mathematical. “It’s a circle because it looks like a tire.” A tire isn’t, strictly speaking, a circle, but telling young children this before they begin to understand shape attributes can lead to confusion. Approximation is normal as students develop geometric understanding. Instead of correcting a lack of precision, focus on the mathematical attributes that likely inspired the connection between a real-world object and a shape, “Tires and circles are both round, aren’t they?”

The sorting activities and games in this module provide plenty of opportunity to help students develop a more descriptive concept of shape based on defining attributes. Once students become more precise by identifying disqualifiers like openness or curved corners, they are ready to consider why a real-world object might not fit into a shape category, “How are the corners of the train window different from a rectangle’s corners?”

When students are constructing and drawing shapes, motor skills and materials significantly impact their precision. Look for intent to create features like straight lines and right angles rather than actual production when assessing student understanding. Gentle pushes toward precision may be appropriate depending on the individual’s motor skill development. “Is there a tool you could use to make straight lines?” “Is your shape open or closed? How could you close it?”



Terminology

The following terms are critical to the work of grade K module 2. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

circle

A circle is a closed, totally round, curved shape. The space from the middle to the curve is always the same. (Lesson 3)

cone

A cone is a solid shape with a circular face and a point. (Lesson 7)

cube

A cube is a solid shape that looks like a box. All of a cube's faces are square rectangles. (Lesson 7)

cylinder

A cylinder is a solid shape that has 2 circular faces and is curved around the side. (Lesson 7)

flat shape

When placed on a table, flat shapes lay down (or do not rise above the table top). Triangles, rectangles, and circles are examples of flat shapes. (Lesson 6)

hexagon

A hexagon is a closed shape with 6 straight sides and 6 corners. (Lesson 3)

pyramid

A pyramid is a solid shape that has square and triangular faces and a point. (Lesson 7)

rectangle

A rectangle is a closed shape with 4 straight sides and 4 corners. If we sit any side of a rectangle on the floor, there are 2 sides that go straight up and down. (Lesson 4)

rectangular prism

A rectangular prism is a solid shape that looks like a brick. All of a rectangular prism's faces are rectangles. (Lesson 7)

solid shape

When placed on a table, solid shapes stand tall (or rise above the table top). Cubes, cylinders, and rectangular prisms are examples of solid shapes. (Lesson 6)

sphere

A sphere is a round, solid shape that looks like a ball. (Lesson 7)

square rectangle

A square rectangle is a special kind of rectangle. It has 4 sides and 4 corners, and all of the sides are the same. (Lesson 4)

triangle

A triangle is a closed shape with 3 straight sides and 3 corners. (Lesson 2)

Familiar**count****number****sort****Academic Verb****describe**

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	20-bead demonstration rekenrek	2	Modeling clay, containers
3	3-D attribute solids, set of 16	25	Pencils
1	Ball	24	Personal whiteboards
1	Box	24	Personal whiteboard erasers
1	Brown paper bag	3	Plastic pattern block sets, 0.5 cm
1	Chart paper, tablet	1	Projection device
25	Crayon sets	1	Puppet or stuffed animal
1	Dot dice, set of 12	25	Scissors
24	Dry-erase markers	24	Small resealable plastic bags
1	<i>Eureka Math</i> ^{2™} 2D shapes, foam, set of 30	12	Smiley face stickers
1	<i>Eureka Math</i> ^{2™} Geometry cards	200	Straw-style plastic coffee stirrers
24	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, student set	1	Teacher computer or device
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, demonstration set	120	Two-color beans, red and white
12	<i>Eureka Math</i> ^{2™} Match cards	24	Two-color counters
1	<i>Eureka Math</i> ^{2™} Pattern block puzzles, set of 46	1	Unifix [®] Cubes, set of 1,000
12	Folders	1	White paper, ream
1	Geometric solids set, 40 pieces	1	Yarn, skein
24	<i>Learn</i> books		

Visit <http://eurmath.link/materials> to learn more.

Daily Tool Kits

In module 2, students and teachers work with hands-on materials to explore the math concepts introduced in each lesson. The list of materials below includes the most frequently used items in module 2. Consider creating a tool kit for each student to minimize materials preparation for each lesson. Having student and teacher tool kits on hand each day allows for smooth transitions and drastically decreases lesson prep time.

Student Daily Tool Kit

- Two-colored beans in resealable bag (10)
- Geometric solids (cube, cylinder, sphere, cone, pyramid, and rectangular prism) in resealable bag
- Pencil
- Scissors
- Crayons



Teacher Daily Tool Kit

- Two-colored beans in resealable bag (10)
- Geometric solids (cube, cylinder, sphere, cone, pyramid, and rectangular prism) in resealable bag
- 2D shapes in a resealable bag
- Puppet
- Marker
- Scissors



Before This Module

Young children have an intuitive understanding of size comparison from their daily experiences in the world. This module develops their ability to talk about their understanding. It also provides tools and encourages behaviors to make more precise comparisons.

Kindergarten Module 1

In the second half of the module, students build on the comparisons they made in module 1 when they used one-to-one correspondence to find out if there were enough. They use their understanding of number to make increasingly accurate comparisons.

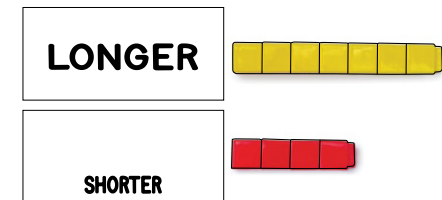
Overview

Comparison

Topic A

Compare Heights and Lengths

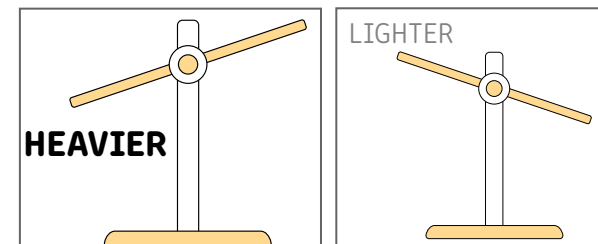
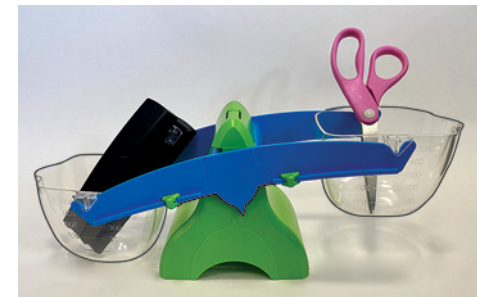
Students develop their understanding of size by focusing on height and length as measurable attributes. Instead of using generic terms such as *bigger* and *smaller* when describing objects, students learn to accurately compare the length of two objects and use the specific terms *taller*, *longer*, and *shorter*. They begin to relate number and length through comparing sticks of Unifix® Cubes.



Topic B

Compare Weights

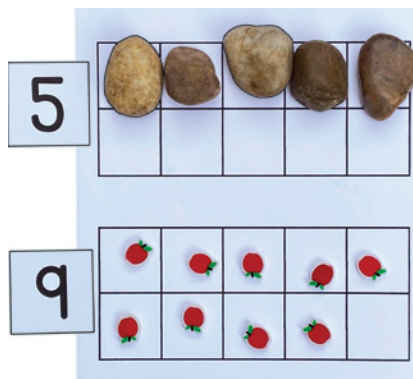
Students learn to use a balance scale to compare the weights of two objects or groups of objects. They make comparison statements by using the terms *heavier than*, *lighter than*, and *about the same weight as*. In the last half of the topic, the balance scale serves as a tool to explore units and conservation. Students consider what happens when you use different units to balance the same object. They also investigate what happens to the weight of an object when you change its shape.



Topic C

Compare Sets Within 10

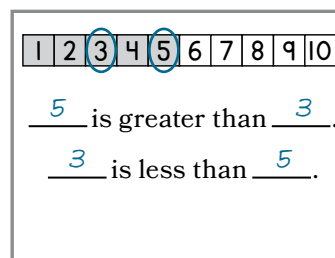
Students compare the number of objects in a set and describe the comparison by using the terminology *more* and *fewer*. Students develop a toolbox of comparison strategies that vary in complexity and precision. As they experience more complicated comparison situations, students find that matching and counting strategies are reliable ways to compare sets. They have opportunities to self-select tools to use for comparison and they evaluate the tools' effectiveness.



Topic D

Compare Numbers Within 10

Because of their experiences connecting number with direct comparisons in topics A, B, and C, students are ready to compare numbers in topic D. They use prior experience with tools like the number stairs and the number path to relate number sequence to quantity. Students use the terms *greater*, *less*, and *equal* to describe the relationship between numbers.



After This Module

Kindergarten Module 4

Students continue to consider the relationship between numbers, now with a focus on composition and decomposition. They discover that both numbers and shapes can be decomposed in more than one way.

Kindergarten Module 5

Number comparison is foundational to work with operations. Awareness of *more*, *less*, and *fewer* helps students reason about story contexts.

Grade 1 Module 4

Students explore indirect comparison by using the length of one object to compare two other objects. They begin to measure length by using the standard units of centimeter cubes.

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Compare Heights and Lengths

Lesson 1	14
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Align endpoints to compare lengths by using *taller than* and *shorter than*.

Lesson 2	24
-----------------------	----

Compare lengths of simple straight objects by using *longer than*, *shorter than*, and *about the same length as*.

