



SPRINGFIELD PUBLIC SCHOOLS
ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21ST CENTURY GLOBAL SKILLS

Springfield Public Schools **Springfield, New Jersey 07081**

Multivariable Calculus

Curriculum Guide

2024

This curriculum may be modified through varying materials, techniques, and strategies as per an individual student's IEP.

Springfield Board of Education
Approved August 26, 2024



SPRINGFIELD PUBLIC SCHOOLS
ENGAGING STUDENTS • FOSTERING ACHIEVEMENT • CULTIVATING 21ST CENTURY GLOBAL SKILLS

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Introduction

This template is the product of an initiative led by the New Jersey Department of Education, Office of Academic Standards, in partnership with the New Jersey Association for Supervision and Curriculum Development (NJASCD) and the New Jersey Principals and Supervisors Association (NJPSA). The project was undertaken with the assistance of New Jersey content area supervisors, curriculum coordinators, and teachers during spring-summer 2009. The goal of the project was to design exemplar unit plans and accompanying lesson plans aligned with the New Jersey Student Learning Standards that may be used by districts as models for the development of local curricula.

The Unit Overview Template, which follows, is a tool that can help districts define one unit plan of several that comprise a specific course in the curriculum for any content area. It is designed, once completed, to summarize the content and objectives for the unit, outline lessons and assessments that support the unit, and provide links to documents that facilitate delivery of the unit, such as student materials, background information, resources, and performance rubrics. To see how the units fit together in the design of a course, see the Curriculum Design Template included at the end of this document.

Seven content area Unit Overviews were created as part of the project to serve as exemplar unit plans that are aligned with the New Jersey Student Learning Standards and that assist in achieving the goal of engaging students, fostering achievement, and cultivating 21st century global skills. The units stress the integration of 21st century themes and skills, global perspectives, technology, and cross-curricular connections.

We encourage districts to use the exemplars in conjunction with the curriculum development tools and templates that emanated from this project as models for teaching and learning based on the New Jersey Student Learning Standards.



Unit Overview Template

Content Area: Math **Grade Level:** 12 **Unit Title:** Vector Functions & The Geometry of Space

Unit Summary: Students will manipulate vectors in 3-space. Students will apply the dot and cross products to analyze scenarios and solve problems. Students will calculate lines and planes in R^3 , and use cylindrical and spherical coordinates. Students will apply vector functions to solve problems.

Primary Interdisciplinary Connections: *Science and Engineering*

21st Century Themes ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ ICT Literacy
☐ Information Literacy ☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy

Unit Rationale: Knowledge of Vectors and Space are the building blocks for the rest of the course.

Career Readiness (Standard 9):

- 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.
- 9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
- 9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
- 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

Learning Targets

Standards: N.VM; F.IF; F.BF; F.TF

Content Statements: Represent and model with vector quantities; Perform operations on vectors; Perform operations on matrices and use matrices in applications; Understand the concept of a function and use function notation; Build a function that models a relationship between two quantities; Model periodic phenomena with trigonometric functions.

CPI#:	Cumulative Progress Indicator (CPI):
N.VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $ \mathbf{v} $, $\ \mathbf{v}\ $, v).
N.VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point



	from the coordinates of a terminal point.
N.VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
N.VM.B.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N.VM.B.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple cv using $cv = c v$. Compute the direction of cv knowing that when $c v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).
N.VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N.VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N.VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.
N.VM.C.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N.VM.C.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N.VM.C.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N.VM.C.12	Work with 2×2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.
F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.BF.A.1	Write a function that describes a relationship between two quantities.



	a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
F.TF.B.5	(+) Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Essential Questions:

- How do we draw vectors?
- How do we add or subtract vectors?
- How do we interpret the different ways to find a product of vectors?
- How do we write equations of lines and planes?
- How do we write vector functions?

Unit Enduring Understandings:

- Classification of given quantities as either vector quantities or scalar quantities.
- Writing vector or plane equations in $\mathbb{R}^3$.
- Drawing vectors and understanding their physical meanings.

Unit Learning Targets:

Students will

...develop an understanding of vectors and vector functions in $\mathbb{R}^3$.

Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m , where $c(m)$ is the number of molecules of carbon dioxide.



Evidence of Learning

Summative Assessment: 2 months

Equipment Needed: Whiteboard, laptop, projector

Teacher Resources: Whiteboard, laptop, projector

Formative Assessments:

- Homework
- Class discussions
- Class work
- Quizzes
- Pop Quizzes
- Exam
- Workshop Problem Sets



Lesson Plans

Lesson	Time Frame
Lesson 1 Adding and Subtracting Vectors	1 day
Lesson 2 The Dot Product	2 days
Lesson 3 The Cross Product	2 days

Teacher Notes: Students should draw diagrams in their notes for most examples until they understand the physical meaning of each of the products.

