

Course Title: Mathematics	Full Year	Required
<p>Course Description: The mathematical work for Grade 4 is partitioned into 9 units:</p> <ul style="list-style-type: none"> ● Understanding Factors and Multiples ● Fraction Equivalence and Comparison ● Extending Operations to Fractions ● From Hundredths to Hundred-thousands ● Multiplicative Comparison and Measurement ● Multiplying and Dividing Multi-digit Numbers ● Angles and Angle Measurement ● Properties of Two-dimensional Shapes ● Putting it All Together 		
<p>Additional Course Information: The big ideas in Grade 4 include:</p> <ul style="list-style-type: none"> ● generalizing place value understanding for multi-digit whole numbers. ● using place value understanding and properties of operations to perform multi-digit arithmetic and solve problems. ● developing understanding and fluency with multi-digit multiplication ● developing understanding of dividing to find quotients involving multi-digit dividends ● building fractions from unit fractions by applying and extending previous understandings of operations with whole numbers. ● developing an understanding of fraction equivalence and ordering, as well as addition and subtraction of fractions with like denominators ● multiplication of fractions by whole numbers ● understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. ● Required fluency: Add and subtract within 1,000,000 (4.NBT.B.4) 	<p>Core Resources:</p> <p>Illustrative Mathematics</p> <p>Instructional Routines and Math Language Routines</p> <p>Grade 4 Glossary</p> <p>Grade 4 Unit 6 Glossary</p> <p>Required Materials</p> <p>IM en Español: Grade 4 en Español</p> <p>Developing a Mathematical Community</p>	<p>Are there any attachments <u>at the course level</u> that teachers will need?</p> <p>Grade 4 Scope and Sequence</p> <p>Pacing Guide and Dependency Diagrams K-5</p>

Unit 6: Multiplying and Dividing Multi-Digit Numbers

Duration: (26 - 27 Days)

Unit Overview - FOCUS:

In this unit, students extend their knowledge of multiplication and division to find products and quotients of multi-digit numbers.

Students begin by describing features of geometric and numerical patterns using ideas and language related to multiplication and multiplicative relationships (such as factors, multiples, double, and triple).

Next, students reason about products of multi-digit numbers. They transition from using diagrams to using algorithms to record partial products.

Students learn that they can multiply the factors by place value, one digit at a time, and then organize the partial products vertically. Here are two ways to show partial products for $3,419 \times 8$.

$$\begin{array}{r} 3,419 \\ \times 8 \\ \hline 72 \\ 80 \\ 3,200 \\ + 24,000 \\ \hline \end{array} \qquad \begin{array}{r} 3,419 \\ \times 8 \\ \hline 24,000 \\ 3,200 \\ 80 \\ + 72 \\ \hline \end{array}$$

Later, students divide dividends up to four-digit by single-digit divisors. Students see that it helps to decompose a dividend into smaller numbers and find partial quotients, just as it helped to decompose factors and find partial products.

They also recognize that sometimes it is most productive to decompose a dividend by place value. For instance, to find $465 \div 5$ we can divide each 400, 60, and 5 by 5.

$$\begin{array}{r} 400 \div 5 = 80 \\ 60 \div 5 = 12 \\ 5 \div 5 = 1 \\ \hline \end{array} \qquad \begin{array}{r} \boxed{93} \\ 1 \\ 12 \\ 80 \end{array}$$

- **Section A: Features of Patterns**
 - Generate a number or shape pattern that follows a given rule.
 - Identify apparent features of a number pattern that were not explicit in the rule itself.
- **Section B: Multi-digit Multiplication**
 - Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.
- **Section C: Multi-digit Division**
 - Divide numbers of up to four digits by one-digit divisors to find whole-number quotients and remainders, using strategies based on place value, properties

Students encounter various ways to record the division process, including an algorithm that records partial quotients in a vertical arrangement.

At the end of the unit, students apply their expanded knowledge of operations to solve multi-step problems about measurement in various contexts—calendar days, distance, and population.

of operations, and the relationship between multiplication and division.

- **Section D: Let’s Put It to Work: Problem Solving with Large Numbers**
 - Use the four operations to solve problems that involve multi-digit whole numbers and assess the reasonableness of answers.

Coherence: How does this unit build on and connect to prior knowledge and learning?

In grade 3, students learned that they could find the value of a product by decomposing one factor into smaller parts, finding partial products, and then combining them. To support this reasoning, they used base-ten diagrams (decomposing two-digit factors into tens and ones) and area diagrams (decomposing one side length into smaller numbers). Here, students use those understandings to multiply up to four digits by single-digit numbers, and to multiply a pair of two-digit numbers.

Essential Questions:

1. In what ways do patterns help me to make sense of mathematical concepts?
2. How do the different multiplication and division strategies connect to one another?
3. How can we find solutions to problems with large numbers?

Enduring Understanding:

1. **There are patterns in the world around us that demonstrate multiplication pictorially and numerically.** Studying visual, picture, and numerical patterns allows us to identify key features of the pattern and to predict unknown sequences.
2. **We can use many strategies when multiplying and dividing numbers, but all of these strategies are connected to our base-ten place value system.** Such strategies include partial products, partial quotients, and the standard algorithm. All of these strategies reinforce our place value system, and when utilized correctly, can provide accurate products and quotients. However, some strategies may be more efficient than others.

	<p>3. Adding, subtracting, multiplying, and dividing help us to find solutions. We may face a task in which we need to figure out an unknown. In some cases, we may need to use multiple operations in order to find missing information before determining a solution. We may need to use evidence to determine the reasonableness of a product or quotient. These problems can be approached in many ways, but we can choose our strategies and representations strategically.</p>	
<p>What Students Will Know: This should be based on the competencies.</p> <ul style="list-style-type: none"> ● We can use patterns to draw conclusions about the rule the pattern follows and the subsequent steps ● We can represent visual patterns numerically and by using operations and observing properties of numbers ● We can show our reasoning using words, numbers, expressions, and equations ● Side lengths, area, perimeter and other features of a rectangle change when the rectangle changes by a rule ● We can use our understanding of operations and place value to make sense of and explain patterns in multiples of numbers ● A larger array can be decomposed into smaller arrays because of the distributive property ● We can use doubling and tripling to find a product ● When factors are large, it may be inconvenient to use a specific strategy ● When rectangles no longer accurately represent area, the term “area diagrams” is not used. Instead, “rectangular diagrams” is used. 	<p>What students will do: This should be based on the competencies.</p> <ul style="list-style-type: none"> ● Analyze, describe, and extend visual patterns in which one or more shapes grow by a rule. ● Analyze, describe, extend, and generate visual patterns in which a series of symbols or shapes repeat by a rule, using structure and mathematical reasoning to do so. ● Analyze, describe, and extend numerical patterns that follow a rule. ● Analyze numerical patterns and use their understanding of place value and operations to find a rule and explain features of the pattern. ● Find the product of a one-digit number and a two-digit number in ways that make sense to them. ● Multiply a two-digit number and a one-digit number using place value understanding. ● Multiply a whole number of up to four digits by a one-digit number by decomposing factors by place value, finding partial products, and using properties of operations. ● Multiply 2 two-digit numbers. ● Multiply a multi-digit number by a one-digit number using an algorithm that uses partial products, while making connections between this algorithm, rectangular diagrams, and equations. ● Use partial products in an algorithm to multiply 2 two-digit numbers. ● Analyze the standard algorithm for multiplication 	<p>Unit Specific Vocabulary:</p> <p>Academic vocabulary</p> <p>Pattern Rule Relationship Feature Extend Numerical pattern Expression Increase Predict Observation</p> <p>Expression Multiplication Value Product Decompose Diagram Estimate Partitioning Represent Factor Equivalent Notation Partial product Algorithm Calculation</p>