Lesson 3	34
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Compare lengths of complex objects by using *longer than*, *shorter than*, and *about the same length as*.

Lesson 4	44
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Compare the lengths of cube sticks to flat shapes.

Lesson 5	56
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Compare the lengths of two cube sticks.

Lesson 6	68
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Compose cube sticks that are the same length.

Topic B	78
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Compare Weights

Lesson 7	82
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Compare weights by using *heavier than*, *lighter than*, and *about the same weight as*.

Lesson 8	92
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Use a balance scale to compare two objects.

Lesson 9	102
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Use a balance scale to compare an object to a group of cubes.

Lesson 10	112
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Use a balance scale to compare an object to different units.

Lesson 11	124
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Observe conservation of weight on the balance scale.

Topic C 132**Compare Sets Within 10****Lesson 12** 136Relate *more* and *fewer* to length.**Lesson 13** 148Compare sets by using *more than*, *fewer than*, and *the same number as*.**Lesson 14** 160

Use number to compare sets with like units.

Lesson 15 170

Classify flat shapes into groups and compare the number of shapes in each group.

Lesson 16 182

Count and compare sets with unlike units.

Lesson 17 196

Count and compare sets in pictures.

Topic D 208**Compare Numbers Within 10****Lesson 18** 212

Compare the capacity of containers by using numerals.

Lesson 19 222Compare numbers by using *greater than*, *less than*, and *equal to*.**Lesson 20** 234

Compare two numbers in story situations.

Lesson 21 246

Describe and compare several measurable attributes of objects and sets.

Lesson 22 (Optional) 256

Organize, count, and represent a collection of objects.

Resources**Observational Assessment Recording Sheet** 265**Module Assessment** 267**Standards** 272**Achievement Descriptors: Proficiency Indicators** 274**Terminology** 278**Math Past** 280**Materials** 282**Works Cited** 286**Credits** 288**Acknowledgments** 289

Why

Comparison

Can I switch the order of topics to maximize materials?

Unlike the topics in most modules, topics A and B can switch order. These topics are designed this way so that two classrooms can share balance scales, maximizing the number available for topic B. Partner with another kindergarten classroom and determine which class will start with topic A and which will start with topic B. The topics can be taught in either order without disrupting the development of conceptual understanding. However, in making adjustments, keep in mind the progression of Fluency across topics. For example, if you teach topic B first, then shift *Building up to the Sprint routine* to topic A so that this activity flows into the next steps given in topics C and D.



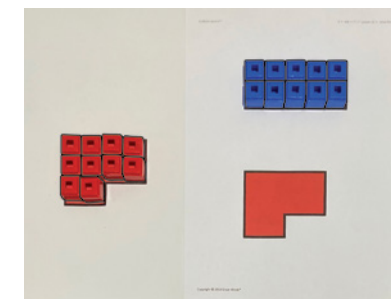
Are capacity and area addressed in *Eureka Math*² kindergarten?

Capacity and area concepts are included in modules 3 and 6. Although these measurement concepts are not always part of kindergarten standards, they are complex and develop best through experience over time. Weaving capacity and area into work with number provides authentic and engaging contexts for the major work of the grade, and it exposes students to new types of measurable attributes.



For example, in module 3 students build number comparison concepts when they compare the capacity of different containers. A container's shape significantly influences young children's ideas about its capacity, or how much it can hold. Filling containers with equal-size scoops, or units, helps students accurately quantify and compare the amount of three-dimensional space in each container. Students compare the capacities of each container by applying number comparison strategies. They find that capacity comparisons based on number are more accurate than those based on appearance.

In module 6 working with area provides another opportunity for students to use number comparison strategies. They cover flat shapes with cubes, or units, and compare the number of cubes on each shape. Repeated experience with capacity and area concepts throughout the primary years prepares students for formal lessons on volume, area, and perimeter beginning in grade 3.



How does understanding of number and units develop in this module?

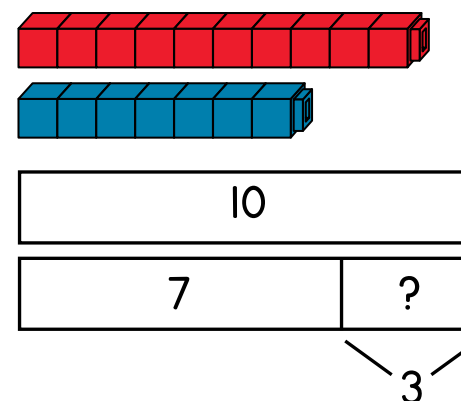
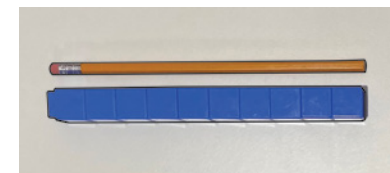
Module 3 helps students understand that, in addition to counting or telling how many, they can also use numbers to make sense of measurable attributes.

For example, students compare the lengths of a pencil and a 10-stick of cubes. They see that the pencil and the 10-stick are the same length because the endpoints on both sides align. They reason about the comparison in more than one way. On one hand, the number of items that make each object is different: 1 pencil versus 10 cubes. On the other hand, the objects are the same: The pencil is *the same length as* the 10-stick.

Module 3 invites students to explore the idea that as units get smaller, more units are needed to measure the same object. This reasoning expands their understanding of equality and lays the foundation for later comparisons such as, “The pencil is about the same length as 19 centimeters, and about the same length as 7 inches.” Students practice this kind of reasoning in terms of both length and weight comparison.

Kindergarten students use different numbers of cubes to make sticks and compare their lengths. The cubes are all the same size, so students see that longer sticks have more cubes, and sticks with more cubes are longer.

Students’ cube stick comparisons are similar to the tape diagrams that they draw to model comparison word problems in grades 1 and 2. The early understanding built by cube stick comparisons sets students up for success with the “How many more?” and “How many fewer?” questions that comparison word problems ask.



Terminology

The following terms are critical to the work of grade K module 3. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

equal

When we compare numbers and they are exactly the same, we say that they are equal. 6 is equal to 6. (Lesson 18)

fewer

When you count two groups, the group with the smaller number has fewer objects. (Lesson 12)

greater

When we compare numbers, we say the bigger number is greater. 8 is greater than 3. (Lesson 18)

heavy

An object that weighs a lot is heavy. The rock is heavier than the feather. (Lesson 7)

height

Height is the length of an object from bottom to top. When we say something is tall or short, we are talking about its height. (Lesson 1)

length

Length is how long an object is from one end to the other. When we say something is long or short, we are talking about its length. (Lesson 2)

less

When we compare numbers, we say the smaller number is less. 3 is less than 8. (Lesson 18)

light

An object that does not weigh a lot is light. The feather is lighter than the rock. (Lesson 7)

long

An object is long if it stretches out farther than other objects. A snake is long. My shoe is longer than a pencil. (Lesson 2)

more

When you count two groups, the group with the bigger number has more objects. (Lesson 12)

short

An object is short if other objects stand above it. Dogs and flowers are short. A flower is shorter than a tree. (Lesson 1)

An object is short if other objects stretch out farther than it does.

An ant is short. The pencil is shorter than my shoe. (Lesson 2)

tall

An object is tall if it stands above other objects. Buildings and trees are tall. A building is taller than a dog. (Lesson 1)

weight

Weight is how much an object presses down. When you pick up a book and feel it pressing down on your hand, you are feeling the weight of the book. (Lesson 7)

Familiar**circle****cone****count****cube****enough****flat shape****hexagon****line****number****rectangle****sort****square****square rectangle****strategy****triangle****Academic Verbs**

Module 3 does not introduce any academic verbs from the grade K list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	100-bead demonstration rekenrek	5	Index cards
5	3" x 3" construction paper squares	24	<i>Learn</i> books
12	Bags of units (blocks, pennies, or beans)	24	Markers
1	Bell	1	Masking tape, roll
1	Bracelet or necklace	1	Music (any)
3	Chart paper, tablets	24	Paper bags
12	Classroom items	5	Paper clips
4	Colored pencils	2	Paper plates
1	Computer or device	1	Paper, white, ream
6	Containers	25	Pencils
2	Cookies	24	Personal whiteboards
10	Cotton balls	24	Personal whiteboard erasers
12	Counting collections	1	Plastic pattern blocks set, 0.5 cm
25	Crayon sets	6	Plastic tubs
1	Dot dice, set of 12	1	Projection device
24	Dough or clay, balls	1	Puppet or stuffed animal
24	Dry-erase markers	30	Resealable plastic bags (small)
2	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, basic student set of 12	10	Rocks, similar-size
1	<i>Eureka Math</i> ^{2™} 2D shapes, foam, set of 30	6	School rocker scales
25	Glue sticks	24	Scissors

6	Scoops	1	<i>Teach</i> book
25	Sentence strips	1	Teacher computer or device
24	Shoes	1	Unifix [®] Cubes, set of 1,000
1	Sorting bag	24	Work mats
1	Stapler	24	Yarn, skeins (3 ft each)
5	Sticky notes, pads		

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 17 for a list of organizational tools (cups, rubber bands, graph paper, etc.) suggested for the counting collection.

Daily Tool Kits

In module 3, students and teachers work with hands-on materials to explore the math concepts introduced in each lesson. The list of materials below includes the most frequently used items in module 3. Consider creating a tool kit for each student to minimize materials preparation for each lesson. Having student and teacher tool kits on hand each day allows for smooth transitions and drastically decreases lesson prep time.

