

Algebra I Instructional Focus Documents

Introduction:

As districts adopt and implement high-quality instructional materials (HQIM) in mathematics, these Instructional Focus Documents (IFD) are intentionally designed to provide a lens into what effective mathematics instruction looks and sounds like in Tennessee classrooms. They are written to support all levels of leadership within a district and complement both the Math Implementation Framework and the Tennessee-specific Instructional Practice Guide (IPG). When used as a suite of resources, the IFDs, the Math Implementation Framework, and the IPG provide guidance and aligned measures with which to track and support district implementation of HQIM in mathematics.

Mathematical rigor does not simply mean increased difficulty or complexity of problems. Rigorous mathematical instruction and learning means deep thinking and exploring at a greater depth. The three aspects of rigor are Conceptual Understanding, Procedural Skill and Fluency, and Application. Each aspect is equally important and necessary for deep mathematical understanding and mastery. These aspects of rigor work in conjunction with the HQIM to provide a meaningful learning experience for students.

Aspects of Rigor:

Conceptual Understanding helps students understand the “how” and the “why” of mathematics. This aspect of rigor focuses on mathematical thinking and reasoning as opposed to answer-getting. Students should understand how and why the math works using mathematical models and manipulatives to aid in achieving conceptual understanding. Instruction should connect prior learning to new ideas and concepts. Opportunities for discussion and reflection may correct and unscramble common misconceptions. Flexible reasoning and fluency grow from conceptual understanding.

Procedural Skill and Fluency is the ability to apply mathematical knowledge accurately, flexibly, and efficiently. It is important to note that the phrase “procedural skill and fluency” is inclusive. The inclusive definition of procedural skill and fluency is *not* the rote use of an algorithm or the recall of facts, but a continuum of understanding. The continuum involves learning or developing algorithms and strategies, executing procedures accurately and efficiently, and learning how to use models and tools. Fluent mastery of a mathematical concept involves the ability to connect and use the Standards for Mathematical Practice while using algorithms and strategies for problem-solving. Students who have achieved fluency can link learned or developed algorithms and strategies to conceptual understanding to explain the “why” behind the procedures. Mathematically proficient students can understand the approaches to solving complex problems and identify correspondences between different approaches to select and use the most appropriate strategy to form an accurate solution path.

Application refers to applying prior knowledge in new and unique situations, other subject areas, and mathematical and contextual problems. Application also includes intentionally integrated content that provides learning opportunities for students to apply and extend their knowledge of multiple standards, clusters, and/or domains within the grade level. The goal is for students to activate their prior knowledge in order to bring a sense of understanding to new mathematical and/or contextual situations.

Evidence of Learning Statements:

The evidence of learning statements provide guidance to connect the Tennessee Mathematics Standards with evidence of learning outcomes that can be collected through classroom activities, observations, or assessments, providing an indication of how students are tracking towards the grade-level expectations that are encompassed within the Tennessee Mathematics Standards. Within the evidence of learning statements, level 3 statements demonstrate on-grade level expectations for all Tennessee students.

The statements are designed to provide a continuum of concrete examples demonstrating what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting. Further, they provide a lens to help offer scaffolding to move a student with unfinished learning up to grade level expectations.

When used alongside high-quality instructional materials, these concrete examples serve to reinforce the grade level expectations and rigor that should be present within the materials and reinforce their inclusion within instruction, ensuring all students have access to on-grade level activities.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance with a focus on Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

When used in conjunction with HQIM, instructional focus statements support teacher understanding as they plan and implement HQIM to the depth and rigor of the Tennessee mathematics standards. Additionally, they serve as a benchmark for district and school leaders to use alongside the IPG as they are monitoring HQIM implementation.

Number and Quantity Quantities*(N.Q)

Standard A1.N.Q.A.1 **Cluster Heading: A. Reason quantitatively and use units to understand problems.**

Use units as a way to understand real-world problems.*

A1.N.Q.A.1a Choose and interpret the scale and the origin in graphs and data displays.*

A1.N.Q.A.1b Use appropriate quantities in formulas, converting units as necessary.*

A1.N.Q.A.1c Define and justify appropriate quantities within a context for the purpose of modeling.*

A1.N.Q.A.1d Choose an appropriate level of accuracy when reporting quantities.*

Scope and Clarifications: (Modeling Standard)

Apply this standard to any real-world problems studied within the scope of this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X	X*	X

*procedural skill and fluency only applies to part b.

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify units appropriately when solving contextual problems. Choose a graph or data display, with appropriate scale and/or units, to represent a real-world problem. Identify individual quantities in context, label them with appropriate units, and connect the units to the values in the real-world problem. Explain the difference between precision and accuracy.	Choose units appropriately when solving contextual problems, including problems that contain real-world formulas. Choose an interpretation of a graph and/or data display, with appropriate scale and/or units, that represents a real-world problem. Identify and interpret necessary information, recognize any extraneous information, and attend to unit selection and/or conversions	Choose and interpret units, scale, and the origin in graphs and data displays that represents a real-world problem, including problems that contain real-world formulas. Determine the most appropriate data display based on the units given in a problem. Use appropriate quantities in formulas, converting units when necessary.	Explain if the information is represented appropriately using mathematical justification, given a numerical and/or a graphical representation of a real-world problem. Explain the interpretation of the units, scale, and origin with respect to the contextual situation using precise mathematical vocabulary. Create a real-world problem involving formulas and data, represented either in a table or

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	in order to select a quantity that models a real-world situation. Choose a solution that is both accurate and reasonable with respect to the contextual situation. Describe how using inaccurate measurements when reporting quantities can affect the solution.	Define and justify with precision the appropriate quantities, and their units, within a context to model a real-world problem. Make observations about extraneous information embedded in context and explain why information is extraneous in a real-world problem. Choose an appropriate level of accuracy, including appropriate units, when reporting quantities in a real-world context. Explain the reasonableness of answers with respect to the context of the problem when reporting quantities as a result of solving the contextual problem. Describe the most common causes of inaccuracies in contextual problems (e.g., when using measurement tools).	graph, in which the data must be analyzed for appropriate units and scale. Flexibly represent quantities in modeling situations with data displays, numerically, algebraically, and graphically and explain relationships among representations. Describe the accuracy of a measurement embedded in a real-world context by stating the possible error when appropriate. Explain why it is important to choose an appropriate level of accuracy and what limitations exist on measurement when reporting quantities in contextual problems.

Instructional Focus Statements

Level 3:

In grades K-8, students developed an understanding of measuring, labeling values, and understanding how the value of a number relates to the described quantity. In the high school Numbers and Quantity (N) domain, students develop an understanding of reasoning quantitatively (MP 2) and solving

problems requiring the evaluation of the appropriateness of the form in which quantities are provided and reported. This standard is important throughout Algebra 1, Geometry, and Algebra 2 and should be integrated with a wide variety of standards throughout the course.

When choosing and interpreting the scale and the origin in graphs and data displays, instruction should focus on the importance of correctly interpreting units and representing contextual problems using appropriate graphs and data displays. Instruction should be centered around tasks that provide real-world, multi-step problems, where interpreting units appropriately is critical. Students should be able to identify and interpret necessary information, in order to select a quantity, with correct units, that models a real-world situation. Students should explore the importance of correctly interpreting units and designing displays of data to most appropriately represent a contextual problem. Students should understand that a key relationship exists between units and appropriate representation of units. Understanding this relationship is beneficial in developing students' conceptual understanding of units in contextual problems.

Students should be able to recognize the relationship between the units for all variables in a formula, use appropriate quantities in formulas, and convert units as necessary. Students should be engaged in classroom discourse (MP 8) that promotes explaining their reasoning and justifying quantities (MP 3) as a result of a solution pathway. Students should be given ample opportunity to work with data representations where students must think critically to set an appropriate scale that displays the data to show key features. It is imperative that students attend to precision (MP 6) in using, interpreting, and reporting units.

Students should extend their understanding of using appropriate quantities to defining and justifying appropriate quantities in modeling situations. Students should be able to select appropriate quantities embedded in real-world contextual problems, identify extraneous information, choose appropriate units, etc. Instruction should focus on providing opportunities for students to attend to precision (MP 6) by describing the quantities in modeling situations. Furthermore, students must be given opportunities to write and create appropriate labels for quantities and explain the meaning of the quantities in a context.

In middle grades, students have had experience interpreting and reporting quantities that involve area, volume, and rates. Students at this level are able to discern an appropriate level of accuracy when reporting quantities of a real-world problem. As this is a modeling standard, students should solve contextual problems and be able to choose a level of accuracy of measurement quantities that is reasonable and makes sense to the contextual situation. For example, when solving a multi-step problem or using graphing technology, students should determine when it is appropriate and not appropriate to use precise values (values that are not rounded or truncated), rounded values, or truncated values. Students should be able to justify their reasoning (MP 3) for using certain values and explain why their choice is important with respect to the context. Additionally, students should experience different solution paths that involve using different forms of values and explain how accuracy does or does not have an impact on the solution within the context of the problem.

This standard builds upon the opportunities students have been given to explain values of numbers in terms of units in previous grades. In middle school, students focused on ratios and proportional relationships and also using quantities to describe data from a statistical lens. Both fields help prepare students for choosing a level of accuracy when reporting quantities. Instruction should focus on providing students a plethora of opportunities to use a

variety of measurements including measuring tools and graphing technology (MP 5). Students should have ample time to explore traditional, physical tools as well as electronic, and digital tools. During this exploration, class discussion should focus on helping students ascertain the difference between precision and accuracy and when it is appropriate to apply each of them in certain problems. Furthermore, instruction should be infused with a broad spectrum of different types of units that describe tiny to very large quantities. This is a modeling standard and students should make connections to other disciplines such as science. Being able to identify, interpret, and justify quantities is a skill that will serve students well to have mastered during this course as this standard lays the foundation for using units as a way to understand problems.

Level 4:

Students should be able to interpret, solve, and represent contextual information using appropriate units and attend to precision when interpreting and using units. One extension of this standard is for students to differentiate between data that is appropriately displayed versus data that is not appropriately displayed. Students can then analyze the difference between both displays providing a critique of the representation. Students should use logical arguments, including explaining how to improve the representation so that all key features and units are displayed appropriately. Data should be represented graphically, numerically, algebraically, and verbally. Furthermore, students should be given opportunities to synthesize information from multiple sources and produce a model that represents the contextual situation.

Instruction should focus on providing opportunities for students to work with problems that have a variety of descriptors and units embedded in the context. Students should be asked to extend their knowledge of quantities by representing them in multiple formats such as a graphical representation of the given information, algebraic representation of the quantities, and multiple representations to predict or draw conclusions about the solution of the real-world problem. Instruction should provide opportunities for students to analyze and critique the interpretation of quantities in a modeling situation. Additionally, students should be given ample opportunities to design their own contextual problem in which they would have to use quantities appropriately in order to describe the modeled contextual situation.

Students have a great opportunity to support their understanding of this standard through the lens of a wide variety of other disciplines. Instruction should focus on providing students with experiences involving problem situations that interest them. Teachers should include opportunities for inquiry and experimentation with ample time for students to explore. For example, students could investigate repeated measurement by exploring the level of precision and/or accuracy provided by various measurement tools in science experiment to determine an acceptable level of accuracy when reporting quantities in real-world context (MP 6). Also, instruction should provide the opportunity for students to analyze and critique (MP 3) the level of accuracy chosen by others' reported quantities.

Algebra

Seeing Structure in Expressions* (A.SSE)

Standard A1.A.SSE.A.1 Cluster Heading: A. Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.*

A1.A.SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.*

A1.A.SSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.*

Scope and Clarifications: (Modeling Standard)

For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P . For example, one truck can transport A cubic feet, and a second truck can transport B cubic feet. The first truck makes x trips to a job site, while the second makes y trips. Interpret the expression $Ax + By$ in terms of the context.

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify parts of an expression (i.e., factor, coefficient, term). Define the formal definition of the terms: <i>factor</i>, <i>coefficient</i>, and <i>term</i>. Define the formal definition of the term <i>expression</i>. Label the single entities in an expression.	Recognize arithmetic operations in an expression in order to see the structure of the expression. Understand and use the definitions of <i>terms</i>, <i>factors</i>, <i>coefficients</i>, and <i>like terms</i> in order to describe the structure of the individual parts of the expression. Identify parts of an expression as a single entity. Recognize that individual parts of an expression affect the whole expression.	Interpret parts of an expression (i.e., term, factor, coefficient) embedded in a real-world situation and explain each part in terms of the context. Interpret parts of an expression (i.e., term, factor, and coefficient) and explain each part in terms of the function the expression defines. Explain the structure of an expression and how each term is related to the other terms by interpreting the arithmetic meaning of each term in the expression and	Interpret expressions in a variety of forms by explaining the relationship between the terms and the structure of the expression. Interpret parts of complicated expressions with varying combinations of arithmetic operations and exponents by viewing one or more of their parts as a single entity. Write and interpret expressions that represent a real-world context and use the expressions to solve contextual problems.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	State arithmetic operations performed within an expression.	recognizing when combining like terms is appropriate. Interpret an expression by describing each individual term as a single entity and the relationship to the expression.	Write expressions in a wide variety of formats and then for each, describe the effects each term has considering them first individually and then considering them as a part of the expression. Identify and explain structure in patterns represented pictorially or graphically and write an algebraic expression to represent the pattern.

Instructional Focus Statements

Level 3:

Seeing structure in expressions is the connecting bridge between arithmetic operations and algebraic thinking. Instruction should build on students understanding of the relationship between arithmetic operations in expressions and equations. Students should explore a variety of expressions in equivalent forms to see and evaluate the structure in each form. Students should be exposed to exponents of varying degree. This allows them to recognize the attributes of a term in order to combine it appropriately with other like terms. Instruction should expose students to a variety of multiple representations and require students to interpret and explain the relationship between the representations. Students should be challenged with complicated, multi-variable expressions to interpret.

Furthermore, students must be able to explain individual terms and interpret that term as a single entity and as a whole expression. Instruction should focus on using the structure of the expression to uncover the attributes of the function it defines. Students should also be able to use precise language to explain the relationship between a verbal description and an algebraic representation. Particular focus needs to be placed on translating words into mathematical expressions and vice versa.

This standard appears in both Algebra 1 and Algebra 2. Tasks are limited to linear, quadratic and exponential in Algebra 1. In Algebra 2, students will extend this understanding with quadratics and cubics. In future courses, students will experience this standard with radical and trigonometric expressions, solidifying students' comprehension of the structure of expressions and interpreting the meaning of terms as single entities is imperative.

In Algebra 1 the focus is on linear, quadratic and exponential expressions and functions. For example, in the linear function $f(x) = 3x + 4$, students should recognize that 4 is the y-intercept and 3 is the rate of change. In the quadratic function $f(x) = -x^2 + 3x + 100$, students should recognize that the graph of the function will have a maximum because the coefficient of x^2 is negative and the y-intercept is 100. For the exponential function $f(x) = 3(0.5)^x$, students should recognize that the y-intercept is 3 and this function represents exponential decay. Instruction should include real-world situations where students can relate the structure of the function to the situation it is modeling.

Level 4:

Students need to be presented with complicated expressions that include a combination of different arithmetic operations and interpret in terms of a real-world context. The pinnacle of level 4 understanding is being able understand, interpret, and explain the relationship between equivalent representations of an expression. Students should be able to explain not only the expression in terms of a contextual situation, but also how each term within the expression connects back to the contextual situation.

Additionally, instruction should focus on relating expressions to real-world contexts. For example, students should be given problems that describe contextual situations from multiple perspectives. Students should interpret the contextual situation for each individual perspective and write an expression that represents the context for each. Students should be challenged to interpret the meaning of the expressions created and use them to predict outcomes and solve problems. Instruction should expose students to multiple representations of the expressions by making connections between the equivalent expressions, which will in turn help students recognize the most useful form of an expression depending on context. Students should be challenged to justify why other formats are equivalent and which format is most relevant given the context of the problem.

Algebra

Arithmetic with Polynomials and Rational Expressions (A.APR)

Standard A1.A.APR.A.1 **Cluster Heading: A. Perform arithmetic operations on polynomials.**

Add, subtract, and multiply polynomials. Use these operations to demonstrate that polynomials form a closed system that adhere to the same properties of operations as the integers.

Scope and Clarification:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Add and subtract linear expressions with rational coefficients.	Add two polynomial expressions where both expressions are comprised of the same degree (e.g., $(x^2+x+3) + (2x^2-3x-4)$). Subtract two polynomial expressions where both expressions are comprised of the same degree (e.g., $(x^2+x+3) + (2x^2-3x-4)$). Multiply two binomial expressions.	Add polynomial expressions. Subtract two polynomial expressions. Multiply polynomial expressions. Explain what it means for polynomials to be closed under the operations of addition, subtraction, and multiplication.	Add, subtract, and multiply multiple polynomial expressions, including situations involving more than one operation. Explain the similarities that exist between adding, subtracting, and multiplying integers and adding, subtracting, and multiplying polynomials.

