

Geometry

Instructional Focus Documents

Introduction:

As districts adopt and implement high-quality instructional materials (HQIM) in mathematics, these Instructional Focus Documents (IFD) are intentionally designed to provide a lens into what effective mathematics instruction looks and sounds like in Tennessee classrooms. They are written to support all levels of leadership within a district and complement both the Math Implementation Framework and the Tennessee-specific Instructional Practice Guide (IPG). When used as a suite of resources, the IFDs, the Math Implementation Framework, and the IPG provide guidance and aligned measures with which to track and support district implementation of HQIM in mathematics.

Mathematical rigor does not simply mean increased difficulty or complexity of problems. Rigorous mathematical instruction and learning means deep thinking and exploring at a greater depth. The three aspects of rigor are Conceptual Understanding, Procedural Skill and Fluency, and Application. Each aspect is equally important and necessary for deep mathematical understanding and mastery. These aspects of rigor work in conjunction with the HQIM to provide a meaningful learning experience for students.

Aspects of Rigor:

Conceptual Understanding helps students understand the “how” and the “why” of mathematics. This aspect of rigor focuses on mathematical thinking and reasoning as opposed to answer-getting. Students should understand how and why the math works using mathematical models and manipulatives to aid in achieving conceptual understanding. Instruction should connect prior learning to new ideas and concepts. Opportunities for discussion and reflection may correct and unscramble common misconceptions. Flexible reasoning and fluency grow from conceptual understanding.

Procedural Skill and Fluency is the ability to apply mathematical knowledge accurately, flexibly, and efficiently. It is important to note that the phrase “procedural skill and fluency” is inclusive. The inclusive definition of procedural skill and fluency is *not* the rote use of an algorithm or the recall of facts, but a continuum of understanding. The continuum involves learning or developing algorithms and strategies, executing procedures accurately and efficiently, and learning how to use models and tools. Fluent mastery of a mathematical concept involves the ability to connect and use the Standards for Mathematical Practice while using algorithms and strategies for problem-solving. Students who have achieved fluency can link learned or developed algorithms and strategies to conceptual understanding to explain the “why” behind the procedures. Mathematically proficient students can understand the approaches to solving complex problems and identify correspondences between different approaches to select and use the most appropriate strategy to form an accurate solution path.

Application refers to applying prior knowledge in new and unique situations, other subject areas, and mathematical and contextual problems. Application also includes intentionally integrated content that provides learning opportunities for students to apply and extend their knowledge of multiple standards, clusters, and/or domains within the grade level. The goal is for students to activate their prior knowledge in order to bring a sense of understanding to new mathematical and/or contextual situations.

Evidence of Learning Statements:

The evidence of learning statements provide guidance to connect the Tennessee Mathematics Standards with evidence of learning outcomes that can be collected through classroom activities, observations, or assessments, providing an indication of how students are tracking towards the grade-level expectations that are encompassed within the Tennessee Mathematics Standards. Within the evidence of learning statements, level 3 statements demonstrate on-grade level expectations for all Tennessee students.

The statements are designed to provide a continuum of concrete examples demonstrating what a student who has a particular level of conceptual understanding of the Tennessee mathematics standards will most likely be able to do in a classroom setting. Further, they provide a lens to help offer scaffolding to move a student with unfinished learning up to grade level expectations.

When used alongside high-quality instructional materials, these concrete examples serve to reinforce the grade level expectations and rigor that should be present within the materials and reinforce their inclusion within instruction, ensuring all students have access to on-grade level activities.

Instructional Focus Statements:

Instructional focus statements provide guidance to clarify the types of instruction that will help a student progress along a continuum of learning. These statements are written to provide strong guidance with a focus on Tier I, on-grade level instruction. Thus, the instructional focus statements are written for level 3 and 4.

When used in conjunction with HQIM, instructional focus statements support teacher understanding as they plan and implement HQIM to the depth and rigor of the Tennessee mathematics standards. Additionally, they serve as a benchmark for district and school leaders to use alongside the IPG as they are monitoring HQIM implementation.

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Number and Quantity Quantities* (N.Q)

Standard G.N.Q.A.1 **Cluster Heading: A. Reason quantitatively and use units to solve problems.**

Use units as a way to understand real-world problems.*

G.N.Q.A.1a Use appropriate quantities in formulas, converting units as necessary.*

G.N.Q.A.1b Define and justify appropriate quantities within a context for the purpose of modeling.*

G.N.Q.A.1c Choose an appropriate level of accuracy when reporting quantities.*

Scope and Clarifications: (Modeling Standard)

Apply this standard to any real-world problems studied within the scope of this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X	X*	X

**Procedural skill and fluency only applies to part b.*

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify units appropriately when solving contextual problems.</p> <p>Identify individual quantities in context, label them with appropriate units, and connect the units to the values in the real-world problem.</p> <p>Explain the difference between precision and accuracy.</p>	<p>Choose units appropriately when solving contextual problems, including problems that contain real-world formulas.</p> <p>Identify and interpret necessary information, recognize any extraneous information, and attend to unit selection and/or conversions in order to select a quantity that models a real-world situation.</p> <p>Choose a solution that is both accurate and reasonable with respect to the contextual situation.</p>	<p>Use appropriate quantities in formulas, converting units when necessary.</p> <p>Define and justify with precision the appropriate quantities, and their units, within a context to model a real-world problem.</p> <p>Make observations about extraneous information embedded in context and explain why information is extraneous in a real-world problem.</p>	<p>Create a real-world problem involving formulas that must be analyzed for appropriate units.</p> <p>Flexibly represent quantities, with appropriate units, in modeling situations and explain relationships among representations.</p> <p>Describe the accuracy of a measurement embedded in a real-world context by stating the possible error when appropriate.</p> <p>Explain why it is important to choose an appropriate level of</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	Describe how using inaccurate measurements when reporting quantities can affect the solution.	<p>Choose an appropriate level of accuracy, including appropriate units, when reporting quantities in a real-world context.</p> <p>Explain the reasonableness of answers with respect to the context of the problem when reporting quantities as a result of solving the contextual problem.</p> <p>Describe the most common causes of inaccuracies in contextual problems (e.g., when using measurement tools).</p>	accuracy and what limitations exist on measurement when reporting quantities in contextual problems.

Instructional Focus Statements

Level 3:

In grades K-8, students developed an understanding of measuring, labeling values, and understanding how the value of a number relates to the described quantity. In the high school Numbers and Quantity (N) domain, students develop an understanding of reasoning quantitatively (MP 2) and solving problems requiring the evaluation of the appropriateness of the form in which quantities are provided and reported. This standard is important throughout Algebra 1, Geometry, and Algebra 2 and should be integrated with a wide variety of standards throughout the course.

Students should be able to recognize the relationship between the units for all variables in a formula, use appropriate quantities in formulas, and convert units as necessary. Students should be engaged in classroom discourse (MP 8) that promotes explaining their reasoning and justifying quantities (MP 3) as a result of a solution pathway. Students should be given ample opportunity to work with data representations where students must think critically to set an appropriate scale that displays the data to show key features. It is imperative that students attend to precision (MP 6) in using, interpreting, and reporting units. Students should extend their understanding of using appropriate quantities to defining and justifying appropriate quantities in modeling situations. Students should be able to select appropriate quantities embedded in real-world contextual problems, identify extraneous information, choose appropriate units, etc. Instruction should focus on providing opportunities for students to attend to precision (MP 6) by describing the quantities in

modeling situations. Furthermore, students must be given opportunities to write and create appropriate labels for quantities and explain the meaning of the quantities in a context.

In middle grades, students have had experience interpreting and reporting quantities that involve area, volume, and rates. Students at this level are able to choose an appropriate level of accuracy when reporting quantities of a real-world problem. As this is a modeling standard, students should solve contextual problems and be able to choose a level of accuracy of measurement quantities that is reasonable and makes sense to the contextual situation. For example, when solving a multi-step problem or using graphing technology, students should determine when it is appropriate and not appropriate to use precise values (values that are not rounded or truncated), rounded values, or truncated values. Students should be able to justify their reasoning (MP 3) for using certain values and explain why their choice is important with respect to the context. Additionally, students should experience different solution paths that involve using different forms of values and explain how accuracy does or does not have an impact on the solution within the context of the problem.

This standard builds upon the opportunities students have been given to explain values of numbers in terms of units in previous grades. In middle school, students focused on ratios and proportional relationships and using quantities to describe data from a statistical lens. Both fields help prepare students for choosing a level of accuracy when reporting quantities. Instruction should focus on providing students with a plethora of opportunities to use a variety of measurements including measuring tools and graphing technology (MP 5). Students should have ample time to explore traditional, physical tools as well as electronic, and digital tools. During this exploration, class discussion should focus on helping students ascertain the difference between precision and accuracy and when it is appropriate to apply each of them in certain problems. Furthermore, instruction should be infused with a broad spectrum of different types of units that describe tiny to very large quantities. This is a modeling standard and students should make connections to other disciplines such as science. Being able to identify, interpret, and justify quantities is a skill that will serve students well to have mastered during this course as this standard lays the foundation for using units to understand problems.

Level 4:

Students should be able to interpret, solve, and represent contextual information using appropriate units and attend to precision when interpreting and using units. Instruction should focus on providing opportunities for students to work with problems that have a variety of descriptors and units embedded in the context. Instruction should provide opportunities for students to analyze and critique the interpretation of quantities in a modeling situation. Additionally, students should be given ample opportunities to design their own contextual problem in which they would have to use quantities appropriately in order to describe the modeled contextual situation.

Students have a great opportunity to support their understanding of this standard through the lens of a wide variety of other disciplines. Instruction should focus on providing students with experiences involving problem situations that interest them. Teachers should include opportunities for inquiry and experimentation with ample time for students to explore. For example, students could investigate repeated measurement by exploring the level of precision and/or accuracy provided by various measurement tools in science experiment to determine an acceptable level of accuracy when reporting quantities in real-world context (MP 6). Also, instruction should provide the opportunity for students to analyze and critique (MP 3) the level of accuracy chosen by others' reported quantities.

Geometry

Congruence (G.CO)

Standard G.CO.A.1 Cluster Heading: A. Experiment with transformations in the plane.

Describe transformations as functions that take points in the plane (pre-image) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not, by hand for basic transformations and using technology for more complex cases.

Scope and Clarifications:

For example, a translation will preserve both distances and angle measures associated with a figure, a compression based around a point preserves angle measures but not distances, and a horizontal stretch does not preserve either except in the cases of a vertical line, ray, or line segment.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify when a figure has been rotated, reflected, or translated in a picture or on a graph.</p> <p>Identify when a figure has been dilated, stretched, or compressed in a picture or on a graph.</p>	<p>Generate a single rigid transformation of a figure on or off a coordinate plane.</p> <p>Generate a dilation of a simple figure on or off a coordinate plane.</p> <p>Verify equal distances and congruent angle measures in given figures.</p>	<p>Describe transformations as functions in which points in the pre-image are input values that result in points in the image as output values.</p> <p>Perform a given transformation on a figure by hand and/or using technology and identify the effects of the transformation(s).</p> <p>Verify that a given transformation results in a congruent figure, a similar figure, or neither.</p>	<p>Explain why a given transformation results in a congruent figure, a similar figure, or neither.</p> <p>Critique others' description of the effects of a transformation using precise mathematical vocabulary.</p>

Instructional Focus Statements

Level 3:

The focus of this standard is for students to examine the effects of basic motions involved in transformations, including both rigid (translations, reflections, and rotations) and non-rigid motion (dilations, stretches, and compressions). When transforming polygons, each point of the pre-image acts as an input value, which results in a corresponding output value in the image. It is important for students to understand that this is true for every point within the figure. Allowing students to explore transformations by hand using tools, such as patty paper, tracing paper, transparencies, graph paper, manipulatives, mirrors, and using technology such as dynamic Geometry software helps them see the movement of the transformation.

In grade 8, students discovered that rigid motions result in congruent figures, while dilations result in similar figures. In this course, students can also explore what happens when a figure is stretched or compressed in a single direction to discover that these transformations do not result in either congruent or similar figures. In Algebra I (A1.F.BF.B.2), they identified the effects on the graphs of a function by replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$. In this course, they should connect these concepts by recognizing that $f(x) + k$ is a vertical translation, $f(x + k)$ is a horizontal translation, $kf(x)$ is a vertical stretch or compression, and $f(kx)$ is a horizontal stretch or compression. These transformations are applied to other function types in Algebra II (A2.F.BF.B.3).

To deepen this understanding, students should continue to explore these properties of transformations and be able to distinguish between a transformation that is a rigid motion and one that is not. Student discourse should be focused on which line segments and angles in the pre-image map to which lines and angles in the image and whether they maintain congruency. Students should recognize that congruence is a subset of similarity. Students should be exposed to multiple ways to represent all transformations including words, coordinate notation, function notation, graphs, and technology.

Level 4:

Students at this level should be able to explain using precise mathematical vocabulary why some transformations result in congruent figures, while others do not. As they extend their understanding of comparing transformations, they should be able to analyze others' descriptions of transformations. This will help develop a deep understanding of the connections and comparisons between the different transformation types.

Geometry

Congruence (G.CO)

Standard G.CO.A.2 Cluster Heading: A. Experiment with transformations in the plane.

Given a rectangle, parallelogram, trapezoid, or regular polygon, determine the transformations that carry the shape onto itself and describe them in terms of the symmetry of the figure.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Verify if a figure has a line of symmetry by folding a paper model of the figure.</p> <p>Rotate a figure 90°, 180°, and 270°.</p>	<p>Draw line(s) of symmetry when given a rectangle, parallelogram, trapezoid, or regular polygon, if they exist.</p> <p>Verify if a figure has rotational symmetry by using a manipulative such as tracing paper.</p>	<p>Determine line(s) of symmetry and/or the degree of rotational symmetry that exist in a rectangle, parallelogram, trapezoid, or regular polygon and describe them with precision.</p> <p>Determine the transformations that carry a rectangle, parallelogram, trapezoid, or regular polygon onto itself and describe them in terms of the symmetries of the figure.</p> <p>Determine symmetries and transformations that map a figure onto itself both on and off a coordinate plane.</p>	<p>Synthesize and explain the relationships that exist between the symmetries of a figure and its geometric attributes.</p> <p>Create figures based on a given set of transformations that allows any two-dimensional figure to be carried onto itself.</p> <p>Describe a series of transformations that result in a figure being mapped back onto itself.</p>

Instructional Focus Statements

Level 3:

Students begin to explore lines of symmetry in various figures in grade 4 (4.G.A.3) and consider symmetry of graphs in Algebra 1 (A1.F.IF.C.8). In Geometry, students expand on the concept of symmetry to include rotational symmetry. Instruction should allow time for students to experience the symmetry by having them fold paper models to verify a line of symmetry and using tracing paper or patty paper to physically check for rotational symmetry. Students should then experiment with transformations on symmetrical figures. Instruction should initially focus on having students explore transformations with rectangles, parallelograms, trapezoids, and regular polygons enabling them to discover which transformations result in the image exactly matching up to the pre-image. Students should be provided with appropriate tools to aid with their explorations such as tracing paper, patty paper, graph paper, mirrors, and/or technology. These explorations should lead students to define a set of transformations in which the image ends up exactly matching the pre-image, ultimately leading students to recognize the mathematical connection between transformations and the symmetries within a figure. For example, while exploring rectangles, students discover that a reflection along a line of symmetry will result in the figure mapping onto itself, and a 180-degree rotation about a specific point at the center of the rectangle will also map it back onto itself. Students should be able to compare the symmetries of a rectangle with quadrilaterals that are not rectangles to better understand the attributes of each quadrilateral and each transformation. This standard helps to lay the groundwork for G.CO.A.3.