Student Daily Tool Kit

- Glue stick
- Pencil
- Crayons
- Unifix® Cubes (16)
Include 8 cubes of one color and 8 cubes of another color.
- Dot dice, 1 set of 12



Teacher Daily Tool Kit

- Bag of classroom items
(marker, crayon, book, pencil, colored pencils)
- Container of Unifix® Cubes (16)
Include 8 cubes of one color and 8 cubes of another color.
- School rocker scale



Before This Module

Kindergarten Module 1

Students decompose through sorting activities in module 1, including sorting in more than one way. They represent story problems by using objects and number sentences.

Kindergarten Module 2

Students informally play with pattern block puzzles beginning with the introduction of shape composition and decomposition at the end of module 2.

Kindergarten Module 3

Students explore the relationships between numbers through the lens of comparison in module 3. This segues into exploring part-total relationships.

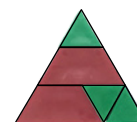
Overview

Composition and Decomposition

Topic A

Explore Composition and Decomposition

Topic A is an exploration of both shape and number. Students begin the topic putting together simple shapes to make a larger composite shape and end the topic putting together smaller numbers to make larger numbers. They develop language to describe part-whole and part-total relationships.

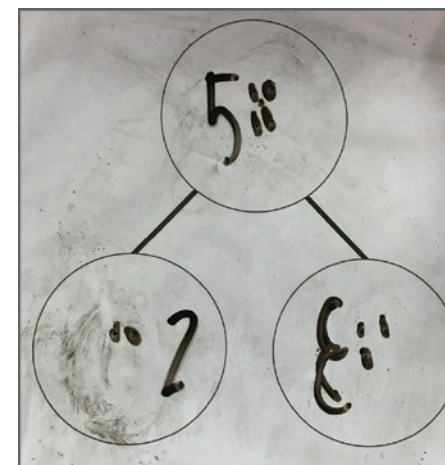


There are 5 parts.

Topic B

Record Composition and Decomposition

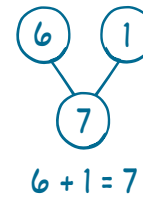
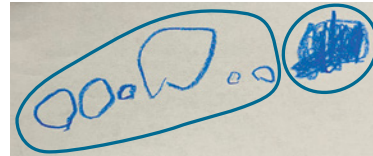
Topic B provides space for students to decompose shapes and numbers in more than one way. Students record their decompositions and analyze them to answer questions such as, How can the same total be made of different parts? Which numbers can be part of 5? Which numbers cannot be part of 5?



Topic C

Model Composition and Decomposition in Story Problems

Topic C expands part-total thinking to include decomposition and composition story contexts. Students identify and choose math tools to help them show the situation and explain their thinking. The teacher uses student work to demonstrate how to write a matching number sentence with symbols. For example, $6 + 1 = 7$.



$$6 + 1 = 7$$

After This Module

Kindergarten Module 5

Students solve addition and subtraction story problems and represent them with number sentences. They continue to use part-total thinking to relate number sentences to number bonds and make sense of different addition and subtraction situations. Emerging competence with operations leads students to develop fluency with sums and differences to 5. Students grow increasingly comfortable with decomposing 10 through continued exploration.

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Composition and Decomposition

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Explore Composition and Decomposition

Lesson 1	18
Compose flat shapes and count the parts.	

Lesson 2	28
Decompose flat shapes and count the parts.	

Lesson 3	40
Decompose a group to identify parts and total.	

Lesson 4	52
Decompose a group and record parts and total by using a number bond.	

Topic B	63
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Record Composition and Decomposition

Lesson 5	66
Sort to decompose a number in more than one way.	

Lesson 6	78
Decompose a number in more than one way and record.	

Lesson 7	88
Find partners to 5.	

Lesson 8	98
Find partners to 10.	

Lesson 9	108
Compose shapes in more than one way.	

Lesson 10	118
Sort and record the decomposition with a number bond.	

Topic C	127
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Model Composition and Decomposition in Story Problems

Lesson 11	132
Model <i>put together with total unknown</i> story problems.	

Lesson 12	148
Draw to represent <i>put together with total unknown</i> story problems.	

Lesson 13	158
Choose a math tool to solve <i>put together with total unknown</i> story problems.	

Lesson 14	170
Model <i>take apart with both addends unknown</i> situations.	

Lesson 15	182
Choose a math tool to solve <i>take apart with both addends unknown</i> situations.	

Lesson 16	194
Compose and decompose numbers and shapes.	

Lesson 17 (Optional)	206
Organize, count, and represent a collection of objects.	

Lesson 18 (Optional)	214
Use the structure of 5 and 10 to build a rekenrek.	

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Why

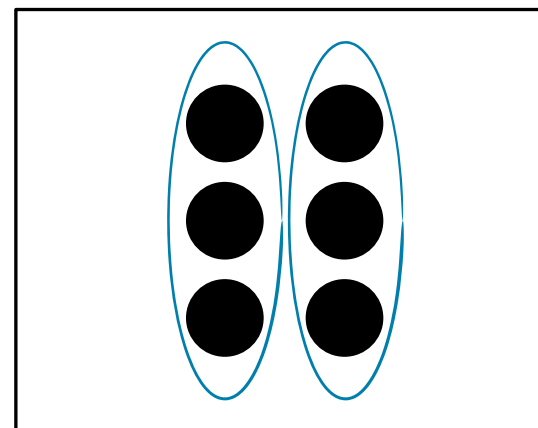
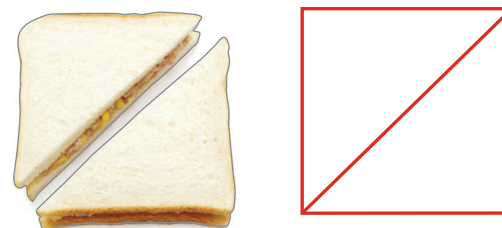
Composition and Decomposition

Why does this module combine geometry and number?

Research suggests that experience with shape composition and decomposition corresponds with a student's ability to compose and decompose numbers.¹ If students first explore the nature of composition and decomposition in a visual context, as with shapes, they can apply that understanding to new contexts such as numbers.

Students benefit from concrete and pictorial experiences with both shape and number decomposition. Staples of early childhood classrooms such as unit blocks and pattern block puzzles provide playful experiences with composition and decomposition that make entry points for discussing part-whole relationships in shapes. Sorting and other familiar hands-on experiences give context for discussing part-total relationships in numbers.

The study of shapes and numbers is linked by the language used to describe part-whole relationships. First students consider their everyday experiences with part-whole relationships: *That is part of the whole sandwich.* They use familiar language to describe composite shapes: *The triangle is part of the whole square.* Then they learn to use *part* and *total* as mathematical terminology when they explore relationships between numbers: *3 and 3 are the parts. 6 is the total.* In both shape and number contexts, students find that there are multiple ways to decompose the whole or total.

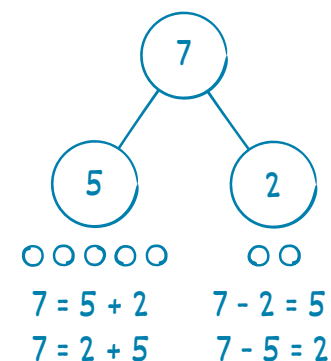


¹ Sarama, J. and Clements, D.H. (2009). *Early Childhood Mathematics Educational Research: Learning Trajectories for Young Children* New York: Routledge. Referencing Clements, D.H., Battista, M.T., Sarama, J., and Swaminathan, S. (1996). "Development of Turn and Turn Measurement Concepts in a Computer-Based Instructional Unit." *Educational Studies in Mathematics*, 30, 313-337

Why teach students to use the number bond model?

Number bonds are one of the most effective tools for modeling and recording composition and decomposition. They show the relationships between numbers, which is a key element of number sense. Number bonds serve at least three purposes:

1. **Number bonds provide a visual structure that helps students learn basic facts more easily.** In kindergarten, students first use number bonds to model the relationship between parts and total. Later they relate the relationships the model shows to addition and subtraction situations. Students in grade 1 explore the inverse relationship between addition and subtraction. They see that the same number bond can represent both addition and subtraction facts. Fluency with basic facts prepares them to solve problems by using mental math or algorithms with fewer errors in years to come.
2. **Number bonds help students think flexibly about numbers by providing a reliable visual structure.** Students who are comfortable representing decomposition with number bonds can break down challenging problems and make them easier to solve. This type of thinking, known as making an easier problem or a Level 3 strategy, generally begins in grade 1 with whole numbers and extends through elementary school as students learn about fractions.
3. **Number bonds prepare students to do quick calculations in their head.** The early decomposition and composition work that students do with the number bond increases their ability to do mental math. Decomposition can make any problem easier to solve, regardless of operations involved in solving.



$$5 + 7 = 12$$

$$5 + 5 = 10$$

$$10 + 2 = 12$$

$$1\frac{6}{8} + \frac{7}{8} = 2\frac{5}{8}$$

$$1\frac{6}{8} + \frac{2}{8} = 2$$

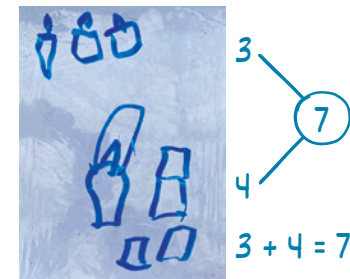
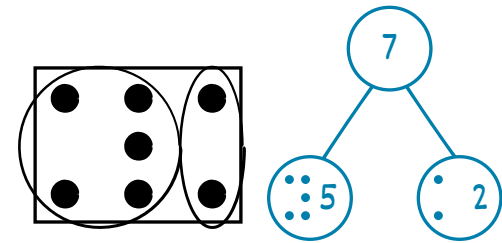
$$2 + \frac{5}{8} = 2\frac{5}{8}$$

Does the orientation of the number bond matter? Why?