Curriculum Development Resources/Technology Resources:

Click the links below to access additional resources used to design this unit:

math.rutgers.edu/~zeilberg/math251.html

<http://tutorial.math.lamar.edu/Classes/CalcII/VectorsIntro.aspx>

<https://www.khanacademy.org/science/physics/magnetic-forces-and-magnetic-fields/electric-motors/v/dot-vs-cross-product>

<https://www.geogebra.org/3d?lang=en>



Lesson Plan 1 Template

Content Area: Math Lesson Title: Adding & Subtracting Vectors Time Frame: 1 day

Lesson Components

21st Century Themes

- ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy
☐ Civic Literacy ☐ Health Literacy

21st Century Skills

- ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy
☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy

Interdisciplinary Connections:

Science and Engineering

Integration of Technology:

Graphing Calculators, Desmos

Equipment Needed:

Laptops, graphing calculators

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: • Draw vectors. • Identify x, y, and z components of vectors. • Add vector components. • Subtract vector components.	Lesson Sequence • Students define a vector and draw by hand. • Students use GeoGebra to graph, add and subtract vectors online. • Students use GeoGebra to illustrate the concept of component vectors. • Students complete a problem set and validate their answers using GeoGebra.	• Can students draw vectors? • Can students label vectors? • Can students add/subtract vectors? • Homework
Differentiation: Use of technology to assist drawings		
Resources Provided: • Laptops with Internet access		



Lesson Plan 2 Template

Content Area: Math Lesson Title: The Dot Product Time Frame: 2 days

Lesson Components

21st Century Themes

- ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy
☐ Civic Literacy ☐ Health Literacy

21st Century Skills

- ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy
☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy

Interdisciplinary Connections:

Science and Engineering

Integration of Technology:

Graphing Calculators, Desmos

Equipment Needed:

Laptops, graphing calculators

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Identify the difference between a scalar quantity and a vector. - Compute the dot product of two vectors. - Draw the dot product of two vectors. - Use alternate definitions of the dot product. - Prove alternate definitions.	Lesson Sequence - Teacher demonstrates computation of the dot product. - Given the equation for the angle between two vectors, students resolve the proof (they may be given a hint to use the Law of Cosines). - Students prove vectors orthogonal using the dot product. - Students verify that vectors with dot product of 0 are orthogonal using GeoGebra. - Students complete a problem set from Stewart text.	- Classwork - Draw two dotted vectors - Discussion - Proof of angle between vectors equation - Use of GeoGebra to verify orthogonality of vectors - Homework
Differentiation: Drawings on the laptop using Grapher or GeoGebra.		



Resources Provided:

- Laptops
- Stewart's Calculus 5th edition



Lesson Plan 3 Template

Content Area: Math Lesson Title: The Cross Product Time Frame: 2 days

Lesson Components

21st Century Themes

- ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy
☐ Civic Literacy ☐ Health Literacy

21st Century Skills

- ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy
☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy

Interdisciplinary Connections:

Science and Engineering

Integration of Technology:

Graphing Calculators, Desmos

Equipment Needed:

Laptops, graphing calculators

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Recall from matrix algebra the determinant. - Compute the cross product. - Draw two crossed vectors. - Understand what kind of products can and cannot be computed. - Alternate definitions of the cross product.	Lesson Sequence - Teacher demonstration of computation of determinant for 2x2 and 3x3 matrices. - Students use technology to graph two vectors and their cross-product vectors to discover properties of cross products. - Students work together with teacher direction to discover connection between cross products and area/volume of parallelograms/ parallelepipeds. - Students and teachers collaboratively explore connections to physics (torque). - Students complete a problem set from Stewart text.	- Homeworks - Class discussion
Differentiation: Review of matrix concepts, technology for diagrams		
Resources Provided: - Laptops - Stewart's Calculus, 5th edition		



Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly*, *Moderately* or *Weakly* meets the criteria below.