Instructional Focus Statements

Level 3:

The development of an understanding of polynomials in high school parallels the development of numbers in elementary and middle grades. In elementary school, students might initially see expressions like $7+4$ and 11 as referring to different entities: $7+4$ might be seen as a calculation and 11 as its answer. They come to understand that different expressions are different names for the same numbers, that properties of operations allow numbers

to be written in different but equivalent forms, and that there are often benefits from writing numbers in various forms. They come to see numbers as forming a unified system: the number system. A similar evolution takes place in algebra. At first, algebraic expressions are simply numbers in which one or more letters are used to stand for a number which is unknown. Students learn to use the properties of operations to write expressions in different but equivalent forms. At some point they see equivalent expressions as naming some underlying quantity. Additionally, they reach the understanding that polynomials form a system in which they can be added, subtracted, and multiplied and that this system is closed for these operations. It is important that students understand this system beyond simply a rote set of steps which allows students to add, subtract, and multiply polynomials.

Level 4:

Students with a deep understanding of polynomials should be able to simplify complex expressions that involve multiple operations. Additionally, they will be able to explain in both verbal and written form the similarities that exist between calculating with integers and adding, subtracting, and multiplying polynomials. For example, students may see a connection between multiplying integers using an area model (from previous learning) and multiplying polynomials using a box method.

Algebra

Creating Equations* (A.CED)

Standard A1.A.CED.A.1 **Cluster Heading: A. Create equations that describe numbers or relationships.**

Create equations and inequalities in one variable and use them to solve problems in a real-world context.*

Scope and Clarifications: (Modeling Standard)

Tasks are limited to linear, quadratic, and absolute value equations and inequalities.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear equation or inequality in one variable that represents a real-world situation. Solve a one-variable linear equation or inequality. Represent real-world problems using equations in the form $px + q = r$ or $p(x + q) = r$.	Solve a one-variable linear equation or inequality that represents a real-world situation. Choose a quadratic or absolute value equation or inequality to represent a real-world situation. Solve a one-variable quadratic or absolute value equation or inequality.	Create and solve a one-variable linear, quadratic, or absolute value equation that represents a real-world situation. Create and solve a one-variable linear inequality that represents a real-world situation. Create and solve a one-variable quadratic or absolute value inequality that represents a simple real-world situation.	Create a real-world situational problem to represent a given linear, quadratic, or absolute value equation or inequality. Interpret and validate results by comparing with the situation and improve upon when necessary.

Instructional Focus Statements

Level 3:

In grade 7 (7.EE.B.4), students used variables to represent quantities in a real-world or mathematical problem and constructed simple equations and inequalities to solve problems by reasoning about the quantities. Students continued to analyze and solve linear equations in grade 8 (8.EE.C). In Algebra 1,

students expand their learning to other equation types including absolute value equations and inequalities in one variable (A1.A.REI.B.2) and quadratic equations and inequalities (A1.A.REI.B.3).

Students should write and solve equations and inequalities in one variable to answer questions about a real-world situation. Students should understand that equations and inequalities are examples of mathematical models. They are mathematical representations of a situation in life that can be used to make sense of problems and solve them (MP1). Facilitate classroom discussion where students make meaning of the context of a real-world problem in order to connect it to the structure of a mathematical equation. A common mistake is that students look only at the numbers and focus on key words that may change from one situation to another rather than the meaning of the given context. Provide opportunities for students to discover if a real-world situation can be represented by a linear, quadratic, or absolute value equation or inequality. They consider not only whether the equation or inequality appropriately models the situation, but also reflect on whether the solution or solution sets make sense in context. Students should practice reasoning quantitatively and abstractly (MP2) to make sense of quantities and their relationships in problem situations. Students should also engage in aspects of mathematical modeling (MP4) to be able to identify important quantities in a practical situation, solve problems arising in everyday life, and interpret the results in the context of the situation.

As this is a modeling standard, students need to encounter equations and inequalities in one variable that evolve from real-world situations. Students should be formulating equations and inequalities, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions to justify solutions to real-world problems. Real-world situations should elicit equations and inequalities from situations which are linear, quadratic, and absolute value in nature. It is imperative that students have the opportunity to work with each of these equation types equally.

Level 4:

When given an equation or inequality in one variable, students can generate a real-world situation that could be solved by a provided equation or inequality demonstrating a deep understanding of the interplay that exists between the situation and the equation or inequality used to solve the problem.

Additionally, students should continue to encounter real-world problems that are increasingly more complex. Students should be using the modeling cycle to solve real-world problems. The modeling cycle involves identifying variables in a situation, model by creating an algebraic representation, performing operations, interpreting the results in terms of the context, validating conclusions by comparing them with the original situation and then either improving upon the model or reporting on the reasoning behind the conclusions.

Algebra

Creating Equations* (A.CED)

Standard A1.A.CED.A.2 Cluster Heading: A. Create equations that describe numbers or relationships.

Create equations and inequalities in two variables to represent relationships between quantities and use them to solve problems in a real-world context. Graph equations with two variables on coordinate axes with labels and scales and use the graphs to make predictions.*

Scope and Clarifications: (Modeling Standard)

Tasks are limited to linear, quadratic, and absolute value equations and inequalities.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear equation that represents a real-world situation. Choose a linear graph to represent a real-world situation. Identify appropriate units of measure within a given real-world context.	Create and solve a two-variable linear equation that represents a real-world situation. Graph a two-variable linear equation that represents a real-world situation. Choose a two-variable quadratic or absolute value equation or inequality to represent a real-world situation. Solve a two-variable quadratic or absolute value equation or inequality.	Create and solve a two-variable linear, quadratic, or absolute value equation or inequality that represents a real-world context. Graph a two-variable linear, quadratic, or absolute value equation or inequality that represents a real-world context. Make predictions using the graph of a two-variable linear, quadratic, or absolute value equation or inequality that represents a real-world context.	Create a real-world situational problem to represent a given linear, quadratic or absolute value equation or inequality. Interpret and validate results by comparing with the situation and improve upon when necessary.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Create a graph with appropriate scales and labels for a given real-world context.		

Instructional Focus Statements

Level 3:

In grade 8 (8.F.B.4), students constructed linear functions to model the relationship between two quantities. In Algebra I (A1.REI.D.5), students understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

This standard is closely related to the interpreting functions standards (A1.F.IF), since a two-variable equation can be meaningfully reinterpreted as a function with a focus on the relationship between the variables. Students should graph equations and inequalities on coordinate axes, clearly labeling the axes defining what the values on the axes represent and the unit of measure. Students should also select intervals for the scale that are appropriate for the context and display adequate information about the relationship. Facilitate classroom discussion where students interpret the context and choose appropriate minimum and maximum values for a graph. To make predictions from graphs, students can use the graph to visually follow trends that appear in the data and use those trends to make predictions.

Students should write and solve equations and inequalities in two variables to answer questions about a real-world situation with attention paid to the relationships between the quantities. Students should understand that equations and inequalities are a type of mathematical model. They are mathematical representations of a situation in life that can be used to make sense of problems and solve them (MP1). Provide opportunities for students to analyze if a real-world situation should be represented by a linear, quadratic, or absolute value equation or inequality. They consider not only whether the equation or inequality appropriately models the situation, but also reflect on whether the solution or solution sets make sense in context. Students should practice reasoning quantitatively and abstractly (MP2) to make sense of quantities and their relationships in problem situations. Students should also engage in aspects of mathematical modeling (MP4) to be able to identify important quantities in a practical situation, solve problems arising in everyday life and interpret the results in the context of the situation.

As this is a modeling standard, students need to encounter equations and inequalities in two variables that evolve from real-world situations. Students should be formulating equations and inequalities, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions to justify solutions to real-world problems. Real-world situations should elicit equations and inequalities from situations which are linear, quadratic, and absolute value in nature. It is imperative that students have the opportunity to work with each of these equation types equally.

Level 4:

One of the most natural situations for students to create an equation, inequality, or graph from is a real-world situation. Students need to be exposed to a variety of real-world situations that elicit the function types embedded within the Algebra I course. Students should encounter real-world problems that are increasingly more complex over time. They should be using the modeling cycle to develop and provide justifications for their solutions. The modeling cycle involves identifying variables in a situation, model by creating an algebraic representation, performing operations, interpreting the results in terms of the context, validating conclusions by comparing them with the original situation and then either improving upon the model or reporting on the reasoning behind the conclusions.

Additionally, students should be asked to generate a real-world situation that could be solved by a provided equation or inequality. Students with this capability are demonstrating a deep understanding of the interplay that exists between the situation and the equation or inequality used to solve the problem.

Algebra

Creating Equations* (A.CED)

Standard A1.A.CED.A.3 Cluster Heading: A. Create equations that describe numbers or relationships.

Create individual and systems of equations and/or inequalities to represent constraints in a contextual situation and interpret solutions as viable or non-viable.*

Scope and Clarifications: (Modeling Standard)

For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify unknown values in a real-world situation. Describe the difference of a viable solution and a non-viable solution.	Choose an equation or inequality that models the constraint on a variable given a contextual problem. Determine when a solution would be viable or non-viable, given an equation or inequality that represents a real-world problem. Determine the viability of each solution, given an equation or inequality that represents a contextual situation and a set of possible solutions.	Write an equation or inequality that models the constraint on a variable given a contextual situation. Write a system of equations or inequalities that models the constraint on a variable given a contextual situation. Explain constraints on a variable in the context of a real-world situation. Justify solutions that model a real-world situation when there are limitations on a variable. Interpret solutions as viable or non-	Create and provide a solution to a real-world problem that has natural limitations on variables. Explain the solution and its viability using multiple representations (i.e., table, graph, equation) and precise mathematical language. Explain examples of both viable and non-viable solutions in context of a real-world problem. Use multiple representations to justify a solution's viability and explain when one representation elicits a more efficient justification.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		viable options in a contextual situation.	

Instructional Focus Statements

Level 3:

Students begin to develop a conceptual understanding of creating equations that represent contextual problems by using a symbol for the unknown as early as kindergarten. In Grade 8 (8.EE.C), students analyze and solve linear equations, and systems of two linear equations. In Algebra I, students expand on that knowledge of variables and equations by identifying and quantifying any limitations present on the value of variable amounts within real-world problems prior to solving the problem. Instruction for this standard should be integrated with the reasoning with equations and inequalities cluster of standards (A1.A.REI).

Through this modeling standard, students should explore the impact of constraints on variable amounts. A constraint is something that limits what is possible or what is reasonable in a situation. Students should recognize that real-life situations often involve constraints and that they can use expressions, equations, and inequalities to represent the constraints. For example, distance above the ground d , in meters, might be constrained to be non-negative, expressed by $d \geq 0$. Students should also be able to define constraints and write an equation, inequality, or system of equations or inequalities that represents the constraint. Students often have a difficult time creating equations or inequalities that represent constraints. They often can verbalize the constraint but have a difficult time translating it into a mathematical representation. Using multiple representations of the contextual situation can help students see how the constraint effects the problem. For example, students should have the opportunity to explore the impact of the constraint algebraically, graphically, and numerically.

Instruction should provide a variety of contexts with variable limitations and allow students to explain the solutions in context of the real-world problem. Students should experience both viable and non-viable solutions and make sense of them with respect to the contextual problem (MP1). Students should experience problems containing exclusively viable solutions, exclusively non-viable solutions, and problems that generate both viable and non-viable solutions. Students should experience problems where they decide the best way to report the solution. For example, when a student obtains a solution to a problem involving the number of animals and the answer includes a fractional part, the student should make sense of the fractional part and determine the best way to report the number of animals. Instruction should focus on students reporting solutions in context of the problem as this encourages students to make sense of and justify the viability of the solution (MP3). Since this is a modeling standard, students should have ample opportunity working with applications of equations and inequalities with real-world constraints (i.e., volume and linear programming).

Level 4:

Instruction at this level should provide opportunities for students to explore in-depth the impact of constraints and natural limitations on variables for a real-world problem. Students at this level of understanding should be given the opportunity to design their own real-world problems that would have constraints on the unknown variables. Furthermore, students should be provided with the opportunity to critique the solutions of others (MP3). Students need multiple opportunities to justify their thinking by developing and providing logical arguments for the viability of solutions.

Algebra

Creating Equations* (A.CED)

Standard A1.A.CED.A.4 **Cluster Heading: A. Create equations that describe numbers or relationships.**

Rearrange formulas to isolate a quantity of interest using algebraic reasoning.*

Scope and Clarifications: (Modeling Standard)

Tasks are limited to linear, quadratic, and absolute value expressions.

For example, rearrange the formula for the perimeter of a rectangle to isolate the length or width.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve linear equations with rational coefficients. Solve equations that require finding the square root of a number.	Choose equivalent forms of a given linear real-world formula. Solve linear, quadratic, and absolute value equations and explain the solution path.	Describe the process that allows an equation to be written in an equivalent form. Rearrange real-world linear, quadratic, or absolute value formulas to highlight a quantity of interest.	Rearrange real-world linear, quadratic, and absolute value formulas and explain the benefit of solving the formula for the various variables.

Instructional Focus Statements

Level 3:

Instruction for this standard should be integrated with A1.A.REI.A.1 where students understand solving equations as a process of reasoning and explain the reasoning. The properties used to solve equations apply regardless of how many variables appear in an equation or formula. Variables are placeholders for numbers and as such have the same properties.

Students understand that a relationship between two or more quantities can be expressed in multiple ways by writing equivalent equations. Students should discover that depending on the quantity in which they are interested, one form of equation might be more useful than others. To highlight a quantity of interest may mean manipulating or rearranging a given equation. The rearrangement may involve solving for a variable - isolating it and

defining it in terms of the other variables. Students should notice that solving for a variable can be an efficient way to solve problems. Students should reason repeatedly and look for regularity as they manipulate equations and solve for a variable (MP8).

In Algebra 1, focus on real mathematical and scientific formulas including geometric formulas for area or perimeter or scientific formulas for energy or temperature. For example, rearrange the formula for the perimeter of a rectangle ($P = 2l + 2w$) to isolate the length or width. Allow students to discover that there may be multiple solution paths to arrange a formula and more than one equivalent equation to isolate a quantity of interest. Width can be isolated in the formula for the perimeter of a rectangle by factoring first and generating the equation $w = \frac{P}{2} - l$ or subtracting a term first and generating the equivalent equation $w = \frac{P-2l}{2}$. Facilitate classroom discussion around the different ways that the formula can be viewed as defining one quantity as a function of the other. Students should develop conceptual understanding of why they might need to write formulas in different ways and what the benefit would be to these various representations of the same real-world formula. This may be a good opportunity to collaborate with science teachers and ask them for formulas that they use often.

Level 4:

Students should be exposed to a wide variety of real-world formulas increasing in complexity over time. Additionally, it is imperative that they can explain why formulas might need to be expressed in different ways and the benefit that each form provides. At this level, facilitate discussion where students to put into words what the original equation and rearranged equations say in relation to the given context. Students should construct a viable argument as to why they might be interested in highlighting a particular quantity of interest and how this is beneficial within the context of the formula (MP3).

Algebra

Reasoning with Equations and Inequalities (A.REI)

Standard A1.A.REI.A.1 **Cluster Heading: A. Understand solving equations as a process of reasoning and explain the reasoning.**

Understand solving equations as a process of reasoning and explain the reasoning. Construct a viable argument to justify a solution method.

Scope and Clarifications:

Tasks are limited to linear, quadratic, and absolute value equations.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the inverse operations used in solving the equation, given a linear, quadratic, or absolute value equation and a list of steps representing the solution path. Choose a possible next step to solve the equation, given a linear, quadratic, or absolute value equation and a partial list of steps representing the solution path. Arrange steps in the order they should be applied to solve an equation, given a linear, quadratic, or absolute value equation and a list of unordered steps representing the solution path.	Explain the reasoning for each step, given a linear equation and a list of steps representing the solution path. Explain the reasoning for each step, given a quadratic equation and a list of steps representing the solution path. Explain the reasoning for each step, given an absolute value equation and a list of steps representing the solution path.	Solve linear, quadratic, and absolute value equations using multiple solution strategies and explain the solution path. Construct a viable argument to justify a chosen solution path used to solve a linear, quadratic, and absolute value equation. Determine an efficient solution path, given a linear, quadratic, or absolute value equation with multiple solution paths.	Solve the problem and justify the solution path chosen, given a real-world problem and an equation that represents the contextual situation. Compare and contrast two given solution paths to a contextual problem and construct a viable argument on which method is most efficient. Correct the mistakes in the solution path and provide an explanation of the misconception using precise mathematical vocabulary, given a solution for a linear, quadratic, and absolute value equation.

Instructional Focus Statements

Level 3:

In grade 8 (8.EE.C.7), students solve linear equations in one variable, including equations with one solution, infinitely many solutions, and no solutions. In Algebra I (A1.REI.B.2 and A1.REI.B.3) students begin working with quadratic and absolute value equations. Students should develop a conceptual understanding of solving equations as a reasoning process to determine a solution that satisfies the equation rather than a procedural list of steps. Solving an equation involves writing a series of equivalent equations that eventually isolates the variable on one side. Instruction should focus on students creating and determining solution paths for each unique equation and providing a viable argument to justify the chosen solution path (MP3). Students should also be able to explain how, when, and why equations have no solution or infinitely many solutions. To give meaning to these solution types, discussion should focus on the solution being a value of the variable that makes the equation true. This will help students make the connections that an equation has no solution because there is no value that can maintain equivalency and an equation has infinitely many solutions because all values used for the variable create a true equivalency statement.