The part-total relationship between the numbers remains the same no matter how the number bond is oriented. Students can write a number bond with the total above, below, or to either side of the parts so long as the arms correctly connect the parts to the total.

However, thoughtful orientation can help students make sense of a situation or story problem. Students usually find it easier to think about known quantities first. The dot card, for example, shows a total of 7. Use the directionality rules for reading and writing English to put the known quantity first: write 7 at the top or to the left. Students decompose the total in different ways by recording parts on the bottom or to the right.

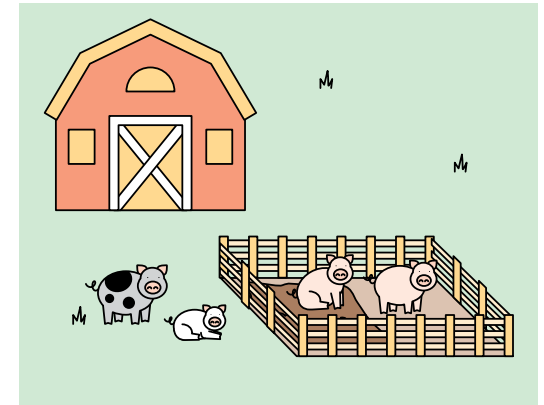
Orient the number bond to represent the way students think about (or are likely to think about) a situation. For example, if there are 3 thin markers and 4 thick markers and the total is unknown, students tend to first create groups of 3 and 4 (the parts). Then they put the groups together to compose 7 (the total). Write the known parts to the left or at the top. Show the total at the bottom or to the right. When students write their own number bonds, the orientation may vary.

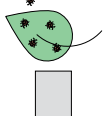
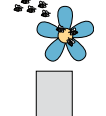


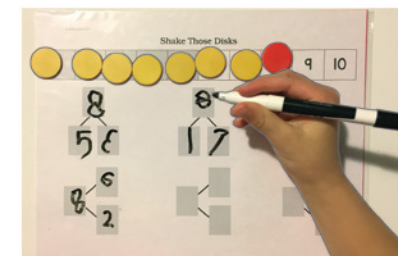
Why do some student pages provide full number bonds while others have only a writing rectangle for the total?

Whenever possible, items in the student book allow students to choose the number of parts for their number bond. Most items provide only a writing rectangle for the total. That way students can represent the situation as they see it, with two parts or many parts. For example, students may see the farm picture as having three groups based on color or two groups based on position. Providing a writing rectangle for the total helps students focus on the parts and drawing a line to connect each part to the total.

In some cases, the number of parts is limited by the activity. For instance, when students play Shake Those Disks, the goal is for them to focus on two parts: red and yellow. There is no need for flexibility in the number of parts, so the recording sheet provides writing rectangles for the entire number bond.



Name _____		11
Walk to a picture. Fill in the number bond.		
		
		



Which word problem types, or addition and subtraction situations, are used in this module?

The table shows examples of addition and subtraction situations.² Darker shading in the table indicates the four kindergarten problem types. Students in grades 1 and 2 work with all problem types. Grade 2 students reach proficiency with the unshaded problem types.

Students solve *put together with total unknown* and *put together with both addends unknown* problem types in topic C. Students continue to solve these problem types in module 5, along with *add to with result unknown* and *take from with result unknown* problem types.

- **Add to with result unknown:** Both parts are given. An action joins the parts to form the total.

Auntie had 3 apples at home. Then she went to the store and bought 5 apples. How many apples does she have now? (Module 5, Lesson 1)

- **Take from with result unknown:** The total and one part are given. An action takes away one part from the total.

I bought 9 oranges. I ate 5 oranges. How many oranges do I have now? (Module 5, Lesson 9)

- **Put together with total unknown:** Both parts are given. No action joins or separates the parts. Instead, the parts are distinguished by an attribute such as type, color, size, or location.

There are 6 baby ducks and 1 adult duck. How many ducks are there? (Module 4, Lesson 12)

- **Take apart with both addends unknown:** Only the total is given. Students take apart the total to find both parts. This situation is the most open ended because the parts can be any combination of numbers that make the total.

There are 8 meerkats moving to a new zoo. Two trucks drive them to their new home. How could the zookeeper put the meerkats in the trucks? (Module 4, Lesson 15)

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
Put Together / Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
Compare	"How many more?" version. Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? $A + \square = C$ $C - A = \square$	"More" version suggests operation. Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? $A + B = \square$	"Fewer" version suggests operation. Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

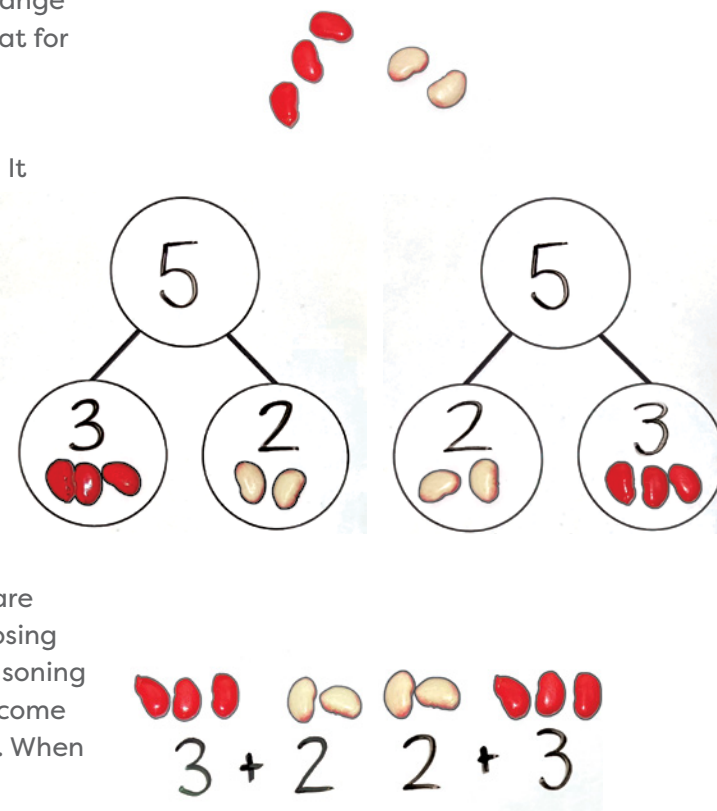
² Common Core Standards Writing Team, *Progressions for the Common Core State Standards in Mathematics: Grades K–5, Counting and Cardinality & Operations and Algebraic Thinking* (Tucson, AZ: Institute for Mathematics and Education, University of Arizona, May 29, 2011), 9, https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_0a_k5_2011_05_302.pdf.

What is the commutative property and how do kindergarten students understand it?

The commutative property of addition says that in an addition equation, we can change the order of the addends without changing the total. Put more formally, it states that for any numbers a and b , $a + b = b + a$.

The commutative property is not immediately clear to most kindergarten students, but early understanding can develop out of their work with part-total relationships. It builds on their understanding of conservation, that rearranging a group of items doesn't change how many there are, which young children gain through repeated experience with counting sets of objects. For example, when counting a group of white and red beans, students see that they get the same total whether they count the white group or the red group first. Students do not initially connect conservation to number bonds and addition sentences. They often see $3 + 2$ and $2 + 3$ as unique partners of 5. Only later do they come to see that the expressions are equivalent because they have the same total.

Students can advance their understanding of the commutative property through strategically sequenced problems. For example, if students first find the total for $3 + 2$ and then find the total for $2 + 3$, some will notice that the parts and total are the same and that the only difference is the order of the addends. Repeatedly exposing students to situations like this can prompt them to notice regularity in repeated reasoning (MP8). This helps students expand their early understanding in grade 1, when they come to understand the commutative property as the idea that you can add in any order. When presented with $2 + 7$, grade 1 students learn that they can count on from 7 rather than counting on from 2 and still get the same total.



Terminology

The following terms are critical to the work of kindergarten module 4. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

part

A piece that is used to make a larger whole or total is called a part.

When we make a trapezoid out of 3 triangles, each triangle is a part. (Lesson 1)

When we make 5 by putting together 2 and 3, we call 2 and 3 parts. (Lesson 3)

In module 4, part is introduced in two different contexts. First, in lesson 1, it is introduced in the context of composite shapes. Then, in lesson 3, it is introduced in the numerical context, for example, the parts in a number bond or number sentence.

partners to x

Two numbers that add to x are called partners to x. For example, 2 and 4 are partners to 6. (Lesson 6)

Since any whole number can be substituted for x, this is defining several different terms: partners to 5, partners to 6, partners to 7, etc.

total

A number made out of smaller numbers is called the total. When we make 5 by putting together 2 and 3, we call 5 the total. (Lesson 3)

whole

Something that is made out of smaller pieces is called a whole. When we make a trapezoid out of 3 triangles, the trapezoid is the whole. (Lesson 1)

Whole and total have similar meanings. Kindergarten uses whole as a general term meaning “the entire thing” and total exclusively when referring to a numerical quantity.