Lesson Activities	Strongly	Moderately	Weakly
Are challenging and require higher-order thinking and problem-solving skills	☒	☐	☐
Allow for student choice	☐	☒	☐
Provide scaffolding for acquiring targeted knowledge/skills	☐	☒	☐
Integrate global perspectives	☐	☒	☐
Integrate 21st-century skills	☒	☐	☐
Provide opportunities for interdisciplinary connection and transfer of knowledge and skills	☒	☐	☐
Foster student use of technology as a tool to develop critical thinking, creativity and innovation skills	☒	☐	☐
Are varied to address different student learning styles and preferences	☒	☐	☐
Are differentiated based on student needs	☒	☐	☐
Are student-centered with teacher acting as a facilitator and co-learner during the teaching and learning process	☒	☐	☐
Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives	☒	☐	☐
Provide opportunities for student reflection and self-assessment	☒	☐	☐
Provide data to inform and adjust instruction to better meet the varying needs of learners	☒	☐	☐

Curriculum Design Template

Content Area: Math	Grade Level: 12	Unit Title: Vector Functions & The Geometry of Space
Created: 2024		Board Approved on:



Unit Overview Template

Content Area: Math **Grade Level:** 12 **Unit Title:** Partial Derivatives

Unit Summary: Students will take partial derivatives and use them to analyze functions in R3. Students will analyze functions of several variables, analyze limits and continuity, calculate partial derivatives, determine tangent planes and their approximations, and apply the Chain Rule. Students will use directional derivatives and the gradient vector to determine maxima and minima. Students will apply Lagrange Multipliers to solve problems.

Primary Interdisciplinary Connections: *Science and Engineering*

21st Century Themes ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ ICT Literacy
☐ Information Literacy ☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy

Unit Rationale: *Applications of partial derivatives are found in all the sciences and economics.*

Career Readiness (Standard 9):

- 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.
- 9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12.prof.CR2b, 2.2.12.LF.8).
- 9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
- 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
- 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
- 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

Learning Targets

Standards: N.VM; F.IF; F.BF; F.TF

Content Statements: Represent and model with vector quantities; Perform operations on vectors; Perform operations on matrices and use matrices in applications; Understand the concept of a function and use function notation; Build a function that models a relationship between two quantities; Interpret functions that arise in applications in terms of the context; Model periodic phenomena with trigonometric functions.

CPI#:	Cumulative Progress Indicator (CPI):
N.VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent



	vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $ \mathbf{v} $, $\ \mathbf{v}\ $, v).
N.VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N.VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
N.VM.B.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N.VM.B.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\ c\mathbf{v}\ = c \mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $c \mathbf{v} \neq \mathbf{0}$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$).
N.VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N.VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N.VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.
N.VM.C.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N.VM.C.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N.VM.C.12	Work with 2×2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.
F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.B.4	For a function that models a relationship between two quantities, interpret key



	features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
F.IF.B.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F.BF.A.1	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Essential Questions:

- How do you evaluate functions in two variables?
- How do you find a limit using the Squeeze Theorem or show that a limit does not exist?
- How do you compute partial derivatives?
- How do you write equations of tangent planes and use the chain rule?
- How do you compute directional derivatives, draw gradient vectors, and find LaGrange multipliers?
- How do you find the maximum or minimum values of a function?

Unit Enduring Understandings:

- Using partial derivatives.
- Calculating tangent planes and gradient vectors.
- Sketching cross sections of functions in 2 variables.

Unit Learning Targets:

Students will

....develop ways to apply partial derivatives in $\mathbb{R}^3$.

Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m , where $c(m)$ is the number of molecules of carbon dioxide.

Climate Change Example: Students may calculate the average rate of change of a function $c(m)$ presented symbolically or as a table, where $c(m)$ represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).



Evidence of Learning

Summative Assessment: 2 months

Equipment Needed: Laptop, LCD Projector, whiteboard

Teacher Resources: Laptop, LCD Projector, whiteboard

Formative Assessments:

- Homework
- Class discussions
- Class work
- Quizzes
- Pop Quizzes
- Exam
- Workshop Problem Sets



Lesson Plans

Lesson	Time Frame
Lesson 1 Finding Limits	3 days
Lesson 2 Calculating Partial Derivatives	3 days
Lesson 3 The Multivariable Chain Rule	2 days

Teacher Notes: Students should draw diagrams in their notes so they get used to drawing cross sections.