<ul style="list-style-type: none"> ● The results of multiplying a part of one factor by the other factor can be called “partial products” ● We can use our place value reasoning to decompose multi-digit factors ● Decomposing factors into single-digit numbers or multiples of 10 can help us find products mentally ● An algorithm helps us to keep track of partial products that come from multiplying the digit of the factors ● An algorithm can represent base-ten diagrams and rectangular diagrams, but is more efficient for keeping track of and recording partial products ● We can apply our multiplication strategies that we have learned to solve various contextual problems involving measurement ● Division can be seen as a way to find the size of a group and as a way to find the number of groups ● We can use our knowledge of the relationship between multiplication and division to find an unknown factor ● We can use base-ten representations to help us reason about division ● We can exchange or decompose one or more units of a higher place value for 10 units of the next lower place value in order to have enough units to put into equal groups ● We can find the quotient by decomposing the dividend and finding the quotient for each decomposed part until all of the dividend is divided 	<p>and compare it to an algorithm that uses partial products they saw in earlier lessons.</p> <ul style="list-style-type: none"> ● Solve contextual problems that involve multiplication of a single-digit number and a whole number of up to four digits, or multiplication of 2 two-digit numbers. ● Solve division problems in context, and recall the two meanings of division: “how many in each group” and “how many groups.” ● Solve division problems that involve finding unknown factors by reasoning with partial quotients and by decomposing a dividend into familiar multiples of the divisor. ● Use partial quotients to solve division problems that involve tiling squares and finding a side length of a rectangle with a known area. ● Make sense of base-ten representations for division. ● Find the quotients of two-digit and three-digit dividends and one-digit divisors by decomposing the dividend by place value—decomposing a larger unit to 10 of a smaller unit—and by reasoning in terms of equal-size groups. ● Introduce students to ways to record partial quotients when dividing multi-digit numbers. ● Use an algorithm that uses partial quotients to find whole number quotients and remainders with up to four-digit dividends and one-digit divisors and analyze some common errors when using an algorithm that uses partial quotients ● Represent and solve contextual problems that involve dividing a whole number of up to four-digits by a single-digit divisor, resulting in a number with or without a remainder and interpret the result and remainder given a situation. ● Represent and solve multi-step contextual problems involving multiplication and division, including 	<p>Standard Algorithm Strategies Situation Measurement Solution</p> <p>Estimate Dividend Mathematical question Situation Expression Remainder Division Value Strategy Equation Multiple Factor Length Partial quotient Relationship Base-ten diagram Representation Mental math Diagram Decompose Method Algorithm Pattern Calculations Efficient</p> <p>Expression Strategy Decompose Partition</p>
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<ul style="list-style-type: none"> ● We can use our knowledge of partial products to find the quotient ● There are various strategies that we can use to divide multi-digit numbers by single-digit divisors ● We can use our knowledge of multiplication and division, including the ideas of factors and multiples, to represent situations ● We can use multiplication and division to convert units of measurements ● Sometimes more than one step is needed to solve a problem and we may need to use more than one operation ● We can use our knowledge of addition, subtraction, multiplication, and division to analyze data sets 	<p>division with remainders.</p> <ul style="list-style-type: none"> ● Apply what they know about multiplication and division to convert units of measurement and solve multi-step problems involving perimeter and area. ● Use the four operations to solve problems involving multi digit numbers and use the standard algorithm for addition and subtraction to solve problems. ● Solve multi-step word problems by analyzing data, estimating, reasoning, and performing multiple operations. ● Create and analyze patterns in a real-world context and to solve multi-step problems. 	<p>Situation Visualize Operation Measurements Statement Equation Algorithm Question Reasonable Predict Pros Cons Standard algorithm Suggestion Clearer Revise Multi-step problem</p>
<p>Entry Level Assessment and Connection to Unit:</p> <p>Grade 4 Unit 6 Entry Level Assessment</p> <p>Task: Provide each student with the pre-unit assessment document. These questions should be completed independently by students in order to understand what prerequisite concepts and skills students have mastered and which concepts and skills students need additional support with. .</p> <p>Purpose: These problems address prerequisite concepts and skills for the unit. Teachers can use these problems to identify unfinished learning that can be carefully addressed during the unit.</p>	<p>Unit Materials, Resources and Technology:</p> <ul style="list-style-type: none"> ● Illustrative Mathematics: ● Instructional Routines and Math Language Routines ● Grade 4 Glossary ● Grade 4 Unit 6 Glossary ● Required Materials ● IM en Español: ● Grade 4 en Español) ● Developing a Mathematical Community 	

Opportunities for Interdisciplinary Connections:

Much of this unit involves students analyzing patterns (both visually and numerically). Consider what patterns you may alert students to around the classroom as well as in other content areas. Such patterns can include:

- Patterns on a clock
- Patterns within a calendar
- Desk patterns
- Color patterns around the room
- Patterns found in nature

Any links, attachments and resources:

[Instructional Routines Document](#)

[Family Support Materials](#)

Planning Ideas:

[Components of a Typical IM Lesson](#)

[What To Know About IM When Planning](#)

[Gr 4 Where to Find the Mathematical Practices in the Units](#)

[Assessing the Mathematical Practices](#)

Topic # 1 (Section A)	Topic Name: Section A: Features of Patterns	Duration: Recommended: 4 days (4 lessons)
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Topic Description:

Section A Learning Goals

- Generate a number or shape pattern that follows a given rule.
- Identify apparent features of a number pattern that were not explicit in the rule itself.

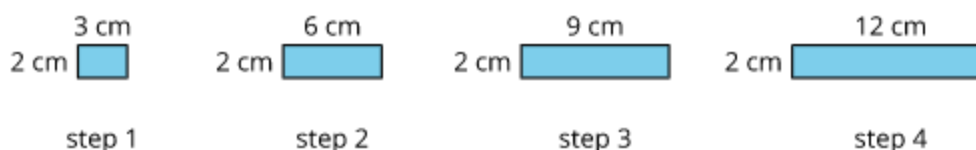
In this section, students observe and describe features of geometric and numerical patterns. Given the rule of a pattern, they predict the values or features of future terms in a pattern sequence. To make predictions, students use their understanding of operations and place value.

The section begins with patterns that are more concrete—such as shapes with features that change quantitatively and thus elicit addition or multiplication. It then moves toward patterns with repeating objects or numbers, which require students to reason more abstractly.

Later, students explore patterns in the features of rectangles—side length, perimeter, and area—that change by a rule. Along the way, students apply their knowledge of factors and multiples.

If the pattern continues, could 50 represent the longer side length or the area of one of the rectangles?

If so, which step? If not, why not?



Competencies Addressed:

Operations and Algebraic Thinking

1. I can generate, analyze, and explain patterns. **4.OA.C.5**

Essential Question and Enduring Understanding Addressed in this Topic:

Essential Question

In what ways do patterns help me to make sense of mathematical concepts?

	<p>Enduring Understanding There are patterns in the world around us that demonstrate multiplication pictorially and numerically. Studying visual, picture, and numerical patterns allows us to identify key features of the pattern and to predict unknown sequences.</p>
<p>In this Topic, students will know:</p> <ul style="list-style-type: none"> ● We can use patterns to draw conclusions about the rule the pattern follows and the subsequent steps ● We can represent visual patterns numerically and by using operations and observing properties of numbers ● We can show our reasoning using words, numbers, expressions, and equations ● Side lengths, area, perimeter and other features of a rectangle change when the rectangle changes by a rule ● We can use our understanding of operations and place value to make sense of and explain patterns in multiples of numbers 	<p>Topic Vocabulary:</p> <p>Academic vocabulary</p> <p>Pattern Rule Relationship Feature Extend Numerical pattern Expression Increase Predict Observation</p>
<p>In this Topic, students will be able to:</p> <ul style="list-style-type: none"> ● Analyze, describe, and extend visual patterns in which one or more shapes grow by a rule. ● Analyze, describe, extend, and generate visual patterns in which a series of symbols or shapes repeat by a rule, using structure and mathematical reasoning to do so. ● Analyze, describe, and extend numerical patterns that follow a rule. ● Analyze numerical patterns and use their understanding of place value and operations 	<p>Plan for Student Reflection:</p> <p>Student Journal Prompts and Reflection Practices</p> <hr/> <p>Plan for Teacher Reflection:</p>

to find a rule and explain features of the pattern.

Teacher Journal Reflection Questions:

Lesson 1: What prompts or structures might better enable unheard students to share their voices and reasoning?

Lesson 2: What do you notice in students' work from today's lesson that you might leverage in that future lesson about multiplying multi-digit numbers?

Lesson 3: Make a note of students whose ideas have not been shared and look for an opportunity for them to share their thinking in tomorrow's lesson.


Lesson 4: What questions did you ask to help make the students' connections more visible?

Utilize additional strategies for Teacher Reflection:

- Reviewing formative assessments
- Developing scaffolds
- Collaborative scoring
- PLCs
- Planning for small groups

Topic 1 (Section A) Task Development

Each Topic has its own Task that serves as a roadmap for instruction during the unit. The task follows the [Learning Cycle Model](#) that drives teaching and learning in Naugatuck Public Schools.

Task Title: Topic 1 - Features of Patterns	Grade Level and Unit: Fourth Grade, Unit 6
Description of Task: In this optional activity, students investigate patterns in multiples of 15 and analyze and describe features of the digits in the tens and ones place. The activity also prompts them to consider why those features exist and to predict whether a given number could be a multiple of 15.	Purpose of Task: The goal here is not to elicit clear justifications, but rather to encourage students to use their understanding of place value and numbers in base-ten to reason more generally about patterns in numerical patterns.
Background of Students/Learning Progression: In grade 3, students identified patterns in numbers and used mathematical operations to explain them. In these lessons, students continue the work of analyzing and describing shape and number patterns, looking for features of patterns that are not apparent in the rule.	Ensure all competencies are addressed in the task: <input type="checkbox"/> Yes, all competencies are addressed <input type="checkbox"/> No - Task needs modification
Getting Started: Display the following image to students: 	
Explain to students, "These are solar panels. They use the sun's light or heat to create electricity."	

Ask students, “What pattern(s) do you see in the solar panels?”

Have students share their answers with the whole class. Record student responses.

Ask students, “Now that you have noticed some patterns in the solar panels, do you think a solar panel array would be more likely to have 50, 55, or 72 panels? Why?”

Provide students with 1-2 minutes of quiet think time. Have students share their thinking with a partner for 2-3 minutes. Ask students to share their thoughts with the entire class. Record responses as students share.