Students should understand that a problem can have multiple entry points and instruction should be focused on solving equations using a reasoning process centered around inverse operations, order of operations and properties of operations. Students develop a conceptual understanding of operations in previous grades and they should deepen their understanding of the interplay that exist between the operations. Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. To illustrate maintaining equivalency, a visual and/or concrete model of a balance scale can be used to aid students in understanding that the same inverse operations are being applied to each side of an equation to maintain equivalency. For example, adding the same term to both sides of an equation or multiplying both sides of an equation by a non-zero constant produces an equation with the same solution. Emphasizing equivalency is vital in developing a conceptual understanding of solving equations and preventing common misconceptions.

A common misconception when solving absolute value equations is to not isolate the absolute value expression on one side of the equation before solving. Absolute value equations are split into two cases to solve, one each for if the expression within the absolute value symbols is negative and non-negative because absolute value refers to the distance of a number from zero, regardless of direction. Therefore, the absolute value expression needs to be isolated first before splitting the equation into two cases. For example, when solving the absolute value equation $|2x - 3| - 4 = 3$, students should first apply an inverse operation by adding 4 to each side of the equation, resulting in $|2x - 3| = 7$, before splitting into two cases and continuing to use reasoning to solve for x .

Students should understand that the solution path they choose to solve any equation must result in a series of equivalent equations all of which have the same solution set. As students apply mathematical properties and inverse operations to solve equations, they should be able to explain why equality holds true when performing the selected operation to both sides of the equation. In this course, students should be exposed to linear, quadratic, and absolute value equations, as other function types will be explored in Algebra II.

Level 4:

As students develop a deeper understanding of solving equations and explaining their solution methods, instruction can be integrated with the application in contextual situations. Students should be able to construct equations that represent a contextual situation as well as create contextual situations to represent a given equation. As students develop a deep understanding of the relationship that exists between the type of equation and the context, they can be given equations embedded in real-world situations. When they are given a contextual situation and an equation, students should be able to determine what each part of the equation represents as it relates to the context. They should also be able to solve the equation and create a viable argument to justify their solution path. Students should understand that there are various ways to solve problems and justifying their steps will help them solidify their understanding of solving equations as well as the most efficient solution path. This standard pairs nicely with A1.A.CED.A.1, as it supports the idea of making connections between an equation and its context.

To challenge students to follow a thought process other than their own, they can be asked to critique or correct the solution paths of others. Students will develop a deeper level of understanding if they are given solution paths with incorrect steps in the process or invalid justifications and asked to correct the process or write justifications and defend them.

Algebra

Reasoning with Equations and Inequalities (A.REI)

Standard A1.A.REI.B.2 **Cluster Heading: B. Solve equations and inequalities in one variable.**

Solve linear and absolute value equations and inequalities in one variable.

A1.A.REI.B.2a Solve linear equations and inequalities, including compound inequalities, in one variable. Represent solutions algebraically and graphically.

A1.A.REI.B.2b Solve absolute value equations and inequalities in one variable. Represent solutions algebraically and graphically.

Scope and Clarification:

Equations and inequalities should include integer, rational, and/or irrational coefficients. If coefficients are irrational, rationalization of a denominator is not required. Tasks may or may not have a real-world context.

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Solve equations in the form $x + p = q$, $px = q$, $x - p = q$, or $x/p = q$ when p, q, and x are all non-negative rational numbers. Determine the solution set to a linear inequality in one variable by graphing on a number line.	Solve linear equations in one variable with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. Determine if a linear equation in one variable has one solution, infinitely many solutions, or no solutions. Solve linear inequalities in the form $px + q > r$ or $px + q < r$.	Solve multi-step linear equations and inequalities in one variable with integer, rational, and/or irrational coefficients. Solve compound inequalities in one variable. Represent solutions algebraically and graphically on a number line. Solve absolute value equations and inequalities in one variable. Represent solutions algebraically and graphically on a number line.	Create a linear or absolute value equation in one variable with a determined number of solutions and provide mathematical justification as to why. Explain the difference between an equation and an inequality and provide examples when each would be used in a real-world context.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Solve linear and absolute value equations in one variable and determine the number of solutions.	

Instructional Focus Statements

Level 3:

In grade 8 (8.EE.C.7), students solved linear equations in one variable with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. Students also determined if a linear equation in one variable has one solution, infinitely many solutions, or no solutions. In Algebra 1, students extend their understanding of linear equations and inequalities to solve absolute value equations and inequalities. While this standard calls for procedural fluency, students should continue to apply the conceptual understanding developed in A1.A.REI.A.1 that solving an equation is a process of reasoning. With this understanding, students can organize the various strategies for solving equations and inequalities into a coherent picture. Students should be presented with a variety of equations and inequalities, with coefficients that include rational numbers such as integers, fractions, and decimals, as well as irrational coefficients. Rationalization of a denominator is not required in Algebra 1.

Students should apply what they know about solving linear equations in one variable to solving equations containing an absolute value. In grade 6 (6.EE.C.7c), students learned that the absolute value refers to the distance of a number from zero, regardless of direction. Therefore $|x| = 5$, has two solutions, $x = 5$ and $x = -5$, because both solutions represent the same distance from zero. To solve an absolute value equation such as $|x + 2| = 7$, students should split the equation into two cases, one each for if the expression within the absolute value symbols is negative and non-negative (meaning positive or zero). This creates two new equations, $x + 2 = -7$ and $x + 2 = 7$, which can each be solved as linear equations, making the solution $x = -9, 5$. Graphically, the solution can be represented using two points on a number line. Students should be able to explain how, when, and why absolute value equations in one variable have no solution or infinitely many solutions. When solving absolute value equations, if the result is a positive number or zero, then there is at least one solution. An absolute value equation has no solution if the absolute value expression equals a negative number since an absolute value can never be negative.

In grade 8 (8.EE.C.9), students determined the solution set to a linear inequality in one variable by graphing on a number line. In Algebra 1, students extend their understanding of inequalities to solve absolute value and compound inequalities. Where the solution to an absolute value equation is points on a number line, the solution to an absolute value inequality will be intervals on a number line. For the inequality $|x| < 3$, students are finding all of the x -values that are less than three units away from zero in either direction, so the solution is the set of all points that are less than three units away from zero. The solution can be written as the compound inequality, $-3 < x < 3$.

Students should understand that a compound inequality is an inequality that combines two simple inequalities that are separated by either “and” or “or.” A compound inequality with an “and” can include the word “and”, such as $x < 3$ and $x > -3$, as well as written as $-3 < x < 3$. The graph of a compound inequality with an “and” represents the intersection, or overlap, of the solutions for each inequality. A value is a solution to the compound inequality only if the value is a solution to both inequalities. The graph of a compound inequality with an “or” represents the union of the graphs of the inequalities. A value is a solution to the compound inequality if the value is a solution to at least one of the inequalities. The solution is the combination, or union, of the two individual solutions.

To build procedural fluency from conceptual understanding to solve absolute value inequalities, students should be given opportunities to connect the visual representation of the number line to the verbal concept of distance to the abstract symbolic form, therefore preventing the process from becoming simply a series of steps to follow. For example, $|x - 3| > 4$ can be read verbally as “the distance from x to 3 is greater than 4.” From the visual number line representation, students can determine that the solution is $x > 7$ and $x < -1$. When solving an absolute value inequality, it's necessary to first isolate the absolute value expression on one side of the inequality before solving the inequality.

These processes should be taught with the mathematical reasoning found in A1.A.REI.A.1. Students should not be presented with a list of steps to solve equations or inequalities. It is more effective for students to be taught the mathematical reasoning for the solving process as these concepts can be applied to all types of equations and inequalities. The process of solving equations and inequalities integrates nicely with A1.A.CED.1 where students learn how to create equations and inequalities in context. Connecting these standards deepens students' knowledge of the purpose for solving equations and inequalities in one variable.

Level 4:

Students should solidify their understanding of the many ways to solve equations and inequalities and supply mathematical justification to the decisions they make. Ask students to articulate the difference between an equation and an inequality and provide real-world examples of when each might be used to solve problems. At this level, students can not only solve equations and inequalities, but they can articulate the mathematical underpinnings of the various strategies of how to solve. Additionally, they can make connections the various strategies, as well as justify how the algebraic and graphical representations of solutions are related. Challenge students to create their own linear or absolute value equations in one variable with one solution (or two possible if absolute value), no solution, or infinitely many solutions and mathematically justify the number of solutions.

Algebra

Reasoning with Equations and Inequalities (A.REI)

Standard A1.A.REI.B.3 **Cluster Heading: B. Solve equations and inequalities in one variable.**

Solve quadratic equations and inequalities in one variable.

A1.A.REI.B.3a Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when a quadratic equation has solutions that are not real numbers.

A1.A.REI.B.3b Solve quadratic inequalities using the graph of the related quadratic equation.

Scope and Clarifications:

Tasks may or may not have a real-world context.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the relationship between squaring a number and taking the square root of a number as inverse operations. Understand that a square root generates two values (positive and negative) when taking the square root of a non-zero number. Solve equations that require finding the square root of a number in the form $x^2 = p$, where p is a positive rational perfect square number. Determine if a number is rational or irrational.	Factor a quadratic expression to generate an equivalent expression. Identify equivalent forms of quadratic expressions. Use coefficients from the standard form of a quadratic equation appropriately in the quadratic formula. Choose the possible intervals that could be solutions given a graph to a quadratic inequality in one variable.	Solve a quadratic equation in one variable by inspection or taking square roots. Know and apply the quadratic formula to solve a quadratic equation in one variable. Use factoring to solve a quadratic equation in one variable. Recognize when a quadratic equation in one variable has real solutions or nonreal solutions. Use the graph of the related	Solve one variable quadratic equations, identify the strategy chosen, and explain why the chosen strategy best suits the initial form of quadratic equation. Explain why an infinite set of solutions bounded within an interval is the solution to a quadratic inequality. Create an example of when a quadratic equation or inequality would be used in a real-world context.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the square root of an irrational non-perfect square.		quadratic equation to solve a simple quadratic inequality in one variable. Solve quadratic equations and inequalities within a real-world context.	

Instructional Focus Statements

Level 3:

In grade 8, students used square root symbols to represent solutions to equations in the form $x^2 = p$ (8.EE.A.2). In Algebra I, students build on this understanding to solve quadratic equations. It is important that, just like for linear equations, students apply the understanding developed in A1.A.REI.A.1 that solving any equation is a process. Students should interact with the multiple strategies for working with quadratic equations internalizing when each strategy is most appropriate and discovering when each strategy is most efficient. The strategies should not live in isolation but as a part of a larger conversation around the merits of each. A quadratic equation is an equation with a variable taken to the second degree. Students should use their knowledge of perfect squares to solve by inspection or using the square root as an inverse operation and understand that a square root generates two values (positive and negative).

Students first learn to factor quadratic expressions in Algebra I. Instruction for this standard should be connected to A1.F.IF.C.8 where students rewrite quadratic functions in equivalent forms to reveal properties of the function and its graphical representation. Student understanding of factoring should draw on connecting that a quadratic expression and a factored form of the same expression are equivalent. The factored form of a quadratic expression can be used to identify the zeros of the quadratic function, tied to the graphical representation, reveals the x-intercepts of the graph as well as the solutions to the quadratic equation. Students should also be able to explain the meaning of the solutions if working within a real-world context. When solving quadratic equations by factoring, ensure that students understand why they must first rewrite the equation as a quadratic expression equal to zero.

In Algebra I, students should know the quadratic formula and be able to apply it to solve a quadratic equation. At this level, students are not expected to use the process of completing the square to derive the quadratic formula, however, teachers may provide a demonstration to support student understanding of the formula. Students should understand how to use coefficients from the standard form of a quadratic equation ($y = ax^2 + bx + c$) appropriately in the quadratic formula. Students should be able to recognize that quadratic equations may have one solution, two solutions, or no real solution. When solving $x^2 = 49$ by inspection or using square roots, students understand that there are two real solutions, positive 7 and negative 7. When

applying the quadratic formula, the radicand ($b^2 - 4ac$) is called the “discriminant” because by using its value, students can “discriminate” between various solution types. For example, if the value of the discriminant is zero, the quadratic equation has one solution. When the discriminant is negative, students can determine that there are no real solutions. Since a value taken to the second degree is either positive or zero, negative numbers do not have real number square roots. At this level, students are not required to find complex square roots, but they should be able to recognize when a quadratic equation has nonreal solutions. When given a quadratic equation within a real-world context, students should understand that any solution(s) must be real numbers.

In grade 8 (8.EE.C.9), students developed an understanding of linear inequalities utilizing both a number line for one variable inequalities and the coordinate plane for two variable inequalities. Students should build on both their prior knowledge of linear inequalities and their developing understanding of quadratic equations to begin developing a conceptual understanding of how to solve quadratic inequalities in one variable, what the solutions to quadratic inequalities look like graphically, and why they look that way. Students should use these same strategies for solving quadratic equations as they are exploring the boundaries of solution intervals for quadratic inequalities. Instruction should focus on student exploration of solutions for simple quadratic inequalities in one variable where they discover the points that segment the graph of the inequality into regions and connect that back to the quadratic inequality. Students should examine the graph to determine which intervals hold viable solutions for the inequality. Students should develop an understanding that the solutions to a quadratic inequality are an infinite number of points within intervals. If presented in a real-world context, students should consider whether the solutions are reasonable for the given situation. The intent of this standard is for students to examine the graph and determine the intervals in which the graph is above or below a given line.

Level 4:

Students should solidify their understanding of the many ways to solve quadratic equations by supplying mathematical justification to the decisions they make. At this level, students can not only solve quadratic equations, but they should be able to articulate the mathematical underpinnings of the various strategies. Additionally, students should be able to make connections between the various strategies in solving quadratic equations and explain why a chosen strategy best suits the initial form of quadratic equation. Students can be challenged to create their own real-world contexts that would require solving quadratic equations and inequalities.

To solidify the understanding of solving quadratic inequalities, students should be able to explain how the solution(s) to the related quadratic equation are connected to the solution(s) to the quadratic inequality. Students should also be able to explain why an infinite set of solutions bounded within an interval is the solution to a quadratic inequality.

Algebra

Reasoning with Equations and Inequalities (A.REI)

Standard A1.A.REI.C.4 **Cluster Heading: C. Solve systems of equations.**

Write and solve a system of linear equations in real-world context.*

Scope and Clarifications: (Modeling Standard)

Systems are limited to at most two equations in two unknowns.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the solution to a system of linear equations in two variables given a graphical representation of the system. Identify the solution to a system of linear equations in two variables given a tabular representation of the system. Use substitution to determine if a given solution satisfies a given system of linear equations.	Solve a system of linear equations by graphing. Determine if two linear equations create parallel lines, the same line, or intersecting lines. Analyze a system of linear equations to determine if there is one solution, no solution or infinitely many solutions.	Solve a system of linear equations in two unknowns algebraically using the substitution method. Solve a system of linear equations in two unknowns algebraically using the elimination method. Write and solve a system of linear equations in two unknowns given a real-world context. Interpret the solution of a system of linear equations in two unknowns in relationship to a context.	Create a real-world scenario to represent a system of linear equations in two variables. Determine if the solution to a system of linear equations in two variables is reasonable in relationship to a real-world context. Justify whether the substitution or elimination method would be most efficient to solve a system of linear equations in two variables.

Instructional Focus Statements

Level 3:

In grade 8 (8.EE.C.8), students analyzed and solved systems of two linear equations graphically. Students should understand that the solution to a system of linear equations is the point (ordered pair) at which two linear graphs intersect. In Algebra 1, students will build on this understanding to begin solving systems of linear equations algebraically. Instruction should include the use of multiple representations consisting of graphs and tables to help students conceptualize the solution and support their ability to solve a system of linear equations in two variables algebraically. When focusing on the graphical representation, students should be able to approximate the intersection of the graphs created by hand and with technology. When creating a graph by hand, students should begin with solutions that lie on integer points on the graph and move on to solutions that do not lie on integer points and discuss approximate solutions that could exist. The calculation of the intersection on a graphing calculator may be used to allow students to continue this exploration of approximate points of intersection. Additionally, students should solidify their solutions by substituting their approximated solutions into the system of linear equations in two variables, which will foster the importance of accuracy in the solutions.

Algebraically, students should have opportunities to utilize multiple methods of solving a system of linear equations in two variables. Students should be introduced to the substitution method by relating to their understanding of evaluating expressions at specific values for their variables. Building on this idea can help solidify the understanding of solving for a variable and then substituting the equivalent expression in for that variable in the other equation. The elimination method should be introduced based on the concept that we cannot solve for more than one unknown value at a time. Instruction should begin with a system of linear equations where one variable has opposite values for the coefficients and results in zero when added together. This will allow students to understand how to "eliminate" one variable in order to solve for the other variable. Students should then engage in problems where a variable cannot be initially eliminated and make use of rewriting the equations in equivalent forms that will lend to one variable having opposite value coefficients by using the multiplication property of equality. Emphasis needs to be placed on the importance of finding both variables since the solution is an ordered pair which correlates to the point where the two equations intersect on the graph. Students should solidify their understanding that the solution is the value of the variables that satisfies both equations. It's important for students to understand why there is a need for multiple approaches to solving a system of equations. Encourage students to look for and assess the structure of the system of equations to determine whether using the substitution method, elimination method or solving graphically might be the most efficient and justify their reasoning.

