

	Name of Course: Pre-Calculus Massachusetts and National Standards: Link to Standard Additional Standards for specific subject Link to Syllabus
	Unit ONE: Functions
Essential Questions	How can translating among different representations of a function (graphically, algebraically, numerically, verbally) help to solve problems?
Standards	F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y=f(x)$. F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function in terms of a context. F-IF.B.4 For each function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given verbal description of the relationship. Key features include: Intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums; symmetries; end behavior; and periodicity. F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F-IF.C.7.b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F-IF.C.7.d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

	F-IF.C.9 Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. F-IF.C.10 Given algebraic, numeric, and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, and trigonometric. F-BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. (Include linear, quadratic, exponential, absolute value, simple rational and radical, logarithmic and trigonometric functions.) Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. (Include recognizing even and odd functions from their graphs and algebraic expressions for them.) F-BF-B.4 Find inverse functions algebraically and graphically.	
Concepts and skills	<i>Students will know...</i> • Domain, Range, Symmetry, Relation, Function, 1-1 Function • Evaluating Functions including Compositions • Transformations of Functions • Piecewise Functions, Absolute Value Functions • Domain restrictions • Increasing, Decreasing, Relative Extrema • Odd, Even (Algebraically and Graphically) • Inverse Functions • Library of Functions	<i>Mathematical Practices</i> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. ~~4. Model with mathematics.~~ 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Content Objectives "I can Statements"	<i>I can....</i> • identify the difference between a relation, function and 1-1 function. • identify the domain and range of a relation, function and 1-1 function. • describe, evaluate, and/or graph a transformation of a function. • graph a piecewise and absolute value function. • analyze the domain restrictions on a curve given algebraically and/or graphically.	

	- analyze a graph by describing its properties (increasing/ decreasing, even/odd, end behavior, max/min) - describe the role of an inverse and/ or find the equation of an inverse function. - identify and describe the properties of the Library of Functions. - Including the following: constant, linear, quadratic, cubic, absolute value, square root, exponential, reciprocal and logarithmic.
Assessments, products, vocabulary, practice	- Functions Quiz - Vocabulary: Domain, Range, Relation, Function, 1-1 Function, Inverse, Even, Odd, Symmetry, Composition, Transformation, Piecewise Function, Absolute Value, Increasing, Decreasing, Relative Minimum, and Relative Maximum.
Text, Materials, and Resources	- Precalculus with Limits - A Graphing Approach - Desmos - Graphing Calculator

	Name of Course: Pre-Calculus Massachusetts and National Standards: Link to Standard Additional Standards for specific subject Link to Syllabus
	Unit TWO: Introduction to Trigonometric Functions
Essential Questions	How can trigonometric functions be applied to evaluate and solve for missing angles and side lengths on a right triangle?
Standards	F-TF.A.1 Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

	F-TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F-TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	
Concepts and skills	<i>Students will know...</i> - Angles Terminology: vertex, initial side, terminal side, clockwise, counterclockwise, coterminal, standard position - Radians, Degrees - Arc Length - Definition of Six Trig Functions: sin, cos, tan, csc, sec, cot - Given point or ratio determine remaining trig functions both on and off the unit circle - SOH-CAH-TOA - Applications of Right Triangle Trigonometry - Unit Circle - Reference Angle - Solving Basic Trig Equations Involving Inverse Functions	<i>Mathematical Practices</i> - Make sense of problems and persevere in solving them. - Reason abstractly and quantitatively. - Construct viable arguments and critique the reasoning of others. - Model with mathematics. - Use appropriate tools strategically. - Attend to precision. - Look for and make use of structure. - Look for and express regularity in repeated reasoning.
Content Objectives "I can Statements"	<i>I can....</i> - draw an angle given the characteristics of the angle. - convert between degrees and radians. - apply the arc length formula to determine the unknown quantity in a given arc. - determine the value of each of the six trigonometric functions given a point and/ or ratio. - apply right triangle trigonometry to solve for missing sides and angles in a right triangle. - determine the values of each trigonometric functions for 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, and their multiples. - solve basic trigonometric equations.	