Students may assume some figures have symmetry when they do not. For example, stating incorrectly that a parallelogram that is not a square or rectangle has lines of symmetry or stating incorrectly that a figure has rotational symmetry at 360° . When these common mistakes occur, it is important to have students use their tools to verify their claim and self-correct.

Students should work with figures both on and off a coordinate plane. When describing lines of symmetry, students should be specific about where the lines of symmetry exist within the figure and consider all possible lines of symmetry. When describing rotational symmetry, students should be specific about how many degrees the figure must be rotated to result in that symmetry. For example, a regular hexagon has rotational symmetry every 60° .

Level 4:

Students should have ample opportunities to compare multiple figures that do and do not have symmetry. They should be encouraged to use these comparisons to explain the connection between the symmetries and the geometric attributes of a figure. To solidify this understanding, students should be able to create their own examples of figures with none, one, or more types of symmetry. Additionally, students should be able to explain their reasoning with precise mathematical vocabulary.

Students can also explore multiple series of transformations that result in a figure mapping back onto itself. For example, a reflection in the x-axis and the y-axis, followed by a 180° rotation. Students can be challenged to consider how symmetry in some figures impacts the series.

Geometry

Congruence (G.CO)

Standard G.CO.A.3 Cluster Heading: A. Experiment with transformations in the plane.

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify rotations, reflections, and translations in a picture or graph.	<p>Informally describe rotations, reflections, and translations in general terms.</p> <p>Measure and construct angles, line segments, and circles both on and off a coordinate plane.</p>	<p>Develop and give a student-generated definition of a rotation in terms of distances, angles, and arcs.</p> <p>Develop and give a student-generated definition of a reflection in terms of distance, and parallel and perpendicular lines.</p> <p>Develop and give a student-generated definition of a translation in terms of distance and parallel lines.</p> <p>Generate precise definitions of rotations, reflections, and translations using appropriate mathematical vocabulary such that</p>	Critique others' definitions of rotations, reflections, and translations, paying particular attention to the level of precision and providing counterexamples when they exist.

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		they are unique to the given transformation.	

Instructional Focus Statements

Level 3:

In grade 8, students explored the effect of transformations on two-dimensional figures (8.G.A.1). In Geometry, students continue to explore transformations both on and off the coordinate plane as they experiment with transformations in this cluster of standards.

One of the most difficult aspects of this standard for students is the level of precision required to generate definitions for rotations, reflections, and translations. Therefore, students should have ample time to experiment with transformations as they develop these definitions. These hands-on experiments can involve using transparencies or tracing paper to visualize and describe the movements of specific angles or points during each transformation. Students should also be challenged to use a straightedge and a compass to perform transformations (an application of G.CO.D.11) to help students focus in on what specific movements occur during each transformation and develop more precise definitions.

It is important that students be allowed adequate time to explore transformations so that they have the appropriate conceptual understanding that allows them to develop the definitions on their own. Students should be encouraged to use the properties of each transformation and focus on the relationships that exist between the lines and angles in the pre-image and the lines and angles in the resulting image. Student discourse should focus on observations of the movements that occur during each transformation and measurements taken between the pre-image and resulting image. They should use appropriate tools such as rulers, protractors, compasses, and/or dynamic geometry software to precisely measure the distances and angles and construct any arcs which occur during the transformations. They may also use the coordinates on a graph to verify geometric relationships algebraically (G.GPE.A cluster).

Students should be provided examples of transformed figures and asked to explain why or why not a relationship can be described as a specific rigid motion. It is vital that students attend to precision when writing their definitions to ensure they are accurate and unique to the given transformation.

Level 4:

Critiquing others' definitions will help students discover the necessary level of precision needed to accurately define these transformations. Providing students with both examples and non-examples of rigid motions is vital to this process. Likewise, students who can self-generate a counterexample demonstrate a much deeper understanding of the properties of these transformations. As students create their own student-developed definitions and critique others' definitions, they should use examples, counterexamples, and precise mathematical vocabulary to justify their conclusions.

Geometry

Congruence (G.CO)

Standard G.CO.A.4 Cluster Heading: A. Experiment with transformations in the plane.

Given a geometric figure, draw the image of the figure after a sequence of one or more rigid motions, by hand and using technology. Identify a sequence of rigid motions that will carry a given figure onto another.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Identify a single rigid motion that carries a given figure onto another.	Draw the image on or off a coordinate plane, or by using technology such as dynamic geometry software, given a geometric figure and a rigid motion (rotation, reflection, or translation).	<p>Draw the image, given a geometric figure and a sequence of rigid motions.</p> <p>Describe a sequence of rigid motions that will carry a given figure onto another.</p> <p>Recognize and explain that there can be more than one correct sequence that will map a given pre-image onto an image.</p>	<p>Describe multiple sequences that will map one to the other, given a pre-image and image.</p> <p>Critique others' drawings or descriptions, recognizing mistakes and correcting them by explaining their justifications using precise mathematical language.</p>

Instructional Focus Statements

Level 3:

Students began exploring the rigid motions of translations, reflections, and rotations in grade 8 (8.G.A.1) both on and off the coordinate plane using appropriate tools to help them visualize each transformation, such as patty paper, tracing paper, transparencies, graph paper, manipulatives, mirrors,

compasses, protractors, rulers, and technology. In this course, students explore the effects of applying a sequence of transformations with the same appropriate tools. While they are developing their understanding of the effects of these transformations, it is vital students experience performing the transformations by hand using these tools to ensure they see them as actions both on and off a coordinate plane.

As students are drawing each transformation in the sequence, discussion should focus on what students notice about the new figure in comparison to both the prior iteration as well as the original figure. Once students have had ample experience drawing a series of transformations, they should be able to determine the sequence applied to a given pre-image that would result in a given image and verify their sequence by showing and/or explaining how they know it is correct. Students can then be challenged to find a different sequence that would result in the same image. Allowing students to share their answers will help all students recognize and explain that there can be more than one correct sequence that will map a given pre-image onto an image.

A common misconception about the order of transformations occurs when students are working with a sequence of two or more transformations. The order the transformations are applied can make a difference on the resulting image. Allowing students to explore the same transformations within a sequence, but applied in a different order, will help students understand the importance of listing the transformations in a specific order.

Level 4:

Students should continue with their exploration of the effects on a figure when a sequence of rigid motions are applied. Finding alternate sequences that will map a pre-image onto an image and recognizing that the order of the transformations may change the result will deepen their understanding of the properties of each transformation type. Additionally, students will solidify their understanding by critiquing others' applications of a specified sequence or description of a sequence provided a pre-image and image, analyzing their mistakes, if any, and providing corrections as needed.

Students at this level can compare the relationship between a sequence of transformations and a single transformation to build an understanding that one transformation can often more efficiently result in the same image. For example, a sequence of two reflections across parallel lines results in a translation or a reflection in the y-axis followed by a reflection in the x-axis results in the same image as a 180-degree rotation about the origin.

Geometry

Congruence (G.CO)

Standard G.CO.B.5 Cluster Heading: B. Understand congruence in terms of rigid motions.

Given two figures, use the definition of congruence in terms of rigid motions to determine informally if they are congruent.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Choose the correct term to describe the rigid transformation given a picture.	Determine what transformation(s) will map one figure onto another. Tell which transformations are considered rigid motions and informally explain why.	Use the definition of congruence in terms of rigid motions to determine informally if two given figures are congruent.	Justify why translations, rotations, and reflections are called rigid motions using precise mathematical vocabulary. Use the definitions of rigid motion and congruence to construct a viable argument on why a rigid motion results in congruent figures.

Instructional Focus Statements

Level 3:

In grade 8, students explored the effects of transformations on two-dimensional figures. Through this exploration, students began to develop an informal understanding of the concept of congruence. In this high school geometry cluster, students will further explore rigid transformations as a method to determine if two figures are congruent. The concept of congruence in terms of rigid motion developed in this standard is foundational to the other standards in this cluster.

Students should recognize that a rigid motion transformation results in an image that is congruent to the pre-image. Student discourse should include discussion about the meaning of the term rigid motion in relation to transformations. They can draw on their prior learning to recall that translations, reflections, and rotations are rigid motions, while dilations are not considered rigid motions. Revisiting this comparison can help students solidify their understanding of congruence in terms of rigid motion.

Students may generalize that two figures are congruent if they have the same shape and size. To refine the vagueness of this definition, this standard focuses on the concept of congruence in terms of rigid motions. Therefore, the abstract concept of congruence can be grounded in kinesthetic experiences. In the G.CO.A cluster, students explore transformations using multiple tools. Using transparencies or tracing paper to perform transformations clearly represents the concept that rigid motions preserve distances and angle measures because the figure does not become distorted in any way. Therefore, two figures can be determined congruent if a rigid motion exists that maps one figure onto the other.

Level 4:

Students will deepen their understanding of this standard by developing a viable argument on why the image of a rigid motion results in a figure congruent to its pre-image. Providing students with examples of transformations that are and are not rigid motions and having them explain why the figures are congruent or not using the definitions will help students build their arguments. These examples can come from other student work or teacher created examples.

Geometry

Congruence (G.CO)

Standard G.CO.B.6 **Cluster Heading: B. Understand congruence in terms of rigid motions.**

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify which transformations are considered rigid motions.</p> <p>Choose the correct term to describe the transformation that maps one triangle onto another from a picture.</p> <p>Find the measures of sides and angles of a triangle both on a blank space and on a coordinate plane using appropriate tools such as a ruler, protractor, Pythagorean Theorem, distance formula, or dynamic geometry software.</p>	<p>Identify corresponding pairs of sides and angles given a drawing of two figures that are marked or a congruence statement describing two figures.</p> <p>Perform a rigid motion on a given triangle both on a blank space and on a coordinate plane using appropriate tools such as a mirror, tracing paper, or dynamic geometry software.</p>	<p>Show that two triangles are congruent using the definition of congruence in terms of rigid motions.</p> <p>Show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.</p> <p>Verify that corresponding sides have equal lengths and corresponding angles have equal measures, given congruent triangles.</p> <p>Use properties, such as the reflexive property, the symmetric property, and the transitive property, to</p>	<p>Explain that rigid motions result in congruent corresponding pairs of sides and angles and thus congruent figures.</p> <p>Create examples of non-congruent triangles that have some congruent corresponding parts and explain why they are not congruent using the definition of congruence in terms of rigid motion.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		<p>justify corresponding parts of two triangles are congruent.</p> <p>Read and use correct notation that shows corresponding parts are congruent and triangles are congruent.</p> <p>Write a triangle congruence statement using correct notation.</p>	

Instructional Focus Statements

Level 3:

In G.CO.B.5, students make the connection that a rigid motion results in a congruent figure. Standard G.CO.B.6 focuses on the understanding that in order for two figures to be congruent, their corresponding parts must also be congruent. Studying this concept with triangles allows students to deepen their understanding while preparing them for the exploration of the converse in G.CO.B.7.

Instruction should require students to develop a strong understanding of what is meant by two congruent figures. Students have developed an informal understanding by exploring rigid motions but should solidify what makes two figures congruent by comparing the distances and angle measures in each figure. It is important for students to make the connection between congruent figures and rigid motions and use measurements to verify that connection. It is best for students to discover this connection by comparing side and angle measurements of figures that have been transformed by a rigid motion with those that have not. Students must practice precision in their measurements to verify these results.

Students can also use relationships between the triangles to justify that corresponding parts are congruent. For example, if the triangles share side $\overline{AB}$, they can justify that $\overline{AB} \cong \overline{AB}$ because of the reflexive property. Other properties such as the symmetric property and the transitive property may also apply. Students are expected to understand and use these properties by name in their justifications in this course.

It is important that students also know how to read and correctly notate congruent triangles (ex. $\triangle ABC \cong \triangle DEF$) and their corresponding parts (ex. $\angle A \cong \angle D$; $\overline{AB} \cong \overline{DE}$). They must attend to precision when setting up these congruence statements and understand that the statement itself will clearly indicate the congruent corresponding parts when written correctly.

Level 4:

Students must use their critical thinking skills to create their own examples of triangles that have some congruent corresponding parts but do not result in congruent triangles. To extend this further, students need to use their communication skills to explain why this happens using the definition of congruence in terms of rigid motion.

Geometry

Congruence (G.CO)

Standard G.CO.B.7

Cluster Heading: B. Understand congruence in terms of rigid motions.

Explain how the criteria for triangle congruence (ASA, SAS, AAS, SSS, and HL) follow from the definition of congruence in terms of rigid motions.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify corresponding parts in two triangles.</p> <p>Write the congruence statement for each corresponding part (i.e., $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$)</p> <p>Identify the included angle between two given sides within a triangle.</p> <p>Identify the included side between two given angles within a triangle.</p>	<p>Determine if two triangles are congruent in terms of rigid motion using appropriate tools such as mirrors or tracing paper.</p> <p>Determine if two triangles are congruent in terms of congruent corresponding parts using tools such as rulers, protractors, Pythagorean Theorem, distance formula, etc.</p>	<p>Determine which combinations of congruent corresponding parts must be known to verify that two triangles are congruent.</p> <p>Explain how knowing SSS, SAS, ASA, AAS, or HL is enough to say that two triangles are congruent using the definition of congruence in terms of rigid motions.</p> <p>Read and use correct notation that shows corresponding parts are congruent and triangles are congruent.</p> <p>Provide instructions to another student giving only three measurements of triangle sides and/or angles so that they can</p>	<p>Provide examples and explain why AAA and SSA are not enough evidence to verify that two triangles are congruent.</p> <p>Critique the work of others in determining congruent triangles based on SSS, SAS, ASA, AAS, or HL and explain why that work is correct or incorrect.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		accurately draw a triangle congruent to their own drawing.	

Instructional Focus Statements

Level 3:

In G.CO.B.6, students show that two triangles are congruent if and only if all corresponding parts are congruent. In G.CO.B.7, students will explore situations where they are only given that some (but not all) of the corresponding parts of two triangles are known to be congruent. Is that enough to verify that the triangles are congruent? Students should explore examples of when this is and is not the case to lead to the discovery that if specific combinations of corresponding parts in two triangles are congruent, then the rest of the parts must also be congruent, which will verify that the triangles are congruent. Therefore, students only need to determine specific criteria to justify that two triangles are congruent using ASA, SAS, AAS, SSS, and HL.

Instruction should focus on allowing students to explore combinations of congruent sides and/or angles to determine which combinations work and which ones do not work. Physical manipulatives allow students to create triangles based on given congruent corresponding parts and then physically compare the triangles to determine if they are congruent. For example, a pair of students can each cut three straws to the same specified lengths (or use a geometry construction manipulative), assemble them into a triangle, and then hold the triangles next to each other to see if they are congruent by SSS. Students can also measure the angle between two straws and tape them to hold that angle measure along with other given information to compare triangles using SAS, AAS, or ASA. Guide students to see that by holding the triangles they are comparing next to each other, they are physically performing a rigid motion by answering the question “will one triangle map onto the other?” This will help them see that the congruence criteria (ASA, SAS, AAS, SSS, and HL) can be verified by the definition of congruence in terms of rigid motion. Dynamic geometry software is also a useful tool to help students see how this works.

Students often struggle to understand the difference between ASA and AAS or SAS and SSA (which does not work). Guide students to see that ASA and SAS indicate the included corresponding side or angle must be congruent, whereas AAS and SSA indicate a non-included side or angle. Allowing students to explore the combinations using physical manipulatives or dynamic geometry software can help. To solidify their understanding, challenge students to draw a triangle on paper and then give a partner a combination of 3 side lengths and/or angle measures such that they can draw a triangle that is congruent to their own. For example, draw a 3-inch segment with a 40-degree angle on one end and a 60-degree angle on the other end.