Familiar

circle

cone

count

cube

enough

equal

fewer

greater

hexagon

less

long

more

number

number sentence

rectangle

short

sort

sphere

square

strategy

tall

triangle

Academic Verbs

Module 4 does not introduce any academic verbs from the kindergarten list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	100-bead demonstration rekenrek	25	Personal whiteboards
1	Chart paper, tablet	25	Personal whiteboard erasers
1	Colorful masking tape or ribbon	1	Plastic pattern blocks set, 0.5 cm
25	Crayons, set of 8	1	Projection device
25	Cups	1	Puppet or stuffed animal
25	Dry-erase markers	25	Scissors
1	<i>Eureka Math</i> ^{2™} 5-group™ cards, demonstration set	25	Sorting bags
1	<i>Eureka Math</i> ^{2™} Pattern block puzzles	1	<i>Teach</i> book
1	<i>Eureka Math</i> ^{2™} Story cards	1	Teacher computer or device
25	Glue sticks	2	Teddy bear counters, set of 96
24	<i>Learn</i> books	100	Two-color beans, red and white
3	Paper plates	1	Two-color counters, 200 pieces
10	Paper, sheets	1	Unifix® Cubes, set of 1,000
25	Pencils		

Visit <http://eurmath.link/materials> to learn more.

Daily Tool Kits

In module 4, students and teachers work with hands-on materials to explore the math concepts introduced in each lesson. The list of materials below includes the most frequently used items in module 4. Consider creating a tool kit for each student to minimize materials preparation for each lesson. Having student and teacher tool kits on hand each day allows for smooth transitions and drastically decreases lesson prep time.

Student Daily Tool Kit

- Unifix® Cubes (10)
- Red and yellow counters (10)
- Pencil
- Dry-erase marker
- Crayon set



Teacher Daily Tool Kit

- Unifix® Cubes (10)
- Red and yellow counters (10)
- Pencil
- Dry-erase marker
- Crayon set



Before This Module

Kindergarten Module 4

Students explore number relationships through sorting, games, and math stories. They use number bonds and their teacher writes number sentences to record part-total relationships. They also practice different methods for representing story situations including *put together with total unknown* and *put together with both addends unknown*.

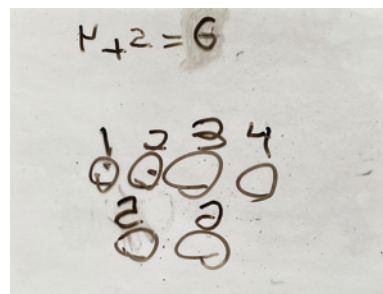
Overview

Addition and Subtraction

Topic A

Represent Addition

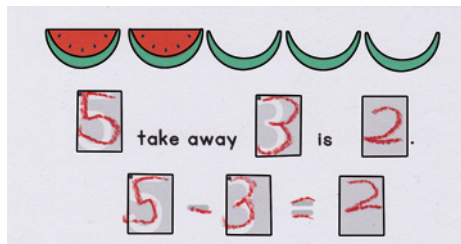
Students make a major leap forward in their understanding of number sentences. They represent addition stories by using number sentences with mathematical symbols, such as $4 + 2 = 6$, and read their work by using mathematical language, “4 plus 2 equals 6.” They also recontextualize the number sentence by using it to retell the story. Students work with three problem types in this topic: *add to with result unknown*, *put together with total unknown*, and *take apart with both addends unknown*. By the end of the topic, students can find the total of an addition sentence without a story context.



Topic B

Represent Subtraction

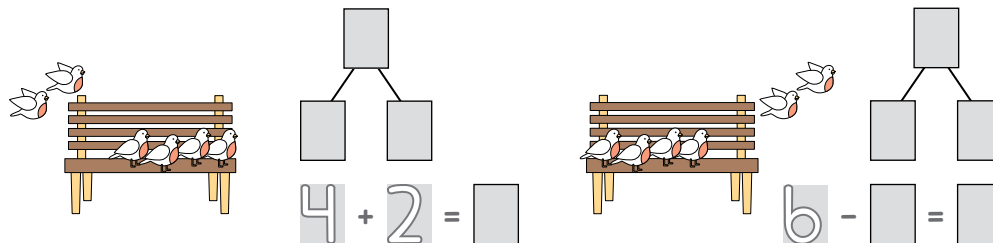
Students solve subtraction problems and represent their thinking by using number sentences with mathematical symbols. For example, $5 - 3 = 2$. They work with a single problem type, *take from with result unknown*, to understand subtraction as taking away. Students use familiar story structure (beginning, middle, and end) to write and understand subtraction sentences: “I had 5 watermelon slices. I ate 3 of them. Now there are 2 watermelon slices. 5 minus 3 equals 2.”



Topic C

Make Sense of Problems

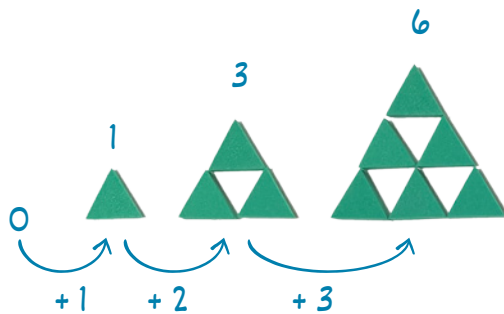
Students transition from working with addition and subtraction separately to thinking about both as they determine which operation can be used to solve. As they focus on story context and the information presented, they build their capacity for sense-making and perseverance. This topic also presents a number of opportunities for students to practice finding partners to 10 through story and measurement contexts.



Topic D

Make Use of Structure

Students look for and make use of structure by looking for and making use of patterns. They learn what a pattern is and how to recognize and extend patterns, and they use patterns to extend their work with numbers. Students apply their knowledge of patterns to help them answer *how many* questions.



After This Module

Kindergarten Module 6

Students extend counting and work with part-total relationships to numbers to 20. Increasing students' familiarity with teen numbers as 10 ones and some ones lays the groundwork for place value strategies in grade 1. Students continue to look for and make use of structure as they extend the counting sequence to 100. They uncover patterns in number words and in the digits used to write them. The utility of 10 comes to light when students find efficient methods for counting out a set of objects to 100.

Grade 1 Module 1

Grade 1 marks the shift from counting all to counting on to solve addition and subtraction problems. Students use counting on as a more efficient way to find the total or a missing part.

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Represent decomposition situations by using number bonds and addition sentences.

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Find the total in an addition sentence.

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Why

Addition and Subtraction

What are the levels of development as students learn to solve addition and subtraction problems?

In their first years of school, students generally move through three levels of development as they solve addition and subtraction problems.

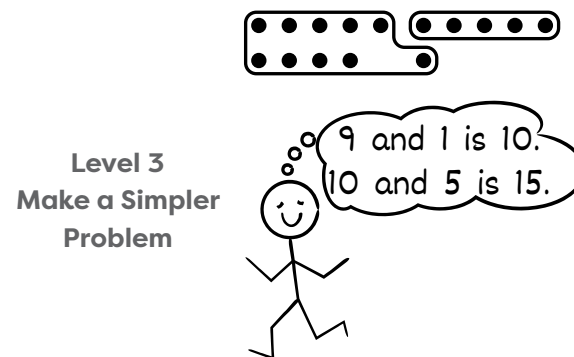
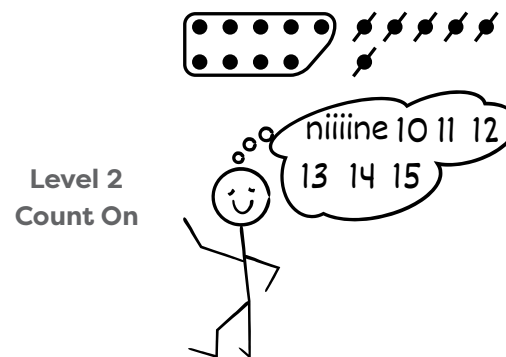
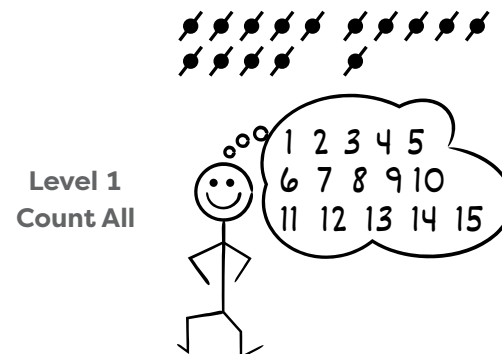
- Level 1: Count all
- Level 2: Count on
- Level 3: Make a simpler addition or subtraction problem

Many students rely on direct modeling to count all throughout the kindergarten year. To add, they represent the parts by using objects or drawings and then count all to find the total. To subtract, they first count out the total, then count to take away the known part, and finally count the remaining part.

Kindergarten students often spend the full year at Level 1 because they are developing conceptual understanding of what it means to add and subtract. They are learning many different ways to represent those actions, including using concrete objects, drawings, mental images, and number sentences. They are also learning which situations call for each operation. As students build conceptual understanding of addition and subtraction through counting all, they increasingly see that parts are embedded in the total. This is foundational for counting on to add or subtract.