Students should have opportunities to work with equations and context that include rational numbers in a real-world situation. Students should be expected to define variables and write pairs of linear equations in two variables to represent a given context. Students should have opportunities to engage in real-world problems in which they must determine if an approximation of a solution using a graph or an exact solution using other methods is most appropriate for the problem. Students should differentiate among problems where there is one solution, no solutions, or infinitely many solutions and justify the meaning for these results. Additionally, students should understand that one solution is represented by two graphs that intersect at one point, no solution is represented by two graphs that are parallel and never intersect, and many solutions is represented by two graphs that lie on the same line and are infinitely intersecting. In Algebra II, students will expand their work to include a system consisting of a linear equation and a quadratic equation (A2.A.REI.B.3), as well as a system of three linear equations with three variables (A2.A.REI.B.4). Students will use matrices to solve linear systems in Algebra 2, but Algebra 1 does NOT include solving using augmented matrices/technology.

Level 4:

Given a system of linear equations in two variables, students should be able to write a real-world scenario to represent the system and construct a graphical model either by hand or using technology. Students should also be able to determine the reasonableness of the solution and then justify their thinking. Instruction should also allow students to demonstrate a deeper level of understanding by choosing the most appropriate algebraic or graphical method for solving a system of linear equations. Students should be expected to justify their reasoning and explain the steps in their chosen solution path.

Algebra

Reasoning with Equations and Inequalities (A.REI)

Standard A1.A.REI.D.5 **Cluster Heading: D. Represent and solve equations and inequalities graphically.**

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Plot the ordered pairs on a coordinate plane, given a table of values. Determine if points on a graph form a line. Represent the set as a table of values, given a set of ordered pairs. Use technology to produce the graph of a given equation. Give an equation in the form $y = mx + b$ or $y = mx$ to represent a line graphed on a coordinate plane.	Find corresponding values of y, given an equation and a set of values of x. Graph ordered pairs to identify whether they represent a linear or non-linear equation. Understand that the solution to an equation in two variables is an ordered pair. Represent a linear equation with an integer-valued slope in two variables graphically on a coordinate plane.	Find a set of solutions that can be used to create the graph, given an equation. Interpret the graph of an equation as the solution set to the equation with two variables. Understand how the coordinates of the points on a graph are related to a given equation. Explain the relationship between the graphical representation and the solutions (ordered pairs) to the equation, given a real-world situation.	Analyze the pattern of the points to determine the type of equation represented by the graph, given a set of ordered pairs. Determine if the graphical representation for a real-world situation is continuous or discrete and justify the reasoning.

Instructional Focus Statements

Level 3:

Students began understanding graphs of linear equations in two variables in grade 8 (8.EE.B.6 and 8.F.A.3) and build on that knowledge to include quadratic, absolute value, and other non-linear equations in Algebra 1. As students increase their understanding of graphs, they should begin to make the connection between the solutions of an equation and its graphical representation. Students should understand that the graph of an equation is the set of all ordered pairs that make that equation a true statement. Students can explain and verify that every point (x, y) on the graph of an equation represents all values for x and y that make the equation true. Students should understand that an equation in two variables has many possible solutions, all of which lie on the curve or line of the graph of that equation.

It is imperative that instruction focus on the multiple representations to make connections between the equation, the table, and the graph throughout the learning process. From an equation, students should be able to find the ordered pairs that are solutions to the equation and generate a table and a graph representing those solutions. Allowing students to choose their own input values can create a variety of different tables that all represent the same equation. This can foster classroom discourse to help students to develop the understanding of the infinite amount of points that lie on a line or curve. In connection with the Interpreting Functions cluster of standards, students can begin to understand how the context of a problem determines whether the solutions should be graphed discretely or continuously. Students often interpret solutions at only integer values but should be encouraged to work with all types of rational numbers. Instruction should provide opportunities for discussion of all solution sets and ways to represent these values on the coordinate plane. As students evaluate graphs, they should be able to determine if all possible solutions are represented to discover the meaning of a continuous line or curve.

As students solidify their understanding of graphical representations, they should extend their learning to include real-world problems. When solutions are graphed, they should be able to determine if the graphical representation of the real-world problem is continuous or discrete and justify their reasoning.

Level 4:

Instruction can extend students learning by making connections to real-world problems. When given a table that represents real-world data, discussion could be extended beyond possible solutions to solutions that fit the context. Students should have experience with recognizing limits of an equation within context and making connections to the context effecting whether solutions would be continuous or discrete. For example, if the original input represents time, it should be understood that time cannot be negative thus the equation could not be used for negative x -values.

Algebra

Reasoning with Equations and Inequalities (A.REI)

Standard A1.A.REI.D.6 **Cluster Heading: D. Represent and solve equations and inequalities graphically.**

Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. Find approximate solutions by graphing the functions or making a table of values, using technology when appropriate.*

Scope and Clarifications: (Modeling Standard)

Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, absolute value, and exponential functions. For example, $f(x) = 3x + 5$ and $g(x) = x^2 + 1$.

When finding solutions approximately, students may be expected to produce graphs of functions that are linear, quadratic, exponential, or absolute value, but may be given graphs of other function types.

Exponential functions are limited to domains in the integers.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent a linear equation with an integer-valued slope in two variables graphically on a coordinate plane. Estimate solutions to a system of equations by graphing. Identify solutions to a system of linear equations by inspecting graphs.	Identify the solution(s) for $f(x) = g(x)$ when $f(x)$ and $g(x)$ are linear, quadratic, absolute value or exponential, given graphs of two equations $f(x)$ and $g(x)$. Graph linear, quadratic, absolute value and/or exponential functions (with and without technology). Generate a table of values for linear, quadratic, absolute value and/or exponential functions (with and without technology).	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. Approximate the solution(s) for $f(x) = g(x)$ by graphing, making a table of values or using technology. Approximate the solution(s) for $f(x) = g(x)$ when $f(x)$ and $g(x)$ are linear, quadratic, absolute value or exponential, given two equations $f(x)$ and $g(x)$.	Approximate the solution(s) for $f(x) = g(x)$ and explain the meaning of the solution(s) in terms of a real-world context when given two equations $f(x)$ and $g(x)$ embedded in a real-world situation. Create a contextual situation for the equations $y = f(x)$ and $y = g(x)$, approximate the solution(s) for $f(x) = g(x)$ and explain the meaning of the solution(s) in terms of a real-world context.

Instructional Focus Statements

Level 3:

In grade 8 (8.EE.C.8), students analyze and solve systems of two linear equations graphically. They understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. Students expand on this understanding in Algebra I to work with various function types, including linear, quadratic, absolute value, and exponential functions.

To conceptually understand what it means to find the solution to two equations using graphing, students should make connections between visual (graph and tables) and symbolic (algebraic) representations. For example, provide opportunities for students to see the connection between graphs and tables of two functions and the coordinate points they have in common. Students can then connect the idea of two equations in two variables to how that relates to a single equation in one variable to understand how both connect to a point(s) on a coordinate plane which is built around two variables. This leads to explaining why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. Draw students' attention to the point where the two graphs intersect and its corresponding representation in function notation. As this is a modeling standard, students should be formulating equations, producing graphs, computing solutions, interpreting findings to validate their thinking and understanding how the various representations are related.

In Algebra I, students write and solve linear systems of equations algebraically (A1.A.REI.C.4). Students should find approximate solutions to all other systems using graphs, tables, and technology, when appropriate. Students should understand that they can graph the functions created by the expressions on the left side and the right side of an equation as two curves (or lines). Students should produce graphs of functions that are linear, quadratic, exponential or absolute value. For other function types, students will be given graphs and asked to find approximate solutions from the provided graphical representation. In developing an understanding of what it means to find the solution to two equations using graphing, it is important that students are provided opportunities to reason through process utilizing multiple representations.

Level 4:

Students should continue to be exposed to a wide variety of linear, quadratic, absolute value, and exponential functions with increasing difficulty embedded in real-world situations. Facilitate classroom discussion that asks students to make connections between the point where the two graphs intersect, its meaning in context, and its corresponding representation in function notation. Given equations embedded in a real-world context, students should be able to explain the meaning of the solution in terms of the context and justify the reasonableness of attained solutions within the context of real-world situations. Students should also be challenged to create and solve their own contextual situations for the equations $y = f(x)$ and $y = g(x)$.

Algebra

Reasoning with Equations and Inequalities (A.REI)

Standard A1.A.REI.D.7 **Cluster Heading: D. Represent and solve equations and inequalities graphically.**

Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify if a point is a solution to a linear inequality in two variables. Identify if a point is a solution to a system of linear equalities in two variables.	Choose the graphical representation of the solution to a linear inequality in two variables. Choose the graphical representation of the solution set to a system of linear inequalities in two variables.	Graph the solution set to a system of two linear inequalities in two variables as the intersection of the corresponding half-planes.	Graph the solution set to a system of more than two linear inequalities in two variables.

Instructional Focus Statements

Level 3:

In grade 8 (8.EE.C.9), students learned to graph solutions to linear inequalities in two variables on a coordinate plane. Students also learned to solve systems of two linear equations graphically in grade 8 (8.EE.C.8). Instruction in Algebra I should focus on extending a student's understanding of graphing linear equations and inequalities in two variables on a coordinate plane to graphing the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. Students should extend their understanding of solutions to systems of linear equations (A1.A.REI.C.4) to understand that the solutions to a system of linear inequalities are all points in the region where the graphs of the two inequalities overlap, because those points represent all pairs of values that make both inequalities true.

Students should make the connection as to why inequalities have multiple solutions. If students are unsure whether an ordered pair represents a solution to the system of inequalities, ask students to pick a point on the graph and consider whether it satisfies the first inequality, and then whether it satisfies the second inequality. Remind students that a solution to a system of inequalities needs to satisfy both. It is important that students understand why they are shading not simply following a set of steps without conceptual understanding. Students may have a misconception that an inequality with a $<$ symbol will be shaded below the boundary line and that an inequality with a $>$ symbol will be shaded above it. Emphasize that we cannot assume that the $<$ or $\leq$ symbol means shading below a line when the equation is not presented in slope-intercept form. It is important to test points on either side of the line to see if the pair of values make the inequality true, or to reason carefully about the inequality statement and think about pairs of values that would satisfy the inequality.

Facilitate classroom discussion regarding points on the boundary lines and how students determined if they are or are not solutions to the system. Highlight explanations that state that a solution to a system of linear inequalities must be a solution to every inequality in the system. If a point on the boundary line is not included in the solution set of one inequality (so the graph is a dashed line), then it is also not included in the solution set of the system.

Level 4:

As students develop a strong command of systems of linear inequalities in two variables, they need to experience a wide variety of systems increasing in difficulty including those with more than two inequalities including those having no solution. A system of linear equations has no solutions if the graphs of the equations are two parallel lines that never intersect, therefore, a system of linear inequalities has no solutions if their regions are bound by two parallel lines and the solution region of each one is on the "outside" of the parallel lines.

Functions

Interpreting Functions (F.IF)

Standard A1.F.IF.A.1 **Cluster Heading: A. Understand the concept of a function and use function notation.**

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

Scope and Clarification:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine if a given table of values represents a function. Identify the domain as the set of input values. Identify the range as the set of output values.	Determine if a given graph represents a function. Find $f(a)$ where a is a real number when given a function f.	Create an example of a function using a set of ordered pairs, a graph, and a table of values to show the correspondence between one input value (domain) and one output value (range). Explain the meaning of a function using correct mathematical vocabulary.	Identify the domain and the range and determine if the relationship represents a function, given a real-world situation. Determine if the domain and range is continuous or discrete and explain your reasoning, given a real-world situation. Create a real-world situation that does not represent a function and explain the reasoning.

Instructional Focus Statements

Level 3:

In grade 8 (8.F.A.1), students used informal notation for functions and referred to the values used in those functions as input and output values. In Algebra I, students build on that understanding and begin using function notation and the mathematical language that describes a function. Students should make the connection that input and output correlate respectively with the mathematical terms of domain and range. Students should understand that a function is a special relationship that assigns one input value (domain) to exactly one output value (range) in a pair of elements. Students should also recognize that in a function each element of the domain is unique, however elements of the range may repeat themselves. A common misconception is that students often interpret a function as each value of the range can only be paired with one domain value. It is important that students have multiple opportunities to explore functions and their ordered pairs as well as their graphs to understand a function.

Students should be able to determine if a relation is a function by examining sets of ordered pairs, graphs, or tables of values. As students begin to examine graphs to determine if they represent functions, using the vertical line test may be helpful. This does not provide a robust definition of a function that transfers well into other situations in future courses but provides a concrete tool for students to use as they begin their study of graphs of functions. As students further develop their understanding of functions, they should understand that $f(x) = y$ and can construct a viable argument using correct vocabulary to explain the meaning of a function. This standard can be integrated with A1.F.IF.A.2 which includes function notation in a real-world context.

Level 4:

Students should solidify their understanding by understanding that the domain of a function represents the input values and the range represents the output values. They should be able to identify the domain and the range in a real-world situation and understand that each element in the domain is unique and is paired with an element in the range. Students should construct a viable argument explaining why real-world situations can be represented as a function. As students solidify their understanding of what kind of real-world situations create functions, they should begin to describe real-world situations that are not functions and explain their reasoning.

Functions

Interpreting Functions (F.IF)

Standard A1.F.IF.A.2 **Cluster Heading: A. Understand the concept of a function and use function notation.**

Use function notation.*

A1.F.IF.A.2.a. Use function notation to evaluate functions for inputs in their domains, including functions of two variables. *

A1.F.IF.A.2.b. Interpret statements that use function notation in terms of a context. *

Scope and Clarification (Modeling Standard):

There are no assessment limits for this standard. The entire standard is assessed in this course.

Use function notation with various functions of two variables. See functions as defined symbolically (e.g., $f(a,b) = 3ab - a$ or a newly defined symbol like a % $b = 3ab - a$).

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine the input-output table that represents a given function. Write an ordered pair given the function notation (e.g., $f(2) = 10$ is the coordinate (2, 10)). Construct an input-output table given a set of ordered pairs.	Substitute input values into a given function of one variable to reveal output values. Substitute input values into a given function of two variables to reveal output values.	Given a function that represents a real-world problem, determine what each variable represents. Given a function that represents a real-world problem, interpret the meaning of output values when given input values and vice versa. Use multiple representations to model a function in a real-world situation.	Explain situations when it is imperative to use function notation. Construct a viable argument to explain the solution of a function in a real-world situation. Given a real-world situation, identify and explain possible restrictions on the domain and range.

Instructional Focus Statements

Level 3:

As students begin to use function notation, they should understand that $f(x)$ is another name for y at a given x and is read as “the value of f at x ” or “ f of x ”. Students should also understand that a variety of letters can be used for function notation such as $g(x)$, $h(x)$ or $d(t)$. Instruction can begin by giving students values of x and an equation written in function notation. Students should use substitution to determine the value of the function at the given x value. For example, when given $f(x) = 5x - 8$, students should be able to find $f(-4)$ by substituting (-4) into the equation. This is helpful to build students’ understanding that function notation is not an arithmetic operation. Graphing would be a natural progression to help students understand that a table of values can be constructed from an equation written in function notation form and a graph can then be produced from the table. In addition, students should be given a function embedded in a real-world situation where they are to explain what the variables represent and what a solution means in the given context. For example, in a contextual problem where a consultant earns a flat fee of \$25 plus \$40 per hour for a contracted job, students should be able to determine how much she will earn in 16 hours. Students should also be able to write the function in function notation, construct a table of values, graph the function, and determine the meaning of the key features of the graph in context. When a question related to the problem is asked, students should be able to solve for the numerical answer and provide an explanation as to what the answer means and why it is or is not viable. Instruction should also focus identifying the domain and the range that is represented in the situation and the reasoning why this is correct for the context of the problem. This standard pairs nicely with A1.F.IF.A.1.

Students should also have the opportunity to explore functions in two variables. For example, the area of a rectangle is a function of the length and the width of the rectangle. This relationship may be written in function notation, $A(l,w) = l \times w$. Students with an understanding of this standard should be able to explain that a change in the length and/or width of the rectangle has an effect on the area of the rectangle. Students should also be introduced to functions that involve newly defined symbols. For example, $f(a \div b) = ab - b$ means that for any value of a and b substituted in this function the output is the difference between the product of a and b , and b . And, $f(3 \div 4) = 3 \times 4 - 4$, so $f(3 \div 4) = 3 \times 4 - 4$, $f(3 \div 4) = 8$. This standard is prerequisite understanding for A1.F.BF.B.2. This standard also is prerequisite understanding for A2.F.BF.B.3.

Level 4:

With a deeper understanding, students can use functions and function notation in a real-world context. An example of this might be using the function $h(t) = -16t^2 + 32t + 48$ where h represents the height of a baseball and t represents time. When the ball is thrown, students can determine how long it takes for the baseball to reach its maximum height or how long it takes for the ball to hit the field. Real-world problems such as this should be represented in multiple ways. Students should be able to write the function in function notation, construct a table of values, graph the function, and determine the meaning of the key features of the graph in context. When a question related to the problem is asked, students solve for the numerical answer and provide an explanation as to what the answer means and why it is or is not viable. Students should also be able to identify the domain and the range that is represented in the situation and explain why this is correct for the context of the problem. Additionally, students can apply their knowledge of functions to a real-world problem such as showing trends across time with a scatter plot and show functional reasoning between two sets of information such as a name and Social Security number or a name and a cell phone number.

