AP Calculus Summer Assignment **Name:______________**

Due Date: The last class day of the second week of school.

The purpose of this assignment is to have you practice the mathematical skills necessary to be successful in AP Calculus. Many of the skills covered in this packet are skills from Algebra 2 and Pre-Calculus, others are skills from the beginnings of Calculus. If you need to, you may use reference materials to assist you and refresh your memory (old notes, textbooks, online resources, etc.). While graphing calculators will sometimes be used in class, you should be able to do everything in this packet without a calculator.

The AP Calculus courses are fast paced and taught at the college level. There is a lot of material in the curriculum that must be covered before the AP exam in May. Therefore, we cannot spend a lot of class time re-teaching prerequisite skills. This is why you have this packet. Spend some time with it and make sure you are clear on everything covered in the packet so that you will be successful in AP Calculus. Of course, you are always encouraged to seek help from your teacher if necessary.

Your assignment is to do the following:

- **Sections I through III: at least the odd-numbered problems. Do more if you feel you need the practice.**
- **Section IV: all problems**

This assignment will be collected and graded the last class day of the second week of school. Be sure to show all appropriate work to support your answers. In addition, there may be a quiz on this material during the first quarter.

Good Luck!

Ms. Falletta

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Show all work – no credit will be awarded for answers missing appropriate work.
No calculators!

Section I: Algebra Review

Identify the following statements as true or false.

1. $\frac{x+y}{2}=\frac{x}{2}+\frac{y}{2}$ _____
2. $\frac{1}{p+q}=\frac{1}{p}+\frac{1}{q}$ _____
3. $\frac{2k}{2x+h}=\frac{k}{x+h}$ _____
4. $3\bullet\frac{a}{b}=\frac{3a}{b}$ _____
5. $3\bullet\frac{a+b}{c}=\frac{3a+b}{c}$ _____
6. $\sqrt{a^2+b^2}=a+b$ _____

Identify the following statements as true or false over the set of real numbers. Give a counter example for any false statement.

7. $x^3+1>x^3$ _____
8. $x^3+x>x^3$ _____
9. $x^2\geq 0$ _____
10. $x^2\geq x$ _____
11. $2x\geq x$ _____
12. $\sqrt{x}\geq 0$ _____
13. $-x\leq 0$ _____
14. $\frac{1}{x}\leq x$ _____
15. $x\leq |x|$ _____

16. Solve $xy'+y+1=y'$ for y'.

17. Solve $\ln y=kt$ for y.

16.______________

17.______________

18. Factor: y^3+27

19. Factor: $x^2(x-1)-4(x-1)$

18.______________

19.______________

Simplify each expression.

20. $\frac{\left(x^2\right)^3 x}{x^7}$ ________________

21. $\sqrt{x}\bullet\sqrt[3]{x}\bullet x^{\frac{1}{6}}$ ________________

22. $\frac{5(x+h)^3-5x^3}{h}$ ________________

23. $\frac{3(x+h)^2-3x^2}{h}$ ________________

24. $\frac{\frac{x^2-1}{x}}{\frac{x+1}{x^3}}$ ________________

25. $\frac{\frac{1}{x}+\frac{4}{x^2}}{3-\frac{1}{x}}$ ________________

26. $\frac{\frac{a}{2x+h}-\frac{a}{2x}}{h}$ ________________

27. $\frac{1}{1-2a}-\frac{2}{1+2a}+\frac{6a+2}{4a^2-1}$ ________________

Simplify, using factoring of binomial expressions. Leave answers in factored form.

Example:

$$\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)} = \frac{(x+1)^2\left[(x+1)(4x-9)-(16x+9)\right]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2(4x^2-5x-9-16x-9)}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2(4x^2-21x-18)}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2(4x+3)(x-6)}{(x-6)(x+1)}$$

$$= (x+1)(4x+3)$$

28. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$ ______________________________

29. $\dfrac{(x-1)^2(3x-1)-2(x-1)\bullet 3}{(x-1)^4}$ ______________________________

30. $\dfrac{(x-1)^3(2x-3)-(4x-1)(x-1)^2}{(x-1)^2(2x-1)}$ ______________________________

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x}\bullet\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x\left(\sqrt{x+4}+2\right)} = \frac{x}{x\left(\sqrt{x+4}+2\right)} = \frac{1}{\sqrt{x+4}+2}$$

31. $\dfrac{\sqrt{x+9}-3}{x}$ ______________________

32. $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$ _________________________

Solve each equation or inequality for x over the set of real numbers.