Some students begin to use counting on to solve addition problems in kindergarten. Module 5 includes teacher notes and lessons to support these students. The lessons include examples of student strategies from Levels 1 and 2, including questions to advance student thinking from one level to the next. Comparing and connecting different student work can help them make sense of more sophisticated strategies and relate them to their own thinking.

Students spend much of their first and second grade years in the third developmental level, using what they know to make simpler problems. Once they acquire several strategies, students reason about which strategy best fits the problem they are solving. The goal is to empower them to continue developing number sense and flexibility in problem solving.

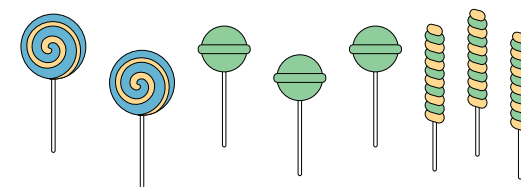


What is the associative property and how do kindergarten students understand it?

The associative property of addition says that in an addition equation, we can choose to start by adding any two numbers that are next to each other, rather than working left to right. For example, to find $3 + 4 + 6$, we can first add $4 + 6$ to make 10, resulting in the simpler problem $3 + 10 = 13$. Put more formally, the associative property states that for any numbers a , b , and c , $(a + b) + c = a + (b + c)$.

As with the commutative property, students' early understanding of the associative property develops from their work with part-total relationships and builds on their understanding of conservation. For example, students are presented with a picture of lollipops and asked to find the total. Some students will count the lollipops from left to right. Others may notice that they can use the doubles fact $3 + 3 = 6$ if they start with the lollipops on the right and then add the 2 lollipops on the left.

In grade 1 students use the associative property, particularly when practicing the make 10 strategy. For example, when presented with $5 + 7$, students may decompose 5 into $2 + 3$, resulting in a new problem: $2 + 3 + 7$. Then students add 3 and 7 first, making use of the associative property to create the simpler problem $2 + 10$.



$$\begin{array}{l} 2 + 3 + 3 = \square \\ \swarrow \quad \searrow \\ 5 + 3 = \boxed{8} \end{array} \qquad \begin{array}{l} 2 + 3 + 3 = \square \\ \swarrow \quad \searrow \\ 2 + 6 = \boxed{8} \end{array}$$

$$\begin{array}{l} 5 + 7 = 12 \\ \swarrow \quad \searrow \\ 2 \quad 3 \end{array} \qquad \begin{array}{l} 10 + 2 = 12 \end{array}$$

Why is it important for students to interpret number sentences in different ways?

In module 4 students describe the relationships between numbers by using everyday language: *and*, *make*, *take away*, and *is*. Everyday language precedes academic language because experiences of making things and taking away are relatable to young students. Statements such as *10 take away 3 is 7* align neatly with the numbers and symbols in an equation, creating a smooth transition to the mathematical terminology of *plus*, *minus*, and *equals*: *10 minus 3 equals 7*. In module 5 reading number sentences using everyday and mathematical terminology helps students make sense of how numbers and symbols work together in a number sentence.

Another way that students read number sentences is called reading like a storyteller: *The baker made 10 muffins. He sold 3 of them. There are 7 muffins left.* By using story language after solving, students move from computation back to context. Rather than saying, “the answer is 7,” they can more specifically say, “there are 7 muffins left.” Recontextualizing the entire number sentence as a story shows that students understand the meaning of each quantity, as well as how the actions or relationships correlate to the symbols.

Saying the number sentence by using mathematical and story language prepares students for the Read-Draw-Write (RDW) process. Beginning in grade 1, students write both a number sentence and a statement in the last step of the RDW process.

Which word problem types, or addition and subtraction situations, must be mastered in kindergarten?

See the table for explanations and examples of some problem types.¹ Darker shading in the table indicates the four kindergarten problem subtypes. Students in grades 1 and 2 work with all problem subtypes and variants. Grade 2 students master the unshaded problem types.

Students solve all four of the kindergarten problem subtypes in module 5.

- **Add to with result unknown:** Both parts are given. An action joins the parts to form the total.

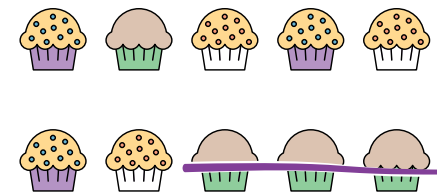
Auntie had 3 apples at home. Then she went to the store and bought 5 apples. How many apples does she have now? (Module 5, Lesson 1)

- **Take from with result unknown:** The total and one part are given. An action takes away one part from the total.

I bought 9 oranges. I ate 5 oranges. How many oranges do I have now? (Module 5, Lesson 9)

- **Put together with total unknown:** Both parts are given. No action joins or separates the parts. Instead, the parts are distinguished by an attribute such as type, color, size, or location.

There are 6 baby ducks and 1 adult duck. How many ducks are there? (Module 4, Lesson 12)



$$10 - 3 = 7$$

Everyday language: 10 take away 3 is 7.

Mathematical language: 10 minus 3 equals 7.

Story language: The baker made 10 muffins. He sold 3 of them. There are 7 muffins left.

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	<i>A</i> bunnies sat on the grass. <i>B</i> more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	<i>A</i> bunnies were sitting on the grass. Some more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies hopped over to the first <i>A</i> bunnies? $A + \square = C$	Some bunnies were sitting on the grass. <i>B</i> more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	<i>C</i> apples were on the table. I ate <i>B</i> apples. How many apples are on the table now? $C - B = \square$	<i>C</i> apples were on the table. I ate some apples. Then there were <i>A</i> apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate <i>B</i> apples. Then there were <i>A</i> apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	Both Addends Unknown ¹	Addend Unknown ²
Put Together / Take Apart	<i>A</i> red apples and <i>B</i> green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has <i>C</i> flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	<i>C</i> apples are on the table. <i>A</i> are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	<i>"How many more?"</i> version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many more apples does Julie have than Lucy? $A + \square = C$ $C - A = \square$	<i>"More"</i> version suggests operation. Julie has <i>B</i> more apples than Lucy. Lucy has <i>A</i> apples. How many apples does Julie have? $A + B = \square$	<i>"Fewer"</i> version suggests operation. Lucy has <i>B</i> fewer apples than Julie. Julie has <i>C</i> apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

¹ Common Core Standards Writing Team. *Progressions for the Common Core State Standards in Mathematics, Operations and Algebraic Thinking Progression.*

- **Take apart with both addends unknown:** Only the total is given. Students take apart the total to find both parts. This situation is the most open ended because the parts can be any combination of numbers that make the total.

There are 8 meerkats moving to a new zoo. Two trucks drive them to their new home. How could the zookeeper put the meerkats in the trucks? (Module 4, Lesson 15)

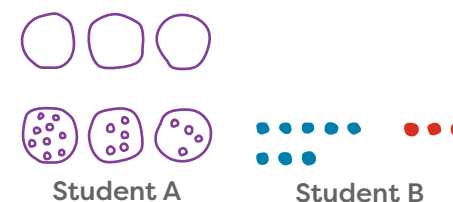
What are numberless word problems? Why are they used in kindergarten?

Numberless word problems are math stories told without numbers. For example: *I bake some sugar cookies. My friend brings over some chocolate chip cookies.* These problems are used in two different ways in module 5.

The first lesson opens with the cookies problem. Students visualize the story in their minds and then make a math drawing to show what they see. They choose the numbers for the math story. One student may see 3 of each type of cookie while another sees 8 sugar cookies and 3 chocolate chip cookies. Numberless word problems build in choice, validate emerging visualization skills, and naturally create engagement and differentiation.

Once students have more experience using addition and subtraction to solve problems, numberless word problems serve a new purpose. They cause students to analyze action and relationships, which provides a scaffold as students make sense of story problems. Consider the following numberless word problem: *Some students are reading in the library. Some of those students go to the computer center.* The class can discuss whether students are coming or going from the library and whether there are more or fewer students in the library. They consider the relationship between quantities before they know the exact numbers. Once students make sense of the situation, numbers are inserted and they choose a solution path to solve the problem.

Sometimes problems are presented with only one number given: *5 students are reading in the library. Some of those students go to the computer center.* Students can model or visualize to figure out which numbers make sense in the story. They might use 5 fingers to show the students in the library and reason that, at most, 5 students can leave because that's how many fingers are showing. Numberless word problems focus students on reasoning about and understanding the context and relationships between quantities before they select an operation or solution strategy.



There are 5 students. I can take away 1, 2, 3, 4, or 5 fingers, but I can't take away 6 fingers.