Curriculum Development Resources/Technology Resources:

Click the links below to access additional resources used to design this unit:

math.rutgers.edu/~zeilberg/math251.html

<https://www.geogebra.org/3d?lang=en>

<http://tutorial.math.lamar.edu/Classes/CalcIII/PartialDerivsIntro.aspx>

<https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives>



Lesson Plan 1 Template

Content Area: Math Lesson Title: Taking Limits Time Frame: 3 days

Lesson Components

21st Century Themes

- ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy
☐ Civic Literacy ☐ Health Literacy

21st Century Skills

- ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy
☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy

Interdisciplinary Connections:

Science and Engineering

Integration of Technology:

Graphing Calculators, Desmos

Equipment Needed:

Laptops, graphing calculators

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Analyze functions. - Take limits one variable at a time. - Find limits using the Squeeze Theorem. - Demonstrate algebraically that limits do not exist. - Verify limits graphically using technology.	Lesson Sequence - Teacher demonstrates/ proves a basic trig limit exists using the Squeeze Theorem. - Students use the Squeeze Theorem to prove limits of multivariable functions exist. - Teacher demonstrates finding limits of functions along different lines to show limits do not exist. - Students complete the problem set involving limits that do not exist. - Students verify all limits graphically using GeoGebra or Grapher.	- Question: Can students find limits that do not exist? - Question: Can students find limits that do exist? - Class discussion - Homework
Differentiation: Use of technology to assist drawings		
Resources Provided: - Laptops - Stewart's Calculus, 5th edition		



Lesson Plan 2 Template		
Content Area: Math	Lesson Title: Finding Directional Derivatives	Time Frame: 3 days
Lesson Components		
<u>21st Century Themes</u> ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy ☐ Civic Literacy ☐ Health Literacy		
<u>21st Century Skills</u> ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy ☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy		
Interdisciplinary Connections: Science and Engineering		
Integration of Technology: Graphing Calculators, Desmos		
Equipment Needed: Laptops, graphing calculators		

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Analyze a topographical map or weather map to find altitudes/temperature. - Identify changes in altitudes or temperatures in various directions from a point. - Estimate directional derivatives. - Find the direction where the directional derivative is maximized.	Lesson Sequence - Distribute maps to students. - Students locate various locations on the maps. - Students estimate the rates of change of temperature or altitude in various directions from given locations. - Teacher delivers a mini-lesson on directional derivatives and gradient vectors. - Students complete a problem set from Stewart text.	- Classwork - Quiz - Discussion - Completion of map activity - Homeworks - Workshop
Differentiation: Students locate other types of maps online (barometric pressure, etc) that could be analyzed with similar methods.		



Resources Provided:

- Various maps
- Textbook
- Laptops



Lesson Plan 3 Template

Content Area: Math Lesson Title: The Chain Rule Time Frame: 2 days

Lesson Components

21st Century Themes

- ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy
☐ Civic Literacy ☐ Health Literacy

21st Century Skills

- ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy
☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy

Interdisciplinary Connections:

Science and Engineering

Integration of Technology:

Graphing Calculators, Desmos

Equipment Needed:

Laptops, graphing calculators

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Recall the chain rule for single variable calculus. - Compute chain rule for different cases in 2 variables. - Use the multivariable chain rule to solve implicit differentiation problems.	Lesson Sequence - Students will solve a warm-up problem set of single variable derivative problems using the Chain Rule. - Students will be presented with a multivariable problem that requires Chain Rule to differentiate. - Students will brainstorm possible solution methods. - Students will discover (possibly with assistance) the general form of the Chain Rule. - Students will explore the use of the Chain Rule to perform implicit differentiation. - Students complete the problem set.	- Homework - Class discussion - Quiz - Workshop
Differentiation: Review of matrix concepts, technology for diagrams, and tree diagrams for showing the chain rule		



Resources Provided:

- Stewart's Calculus, 5th ed.
- Laptops



Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly*, *Moderately* or *Weakly* meets the criteria below.

Lesson Activities	Strongly	Moderately	Weakly
Are challenging and require higher-order thinking and problem-solving skills	☒	☐	☐
Allow for student choice	☐	☒	☐
Provide scaffolding for acquiring targeted knowledge/skills	☒	☐	☐
Integrate global perspectives	☐	☒	☐
Integrate 21st-century skills	☐	☒	☐
Provide opportunities for interdisciplinary connection and transfer of knowledge and skills	☒	☐	☐
Foster student use of technology as a tool to develop critical thinking, creativity, and innovation skills	☒	☐	☐
Are varied to address different student learning styles and preferences	☒	☐	☐
Are differentiated based on student needs	☒	☐	☐
Are student-centered with the teacher acting as a facilitator and co-learner during the teaching and learning process	☒	☐	☐
Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives	☒	☐	☐
Provide opportunities for student reflection and self-assessment	☒	☐	☐
Provide data to inform and adjust instruction to better meet the varying needs of learners	☒	☐	☐