Functions

Interpreting Functions (F.IF)

Standard A1.F.IF.A.3 **Cluster Heading: A. Understand the concept of a function and use function notation.**

Understand geometric formulas as functions.*

Scope and Clarification (Modeling Standard):

Limit to linear functions. For example, see geometric formulas such as interior angle sum, perimeter of a square, and circumference of a circle as linear functions.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Understand the definition of <i>function</i>. Apply geometric formulas to find a given characteristic of a shape (such as the perimeter of a square, the circumference of a circle, or the measure of an interior angle).	Explain how the parts of a geometric formula relate to the shape that it is describing. Graph the function related to the geometric formula. Interpret the parts of a linear function, such as intercepts and the rate of change.	See a geometric formula as a function that has characteristics such as intercepts and a rate of change. Apply the characteristics of a linear function to geometric formulas. Explain how the parts of a geometric formula relates to the graph of the related function (including domain and range). Interpret the parts of a geometric formula as it relates to a real-world situation. Write a geometric formula as a function of the variable measure (i.e., the perimeter of a square is a	Create a real-world situation that can be solved using a geometric function.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		function of the side lengths of the square $P(s) = 4s$.	

Instructional Focus Statements

Level 3:

Students were introduced to functions in grade 8. Students have also used geometric formulas beginning in grade 3 (3.MD.D.8). In Algebra 1 students will be expected to think about geometric formulas as functions. They will apply the characteristics of linear functions to linear geometric formulas. For example, students may be asked to create the graph of the function for the perimeter of a square, $P(s) = 4s$, where P is the perimeter of the square, s is the length of a side, and $P(s)$ indicates the perimeter as a function of the length of the side of the square. Discussion should include finding the key characteristics of the perimeter function and how they relate to the square it describes. The discussion should lead students to recognize that y -intercept of 0 is reasonable, because if the side length is 0, there is no perimeter, and the rate of change is 4, because for every 1 unit increase in the side length, the perimeter increases by 4 units. This standard pairs well with A1.F.IF.B.4 and A1.F.BF.A.1.

It is imperative that the students build a conceptual understanding of functions. This standard affords an opportunity for students to gain that conceptual understanding. Instruction should focus on the idea that linear geometric formulas are functions. An example of this could be the circumference of a circle, $C(r) = 2\pi r$, where r is the radius and $C(r)$ is the circumference. Discussion and questioning should lead students to view the circumference as a measure that changes as the radius changes. The circumference is a function of the radius. A common misconception is that the graph of the circumference of a circle will be a circle. A good way to avoid this misconception is to ask students to sketch what they think the graph of $C(r) = 2\pi r$ looks like. Allow students to discuss and justify their reasoning. After this discussion have students create a table of values generated by the function, then graph.

This standard is prerequisite understanding for the Algebra 2 standard A2.F.IF.A.3 where students will apply this knowledge to quadratic and cubic geometric relationships.

Level 4:

Students who have a level 4 understanding of this standard will be able to create a real-world situation that can be solved using a geometric function. Students should be given opportunities to create problems that can be solved by thinking of linear geometric formulas as linear functions. For example, a company creates labels to be used on aluminum food cans. These cans vary in size. Write and use a linear function that would determine the circumference of cans so the printed labels are the correct sizes to fit the cans. Students should see this situation as one where the circumference is a function of the radius, as the radius changes so does the circumference.

Functions

Interpreting Functions (F.IF)

Standard A1.F.IF.B.4 **Cluster Heading: B. Interpret functions that arise in applications in terms of the context.**

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.*

Scope and Clarification: (Modeling Standard)

Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

Tasks are limited to linear functions, absolute value functions, quadratic functions, and exponential functions with integer exponents.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify intercepts, maximums and minimums when provided a graphical representation of the function. Identify intervals where a given function is increasing, decreasing, positive or negative when provided a graphical representation of the function.	Identify key features of the graph or table of values and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing a linear function embedded in a real-world context. Identify evident intercepts, maximums and minimums when provided a table of values representing a linear, quadratic, or absolute value function.	Identify all evident intercepts, maximums and minimums when provided a table of values representing an exponential function with integer exponents. Identify all evident key features when provided a table of values representing a linear, quadratic, exponential or absolute value function. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values	Identify key features of the graph and interpret the meaning of the key features in relationship to the context of the problem, given a quadratic function embedded in a real-world context and graph the function. Identify key features of the graph and interpret the meaning of the key features in relationship to the context of the problem, given an absolute value function embedded in a real-world context and graph the function.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Sketch a graph of a linear function, given a verbal description of its key features.	representing a quadratic function embedded in a real-world context. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values representing an absolute value function embedded in a real-world context. Identify key features of the graph or table and interpret the meaning of the key features in relationship to the context of the problem, given a graph or table of values of an exponential function with domain in the integers embedded in a real-world context. Sketch a graph of the function, given a verbal description of the key features of a quadratic function, absolute value function and an exponential function with integer exponents.	Create a real-world context that would generate a function with the provided attributes, given key features of a linear, quadratic, or absolute value function.

Instructional Focus Statements

Level 3:

Functions are often described and understood in terms of their key features and behaviors. Instruction for this standard should, in part, focus on helping students develop an understanding of how to identify key features and behaviors from both graphs and tables. That said, instruction should extend beyond simple identification from isolated graphs and tables. As this is a modeling standard, students need opportunities to develop an understanding of the relationship between key features/behaviors and the real-world situation that the function models. The focus should be on developing an understanding of what key features/behaviors are, while also developing a strong understanding of their relationship and meaning to real-world situations. Additionally, instruction should provide students with an opportunity to develop an understanding of not only how to identify key features/behavior in graphs and tables, but also how to generate a graph when given the key features/behaviors.

Instruction can be paired with standard A1.F.IF.C.7 where students generate linear, absolute value, exponential and quadratic graphs from real-world situations. This pairing allows students the opportunity to generate a graph from a real-world situation, identify key features/behaviors, and then discuss their meaning as related to the real-world situation.

Level 4:

As students develop a deep understanding of this standard, they should be exposed to increasingly more complex real-world situations. Students should begin to create their own real-world scenarios that generate functions with a pre-determined list of key features/behaviors. Additionally, students with a deep understanding of this standard can interpret key features/behaviors from non-traditional linear, absolute value, quadratic, and exponential functions embedded in real-world situations.

Functions

Interpreting Functions (F.IF)

Standard A1.F.IF.B.5 **Cluster Heading: B. Interpret functions that arise in applications in terms of the context.**

Relate the domain of a function to its graph and, where applicable, to the context of the function it models.*

Scope and Clarification (Modeling Standard):

For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Tasks are limited to linear functions, piecewise functions (including step and absolute value functions), quadratic functions, and exponential functions with integer exponents.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Explain the difference between domain and range. Identify the domain and range from a table of values. Identify the domain and range from a discrete graph. Identify the domain and range, given a mapping. Identify the domain and range from a set of ordered pairs. Explain the difference between a	Explain how domain relates to the definition of a function. Identify the domain, given the continuous graph of the function. Explain why a function is continuous or discrete given its graph.	Explain how the domain relates to the graph of a function. Explain why a function is continuous or discrete given an equation. Describe how a function's domain is affected when situated within a context. Explain if a function is continuous or discrete, given a context.	Determine an appropriate domain and range, given a function in context. Create a contextual situation to describe a function with a given domain and range. Using the definition of discrete and continuous, compare and contrast sequences and the functions used to model them.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
continuous function and a discrete function.			

Instructional Focus Statements

Level 3:

As students begin their work with this modeling standard, instruction should start by making sure the students have a good understanding of domain and range. Providing tables, graphs, ordered pairs and mappings for students to identify the domain and range will lead to rich classroom discussion and the multiple representations will give students a deep understanding of the definitions of domain and range. Students should be able to explain how and why they have identified the domain and range in these varied forms. After students have a clear understanding of how to identify the domain and range, they should have experience relating the domain to the definition of a function. That is, every input must correspond to exactly one output value. They should also identify the domain given the graph of a function. It is important that students have a conceptual understanding of domain and range. Their experiences should not be limited to listing the domain and range as a set of ordered pairs or a table. Discussion should include comparing and contrasting discrete and continuous functions and identifying them from a graph and real-world situations.

An example that could be used to help students apply this standard might include small packs of Skittles in the check-out line at a convenient store. If one bag contains 10 Skittles and 2 bags contain 20 Skittles, students can create a table of values and then determine if the graph is continuous or discrete. In this particular case, they should relate the number of packs to the domain and the number of Skittles to the range. Students may ask if there has to be at least 1 bag at the check-out line and/or is there a limit of bags that could be placed there. After students have determined whether the domain is continuous or discrete, they should graph the function. In addition, careful attention must be paid to real-world problems where the domain might be continuous, but also restricted because of the context. An example might be a ball's height when thrown, $h(t)$, with respect to time. The domain would be continuous, but given the context, t must be greater than or equal to 0. As students solidify their understanding of different function families, they should begin to realize that the domain and range do not both have to be continuous or discrete. This is a good place to introduce step functions and explain how the domain might be continuous while the range is discrete.

Level 4:

As students are exposed to a variety of real-world problems, they should begin to realize how unique and different every problem can be, but that every modeling situation will have a domain and range. At a level 4 understanding, students should be able to determine the domain and range of real-world problems and have practice with a variety of scenarios. They should also have practice with identifying whether the function is continuous or discrete and explain why. As they develop a good understanding of domain and range, opportunities should be provided for students to create their own scenarios

when given a domain and range. In addition, students should construct an argument explaining why their scenario represents the given domain and range. Instruction should also focus on connecting arithmetic and geometric sequences with the functions that model them. Students should justify why these sequences are discrete, while linear and exponential functions are continuous.

Examples which might be used to support understanding of this standard would include a scenario representing the cost of movie tickets per person. This is a discrete function and the domain and range are restricted to positive integer values. A continuous example might be a punter kicking a football over time (x) compared to height (y). This example also restricts the domain and range to positive values. As students understand how to determine the domain and range and graphing functions, they should also be able to create their own scenarios with both discrete domain and range and continuous domain and range.

Functions

Interpreting Functions (F.IF)

Standard A1.F.IF.B.6 **Cluster Heading: B. Interpret functions that arise in applications in terms of the context.**

Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval. Estimate and interpret the rate of change from a graph.*

Scope and Clarification (Modeling Standard):

Tasks are limited to linear functions, piecewise functions (including step and absolute value functions), quadratic functions, and exponential functions with integer exponents.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the average rate of change for a linear function when given a symbolic representation, table, or graph. Choose the estimated rate of change when given a graph of a linear function.	Calculate the average rate of change of a linear function when given a graph. Calculate the average rate of change of a linear function when given a table. Calculate the average rate of change of a linear function when given the function. Interpret the rate of change for a linear function in terms of a real-world context.	Calculate the average rate of change when given an equation or table of a quadratic, absolute value, piecewise, or exponential function, where exponential functions are limited to domains in the integers. Interpret the average rate of change of a quadratic, absolute value, piecewise, or exponential function, where exponential functions are limited to domains in the integers. Estimate the average rate of change for a specific interval of a quadratic, absolute value, piecewise, or exponential function when given a graph, where exponential functions	Identify the average rate of change for specific intervals of a function as being greater or less than other intervals of the same function. Compare the average rate of change of multiple intervals of the same function and make connections to the real-world situation. Create a contextual situation and identify and interpret the average rate of change with a specific interval.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Choose the estimated rate of change for a specific interval when given a quadratic function.	are limited to domains in the integers.	

Instructional Focus Statements

Level 3:

In grades 6 (6.RP.A.2) and 7 (7.RP.A.2.b), students began developing an understanding of ratios and proportional relationships. Their understanding of rate of change involved both ratios and proportions using similar triangles to show the additive and multiplicative conceptual underpinnings of the concept. In grade 8 (8.F.A.2), students extended this understanding to functions by examining the rate of change in linear functions. In high school, students should solidify this understanding for linear functions and generalize this concept to additional function types. Students should make the connection that the rate of change is the ratio of the change between the dependent and independent variable. For linear functions, students have discovered that this ratio of change is constant between any two points on the line. Students should now make the connection that, for non-linear functions, the ratio of change is not constant due to the curvature of the function. For example, for the quadratic function $f(x) = x^2$, the average rate of change from $x = 1$ to $x = 4$ is $\frac{f(4)-f(1)}{4-1} = \frac{16-1}{4-1} = \frac{15}{3} = 5$. This is the slope of the line from $(1, 1)$ to $(4, 16)$ on the graph f . If f is interpreted as the area of a square of side length x , then this calculation means that over this interval the area changes, on average, 5 square units for each unit increase in the side length of the square.

It is imperative that students gain a conceptual understanding of the average rate of change for a specified interval for non-linear functions. To grasp this idea, students should draw illustrations of the graph and the secant line connecting the intended endpoints. Students should not only be able to calculate the average rate of change, but they should also be able to generate a visual representation and use the visual representation to estimate the average rate of change over a specified interval. Students will gain a deeper conceptual understanding when they compare their estimations to the actual average rate of change for a non-linear function. As students solidify their understanding, they should be able to explain what the average rate of change means in the context of a problem when given symbolic representations, tables, graphs, or contextual situations. As students use multiple representations to evaluate the average rate of change, they should be able to explain the relationship between the multiple representations using both appropriate mathematical language and appropriate justifications.

Level 4:

Students should extend their understanding of the average rate of change by comparing the average rate of change of one interval to another interval of the same function. Students should also further their understanding by creating their own contextual situations and interpreting the average rate of change for a significant interval. Students should be intentional in determining which interval or intervals they select and explain the importance of the interval(s) with respect to the context using both precise mathematical vocabulary and precise justifications.

Functions

Interpreting Functions (F.IF)

Standard A1.F.IF.C.7 **Cluster Heading: C. Analyze functions using different representations.**

Graph functions expressed algebraically and show key features of the graph by hand and using technology.*

Scope and Clarification (Modeling Standard):

Function types are limited to linear functions, quadratic functions, absolute value functions, and exponential functions with integer exponents.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Represent a constant rate of change between two variables as slope of a line. Use characteristics of the algebraic representation of a function to determine the function type and behavior of the graph. Identify $y=x$ as the parent function for linear function Explain the effects of slope and intercepts on a linear function.	Graph a linear function by hand and using technology. Sketch the graph of a quadratic function given intercepts and extrema. Explain how the absolute value function is an example of a piecewise-defined function. Identify key features, such as extrema and intercepts, from a graph of a piecewise-defined function. Infer restrictions on the domain and range from a graph.	Graph a linear function by hand and using technology and identify the slope and intercepts. Graph a quadratic function by hand and using technology identifying intercepts, maxima, and minima. Graph an absolute value function by hand and using technology. Attend to precision when illustrating intercepts, maxima, and minima and determine the domain and range of the function.	Explain the relationship that exists between a contextual problem and the key features of a graph for linear, quadratic, and absolute value functions. Critique graphs drawn by others to ensure key features are shown efficiently and appropriately. Given a graph, write the corresponding function algebraically. Explain restrictions on domain and range in context.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Identify key features, such as intercepts and extrema, from a graph of a quadratic function.		

Instructional Focus Statements

Level 3:

In grade 8 (8.F.B.5), students used functions to model relationships between quantities and construct a function to model a linear relationship. Students also determined if a function is linear or nonlinear, and they had experience interpreting and representing functions algebraically, numerically, graphically, and verbally. In Algebra I, students will be introduced to additional function families and more key features such as extrema and end behavior. Intercepts, shape, domain, and range take on greater meaning to students through the exploration of a variety of functions. Therefore, it is important that instruction provides tasks that allow students to explore the behavior and varying parameters of functions. To meet the rigor of this standard, students should be given the opportunity to work with functions that vary in their algebraic representation. For example, students should experience point-slope, slope-intercept, and standard form of a linear function as well as standard form, factored form, and vertex form of a quadratic function. This will help students have access to the problem regardless of the algebraic representation which is further developed in standard A1.F.IF.C.8 in which students will write functions in different forms to reveal different properties of the function.

Students should be able to graph functions by hand and with the use of technology. It is imperative to model how to graph with a graphing calculator or other graphing technology. This is the first time students will use technology to graph a function type other than a linear function. Furthermore, ample time must be given for students to explore how a table of values can be helpful in identifying key features, domain, and range from a graph, which is supported by standard A1.F.IF.B.4. The use of technology allows students to explore functions whose key features are irrational values.

This standard appears in Algebra 2 (A2.F.IF.B.4) as well. In Algebra 2 students will be required to graph and interpret key features of additional function types. Therefore, instruction should introduce the concept of parent functions. This will help students make the connection of how transformations affect the graph, equation, and table of a function, which is explored in A1.F.BF.B.2. This standard can be integrated in instruction as students are presented with problem types whose symbolic representation varies and asked to identify the parent function and describe the transformation from its original, non-transformed graph. This will help students attend to precision as they graph functions of many types and use their understanding of transformations to support the reasonableness of their graph. Instruction should provide ample opportunity for students to compare and contrast the graphs of functions, and it should help them efficiently recognize a parent function when expressed symbolically and graph it fluently.