Learning Cycle Unit 6 Section A

IM Lesson	L1: Patterns that Grow	L2: 2: Patterns that Repeat	L3: From Visual Patterns to Numerical Patterns	L4: Numerical Patterns
Learning Cycle Model	Making Meaning	Making Meaning	Investigation	Create & Produce
Naugatuck Math Competency	4.OA.1	4.OA.1	4.OA.1	4.OA.1
Math Practice Standards	MP7	–	MP6	MP 6, 7, 8
Lesson Purpose	The purpose of this lesson is for students to analyze, describe, and extend visual patterns in which one or more shapes grow by a rule.	The purpose of this lesson is for students to analyze, describe, extend, and generate visual patterns in which a series of symbols or shapes repeat by a rule, using structure and mathematical reasoning to do so.	The purpose of this lesson is for students to analyze, describe, and extend numerical patterns that follow a rule.	The purpose of this lesson is for students to analyze numerical patterns and use their understanding of place value and operations to find a rule and explain features of the pattern.
Vocabulary Focus	Pattern, rule, relationship	Pattern, features, rule, extend, numerical pattern	Expression, pattern, increase, feature, predict	Feature, pattern, observation, extend
Lesson Materials/ Resources	Lesson 1 Slides Teacher Presentation Materials Student Pages Activity 2:	Lesson 2 Slides Teacher Presentation Materials Student Pages No additional materials needed	Lesson 3 Slides Teacher Presentation Materials Student Pages For Activity 1:	Lesson 4 Slides Teacher Presentation Materials Student Pages No additional materials needed

	<ul style="list-style-type: none"> Consider preparing a set of pattern blocks for building the first two or three steps of the giraffe pattern. The set should include 6 hexagons, 6 triangles, 3 trapezoids, and 24 squares <p>Lesson 1 Cool-Down Andre's House Pattern</p>	<p>Lesson 2 Cool-Down Happy Faces</p>	<ul style="list-style-type: none"> Provide students access to graph paper, in case requested <p>Lesson 3 Cool-Down Another Set of Rectangles</p>	<p>Lesson 4 Cool-Down Count by 8</p>
Assessment	Formative Assessment Strategies: observation, questioning, student discourse: Monitoring Sheet See Section A Checkpoint Assessment , Section A Checkpoint Teacher's Guide			
				Section A Practice Problems
Centers	<p>Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)</p> <p>Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)</p>	<p>Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)</p> <p>Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)</p>	<p>Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)</p> <p>Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)</p>	<p>Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)</p> <p>Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)</p>

Making Meaning:

In these lessons, students continue the work of analyzing and describing shape and number patterns, looking for features of patterns that are not apparent in the rule.

The patterns in Lesson 1 consist of shapes that use an increasing number of objects in each step. Students describe not only the rule of the pattern (that is, how the number of objects is changing), but also any features of the patterns that are not explicit in the rule. They also extend patterns and make predictions by looking for and making use of structure (MP7), rather than by drawing or writing out each step along the way.

In Lesson 2, students analyze and describe features of patterns with designs with shapes that repeat according to a rule. Students begin by examining the patterns visually. They look for structure and make use of it to extend the patterns (MP7). Later, they represent each shape in the

pattern with numbers and reason about the repetition mathematically—by using operations and observing the properties of the numbers (MP2). The third activity is optional as it provides an opportunity for extra practice.

Lesson 1: Patterns that Grow

- The purpose of this lesson is for students to analyze, describe, and extend visual patterns in which one or more shapes grow by a rule.
- [Teacher Presentation Materials](#)
- [Lesson 1 Slides](#)

Lesson 2: Patterns that Repeat

- The purpose of this lesson is for students to analyze, describe, extend, and generate visual patterns in which a series of symbols or shapes repeat by a rule, using structure and mathematical reasoning to do so.
- [Teacher Presentation Materials](#)
- [Lesson 2 Slides](#)

Investigation:

Previously, students explored growing and repeating patterns and reasoned about the patterns using words, numbers, and operations. In this lesson, students investigate patterns in a geometric context and explore how the side lengths, area, perimeter, and other features of a rectangle change when the rectangle changes by a rule. In doing so, students practice looking for and making use of structure (MP7).

Students also practice reasoning quantitatively and abstractly (MP2) as they interpret the values in number sequences that represent geometric features of rectangles, and vice versa. (For example, 6, 8, 10, 12, . . . may represent the area, in square centimeters, of a series of rectangles whose width is 2 centimeters and whose length grows by 1 centimeter each time.)

The second activity in this lesson is optional as it allows students more time to work with the ideas from the first activity.

Lesson 3: From Visual Patterns to Numerical Patterns

- The purpose of this lesson is for students to analyze, describe, and extend numerical patterns that follow a rule.
- [Teacher Presentation Materials](#)
- [Lesson 3 Slides](#)

Create and Produce:

Previously, students examined numerical patterns alongside visual patterns (diagrams of pattern blocks, arrangements of shapes, attributes of

rectangles, and so on). In this lesson, they focus solely on numerical patterns, without a visual representation. Students use their understanding of operations and place value to make sense of and explain patterns in multiples of numbers. Along the way, students have multiple opportunities to look for and make use of structure and regularity (MP7, MP8) to solve problems.

The reasoning in this lesson helps to transition students to the work in the next section, in which students explore strategies for multiplying single-digit numbers and multi-digit numbers up to four digits, and for multiplying 2 two-digit numbers. The third activity is optional as it provides an opportunity for extra practice.

Lesson 4: Numerical Patterns

- The purpose of this lesson is for students to analyze numerical patterns and use their understanding of place value and operations to find a rule and explain features of the pattern.
- [Teacher Presentation Materials](#)
- [Lesson 4 Slides](#)

Communicate and Present:

Invite students to share the patterns they noticed and their explanations for the patterns. Record them for all to see.

- Select other students to share their explanation on whether 250 could be a number that Elena calls. Highlight explanations that make use of the structure in the numbers.

Reflection:

“Today we saw different features of patterns in the numbers that we get when counting by 9, 10, 99, and 100.” (Include 15, if students completed the optional activity).

“What new ideas did you have about patterns in this section?”

“What are you still wondering about patterns?”

- [List of terms to cut and paste](#)
- [IM Reflection Practices](#)

Notes:

Complete File with Resources and Task:

Task-Based Learning Plan Format for Topic 1

Topic # 2 (Section B)	Topic Name: Section B: Multi-digit Multiplication	Duration: Recommended: 8 days (8 lessons)
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Topic Description:

Section B Learning Goals

- Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.

In this section, students use their knowledge of multiplication, place value, and area of rectangles to multiply one-digit numbers and numbers up to four digits, and to multiply pairs of two-digit numbers.

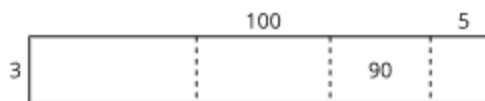
A key thread here is the idea of decomposing factors—particularly by place value—as a productive way of finding products. Students explore this idea with concrete and visual representations: arrays, base-ten diagrams, and rectangles with grids. As they decompose larger factors, they see the limits of these representations, motivating more efficient representations and strategies.

In grade 3, students saw that rectangles can help us reason about multiplication—the side lengths of a rectangle can represent the two factors and its area can represent the product. As the factors become larger (for instance, $3 \times 2,135$), it becomes necessary to draw rectangles whose side lengths are not to scale. When rectangles no longer accurately represent area, the term “area diagrams” is not used. Instead, “rectangular diagrams” is used in teacher materials and “diagrams” in student materials.

Students use such diagrams as a visual tool to decompose factors by place value and to organize partial products.

Lin drew a diagram to represent

$$3 \times 2,135.$$

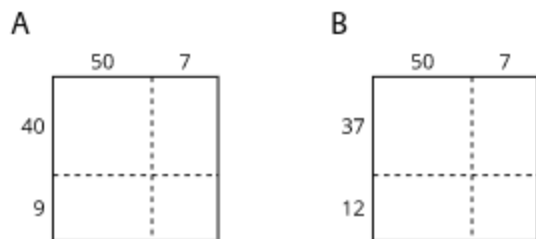


Complete the diagram.

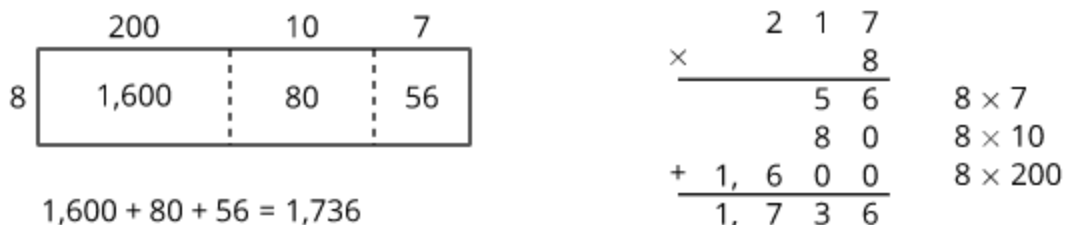
Use it to find the value of $3 \times 2,135$.

The benefits of decomposing factors by place value become more apparent as students multiply pairs of two-digit numbers.

They consider, for example, why diagram A may be more helpful than diagram B for finding the value of 49×57 .



Later, students encounter an algorithm that uses partial products, a different way to record the reasoning they used with diagrams. They learn that the partial products can be listed vertically, instead of inside the boxes of a rectangular diagram.



Students use this algorithm to multiply two-digit numbers, likewise connecting the partial products to the values in a corresponding diagram.

Algorithms that use partial products prepare students to make sense of the standard algorithm for multiplication, which students preview in this unit but will study closely in grade 5.

Competencies Addressed:

Measurement and Data Investigations

1. I can solve problems involving measurement (time, money, customary and metric, are, perimeter, and conversion of measurements). **4.MD.A.2**

Understanding and Applying Number Systems

3. I can add and subtract whole numbers. **4.NBT.B.4**

Essential Question and Enduring Understanding Addressed in this Topic:

Essential Question

How do the different multiplication and division strategies connect to one another?