Assessments, products, vocabulary, practice	2 Quizzes Vocabulary: vertex, initial side, terminal side, clockwise, counterclockwise, coterminal, standard position, radians, degrees, arc length, sine, cosine, tangent, cosecant, secant, cotangent, angle of elevation, angle of depression, reference angle.
Text, Materials, and Resources	- Precalculus with Limits - A Graphing Approach - Desmos - Graphing Calculator

	Name of Course: Pre-Calculus Massachusetts and National Standards: Link to Standard Additional Standards for specific subject Link to Syllabus
	Unit THREE: Graphing Trigonometric Functions
Essential Questions	How can the trigonometric functions be represented graphically?
Standards	F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. (Include linear, quadratic, exponential, absolute value, simple rational and radical, logarithmic and trigonometric functions.) Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. (Include recognizing even and odd functions from their graphs and algebraic expressions for them.) F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F-TF.B.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

Concepts and skills	<i>Students will know...</i> - Graphing Sine and Cosine - Domain, Range, Period, Amplitude, Vertical and Horizontal Shifts (phase shift), Horizontal and Vertical Compressions/ Stretch, Reflections, Symmetry - Writing an Equation of a Trig Function Given a Graph or Descriptors - Graphing Tangent, Cotangent, Secant and Cosecant - Vertical Asymptotes	<i>Mathematical Practices</i> - Make sense of problems and persevere in solving them. - Reason abstractly and quantitatively. - Construct viable arguments and critique the reasoning of others. ~~Model with mathematics.~~ - Use appropriate tools strategically. - Attend to precision. - Look for and make use of structure. - Look for and express regularity in repeated reasoning.
Content Objectives "I can Statements"	<i>I can....</i> - graph the sine and cosine functions. - apply transformations on the sine and cosine functions. - verbally describe the properties of a given sine and cosine function. - write an equation (or equations) of a sine and/or cosine function. - graph the tangent, cotangent, secant and cosecant functions and describe their vertical asymptotes.	
Assessments, products, vocabulary, practice	2 Quizzes - Vocabulary: phase shift (or horizontal shift), amplitude, period, midline, vertical shift, vertical compressions/ stretches, horizontal compressions/ stretches	
Text, Materials, and Resources	- Precalculus with Limits - A Graphing Approach - Desmos - Graphing Calculator - Graphing Trig Functions Intro Activity	

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	Massachusetts and National Standards: Link to Standard Additional Standards for specific subject Link to Syllabus	
	Unit FOUR: Analytic Trigonometry	
Essential Questions	How can the trigonometric functions be represented in different ways and how can I utilize these relationships to choose the most efficient way to solve an equation?	
Standards	F-TF.B.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. F-TF.C.8 Prove the Pythagorean identity and use it to find $\sin$, $\cos$, or $\tan$ given $\sin$, $\cos$, or $\tan$ and the quadrant. F-TF.C.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. G.SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
Concepts and skills	<i>Students will know...</i> - Simplifying Trigonometric Expressions using Pythagorean Identities, Quotient/Ratio Identities and Reciprocal Identities - Proving Trigonometric Identities - Solving Trigonometric Equations	<i>Mathematical Practices</i> - Make sense of problems and persevere in solving them. - Reason abstractly and quantitatively. - Construct viable arguments and critique the reasoning of others. - Model with mathematics. - Use appropriate tools strategically. - Attend to precision.