Students also often confuse HL with SSA. Challenge students to explain why referring to one of the sides as a hypotenuse provides additional information about the triangle – that it must be a right triangle and therefore have a right angle. Also, allowing students to use manipulatives or dynamic geometry

software to manipulate and explore pairs of triangles with only two corresponding congruent sides and a non-included congruent angle will help them to better understand why the right triangle is the exception to the SSA combination.

Level 4:

Challenge students to demonstrate by creating examples using precise drawings, geometry manipulatives, or dynamic geometry software that show why AAA and SSA are not enough evidence to verify that triangles are congruent. Students at this level should also critique the work of others. This can be student work or teacher created examples. Some should be correct and some incorrect. Have students explain why the conclusion is correct or incorrect and verify using the definition of congruence in terms of rigid motions.

Geometry

Congruence (G.CO)

Standard G.CO.C.8 Cluster Heading: C. Use geometric theorems to justify relationships.

Use definitions and theorems about lines and angles to solve problems and to justify relationships in geometric figures.

Scope and Clarifications:

“Justification” may take a variety of forms. Theorems include but are not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify special lines, such as parallel, perpendicular, segment bisectors, and angle bisectors.</p> <p>Identify special pairs of angles, such as supplementary, complementary, vertical, and those formed by lines crossed by a transversal.</p>	<p>Give an informal explanation of the relationship of pairs of lines and/or angles.</p> <p>Define congruence in terms of rigid motion.</p>	<p>Form conjectures about the relationships between lines and/or angles in geometric figures.</p> <p>Use definitions and theorems about lines and angles to solve problems.</p> <p>Use definitions and theorems about lines and angles to justify relationships in geometric figures.</p>	<p>Analyze and critique others’ justifications by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Improve their own justifications based on what they have seen or heard from others.</p>

Instructional Focus Statements

Level 3:

Theorems introduced in this course about lines and angles build from basic understandings of their relationships. For example, students classified shapes based on the presence or absence of parallel and perpendicular lines in grade 4 (4.G.A.2), explored the relationships between specific pairs of angles in grade 7 (7.G.B.4), and learned which transformations result in congruent figures in grade 8 (8.G.A.1). This standard, G.CO.C.8, calls for students to make

connections to this prior learning and apply it to other figures such as a pair of parallel lines cut by a transversal. This will help students conceptualize and mathematically justify these new relationships and use them to solve problems.

Formal proofs are not required in this course. Instead, the focus is on providing reasoning based on definitions and theorems to justify relationships and solutions. Justifications using precise mathematical language and reasoning require practice and should be required of students throughout this course. This can include asking students “why?”, “how do you know?”, or to “explain your strategy.” Asking students to informally explain properties and relationships of lines and angles to justify answers to problems will help students learn to think more strategically and lead them to be more precise in their justifications.

Students often struggle with where to start with their explanation. Encouraging them to begin by stating what they know and identifying or making conjectures about any relationships that exist between the lines and/or angles in the figures can help them to get started. Other questions that can help spur their thinking include, but are not limited to:

- What information are you given?
- What does the picture/drawing/situation show?
- What are you trying to solve?
- How did you begin to think about this problem?
- What definitions or theorems apply?
- What is the relationship between...?
- Can you sketch a picture that demonstrates your reasoning?
- Is the answer/solution reasonable? Why or why not?

Students should draw on their experiences with transformations and congruence as well as definitions and theorems within the scope of this course to both develop and justify conjectures about relationships of lines and angles. For example, students can apply transformations to form their own conjectures about pairs of alternate interior angles formed by parallel lines cut by a transversal and justify them using the definition of congruence in terms of rigid motion. Instruction should reinforce the need for precise mathematical language when sharing their reasoning both orally and in written form. It is particularly helpful to allow students to explain their reasoning to another student to verify their language before writing it down. This discussion will be mutually beneficial since each student will have the opportunity to analyze another’s reasoning as well as talk through their own. Sharing their reasoning and justifications whole group also allows students to hear and learn from the reasoning of others.

Level 4:

To strengthen students’ justifications, they need to analyze and critique others’ reasoning. Students should be exposed to a combination of accurate justifications and some with mistakes or misunderstandings. Students should provide counterexamples with their explanation when disproving these arguments. These critiques will help students develop a deeper understanding of the logic of the relationships and theorems and the need for clear and precise language that they can then apply to their own reasoning.

Geometry

Congruence (G.CO)

Standard G.CO.C.9 **Cluster Heading: C. Use geometric theorems to justify relationships.**

Use definitions and theorems about triangles to solve problems and to justify relationships in geometric figures.

Scope and Clarifications:

“Justification” may take a variety of forms. Theorems include but are not limited to: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define congruent. Identify congruent figures in a picture given markings or notation.</p> <p>Define complementary and supplementary angles.</p> <p>Define parallel lines. Identify parallel lines in a picture using symbols or given notation.</p> <p>Identify the angle opposite a specified side in a triangle and the side opposite a specified angle in a triangle.</p>	<p>Know that the sum of the angles in a triangle is 180 degrees.</p> <p>Define a median of a triangle and identify or draw it with precision in a picture.</p> <p>Define congruence in terms of rigid motion and use it to verify congruent figures.</p> <p>Find the midpoint of a line segment.</p> <p>Calculate the length of a line segment on a graph using the distance formula or the Pythagorean Theorem.</p>	<p>Form conjectures about triangles.</p> <p>Use definitions and theorems about triangles to solve problems.</p> <p>Use definitions and theorems about triangles to justify relationships in geometric figures.</p>	<p>Analyze and critique others' justifications by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Improve their own justifications based on what they have seen or heard from others.</p>

Instructional Focus Statements

Level 3:

Theorems about triangles in this course build on previous learning. For example, in grade 7 (7.G.A.2), students explore the properties of triangles to reason about combinations of side lengths or angle measures that result in a unique triangle, more than one triangle, or no triangle. In grade 8 (8.G.A.1), students learn which transformations result in congruent figures. This standard, G.CO.C.9, calls for students to make connections to this prior learning as they formalize the triangle sum theorem and triangle inequality theorem, among other theorems. They can then use this understanding to solve problems and justify their reasoning.

Students should draw on their prior knowledge and experiences with triangles along with the concepts in standard G.CO.C.8 as they continue to form their own conjectures about triangles including but not limited to concepts like finding angle bisectors and perpendicular bisectors, and using (not finding) all points of concurrency. Students should be allowed to continue exploring triangles and their properties using appropriate tools such as a compass, protractor, tracing paper, or dynamic geometry software as they grapple with the relationships of the figures and their components to develop those conjectures.

Note that this is the first time students will be introduced to a median of a triangle. Instruction should connect to geometric constructions (G.CO.D.11 and G.CO.D.12) and allow students to use appropriate tools to precisely construct the medians in a triangle and discover their point of concurrency, the centroid. Students should be encouraged to start with different types of triangles (acute, obtuse, right, etc.), attend to precision in their construction, and make observations to lead them to the conjecture that the medians will always meet at a single point and how they are different from an altitude of a triangle. It is important for students to see that points of concurrency in triangles have a practical use. For example, having students construct the medians in a cut out triangle to find the centroid and balancing the triangle on their pencil helps students to recognize real world applications.

Formal proofs are not required in this course. Instead, the focus is on providing reasoning based on definitions and theorems to justify relationships and solutions. Justifications using precise mathematical language and reasoning require practice and should be required of students throughout this course. This can include asking students “why?”, “how do you know?”, or to “explain your strategy.” Asking students to informally explain properties and relationships of triangles to justify answers to problems will help students learn to think more strategically and lead them to be more precise in their justifications. Additional questions to consider are included in the Instructional Focus Document for standard G.CO.C.8. Instruction should focus on using precise language and a logical order for their reasoning. It is particularly helpful to allow students to talk through their reasoning with another student to verify their language and logic before writing it down. Classroom discussions can help students broaden their thinking by making them consider different strategies and thought processes. It will also help students recognize the need to develop more precise mathematical language.

Level 4:

To strengthen students' justifications, they need to analyze and critique others' reasoning. Students should be exposed to a combination of accurate justifications and some with mistakes or misunderstandings. Students should provide counterexamples with their explanation when disproving these arguments. These critiques will help students develop a deeper understanding of the logic of the relationships and theorems and the need for clear and precise language that they can then apply to their own reasoning.

Geometry

Congruence (G.CO)

Standard G.CO.C.10 Cluster Heading: C. Use geometric theorems to justify relationships.

Use definitions and theorems about parallelograms to solve problems and to justify relationships in geometric figures.

Scope and Clarifications:

“Justification” may take a variety of forms. Theorems may include, but are not limited to, opposite sides of a parallelogram are congruent, opposite angles of a parallelogram are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define parallel and perpendicular lines and identify them in a figure with markings or notation.</p> <p>Define supplementary angles and recognize them in a figure with markings or notation.</p>	<p>Define a parallelogram as a quadrilateral with two pairs of opposite parallel sides.</p> <p>Define congruence in terms of rigid motion.</p> <p>Define bisect in terms of a line segment or angle and identify bisectors in a figure given markings or notation.</p> <p>Identify diagonals of a polygon and draw them in a given figure using appropriate markings or notation.</p>	<p>Form conjectures about the properties of parallelograms, including rectangles, rhombuses, and squares.</p> <p>Use definitions and theorems about parallelograms to solve problems.</p> <p>Use definitions and theorems about parallelograms to justify relationships in geometric figures.</p>	<p>Analyze and critique others’ justifications by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Improve their own justifications based on what they have seen or heard from others.</p>

Instructional Focus Statements

Level 3:

Students began exploring parallelograms in grade 6 (6.G.A.1). In this standard, G.CO.C.10, students should explore parallelograms to discover their properties using a variety of tools such as rigid motion, constructions, and measuring tools such as rulers and protractors.

Students can construct a parallelogram using skills developed in G.CO.D.11, given that it is a quadrilateral with two pairs of opposite parallel sides. They can then explore the relationships that exist between its sides, angles, and diagonals to discover its properties. For example, apply rigid motions to each side and angle to confirm which pairs are congruent, rotate one angle using tracing paper so that it is adjacent to its consecutive angle to discover that consecutive angles are supplementary, and use reflections to discover that the diagonals bisect each other. It is important to allow students to work together to discover these properties through exploration and form conjectures based on their findings.

Once they have solidified their understanding of the properties of parallelograms, students can explore other special quadrilaterals and compare them to the properties of the parallelogram. Instruction should guide students to recognize that rectangles, rhombuses, and squares are special parallelograms because they have the same properties plus a few extra. In grade 5 (5.G.B.3), students began to classify two-dimensional figures in a hierarchy based on properties. However, even in high school, they often struggle with the concept that all squares are rectangles, but not all rectangles are squares. When considering the hierarchy that includes the parallelogram, there are other relationships to consider based on properties. For example, rectangles and rhombuses are always parallelograms, but only sometimes squares; rectangles are rhombuses, but only when they are squares; and all squares are rectangles, rhombuses, and parallelograms. These relationships are all based on the properties of each figure. Therefore, allowing students to explore the properties of each shape and put them into a hierarchy based on their findings can help them better conceptualize these relationships. Students should then be able to justify their hierarchy by explaining the relationships between these figures based on their properties.

To deepen their understanding, have students create accurate representations of parallelograms, rectangles, rhombuses, and squares both on the coordinate plane as well as using the construction tools developed in G.CO.D.11. Instruction should challenge students to draw each figure on the coordinate plane such that no sides are vertical or horizontal. This will help students conceptualize and mathematically justify these relationships and use them to solve problems.

Formal proofs are not required in this course. Instead, the focus is on providing reasoning based on definitions and theorems to justify relationships and solutions. Justifications using precise mathematical language and reasoning require practice and should be required of students throughout this course. This can include asking students “why?”, “how do you know?”, or to “explain your strategy.” Asking students to informally explain properties and relationships of triangles to justify answers to problems will help students learn to think more strategically and lead them to be more precise in their justifications. Additional questions to consider are included in the Instructional Focus Document for standard G.CO.C.8. Instruction should focus on using precise language and a logical order for their reasoning. It is particularly helpful to allow students to talk through their reasoning with another student to

verify their language and logic before writing it down. Classroom discussions can help students broaden their thinking by making them consider different strategies and thought processes. It will also help students recognize the need to develop more precise mathematical language.

Level 4:

To strengthen students' justifications, they need to analyze and critique others' reasoning. Students should be exposed to a combination of accurate justifications and some with mistakes or misunderstandings. Students should provide counterexamples with their explanation when disproving these arguments. These critiques will help students develop a deeper understanding of the logic of the relationships and theorems and the need for clear and precise language that they can then apply to their own reasoning.

Congruence (G.CO)

Standard G.CO.D.11 Cluster Heading: D. Perform geometric constructions.

Perform formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Scope and Clarifications:

Constructions are limited to: bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X	X	

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Define congruency and recognize symbols that indicate congruent line segments or symbols in a picture.</p> <p>Define bisect and recognize symbols that indicate a bisected line segment or angle in a picture.</p> <p>Define parallel and perpendicular and recognize symbols that indicate parallel and perpendicular lines in a picture.</p>	<p>Use a compass to construct circles and arcs.</p> <p>Bisect an angle and bisect a line segment by drawing the figure on tracing paper and folding it.</p>	<p>Bisect an angle using a compass.</p> <p>Construct perpendicular lines, including the perpendicular bisector of a line segment.</p> <p>Construct a line parallel to a given line through a point not on the line.</p> <p>Use the virtual compass and line tool in dynamic geometry software to construct various geometric objects.</p> <p>Develop methods using a variety of appropriate tools (compass, straightedge, string, reflective device, paper folding, etc.) to</p>	<p>Compare various constructions to determine similarities and differences.</p> <p>Critique others' construction methods using the properties of the construction and precise math vocabulary.</p> <p>Justify why accurate construction methods work using the geometric relationships between the objects in the construction with precise math vocabulary.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		<p>perform precise geometric constructions.</p> <p>Explain informally why and how these construction methods work.</p> <p>Understand the importance of precision in these constructions and attend to precision when performing geometric constructions.</p>	

Instructional Focus Statements

Level 3:

In grade 7 (7.G.A cluster), students begin to experiment with mathematical tools to construct geometric figures and explore their relationships. In this course, students learn to use these and additional tools to perform constructions to explore and demonstrate geometric properties and help students visualize geometric theorems. It is important that students understand that constructions serve a purpose. Therefore, pairing this standard with others throughout this course, including G.CO.A.3 and G.CO.D.12, will help students see the why behind these valuable skills.

Students must be allowed to experiment with the construction tools to develop their own method to perform these constructions rather than just be given specific instructions to follow. They will need a basic understanding of the expected outcome. However, it is through the process of the construction and particularly discovering the method that students will develop a deeper understanding of the properties of these objects. Students will want to use a ruler to bisect a line segment or a protractor to bisect an angle, but when performing these formal constructions, students should not use tools that measure. Instead, they need to focus on the properties of the figures in the construction. Likewise, when students are using dynamic geometry software, they should avoid using automatic commands for bisecting and performing other constructions and use the virtual compass and line tool instead.