Curriculum Design Template

Content Area: Math	Grade Level: 12	Unit Title: Partial Derivatives
Created: 2024	Board Approved on:	



Unit Overview Template

Content Area: Math	Grade Level: 12	Unit Title: Multiple Integrals
Unit Summary: Students will complete double and triple integrals in R_3 over rectangles, general regions, and in polar coordinates. Students will apply double integrals to solve problems. Students will extend to triple integrals, and convert to spherical or cylindrical coordinate systems based on the problem. Students will apply a change of variables in multiple integrals. Primary Interdisciplinary Connections: <i>Science and Engineering</i> 21st Century Themes ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ ICT Literacy ☐ Information Literacy ☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy		
Unit Rationale: Applications of multiple integrals are found in engineering and physics.		
Career Readiness (Standard 9): 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth. 9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills. 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. 9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8). 9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1). 9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice. 9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data. 9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6). 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).		

Learning Targets

Standards: N.VM; F.IF; F.BF; F.TF	
Content Statements: Represent and model with vector quantities; Perform operations on vectors; Perform operations on matrices and use matrices in applications; Understand the concept of a function and use function notation; Build a function that models a relationship between two quantities; Model periodic phenomena with trigonometric functions.	
CPI#:	Cumulative Progress Indicator (CPI):
N.VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for



	vectors and their magnitudes (e.g., v , $ v $, $ v $, v).
N.VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N.VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
N.VM.B.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N.VM.B.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple cv using $cv = c v$. Compute the direction of cv knowing that when $c v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).
N.VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N.VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N.VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.
N.VM.C.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N.VM.C.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N.VM.C.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N.VM.C.12	Work with 2×2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.
F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and



	interpret statements that use function notation in terms of a context.
F.BF.A.1	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
F.TF.B.5	(+) Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Essential Questions:

- How do you compute and interpret double/triple integrals?
- How do you compute iterated integrals?
- How do you compute integrals in polar, cylindrical, and spherical coordinates?
- How do you apply double/triple integrals to engineering, physics, and statistics?
- How do you calculate the surface area with double integrals?
- How do you use a change of variable with multiple integrals?

Unit Enduring Understandings:

- Methods of computing higher integrals.
- Using different coordinate systems.
- Sketching regions of integration.

Unit Learning Targets:

Students will

....develop ways to apply higher integrals in $\mathbb{R}^3$.

Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m , where $c(m)$ is the number of molecules of carbon dioxide.



Evidence of Learning

Summative Assessment: 2 months

Equipment Needed: Laptop, whiteboard, projector, textbooks

Teacher Resources: Laptop, whiteboard, projector

Formative Assessments:

- Homework
- Class discussions
- Class work
- Quizzes
- Pop Quizzes
- Exam
- Workshop Problem Sets



Lesson Plans

Lesson	Time Frame
Lesson 1 Double Integrals Over Rectangles	3 days
Lesson 2 Double Integrals Over General Regions	3 days
Lesson 3 Triple Integrals	4 days

Teacher Notes: Students should continue to draw diagrams in their notes so they get used to drawing regions of integration.

Curriculum Development Resources/Technology Resources:

Click the links below to access additional resources used to design this unit:

math.rutgers.edu/~zeilberg/math251.html

<https://www.geogebra.org/3d?lang=en>

<http://tutorial.math.lamar.edu/Classes/CalcIII/MultipleIntegralsIntro.aspx>

<https://www.khanacademy.org/math/multivariable-calculus/integrating-multivariable-functions>

<https://www.symbolab.com/solver/double-integrals-calculator>

<https://www.symbolab.com/solver/triple-integrals-calculator>



Lesson Plan 1 Template

Content Area: Math Lesson Title: Double Integrals Over Rectangles Time Frame: 3 days

Lesson Components

21st Century Themes

- ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy
☐ Civic Literacy ☐ Health Literacy

21st Century Skills

- ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy
☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy

Interdisciplinary Connections:

Science and Engineering

Integration of Technology:

Graphing Calculators, Desmos

Equipment Needed:

Laptops, graphing calculators

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Recall the definition of integration. - Recall how to take definite and indefinite integrals in 1 variable. - Extend to two variables over a rectangular region.	Lesson Sequence - Review computation and graphical significance of a single variable integral. - Draw a region in the xy-plane and ask students to break it into squares and connect the drawing to integral notation using 1 as the integrand. - Demonstrate “inside-out” method of evaluating double integrals. - Extend and discuss the geometric meaning of using an integrand other than 1--guide students as necessary toward the concept of volume.	- Workshop - Class discussion - Homework - Quiz



- Students complete a problem set on double integrals.