33. $2x^4 + 3x^3 - 2x^2 = 0$ ______________________

34. $\frac{2x-7}{x+1} = \frac{2x}{x+4}$ ______________________

35. $\frac{3x+5}{(x-1)(x^4+7)} = 0$ ______________________

36. $\sqrt{x^2-9} = x-1$ ______________________

37. $|2x-3| = 14$ ______________________

38. $x^2 - 2x - 8 < 0$ ______________________

Solve each of the systems.

39. $x + y = 8$
$2x - y = 7$ ______________________

40. $y = x^2 - 3x$
$y = 2x - 6$ ______________________

Section II: Pre-Calculus Review

Use your knowledge of the unit circle to evaluate each of the following. Leave your answers in radical form.

41. $\sin(30°)$ _____

42. $\cos\frac{2\pi}{3}$ _____

43. $\tan 45°$ _____

44. $\sin\left(-\frac{\pi}{6}\right)$ _____

45. $\tan\pi$ _____

46. $\csc\frac{5\pi}{6}$ _____

47. $\cos(90°)$ _____

48. $\cos\frac{3\pi}{4}$ _____

49. $\tan\frac{\pi}{6}$ _____

50. $\cos^{-1}\left(\frac{1}{2}\right)$ _____

51. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ _____

52. $\tan^{-1}(1)$ _____

Solve each trigonometric equation for $0 \le x \le 2\pi$.

53. $\sin x = \frac{\sqrt{3}}{2}$ ____________

54. $\tan^2 x = 1$ ____________

55. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$ ____________

56. $2\sin^2 x + \sin x - 1 = 0$ ____________

For each trigonometric function identify the amplitude and any horizontal or vertical shifts from the basic function.

57. $y = \frac{1}{2}\cos\frac{x}{2} - 3$ amplitude:_________ period:________ vertical shift:________

58. $y = 2\sin(2x - \pi)$ amplitude:_________ period:________ horizontal shift:_______

59. $y = \tan 3x$ period:___________

Solve each exponential or logarithmic equation.

60. $5^x = 125$ _________ 61. $8^{x+1} = 16^x$ _________ 62. $81^{\frac{3}{4}} = x$ ________

63. $8^{\frac{-2}{3}} = x$ _________ 64. $\log_2 32 = x$ _________ 65. $\log_x \frac{1}{9} = -2$ _________

66. $\log_4 x = 3$ _________ 67. $\log_3(x+7) = \log_3(2x-1)$ ___________

Expand each of the following using the laws of logs.

68. $\log_3 5x^2$ ______________________ 69. $\ln\frac{5x}{y^2}$ _____________________

Complete each of the following using trigonometric identities and formulas.

70. $\sin\left(\frac{\pi}{2} - x\right) =$ __________ 71. $\sin^2 x + \cos^2 x =$ ________ 72. $\sin 2u =$ ________________

73. $\tan x =$ ______________ 74. $1 + \cot^2 x =$ ___________ 75. $1 - \cos^2 x =$ ______________

76. A right triangle has a base of 5 and a hypotenuse of 7. Find the height.

Section III: Graphing Review

Sketch the following functions. State the domain and range of each. Draw and label your own axes.

77. $f(x) = x$

78. $f(x) = x^2$

79. $f(x) = x^3$

80. $f(x) = |x|$

81. $f(x) = [x]$ (Greatest integer function)

82. $f(x) = \frac{1}{x}$

83. $f(x) = \sqrt{x}$

84. $f(x) = e^x$

85. $f(x) = \ln x$

86. $f(x) = \sqrt{9 - x^2}$

87. $f(x) = \sin x$

88. $f(x) = \cos x$

89. $f(x) = \tan x$

90. $f(x) = \csc x$

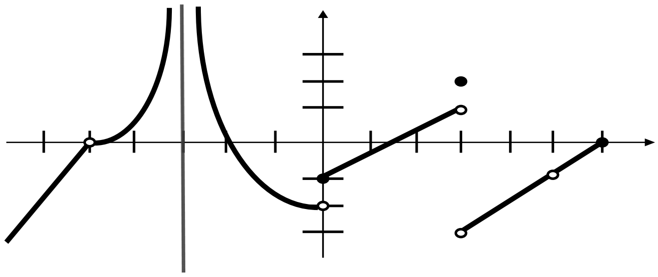
91. $f(x) = \sec x$

92. $f(x) = \cot x$

Section IV: Intro to Calculus Review – Limits/Continuity and Derivatives

Limits/Continuity

93. **Answer the following questions using the graph of *f*(x) given below.**



(a) Find *f*(0)