Requiring students to perform constructions by hand will help them discover the need for precision, which is essential in performing these constructions or they will not work. For example, a perpendicular bisector construction may not end up exactly in the middle or exactly perpendicular if the student does not use the same holes in the compass during the construction. Dynamic geometry software may help students perform the constructions precisely,

particularly for students who struggle with using the tools precisely, but it is important that students also experience performing constructions by hand. Developing the process of the methods leads to a deeper understanding of why and how each method works. Therefore, it is important that students be required to show their understanding by informally explaining what their chosen method does and why it works.

Some students may need some guidance to get started. Providing questions for them to consider may be helpful. For example, when bisecting an angle, students may be asked to consider what their end goal should look like and how they can use the compass to ensure they are getting congruent measurements. Likewise, when constructing parallel lines, students may be asked to consider the properties of parallel lines. Students should share and explain their methods that work with other students and be open to refining their constructions to ensure they will work in every situation.

Level 4:

Different constructions have some similarities. Allowing students to compare the methods for different constructions, such as constructing parallel lines and constructing perpendicular lines, to find those similarities will help them develop a deeper understanding of what makes them work. It will also help students better realize the importance of precision. This will in turn help students develop a more mathematically precise justification of how and why each method works. Students can use their reasoning to critique others' work by basing their feedback on the properties of the figures and purpose of the construction. Teachers may also consider allowing students at this level to experiment with the tools to discover additional geometric constructions that can be used to solve geometric problems.

Geometry

Congruence (G.CO)

Standard G.CO.D.12 **Cluster Heading: D. Perform geometric constructions.**

Use geometric constructions to solve geometric problems in context, by hand and using technology.*

Scope and Clarifications: (Modeling Standard)

For example, find the point equidistant from three given points by constructing two perpendicular bisectors and locating their point of intersection.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Define and identify an angle bisector and a perpendicular bisector of a line segment.	Construct an angle bisector and a perpendicular bisector of a line segment using math construction tools or dynamic geometry software.	<p>Analyze a contextual problem and identify a geometric construction that can be used to solve it.</p> <p>Use geometric constructions to solve geometric problems in context by hand.</p> <p>Use geometric constructions to solve geometric problems in context using technology.</p>	<p>Justify why a specific geometric construction can be used to solve a contextual problem using the geometric relationships between the objects in the construction with precise math vocabulary.</p> <p>Create and solve a real-world problem that can be solved using geometric constructions.</p>

Instructional Focus Statements

Level 3:

One purpose for studying geometry is to help students learn how to visualize a mathematical concept or problem. This standard along with G.MG.A.1 address this concept of using geometry to visualize a situation for the purpose of solving a problem. The use of mathematical tools can lead to different insights and understandings and can be used throughout the course to explore and develop conceptual understanding of geometric properties and relationships. They should be required to do constructions both by hand and using technology. In G.CO.D.11, students expand on their prior experiences

with math tools to explore how geometric constructions can be used to demonstrate geometric properties. In this standard, they apply these construction methods to contextual problems to solve them.

This standard draws on several math practices. Mathematically proficient students consider the available tools when solving a mathematical problem (MP5). This includes modeling a situation (MP4) using geometric constructions to make sense of the problem and look for entry points to its solution (MP1). This requires precision (MP6) and identifying and using any properties or structures that are discovered (MP7).

For example, a city wants to build a ballpark that can be used by all three of their schools. They want to find a location for the park so that it lies equidistant to each school. Students can begin by creating a triangle from the three locations on a map of the city. Then through experimentation by applying the construction methods they learned in G.CO.D.11, they can discover that finding the point of intersection of the perpendicular bisectors of each side of the triangle will reveal a location that is equidistant to each vertex of the triangle and thus each school.

Another example could be that the same city wants to build a new fire station that has equal access to the three main highways that run through the city. The highways intersect with each other to form a triangle. Through experimentation of their construction methods, students can discover that finding the point of intersection of the angle bisectors will reveal a location that is equidistant to each side of the triangle and thus each highway.

Students can also use constructions to draw a precise polygon or circle and develop their definitions, explore properties of a figure, explore a new theorem, or solve a design problem. There are many real-life applications to geometric constructions including architecture, computer graphics, nanotechnology, manufacturing, cartography, and astronomy.

Level 4:

Students at this level should be encouraged to explore geometric properties and relationships in a variety of applications throughout the course using constructions. Students should be able to justify why a specific geometric construction can be used to solve a contextual problem using the geometric relationships between the objects in the construction with precise math vocabulary. They can also be challenged to create a real-world situation that can be solved using a geometric construction.

Geometry

Similarity, Right Triangles, and Trigonometry (G.SRT)

Standard G.SRT.A.1 **Cluster Heading: A. Understand similarity in terms of similarity transformations.**

Use properties of dilations given by a center and a scale factor to solve problems and to justify relationships in geometric figures.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Distinguish between a dilation and a translation, reflection, or rotation.</p> <p>Perform a dilation on the coordinate plane given a scale factor and a center at the origin.</p>	<p>Perform a dilation both on and off the coordinate plane given a scale factor and a center.</p> <p>Recognize when a scale factor results in a reduction or enlargement of the pre-image.</p>	<p>Form conjectures about the properties of dilations and verify them.</p> <p>Solve problems using the properties of dilations given by a center and a scale factor.</p> <p>Justify relationships in geometric figures using the properties of dilations.</p> <p>Find the pre-image of a dilated figure when given the image, scale factor, and a center of dilation.</p> <p>Find the center of dilation and scale factor when given the pre-image and image of a dilation.</p>	<p>Analyze and critique others' justifications by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Improve their own justifications based on what they have seen or heard from others.</p>

Instructional Focus Statements

Level 3:

In grade 8 (8.G.A.1), students explored the effects of transformations including dilations on a coordinate plane with a center at the origin and began to think about which transformations result in congruent or similar figures. In this high school course, students explore transformations both on and off the coordinate plane and compare rigid motions with non-rigid motions to a greater depth (G.CO.A.1). In this standard (G.SRT.A.1), the focus is on dilations. Students will form conjectures about the properties of dilated figures and verify them. They will then use those properties to solve problems and justify relationships in geometric figures.

Instruction should provide students opportunities to perform and explore dilations both on and off the coordinate plane. They should use a variety of tools, including but not limited to, paper and pencil, rulers, grid paper, dynamic geometry software, and the coordinate plane. When performing dilations on the coordinate plane, students should start with the center at the origin and form conjectures based on the resulting coordinates. They can then justify those conjectures by applying those conjectures to a dilation with the center of dilation away from the origin and verifying that the resulting dilation is an image that is mathematically similar to the pre-image. For example, if applying a scale factor of 2 to a triangle with vertices at $(-1, -2)$, $(2, 3)$, and $(3, -2)$ and center of dilation at $(2, -5)$, the students can double the distance from the center to each vertex of the pre-image along the same line to find the coordinates of the vertices of the image of the dilated triangle to be $(-4, 1)$, $(2, 11)$, and $(4, 1)$. They can then verify the image is similar to the pre-image using transformations or by comparing the corresponding side lengths of the triangles to ensure they are proportional (G.SRT.A.2). Students who struggle with calculating this distance on the coordinate plane should be encouraged to count the horizontal and vertical distances (often referred to as slope triangles) and applying the scale factor to that distance rather than using a ruler to measure. Students should be able to reason through why this works by making connections to the Pythagorean Theorem (G.GPE.A.3). Students should also perform dilations off a grid or coordinate plane, so that their focus changes to exploring distances and relationships between lines and angles between the pre-image and dilated image using appropriate tools.

Students should use precise mathematical justifications to explain that dilations preserve angle measure, betweenness, and collinearity. Students should also be able to understand and demonstrate that dilations map a line segment (the pre-image) to another line segment (the image) whose length is the product of the scale factor and the length of the pre-image. Additionally, students should be able to understand and demonstrate that dilations map a line not passing through the center of dilation to a parallel line and leave a line passing through the center unchanged.

Students should explore dilations with a range of scale factors, including those both between 0 and 1 and greater than 1, to gain a full understanding of how dilations can shrink as well as expand figures. They should be able to reason through when a scale factor will result in a reduction or enlargement and justify their reasoning using precise mathematical language.

Students at this level should be challenged to reason through how to find a center of dilation and a scale factor given the pre-image and image of a dilated figure. They can also be given an image, a scale factor, and a center of dilation, and be challenged to find the pre-image. Students should be allowed to discover their own methods for each of these situations through productive struggle using the properties of dilations rather than being given a series of steps to follow.

Level 4:

Once students have solidified their understanding of the properties of dilations, they can begin to expand their exploration of dilations and scale factors by exploring 0, 1, or negative scale factors both on and off the coordinate plane. Students should recognize and explain why 0 cannot serve as a scale factor and that a scale factor of 1 results in a congruent figure. They can also explore what happens when a negative scale factor is applied and describe the results by making connections to other transformations, such as by recognizing that the effect is a combination of a rotation and a dilation.

Geometry

Similarity, Right Triangles, and Trigonometry (G.SRT)

Standard G.SRT.A.2 Cluster Heading: A. Understand similarity in terms of similarity transformations.

Define similarity in terms of transformations. Use transformations to determine whether two figures are similar.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Distinguish between a dilation and a translation, reflection, or rotation.</p> <p>Identify when figures are congruent, similar, or neither.</p>	<p>Tell which transformations are considered rigid motions and which are not and informally explain why.</p> <p>Describe the properties of a dilation.</p> <p>Recognize that the image of a dilation is similar to the pre-image.</p>	<p>Define similarity in terms of transformations.</p> <p>Describe the properties of similar figures.</p> <p>Use transformations to determine whether two figures are similar.</p> <p>Explain the AA triangle similarity postulate.</p>	<p>Justify the triangle similarity postulates (AA, SSS, and SAS).</p> <p>Compare and contrast the triangle similarity postulates to the triangle congruence theorems.</p> <p>Explain why all congruent figures are similar, but similar figures are only sometimes congruent using the definitions of congruence and similarity.</p>

Instructional Focus Statements

Level 3:

Students explored scale drawings and scale factors in grade 7 (7.G.A.1) and the effects of dilations on figures in grade 8 (8.G.A.1). Additionally, in grade 8, students discovered that the corresponding angles of dilated figures are congruent. In this high school standard (G.SRT.A.2), students should recognize the connection between dilations and similarity and realize that they can use dilations to determine whether any two figures are similar by verifying if one is a dilation of the other.

As students are exploring the properties of dilations in G.SRT.A.1, they should look for patterns when examining sequences of dilations (MP8). To develop a working definition of similarity, students will need ample opportunity to explore and discover the relationships between similar triangles. They may recall from grade 8 that all pairs of corresponding angles in similar figures are congruent. They should be able to verify that property through this exploration. Additionally, they should discover that all pairs of corresponding sides are proportional.

When exploring triangles, students should apply their understanding of the properties of triangles to determine the criteria necessary to say two triangles are similar. For example, students may be asked to draw or create two triangles with congruent corresponding angles to determine if they will always satisfy the properties of similar figures. They should recognize that knowing the corresponding angles are congruent is enough to justify that the triangles are similar without the need to check the side lengths. They can then reason that only knowing two congruent corresponding angles is actually enough to verify that the triangles are similar (AA), since the third corresponding angles must also be congruent to each other using the triangle sum theorem (G.CO.C.9). This is the discovery of the AA Similarity Postulate, which students should be able to explain.

Instruction should include exposure to different types of polygons for students to explore. For example, using the understanding that similar figures have congruent angles, does that mean that all rectangles are similar? Why or why not? What about all squares? What conjectures can be made about similarity of regular polygons vs. non-regular polygons? How can you use transformations to verify your conjecture?

Once similarity of polygons is solidified, students can consider similarity of figures that are not polygons. For example, they should recognize that any two circles are similar since one circle can always be dilated to result in the other.

Level 4:

Students can explore criteria other than angle-angle (AA similarity) to justify that triangles are similar. They should discover that when all three corresponding sides are proportional, the triangles are similar without the need to check the angle measures (SSS similarity). Side-angle-side similarity (SAS similarity) also works when two sides are proportional and their included angle is congruent. For each of these criteria, transformations can be used to verify similarity. Students should be able to justify these criteria using the properties of triangles as well as transformations. They can also compare and contrast these similarity theorems to the triangle congruence criteria for SSS, SAS, ASA, AAS, and HL (G.CO.B.7).

Students at this level can consider the relationship between congruent figures and similar figures. If two similar figures have a scale factor of 1, they will be congruent. Therefore, congruent figures satisfy the definition of similar figures. However, similar figures are only congruent if they have a scale factor of 1. Therefore, similar figures are only sometimes congruent. Instruction should have students explore and compare the properties of each to discover this relationship.

Geometry

Similarity, Right Triangles, and Trigonometry (G.SRT)

Standard G.SRT.B.3

Cluster Heading: B. Use similarity to solve problems and justify relationships.

Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures.

Scope and Clarifications:

“Justification” may take a variety of forms. For example, tasks could include, but are not limited to: a line parallel to one side of a triangle divides the other two proportionally, and conversely; finding the area of a kite by partitioning it into two congruent triangles and finding the area of one triangle.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Distinguish between a dilation and a rigid motion.</p> <p>Identify when figures are congruent, similar, or neither.</p> <p>Informally describe the concept of proportional relationships.</p>	<p>Define similarity and congruence in terms of transformations.</p> <p>Recognize congruence criteria for geometric figures.</p> <p>Recognize similarity criteria for geometric figures.</p> <p>Apply their understanding of proportional relationships to determine if the side lengths of two figures are proportional.</p>	<p>Use congruence and similarity criteria for triangles to solve problems.</p> <p>Use congruence and similarity criteria for triangles to justify relationships in geometric figures.</p>	<p>Apply congruence or similarity criteria to solve complex problems, and explain the geometric reasoning involved.</p> <p>Analyze and critique others' justifications by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Improve their own justifications based on what they have seen or heard from others.</p>

Instructional Focus Statements

Level 3:

In grades 7 and 8, students explored the concepts of similarity and congruence through transformations. Students explored scale drawings and scale factors in grade 7 (7.G.A.1) and the effects of transformations on figures in grade 8 (8.G.A.1). Additionally, in grade 8, students discovered that the corresponding angles of both congruent and similar figures are congruent (8.G.A.1). In this high school course, students solidify their understanding of the criteria for both congruent (G.CO.B.5) and similar (G.SRT.A.2) geometric figures. In this standard (G.SRT.B.3), they apply that understanding to solve problems and justify relationships in geometric figures.

Students will need to draw on their understanding of proportional relationships from grade 7 (7.RP.A.2) when solving problems and justifying relationships in similar geometric figures. For example, when determining if a trapezoid is similar to another trapezoid, students will need to verify if the corresponding sides are proportional in addition to establishing that the corresponding angles are congruent. To do this, they may set up a table or a set of equations and test for equivalent ratios (a constant of proportionality), which they can also relate to the scale factor for a dilation.

Students should work with a broad range of situations to make conjectures about geometric relationships using triangle similarity and congruence. As students form connections between pairs of similar and congruent triangles, they should use geometric transformations to explain how the triangles are related and establish the correspondence between them. Many real-world examples can be explored such as finding the height of a building using similar triangles. For example, a person stands in the shadow of the building so that the end of his or her shadow lines up with the end of the shadow of the building. By measuring the length of the shadow of the building and the length of the shadow of the person as well as his or her height, the height of the building can be calculated using the proportional relationship between the similar triangles formed by each object and their shadows. This example utilizes the theorem that states that a line parallel to one side of a triangle divides the other two sides proportionally since the smaller triangle is embedded inside the larger triangle such that they share an angle and have two adjacent sides.