<p>4. I can use strategies to multiply and divide whole numbers. 4.NBT.B.5</p> <p>Operations and Algebraic Thinking</p> <p>2. I can use the four operations to solve multi-step problems. 4.OA.A.3</p>	<p>Enduring Understanding</p> <p>We can use many strategies when multiplying and dividing numbers, but all of these strategies are connected to our base-ten place value system. Such strategies include partial products, partial quotients, and the standard algorithm. All of these strategies reinforce our place value system, and when utilized correctly, can provide accurate products and quotients. However, some strategies may be more efficient than others.</p>
<p>In this Topic, students will know:</p> <ul style="list-style-type: none"> ● A larger array can be decomposed into smaller arrays because of the distributive property ● We can use doubling and tripling to find a product ● When factors are large, it may be inconvenient to use a specific strategy ● When rectangles no longer accurately represent area, the term “area diagrams” is not used. Instead, “rectangular diagrams” is used. ● The results of multiplying a part of one factor by the other factor can be called “partial products” ● We can use our place value reasoning to decompose multi-digit factors ● Decomposing factors into single-digit numbers or multiples of 10 can help us find products mentally ● An algorithm helps us to keep track of partial products that come from multiplying the digit of the factors ● An algorithm can represent base-ten diagrams and rectangular diagrams, but is more efficient for keeping track of and recording partial products ● We can apply our multiplication strategies that we have learned to solve various contextual problems involving measurement 	<p>Topic Vocabulary:</p> <p>Academic vocabulary</p> <p>Expression Multiplication Value Product Decompose Diagram Estimate Partitioning Represent Factor Equivalent Notation Partial product Algorithm Calculation Standard Algorithm Strategies Situation Measurement Solution</p>

In this Topic, students will be able to:

Find the product of a one-digit number and a two-digit number in ways that make sense to them.

- Multiply a two-digit number and a one-digit number using place value understanding.
- Multiply a whole number of up to four digits by a one-digit number by decomposing factors by place value, finding partial products, and using properties of operations.
- Multiply 2 two-digit numbers.
- Multiply a multi-digit number by a one-digit number using an algorithm that uses partial products, while making connections between this algorithm, rectangular diagrams, and equations.
- Use partial products in an algorithm to multiply 2 two-digit numbers.
- Analyze the standard algorithm for multiplication and compare it to an algorithm that uses partial products they saw in earlier lessons.
- Solve contextual problems that involve multiplication of a single-digit number and a whole number of up to four digits, or multiplication of 2 two-digit numbers.

Plan for Student Reflection:

[Student Journal Prompts and Reflection Practices](#)

Plan for Teacher Reflection:

- Reviewing formative assessments
- Developing scaffolds
- Collaborative scoring
- PLCs
- Planning for small groups

-What was the best question you asked students today?

-What strategy did most students use in their work today? Which did you anticipate?

-How can you leverage each of your student's ideas to support them in being seen and heard in tomorrow's math class?

-In a future lesson, students will be analyzing partial products from rectangular diagrams and making connections to the traditional algorithm notation. How do rectangular diagrams support this thinking?

-What part of the lesson went really well today in terms of students learning?

-How did understanding the cool-down of the lesson before you started teaching today help you synthesize that learning?

-How does the way they analyzed two different algorithms build toward the use of standard algorithm in fifth grade?

-How did you ensure all students' voices were heard and valued as an important part of the collective learning?

Topic 2 (Section B) Task Development

Each Topic has its own Task that serves as a roadmap for instruction during the unit. The task follows the [Learning Cycle Model](#) that drives teaching and learning in Naugatuck Public Schools.

Task Title: Topic 2 - Multi-digit Multiplication	Grade Level and Unit: Fourth Grade, Unit 6
Description of Task: In this activity, students use what they learned about multiplication of multi-digit numbers and unit conversion to solve problems involving measurements. Students may choose to represent the situations in a number of ways—concretely or visually (by drawing diagrams) or abstractly (by writing expressions and equations). While some problems can be reasoned additively, students may opt to reason multiplicatively for practical reasons.	Purpose of Task: Regardless of their chosen representations and reasoning strategy, students reason quantitatively and abstractly when they interpret and solve the questions about different units of time (MP2).
Background of Students/Learning Progression: In grade 3, students learned about multiplication and learned to find products within 100. Earlier in this course, students identified factors and multiples, performed multiplicative comparison with whole numbers and fractions, and used the structure of base-ten numbers and properties of operations to find multiples of 10, 100, 1,000, and so on.	Ensure all competencies are addressed in the task: <input type="checkbox"/> Yes, all competencies are addressed <input type="checkbox"/> No - Task needs modification
Getting Started: Display one expression at a time. Ask students, “Give me a signal when you have an answer and can explain how you got it.” Provide one minute of quiet think time. As students share their thinking, record their answers and strategies. Keep their expressions and work displayed. Repeat with each expression. 8×30 5×30 10×30 15×30 After, synthesize the activity by asking the following question: how did the first three expressions help you to find the value of 15×30 ?	

Learning Cycle Unit 6

Section B

IM Lesson	L5:Products Beyond 100	L6: Multiply Two-digit Numbers and One-digit Numbers	L7: Multiply Three- and Four-digit Numbers by One-digit Numbers	L8: Multiply 2 Two-digit Numbers	L9:Recording Partial Products: One-digit and Three- or Four-digit Factors	L10: Using Algorithms with Partial Products: 2 Two-digit Numbers	L11: Partial Products and the Standard Algorithm	L12: Solve Problems Involving Multiplication
Learning Cycle Model	Making Meaning	Making Meaning	Making Meaning	Making Meaning	Investigation	Investigation	Investigation	Create & Produce
Naugatuck Math Competency	4.NBT.B.5	4.NBT.B.5	4.NBT.B.5	4.NBT.B.4, 4.NBT.B.5	4.NBT.B.5	4.NBT.B.5	4.NBT.B.5	4.MD.A.2, 4.NBT.B.5, 4.OA.A.3
Math Practice Standards	MP7	–	MP3	MP7	MP3	–	MP3	MP1, 2
Lesson Purpose	The purpose of this lesson is for students to find the product of a one-digit number and a two-digit number in ways that make sense to them.	The purpose of this lesson is for students to multiply a two-digit number and a one-digit number using place value understanding.	The purpose of this lesson is for students to multiply a whole number of up to four digits by a one-digit number by decomposing factors by place value, finding partial products, and using properties of operations.	The purpose of this lesson is for students to multiply 2 two-digit numbers.	The purpose of this lesson is for students to multiply a multi-digit number by a one-digit number using an algorithm that uses partial products. Students make connections between this algorithm, rectangular diagrams, and equations.	The purpose of this lesson is for students to use partial products in an algorithm to multiply 2 two-digit numbers.	The purpose of this lesson is for students to analyze the standard algorithm for multiplication and compare it to an algorithm that uses partial products they saw in earlier lessons.	The purpose of this lesson is for students to solve contextual problems that involve multiplication of a single-digit number and a whole number of up to four digits, or multiplication of 2 two-digit numbers.
Vocabulary Focus	Expression, multiplication, value, product, decompose	Value, diagram, product, expression, multiplication	Estimate, diagram, expression, value,	Expression, value, diagram, product, partitioning,	Expression, equivalent, diagram, representation,	Expression, algorithm, value, diagram, calculations, partial products	Expression, algorithm, value, product, standard	Strategies, situation, value, measurement, solution

			represent, representation	decompose, factor	notation, partial products		algorithm, partial product	
Lesson Materials/ Resources	Lesson 5 Slides	Lesson 6 Slides	Lesson 7 Slides	Lesson 8 Slides	Lesson 9 Slides	Lesson 10 Slides	Lesson 11 Slides	Lesson 12 Slides
	Teacher Presentation Materials	Teacher Presentation Materials	Teacher Presentation Materials	Teacher Presentation Materials	Teacher Presentation Materials	Teacher Presentation Materials	Teacher Presentation Materials	Teacher Presentation Materials
	Student Pages	Student Pages	Student Pages	Student Pages	Student Pages	Student Pages	Student Pages	Student Pages
	Activity 1: For each group of 2, give each group tools for creating a visual display (such as chart paper, markers, whiteboard space and markers, shared online drawing tool, or access to a document camera)	No additional materials needed	No additional materials needed	No additional materials needed	No additional materials needed	No additional materials needed	No additional materials needed	Activity 1: For each group of 2, give each group tools for creating a visual display (such as chart paper, markers, whiteboard space and markers, shared online drawing tool, or access to a document camera)
	For Activity 2: <ul style="list-style-type: none"> • Create 4 posters showing the 4 representations shown in the activity narrative 							
	Lesson 5 Cool-down: Rows of Seats	Lesson 6 Cool-down: Represent the Product	Lesson 7 Cool-down: The Value of the Product	Lesson 8 Cool-down: What's the Product?	Lesson 9 Cool-down: Partial Products	Lesson 10 Cool-down: Choose Your Own Strategy	Lesson 11 Cool-down: Choose a Way to Multiply	Lesson 12 Cool-down: Represent Multiplicative Comparison

Assessment	Formative Assessment Strategies; observation, questioning, student discourse: Monitoring Sheet See Section B Checkpoint Assessment , Section B Checkpoint Teacher's Guide							
								Section B Practice Problems
Centers	Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting) Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)	Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting) Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)	Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting)	Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting)	Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting)	Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting)	Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting)	Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing) Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Making Meaning:

Lesson 5 is the first in a series focused on finding whole-number products beyond 100. Here, students reason about equal-group situations involving one-digit and two-digit numbers in any way that makes sense to them. In the first activity, students work with an array of objects to build on a familiar representation. In the second activity, no visual representation is provided. Students may find products by creating arrays or diagrams, decomposing a factor into smaller numbers or place value, and using their understanding of properties of operations.

In Lesson 6, students extend the ideas of Lesson 5 to find the value of products beyond 100, focusing on representations and strategies based on place value and the properties of operations, which are familiar from grade 3.

Students analyze base-ten diagrams and diagrams that involve rectangles, some of which are partitioned by place value. They explain how the diagrams represent multiplication and make connections between them, deepening their understanding of place value and properties of operations. At the end of the lesson, students consider a rectangular diagram that will be used through the rest of the section.

In Lesson 7, students use rectangular diagrams and expressions to multiply up to four-digit numbers by one-digit numbers. They continue to use place value reasoning to decompose the multi-digit factor and to use partial products in their computation.

Students should have multiple opportunities to hear the term “partial products” as referring to the results of multiplying a part of one factor and the other factor (or a part of one factor and a part of the other factor).

