Fox Lane High School
Department of Mathematics

June 2025

Hello Future AP Calculus AB Student!

This is the summer assignment for all students taking AP Calculus AB next school year. It contains a set of problems along with their answers. These problems focus on skills that you have previously learned and are essential to your success in AP Calculus. The first few days of class will be devoted to answering questions on this material. You will then have a **test** that assesses your mastery of this content. Your assignment is to complete **ALL** of these questions to ensure you have the necessary prerequisite skills to be successful in AP Calculus. Seek additional help, if necessary. The first test of the year is an excellent opportunity to start the year off well, so make sure you study until you are confident on these questions. You may email me at the address shown below if you have any questions. Please keep in mind that it may take me some time to get back to you depending on my schedule over the summer, so please don't wait until the last minute to ask for help.

PLEASE NOTE: This assignment is to be done **WITHOUT** the use of a **calculator**. Over half of the AP Calculus exam needs to be completed without a calculator, so start getting used to that now. In addition, a calculator will not be allowed on the test you will take on this material.

You may also find it helpful to use the following websites as a resource while working through the problems:

- https://www.jmap.org/
- http://www.themathpage.com/aPreCalc/precalculus.htm
- http://www.purplemath.com
- Online Math Videos can be found at the following sites:
- https://www.youtube.com/user/patrickJMT
- https://www.khanacademy.org/#browse

Enjoy your summer and see you in September!

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Required Skills (note: sample questions on these skills are found on the following pages)

1. Simplifying complex fractions
2. Working with rational (fraction) and negative exponents without a calculator
3. Simplifying rational expressions (adding, subtracting, multiplying, and dividing algebraic fractions)
4. Factoring quadratic expressions
5. Using function notation and composite function notation
6. Finding x and y intercepts of a function
7. Solving systems of equations (i.e., finding intersection points) algebraically
8. Finding the domain and range of functions and expressing answers in interval notation
9. Finding the inverse of a function
10. Writing the equation of a line using point slope form
11. Solving all types of equations including:
    a. Rational (fraction) equations
    b. Equations with square roots and cube roots
    c. Quadratic equations (when factorable and not factorable)
    d. Exponential equations (including those with a base of e)
    e. Logarithmic equations (including those with ln)
12. Evaluating trig and inverse trig functions without a calculator
13. Proving trig identities
14. Sketching basic functions without a calculator. These include:

    $$y=x^2,\ y=x^3,\ y=\sqrt{x},\ y=2^x,\ y=e^x,\ y=\ln x,\ y=|x|,\ y=\frac{1}{x}$$
15. Graphing the functions listed in item 14 after a shift has been applied.
16. Graphing and evaluating piecewise functions
17. Rationalizing a binomial denominator (i.e., get the radical out of a denominator of a fraction)

Complex Fractions

When simplifying complex fractions, multiply by a fraction (equal to 1) which has the numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7(x+1)-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

6. $\dfrac{\frac{2}{x^3}-\frac{5}{x^2}}{\frac{3}{x^2}+4}$

Make sure you are very comfortable manipulating exponents, positive/negative, fractional. Also, know the relationship between exponents and radicals; the appropriate radical will "undo" an exponent.

Examples:

$\sqrt[3]{(2)^3} = 2$

$\sqrt[3]{8} = 8^{1/3} = 2$

$\left(\sqrt[10]{25}\right)^5 = \left(25^{1/10}\right)^5 = 25^{5/10} = 25^{1/2} = \sqrt{25} = 5$

$x^{-4} = \dfrac{1}{x^4}$

$\dfrac{1}{x^{-3}} = x^3$

$(3x)^{-2} = \dfrac{1}{(3x)^2} = \dfrac{1}{9x^2}$

$x^6 x^5 = x^{11}$

$\dfrac{x^3}{x^9} = x^{-6} = \dfrac{1}{x^6}$

$\left(x^2\right)^3 = x^6$

Simplify each expression using only positive exponents where applicable.