Students should also apply the criteria for similar and congruent triangles to verify properties of other figures. They should be able to provide mathematical justifications explicitly linking similarity and congruence of triangles to other geometric figures. For example, they can use triangle congruency criteria to determine and justify properties of parallelograms and special parallelograms, which supports G.CO.C.10, and they can use triangle similarity criteria to discover that the parts of the intersecting diagonals in a trapezoid are proportional. Additional examples are included in the scope and sequence for this standard.

Level 4:

Students at this level should be given increasingly complex problems to solve using the criteria of congruent and similar triangles and be expected to provide justifications of their reasoning. To strengthen students' justifications, they need to analyze and critique others' reasoning. Students should be exposed to a combination of accurate justifications and some with mistakes or misunderstandings. Students should provide counterexamples with their explanation when disproving these arguments. These critiques will help students develop a deeper understanding of the logic of the relationships and theorems and the need for clear and precise language that they can then apply to their own reasoning.

Geometry

Similarity, Right Triangles, and Trigonometry (G.SRT)

Standard G.SRT.C.4 **Cluster Heading: C. Define trigonometric ratios and solve problems involving triangles.**

Use side ratios in right triangles to define trigonometric ratios.

G.SRT.C.4a Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G.SRT.C.4b Explain and use the relationship between the sine and cosine of complementary angles.

Scope and Clarifications:

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Identify and label the hypotenuse and legs in a right triangle.</p> <p>Identify and label the leg opposite to and the leg adjacent to a given acute angle in a right triangle.</p> <p>Find the measure of a missing angle in a triangle using the triangle sum theorem.</p> <p>Define complementary angles.</p>	<p>Understand and apply the proportional relationship for the lengths of the sides of similar triangles.</p> <p>Understand that similar triangles have congruent angles.</p> <p>Choose which trigonometric ratio is appropriate to use to find missing sides and angles of a right triangle.</p> <p>Recognize that the acute angles in a right triangle are complementary.</p>	<p>Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle.</p> <p>Understand and apply the definitions of trigonometric ratios for acute angles to find missing sides and angles of a right triangle, given other sides and angles.</p> <p>Explain and use the relationship between the sine and cosine of complementary angles in right triangles.</p>	<p>Explain the relationship that exists between the ratios of the sides of similar right triangles.</p> <p>Create right triangles that have and do not have proportional side lengths and use them to explain how trigonometric ratios are defined.</p> <p>Explain why trigonometric ratios only apply to the acute angles of right triangles.</p>

Instructional Focus Statements

Level 3:

Trigonometry is one of the most widely used mathematical concepts and serves a variety of applications in many fields, so building the foundational knowledge of the special relationships in right triangles is crucial for students to make sense of trigonometry and apply trigonometric concepts fluently. In grade 8 (8.G.B.4), students solved for a missing side of a right triangle given two side lengths using the Pythagorean Theorem. In this high school course, they will explore how to find missing side lengths of a right triangle without knowing two side lengths and find an acute angle measure in a right triangle, using trigonometric ratios and their inverses. While the solving is a focus of G.SRT.C.5, the focus of this standard (G.SRT.C.4) is to build a conceptual understanding of the foundation of trigonometric ratios in similarity. It is important to note that for this course, students are only expected to work with sine, cosine, and tangent ratios and their inverses. They are not expected to know or use the reciprocal trig ratios of secant, cosecant, and cotangent. These will be introduced in pre-calculus.

Students should have opportunities to explore different right triangles that have and do not have proportional side lengths (similar and non-similar) to understand that corresponding sides of right triangles with the same acute angle measure will be proportional in length. Students should use this understanding to explore and understand the definitions of trigonometric ratios for acute angles.

Students can apply their understanding of the criteria of similar triangles (G.SRT.A.2 and G.SRT.A.3) to recognize that all right triangles with one congruent acute angle will be similar by the AA similarity postulate, since by definition right angles will always be congruent. Because they are similar, lengths of the corresponding sides will be proportional. Therefore, any right triangle with a given acute angle measure will have a specific ratio between each pair of sides. For example, all right triangles with a 32-degree angle will have a common ratio between the opposite side and the hypotenuse of 0.5299. These specific ratios are defined as trigonometric ratios. To develop a deep conceptual understanding of trigonometric ratios, students should be allowed to discover these properties through a scaffolded task with discourse focused on patterns that they find.

Once students have defined the trigonometric ratios in right triangles, they should pay attention to patterns that appear within the ratios. In particular, they should discover a unique relationship between the sine and cosine of the complementary angles, specifically, that the sine of one acute angle is equivalent to the cosine of the other acute angle in any right triangle. This discovery can be extended to recognizing that this relationship exists both within a single right triangle, as well as when complementary angles exist between two different right triangles. Students should be able to explain why this relationship exists and use this concept to solve problems. It is important to make the connection that cosine literally means "Sine of the complement" and the "co" means complement. This will help students remember that $\cos(x) = \sin(90-x)$.

A common misunderstanding is that trigonometric ratios apply to any angle in any triangle. Through exploration, students should discover that they only apply to the acute angles in a right triangle. Allowing students to explore what happens when they try to apply trig ratios to the right angle will help them understand why it is not useful. Additionally, further exploration should reveal that non-right similar triangles will have different ratios of the lengths of the sides that are unique to each set of similar triangles. For example, a triangle with angle measures of 32° , 65° , and 83° will have different side length ratios than a triangle with angle measures of 51° , 62° , and 67° . This discovery connects to G.SRT.C.5c and the use of the Law of Sines and Cosines to solve non-right triangles.

Level 4:

Students at this level should be able to explain how trigonometric ratios are defined. As part of their explanation, they may be asked to create examples of right triangles that do and do not have proportional side lengths. They should also be challenged to explain why trigonometric ratios only apply to the acute angles in a right triangle by exploring the properties of a right triangle and the ratios of the sides in non-right similar triangles. They can use these explanations to justify the application of trigonometric ratios in appropriate mathematical or real-world problems. All explanations and justifications of their reasoning should use precise mathematical language.

Geometry

Similarity, Right Triangles, and Trigonometry (G.SRT)

Standard G.SRT.C.5 Cluster Heading: **C. Define trigonometric ratios and solve problems involving triangles.**

Solve triangles.*

G.SRT.C.5a Know and use the Pythagorean Theorem and trigonometric ratios (sine, cosine, tangent, and their inverses) to solve right triangles in a real-world context.*

G.SRT.C.5b Know and use relationships within special right triangles to solve problems in a real-world context.*

G.SRT.C.5c Use the Law of Sines and Law of Cosines to solve non-right triangles in a real-world context.*

Scope and Clarifications: (Modeling Standard)

For part C: exclude ambiguous cases.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Use the triangle sum theorem to find a missing angle measure in a triangle.</p> <p>Recognize that the Pythagorean Theorem can be used to find missing side lengths in a right triangle.</p> <p>Solve simple quadratic equations.</p>	<p>Use the Pythagorean Theorem to find a missing side length of a right triangle.</p> <p>Choose which trigonometric ratio is appropriate to use in solving a right triangle.</p> <p>Reason through solving equations, including quadratics and simple equations with radical coefficients.</p>	<p>Use the Pythagorean Theorem and trigonometric ratios to solve mathematical and real-world problems.</p> <p>Use relationships within special right triangles to solve mathematical and real-world problems.</p> <p>Use the Law of Sines and Law of Cosines to solve mathematical and real-world problems.</p> <p>Recognize when it is appropriate to use each method to solve</p>	<p>Justify the method they chose for solving a triangle using precise mathematical language.</p> <p>Critique the reasoning of others and use others' justifications to improve their own.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		mathematical and real-world problems and explain their reasoning.	

Instructional Focus Statements

Level 3:

When asked to solve triangles, students are expected to find all side lengths and angle measures that are not given. This is a modeling standard; therefore, it is essential that students make direct connections between their solution methods and the triangles for which they are solving.

Students began applying the Pythagorean Theorem to right triangles in grade 8 (8.G.B cluster). In this high school course, students will encounter right triangles to solve that don't provide two side lengths, and thus cannot be solved using the Pythagorean Theorem. They should recognize that in these cases, as well as when they are working with non-right triangles, there are other methods available to use to find missing measurements including trigonometric ratios, special right triangles, and the Law of Sines and Law of Cosines. They should also develop an understanding of when it is appropriate to use each method.

If working with a 45-45-90 triangle or a 30-60-90 triangle, students only need to know one side length to find the other two using the unique properties of these special right triangles. Instruction should allow students to discover the relationships between the side lengths within each special right triangle. To discover the relationships between the side lengths of a 45-45-90, they can start with a square and draw in one diagonal to create two 45-45-90 triangles. Recognizing that the sides of the square are congruent, they can apply the Pythagorean Theorem to find the length of the diagonal. Using the equation $x^2 + x^2 = c^2$, they can solve to find that $c = x\sqrt{2}$. To discover the relationships between the side lengths of a 30-60-90 triangle, they can begin with an equilateral triangle with sides that are 2 units in length and draw in the altitude. Recognizing that the altitude by definition is perpendicular to the base of the triangle, and since it is an equilateral triangle, the perpendicular bisector of the base, then they can see that the side of the 30-60-90 triangle that is opposite the 30-degree angle (the short leg) is 1 unit, while the hypotenuse is 2 units. Therefore, they can apply the Pythagorean Theorem to find the length of the altitude (long leg). Using the equation $x^2 + b^2 = (2x)^2$, they can solve to find that $b = x\sqrt{3}$.

Once students have discovered those properties, they need only consider a square or equilateral triangle to remember the relationships if they forget them. Students can then apply those relationships to find any two side lengths in a 45-45-90 or a 30-60-90 right triangle when given only one side length. To develop procedural fluency, students should be exposed to problems in which each of the sides of both special right triangles are given as different numerical types including integers, fractions, and radicals. This forms a strong connection between geometry and algebra as they work to solve for the

missing measurements. Occasionally, they may need to solve a simple radical equation, such as $x\sqrt{2} = 5$. Solving radical equations will be new for students in this course, but they can apply their reasoning developed in Algebra 1 (A1.A.REI.A.1) to solve this equation by using the inverse operation of multiplication and applying the division property of equality by dividing both sides by $\sqrt{2}$. For example, they should be challenged to solve a 45-45-90 triangle when given the length of the hypotenuse as an integer or a leg length as a radical. This will result in the missing side lengths being a fraction with a radical in the denominator (i.e., $\frac{5}{\sqrt{2}}$). While it is not an expectation for this course to rationalize denominators, this is an opportunity to demonstrate equivalent fractions with rationalized denominators as they occur (i.e., $\frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$). Simplifying rational expressions will be addressed in Pre-Calculus (P.N.NE.A.4). Special right triangles have multiple applications and are foundational for extended work with trigonometric ratios including the unit circle in Pre-Calculus (P.F.TF.A.2).

To address the common misconception that the Pythagorean Theorem can be applied to any triangle to find missing side lengths, students can explore various types of triangles that are easily measured to test which type triangle it works for. This exploration can also reveal another practical use for the Pythagorean Theorem as a way to classify triangles. For example, given three measurements and substituting the longest length for c , if $c^2 = a^2 + b^2$, then not only is it a right triangle, but they can also tell that the right angle is between sides a and b .

Students work with three trigonometric ratios in this course: sine, cosine, and tangent. The reciprocals of these trig ratios are addressed in Pre-Calculus and are not an expectation of this course. They should develop a conceptual understanding of these three trig ratios as they work to define them in relation to right triangles in G.SRT.C.4. They should solidify the procedural fluency of identifying the hypotenuse and appropriate opposite and adjacent legs of triangles by using the trigonometric ratios as they work to solve mathematical and real-world problems. Instruction should expose students to a variety of problems that require ample application of each trigonometric ratio to solve for side lengths as well as their inverses to solve for angle measures. Students often struggle with setting up the equations to solve for the missing measurements. Applying their conceptual understanding of the definition of trig ratios developed in G.SRT.C.4 should help students reason through the set up. For example, recognizing that the trig ratio is the ratio of the sides in relationship to the referenced angle will help students understand that the sine of the angle is found by the ratio of the opposite leg to the hypotenuse, which leads to setting up the equation as $\sin(\text{angle}) = \text{opposite leg}/\text{hypotenuse}$.

Students should progress from solving right triangles, to solving non-right triangles using the Law of Sines and Law of Cosines. Not only will students need to recognize that these methods apply to non-right triangles, but they will also need to develop a deep understanding of when to use which law when solving a non-right triangle. Instruction should allow students to grapple with the structure of the problems that are given to determine which to use rather than following a given rule. Instruction should exclude ambiguous cases as this is not an expectation in this course. Ambiguous cases will be addressed in Pre-Calculus.

As students solidify their understanding of each of these methods to solve triangles, instruction should focus on identifying which method is appropriate based on the problem and what measurements are given. They should also be able to explain their reasoning for choosing the method they chose.

Level 4:

Students at this level should be expected to justify the method that they chose for solving triangles using precise mathematical language. They should also be able to critique the reasoning of others and use other's justifications to improve their own.

Students may also be given more complex mathematical and real-world problems that require solving triangles including those that occur within three-dimensional solids. Additionally, the exploration mentioned above regarding using the converse of the Pythagorean Theorem to reveal that a triangle is a right triangle, can also reveal that it can be used to classify a triangle as acute, if $c^2 < a^2 + b^2$, or obtuse, if $c^2 > a^2 + b^2$. Students at this level should be able to informally explain why this happens.

Geometry Circles* (G.C)

Standard G.C.A.1 Cluster Heading: A. Find areas of sectors of circles.

Use proportional relationships between the area of a circle and the area of a sector within the circle to solve problems in a real-world context.*

Scope and Clarifications: (Modeling Standard)

Angles are measured in degrees.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Recognize a sector within a circle is formed by the two rays of a central angle and the subtended arc.</p> <p>Define and recognize a central angle on a circle.</p>	<p>Calculate the area of a circle.</p> <p>In a sector within a circle, identify the degree measure of the central angle as a fractional part of the whole 360 degrees in the circle.</p> <p>Recognize the area of a sector within a circle as a fractional part of the whole area of the circle.</p>	<p>Explain how to find the area of a sector within a circle by defining the proportional relationship between the area of the circle and the sector within it.</p> <p>Use proportional relationships between the area of a circle and the area of a sector within the circle to solve problems in a real-world context.</p>	<p>Create a real-world context that requires finding the area of a sector within a circle.</p> <p>Critique others' solutions and identify any errors in reasoning, misconceptions, misrepresentation of the fractional form, or errors in application.</p> <p>Explore how to find the area of a segment of a circle and consider real-world applications.</p> <p>Explore how to find arc length and consider real-world applications.</p>

Instructional Focus Statements

Level 3:

In grade 7, students found the area and circumference of a circle (7.G.B.3) and explored proportional relationships (7.RP.A cluster). In Geometry, students combine these concepts to recognize the proportional relationship between the area of a sector within a circle and the area of the circle. Instruction should guide students to recognize that a sector is simply a fractional piece of the whole circle. Students should reason that since a sector is a fraction of the circle, then the area of a sector is a fraction of the circle's area. The expectation for this standard is for students to use the proportional relationship between a circle and a sector within the circle to find the area of the sector, rather than memorize a separate formula. For example, if given a sector with a 50 degree central angle, students should recognize that the central angle is $\frac{50}{360}$ of the whole circle, and thus, the area of the sector is $\frac{50}{360}$ of the area of the whole circle. Since the area of a circle can be found using πr^2 , then the area of the sector can be found using $\frac{50}{360} \pi r^2$.

Students should see the proportional relationship between the fractional piece of the circle and the circle's whole area and be able to explain it. Students can then use this conceptual understanding to calculate the area of a sector. Students should grapple with problems with a variety of given information, including a drawing, graph, or measurements such as a radius length and central angle measure in degrees. They can also be challenged to find the area of a circle with a sector removed. Students should be provided real-world examples in which they must determine that the area of a sector is needed.