In Lesson 8, students apply the idea of place value to decompose a factor to multiply 2 two-digit numbers. They reason about why it is helpful to decompose both two-digit numbers by place value. As students analyze the connections between expressions and diagrams, they recognize that partial products in which the factors are either single-digit numbers or multiples of 10 can be found mentally, making the rectangular diagram a useful tool for multiplying two-digit numbers.

[Lesson 5: Products Beyond 100](#)

- The purpose of this lesson is for students to find the product of a one-digit number and a two-digit number in ways that make sense to them.
- [Teacher Presentation Materials](#)
- [Lesson 5 Slides](#)

[Lesson 6: Multiply Two-digit Numbers and One-digit Numbers](#)

- The purpose of this lesson is for students to multiply a two-digit number and a one-digit number using place value understanding.
- [Teacher Presentation Materials](#)
- [Lesson 6 Slides](#)

[Lesson 7: Multiply Three- and Four-digit Numbers by One-digit Numbers](#)

- The purpose of this lesson is for students to multiply a whole number of up to four digits by a one-digit number by decomposing factors by place value, finding partial products, and using properties of operations.
- [Teacher Presentation Materials](#)
- [Lesson 7 Slides](#)

[Lesson 8: Multiply 2 Two-digit Numbers](#)

- The purpose of this lesson is for students to multiply 2 two-digit numbers.
- [Teacher Presentation Materials](#)

- [Lesson 8 Slides](#)

Investigation:

In previous lessons, students used diagrams to represent multiplication of a one-digit number and a whole number of up to four digits. They learned to decompose larger factors by place value and used diagrams and corresponding expressions to support them in finding partial products. In Lesson 9, students learn an algorithm for keeping track of partial products that come from multiplying the digits of the factors. This algorithm that uses partial products lays the foundation for the standard algorithm for multiplication.

Students engage in quantitative and abstract reasoning (MP2) as they relate the partial products in a diagram and in an algorithm. Because this lesson offers an initial exposure to the new notation, students are not required to use an algorithm that uses partial products to multiply. They can rely on other methods they have learned so far.

In Lesson 10, students apply the work of Lesson 9 to 2 two-digit factors.

Lesson 11 extends students' analysis to include the standard algorithm for multiplication of multi-digit numbers. In grade 4, the standards focus on understanding place value and how it is represented in different methods for finding products. The work here serves to build the groundwork for making sense of the standard algorithm in grade 5, so students are not expected to use the standard algorithm at this time.

[Lesson 9: Recording Partial Products: One-digit and Three- or Four-digit Factors](#)

- The purpose of this lesson is for students to multiply a multi-digit number by a one-digit number using an algorithm that uses partial products. Students make connections between this algorithm, rectangular diagrams, and equations.
- [Teacher Presentation Materials](#)
- [Lesson 9 Slides](#)

[Lesson 10: Using Algorithms with Partial Products: 2 Two-digit Numbers](#)

- The purpose of this lesson is for students to use partial products in an algorithm to multiply 2 two-digit numbers.
- [Teacher Presentation Materials](#)
- [Lesson 10 Slides](#)

[Lesson 11: Partial Products and the Standard Algorithm](#)

- The purpose of this lesson is for students to analyze the standard algorithm for multiplication and compare it to an algorithm that uses partial products they saw in earlier lessons.
- [Teacher Presentation Materials](#)
- [Lesson 11 Slides](#)

Create and Produce:

This lesson gives students the opportunity to apply the multiplication strategies they have learned to solve various contextual problems involving measurement. The problems vary in format and complexity—some involve a single computation and others require multiple steps to solve. The work here prompts students to make sense of problems and persevere in solving them (MP1) and to reason quantitatively and abstractly (MP2).

[Lesson 12: Solve Problems involving Multiplication](#)

- The purpose of this lesson is for students to solve contextual problems that involve multiplication of a single-digit number and a whole number of up to four digits, or multiplication of 2 two-digit numbers.
- [Teacher Presentation Materials](#)
- [Lesson 12 Slides](#)

Communicate and Present:

- “Most of the problems can be solved by multiplication. What is the same between the solutions and multiplication strategies that you saw? What’s different?”
- 30 seconds quiet think time
- 1 minute: partner discussion
- Record students’ responses, or display students’ diagrams and representations.
- “What are some common strategies for multiplying a multi-digit number by a one-digit number (48 and 7, or 366 and 3)?”
- “What are some strategies you saw for multiplying 2 two-digit numbers (12 and 31)?”

Reflection:

“Today we used what we have learned about multiplication to solve problems involving measurement.”

Select students to share their responses to the problems in the last activity. As each student shares, ask if others in the class solved it the same way and if they approached it differently.

Prompt students to explain what their numerical solutions represent in each situation.

“How would you know if your solutions were correct?”
(Sample responses: I used another strategy to see if I got the same answer. I estimated first so that I had an idea

	<p>how big or small the answer would be. I checked with my groupmates.)</p> <p>Consider asking: “When you had to multiply numbers, which method did you rely on the most? What made you choose that method?”</p> <ul style="list-style-type: none">• IM Reflection Practices
Notes:	Complete File with Resources and Task: Task-Based Learning Plan for Topic 2

Topic # 3 (Section C)	Topic Name: Section C - Multi-digit Division	Duration: Recommended: 8 days (8 lessons)
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Topic Description:

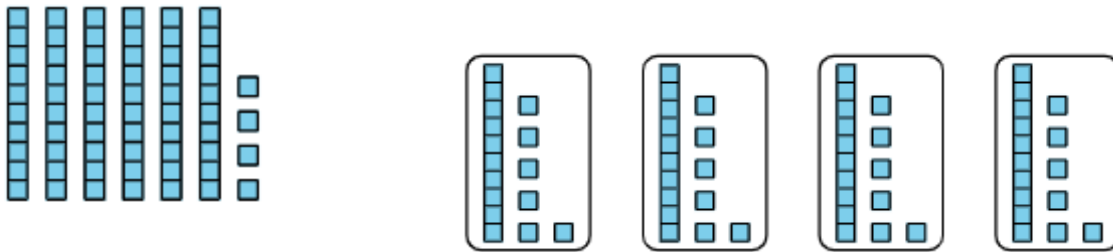
Section C Learning Goals

- Divide numbers of up to four digits by one-digit divisors to find whole-number quotients and remainders, using strategies based on place value, properties of operations, and the relationship between multiplication and division.

In grade 3, students made sense of division in relation to multiplication and equal-size groups. They reasoned about division problems in context and found whole-number quotients from two-digit dividends and one-digit divisors. Here, students find quotients from larger dividends (up to four digits), investigate new division strategies and ways to represent them, and interpret division situations that involve remainders.

Students begin by solving problems in various situations, including those about equal-size groups, factors and multiples, and area of rectangles. These experiences reinforce students' understanding of the relationship between multiplication and division. They also build students' intuition for the kinds of situations that involve division (including those where a remainder may be involved), before focusing on finding the value of quotients.

Students first reason about division problems in any way that makes sense to them, and later use base-ten representations. They recall that to find the value of $64 \div 4$, for instance, they could first put 4 tens and 4 ones into 4 groups (1 ten and 1 one in each group), and then decompose the remaining 2 tens into 20 ones and put 5 ones in each group.



Students see that, just as they can distribute blocks of tens and ones into groups incrementally, they can decompose a dividend into parts and find partial quotients.

While there is not a single way to decompose a dividend, doing so by place value is often helpful, as was the case when finding partial products.

Students learn to use a series of equations and a vertical recording method to organize partial quotients.

$$\begin{array}{r} 720 \div 9 = 80 \\ 18 \div 9 = 2 \\ \hline 738 \div 9 = 82 \end{array}$$
$$\begin{array}{r} \boxed{82} \\ 2 \\ 80 \\ 9 \overline{)738} \\ - 720 \quad 9 \times 80 \\ \hline 18 \\ - 18 \quad 9 \times 2 \\ \hline 0 \end{array}$$

Later in the section, students take a closer look at division problems that do not have whole-number quotients and interpret the remainders in the context of the problem.

Competencies Addressed:

Measurement and Data Investigations

1. I can solve problems involving measurement (time, money, customary and metric, are, perimeter, and conversion of measurements). **4.MD.A.3**

Understanding and Applying Number Systems

4. I can use strategies to multiply and divide whole numbers. **4.NBT.B6**

Operations and Algebraic Thinking

2. I can use the four operations to solve multi-step problems. **4.OA.A.3**

1. I can generate, analyze, and explain patterns. **4.OA.B.4**

Essential Question and Enduring Understanding Addressed in this Topic:

Essential Question

How do the different multiplication and division strategies connect to one another?