	○ Linear, Quadratic (u-sub), horizontal shift ● Graphical Representation of a Trigonometric Equation ● Additional Identities Explored: Sum and Difference, Double Angle	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Content Objectives "I can Statements"	<i>I can....</i> ● simplify trigonometric expressions. ● prove trigonometric identities. ● solve trigonometric equations.	
Assessments, products, vocabulary, practice	3 Quizzes Vocabulary: Pythagorean Identities, Quotient/Ratio Identities, Reciprocal Identities, Sum/Difference Identities, Double-Angle Identities	
Text, Materials, and Resources	● Precalculus with Limits - A Graphing Approach ● Desmos ● Graphing Calculator	

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	Unit FIVE: Non-Right Triangle Trigonometry

Essential Questions	How can Trigonometric Functions be used to solve real world problems?	
Standards	G-SRT.D.9 Derive the formula $A = \frac{1}{2}ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G-SRT.D.10 Prove the Laws of sines and Cosines and use them to solve problems. G-SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.	
Concepts and skills	<i>Students will know...</i> • Inverse Trigonometric Functions ◦ Evaluating ◦ Graphing • Law of Sines ◦ excluding the ambiguous case • Law of Cosines • Heron's Formula • Area of a non-right Triangle	<i>Mathematical Practices</i> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Content Objectives "I can Statements"	<i>I can....</i> • evaluate an inverse trigonometric function. • graph an inverse trigonometric function. • apply the Law of Sines/Cosines to solve a non-right triangle. • determine the area of a non-right triangle.	
Assessments, products, vocabulary, practice	• 1 quiz • Vocabulary: Inverse, Arcsine, Arccosine, Arctangent,	

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	Unit SIX: Analytic Geometry	
Essential Questions	How can an implicitly defined relation be represented graphically and algebraically?	
Standards	G-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
Concepts and skills	<i>Students will know...</i> • Coordinate Geometry ◦ Slopes of parallel and perpendicular lines ◦ Slope-Intercept, Standard and Point-Slope forms ◦ Midpoint and Distance Formulas • Graphing, Analyzing Graphs and Writing Equations for: ◦ Circles ◦ Parabolas	<i>Mathematical Practices</i> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. ~~Model with mathematics.~~ 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

	○ Ellipses ○ Hyperbolas ● Systems of Quadratic Equations and Inequalities	
Content Objectives "I can Statements"	<i>I can...</i> ● connect the different concepts associated with coordinate geometry. ● identify the correct conic section given an equation or graph. ● manipulate a given equation into the appropriate form for a given conic. ● graph and label each of the conic sections. ● write an equation for a conic given verbal description or a graph. ● solve a system involving quadratic equations or inequalities algebraically and graphically.	
Assessments, products, vocabulary, practice	● 2 quizzes ● Vocabulary: parallel, perpendicular, slope, intercept, standard form, point-slope form, slope-intercept form, midpoint, distance, circle, parabola, ellipse, hyperbola, center, radius, vertex/ vertices, major axis, minor axis, transverse axis, conjugate axis, asymptote, tangent	
Text, Materials, and Resources	● Precalculus with Limits - A Graphing Approach ● Desmos ● Graphing Calculator	

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	Unit SEVEN: Sequences and Series	
Essential Questions	In what ways can a sequence and series be represented formulaically and applied?	
Standards	F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. A-SSE.B.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	
Concepts and skills	<i>Students will know...</i> - Sequences: Arithmetic, Geometric and Neither - Formalizing Sequences in a Formulaic Form - Explicit vs. Recursive - Finite Arithmetic and Geometric Series - Infinite Geometric Series - Summation Notation	<i>Mathematical Practices</i> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Content Objectives "I can Statements"	<i>I can...</i> - identify a sequence as being arithmetic, geometric or neither. - create a formula given a sequence. - identify a series as being arithmetic, geometric or neither. - expand an expression written with summation notation. - write an expression using summation notation to describe a given series. - determine the sum of a series. - determine if an infinite geometric series converges or diverges, and determine the sum if it converges.	
Assessments, products,	1 quiz - Vocabulary: term, sequences, series, arithmetic, geometric, explicit, recursive, summation, infinite geometric,	

vocabulary, practice	converges, diverges
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	Name of Course: Pre-Calculus Massachusetts and National Standards: Link to Standard Additional Standards for specific subject Link to Syllabus
	Unit EIGHT: Limits
Essential Questions	How can a limit be evaluated graphically, algebraically, verbally and numerically to describe a function?
Standards	LIM-1.A: Represent limits analytically using correct notation. LIM-1.B: Interpret limits expressed in analytic notation. LIM-1.C: Estimate limits of functions. LIM-1.D: Determine the limits of functions using limit theorems. LIM-1.E: Determine the limits of functions using equivalent expressions for the function or the squeeze theorem. LIM-2.A: Justify conclusions about continuity at a point using the definition.