Level 4:

Students at this level of understanding should be given the opportunity to see and hear others' solution paths to finding the area of a sector and critique their reasoning. Instruction should provide students with incorrect examples so students can identify errors in reasoning, misconceptions, misrepresentation of the fractional form, or errors in application. Students should be challenged to find their own real-world example that requires finding the area of a sector.

An extension to the concept of this standard is the exploration of finding the area of a segment of a circle. Students should discover that they need to compute the area of the sector and then subtract the area of the included triangle. The proportional relationship also connects to arc length in a circle. Students can reason through finding arc length utilizing their understanding of finding the area of a sector, by recognizing the arc length as a fraction of the circumference of the circle.

Geometry

Expressing Geometric Properties with Equations (G.GPE)

Standard G.GPE.A.1 **Cluster Heading: A. Use coordinates to solve problems and justify simple geometric theorems algebraically.**

Use coordinates to justify geometric relationships algebraically and to solve problems.

Scope and Clarifications:

Examples include but are not limited to:

determine whether four points in the coordinate plane are the vertices of a rectangle, trapezoid, rhombus, square, or parallelogram;

determine whether three points in the coordinate plane are the vertices of a scalene, isosceles, equilateral, or right triangle;

determine the median of a triangle using the midpoint formula;

given the coordinates of the center of a circle and a point on the circle, find the area and/or circumference of the circle.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Know the properties of a given geometric figure.</p> <p>Identify a geometric figure based on its properties.</p>	<p>Find measures, lengths, or distances within a given figure on a coordinate plane.</p> <p>Know the conditions necessary for applying a geometric theorem to a figure.</p>	<p>Identify how properties of geometric figures can be expressed using coordinates and justify them algebraically.</p> <p>Justify geometric relationships between figures or within a single figure on a coordinate plane algebraically.</p> <p>Recognize when a geometric theorem is applicable to a given figure and use it appropriately to justify relationships or solve a problem.</p>	<p>Analyze and critique others' justifications by either verifying the logic and language for accuracy and precision or providing counterexamples that disprove the argument.</p> <p>Improve their own justifications based on what they have seen or heard from others.</p> <p>Use coordinates to justify complex geometric theorems algebraically.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		Solve problems involving geometric relationships on a coordinate plane.	

Instructional Focus Statements

Level 3:

Instruction should provide students with the opportunity to explore geometric theorems on a coordinate plane. Students may struggle with representing important lengths, measures, or coordinates in general terms, so teachers should allow students the opportunity to study the process using numerical values and then grapple with how to represent geometric measures algebraically using general coordinates. Instruction should provide students with a variety of contexts that utilize different properties of geometric figures.

This standard provides an excellent opportunity to help students see the vital connection between algebra and geometry. As gaps in algebraic skills are identified, utilize just-in-time instruction to review as needed rather than reviewing skills in isolation. Application of this standard can be spread throughout the course as appropriate geometric theorems are introduced within the curriculum. For example, it provides an extension to G.CO.C.10 by placing the parallelogram on a coordinate plane and justifying the properties algebraically using coordinates. Using coordinates to represent various contexts may help students develop their conceptual understanding and communicate their reasoning by providing a basis for the relationships or properties on which they are focusing.

Instruction should give students ample opportunity to justify and demonstrate algebraically why a particular geometric theorem holds true on a coordinate plane. During instruction, students should reason quantitatively and make generalizations about coordinates, the algebraic representation of measures of a figure, and geometric theorems. This will help students see that providing a justification has many avenues. It is imperative students be required to use precise mathematical language and make use of the structure on the coordinate plane to aid them in developing their reasoning. This standard requires the application of multiple mathematical practice standards including, but not limited to, making sense of problems and persevering in solving them, using quantitative reasoning, constructing viable arguments, and attending to precision.

Level 4:

Instruction at this level should provide the opportunity for students to fine tune their reasoning skills by analyzing other justifications and critique them to find best practices. Students should then apply what they learned through the analysis to improve their own reasoning and justifications. Furthermore, students should be challenged to justify more complex geometric theorems.

Geometry

Expressing Geometric Properties with Equations (G.GPE)

Standard G.GPE.A.2 **Cluster Heading: A. Use coordinates to solve problems and justify simple geometric theorems algebraically.**

Use the slope criteria for parallel and perpendicular lines to solve problems and to justify relationships in geometric figures.

Scope and Clarifications:

For example, justify why two lines are parallel, perpendicular, or neither.

Create parallel and perpendicular lines to solve problems in context (e.g., given three noncollinear points in the plane, find the coordinates of a fourth point so that the four points are the vertices of a rectangle).

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Find the slope of a line given a graph.</p> <p>Understand the meaning of the mathematical terms opposite and reciprocal and provide examples of each.</p> <p>Identify parallel and perpendicular lines in a picture indicated by symbols or given notation.</p>	<p>Given the slope of a line, provide the slope of line parallel and the slope of a line perpendicular.</p> <p>Sketch a parallel line or a perpendicular line in a drawing given a point and a line.</p>	<p>Determine if lines are parallel, perpendicular, or neither and justify using slope criteria.</p> <p>Apply the properties of parallel lines and perpendicular lines to solve geometric problems.</p> <p>Use slope criteria for parallel and perpendicular lines to justify relationships in geometric figures.</p> <p>Write equations of parallel lines and perpendicular lines given a point and a slope or a graph.</p>	<p>Create and solve a real-world situation that involves parallel and perpendicular lines.</p> <p>Critique the work of others in using the slope criteria to justify relationships in geometric figures and apply their learning to improve their own reasoning.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
		<p>Write equations of parallel lines and perpendicular lines by identifying a point and a slope from a drawing.</p> <p>Use precise mathematical language and symbolic notation to describe parallel and perpendicular lines.</p>	

Instructional Focus Statements

Level 3:

Students began recognizing parallel and perpendicular lines in geometric figures in grade 4 (4.G.A.2) and informally verified parallel lines in grade 8 (8.G.A.1). In this course, students explore parallel and perpendicular lines on the coordinate plane to discover the relationships of their slopes. Then they use the slope criteria to precisely verify the relationship between the lines and solve problems.

Understanding parallelism and perpendicularity is a concept that applies to many mathematical concepts including properties of geometric figures, graphs, and extends into concepts in Calculus where students will be asked to write the equation of a tangent line. Because conceptual understanding of this standard is so important, instruction should focus on allowing the students to explore and discover the relationship between the slopes of parallel lines and perpendicular lines.

One strategy is to build on students' understanding of transformations from 8.G.A.1 when exploring parallel lines. Students informally verified that when translation is applied to a figure, corresponding lines will be parallel. This concept is also explored in G.CO.A.3 as students develop a formal definition of transformations. Therefore, students can translate a line drawn on a coordinate plane and compare the slopes of the image and the pre-image to see that they will always be the same. Likewise, students can rotate a line drawn on a coordinate plane 90 degrees and compare the slopes of the image and the pre-image to discover that they will always be opposite reciprocals of one another.

It is important that students understand the reasoning behind the properties of the slopes of parallel and perpendicular lines and not just memorize these properties. These explorations will provide students the means to justify when lines are parallel, perpendicular, or neither using precise mathematical language and appropriate symbolic representation.

Once students have solidified the properties of the slopes of parallel and perpendicular lines, they should extend this understanding to linear equations. This standard provides an excellent connection between Geometry and Algebra. In Algebra 1, students both created (A1.F.BF.A.1) and graphed (A1.F.IF.C.7) linear equations. Applying the slope criteria for parallel and perpendicular lines, students should write the equation of a line parallel or perpendicular to a given line that passes through a given point. Instruction should provide students with problems where the student can ascertain the slope from an equation, a graph, a table, or from words. The exploration that took place to explore the criteria of the slopes should help students understand that the equation they are writing is for a second line and avoid the common misconception that they just substitute the slope without changing the equation. Graphing both equations can also help students recognize they are writing equations for two different lines with different y-intercepts.

Furthermore, students should experience a variety of problems where they must apply the criteria for slope in order to solve a problem. For example, classify a quadrilateral given the coordinates of its vertices and justify the answer.

Level 4:

Instruction should provide students ample opportunity to explore parallel and perpendicular lines in real-world contexts. Students should be expected to recognize the need for justifying lines as parallel or perpendicular and understand that slope provides the necessary precision in a real-world context. Giving students the opportunity to hear the reasoning of how others' applied slope criteria to solve problems can increase their level of understanding and help them to improve their own reasoning.

Students can be challenged to describe a geometric figure(s) (with increasing complexity over time) using slope criteria and other properties as needed such that another student can draw and/or name the figure(s). This activity will help students develop more precise mathematical vocabulary and deepen their understanding of the properties of geometric figures.

Geometry

Expressing Geometric Properties with Equations (G.GPE)

Standard G.GPE.A.3

Cluster Heading: A. Use coordinates to solve problems and justify simple geometric theorems algebraically.

Understand the relationship between the Pythagorean Theorem and the distance formula and use an efficient method to solve problems on the coordinate plane.

Scope and Clarifications:

For example, compute the radius of a circle given a center and a point on the circle, perimeters of polygons, and areas of triangles and quadrilaterals. Finding the area of a triangle is limited to cases when the triangle is either right or contains a side that is horizontal or vertical.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Find the length of a horizontal or vertical line segment on a coordinate plane. Draw a right triangle using a given line segment on a coordinate plane.	Know and apply the Pythagorean Theorem to find unknown side lengths in right triangles. Find the distance between two points given as coordinates using the Pythagorean Theorem. Find the distance between two points given as coordinates using the distance formula.	Explain the relationship between the Pythagorean Theorem and the distance formula. Choose the most efficient method to find the distance between two points in a coordinate system and use it to solve problems.	Derive the distance formula from the Pythagorean Theorem. Justify a method as the most efficient method to find the length of a line segment on the coordinate plane for a given situation.

Instructional Focus Statements

Level 3:

In grade 8, students were introduced to the Pythagorean Theorem as a method to find a missing side length of a right triangle (8.G.B.4). They also used it to find the distance between two points in a coordinate system (8.G.B.5). In this high school course, students will extend their understanding of the application of Pythagorean Theorem to find a distance and generalize it to find the distance between any two points in a coordinate system and thus

discover the distance formula. Students often mistakenly assume that they can count a diagonal distance on the coordinate plane like they do with horizontal or vertical distances. To avoid this common misconception, have students measure and compare a side length and diagonal of a square and connect this comparison to the square units on a coordinate plane. They can then calculate the length of the diagonal of one square unit using the Pythagorean Theorem ($1^2 + 1^2 = c^2$) to see that the length of the diagonal is actually $\sqrt{2}$ which is longer than 1 unit or approximately 1.41 units.

Instruction should allow students to explore using the Pythagorean Theorem to find the distance between two points graphed on the coordinate plane. It may be easier for students to use numerical coordinates at first. However, to help students generalize the process, the given points can be (x_1, y_1) and (x_2, y_2) . For example, students should recall from grade 8 that they can draw in a right triangle with the segment drawn between the two given points as the hypotenuse of the right triangle. When applying the Pythagorean Theorem, they should discover that to find a, the length of the horizontal leg, they can subtract the x coordinates of the endpoints $(x_2 - x_1)$ which are also the x coordinates of each original point. They should also discover that to find b, the length of the vertical leg, they can subtract the y-coordinates of the endpoints $(y_2 - y_1)$ which are also the y coordinates of the original points. When substituting each expression into the Pythagorean Theorem, $a^2 + b^2 = c^2$ becomes $(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$, with c representing the hypotenuse which is the distance between the original two points. When isolating c, students will see the distance formula: $c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, while in the distance formula, c is usually replaced by d to represent distance.

By allowing students to discover this connection between the two formulas, students should be able to flexibly move between these two methods. They can then explore applications for each method such as finding the area of circles or polygons graphed on the coordinate plane. They can also be challenged to solve problems given coordinates that are not graphed. Students should be given the opportunity to choose which method is the most efficient and explain why.

While exploring applications, students should also be given problems with coordinates that have either the same x- or y-coordinates to help them recognize that the formulas are not always the most efficient method.

Level 4:

Students at this level should be able to show and explain the derivation of the distance formula. This explanation can include words, symbols, and/or drawings to represent the mathematics involved. They should also be able to justify the efficiency of the method they chose to find the distance between 2 points given as coordinates using precise mathematical language. Students can be challenged to provide examples for which each method would be most efficient.

Students at this level can also explore applying these methods to find the length of the radius in a circle, which leads to the standard equation for any circle. Prior to taking the square root of both sides, you get $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$. Replacing d with r to represent the radius, and replacing x_2 and y_2 with h and k, respectively, to represent the coordinates for the center of the circle (h, k), you get $r^2 = (x - h)^2 + (y - k)^2$ which is the standard form of the equation of a circle.

Geometry

Geometric Measurement and Dimension (G.GMD)

Standard G.GMD.A.1 **Cluster Heading: A. Explain volume and surface area formulas and use them to solve problems.**

Understand and explain the formulas for the volume and surface area of a cylinder, cone, prism, and pyramid.

Scope and Clarifications:

Informal arguments are limited to dissection.

Memorizing formulas is not required.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Informally define volume and surface area of a solid figure.</p> <p>Identify the parts of a solid including: edges, bases, and lateral faces.</p> <p>Find area and perimeter of two-dimensional figures including rectangles and triangles.</p> <p>Find area and circumference of circles.</p> <p>Identify a cylinder, cone, prism, and pyramid.</p>	<p>Accurately identify the shape of the base of a prism and a pyramid.</p> <p>Find the volume and surface area of a cylinder, cone, prism, and pyramid given the formula.</p>	<p>Explain how each variable and term in the formula for volume or surface area relates to the solid.</p> <p>Provide an informal argument for the formulas for the volume and surface area of a cylinder, cone, prism, and pyramid.</p>	<p>Compare and explain the relationship between the formulas for surface and volume of a prism with a cylinder and a pyramid with a cone.</p> <p>Analyze and critique others' arguments for the formulas for volume and surface area of a cylinder, cone, prism, and pyramid.</p> <p>Improve their own argument based on what they have seen or heard from others.</p>

Instructional Focus Statements

Level 3:

In grade 6, students explore and apply the formulas $V=Bh$ and $V=lwh$ for the volume of a rectangular right prism (6.G.A.2), and use nets made up of rectangles and triangles to find the surface area of three-dimensional figures (6.G.A.4). In grade 7, students solve problems involving surface area and volume of three-dimensional objects composed of cubes and right prisms (7.G.B.5). In grade 8, students apply the formulas for the volume of cones, cylinders, and spheres to solve problems (8.G.C.6). In this high school course, students use these experiences from the middle grades to develop informal arguments about why the volume and surface area formulas of a cylinder, cone, prism, and pyramid work. This standard may be taught concurrently with G.GMD.A.2 in which students use these formulas to solve problems.

Dissecting the formulas to consider how each variable and each term connects to each part of the solid will help students conceptualize the structure of the formula. Instruction should provide students with models of each solid to help them make these connections. For example, to make connections between the volume formula and a cylinder, students can consider stacking multiple bases on top of each other to represent the height of the solid.