Differentiation:

Students may use technology (graphing calculators, GeoGebra, Grapher, or Desmos) to assist in drawing regions to integrate over. Students may also use a computer-based double integral calculator to assist with integration procedures.

Resources Provided:

- Graphing calculator
- Stewart's Calculus, 5th ed.



Lesson Plan 2 Template

Content Area: Math Lesson Title: Double Integrals Using Polar Time Frame: 3 days

Lesson Components

21st Century Themes

- ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy
☐ Civic Literacy ☐ Health Literacy

21st Century Skills

- ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy
☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy

Interdisciplinary Connections:

Science and Engineering

Integration of Technology:

Graphing Calculators, Desmos

Equipment Needed:

Laptops, graphing calculators

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Recall double derivatives over rectangular regions. - Recall conversion techniques from rectangular to polar. - Interpret double integrals. - Draw regions. - Evaluate double integrals by hand and using technology.	Lesson Sequence - Review the meaning of double integrals over a general region. - Present problem involving double integration over the unit disc. Allow students to identify areas of difficulty with bounds/antiderivatives. - Brainstorm different ideas and methods that could make integration easier--guide students toward using polar coordinates as needed. - Review conversion from rectangular to polar and computation of arc length.	- Classwork - Quiz - Discussion - Homeworks - Workshop



- Discuss the use of arc length to convert $dy dx$ to $r dr d\theta$.
- Students complete a problem set.

Differentiation:

Students may use technology (graphing calculators, GeoGebra, Grapher, or Desmos) to assist in drawing of regions to integrate over. Students may also use a computer-based double integral calculator to assist with integration procedures.

Resources Provided:

- Graphing calculator
- Stewart's Calculus, 5th ed.



Lesson Plan 3 Template

Content Area: Math Lesson Title: Triple Integrals Time Frame: 4 days

Lesson Components

21st Century Themes

- ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy
☐ Civic Literacy ☐ Health Literacy

21st Century Skills

- ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy
☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy

Interdisciplinary Connections:

Science and Engineering

Integration of Technology:

Graphing Calculators, Desmos

Equipment Needed:

Laptops, graphing calculators

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Recall how to compute double integrals. - Visualize what triple integrals mean. - Perform triple integrals over closed boxes. - Perform triple integrals over described regions. - Solve application problems involving triple integrals.	Lesson Sequence - Review double integrals for computation of area with an integrand of 1. Guide students to a similar notion of volume using triple integrals. - Demonstrate calculation of the volume of a rectangular prism using triple integrals. - Review how to generate the area of a plane. - Students work in groups to set up and evaluate triple integral expressions for the volume of a variety of solids. - Guide students toward the notion of computing mass using triple	- Homeworks - Class discussion - Quiz - Workshop



integrals if given a density function.

- Students complete the problem set on the mass of solids.

Differentiation:

Students may use technology (graphing calculators, GeoGebra, Grapher, or Desmos) to assist in drawing of regions to integrate over. Students may also use a computer-based triple integral calculator to assist with integration procedures.

Resources Provided:

- Graphing calculator
- Stewart's Calculus, 5th ed.



Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly*, *Moderately* or *Weakly* meets the criteria below.

Lesson Activities	Strongly	Moderately	Weakly
Are challenging and require higher order thinking and problem solving skills	☒	☐	☐
Allow for student choice	☒	☐	☐
Provide scaffolding for acquiring targeted knowledge/skills	☒	☐	☐
Integrate global perspectives	☐	☒	☐
Integrate 21st century skills	☒	☐	☐
Provide opportunities for interdisciplinary connection and transfer of knowledge and skills	☒	☐	☐
Foster student use of technology as a tool to develop critical thinking, creativity and innovation skills	☒	☐	☐
Are varied to address different student learning styles and preferences	☒	☐	☐
Are differentiated based on student needs	☒	☐	☐
Are student-centered with teacher acting as a facilitator and co-learner during the teaching and learning process	☒	☐	☐
Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives	☒	☐	☐
Provide opportunities for student reflection and self- assessment	☒	☐	☐
Provide data to inform and adjust instruction to better meet the varying needs of learners	☒	☐	☐

Curriculum Design Template

Content Area: Math	Grade Level: 12	Unit Title: Multiple Integrals
Created: 2024	Board Approved on:	



Unit Overview Template

Content Area: Math

Grade Level: 12

Unit Title: Vector Calculus

Unit Summary: Students will complete double and triple integrals in $\mathbb{R}^3$. Students will apply the Fundamental Theorem for Line Integrals, Green's Theorem, Curl, Divergence, Parametric Surfaces, Stokes' Theorem, and Divergence Theorem to solve problems.