Terminology

The following terms are critical to the work of grade K module 5. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

addition

When we put groups or parts together to find the total, that's called addition. (Lesson 1)

equals

When writing a number sentence, instead of saying or writing “makes” or “is,” mathematicians use the equal sign and say “equals.” A mathematician reads $2 + 3 = 5$ as “2 plus 3 equals 5” and $4 - 3 = 1$ as “4 minus 3 equals 1.” (Lesson 1)

Students were initially introduced to the term equals in module 3 in the context of comparing two numbers, e.g., $5 = 5$. In module 5, students advance their understanding of the term to include comparing an expression and a number, e.g., $2 + 3 = 5$.

minus

When writing a number sentence, instead of saying or writing “take away,” mathematicians use the minus sign and say “minus.” A mathematician reads $4 - 3 = 1$ as “4 minus 3 equals 1.” (Lesson 9)

pattern

When we see a pattern, we can decide what comes next. For example, a pattern can help us know what color, size, or number comes next. (Lesson 22)

Students are introduced to two kinds of patterns in module 5, repeating patterns and growing patterns. These patterns are not named, instead students focus their attention on how different patterns help them predict different kinds of things, e.g., color, size, or number.

plus

When writing a number sentence, instead of saying or writing “and,” mathematicians use the plus sign and say “plus.” A mathematician reads $2 + 3 = 5$ as “2 plus 3 equals 5.” (Lesson 1)

subtraction

When something gets taken away and we find how many are left, it's called subtraction. (Lesson 8)

Familiar

count

equal

fewer

greater

hexagon

length

less

line

long

more

number

number sentence

part

partners to x

sort

strategy

total

triangle

whole

Academic Verbs

organize

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	100-bead demonstration rekenrek	24	<i>Learn</i> books
1	10-sided dice, set of 24	1	Marker
10	Cardstock, white sheets	50	Paper plates
1	Chart paper, tablet	96	Paper, sheets
1	Construction paper, sheet	25	Pencils
5	Containers, assorted	5	Pennies
12	Counting collections	25	Personal whiteboards
24	Crayons, set of 8	25	Personal whiteboard erasers
24	Cups	3	Plastic pattern blocks set, 0.5 cm
2	Dot dice, set of 12	1	Projection device
25	Dry-erase markers	1	Puppet or stuffed animal
1	Empty can	1	School rocker scale
2	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, basic student set of 12	25	Scissors
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, demonstration set	24	Sentence strips
2	<i>Eureka Math</i> ^{2™} Match cards, set of 12	4	Sticky notes
1	<i>Eureka Math</i> ^{2™} Story cards	1	<i>Teach</i> book
1	Geometric solids set, 40 pieces	1	Teacher computer or device

- 2 Teddy bear counters, set of 96
- 240 Two-color beans, red and white
- 1 Two-color counters, 200 pieces

- 1 Unifix[®] Cubes, set of 1,000
- 24 Workmat

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 27 for a list of organizational tools suggested for the counting collection.

Daily Tool Kits

In module 5, students and teachers work with hands-on materials to explore the math concepts introduced in each lesson. The list of materials below includes the most frequently used items in module 5. Consider creating a tool kit for each student to minimize materials preparation for each lesson. Having student and teacher tool kits on hand each day allows for smooth transitions and drastically decreases lesson prep time.

Student Daily Tool Kit

Unifix[®] Cubes (10)

Two-color counters (10)

Eureka Math^{2™} Hide Zero[®] cards, basic student set of 12

Crayons (5)

Dry-erase marker

Pencil



Teacher Daily Tool Kit

Puppet

Unifix[®] Cubes (10)

Two-color counters (10)

Eureka Math^{2™} Hide Zero[®] cards, demonstration set

10-sided die

6-sided dot die

Dry-erase marker

Pencil



Before This Module

Kindergarten Module 1

Students learn to integrate all four elements of the number core as they count and create sets. Their tools encourage them to think of numbers relative to 10.

Kindergarten Module 5

Students use addition and subtraction sentences to represent composition and decomposition of numbers to 10. They look for and make use of patterns. They continue to develop fluency with counting within 100. Through counting collections, students discover that grouping objects makes it easier to both track and count.

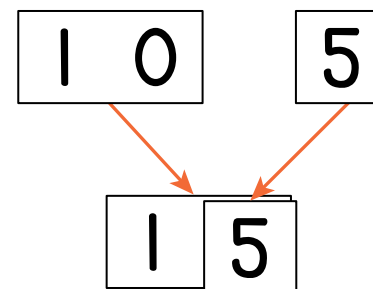
Overview

Place Value Foundations

Topic A

Count and Write Teen Numbers

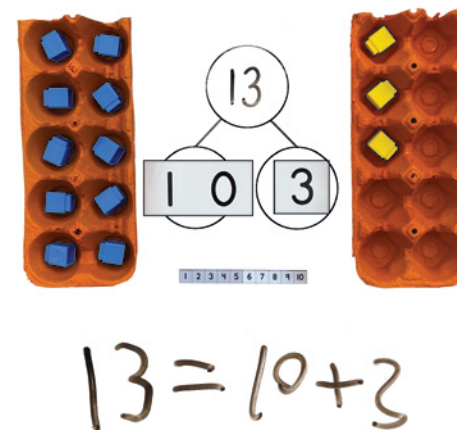
Students dive into place value concepts as they count and write numbers 11 to 20. They learn that each teen number, 11 to 19, is composed of 10 ones and some more ones and that the digit 1 in a teen number represents 10 ones. Throughout the topic, students use and discuss tools that highlight the structure of ten in our number system such as Hide Zero® cards, fingers, 10-frames, and rekenreks. They use the tens and ones structure to organize objects. This helps them count a group of 11 to 20 objects efficiently and accurately.



Topic B

Compose and Decompose Teen Numbers

Students expand their experience with place value and part-total relationships by representing teen numbers with number bonds and number sentences. Story contexts offer an entry point for students to connect real-life situations to abstract mathematical representations. Each story uses 10 as a part to reinforce the understanding that teen numbers are composed of 10 ones and some more ones.



Topic C

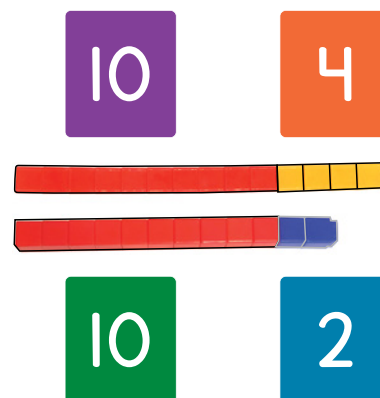
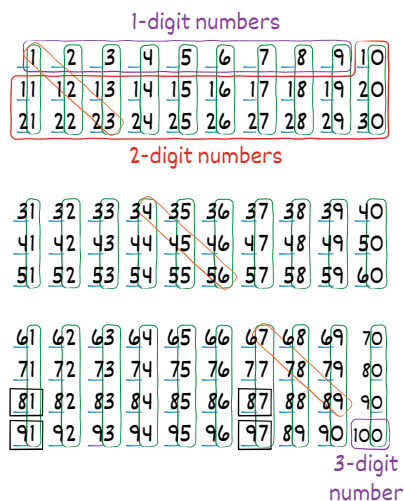
Count to 100

Students master the count to 100 by using pattern and structure. They discover that number words after 20 can be formed by pairing a decade word with a number word one through nine. Students consider efficiency as they organize materials into groups and as they decide when to count by tens or ones.

Topic D

Compare

Students conclude the year by applying comparison strategies to situations involving greater numbers and measurable attributes. Students use their understanding of part-total relationships to decompose teen numbers into 10 ones and some ones before comparing. Measurement scenarios extend student learning and set the stage for comparison in grade 1.



After This Module

Students use tools to organize and count efficiently when they count collections in grade 1.

Grade 1 Module 1

Students use bar graphs they make with number paths to record the data they collect through surveys, observations, and sorting. Because students work within 20, they use emerging place value understanding to compare numbers.

Grade 1 Module 3

Proficiency with $10 + n$ facts, decomposition within 10, and partners to 10 gives students access to the Level 3 strategy of making a simpler problem. For example, to solve $9 + 6$ they decompose 6 into 1 and 5 and then add 1 and 9 to make 10. This converts the problem to the familiar $10 + 5$.

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Resources**Observational Assessment Recording Sheet** 310**Module Assessment** 312**Standards** 316**Achievement Descriptors: Proficiency Indicators** 318**Terminology** 324**Math Past** 326**Materials** 330**Works Cited** 332**Credits** 334**Acknowledgments** 335

Why

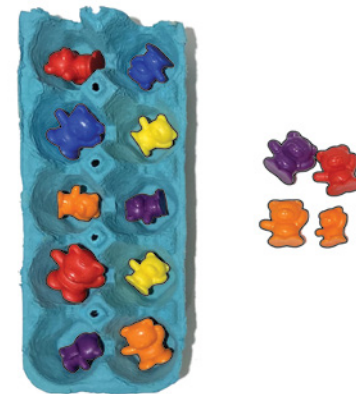
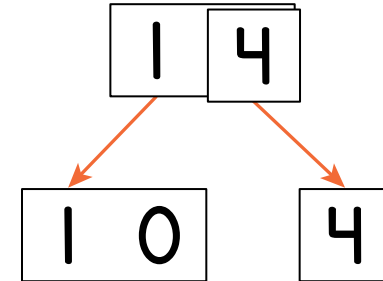
Place Value Foundations

How does the way we name teen numbers impact the way students understand them?

Kindergarten students need to master a critical idea about teen numbers, or numbers 11 to 19. Each teen number is composed of 10 ones and some more ones. The way we name teen numbers in English does not make this clear and can cause confusion for the following reasons.