Students may struggle with domain restrictions and absolute value functions. Technology can support student understanding of domain and range restrictions by graphing multiple functions on the same coordinate plane. For instance, have students graph the functions $y = x^2$, $y = (x + 4)^2$, and $y = x^2 - 4$ simultaneously and make observations about the domain and range by looking at the graph and analyzing the table. Students often struggle with the domain and range of absolute value functions. Instruction should provide examples for students to discover the relationship between the algebraic representation of an absolute value function and the resulting domain and/or range. Instruction should begin by defining a simple absolute value function of two constant functions in context so students can make connections between the graph and the domain. Students will need to connect their prior knowledge of inequalities when working with domain in absolute value functions.

Level 4:

Students should extend their conceptual understanding of key features of graphs by connecting key features to the relationships that exist in contextual problems. Using their knowledge of multiple representations built in standard A1.F.IF.C.8, students should be able to provide a graph, table, equation, and verbal representations of a contextual situation. Instruction should include posing purposeful questions asking them to show and describe key features from their created problem in context. Students should be given the opportunity to look at graphs drawn by others so they can analyze and critique their peers work. Through the analysis of many graphs, students should develop an understanding of when key features are efficiently and effectively represented, and, if not, provide a suggestion for representing them more appropriately.

Functions

Interpreting Functions (F.IF)

Standard A1.F.IF.C.8 **Cluster Heading: C. Analyze functions using different representations.**

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.*

A1.F.IF.C.8a Rewrite quadratic functions to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a real-world context.

Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Understand that the zeros of a quadratic function are located on the x-axis (as the x-intercepts). Identify the zeros from a given graph of a quadratic function. Identify the vertex from a given graph of a quadratic function. Identify the axis of symmetry from a given graph of a quadratic function.	Graph a quadratic function. Factor a quadratic expression. Identify the zeros, given a quadratic function in factored form. Identify an extreme value, given a quadratic in vertex form. Recognize the y-value of the vertex of the graph of a quadratic function as a minimum or maximum.	Rewrite quadratic functions in different but equivalent forms. Interpret the meaning of zeros, extreme values, and symmetry of the graph in the context of a real-world problem. Recognize which form of a quadratic function is most appropriate for revealing certain properties, when given a real-world problem.	Write a quadratic function from a real-world context given zeros or the extreme value, and one additional point. Create a real-world contextual problem that matches a given set of key features for a quadratic graph.

Instructional Focus Statements

Level 3:

In grade 8 (8.F.A.3) students know and interpret linear functions in slope-intercept form to reveal two characteristics of linear functions: slope and y-intercept. In Algebra I, students look for and make use of structure to rewrite quadratic functions to reveal characteristics about their graphs. Instruction for this standard should be connected to A1.A.REI.B.3 where students solve quadratic equations using multiple strategies: by inspection, taking square roots, knowing and applying the quadratic formula and factoring.

Student understanding of factoring a quadratic expression should draw on connecting that a quadratic expression and a factored form of the same expression are equivalent. There are several approaches to rewriting a quadratic expression as the product of two binomials. Instruction may begin with trial and error when rewriting a simple quadratic expression in a different but equivalent form. As the expressions become more complex, for example, the coefficient of the second-degree term is no longer one, students should recognize when there is a greatest common factor among the terms of the quadratic expression. Students should also discover when the quadratic expression represents a difference of perfect squares or a perfect square trinomial to support using factoring to rewrite the expression. The factored form of a quadratic expression can be used to identify the zeros of the quadratic function, which tied to the graphical representation, reveals the x-intercepts of the graph.

The graph of a quadratic function is a parabola. Students should understand the features of a graph of a parabola, such as that the parabola is symmetrical about an axis of symmetry and that the function's roots are equidistant from the axis of symmetry. Students should be able to analyze and compare properties of a function represented in different ways and understand equivalent forms of the function. Students should examine key features of a function, for example, the standard form of a quadratic equation reveals the y-intercept and the coefficient of the quadratic term indicates the direction the graph opens. The factored form of a quadratic function reveals the x-intercepts or zeros of the function. Vertex form reveals an extreme point, also called the vertex of the parabola, as well as the axis of symmetry.

Students should understand how to rewrite quadratic functions in standard, vertex, and factored forms and discover which form is most appropriate for revealing certain key features of the graph. Students are not expected to use the process of completing the square to rewrite quadratic functions in vertex form. Instead, students should make connections to the concept of transformations of functions (A1.F.BF.B.2) in order to understand the properties of a quadratic function revealed through vertex form. Instruction should give students opportunities to connect functions given in various forms with possible graphs that they could represent to help students visualize how each form of a quadratic reveals key features of the function.

Students should also be able to explain the contextual meaning of the key features of the quadratic function when working within a real-world context. The typical key features of quadratic functions are: domain and range, y-intercept, x-intercepts (zeros), intervals of increasing and decreasing, intervals of positive and negative values, maximum/minimum, vertex, axis of symmetry and end behavior. Teachers should provide a variety of contexts in which the type of variables differ so that students can develop a deep understanding of how each component relates to the real-world context. Students can write quadratic functions in vertex form to reveal the vertex and axis of symmetry of the function. Visually, students see that the y-value of the vertex is the maximum or minimum of the function, which is important in a variety of contexts. For example, it can reveal the unit price that maximizes profit, the

time when a projectile reaches its maximum height, or the length of a rectangle that maximizes the area. Teachers should provide a variety of contexts in which the type of variables differ so that students can develop a deep understanding of how each component relates to the real-world context.

Level 4:

Students with a deep understanding of the three forms of a quadratic should be able to create quadratic functions when given information about key features. For example, students could be given one zero and the vertex to write the quadratic function that it represents. In this example using the symmetry of the graph, teachers can focus students' attention on finding an additional zero to build factored form. From there, they can substitute in the x- and y-value of the vertex to find the leading coefficient. Teachers should present students functions within a context so that students continue to see the key features of quadratic functions within various contexts. Additionally, students should further extend their understanding by creating real-world problems that match a given set of key features.

Functions

Interpreting Functions (F.IF)

Standard A1.F.IF.C.9 **Cluster Heading: C. Analyze functions using different representations.**

Compare properties of functions represented algebraically, graphically, numerically in tables, or by verbal descriptions.*

A1.F.IF.C.9.a Compare properties of two different functions. Functions may be of different types and/or represented in different ways.*

A1.F.IF.C.9.b Compare properties of the same function on two different intervals or represented in two different ways.*

Scope and Clarification (Modeling Standard):

Functions may or may not have a real-world context. Function types are limited to linear functions, quadratic functions, absolute value functions, and exponential functions with integer exponents.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the y-intercept of a function from multiple representations. Identify the slope of a linear function from multiple representations. Describe connections among multiple representations of a linear function. Compare properties of two linear functions each represented in a different way.	Identify the zeros of a function from multiple representations. Identify the vertex of a quadratic function from multiple representations. Identify the percent rate of change of an exponential function from multiple representations. Describe connections among multiple representations of a quadratic function. Describe connections among multiple representations of an	Compare properties of two exponential functions each represented in a different way. Compare properties of two absolute value functions each represented in a different way. Compare properties of two quadratic functions each represented in a different way. Compare properties of two functions from different function families each represented in a different way.	Compare properties of two functions within a context. Use precise mathematical vocabulary to explain the relationships of the various representations of a function.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	exponential function. Describe connections among multiple representations of an absolute value function. Move fluently among multiple representations of a function.	Compare properties of the same function on two different intervals or represented in two different ways.	

Instructional Focus Statements

Level 3:

Prior to comparing properties of two functions represented in different ways, students need to first identify properties of functions and make connections between different representations of the same function. Instruction should focus on students representing a function in multiple ways. As students translate between representations, they must consider relationships among quantities and how each representation provides a unique perspective of the function. Teachers can foster this way of seeing mathematics by having students discuss the similarities among representations that reveal the key features of a function that persist regardless of the form. Through these discussions students can determine which representations are most appropriate for revealing certain key features of the function.

In grade 8 (8.F.A.2), students compared properties of two linear functions each represented in different ways. Once students have a strong understanding of the various representations of linear, quadratic, absolute value, and exponential functions in Algebra 1, they can begin to compare properties of two functions represented in different ways. For example, given a graph of one quadratic function and a table of another, a student should be able to compare their y-intercepts. One strategy that can sometimes be useful is to convert one or both to a different form so that both functions are represented the same way. As students begin to grasp this concept, it is important that teachers provide students with examples that include each function type, with some situated within a context. Therefore, comparing properties in different representations further supports students' understanding of each function type, which means this standard can be paired nicely with other standards that focus on properties and graphs of linear, quadratic, absolute value, and exponential functions, such as standards A1.F.IF.B.4 and A1.F.IF.C.7. As students recognize various function types in multiple representations, discussion should lead to the comparison of functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing a quadratic function and a verbal description of an exponential function. Instruction should support students in first recognizing the function family prior to comparing properties.

Students should be able to compare properties of the same function on two different intervals or represented in two different ways. For example, consider the functions $f(x) = x^2 + 7x + 12$ over the interval from $x = -4$ to $x = -2$ and $g(x) = (x+2)(x+4)$ over the interval from $x = -4$ to $x = -3$. Describe how the functions behave over these intervals. How is the behavior the same? How is the behavior different?

Level 4:

Students with a deep understanding of the various function types and representations should also be able to compare functions from different families represented in different ways. For example, compare y-intercepts given a table of values representing a quadratic function and a verbal description of an exponential function. Instruction should support students in first recognizing the function family prior to comparing properties. Once conclusions are formed, teachers can ask further questions related to the context. For example, given a graph of a linear function and an algebraic representation of an exponential function each describing the growth of a saving account, decide which type of account is a better choice for earning the most interest. Students should be given the opportunity to describe how to identify function types and compare the properties of functions in various forms. At this level, teachers should expect students to use precise mathematical vocabulary to describe and justify these relationships and qualities.

Functions

Building Functions (F.BF)

Standard A1.F.BF.A.1 **Cluster Heading: A. Build a function that models a relationship between two quantities.**

Build a function that describes a relationship between two quantities.*

A1.F.BF.A.1.a Determine steps for calculation, a recursive process, or an explicit expression from a context. *

Scope and Clarification (Modeling Standard):

Tasks are limited to linear and exponential relationships. For example, create a function from a visual pattern and describe how each component of their function relates to characteristics of figures in the pattern.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Write a function defined by an expression to model a linear relationship, given a table or graph. Identify the independent and dependent variable in a real-world context. Identify the first term and rate of change of a linear function, given a table or graph. Identify the first term and common ratio of an exponential function, given a real-world context.	Determine whether a real-world context would be represented by a linear or non-linear function. Write a function defined by a recursive process or steps for calculation to model a linear relationship, given a real-world context. Write a function defined by a recursive process or steps for calculation to model an exponential relationship, given a real-world context.	Write a function defined by an expression to model a linear relationship, given a real-world context. Write a function defined by an expression to model an exponential relationship with domain in the integers, given a real-world context. Compare key characteristics of real-world contexts that can be described by various types of functions.	Create a real-world context that would generate the given function, given a function defined by an expression, a recursive process, or steps for calculation. Explain the various ways a function can be defined and in what real-world contexts they would be appropriate. Justify why a specific type of function should be used to describe a given real-world context.

Instructional Focus Statements

Level 3:

In grade 8 (8.F.B.4), students construct a function defined by an expression to model a linear relationship. In Algebra I, students create linear functions given a real-world context defined by a recursive process or steps for calculation in addition to an explicit expression. In many situations, it is natural to use a function defined recursively, which generates values by applying operations on previous terms. For example, mortgage payments and drug dosages can be described with a recursive process. Students also create exponential functions defined in the same three ways. Instruction should focus on creating functions given a real-world context, while recognizing appropriate ways to define functions.

If given a table of values, students should first recognize which type of function is represented by the table. Instruction should focus on the relationship between consecutive points to see if there is a common first difference or constant additive change (linear function) or a common ratio or constant multiplicative change (exponential function). Once students identify the function type, they should begin to write the function given the common first difference or ratio and other information from the table.

To build coherence, it is important for students to make connections between linear functions and arithmetic sequences and between exponential functions and geometric sequences. Thus, instruction can be nicely paired with standard A1.F.LE.A.2, where students generate arithmetic and geometric explicit formulas to model situations. Both linear functions ($y = ax + b$) and arithmetic sequences ($a_n = a_1 + d(n - 1)$) describe additive changes, and students should make connections between the two. For example, b is equivalent to a_0 and a is equivalent to d . Similarly, exponential functions ($y = ab^x$) and geometric sequences ($a_n = a_1 r^{n-1}$) both describe multiplicative changes and a is equivalent to a_0 and b is equivalent to r . Students should understand the similarities, but instruction should also help students realize an important difference: arithmetic and geometric sequences are discrete while linear and exponential functions are continuous. This can be done by comparing the graphs of an arithmetic sequence and a linear function, for example.

Level 4:

As students develop a deep understanding of this standard, they should be able to create a real-world scenario given a function or combination of functions. Moreover, they should be able to describe which characteristics of their scenario correspond to each part of the given function. For example, given $y = 10 + 5x$ a student might create a scenario similar to the following: A ski lodge charges \$10 to rent a snowboard and \$5 for each hour it is used. In this function, x represents time in hours and y represents the final cost. The student should also be able to explain how the $5x$ relates to \$5 per hour and how the 10 in the function relates to the initial charge of \$10. Similar scenarios can be created to describe exponential functions (e.g., sharing news or half-life).

Functions

Building Functions (F.BF)

Standard A1.F.BF.B.2 Cluster Heading: B. Build new functions from existing functions.

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs.

Scope and Clarifications:

Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Tasks are limited to absolute value and quadratic functions.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Describe transformations that would map a geometric figure to its image using precise mathematical vocabulary. Write the function defined by $f(x)+k$, $kf(x)$, $f(x+k)$, and $f(kx)$ given the parent function and a positive value of k. Graph the parent functions $y = x$ and $y = x^2$ using technology.	Know and graph the parent functions for absolute value and quadratic functions. Use function notation to identify the difference between $f(x) + k$ and $f(x+k)$ and the difference between $kf(x)$ and $f(kx)$.	Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $f(x) + k$. Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $f(x + k)$. Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $f(kx)$. Describe the effect on the graph for specific values of k, given two functions, $f(x)$ and $kf(x)$. Describe the effects on a graph for more than one transformation	Given a graph of a function, write the equation by identifying the transformation(s) to the parent function. Apply transformations to a function that has already been transformed. Explain why changes to the argument of $f(x)$ affect the input values and changes outside the function affect the output values.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		using specific values of a, h, and k given two functions, $f(x)$ and $af(x + h) + k$. Determine the value of k for vertical and horizontal translations, stretches, and compressions, given the graphs of the image and pre-image.	

Instructional Focus Statements

Level 3:

In grade 8 (8.G.A.1), students are introduced to the properties of rotations, reflections, translations, and dilations of geometric figures. Students expand on this concept in Algebra 1 by exploring the effects of translations, stretches, and compressions on functions and describing the transformations using function notation. To master this standard, students need to understand function notation and how to write a function defined by function notation. For example, given $f(x) = x^2$, write the function $f(x + 3)$ as $f(x + 3) = (x + 3)^2$. Therefore, pairing A1.F.BF.B.2 with A1.F.IF.A.2 will allow students to see how function notation can be used to generalize transformations to all functions.

Students can use technology (i.e., graphing calculator or online graphing tool) to compare a parent function $f(x)$ with each of the transformation types $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$, to discover the effects of each transformation on the parent function's graph. As students vary one value of k at a time, they can begin to discern how each component affects the graph of the parent function, $f(x)$. For example, students can graph $f(x) = x^2$ and then graph $f(x) = x^2 + 4$. By examining the graph, they can see that the $+ 4$ translated the graph up 4 units. They can then predict what would happen if the equation was $f(x) = x^2 - 4$ and then graph it to see that the $- 4$ translated the graph down 4 units.

When exploring what happens to the parent function for $f(x + k)$, student discourse should also focus on why k translates the graph $-k$ units horizontally. One strategy is to compare $f(x)$ with $f(x - 5)$, when $f(x)$ is defined at $f(3)$. In this example, an input of 3 into $f(x)$ is equivalent to an input of 8 into $f(x - 5)$, and 8 is 5 units to the right of 3, not left. Students can think about it as having to undo what has been done to x inside the argument, in this situation a translation 5 units to the right of the parent function will “undo” the -5 . Likewise, when exploring the effects of k on $f(kx)$, students will need to see that if $k > 1$ the graph will be compressed horizontally by $1/k$, and if $k < 1$, it will be stretched by $1/k$.

After students have had the opportunity to explore the effects of each transformation visually using technology, instruction should guide students to examine the effects of each transformation on the coordinates. Instruction should connect to the concepts of transformation from grade 8 to examine the effects of the transformation on each point in the parent function. For example, when exploring $f(kx)$ with $k = \frac{1}{2}$, upon closer examination of the coordinates, students will find that the x coordinates of the parent graph (input value) will be multiplied by $\frac{1}{\frac{1}{2}}$ or 2 in each corresponding point in the resulting image (output value).