7. $4x^{\frac{3}{8}} \cdot 5x^{\frac{5}{8}}$

8. $\dfrac{\left(5x^{\frac{4}{5}}\right)^3}{x^{\frac{2}{3}}}$

9. $r^{\frac{2}{5}} \cdot r^{-\frac{3}{5}}$

10. $\left(b^5\right)^{\frac{2}{5}}$

11. $\dfrac{y^{\frac{3}{4}}}{y^{\frac{1}{4}}}$

12. $\dfrac{12x^{-3}y^2}{18xy^{-1}}$

Simplify and express answer without exponents:

13. $(-125)^{-\frac{2}{3}}$

14. $-8^{-\frac{4}{3}}$

Simplify each of the following:

15. $\dfrac{1}{x+h} - \dfrac{1}{x}$

16. $\dfrac{3}{4/5}$

17. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^3}}$

Factor Completely:

18. $6x^3 + 23x^2 + 20x$

19. $14x^2 - 49x - 28$

Simplify and Express Answer Without Exponents:

20. $\dfrac{15x^2}{5\sqrt{x}}$

21. $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$

22. $\dfrac{5-x}{x^2-25}$

23. $\dfrac{x}{x^2-16} - \dfrac{3}{x^2+5x+4}$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "*f* of *g* of *x*" means: plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each of the following:

24. $f(2) =$

25. $g(-3) =$

26. $f(t+1) =$

27. $f(g(-2)) =$

28. $g(f(m+2)) =$

29. $\dfrac{f(x+h) - f(x)}{h}$

Let $f(x) = \sin x$. Find the exact value of each of the following:

30. $f\left(\dfrac{\pi}{2}\right) =$

31. $f\left(\dfrac{2\pi}{3}\right) =$

Let $f(x) = x^2$, $g(x) = 2x + 5$, $h(x) = x^2 - 1$. Find each of the following:

32. $h(f(-2)) =$

33. $f(g(x-1)) =$

34. $g(h(x^3)) =$

35. If $f(x) = 9x + 3$, find the value of: $\dfrac{f(x+h) - f(x)}{h}$

36. If $f(x) = 5 - 2x$, find the value of: $\dfrac{f(x+h) - f(x)}{h}$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

x − int. (*Let* $y = 0$)

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1$ *or* $x = 3$

x − intercepts $(-1,0)$ *and* $(3,0)$

y − int. (*Let* $x = 0$)

$y = 0^2 - 2(0) - 3$

$y = -3$

y − intercept $(0,-3)$

To find points of intersection, solve the system of equations using either the substitution or elimination methods. Remember to find the coordinates of each intersection point by using one of the original equations. See example below:

Find the intersection point(s) of the two curves:

$$y = \frac{1}{2}x + \frac{3}{2}$$

$$x^2 - y^2 - 9 = 0$$

Using the substitution method and substituting the top equation into the bottom one:

$$x^2 - \left(\frac{1}{2}x + \frac{3}{2}\right) - 9 = 0$$

$$x^2 - \frac{1}{4}x^2 - \frac{3}{2}x - \frac{9}{4} - 9 = 0$$

$$\frac{3}{4}x^2 - \frac{3}{2}x - \frac{45}{4} = 0$$

$$4\left(\frac{3}{4}x^2 - \frac{3}{2}x - \frac{45}{4}\right) = (0)(4)$$

$$3x^2 - 6x - 45 = 0$$

$$3\left(x^2 - 2x - 15\right) = 0$$

$$3(x-5)(x+3) = 0$$

$$x = 5 \quad x = -3$$

If $x = 5$

$$y = \frac{1}{2}(5) + \frac{3}{2} = 4$$

Point = $(5,4)$

If $x = -3$

$$y = \frac{1}{2}(-3) + \frac{3}{2} = 0$$

Point = $(-3,0)$

Find the x and y intercepts for each:

37. $y=2x-5$

38. $y=x^2+x-2$

39. $y=x\sqrt{16-x^2}$

40. $y^2=x^3-4x$

Find the intersection point(s) of the graphs of the given equations:

41. $\begin{matrix} x+y=8 \\ 4x-y=7 \end{matrix}$

42. $\begin{matrix} x^2+y=6 \\ x+y=4 \end{matrix}$

INTERVAL NOTATION: recall that $-2<x\leq 4$ expressed in interval notation is $(-2,4]$ and $x\leq 5$ expressed in interval notation is $(-\infty,5]$

Find the domain and range for each function. Write your answer in INTERVAL notation.