(b) Find *f*(3)

(c) Find $\lim_{x \to -5} f(x)$

(d) Find $\lim_{x \to 0^+} f(x)$

(e) Find $\lim_{x \to 3^-} f(x)$

(f) Find $\lim_{x \to -3^+} f(x)$

(g) List all x-values for which *f*(x) has a removable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

(h) List all x-values for which *f*(x) has a nonremovable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

94. **Find the limit if it exists analytically.**

(a) Find $\lim_{x \to -2} \frac{3x^2 + 21x + 30}{x^3 + 8}$

(b) Find $\lim_{x \to \frac{\pi}{6}} \frac{1 - \cos^2 x}{4x}$

(c) Find $\lim_{x \to 4} \frac{\sqrt{x-3} - 1}{x - 4}$

(d) Find $\lim_{t \to 0} \frac{\sin^2 3t^2}{t^3}$

95. **Suppose** $f(x) = \begin{cases} \frac{\sqrt{2x+1} - \sqrt{3}}{x - 1}, & x \geq 0 \\ 4x^2 + k, & x < 0 \end{cases}$,for what value of *k* will *f* be continuous at *x* = 0?

Explain why this is true. (**Hint: A function is continuous at x =*c* if (1) *f(c)* exists, (2)** $\lim_{x \to c} f(x)$ **exists, and (3)** $\lim_{x \to c} f(x) = f(c)$ **)**

96. **Circle the correct answer and explain why it is the correct one.**

If $f(x) = x^3 + x - 3$, and if *c* is the only real number such that *f(c)* = 0, then by the Intermediate Value Theorem, *c* is necessarily between

(A) -2 and -1

(B) -1 and 0

(C) 0 and 1

(D) 1 and 2

(E) 2 and 3

Hint: The Intermediate Value Theorem states that if *f* is a continuous function on the interval [a, b] and *k* is any number between *f(a)* and *f(b)*, then there must exist at least one number $c \in [a,b]$ such that *f(c) = k*.

Derivatives

97. Use the limit definition to find the derivative of:. $f(x) = 4x^2 + 3x - 5$

98. Find the derivative of each function using the basic, product, quotient and/or chain rules.

(a) $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

(b) $f(x) = \sqrt{x}\sin x$

(c) $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

(d) $f(x) = 3x\csc x + x\cot x$

99. Find an equation of the tangent line to the graph of: $f(x)=\frac{1+\cos x}{1-\cos x}$, $at\ P(\frac{\pi}{2},1)$.

100. Circle the correct answer and explain why the answer is the correct one.

(i) $\lim_{\Delta x\to 0}\frac{\cos(\frac{\pi}{6}-\Delta x)-\cos(\frac{\pi}{6})}{\Delta x}$

(A) Does not exist

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) $\frac{\sqrt{3}}{2}$

(E) $-\frac{\sqrt{3}}{2}$

(ii.) Let *f* and *g* be differentiable functions with values for *f(x), g(x), f′(x), and g′(x)* shown below for *x* = 1 and *x* = 2.

x	*f(x)*	*g(x)*	*f′(x)*	*g′(x)*
1	4	-4	12	-8
2	5	1	-6	4

Find the value of the derivative of $f(x)g(x)$ at *x* = 1.

(F) -96

(G) -80

(H) -48

(I) -32

(J) 0

(iii). Let $f(x)=\begin{cases}3x^2+4, & x<1\\ x^3+3x, & x\geq 1\end{cases}$. Which of the following is true?

I. $f(x)$ is continuous at $x = 1$

II. $f'(x)$ is differentiable at $x = 1$

III. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

(K) I only

(L) II only

(M) III only

(N) I and III only

(O) II and III only

(iv.) An equation of the line <u>normal</u> to the curve $y = \sqrt[3]{x^2 - 1}$ at the point where $x = 3$ is

(P) $y + 12x = 38$

(Q) $y - 4x = 10$

(R) $y + 2x = 4$

(S) $y + 2x = 8$

(T) $y - 2x = -4$

Hint: A normal line to a curve at a point is perpendicular to the tangent line to the curve at the same point.