Primary Interdisciplinary Connections: *Science and Engineering*

21st Century Themes ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ ICT Literacy
☐ Information Literacy ☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy

Unit Rationale: *Vector calculus is used in engineering, physics, and meteorology.*

Career Readiness (Standard 9):

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth.

9.2.12.CAP.6: Identify transferable skills in career choices and design alternative career plans based on those skills.

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12.prof.CR2b, 2.2.12.LF.8).

9.4.12.CI.3: Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.TL.4: Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problem (e.g., 7.1.AL.IPERS.6).

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

Learning Targets

Standards: N.VM; F.IF; F.BF; F.TF

Content Statements: Represent and model with vector quantities; Perform operations on vectors; Perform operations on matrices and use matrices in applications; Understand the concept of a function and use function notation; Build a function that models a relationship between two quantities; Model periodic phenomena with trigonometric functions.

CPI#:

Cumulative Progress Indicator (CPI):

N.VM.A.1

Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $|\mathbf{v}|$, $\|\mathbf{v}\|$, v).

N.VM.A.2

Find the components of a vector by subtracting the coordinates of an initial point



	from the coordinates of a terminal point.
N.VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
N.VM.B.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N.VM.B.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple cv using $cv = c v$. Compute the direction of cv knowing that when $c v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).
N.VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N.VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N.VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.
N.VM.C.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N.VM.C.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N.VM.C.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N.VM.C.12	Work with 2×2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.
F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.BF.A.1	Write a function that describes a relationship between two quantities.



	a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
F.TF.B.5	(+) Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Unit Essential Questions:

- How do you interpret vector fields?
- How do you compute line integrals?
- What are curl and divergence?
- What is a parametric surface?
- What is the relationship between the Fundamental Theorem for Line Integrals, Green's, Stokes', and the Divergence Theorem?

Unit Enduring Understandings:

- Drawing, using, applying, and interpreting vector fields.
- Applications of line integrals.
- How Green's, Stokes', and Divergence theorems work.

Unit Learning Targets:

Students will

....understand the higher dimension versions of the Fundamental Theorem of Calculus and how they are used to interpret vectors and vector fields in $\mathbb{R}^3$.

Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m , where $c(m)$ is the number of molecules of carbon dioxide.



Evidence of Learning

Summative Assessment: 2 months

Equipment Needed: Computer, projector, whiteboard, graphing calculators

Teacher Resources: Computer, projector, whiteboard, graphing calculators

Formative Assessments:

- Homework
- Class discussions
- Class work
- Quizzes
- Pop Quizzes
- Exam
- Workshop Problem Sets



Lesson Plans

Lesson	Time Frame
Lesson 1 Vector Fields	3 days
Lesson 2 Line Integrals	3 days
Lesson 3 Green's Theorem	3 days

Teacher Notes: Students should continue to draw diagrams in their notes.

Curriculum Development Resources/Technology Resources:

Click the links below to access additional resources used to design this unit:

math.rutgers.edu/~zeilberg/math251.html

<https://www.geogebra.org/3d?lang=en>

<http://tutorial.math.lamar.edu/Classes/CalcIII/LineIntegralsIntro.aspx>

<http://tutorial.math.lamar.edu/Classes/CalcIII/SurfaceIntegralsIntro.aspx>

<https://www.khanacademy.org/math/multivariable-calculus/greens-theorem-and-stokes-theorem>

<https://www.symbolab.com/solver/double-integrals-calculator>

<https://www.symbolab.com/solver/triple-integrals-calculator>

<https://www.geogebra.org/m/QPE4PaDZ>



Lesson Plan 1 Template

Content Area: Math Lesson Title: Vector Fields Time Frame: 3 days

Lesson Components

21st Century Themes

- ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy
☐ Civic Literacy ☐ Health Literacy

21st Century Skills

- ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy
☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy

Interdisciplinary Connections:

Science and Engineering

Integration of Technology:

Graphing Calculators, Desmos

Equipment Needed:

Laptops, graphing calculators

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: • Draw vector fields in R^2 and R^3. • Interpret vector fields with real-life data. • Calculate gradient vector fields. • Identify vector fields by their plots.	Lesson Sequence • Students will be given a vector field representation of wind velocities in a region and asked to relate what they see to slope fields from AP Calculus. • Students will answer guided questions about a vector field. • Students will use GeoGebra's Vector Field generator to create vector fields for their own functions and pair with other students to match equations to graphical representations • Students complete a problem set regarding vector fields	• Real-value analysis • Workshop • Class discussion • Homework • Quiz



Differentiation:

Use of technology to assist in drawing vector fields in $\mathbb{R}^3$.