Students should understand that since $f(x)$ represents the output values, any operations applied to $f(x)$ outside the argument only affect the y values, which results in vertical transformations. Any operations applied to $f(x)$ inside the argument only affect the x values, which results in horizontal transformations. Once students understand the effects of each component individually, they should then be able to describe changes to a graph involving multiple transformations at once. Eventually, they should connect each transformation to how a , b , h , and k impact the graph of $f(x)$ in $af(bx + h) + k$. Note that this is vertex form of a quadratic function since it reveals the vertex (h,k) of the parabola.

When working with multiple transformations, teachers should focus students' attention on the order of each transformation. For example, given $-f(x) + 9$, the graph is first reflected across the x-axis, then shifted up 9, rather than shifted up 9 then reflected across the x-axis, due to the order of operations. It is also important that teachers place an emphasis on factoring out b from inside the argument so that the horizontal shift can be found. For example, $f(x) = (2x-6)^2$ becomes $f(x) = (2(x-3))^2$, revealing a horizontal shift of 3 to the right, not 6.

Once students can describe the effects of transformations on the graph of a function, they should be able to graph a given absolute value or quadratic function by applying the concept of transformations. This strategy for graphing functions is applied in A1.F.IF.C.7. They should also be able to describe vertical and horizontal translations, stretches, and compressions and determine the value of k when given the graphs of the image and pre-image.

These transformations will be further explored in Geometry (G.CO cluster) and applied to additional function types in Algebra 2 (A2.F.BF.B.3).

Level 4:

Students with a deep understanding of this standard should be able to write the equation of a function given the graph by identifying the parent function and any transformations that have been applied to it by using the format $f(x) = af(bx + h) + k$.

Teachers should help students understand that transformations can be applied to functions that have already been transformed. For example, given $g(x) = (x+2)^2$, write an equation for $g(x-5)$ and describe the overall change in the graph. The given function $g(x)$ is simply translated to the right 5 units, but since it had already been translated to the left 2 from the parent function $f(x) = x^2$, the transformed function will be $g(x-5) = (x+2-5)^2$ or $(x-3)^2$ and the vertex of the image will land at $(3,0)$, which is a translation from the parent function to the right 3.

In addition, students at this level should be able to explain concepts such as why changes inside the argument of a function have the inverse effect on a graph. Taken collectively, students should understand a function as a process that generates output values from specific input values, which leads to a deeper understanding of functions.

Functions

Linear and Exponential Models* (F.LE)

Standard A1.F.LE.A.1 **Cluster Heading: A. Construct and compare linear and exponential models and solve problems.**

Distinguish between situations that can be modeled with linear functions and with exponential functions.*

A1.F.LE.A.1a Know that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.*

A1.F.LE.A.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*

A1.F.LE.A.1c Recognize situations in which a quantity grows or decays by a constant factor per unit interval relative to another.*

Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize a function as linear from a graph, a table, and an equation. Recognize a function as exponential from a graph, a table, and an equation.	Find the slope of a line from a graph, table of values, or given two coordinate points. Recognize that a line has a constant rate of change, while an exponential function does not.	Know that linear functions have a constant rate of change, while exponential functions do not. Show informally or explain that linear functions grow by adding the same number per unit. Show informally or explain that exponential functions grow by multiplying by the same factor per unit. Determine if a given real-world situation has a constant rate per	Create a real-world example of a situation that can be modeled by a linear function and explain why it is linear. Create a real-world example of a situation that can be modeled by an exponential function and explain why it is exponential, including why it is growth or decay.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		unit interval and can be modeled by a linear function. Determine if a given real-world situation has a constant factor per unit interval and can be modeled by an exponential function. Determine if a given real-world situation that can be modeled by an exponential function represents growth or decay.	

Instructional Focus Statements

Level 3:

Students first learn how to find the slope of a line in grade 7. In grade 8, they begin exploring non-linear functions. In Algebra I, students compare the behavior of linear and exponential functions in terms of rate of change and its effect on end behavior.

As this is a modeling standard, students should be working with linear and exponential situations embedded in real-world applications. Allow students to explore the rate of change of linear and exponential functions from graphs, tables of values, equations, and real-world examples to determine if the rate of change remains constant. Slope should be calculated over equal intervals and compared. This may be easier if students organize the information in a table.

It is important for students to be able to identify when a function is linear vs. exponential, therefore, students should be provided with mixed examples and not just one or the other in isolation.

Students should be expected to use mathematical structure and repeated reasoning through multiple representations to see that the rate of change for a linear function remains constant.

Students may struggle with finding the percent growth or decay in an exponential function. For example, the amount of money earned in an investment is represented by the function $f(t) = 1500(1.07)^t$. By completing a table of values, students can see that the function represents growth, but they may be confused by the 1. It may be easier for them to understand where the 1 comes from in an example of buying a pair of blue jeans at a store. Show students that they can calculate the cost of the blue jeans and add sales tax in one step by taking 100% of the cost of the blue jeans plus the percent of sales tax. For a \$20 pair of blue jeans at 8% sales tax, they can multiply 20 by 1.08 (or 108%) to get the total out of pocket cost. Likewise, if the blue jeans are on sale at 15% off, they can multiply 20 by .85 (or 85%) to get the sale price.

Level 4:

The focus of this standard is on the comparison of the rates of change between linear and exponential functions. To ensure students are making connections between real-world situations and linear functions, have them create their own real-world examples that can be represented by linear functions. They should also explain why the situation is linear.

Similarly, have students create their own real-world examples that can be represented by exponential functions. Students should represent their examples in words, algebraically, in a table, and graphically. They should include in their explanation why the situation represents exponential growth or decay and what the constant factor would be.

Functions

Linear and Exponential Models* (F.LE)

Standard A1.F.LE.A.2 **Cluster Heading: A. Construct and compare linear and exponential models and solve problems.**

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs.*

Scope and Clarifications: (Modeling Standard)

Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Recognize a graph of a function as linear. Recognize a graph of a function as exponential.	Recognize a function as linear from a table, a description, or a set of ordered pairs and justify it using rates of change. Recognize a function as exponential from a table, a description, or a set of ordered pairs and justify it using multiplicative change.	Construct a linear function given a graph. Construct a linear function given a table of values. Construct a linear function given a description of a simple real-world relationship. Construct a linear function given a set of input-output pairs (ordered pairs). Construct an exponential function given a graph. Construct an exponential function given a table of values.	Analyze functions created by others to determine accuracy and explain and correct any errors. Create a real-world situation that may be modeled by a linear function and write the function. Create a real-world situation that may be modeled by an exponential function and write the function. Collect data for a real-world situation that can be represented by a linear or exponential function and write the function that models it. Define the variables and explain

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Construct an exponential function given a description of a simple real-world relationship. Construct an exponential function given a set of input-output pairs (ordered pairs). Construct a function given an arithmetic or geometric sequence or a description of one.	in context why the function models the situation.

Instructional Focus Statements

Level 3:

This standard aligns with several Algebra I standards, including the CED, IF, BF, and SSE clusters, and should be incorporated with those rather than as a stand-alone lesson. Once students have been exposed to both linear and exponential functions, instruction should include examples of both function types in multiple representations including graphs, tables, and descriptions to allow students time to determine whether the function is linear or exponential as well as write the function. This is a good opportunity to relate input and output values with independent and dependent variables.

As this is a modeling standard, real-world examples should be provided, and students should be required to explain how the function models the context.

This standard also allows for students to connect their learning about arithmetic and geometric sequences to creating the functions that model them. These should also come from multiple representations including graphs, tables, and descriptions of real-world situations.

Level 4:

To increase the level of understanding, students should critique examples of functions created by others. One way to do this is to have students work in pairs, with one student constructing a function and the other student checking it. For example, student A constructs a function from a graph while student B constructs a function from a table of values. Then they swap papers and student B graphs student A's function while student A creates a table of values

from student B's function. Then each compares their work with the originals. If they do not match, they must determine where the mistake was made and correct it.

Once students have a good understanding of constructing both linear and exponential functions, they can create their own real-world examples. This can also involve students predicting a situation that would provide data that could be modeled by a linear or exponential function, collecting that data, and writing the function based on that data to test their prediction. Students should show and explain why the real-world example represents a linear or an exponential function, including representing it in multiple ways.

Functions

Linear and Exponential Models* (F.LE)

Standard A1.F.LE.B.3

Cluster Heading: B. Interpret expressions for functions in terms of the situation they model.

Interpret the parameters in a linear or exponential function in terms of a context.*

Scope and Clarifications: (Modeling Standard)

For example, the total cost of an electrician who charges 35 dollars for a house call and 50 dollars per hour would be expressed as the function $y = 50x + 35$. If the rate were raised to 65 dollars per hour, describe how the function would change.

For example, a population is modeled by a function $y = 30000(1.04)^x$. Interpret the value 30000 as the initial population and 1.04 as a 4% increase per year.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define slope as a rate of change. Identify the slope and the y-intercept in a linear function written in slope-intercept form. Identify the coefficient, base, and exponent in an exponential function.	Determine the slope and y-intercept of a linear function in a graph. Calculate the slope of a line that passes through two given points. Calculate the y-intercept of a linear function algebraically. Identify the initial value of an exponential function in a graph. Calculate the initial value of an exponential function algebraically. Calculate the growth rate of an	Explain the meaning of the slope and y-intercept in context of the real-world situation, given a linear function. Explain the meaning of the coefficient, the base, and the exponent in context of the real-world situation, given an exponential function with a domain in the integers. Predict and determine how a linear function is affected by a change in the slope or y-intercept. Explain this change in context.	Create a real-world scenario that can be modeled by a given linear function. Create a real-world scenario that can be modeled by a given exponential function.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	exponential function by finding the ratio of successive terms.	Predict and determine how an exponential function is affected by a change in the coefficient, base, or exponent. Explain this change in context.	

Instructional Focus Statements

Level 3:

This standard is an extension of A1.A.SSE.A.1, in which students interpret parts of expressions. This focus on this standard is on how the different components affect each other. As this standard is a modeling standard, examples should connect to a real-world context. Use questions that ask students to interpret the slope and y-intercept of linear functions in the context of a real-world situation. Likewise, ask students to interpret the coefficient, base, and exponent of exponential functions in context of a real-world situation. Then extend their learning by asking them to determine the effect of changes to the parameters on the function. For example, given an account that is modeled by $A(t)=200(1.005)^{12t}$, determine the initial amount invested, the interest rate, and how often the money is compounded. Then describe what the effect would be if the initial investment is increased by \$100.

Have students make a prediction of the effect of a change in a parameter and then verify if their prediction was correct by applying the change and comparing the results. Repetition of this activity will help students develop a better understanding of the properties of the operations within the function. Connecting the function to the context will help them justify their reasoning for their predictions. This will also help students see the structure of the function and make a connection to its use as a general formula for the given real-world situation. It is important for students to attend to precision in their interpretations and explanations should include units to ensure they are interpreting completely and correctly.

Level 4:

Teachers should provide students examples of linear and exponential functions and ask them to create a real-world scenario that could be modeled by them.

Have students critique others' interpretations of the parameters and correct any mistakes. Students should look for correct and precise mathematical language in others' explanations, explain any mistakes made or lack of precision, and provide accurate corrections.

This standard can be easily integrated with A1.F.IF.B.4 and interpreting key features of a graph in connection with the function and the context.

Statistics and Probability

Interpreting Categorical and Quantitative Data* (S.ID)

Standard A1.S.ID.A.1 **Cluster Heading: A. Summarize, represent, and interpret data on a single count or measurement variable.**

Use measures of center to solve real-world and mathematical problems.*

Scope and Clarifications: (Modeling Standard)

Measures of center should include mean (including weighted averages), median, and mode. For example, a course has 6 tests during the semester. If your average after the first 5 tests is 85, what must you score on the 6th test to have at least an 87 semester average?

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Find the mean, median, and mode given a list of data.	Choose the measure of center most appropriate to the context. Find the weighted average of a data set.	Use mean, median, and mode to solve real-world and mathematical problems, including weighted averages.	Create an example of a real-world problem that requires a specified measure of center to solve and explain the best solution method.

Instructional Focus Statements

Level 3:

In grade 6, students began calculating the mean, median, and mode of a data set. The focus of this Algebra 1 standard is for students to use those measures of center to solve a real-world or mathematical problem. To do that, students must be able to select a measure of center appropriate to the context. Students began to develop this understanding in grade 7 and dig deeper into the relationships between the context or shape of a data set and its measures of center in A1.S.ID.A.2 and A1.S.ID.A.3, therefore these standards can be taught concurrently with this one.

Teachers should guide students to understand the usefulness of measures of center in the real world. For example, mode is useful when the owner of an ice cream store wants to know which flavor or flavors of ice cream are favorites of customers. The result may be unimodal, bimodal, or multimodal. Mean is useful in predicting the number of candies in a snack size pack. Median is useful when determining the average price of homes in a town. Weighted

averages are new to students in Algebra 1, so instruction should include an introduction to this concept such as through calculating a course average using weighted grades.

Questions should extend beyond just finding an appropriate measure of center, to using that measure of center to answer a question. These questions require students to have a clear understanding of how that measure of center is found or calculated in order to solve. An example that requires an understanding of mean is provided in the Scope and Clarifications above. An additional example using that same scenario that requires an understanding of weighted averages is if it is also given that the tests are worth 60% of the grade, quizzes are worth 40% of the grade, and their current quiz average is 88%.

Students should use precise mathematical vocabulary when describing their solution and solution methods. Mean, median, and mode should be used appropriately, while recognizing that an average can refer to any measure of central tendency.

Level 4:

Students at this level should be given more complex and possibly open-ended questions. For example, provide 2 sets of data that show the productivity of seed packets from two different companies. Ask students to find each measure of center and then answer the question, "Which company does the mode suggest is best?" and then "Which company does the median suggest is best?" and the same for the mean. Then ask students which measure of center would provide the clearest picture for the gardener and explain why they chose their answer.

Students should also be challenged to write their own examples of real-world problems that require using each of the measures of center to solve. Require students to explain why the intended measure of center is the best choice for their situation and explain their solution method.

Statistics and Probability

Interpreting Categorical and Quantitative Data* (S.ID)

Standard A1.S.ID.A.2 **Cluster Heading: A. Summarize, represent, and interpret data on a single count or measurement variable.**

Use statistics appropriate to the shape of the data distribution to compare center (median, mean, and/or mode) and spread (range, interquartile range) of two or more different data sets.*

Scope and Clarifications: (Modeling Standard)

Students may be given a numerical data set or a visual and/or verbal depiction of a data set. Shapes of distribution are limited to: uniform, symmetric, right skewed, and left skewed.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine the mean, median, and mode of a single set of data from a table. Determine the range and interquartile range of a single set of data from a table. Define center and spread, when related to a data set.	Recognize and describe different distribution shapes using the appropriate mathematical terms: uniform, symmetric, right skewed, and left skewed. Calculate the mean, median, mode, range, and interquartile range of a data set using technology.	Explain similarities and differences using the mean, median, and/or mode and range and/or interquartile range of two or more data sets and describe how they relate to the shape of the data distribution. Given a numerical data set or a visual and/or verbal depiction of a data set, choose which measure of center and measure of spread are most appropriate to the shape of the data distribution.	Compare and contrast the shape and measures of center and spread of multiple data sets. Relate the shape and the measures of center and spread in a real-world context and write a conclusion using appropriate mathematical vocabulary.

Instructional Focus Statements

Level 3:

One of the major goals of statistics is to summarize, compare, and predict data. In grades 6 and 7, students informally compare the center and spread of two data sets when represented visually, such as in a dot plot, box plot, or histogram. Through this comparison, they begin to build an understanding of how the choice of center and spread relates to the shape and context. In Algebra I, students develop a deep conceptual understanding by comparing two or more different data sets, given as a numerical data set or a visual and/or verbal depiction of a data set. They also begin to formalize their descriptions in this course using mathematically appropriate vocabulary to describe the shape of the visual representation of the data distribution with the terms uniform, symmetric, right skewed, and left skewed.

Students should be given the time to explore the measures of center and the measures of spread in differently shaped distributions and make connections between them. For example, allow students to explore what happens to the mean, median, and mode when the data set is uniform, symmetric, or skewed. Have them compare data sets that have equal measures of spread and different measures of center, or vice versa, and then draw conclusions about which measure of center or spread is most appropriate for the shape and context. While students should continue to practice creating the visual representations of the data by hand, having students use technology to create the graphs allows for a focus on the comparison, particularly between multiple representations such as comparing a box plot to a histogram.

Students should be able to discuss which data set has a greater center and which data set has greater variability, when using appropriate measures of center and spread. The mean better represents the data when it is approximately uniform or symmetrical and does not contain any extreme values. However, extreme values will pull on the mean, therefore the median better represents data that is strongly skewed. Likewise, the range provides an appropriate measure of spread when data is more uniform or symmetrical, and the interquartile range gives a clearer understanding of the measure of spread when extreme values exist. Using statistics appropriate to the shape of the data will allow students to represent their data sets accurately and make stronger comparisons between multiple data sets within given contexts.

Instruction should be paired with A1.S.ID.A.3, in which students explain the advantages and disadvantages of using each parameter and how extreme values impact the mean and median. In Algebra 2, students will study standard deviation as another way to measure the spread of a data set and how it is used in a normal distribution of data.