Enduring Understanding

We can use many strategies when multiplying and dividing numbers, but all of these strategies are connected to our base-ten place value system. Such strategies include partial products, partial quotients, and the standard algorithm. All of these strategies reinforce our place value system, and when utilized correctly, can provide accurate products and quotients. However, some strategies may be more efficient

	than others.
<p>In this Topic, students will know:</p> <ul style="list-style-type: none"> • Division can be seen as a way to find the size of a group and as a way to find the number of groups • We can use our knowledge of the relationship between multiplication and division to find an unknown factor • We can use base-ten representations to help us reason about division • We can exchange or decompose one or more units of a higher place value for 10 units of the next lower place value in order to have enough units to put into equal groups • We can find the quotient by decomposing the dividend and finding the quotient for each decomposed part until all of the dividend is divided • We can use our knowledge of partial products to find the quotient • There are various strategies that we can use to divide multi-digit numbers by single-digit divisors 	<p>Topic Vocabulary:</p> <p>Academic vocabulary</p> <p>Estimate Dividend Mathematical question Situation Expression Remainder Division Value Strategy Equation Multiple Factor Length Partial quotient Relationship Base-ten diagram Representation Mental math Diagram Decompose Method Algorithm Pattern Calculations Efficient</p>

In this Topic, students will be able to:

- Solve division problems in context, and recall the two meanings of division: “how many in each group” and “how many groups.”
- Solve division problems that involve finding unknown factors by reasoning with partial quotients and by decomposing a dividend into familiar multiples of the divisor.
- Use partial quotients to solve division problems that involve tiling squares and finding a side length of a rectangle with a known area.
- Make sense of base-ten representations for division.
- Find the quotients of two-digit and three-digit dividends and one-digit divisors by decomposing the dividend by place value—decomposing a larger unit to 10 of a smaller unit—and by reasoning in terms of equal-size groups.
- Introduce students to ways to record partial quotients when dividing multi-digit numbers.
- Use an algorithm that uses partial quotients to find whole number quotients and remainders with up to four-digit dividends and one-digit divisors and analyze some common errors when using an algorithm that uses partial quotients
- Represent and solve contextual problems that involve dividing a whole number of up to four-digits by a single-digit divisor, resulting in a number with or without a remainder and interpret the result and remainder given a situation.

Plan for Student Reflection:

[Student Journal Prompts and Reflection Practices](#)

Plan for Teacher Reflection:

- Reviewing formative assessments
- Developing scaffolds
- Collaborative scoring
- PLCs
- Planning for small groups

-How readily did students see equal-size situations in terms of division (and see division expressions in terms of equal-size groups)?

-How would you rephrase questions you asked today to improve students’ ability to make connections or to help them better consolidate what they did?

-What surprised you about how students used base-ten blocks to find the value of quotients?

-How did the representations in today’s lesson support students in dividing multi-digit numbers?


-How did students interact with each other’s ideas today in the work?

-Identify one or more ways in which your students’ thinking offered a new insight or a positive surprise today.

-How might you amplify the productive beliefs and address the unproductive ones?

Topic 3 (Section C) Task Development

Each Topic has its own Task that serves as a roadmap for instruction during the unit. The task follows the [Learning Cycle Model](#) that drives teaching and learning in Naugatuck Public Schools.

Task Title: Topic 3 - Multi-digit Division	Grade Level and Unit: Fourth Grade, Unit 6
Description of Task: In this activity, students continue to solve contextual problems that involve division (MP2). Here, the dividends extend to four-digit numbers and the problems demand a greater lift. In the second half of the activity, students are asked to reason in the opposite direction: given a division expression, they are to invent a situation that can represent and interpret the value of the expression in context.	Purpose of Task: This activity encourages students to interpret the quantities in situations, represent them mathematically, use their representations to find solutions, and then interpret their solutions in context (MP2).
Background of Students/Learning Progression: In grade 3, students learned about the relationship between multiplication and division and reasoned about division in terms of equal groups. They also saw two interpretations of division: as a way to find the size of a group and as a way to find the number of groups. They used these understandings to find quotients and to reason about division problems in context, finding whole-number quotients from two-digit dividends and one-digit divisors.	Ensure all competencies are addressed in the task: <ul style="list-style-type: none"><input type="checkbox"/> Yes, all competencies are addressed<input type="checkbox"/> No - Task needs modification
Getting Started: Display the following image for students to see: 	

Ask students, “What shapes do you see in the picture? Draw the shapes and divide them into parts with equal areas. Explain how you know the parts have equal areas.”

Provide 1-2 minutes of quiet think time. After, allow students to share their ideas with a partner for 2-3 minutes. Ask students to share their ideas with the whole class. As students are sharing, record their responses.

Learning Cycle Unit 6 Section C

IM Lesson	L 13: Situations Involving Equal-size Groups	L 14: Situations Involving Factors and Multiples	L 15: Situations Involving Area	L 16: Base-ten Blocks to Divide	L 17: Base-ten Diagrams to Represent Division	L 18: Divide with Partial Quotients	L 19: Division With and Without Remainders	L20: Interpret Remainders in Division Situations
Learning Cycle Model	Making Meaning	Making Meaning	Making Meaning	Making Meaning	Investigation	Investigation	Investigation	Create & Produce
Naugatuck Math Competency	4.NBT.B.6	4.NBT.B.6, 4.OA.A.3	4.MD.A.3, 4.NBT.B.6, 4.OA.A.3	4.NBT.B.6	4.NBT.B.6	4.NBT.B.6, 4.OA.A.3	4.NBT.B.6, 4.OA.A.3, 4.OA.B.4	4.NBT.B.6, 4.OA.A.3
Math Practice Standards	–	–	–	MP7	–	MP3	MP3	MP2
Lesson Purpose	The purpose of this lesson is for students to solve division problems in context, and to recall the two meanings of division: “how many in each group” and “how many groups.”	The purpose of this lesson is for students to solve division problems that involve finding unknown factors. They do so by reasoning with partial quotients and by decomposing a dividend into familiar multiples of the divisor. In the problems, the dividends are greater than 100 and the	The purpose of this lesson is for students to use partial quotients to solve division problems that involve tiling squares and finding a side length of a rectangle with a known area.	The purpose of this lesson is for students to make sense of base-ten representations for division.	The purpose of this lesson is for students to find the quotients of two-digit and three-digit dividends and one-digit divisors. They do so by decomposing the dividend by place value—decomposing a larger unit to 10 of a smaller	The purpose of this lesson is to introduce students to ways to record partial quotients when dividing multi-digit numbers.	The purpose of this lesson is for students to use an algorithm that uses partial quotients to find whole number quotients and remainders with up to four-digit dividends and one-digit divisors. Students also	The purpose of this lesson is for students to represent and solve contextual problems that involve dividing a whole number of up to four-digits by a single-digit divisor, resulting in a number with or without a remainder.

		divisions result in whole numbers with and without a remainder.			unit—and by reasoning in terms of equal-size groups.		analyze some common errors when using an algorithm that uses partial quotients.	Students also interpret the result and remainder given a situation.
Vocabulary Focus	Estimate, Dividend, mathematical question, situation, expression, remainder, division	Expression, value, strategy, equation, multiples, dividend, division, factor	Length, estimate, equation, division, partial quotients, relationship	Base-ten blocks, expression, value, representation, mental math	Diagram, value, base-ten diagram, decompose	Value, expression, method, decompose, partial quotient, subtraction, algorithm	Pattern, algorithm, remainder, calculations, efficient, estimating	Multiple, equation, relationship, quotient, value, situation, partial product, partial quotient
Lesson Materials/ Resources	Lesson 13 Slides Teacher Presentation Materials Student Pages No additional materials needed	Lesson 14 Slides Teacher Presentation Materials Student Pages No additional materials needed	Lesson 15 Slides Teacher Presentation Materials Student Pages Activity 2: <ul style="list-style-type: none"> For groups of 2, give access to grid paper, in case students wish to use it to create an area diagram If doing a gallery walk, create 3-4 posters to display during the activity 	Lesson 16 Slides Teacher Presentation Materials Student Pages Activity 1: <ul style="list-style-type: none"> Each group of 3-4 students needs a set of base-ten blocks that includes 4 hundreds block, 10 ten blocks, and 25 ones blocks 	Lesson 17 Slides Teacher Presentation Materials Student Pages Activity 1: <ul style="list-style-type: none"> Give students access to base-ten blocks Activity 2: <ul style="list-style-type: none"> Give students access to base-ten blocks 	Lesson 18 Slides Teacher Presentation Materials Student Pages Activity 1: <ul style="list-style-type: none"> Give students access to base-ten blocks 	Lesson 19 Slides Teacher Presentation Materials Student Pages No additional materials needed	Lesson 20 Slides Teacher Presentation Materials Student Pages No additional materials needed

	Lesson 13 Cool-down: After the Class Party	Lesson 14 Cool-down: Reaching 161 with Multiples	that show or describe different strategies students are likely to use to solve the problem Lesson 15 Cool-down: Sticky Notes on the Door	Lesson 16 Cool-down: Division Reflection	Lesson 17 Cool-down: Find the Value of a Quotient	Lesson 18 Cool-down: Subtract Groups	Lesson 19 Cool-down: Find a Quotient	Lesson 20 Cool-down: Miscounting?
Assessment	Formative Assessment Strategies: observation, questioning, student discourse : Monitoring Sheet See Section C Checkpoint Assessment , Section C Checkpoint Teacher's Guide							
								Section C Practice Problems
Centers	Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing) Compare (1–5), Stage 4: Divide within 100 (Supporting)	Compare (1–5), Stage 4: Divide within 100 (Supporting) Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)	Compare (1–5), Stage 4: Divide within 100 (Supporting) Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)	Compare (1–5), Stage 4: Divide within 100 (Supporting) Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)	Compare (1–5), Stage 4: Divide within 100 (Supporting) Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)	Compare (1–5), Stage 4: Divide within 100 (Supporting) Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)	Compare (1–5), Stage 4: Divide within 100 (Supporting) Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)	Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing) Compare (1–5), Stage 4: Divide within 100 (Supporting)

Making Meaning

In Lesson 13, students build from the skills and ideas learned in grade 3 as they recall strategies for reasoning about division problems in context (MP2). Students encounter situations that involve equal-size groups and that call for finding the size of a group and the number of groups. The work here prepares students to rely on the relationship between division and multiplication to solve problems involving three-digit dividends in the next lesson.

In Lesson 14, students relate problems about factors and multiples to division. To solve the problems, they rely on the relationship between multiplication and division, and their understanding of division as a way to find an unknown factor.

Students continue to interpret division in terms of finding the number of groups (“If we write multiples of 5, how many numbers will we need to write to get to 105?”) and the size of a group (“What number are we finding multiples of if we get to 112 after writing 7 numbers?”). They may solve the problems by multiplying in parts (finding partial products) or by dividing in parts (finding partial quotients). Through repeated reasoning, they notice that it helps to decompose a dividend into familiar multiples (MP2, MP8).