	LIM-2.B: Determine intervals over which a function is continuous. LIM-2.C: Determine values of x or solve for parameters that make discontinuous functions continuous, if possible. LIM-2.D: Interpret the behavior of functions using limits involving infinity. F-IF.C.7.d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	
Concepts and skills	<i>Students will know...</i> • Limit Notation • Definition of a limit ◦ One-Sided Limits • Evaluate a Limit Graphically • Evaluate a Limit with a Table • Evaluate a Limit Algebraically ◦ Substitution ◦ Factoring ◦ Rationalizing ◦ Reasoning ◦ Trigonometric Functions • Limit of the Difference Quotient • Rational Functions ◦ Discontinuities ◦ Multiplicity • Limits to and from Infinity • Analyzing Rational Functions ◦ Produce a graph from given information • Continuity • Intermediate Value Theorem • Extreme Value Theorem	<i>Mathematical Practices</i> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. ~~4. Model with mathematics.~~ 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Content Objectives "I can"	<i>I can...</i> • write a limit statement from a graph or table. • evaluate a limit statement from a given graph or table.	

Statements"	• evaluate a limit statement algebraically using a variety of different techniques. • apply algebraic techniques to find the limit of the difference quotient. • evaluate a limit involving infinity. • analyze the discontinuities and end behavior of a rational function. • graph a rational function. • state the 3 conditions for continuity at a point. • determine if a function is continuous at a point and justify. • determine if a function is continuous within its domain. • apply the Intermediate Value Theorem. • apply the Extreme Value Theorem.
Assessments, products, vocabulary, practice	• 3 quizzes • Vocabulary: limit, one-sided limit, Difference Quotient, Continuous
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	Unit NINE: Algebra Fun!
Essential Questions	F-BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. (Include linear, quadratic, exponential, absolute value, simple rational and

	radical, logarithmic and trigonometric functions.) Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. (Include recognizing even and odd functions from their graphs and algebraic expressions for them.) F-BF-B.4 Find inverse functions algebraically and graphically. F-BF-B.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). F-LE.A.3 Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, (or more generally) as a polynomial function. F-IF.C.7.e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F-IF.C.9 Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. F-IF.C.10 Given algebraic, numeric, and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, and trigonometric. F-IF.B.4 For each function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given verbal description of the relationship. Key features include: Intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums; symmetries; end behavior; and periodicity.	
Standards	F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	
Concepts and skills	<i>Students will know...</i> ● Solving Equations and Inequalities ○ Polynomial ○ Rational	<i>Mathematical Practices</i> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.

	○ Radical ○ Absolute Value ● Utilize a sign chart effectively ● Analyze a function ○ Increasing, decreasing, negative, positive, zeros, points of intersection and discontinuities ● Graphing Exponential Functions ○ End behavior described with limits ○ Domain, Range, Intercepts, Asymptotes ○ Including e as a base ● Solving Exponential Equations and Inequalities ● Graphing Logarithmic Functions ○ End behavior described with limits ○ Domain, Range, Intercepts, Asymptotes ○ Including natural log ● Properties of Logs ● Solving Logarithmic Equations ○ Given exponential ○ Given logarithmic ○ Domain restrictions	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Content Objectives "I can Statements"	<i>I can...</i> ● solve equations and inequalities involving polynomials, rationals, radicals and absolute value. ● create a sign chart to solve inequalities. ● describe the characteristics of a function. ● analyze and graph exponential and logarithmic functions. ● apply the properties of logarithms to simplify an expression. ● solve exponential and logarithmic equations and inequalities.	

Assessments, products, vocabulary, practice	• 2 quizzes • Vocabulary: Review of prior vocabulary
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