Allow students to recognize that to find the surface area of a prism, pyramid, cylinder, or cone, the formula calls for adding the area of the base(s) to the lateral surface area. Recognizing that the lateral faces of a prism are always rectangular and can be represented in a net as one large rectangle folded around the solid will help students see why the formula includes multiplying the perimeter of the base of the solid (or the base edges of each lateral face added together) by the height of the prism. Recognizing that the lateral faces of a pyramid are always triangular will help students understand why the surface area formula of a pyramid includes $\frac{1}{2}Pl$ by connecting it to the formula for the area of a triangle $\frac{1}{2}bh$. Additionally, looking at models will help students better understand why the actual height of the pyramid is used for volume (think stacking), while the slant height is used for surface area (think calculating the area of the surface).

It is important for students to understand that an uppercase B represents something different than a lowercase b in geometry formulas. Specifically, B represents the area of the base of the solid, while b represents an edge length. This can be particularly confusing when applying the surface area or volume of a triangular prism, since students will have to use $\frac{1}{2}bh$, half the base of the triangle multiplied by the height of the triangle, to find B , the area of the base of the prism. For students who struggle with this distinction, instruction should emphasize the connection between the two rectangular prism volume formulas to help students see that since the shape of the base of a rectangular prism is a rectangle and one way to find the area of that base is length x width ($l \times w$), then $V = lwh$ can be rewritten as $V = Bh$, because lw is equivalent to B for this solid.

Notice that in both examples above, each of these formulas also include h , while one h represents the height of the rectangle or triangle, and the other represents the height of the prism. Instruction should have students focus first on the components within each formula, such as area of the base or perimeter of the base, before applying the formula. For example, when finding the surface area of a triangular pyramid, $S=B+\frac{1}{2}Pl$, students should recognize that they must first find the area of the base, which is a triangle, using $A=\frac{1}{2}bh$, where b is the length of the base of the triangle, and h is the height of the triangle. Then find P , the perimeter of the base, which again is the same triangle, by adding the three edge lengths of that triangle only. Once they

have those components, then they can apply the formula. There is only one base, so take the area of the base and add the product of half the perimeter they found multiplied by the slant height.

Students should also make the connection that the volume of a pyramid is one-third the volume of a rectangular prism with the same base and height resulting in $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the prism. The same argument can be applied to a cone such that the volume of a cone is one-third the volume of a cylinder with the same base and height. This can be easily verified when provided a prism and a pyramid with congruent bases and heights that are fillable with water or sand. Students can then pour 3 full pyramids into the prism, to see the relationship.

As students make these connections, they should be able to explain why the formulas work both in writing and orally using drawings and/or models as a reference. The focus of this standard is on students developing an understanding of how these formulas were created and why they are structured the way they are.

Level 4:

Instruction should allow students to compare the formulas for prisms and cylinders and for pyramids and cones. For example, the volume formula $V=Bh$ can be applied to both a rectangular prism and a triangular prism. The difference is in calculating B , the area of the base, since the shape of the base changes. The formula for the volume of a cylinder, $V=\pi r^2h$, can also be considered $V=Bh$, since πr^2 is the area of the base of a cylinder. This holds true for the surface area formulas. The surface area of prism can be found by $S=2B+Ph$, where B is the area of the base, P is the perimeter of the base, and h is the height of the prism. This connects to the traditional formula for the surface area of a cylinder, $S=2\pi r^2+ 2\pi rh$, since πr^2 is the area of the base and $2\pi r$ is the circumference ("perimeter" of a circle) of the base.

To further deepen a student's understanding of the formulas, students at this level can consider Cavalieri's principal as it applies to both area of two-dimensional figures and volume of three-dimensional figures. It connects to the reason why the area of a rectangle and a parallelogram are the same when they have congruent bases and heights and thus why the formula $A=bh$ works for both a rectangle and a parallelogram. A common example for demonstrating Cavalieri's principal as it relates to volume is by stacking coins in a cylindrical shape and off-setting or shifting the coins such that the height is maintained but the sides are no longer vertical. Since no coins were added, taken away, or distorted, the volume of the stack does not change. Recognize that this example is not exact since, if a cylinder is built around the stack, a little bit of volume is inadvertently added to the "shifted" cylinder along the edges of the coins where they don't quite meet the ones above and below.

To strengthen students' arguments, they need to analyze and critique others' reasoning. Students should be exposed to a combination of accurate arguments and some with mistakes or misunderstandings. Students should attend to precision in their explanation when disproving these arguments. These critiques will help students develop a deeper understanding of formulas and the need for clear and precise language that they can then apply to their own reasoning.

Geometry

Geometric Measurement and Dimension (G.GMD)

Standard G.GMD.A.2 **Cluster Heading: A. Explain volume and surface area formulas and use them to solve problems.**

Use volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems in a real-world context.*

Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
Informally define volume and surface area of a solid figure. Find area and perimeter of two-dimensional figures including rectangles and triangles, Find area and circumference of circles. Identify a cylinder, cone, prism, pyramid, and sphere.	Accurately identify the shape of the base of a prism and a pyramid. Find the volume and surface area of a cylinder, cone, prism, pyramid and sphere given the formula.	Apply volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve mathematical and real-world problems. Modify the formula as needed to solve mathematical and real-world problems involving finding the lateral area of solids.	Create real-world and mathematical problems involving volume and surface area of cylinders, cones, prisms, pyramids, and spheres and apply formulas to solve them.

Instructional Focus Statements

Level 3:

In grade 6, students explore and apply the formulas $V=Bh$ and $V=lwh$ for the volume of a rectangular right prism (6.G.A.2), and use nets made up of rectangles and triangles to find the surface area of three-dimensional figures (6.G.A.4). In grade 7, students solve problems involving surface area and volume of three-dimensional objects composed of cubes and right prisms (7.G.B.5). In grade 8, students apply the formulas for the volume of cones,

cylinders, and spheres to solve problems (8.G.C.6). In standard G.GMD.A.1, students give informal arguments of volume and surface area formulas and their conceptual understandings. In this standard (G.GMD.A.2), students use that developed conceptual understanding as a foundation to solve mathematical and real-world problems.

Instruction should include situations that require students to modify the formula to solve. Students can connect to their learning from G.GMD.A.1 to apply only the part of the formula needed to find the solution. For example, a farmer needs to repaint his cylindrical silo. The silo has a base with a diameter of 10 feet and a height of 25 feet. He only wants to paint the outside wall. How many square feet will he need to paint? Notice in this example that students only need to calculate the lateral area since the bases of the cylinder do not need to be painted. Therefore, students should recognize that they only need to use the portion of the surface area formula that represents the lateral area, Ch .

It is not the intent of this standard for students to memorize the formulas. Instead, the focus is on students applying their understanding of the structure of each formula in relation to the solids so they can use them to solve problems in a real-world context.

Level 4:

As students solidify their understanding of surface area and volume formulas and their applications, they should begin to show evidence of procedural fluency in their efficiency and application of the formulas in real-world contexts.

This standard directly addresses mathematical modeling. Given that geometric solids can be used to approximate many real-life objects, the volume and surface area formulas can be used to address a broad range of contexts. Students should recognize that not all real-world scenarios can be perfectly modeled by geometric solids, but these models can provide approximation that yields useful information about the situation. Students should be able to explain this connection using precise mathematical vocabulary.

Geometry

Modeling with Geometry* (G.MG)

Standard G.MG.A.1 Cluster Heading: A. Apply geometric concepts in modeling situations.

Use geometric shapes, their measures, and their properties to model objects found in a real-world context for the purpose of approximating solutions to problems.*

Scope and Clarifications: (Modeling Standard)

For example, determine which geometric shape best approximates a real-world object.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Given a geometric shape, list real-world objects that can also be described by the shape.</p> <p>Choose which geometric attribute(s) need(s) to be calculated to solve a real-world geometric problem.</p>	<p>Choose a geometric shape that can be used to describe a real-world object.</p> <p>Determine which geometric attribute(s) need(s) to be calculated to solve a real-world geometric problem.</p> <p>Solve mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, right prisms, pyramids, cones, cylinders, and spheres when a visual representation is provided.</p>	<p>Use geometric shapes, their measures, and their properties to describe and approximately model objects in a real-world context.</p> <p>Apply geometric methods to solve real-world problems.</p>	<p>Create a variety of real-world problems that require the application of geometric methods to approximate solutions.</p> <p>Explain how and why variability that can occur in a real-world problem that involves making calculations based on an approximate model should be considered in a real-world situation.</p>

Instructional Focus Statements

Level 3:

This standard along with G.CO.D.12 addresses the concept of using geometry to visualize a situation for the purpose of solving a problem. As students solve real-world problems that involve two- and three-dimensional objects throughout this course, they should recognize that geometric shapes can be used to model real-world objects. This is imperative for students to understand and apply as they solve real-world problems in future course work. As students make this connection, they should be able to assign geometric shapes to describe objects, such as understanding that a cylinder's measure and properties can be used to model a tree trunk or a rocket ship.

Students apply geometric concepts learned in this and previous grades to solve real-world geometric application problems. Throughout the course, students should be exposed to a variety of real-world situations that require the application of geometric concepts to solve. Often, the challenge for students is to identify which concept is needed to address the problem. Therefore, instruction should intentionally provide problems that require students to analyze the context to decide what is needed to solve. Examples may include the need to calculate area, volume, surface area, or verifying parallel lines or angle measures. By modeling the situation with geometric figures, students can more easily recognize an appropriate solution method.

Students should formulate a strategy to solve the problem based on a mathematical understanding of the situation, compute solutions, interpret findings, and validate their thinking and the reasonableness of attained solutions in order to justify solutions to real-world problems.

Level 4:

As students solidify their understanding of using geometric shapes to describe real-world objects, they should also be able to translate this understanding to using a combination of shapes to model real-world situations. Students should continue to work with and justify solutions to real-world geometric problems with increased rigor over the entire course. Students should be challenged to create their own examples that require the application of geometric modeling as a method for solving. They should recognize that these solutions will often be approximate. For example, modeling a tree trunk as a cylinder would provide an approximate measurement since a natural tree trunk is rarely a perfect cylinder. It is still an appropriate method, but there will be situations that will require variations to be considered. Students at this level should reflect on variability and be able to explain how and why variations should be considered.

Statistics and Probability

Conditional Probability and the Rules of Probability* (S.CP)

Standard G.S.CP.A.1 Cluster Heading: A. Understand independence and conditional probability and use them to create visual representations of data.

Use set notation to represent contextual situations.*

G.S.CP.A.1a Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").*

G.S.CP.A.1b Flexibly move between visual models (Venn diagrams, frequency tables, etc.) and set notation.*

Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams.</p> <p>Identify the outcomes in a sample space which compose the event, for an event described in everyday language (e.g., "rolling double sixes").</p> <p>Identify a visual model (Venn diagrams, frequency tables, etc.) or the set notation, which represents a sample space.</p>	<p>Identify the union of two subsets, given a sample space within a context.</p> <p>Identify the intersection of two subsets, given a sample space within a context.</p> <p>Identify the complement of a subset, given a sample space within a context.</p> <p>Identify a visual model (Venn diagrams, frequency tables, etc.) and relate to the appropriate set</p>	<p>Represent sample spaces for compound events as unions, intersections, or complements of other events.</p> <p>Use set notation and the symbols u and n to represent sample spaces for compound events.</p> <p>Move flexibly between visual models (Venn diagrams, frequency tables, etc.) and set notation.</p>	<p>Create a visual model (Venn diagram, two-way frequency table, etc.) to illustrate the union, intersection, or complement of events.</p> <p>Identify more complex subsets (e.g., the complement of $A \cup B$), given a sample space within a context.</p>

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
	notation, which represent a sample space.		

Instructional Focus Statements

Level 3:

In grade 8, students represent sample spaces for compound events using organized lists, tables, and tree diagrams. In Geometry, students are expected to expand their understanding of compound events by describing them as unions, intersections, or complements of other events. Teachers should build on students' prior knowledge of compound events by using organized lists, tables, and tree diagrams to describe the union, intersection, and complements of events.

Before moving on to contextual situations, teachers should ask students to find subsets of simple sets of numbers using set notation. For example, consider set $A = \{1, 2\}$ and set $B = \{2, 3\}$. The union of sets A and B in set notation is $A \cup B = \{1, 2, 3\}$. The intersection of sets A and B in set notation is $A \cap B = \{2\}$. If the universe of all possible events is $\{1, 2, 3, 4\}$, the complement, for example, of the union of sets A and B , in set notation is $\sim (A \cup B) = \{4\}$. NOTE: different curriculum can use different symbol notation to represent complements such as $\sim A$, A' , A^c , or $\bar{A}$. As students become more proficient with set notation and identifying the unions, intersections, and complements of simple sets, teachers can then relate set notation to different contextual situations, for example, a bag containing various colored marbles. Students are expected to describe compound events using set notation and the symbols $\cup$ and $\cap$. Using a Venn diagram can aid students in visualizing compound events. For example, let's assume there are 18 students enrolled in French (F), 45 students enrolled in Algebra 2 (A), 7 in both, and 114 enrolled in neither. How many students are there in $F \cup A$? Creating a Venn diagram and coloring in F, A, and the intersection would describe $F \cup A$.

In this situation, the union of French students and Algebra 2 students would be all those enrolled in French, Algebra 2, or both. Students in both subjects should not be counted twice. Therefore, the intersection should be subtracted, and thus, there are $18 + 45 - 7 = 56$ students in $F \cup A$. In G.S.CP.B.3, students will build off this idea to understand the addition rule with inclusive events. Teachers should remind students to not assume that the complement of F is A. The complement of F is all the students not in French, which would be the total (170) minus those enrolled in French (18), which is 152. Situations with two categories are best described using a two-way table. For example, 65 students travel on a soccer trip. 43 are players and 12 are left-handed. Only 5 of the left-handed students are soccer players. How many students are soccer players or are left-handed?

Students should be expected to apply unions, intersections, and complements of events to calculating probability. Subsets can describe the number of ways an event can occur as well as the total number of possible outcomes. Students should be exposed to the concept of the empty set and notations that can represent the empty set. Therefore, a deep understanding of this standard sets the foundation for all other S.CP standards.

Level 4:

As students develop a deep understanding of this standard, they should be able to fluently use set notation and create their own Venn diagram or two-way table to describe various contextual situations. Therefore, students should be exposed to increasingly more complex real-world situations that are best described with a Venn diagram or two-way table. Additionally, students with a deep understanding of compound events can identify complex subsets. For example, students can find the complement of $A \cup B$ or see that the union of all subsets is the sample space.

Statistics and Probability

Conditional Probability and the Rules of Probability* (S.CP)

Standard G.S.CP.B.2 **Cluster Heading: B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.**

Find the conditional probability of A given B as the fraction of B 's outcomes that also belong to A and interpret the answer in terms of the given context.*

Scope and Clarifications: (Modeling Standard)

Calculating conditional probability may be performed via use of a visual model (Venn diagrams, frequency tables, etc.).

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Recognize conditional probability situations in a real-world context.</p> <p>Identify a visual model (e.g., Venn diagram or frequency table) that represents the relationship of two conditional events.</p>	<p>Recognize conditional probabilities in real-world context and recognize the conditional whole may or may not equal the total.</p> <p>Create a visual model (e.g., Venn diagram or frequency table) that represents the relationship of two conditional events.</p>	<p>Interpret the conditional probability and the conditional whole of two events in terms of the given context.</p> <p>Use a visual model (e.g., Venn diagram, frequency table, etc.), that represents the relationship of two conditional events to find conditional probability in a real-world context.</p>	<p>Explain the conditional probability and the conditional whole of two events in terms of the regions of A and B in a Venn diagram.</p> <p>Fluently choose, create, and use a visual model (e.g., Venn diagram, frequency table, etc.) to find conditional probability of two conditional events in a real-world context.</p>

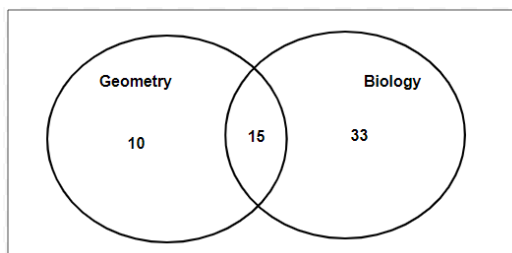
Instructional Focus Statements

Level 3:

To develop students' conceptual understanding of conditional probability, teachers should explicitly make connections to previous learning from Ratio and Proportional Relationships (RP) standards in middle grades, such as ratios, percents, and part-whole relationships, to reason about what constitutes the conditional whole in comparison to the total in a given context.