In these materials, division that results in a whole number with a remainder—for example $145 \div 7$ —is not expressed with an expression such as “20 R 5.” Instead, students will relate this result to a multiplication equation, in that $145 = 7 \times 20 + 5$.

In Lesson 15, students encounter division as they find a side length of a rectangle whose area is a three-digit number and one side is a one-digit number.

The context involves tiling rectangles with square tiles. This enables students to connect the dividend to the number of tiles in the rectangle and the divisor to the number of rectangles along one side. The grid provided in the first activity encourages students to partition the area (the dividend) into smaller parts, which in turn facilitates finding the unknown length (the quotient).

In Lesson 16, students work with larger dividends and represent problems with base-ten blocks. This representation emphasizes place value, which supports the work with division in this section. Students are asked to represent their work with base-ten blocks on paper, but that is not the emphasis of this lesson. In the next lesson, students will make sense of and use base-ten diagrams. In future lessons, they will be able to choose a representation and method that makes sense to them as they go deeper into division work.

[Lesson 13: Situations Involving Equal-size Groups](#)

- The purpose of this lesson is for students to solve division problems in context, and to recall the two meanings of division: “how many in each group” and “how many groups.”
- [Teacher Presentation Materials](#)
- [Lesson 13 Slides](#)

[Lesson 14: Situations Involving Factors and Multiples](#)

- The purpose of this lesson is for students to solve division problems that involve finding unknown factors. They do so by reasoning with partial quotients and by decomposing a dividend into familiar multiples of the divisor. In the problems, the dividends are greater than 100 and the divisions result in whole numbers with and without a remainder.

- [Teacher Presentation Materials](#)

- [Lesson 14 Slides](#)

Lesson 15: Situations Involving Area

- The purpose of this lesson is for students to use partial quotients to solve division problems that involve tiling squares and finding a side length of a rectangle with a known area.

- [Teacher Presentation Materials](#)

- [Lesson 15 Slides](#)

Lesson 16: Base-ten Blocks to Divide

- The purpose of this lesson is for students to make sense of base-ten representations for division.

- [Teacher Presentation Materials](#)

- [Lesson 16 Slides](#)

Investigation:

In grade 3, students used base-ten representations to help them reason about division of a two-digit number into equal-size groups. Lesson 17 builds on that understanding and revisits it in the context of three-digit dividends. Students recall that they can exchange or decompose one or more units of a higher place value for 10 units of the next lower place value in order to have enough units to put into equal groups.

The work here sets the groundwork for students to later decompose a dividend by place value (even when not using base-ten blocks or diagrams). It is also the basis for dividing multi-digit numbers using the standard division algorithm (in grade 5), which relies on dividing by place value, one digit at a time.

In Lesson 18, students use partial quotients and a couple of ways to record them systematically—by writing a series of equations, and by using an algorithm that uses partial quotients.

In Lesson 19, students deepen and apply what they learned about partial quotients to divide four-digit numbers by single-digit divisors. They also deepen their understanding of an algorithm that uses partial quotients—by noticing how the algorithm shows whether a division would result in a remainder, and by analyzing missteps that are commonly made in an algorithm like it.

[Lesson 17: Base-ten Diagrams to Represent Division](#)

- The purpose of this lesson is for students to find the quotients of two-digit and three-digit dividends and one-digit divisors. They do so by decomposing the dividend by place value—decomposing a larger unit to 10 of a smaller unit—and by reasoning in terms of equal-size groups.
- [Teacher Presentation Materials](#)
- [Lesson 17 Slides](#)

[Lesson 18: Divide with Partial Quotients](#)

- The purpose of this lesson is to introduce students to ways to record partial quotients when dividing multi-digit numbers.
- [Teacher Presentation Materials](#)
- [Lesson 18 Slides](#)

[Lesson 19: Division With and Without Remainders](#)

- The purpose of this lesson is for students to use an algorithm that uses partial quotients to find whole number quotients and remainders with up to four-digit dividends and one-digit divisors. Students also analyze some common errors when using an algorithm that uses partial quotients.
- [Teacher Presentation Materials](#)
- [Lesson 19 Slides](#)

Create and Produce:

By now students have developed various strategies to divide multi-digit numbers by single-digit divisors and have used different representations along the way. In this lesson, students apply what they learned to solve a variety of word problems that involve division (MP2).

[Lesson 20: Interpret Remainders in Division Situations](#)

- The purpose of this lesson is for students to represent and solve contextual problems that involve dividing a whole number of up to four-digits by a single-digit divisor, resulting in a number with or without a remainder. Students also interpret the result and remainder given a situation.

- [Teacher Presentation Materials](#)
- [Lesson 20 Slides](#)

Communicate and Present:

- Students find a partner who chose a different expression and take turns presenting their work. The person listening should consider whether the response makes sense and check if the quotient is correct.
- If time permits, students share with a different student or partnership.

Reflection:

“Today we solved problems that involved division. What strategies did you find yourself using to divide numbers?”

Did you:

- use partial products?
- use partial quotients?
- draw diagrams?
- divide by place value (thousands, hundreds, tens, and ones)?
- write a series of equations?
- estimate first?”

[IM Reflection Practices](#)

Notes:

Complete File with Resources and Task:

Task-Based Learning Plan for Topic 3

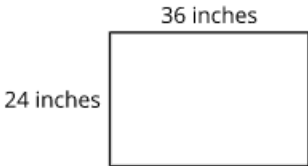
Topic # 4 (Section D)	Topic Name: Section D - Let's Put It to Work: Problem Solving with Large Numbers	Duration: Recommended: 5 days (5 lessons)
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Topic Description:
Section D Learning Goals

- Use the four operations to solve problems that involve multi-digit whole numbers and assess the reasonableness of answers.

In the final section of this unit, students engage with a variety of contextual problems that involve multi-digit numbers and all four operations. The problems can be approached in many ways, presenting students with opportunities to choose their strategies and representations strategically. Many of them also involve multiple steps and justifications, prompting students to practice constructing logical reasoning and critiquing the reasoning of others (MP3).

Jada plans to cut up a sheet of poster paper, rearrange the pieces, and tape them to make a banner that is 8 inches tall and 8 feet long. Does she have enough paper to make the banner?



36 inches

24 inches

Are there more people who only speak English or more people who speak a language other than English?

Show how you know.

language	number of speakers
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English only	1,224,539
Spanish	127,352
Other Indo-European	6,750
Asian	364

Competencies Addressed:

Measurement and Data Investigations
 1. I can solve problems involving measurement (time, money, customary and metric, are, perimeter, and conversion of measurements). **4.MD.A.2,3**

Understanding and Applying Number Systems
 3. I can add and subtract whole numbers. **4.NBT.B.4**
 4. I can use strategies to multiply and divide whole numbers. **4.NBT.B.5-6**

Operations and Algebraic Thinking
 2. I can use the four operations to solve multi-step problems. **4.OA.A.1, 4.OA.A.2, 4.OA.A.3**
 1. I can generate, analyze, and explain patterns. **4.OA.C.5**

Essential Question and Enduring Understanding Addressed in this Topic:

Essential Question
 How can we find solutions to problems with large numbers?

Enduring Understanding
Adding, subtracting, multiplying, and dividing help us to find solutions. We may face a task in which we need to figure out an unknown. In some cases, we may need to use multiple operations in order to find missing information before determining a solution. We may need to use evidence to determine the reasonableness of a product or quotient. These problems can be approached in many ways, but we can choose our strategies and representations strategically.

In this Topic, students will know:

- We can use our knowledge of multiplication and division, including the ideas of factors and multiples, to represent situations
- We can use multiplication and division to convert units of measurements

Topic Vocabulary:

Academic vocabulary

<ul style="list-style-type: none"> • Sometimes more than one step is needed to solve a problem and we may need to use more than one operation • We can use our knowledge of addition, subtraction, multiplication, and division to analyze data sets 	<p>Expression Strategy Decompose Partition Situation Visualize Operation Measurements Statement Equation Algorithm Question Reasonable Predict Pros Cons Standard algorithm Suggestion Clearer Revise Multi-step problem</p>
<p>In this Topic, students will be able to:</p> <ul style="list-style-type: none"> • Represent and solve multi-step contextual problems involving multiplication and division, including division with remainders. • Apply what they know about multiplication and division to convert units of measurement and solve multi-step problems involving perimeter and area. • Use the four operations to solve problems involving multi-digit numbers and use the standard algorithm for addition and subtraction to solve problems. • Solve multi-step word problems by analyzing data, estimating, reasoning, and performing multiple operations. • Create and analyze patterns in a real-world context and to solve multi-step problems. 	<p>Plan for Student Reflection:</p> <p>Student Journal Prompts and Reflection Practices</p> <hr/> <p>Plan for Teacher Reflection:</p> <ul style="list-style-type: none"> • Reviewing formative assessments • Developing scaffolds • Collaborative scoring • PLCs

	<ul style="list-style-type: none">● Planning for small groups-What did students say or do that showed the exercise was effective in expanding their view of problem solving?-What evidence of flexible reasoning, structural thinking, or deep understanding did you see today?-How have you seen each student grow as a young mathematician throughout this work?-What norms or routines during discussions allowed students to engage with other students' ideas?
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Topic 4 (Section D) Task Development

Each Topic has its own Task that serves as a roadmap for instruction during the unit. The task follows the [Learning Cycle Model](#) that drives teaching and learning in Naugatuck Public Schools.