Level 4:

Students should compare and contrast data sets based on real-world situations. Therefore, students should think about how the differences in shape, center, and spread relate to the real-world context accounting for the effects of any extreme data points. They should also be able to describe how the

information can inform decision making. Teachers should guide students to write their conclusions using precise mathematical vocabulary. For example, provide two sets of data that shows the productivity of seed packets from two different companies. Ask students to find each measure of center and then answer the question, “Which company does the mode suggest is best?” and then “Which company does the median suggest is best?” and the same for the mean. Then ask students to describe which measure of center would provide the clearest picture for the gardener and explain why they chose their answer.

Statistics and Probability

Interpreting Categorical and Quantitative Data* (S.ID)

Standard A1.S.ID.A.3 **Cluster Heading: A. Summarize, represent, and interpret data on a single count or measurement variable.**

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points.*

Scope and Clarifications: (Modeling Standard)

Students may be given a numerical data set or a visual and/or verbal depiction of the data set.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Determine the mean, median, mode, range, and interquartile range of a single set of data from a table. Identify unusual or extreme data points in a data set.	Recognize and describe different distribution shapes and their appropriate measures of center and spread. Interpret data displayed in dot plots, box plots, stem plots, back-to-back stem plots, and histograms.	Interpret differences in shape, center, and spread of the data sets in context given a numerical data set or a visual and/or verbal description of the data. Describe the effect of an extreme value on the shape, center, and spread of a data set.	Use justifications about appropriate measures of center and spread to interpret differences between data sets in context. Explain advantages and disadvantages of using each measure of center and spread. Explain and relate in context how extreme values affect the mean and median differently.

Instructional Focus Statements

Level 3:

This standard should be taught along with A1.S.ID.A.2, in which students compare the measures of center and spread in multiple data sets in relation to the shape of the data. The focus of A1.S.ID.A.3 is to use their understanding of those relationships to interpret the differences in context. Students should also be able to describe how unusual or extreme data points affect the shape and measures of center and spread. Students are not expected to classify an

extreme data point as an outlier in this course. Therefore, instruction does not need to include the calculations to determine if a value is an outlier. Outliers will be addressed in a future course.

Instruction should provide opportunities for students to work with multiple data sets of different shapes and allow them time to discover how unusual or extreme data points affect the shape as well as measures of center and spread. These data sets may be given as a numerical data set or a visual and/or verbal depiction of a data set.

They should be able to describe how extreme values impact each measure of center and spread. As students are asked to discuss skewedness, teachers should focus on the relationship of the extreme value to the skew of the data. Teachers should lead students to discover that the mean and the range are less useful with strongly skewed data sets while the median and interquartile range are less impacted when extreme data values do exist. When data sets include extreme values, students can compare the median to the quartiles and draw conclusions about the data set in context based on these relationships.

To help students better understand how extreme values affect the mean and median, teachers can have students compare the median and mean visually. The median of a density curve is the value that splits the area of the distribution in half, while the mean is the balance point (the point at which a fulcrum can be placed). A skew or extreme value affects the balance point (mean) much more than the halfway point of the area.

Level 4:

In A1.S.ID.A.2, students justified which measures of spread were appropriate based on the shape of the data. For this standard, students at a level 4 will use those justifications to interpret the differences between the data sets in context. Teachers can also facilitate a discussion on whether a real-world context can have an impact on determining which measure of center or spread is more appropriate.

Students at this level should be able to explain how the mean and median are affected by unusual or extreme values and the advantages and disadvantages of using each one in relation to the context. Teachers can have students compare the same distribution within a context with and without an extreme value to see how the mean is impacted. Teachers should ask students to explain how the mean shifts towards a skew (given a shape) or extreme value (given a numerical data set) and explain why this tendency is the reason mean is not an appropriate measure of center in these situations and relate it to the context of the situation.

Statistics and Probability

Interpreting Categorical and Quantitative Data* (S.ID)

Standard A1.S.ID.B.4

Cluster Heading: A. Summarize, represent, and interpret data on two categorical and quantitative variables.

Represent data from two quantitative variables on a scatter plot and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.*

Scope and Clarifications: (Modeling Standard)

Fitted functions are limited to linear, exponential, and quadratic functions.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose a linear function to fit a given data set. Informally decide if a given scatter plot is best represented by a linear or non-linear function.	Represent data from two quantitative variables on a scatter plot. Choose which function type (linear, exponential, quadratic) is suggested by the shape of the data. Informally fit a linear function to a given set of data represented in a scatter plot. Use a given linear function to solve a problem.	Represent data from two quantitative variables on a scatter plot and describe how the variables are related. Fit a function (linear, exponential, quadratic) to a given set of data. Use a linear, quadratic, or exponential function fitted to data to solve problems in the context of the data. Recognize that extrapolation may be unreliable for making predictions.	Create a contextual situation with an embedded data set derived from a given function. Explain the relationship between the function, data set, and the contextual situation using precise mathematical language and justifications. Use a given function or function type to explain the relationship between two quantities in context. Explain why extrapolation may be unreliable for making predictions.

Instructional Focus Statements

Level 3:

In Grade 8, students constructed and interpreted scatter plots and informally fit a line to the data in a scatterplot that suggested a linear relationship. In Algebra 1, students will connect that understanding to finding the line of best fit using technology. They will also discover that non-linear functions sometimes provide a better fit to the data. This concept broadens the options students have for modeling data sets, where data sets can be represented in tabular, graphical, or as a discrete set of points.

Students will explore real-world situations using the function types they are learning in Algebra 1 (linear, exponential, and quadratic) to choose an appropriate model that fits the data. They will also learn how to use technology to find the equation for the function that best fits the data. As students progress through the course, it should become apparent to them that many real-world situations produce data that can be modeled using functions that are not linear. Students will learn to use these mathematical models to capture key elements of the relationship between the two variables and explain what the model reveals about the relationship.

Students should develop an understanding of the connection that exists between the model and the contextual situation that it represents and understand that this connection is essential in identifying and building appropriate models. To build this concept, students can start by informally finding the line of fit to data in a given scatter plot as they did with linear models in Grade 8. Then they can find the actual line of best fit using technology. This should lead to an exploration around the similarities and differences between their informal line and the line of best fit. Discourse should focus on why the informal line was chosen and what data points may have affected the line of best fit, taking into consideration both the function type and the placement of the fitted line. Students can use various forms of technology to explore and represent scatterplots to enhance their ability to see the relationship that exists between the variables.

As students solidify their understanding, they should be able to describe how the variables are related within the context of the situation. For example, as a child ages (x-axis variable), she gets taller (y-axis variable) or the further a car travels (x-axis variable), the less gasoline is in the tank (y-axis variable). It is imperative that students discover that sometimes obvious patterns may not tell the whole story. Students should recognize that making predictions outside the given range of data points provided is referred to as extrapolation and can lead to false assumptions. For example, once a child reaches a certain age, she will stop getting taller. Therefore, the function that fits the data in this example cannot be used to predict the child's height at age 30.

Level 4:

As students extend their understanding, they should be able to create a contextual situation with an embedded data set derived from a given function. Students should also be able to explain and provide justifications for the relationships that exist between the function, data set, and the contextual situation using precise mathematical language. Particular attention should be placed on creating situations that differentiate between linear, exponential, and quadratic functions. Students should be able to justify and explain why one function is more appropriate than another function for the contextual situation.

Students at this level should have a deeper understanding of why extrapolation can lead to false assumptions. As they apply models to make predictions about unknown situations, they should justify what domain is appropriate based on the given context. For example, if time is the independent variable, students should recognize that the domain should be restricted to values greater than or equal to zero. Teachers should also expect students to explain why extrapolation is not reliable in some situations and that it cannot be assumed that the existing trend will continue to unknown values beyond the given data.

Statistics and Probability

Interpreting Categorical and Quantitative Data* (S.ID)

Standard A1.S.ID.C.5 **Cluster Heading: C. Interpret linear models.**

Interpret the rate of change and the constant term of a linear model in the context of the data.*

Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify the slope and y-intercept, given a linear function in slope-intercept form. Identify two points on a scatterplot that could be used to build the line of best fit.	Understand that the rate of change in a linear model refers to the slope and the constant term when written in slope-intercept form is the y-intercept. Identify the slope and y-intercept of the graph of a line. Estimate the slope and y-intercept of a linear model that would best fit data on a given scatterplot.	Interpret the rate of change of a linear model in the context of the data. Interpret the constant term of a linear model in the context of the data.	Justify the appropriateness of the slope and y-intercept of a linear model in the context of the data.

Instructional Focus Statements

Level 3:

It is important that students build an understanding that fitting a line to data in a scatter plot reveals possible relationships between two quantities and provides a tool for analyzing the data. Therefore, students should investigate models with the intention of interpreting them in the context of the data. Students were introduced to this concept in grade 8 and continue to investigate and interpret linear models in context in high school.

Instruction should expose students to multiple contextual linear models and ask them to capture key elements of the relationship between the two variables and explain what the model reveals about the relationship. Specifically, students should be interpreting key features of linear models such as slope and y-intercept. Instruction should focus on slope as the rate of change of the function, identifying for every increase of one by the independent variable (i.e., x), the dependent variable (i.e., y) increases or decreases by the given slope. Discussion and questioning should be focused on interpretation of the y-intercept as the value of the dependent variable when the independent variable is zero and that when the independent variable is a unit of time, the y-intercept can be interpreted as the initial value of the dependent variable. For example, given a linear model of $f(t) = 13.04t + 6.79$ where t represents years since 2009 and $f(t)$ represents the height of a tree in inches, students should interpret 13.04 as the number of inches the tree grows per year and 6.79 inches as the initial height of the tree in 2009. Students should be expected to interpret values such as $f(12)$ or $f(0)$, or solve equations like $f(t) = 0$. In the real world, there are instances when the initial value may not be the y-intercept. Providing students examples of such situations can elicit strong discussions around why the y-intercept is not useful for interpretation in that context.

Overall, students should practice modeling with mathematics by using linear models to describe how one real-world quantity of interest depends on another, which will then be expanded to include other function types in future courses.

Level 4:

As students develop a deep understanding of linear models, they should be expected to justify the reasonableness of their model to determine if it makes sense in the given context. Given the example in level 3 above, students would consider if $13.04x + 6.79$ is reasonable for describing tree growth, but students should be challenged to relate life experiences and number sense or research on the internet to make and explain this determination. Teachers should provide examples of scatterplots that do not fit the given context so that students have opportunities to justify why the model does not appropriately fit the context. For example, if the tree model was $y = 275.23x - 5.47$ instead, students should realize that the slope is much larger than expected and that the tree had a negative height in 2009. Although some students might say that the tree was in the ground at that point, the height of the tree would still be positive.

Statistics and Probability

Interpreting Categorical and Quantitative Data* (S.ID)

Standard A1.S.ID.C.6 **Cluster Heading: C. Interpret linear models.**

Use technology to compute the correlation coefficient of a linear model; interpret the correlation coefficient in the context of the data.*

Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X	X	X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Describe the correlation of the data set that it represents as strong or weak, given a correlation coefficient. Describe the correlation of the data set as having a positive or negative direction, given a scatter plot.	Choose a correlation coefficient that represents a data set given a scatter plot suggesting a linear fit. Choose the strength and direction that describes the relationship, given a scatter plot that represents a data set.	Calculate the correlation coefficient of a linear model using technology. Interpret the correlation coefficient of a linear model in the context of the data.	Create a real-world linear situation and calculate and interpret an appropriate correlation coefficient. Explain what the correlation coefficient represents with respect to the contextual problem, using mathematically precise vocabulary and justifications.

Instructional Focus Statements

Level 3:

In grade 8, students developed a conceptual understanding of what it means for a set of data to have a linear association and informally determined a linear model for a set of data. In Algebra 1, students should extend their understanding by exploring the strength of the correlation between the variables. The correlation coefficient, denoted by r , measures how close the data points are to the line fitted to the data with a range of $-1 \leq r \leq 1$. Students should interpret this as the closer r is to 1 or -1 the stronger the correlation of the points is to the fitted line, and the closer r is to 0 the weaker the correlation of the points is to the line. Students should also recognize the rare cases of $r = 1$ or $r = -1$ as meaning a perfect linear correlation exists and

when $r = 0$ as meaning there is no correlation at all between the variables. To help students understand what this means, they should be provided with multiple sample scatter plots of data within context to compare with the respective correlation coefficient for that data.

Students should use technology to calculate the correlation coefficient of data that has a linear relationship and understand that this value indicates that the data set has a correlation that can be described as either strong or weak, and either positive or negative. Therefore, when describing the correlation of the variables, students should use the terms strong positive, weak positive, strong negative, weak negative, or no correlation. Algebra 1 students are not expected to use a formula to find the correlation coefficient. Instead the focus should be on interpreting the correlation coefficient with respect to the data set and contextual situation. Students should develop a conceptual understanding by analyzing graphical representations that illustrate what the correlation coefficient represents with respect to the distance each point is from the line of best fit. They should recognize that a strong correlation exists when the data points are closer to the line of best fit and a weak correlation exists when the points are more widely dispersed with larger distances between each data point and the line of best fit.

Additionally, students should explain both orally and in writing the connection between the correlation coefficient and contextual situation that it represents. It is important that students don't assume causation when a strong correlation exists. This concept is further developed in A1.S.ID.C.7.

Level 4:

As students extend their understanding of what a correlation coefficient means and reveals about the data set, they should be able to illustrate and explain the relationship between the points in the scatter plot and the line of best fit. Students at this level should be able to create their own contextual situations and provide data with a correlation coefficient that appropriately corresponds to the situation. Students could also be challenged to hypothesize the relationship between two real-world variables and create an experiment to collect relevant data. Then they can test their hypothesis by creating a scatter plot representing their data, describing the relationship in context, finding the correlation coefficient, and interpreting it in context.

The emphasis on technology is specifically called out in the standard and students should use technology to not only compute the correlation coefficient but also use technology as a means to compare different models and their respective features. Additionally, students should solidify their understanding by reasoning and making sense of different correlation coefficients and their relationship to their contextual situations. Students at this level may be curious about at what point the correlation coefficient shifts from strong to weak. Pre-defined values for these shifts vary somewhat from source to source. Therefore, students should be encouraged to discuss where they think the shift would be appropriate and explore additional terms to describe the strength of the correlation such as moderate or even moderately strong or moderately weak.

Statistics and Probability

Interpreting Categorical and Quantitative Data* (S.ID)

Standard A1.S.ID.C.7 **Cluster Heading: C. Interpret linear models.**

Explain the differences between correlation and causation. Recognize situations where an additional factor may be impacting correlated data.*

Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Given a correlation coefficient, describe the correlation of the data set that it represents as strong or weak. Given a scatter plot, describe the correlation of the data set as strong or weak.	Define correlation as the association between two variables and provide an example. Define causation between two variables as a cause-and-effect relationship and provide an example.	Explain the differences between correlation and causation. Explain why a strong correlation does not imply causation. Distinguish variables that are correlated because one is a cause of another and justify their reasoning. Recognize situations where an additional factor may be impacting correlated data.	Critique and recognize misinterpretation of correlation as causation in worked examples and explain why the reasoning is incorrect. Create or find real-world data points that suggest a strong linear correlation between two variables, but one is not caused by the other.

Instructional Focus Statements

Level 3:

As students learn about correlation in standard A1.S.ID.C.6, an emphasis should be placed on interpreting a strong correlation appropriately. One of the most common misconceptions when learning statistics is assuming that a strong correlation implies causation. Instruction for this standard should help students recognize that causation cannot be determined by data alone. Therefore, these two standards should be paired together.

Students should understand correlation as the strength of association of two variables, which does not mean that changes in one variable cause changes in the other (causation). Teachers should provide examples of contextual situations in which there exists both a strong correlation with causation, as well as examples with a strong correlation but the implication of causation is obviously incorrect. For example, teachers can show students a scatterplot comparing children's shoe sizes to their level of vocabulary, indicating a strong correlation between the two. Using reasoning, students should realize that this does not indicate that shoe size influences a child's vocabulary by listing other factors or lurking variables that may affect vocabulary instead, such as age or level of reading. Until all other factors can be eliminated, it cannot be implied that shoe size causes changes in vocabulary.

Although students should not imply that correlation means causation, there are situations in which variables are correlated because one is the cause of another. For example, there is a correlation between the number of hours studied for a test and the grade a student received on a test because there is a cause-and-effect relationship between the two. So, a causal relationship between two variables implies the variables will be correlated, but correlation does not imply causation.

Level 4:

As students develop a deep understanding of the relationship between correlation and causation, they should be given opportunities to critique the work of others and justify why their reasoning is correct or incorrect. Students should recognize when a real-world example implies correlation from causation or causation from correlation. Teachers can also ask students to consider the impact that sampling has on causation. Causation is often supported by a strong use of sampling techniques involving randomization to create fair and equal opportunities.

Students at this level should be challenged to find examples of real-world data that suggest a strong linear correlation between two variables, but they are obviously not linked. Alternately, they could conduct their own experiment to collect data on two variables with a strong correlation and be able to justify and explain whether the causation exists between the variables. Experiencing both types of situations solidifies the idea that correlation does not imply causation.