As students reason to find conditional probabilities, problems should be contextual situations, have multiple entry points, and allow for multiple solution paths. In Geometry, initial emphasis on finding conditional probability with a visual model (Venn diagrams, frequency tables, etc.) will help students model real-world situations and strengthen students' conceptual understanding of mathematical concepts. Students will build upon this conceptual understanding further in A2.S.CP.C.4 by exploring connections between visual models, counting techniques, and calculation/formula. It is not the expectation for students to know or use formulas to calculate conditional probability in this Geometry standard.

Venn diagrams aid students in visualizing part-whole relationships by depicting the idea of conditional probability as the intersection of events A and B , $P(A \cap B)$, as a part of the conditional whole. Understanding the population total in comparison to the conditional total is often a common misconception for students. In many cases, students are given the $P(A \cap B)$ and either $P(A)$, $P(B)$, or both and asked to calculate the conditional probability $P(B|A)$ or $P(A|B)$. Discussion during instruction would need to help students recognize that if they are given both $P(A)$ and $P(B)$, one is superfluous information. For example, a Venn diagram can be used to model the number of students that are enrolled in Biology given they are also enrolled in Geometry, $P(\text{enrolled in Biology} | \text{enrolled in Geometry})$. If 25 students are enrolled in Geometry (event A), 48 students are enrolled in Biology (event B), and 15 students are enrolled in both. Students may need support understanding the number of students enrolled in Geometry (25) becomes the conditional whole rather than the population total (58). Then, the conditional probability of students enrolled in Biology, given they are also enrolled in Geometry, can be expressed as a part-whole relationship using the conditional whole $P(B \text{ given } G) = P(\text{enrolled in Biology} | \text{enrolled in Geometry}) = 15/25$ or 60%.



If more than two categories are used, teachers can provide or encourage students to use a table. For example, a frequency table can be used to compare the interest in sports across grade levels. If students are asked to find the probability a randomly selected student likes sports given they are in the 10th grade, $P(\text{student likes sports} | \text{student in grade 10})$, students could be shown how to cover up the rest of the table to highlight only 10th graders. From there, students can use a part-whole relationship to express the conditional probability that a student likes sports as a fraction of the conditional whole. Explaining that the condition given narrowed down the subjects to only 10th graders helps students begin to focus on only the values involved in the problem. Instruction, which is first grounded in visual models, deepens student-made connections to calculations with formulas and strengthens their understanding calculating a percent of a conditional whole.

Level 4:

Students at this level can distinguish between the population whole and the conditional whole in terms of the regions of A , B , and $(A \cap B)$ in a Venn diagram. Using a Venn diagram, students can shade in the regions of A and B and show how it is a fraction of the whole (either A or B) and explain the conditional probability in terms of these regions. Teachers should prompt students to discuss how the population whole is narrowed to a conditional whole and how that impacts the probability of a given event. Likewise, students could be asked to justify why conditional probability narrows a table to a row or column, in which the probability can then be found.

As students become more comfortable using visual models to represent and find conditional probabilities, students should be provided opportunities which allow choice of representation/model and be prompted for justification of that choice. For example, teachers can ask students to choose a model, create that model, and use their model to find the conditional probability of two conditional events in real-world context. Further, students could be asked to write a response about their findings, explaining how the conditional probability was modeled and justifying their thinking. This activity would support the literacy standards in mathematics, as students would be discussing and articulating mathematical ideas while using correct mathematical vocabulary to build their argument.

Statistics and Probability

Conditional Probability and the Rules of Probability* (S.CP)

Standard G.S.CP.B.3 Cluster Heading: B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Understand and apply the Addition Rule.*

G.S.CP.B.3a Explain the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ in terms of visual models (Venn diagrams, frequency tables, etc.).*

G.S.CP.B.3b Apply the Addition Rule to solve problems and interpret the answer in terms of the given context.*

Scope and Clarifications: (Modeling Standard)

There are no assessment limits for this standard. The entire standard is assessed in this course.

Aspect of Rigor Alignment

Conceptual Understanding	Procedural Skill and Fluency	Application
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Represent all possibilities in a sample space for compound events using a visual representation or model such as elements of a set using set notation, organized lists, tables, tree diagrams, or simulations.</p> <p>Recognize when two compound events are mutually exclusive and when they are not.</p> <p>Find $P(A \text{ or } B)$, $P(A \cup B)$, given two events that are mutually exclusive.</p>	<p>Interpret more complex visual representations or models (Venn diagrams, frequency tables) of all possibilities in a sample space for compound events and explain why certain quantities overlap or may lead to double counting.</p> <p>Determine if two events are mutually exclusive or inclusive.</p> <p>Given a sample space within a context, identify both the union and the intersection of two subsets, which may be mutually exclusive or inclusive.</p>	<p>Flexibly use visual representations or models (Venn diagrams, frequency tables, etc.) of a sample space in a real-world context to identify and connect components of the Addition Rule.</p> <p>Understand and apply the Addition Rule $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, in a real-world context for events that are either inclusive or mutually exclusive.</p> <p>Interpret $P(A \text{ or } B)$ in terms of a real-world context to solve problems.</p>	<p>Use multiple representations to explain why $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ in context.</p> <p>Use and explain complements of events, such as $P(A \text{ or } B) = P(A) + P(B \text{ and the complement of } A)$ or $P(A \text{ or } B) = P(A \text{ and the complement of } B) + P(B \text{ and the complement of } A) + P(A \text{ and } B)$, as alternate solution methods to find $P(A \text{ or } B)$.</p>

Instructional Focus Statements

Level 3:

In 8.SP.B.4, students calculate the probability of compound events by examining the sample space using lists, tables, and tree diagrams. In Geometry, students use their prior knowledge from grade 8 and their understanding of subsets from G.S.CP.A.1 to model and calculate $P(A \text{ or } B)$, when A and B are inclusive or mutually exclusive events. Students will continue to use set notation (G.S.CP.A.1) in G.S.CP.B.3. Teachers should make explicit connections to students' past experiences with visual representations and models (elements of a set using set notation, organized lists, tree diagrams, tables/frequency tables, Venn diagrams, etc.) and previous understanding of compound events and independence, to support students' understanding of inclusive events, mutually exclusive events, and each component of the Addition Rule.

Learning experiences should allow students the opportunity to explore and discover the Addition Rule in context rather than simply providing the rule for computation. Instead of giving students a formula, teachers should let students discover why the $P(A \text{ and } B)$ needs to be subtracted out so that outcomes (i.e., $P(A \text{ and } B)$) are not counted twice. It can be helpful to use a Venn diagram so that students can visualize this concept and make the connection to the formula, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. If A and B are inclusive, shading $P(A)$ and $P(B)$ on a Venn diagram reveals to students how the intersection is being shaded twice.

Modeling the standard should be progressive and move from visual representations and models that list all elements, possibilities, or outcomes of events to more concise representations that summarize possibilities. For example, consider two mutually exclusive events, flipping a coin and rolling a die. To find the probability of a coin landing on heads or rolling an even number on a die, $P(\text{Heads or Even})$, students might first create a t-chart to organize all possible outcomes (Figure 1), and then move to a more concise representation using a frequency table (Figure 2). Once students have grappled with various representations, they will begin to uncover the Addition Rule and explain the rule in terms of the visual representations and models.

Figure 1

coin	die
H	1
H	2
H	3
H	4
H	5
H	6
T	1
T	2
T	3
T	4
T	5
T	6

Figure 2

	H	T
odd	3	3
even	3	3

One misconception students tend to have is that "or" always means add, and therefore, simply calculate $P(A \text{ or } B)$ as $P(A) + P(B)$. Teachers can break this misconception by giving students examples of inclusive events where $P(A) + P(B)$ is greater than 1. For example, when rolling a 6-sided number cube, what is the probability of rolling an odd number or a number greater than 2? In this case, $P(A) + P(B) = 3/6 + 4/6 = 7/6$. Discussion should help students recognize that this cannot be correct and realize that by simply adding the probabilities together, they counted 3 and 5 twice. Having students examine the sample space can quickly reveal what the probability should be. At this point, teachers can give students time to discover how to calculate $P(A \text{ or } B)$ by starting with $P(A) + P(B) = 7/6$ and asking students to identify the overlap and see that they need to subtract it to calculate the probability correctly. This should help students understand the need to subtract the outcomes that fit in both categories and were counted twice. Thus, students calculate $P(A \text{ or } B)$ as $7/6 - 2/6 = 5/6$.

Level 4:

A Venn diagram or two-way table can support students in developing a deep understanding of this standard. Instruction should encourage students to explain why $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ using multiple representations, such as both a Venn diagram and a two-way table. By having students create a two-way table, the $P(A \text{ and } B)$ would be separated out from $P(A)$ and $P(B)$ and students can see these four probabilities shown in the table: $P(A \text{ and the complement of } B)$, $P(B \text{ and the complement of } A)$, $P(A \text{ and } B)$, and $P(\text{the complement of } B \text{ and the complement of } A)$. By displaying data in this form, students should see other ways to calculate $P(A \text{ or } B)$, such as $P(A) + P(B \text{ and the complement of } A)$, or $P(A \text{ and the complement of } B) + P(B \text{ and the complement of } A) + P(A \text{ and } B)$. Asking students to separate a Venn diagram into its four parts can also aid in understanding these alternate ways to calculate $P(A \text{ or } B)$ and often better aligns with students' thinking as they examine the sample space to calculate $P(A \text{ or } B)$. Note – the complement of an event, in set notation, may be represented in a variety of ways ($\sim P(A)$, $P(A)^c$, etc.).

Statistics and Probability

Conditional Probability and the Rules of Probability* (S.CP)

Standard G.S.CP.C.4 Cluster Heading: C. Apply geometric concepts to situations involving probability.

Calculate probabilities using geometric figures.*

Scope and Clarifications: (Modeling Standard)

Geometric figures include line segments, two-dimensional shapes, and three-dimensional solids.

Aspect of Rigor Alignment

<u>Conceptual Understanding</u>	<u>Procedural Skill and Fluency</u>	<u>Application</u>
X		X

Evidence of Learning Statements

Students with a level 1 understanding of this standard will most likely be able to:	Students with a level 2 understanding of this standard will most likely be able to:	Students with a level 3 understanding of this standard will most likely be able to:	Students with a level 4 understanding of this standard will most likely be able to:
<p>Solve mathematical problems involving length, area, surface area, and volume of two- and three-dimensional shapes when a visual model is provided.</p> <p>Apply concepts of basic probability to that of geometric figures (e.g., the probability of a shaded region).</p>	<p>Solve mathematical problems involving length, area, surface area, and volume of two- and three-dimensional shapes when a visual model is not provided.</p> <p>Identify which geometric attribute(s) need(s) to be calculated to calculate probabilities using geometric figures.</p>	<p>Calculate probabilities using geometric figures (line segments, two-dimensional shapes, three-dimensional shapes).</p> <p>Apply probability concepts to geometric figures given a real-world situation.</p> <p>Analyze a contextual probability situation to determine an appropriate strategy involving geometric figures.</p>	<p>Calculate probabilities using non-uniform geometric figures (e.g., not evenly divided spinner).</p> <p>Create a real-world situation where geometric figures can be used to calculate probabilities.</p>

Instructional Focus Statements

Level 3:

Standard G.S.CP.C.4 is intended to provide opportunities for students to make connections between geometry concepts and probability. Instruction should draw upon students' middle grades ratio (part-to-whole) understanding and previous experiences with probability (desired outcomes-to-total

possible outcomes) to calculate probabilities with geometric shapes (part of a geometric figure-to-whole geometric figure). Students should be given the opportunity to discover how calculating probability of simple or compound events with a discrete number of possible outcomes (flipping a coin, rolling a die) compares to calculating probabilities of events with a non-discrete or continuous number of possible outcomes (arrow hitting the bullseye on a target). Instruction should focus on developing students' ability to analyze a contextual situation, determine which geometric probability strategy is needed, and apply appropriate mathematical and geometric formulas and calculations to determine the likelihood of the event.

To calculate probability using geometric figures, students will draw on previous experiences with solving real-world and mathematical problems involving length of line segments, area of two-dimensional shapes, surface area and volume of three-dimensional shapes, and associated geometric formulas. Modeling and mathematical tools, such as number lines, rulers, graph paper, nets of geometric figures, 2-D and 3-D geometric models, dynamic geometric software, game spinners, real-world representations of geometric figures, etc. are encouraged to support students' conceptual development of the concepts. Suggested instructional resources are linked below.

To calculate probability using a line segment, student will need to know the length of the whole line segment and the length of the desired line segment. Using Figure 1, assume students are asked to calculate the probability that a point selected at random is on line segment $\overline{CD}$, $P(\overline{CD})$. The length of the whole line segment $\overline{AD}$ is 10 units and the length of line segment $\overline{CD}$ is 4 units. Thus, $P(\overline{CD}) = 4/10$. Note, this is also a good opportunity to make connections between length, integers, and absolute value when the length of the line segment includes both positive and negative values.

Figure 1.



A common misconception for students is to simply count the labeled integer values (C and D are two of four labeled values, A,B,C,D) and inaccurately determine $P(\overline{CD})$ is $2/4$ or $1/2$. Thus, students will need time to affirm the reality that infinitely many points lie on line segment AD. Practical application problems, such as determining arrival times on a bus schedule are linked below.

Likewise, student can determine the probability of two-dimensional shapes by calculating desired areas in comparison to the total area or various probabilities of three-dimensional shapes using surface area or volume. Learning and modeling opportunities should scaffold in complexity. For example, more simplistic examples may provide a figure and specify which calculation is needed (e.g., given the figure of a sphere below, use the volume of a sphere formula to calculate the probability of a dart landing on the top half of the sphere). As students' understanding progresses, scaffolds should be removed, allowing students the opportunity to analyze the context of the problem and determine what tools and strategies are needed to calculate the desired probability (e.g., A bird flying North for the winter collided with a sphere-shaped hot air balloon. If the hot air balloon has a diameter of 60 feet, what is the probability the bird collided with the top half of the hot air balloon?).

Level 4:

Students should be formulating a strategy to solve the problem based on a mathematical understanding of the situation, computing solutions, interpreting findings, and validating their thinking and the reasonableness of attained solutions in order to justify solutions to real-world geometric problems with increased rigor. This modeling standard provides a great opportunity for creativity. Teachers can challenge students to write their own real-world situations where geometric figures can be used to calculate probabilities (e.g., A baker lost an engagement ring while mixing the batter for a three-tier wedding cake. What would the baker need to know to calculate the probability that the ring is in the top layer of the cake? What if a portion of each layer is cut out to account for filling? How would your calculations change?).

In addition, students can expand their knowledge to irregular geometric figures (a spinner with unevenly divided areas) or investigate how to calculate probabilities of events when outcome anomalies occur (spinner arrow lands on a line, which divides two sections; a penny, when flipped, lands on its edge rather than on heads or tails).