Task Title: Topic 4 - Let's Put It to Work: Problem Solving with Large Numbers	Grade Level and Unit: Fourth Grade, Unit 6
Description of Task: The purpose of this activity is for students to make paper flowers, use them to create patterns, and describe the patterns. The experience of making the flowers provides a concrete reference that will be helpful when students make sense of, solve, and create multi-step problems with the same context later in the lesson.	Purpose of Task: When students ask and answer questions that arise from a given situation, use mathematical features of an object to solve a problem, make choices, analyze real-world situations with mathematical ideas, interpret a mathematical answer in context, and decide if an answer makes sense in the situation, they model with mathematics (MP4).
Background of Students/Learning Progression: In the final section of this unit, students engage with a variety of contextual problems that involve multi-digit numbers and all four operations. The problems can be approached in many ways, presenting students with opportunities to choose their strategies and representations strategically. Many of them also involve multiple steps and justifications, prompting students to practice constructing logical reasoning and critiquing the reasoning of others (MP3).	Ensure all competencies are addressed in the task: <input type="checkbox"/> Yes, all competencies are addressed <input type="checkbox"/> No - Task needs modification
Getting Started: Display the following expressions: A. 5×90 B. $90 + 90 + 90 + 90 + 90$ C. $(4 \times 90) + (1 \times 90)$ D. $3 \times 3 \times 10 \times 5$ Provide one minute of quiet think time. Ask students to discuss their thinking with a partner and provide 2-3 minutes for partner discussion. Have students share their ideas with the whole class. As students are sharing, record their responses.	

Learning Cycle Unit 6 Section D

IM Lesson	L21: Different Ways to Solve Problems	L 22:Problems About Perimeter and Area	L 23: Solve Problems with Many Operations	L 24: Assess the Reasonableness of Solutions	L 25: Paper Flower Decorations (Optional)
Learning Cycle Model	Making Meaning	Making Meaning	Investigation	Investigation	Create & Produce
Naugatuck Math Competency	4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3	4.MD.A.2, 4.MD.A.3, 4.NBT.B.5, 4.OA.A.3	4.MD.A.2, 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6	4.NBT.B.4, 4.OA.A.2, 4.OA.A.3	4.OA.A.3, 4.OA.C.5
Math Practice Standards	MP2	MP1	MP2	–	MP4
Lesson Purpose	The purpose of this lesson is for students to represent and solve multi-step contextual problems involving multiplication and division, including division with remainders.	The purpose of this lesson is for students to apply what they know about multiplication and division to convert units of measurement and solve multi-step problems involving perimeter and area.	The purpose of this lesson is for students to use the four operations to solve problems involving multi-digit numbers. Students also use the standard algorithm for addition and subtraction to solve problems. For each problem, they assess the reasonableness of their responses.	The purpose of this lesson is for students to solve multi-step word problems by analyzing data, estimating, reasoning, and performing multiple operations. It also helps students to build fluency in using the standard algorithm to add and subtract multi-digit numbers up to 1 million. In each activity, students assess the reasonableness of their responses.	The purpose of this lesson is for students to create and analyze patterns in a real-world context and to solve multi-step problems.
Vocabulary Focus	Expression, strategy,	Decompose, partition, situation, visualize, strategy, operations, measurements	Statement, equation, algorithm, expression, question, reasonable,	Predict, operation, reasonable, pros, cons, standard algorithm	Suggestion, clearer, revise, multi-step problem
Lesson Materials/Resources	Lesson 21 Slides Teacher Presentation Materials Student Pages	Lesson 22 Slides Teacher Presentation Materials Student Pages	Lesson 23 Slides Teacher Presentation Materials Student Pages	Lesson 24 Slides Teacher Presentation Materials Student Pages	Lesson 25 Slides Teacher Presentation Materials Student Pages

	<p>Activity 1:</p> <ul style="list-style-type: none"> Before the lesson, display posters with the five strategies around the room (shown in Going on a Field Trip) <p>Lesson 21 Cool-down: Big Weekend at the Movies</p>	<p>Activity 1:</p> <ul style="list-style-type: none"> If students do not what a banner is, show an example (if needed) Give students access to grid paper and inch tiles <p>Lesson 22 Cool-down: Paper for a Banner</p>	<p>Activity 1:</p> <ul style="list-style-type: none"> Give students access to grid paper <p>Activity 2:</p> <ul style="list-style-type: none"> Give students access to grid paper <p>Lesson 23 Cool-down: Long-distance Driving</p>	<p>Activity 1:</p> <ul style="list-style-type: none"> Give students access to grid paper <p>Lesson 24 Cool-down: The Children and the Elderly</p>	<p>Activity 1:</p> <ul style="list-style-type: none"> Gather rubber bands or pipe cleaners and 60 sheets of tissue paper that measure 18 inches by 24 inches Cut the tissue paper in the following ways (measurements do not need to be exact): <ul style="list-style-type: none"> 20 sheets cut into strips that are 4 inches by 9 inches 40 sheets cut into strips that are 6 inches by 12 inches (length should be about 2 times the width)
Assessment	<p>Formative Assessment Strategies: observation, questioning, student discourse: Monitoring Sheet See Section D Checkpoint Assessment, Section D Checkpoint Teacher’s Guide Unit 6 End of Unit Assessment, Unit 6 End of Unit Assessment Teacher Guide</p>				
					Section D Practice Problems
Centers	<p>Compare (1–5), Stage 7: Multi-digit Operations (Addressing)</p> <p>Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)</p>	<p>Compare (1–5), Stage 7: Multi-digit Operations (Addressing)</p> <p>Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)</p>	<p>Compare (1–5), Stage 7: Multi-digit Operations (Addressing)</p> <p>Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)</p>	<p>Compare (1–5), Stage 7: Multi-digit Operations (Addressing)</p> <p>Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)</p>	

Making Meaning

In Lesson 21, students analyze and use various strategies and representations to reason about multi-step problems. They use their knowledge of multiplication and division, including the ideas of factors and multiples, to represent situations. Students also interpret products, quotients, and remainders in context (MP2).

Lesson 22 prompts students to apply their reasoning skills and knowledge of all operations to solve problems about area and perimeter. Along the way, students also use multiplication and division to convert units of measurement. Most numbers used here are two- and three-digit numbers. The problems in the lesson may include more than one step and can be solved in multiple ways, offering students opportunities to construct logical arguments to communicate their thinking and to critique the reasoning of others (MP3). As students begin the lesson, remind them of their past experiences with multi-step problems and explain that the problems in this lesson may involve more than one step.

[Lesson 21: Different Ways to Solve Problems](#)

- The purpose of this lesson is for students to represent and solve multi-step contextual problems involving multiplication and division, including division with remainders.
- [Teacher Presentation Materials](#)
- [Lesson 21 Slides](#)

[Lesson 22: Problems About Perimeter and Area](#)

- The purpose of this lesson is for students to apply what they know about multiplication and division to convert units of measurement and solve multi-step problems involving perimeter and area.
- [Teacher Presentation Materials](#)
- [Lesson 22 Slides](#)

Investigation:

In the preceding lessons, students interpret situations and solve them using a variety of reasoning strategies. The computations focus mostly on multiplication and division, and the numbers are mainly two and three digits long.

In Lesson 23, students continue to engage in problem solving—this time in the context of finding distances. Students now use the four operations and work with numbers up to five digits. In the next lesson, they will work with numbers up to 1 million.

In Lesson 24, students apply their knowledge of numbers in base-ten and their estimation and computation skills to solve problems about languages and populations in the United States. The census data used here prompts students to work with large numbers and to interpret them carefully.

[Lesson 23: Solve Problems with many Operations](#)

- The purpose of this lesson is for students to use the four operations to solve problems involving multi-digit numbers. Students also use the standard algorithm for addition and subtraction to solve problems. For each problem, they assess the reasonableness of their responses.
- [Teacher Presentation Materials](#)
- [Lesson 23 Slides](#)

Lesson 24: Assess the Reasonableness of Solutions

- The purpose of this lesson is for students to solve multi-step word problems by analyzing data, estimating, reasoning, and performing multiple operations. It also helps students to build fluency in using the standard algorithm to add and subtract multi-digit numbers up to 1 million. In each activity, students assess the reasonableness of their responses.
- [Teacher Presentation Materials](#)
- [Lesson 24 Slides](#)

Create and Produce:

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling. In this lesson, students build on their prior understanding and experiences with creating and analyzing patterns to solve multi-step problems in a real-world context.

In the first activity, students make different types of paper flowers. In the second activity, they consider patterns and solve problems involving paper flower garlands. In the third activity, students think of their own pattern and multi-step problems inspired by their process of making paper flowers.

When students ask and answer questions that arise from a given situation, use mathematical features of an object to solve a problem, make choices, analyze real-world situations with mathematical ideas, interpret a mathematical answer in context, and decide if an answer makes sense in the situation, they model with mathematics (MP4).

Lesson 25: Paper Flower Decorations

- The purpose of this lesson is for students to create and analyze patterns in a real-world context and to solve multi-step problems.
- [Teacher Presentation Materials](#)
- [Lesson 25 Slides](#)

Communicate and Present:

- Display student work of selected students and invite them to share.

Reflection:

<ul style="list-style-type: none"> ● “What specific part of the work makes their explanation clear?” ● Consider asking: <ul style="list-style-type: none"> ● “What suggestion do you have to make it clearer?” ● “How can you revise your own work to make it more clear or organized?” 	<p>“Today, we solved many different problems about paper flowers and created flowers and problems of our own.”</p> <p>“What are some ways making the flowers helped you write your own math problems?” (I was able to see what kinds of situations I could ask about in my story problem.)</p> <ul style="list-style-type: none"> ● IM Reflection Practices
<p>Notes:</p>	<p>Complete File with Resources and Task:</p> <p>Task-Based Learning Plan for Topic 4</p>