Resources Provided:

- Graphing calculator



Lesson Plan 2 Template		
Content Area: Math	Lesson Title: Line Integrals	Time Frame: 3 days
Lesson Components		
<u>21st Century Themes</u> ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy ☐ Civic Literacy ☐ Health Literacy		
<u>21st Century Skills</u> ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy ☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy		
Interdisciplinary Connections: Science and Engineering		
Integration of Technology: Graphing Calculators, Desmos		
Equipment Needed: Laptops, graphing calculators		

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Recall parametric equations. - Take a line integral over a smooth curve. - Compute line integrals in R^3. - Compute line integrals of vector fields.	Lesson Sequence - Review parametric equations and the single variable method for calculating the length of a curve. - Students use GeoGebra or Grapher on laptops to practice generating graphs of parametric functions. - Use examples involving circles and ellipses to demonstrate how to parameterize a curve. - Discuss the calculation and interpretation of line integrals. - Students work on problem sets involving line integrals.	- Classwork - Quiz - Discussion - Homeworks - Workshop



Differentiation:

Drawings on a laptop using Grapher or Notebook to assist visualization of line integrals in $\mathbb{R}^3$.

Resources Provided:

- Graphing calculator
- Stewart's Calculus, 5th ed.



Lesson Plan 3 Template		
Content Area: Math	Lesson Title: Green's Theorem	Time Frame: 3 days
Lesson Components		
<u>21st Century Themes</u> ☐ Global Awareness ☐ Financial, Economic, Business and Entrepreneurial Literacy ☐ Civic Literacy ☐ Health Literacy		
<u>21st Century Skills</u> ☒ Creativity and Innovation ☒ Critical Thinking and Problem Solving ☐ Information Literacy ☐ Life and Career Skills ☐ Communication and Collaboration ☐ Media Literacy ☐ ICT Literacy		
Interdisciplinary Connections: Science and Engineering		
Integration of Technology: Graphing Calculators, Desmos		
Equipment Needed: Laptops, graphing calculators		

Goals/Objectives	Learning Activities/ Instructional Strategies	Formative Assessment Tasks
Students will: - Define Green's Theorem. - Use Green's Theorem on simple regions. - Use Green's Theorem to calculate areas. - Use Green's Theorem on unions of simple subregions. - Understand applications of Green's Theorem to physics.	Lesson Sequence - Students generate multiple closed curves and determine whether the curves have positive or negative orientation. - Discuss several situations where employing Green's Theorem can simplify more complex integrals. - Provide the necessary fundamentals for using Green's Theorem. - Compare the evaluation of line integrals directly vs. using Green's Theorem. - Students complete a problem set of line integrals, some which require Green's Theorem and some which do not.	- Homeworks - Class discussion - Quiz - Workshop - Exam



Differentiation:

Technology for diagrams to imagine regions in $\mathbb{R}^3$.

Resources Provided:

- Graphing calculator
- Stewart's Calculus, 5th ed.



Lesson Reflection

Reflect on the lesson you have developed and rate the degree to which the lesson *Strongly*, *Moderately* or *Weakly* meets the criteria below.

Lesson Activities	Strongly	Moderately	Weakly
Are challenging and require higher-order thinking and problem-solving skills	☒	☐	☐
Allow for student choice	☒	☐	☐
Provide scaffolding for acquiring targeted knowledge/skills	☒	☐	☐
Integrate global perspectives	☐	☒	☐
Integrate 21st century skills	☒	☐	☐
Provide opportunities for interdisciplinary connection and transfer of knowledge and skills	☒	☐	☐
Foster student use of technology as a tool to develop critical thinking, creativity and innovation skills	☐	☒	☐
Are varied to address different student learning styles and preferences	☒	☐	☐
Are differentiated based on student needs	☒	☐	☐
Are student-centered with the teacher acting as a facilitator and co-learner during the teaching and learning process	☒	☐	☐
Provide means for students to demonstrate knowledge and skills and progress in meeting learning goals and objectives	☒	☐	☐
Provide opportunities for student reflection and self-assessment	☒	☐	☐
Provide data to inform and adjust instruction to better meet the varying needs of learners	☒	☐	☐

Curriculum Design Template

Content Area: Math	Grade Level: 12	Unit Title: Vector Calculus
Created: 2024	Board Approved on:	