- **Absence of ten in the number names:** Numbers 13 through 19 include *teen*, which indicates, but does not say, ten. *Eleven* and *twelve* do not include *teen*, as their origins are Old English words meaning “one left” (after ten) and “two left.”¹ Because the word *ten* is obscured or left out of teen number names, most students do not initially understand that the digit 1 in a teen number represents 10 ones. They see 14 as 1 and 4 instead of 10 and 4.
- **Inconsistent naming structure:** The words *eleven*, *twelve*, *thirteen*, and *fifteen* do not contain the name of the digit in the ones place (i.e., *one*, *two*, *three*, *five*). *Fourteen*, *sixteen*, *seventeen*, *eighteen*, and *nineteen* do contain the name of the digit in the ones place. Inconsistency makes it more difficult for students to discern how many ones there are beyond 10.
- **Order of pronunciation:** In numbers 13 through 19, we say the digit in the ones place first. This can lead students to reverse digits when writing numerals. *Fourteen* is often written as 41 instead of 14.

Clarifying the idea that each teen number is composed of 10 ones and some more ones began with Say Ten counting in module 5. This way of counting establishes 10 as a benchmark and highlights the base-ten structure within teen numbers. When students relate Say Ten counting to regular counting (e.g., *ten 1* is the same as *eleven*), students understand that 11 represents 10 ones and 1 one and 12 represents 10 ones and 2 ones, and so on.



¹ Clements and Sarama, 102.

Module 6 provides repeated experience decomposing teen numbers by using tools that highlight the base-ten structure of our number system, including fingers, 10-frames, and rekenreks. Hide Zero cards are used alongside these concrete representations to help students see that the digit 1 in a teen number represents 10 ones.

Why do students need to represent teen numbers with number sentences?

Throughout module 6, students represent teen numbers in predictable ways such as by using objects, drawings, and written numerals. When students decompose teen numbers as 10 ones and some more ones, they may also record their work with an addition or subtraction sentence. Depending on the context, students might record the decomposition of 13 as $13 = 10 + 3$, $13 = 3 + 10$, $13 - 3 = 10$, or $13 - 10 = 3$.

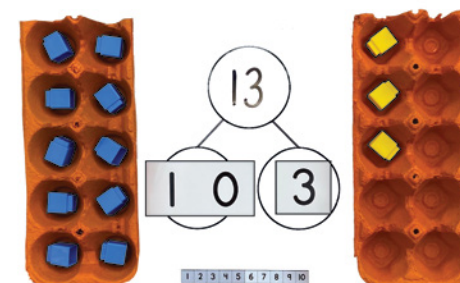
Students who can fluently decompose teen numbers into 10 ones and some more ones and represent that thinking with a number sentence are said to know their $10 + n$ facts. When students fluently know $13 = 10 + 3$, they also know $3 + 10 = 13$, $13 - 10 = 3$, and all of the related facts. In kindergarten, the focus is on number sentences that have 10 as a part. Students may see, but are not expected to work comfortably with, number sentences such as $8 + 5 = 13$ or $13 - 5 = 8$.

Decomposing a teen number into 10 ones and some more ones is foundational to Level 3 make ten and take from ten strategies, which students learn in grade 1 module 3. When students know their $10 + n$ facts, they can simplify problems such as $8 + 5$ or $13 - 5$.

Why are measurement concepts included in a module about counting and place value?

Weaving measurement concepts into work with numbers provides authentic and engaging contexts for the major work of the grade. Topics B and D include investigations of area and length that allow students to test and apply their emerging place value understanding.

In an optional topic B lesson, students investigate ways to decompose teen numbers by using counters to fill shapes of different sizes. Possible decompositions are tied to the area of each shape: *2 purple boxes will fit 16 doughnuts, but I need 4 yellow boxes to fit 16 doughnuts.*



$$13 = 10 + 3$$

$$8 + 5 = \square$$

$$\begin{array}{c} \swarrow \quad \searrow \\ 2 \quad 3 \end{array}$$

$$8 + 2 = 10$$

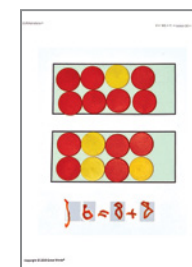
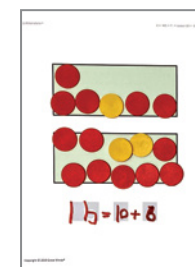
$$10 + 3 = 13$$

$$13 - 5 = \square$$

$$\begin{array}{c} \swarrow \quad \searrow \\ 3 \quad 10 \end{array}$$

$$10 - 5 = 5$$

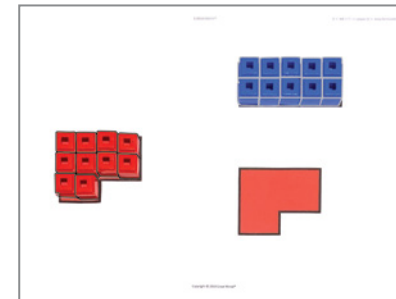
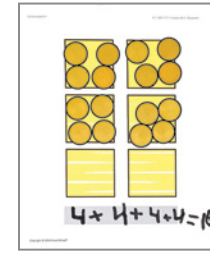
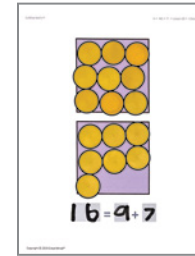
$$5 + 3 = 8$$



In topic D, area serves as a context for comparing numbers. Students explore early area comparison strategies, such as comparing the sides of shapes and covering one shape with the other, before landing on a more reliable strategy. They cover flat shapes with Unifix[®] Cubes and compare the number of cubes needed to cover each shape. With shapes that require 10 to 20 cubes, there is an authentic opportunity to consider place value when comparing the numbers.

A length comparison lesson also provides context for using place value concepts. Lesson 23 engages students with familiar length comparison situations, but now the length of the objects requires use of both 10-sticks and individual cubes. Students have an opportunity to count by tens and then ones to describe their cube sticks: 10, 20, 21, 22, 23. *The backpack is as long as 23 cubes.*

Lessons that weave together number and measurement concepts prepare students for the work of subsequent grades. As students use direct comparison strategies to compare objects with greater lengths and areas, they develop a need for the place value understanding built in grade 1. Repeated experience with area concepts throughout the primary years prepares students for formal lessons on area in grade 3. Module 6 highlights the interconnectedness of all disciplines of mathematics.



Terminology

The following terms are critical to the work of grade K module 6. This resource groups terms into categories called New, Familiar, and Academic Verbs. The lessons in this module incorporate terminology with the expectation that students work toward applying it during discussions and in writing.

Items in the New category are discipline-specific words that are introduced to students in this module. These items include the definition, description, or illustration as it is presented to students. At times, this resource also includes italicized language for teachers that expands on the wording used with students.

Items in the Familiar category are discipline-specific words introduced in prior modules.

Items in the Academic Verbs category are high-utility terms that are used across disciplines. These terms come from a list of academic verbs that the curriculum strategically introduces at this grade level.

New

ones

When we count things one at a time, we call that counting by ones. (Lesson 1)

Familiar

addition

count

equals

fewer

greater

length

less

line

long

minus

more

number

number sentence

part

partners to x

pattern

plus

short

sort

strategy

subtraction

total

whole

Academic Verbs

Module 6 does not introduce any academic verbs from the grade K list.

Materials

The following materials are needed to implement this module. The suggested quantities are based on a class of 24 students and one teacher.

1	10-sided dice, set of 24	12	<i>Eureka Math</i> ^{2™} Numeral Cards
1	20-bead demonstration rekenrek	24	Glue sticks
1	100-bead demonstration rekenrek	55	Index cards
1	Big book	24	<i>Learn</i> books
13	Books	16	Marbles or pom-poms
2	Boxes, small	4	Markers, assorted colors
1	Chart paper, tablet	20	Paper clips
12	Clear cups	25	Pencils
12	Counting collections	5	Pennies
24	Crayons, set of 8	25	Personal whiteboards
2	Dot dice, set of 12	25	Personal whiteboard erasers
25	Dry-erase markers	25	Pipe cleaners
1	Empty can	250	Pony beads (125 red, 125 white)
24	<i>Eureka Math</i> ^{2™} 10-frame carton	1	Projection device
1	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, demonstration set	1	Puppet or stuffed animal
2	<i>Eureka Math</i> ^{2™} Hide Zero [®] cards, basic student set of 12	50	Resealable plastic bags (small)
2	<i>Eureka Math</i> ^{2™} Match cards, set of 12	25	Scissors

- | | | | |
|-----|--------------------------------|----|---|
| 1 | Teach book | 1 | Two-color counters, 200 pieces |
| 1 | Teacher computer or device | 1 | Unifix [®] Cubes, set of 1,000 |
| 250 | Two-color beans, red and white | 24 | Work mats |

Visit <http://eurmath.link/materials> to learn more.

Please see lesson 13 for a list of organizational tools suggested for the counting collection.

Daily Tool Kits

In module 6, students and teachers work with hands-on materials to explore the math concepts introduced in each lesson. The list of materials below includes the most frequently used items in module 6. Consider creating a tool kit for each student to minimize materials preparation for each lesson. Having student and teacher tool kits on hand each day allows for smooth transitions and drastically decreases lesson prep time.

Student Daily Tool Kit

- Unifix[®] Cubes (10)
- Two-color beans (5)
- Dry-erase marker
- Glue
- Pencil
- Personal whiteboard
- Scissors



Teacher Daily Tool Kit

- 100-bead rekenrek
- Chart paper
- Unifix[®] Cubes (15)
- Empty can
- Pencil
- Pennies (5)
- Eureka Math^{2™} Hide Zero[®] cards, demonstration set