43. $f(x)=x^2-5$

44. $f(x)=-\sqrt{x+3}$

45. $f(x)=3\sin x$

46. $f(x)=\dfrac{2}{x-1}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the Inverse of each function:

47. $f(x)=2x+1$

48. $f(x)=\dfrac{x^3}{2}$

In Calculus, it is more common to use the Point-Slope form for the equation of a line. Get comfortable with using that form.

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: $y = c$ (slope is 0)

49. Use point-slope form to write the equation of a line passing through the point $(3,-5)$ with a slope of 7.

50. Find the equation of the line passing through the points $(-3,6)$ and $(9,2)$.

51. Find the equation of a line passing through the point $(2,8)$ and parallel to the line $y = \frac{5}{6}x - 1$.

52. Write the equation of a line that passes through the point $(1,-5)$ and is perpendicular to the line $-7x - 5y = 18$.

53. Determine the equation of a line passing through the point $(5,-3)$ with an undefined slope.

54. Determine the equation of a line passing through the point $(-4,2)$ with a slope of 0.

55. Find the equation of a line with an x-intercept of $(2,0)$ and a y-intercept of $(0,3)$.

You need to be able to solve equations that are in many different forms. Practice by solving the following equations.

56. $\frac{2}{x^2} = \frac{1}{x+6}$

57. $-x + \sqrt{4x-8} = -5$

58. $\frac{9}{x-1} + \frac{x}{x+1} = \frac{11}{x^2-1}$

59. $(4x+1)^2 - 8(4x+1) = -12$

60. $\sqrt[3]{4x+5} + 4 = 0$

61. Solve for x: $8.76^x = 9$

62. Solve: $\log_2\left(\frac{x}{3}\right)=4$

63. Solve: $2\log_b 5+\frac{1}{2}\log_b 36-\log_b 15=\log_b x$

64. Solve: $\log_5(x+1)-2\log_5 3=\log_5 x$

65. Solve: $8\ln(x-2)=40$

66. Solve: $60=10e^x$

67. Solve: $4=e^{-2x}$

68. Solve: $12=x^{\frac{2}{3}}$

69. Solve for a and b: $\begin{aligned}3a+2b&=-6\\ a-5b&=-19\end{aligned}$

70. Find the zero of the function by hand, then check answer with graphing calculator:

$f(x)=e^x-7$

You should be able to find the exact values of trigonometric and inverse trig functions WITHOUT a calculator. You also need to be comfortable with performing algebraic operations with trig functions and using this to prove trig identities.

71. $\sin\left(\frac{5\pi}{3}\right)$

72. $\cos\left(-\frac{5\pi}{4}\right)$

73. $\tan\left(-\frac{2\pi}{3}\right)$

74. $\csc\left(\frac{4\pi}{3}\right)$

75. $\sec\left(\frac{2\pi}{3}\right)$

76. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

77. $\arctan(1)$

78. $\arccos\left(\frac{\sqrt{3}}{2}\right)$

79. $\cos^{-1}\left(-\frac{1}{2}\right)$

80. $\arcsin\left(-\frac{1}{2}\right)$

81. Prove the identity: $\frac{\sin x}{1-\cos x}+\frac{\sin x}{1+\cos x}=2\csc x$

82. Prove the identity: $\cos^2 x-\sin^2 x=1-2\sin^2 x$

83. Prove the identity: $\frac{\sin x}{\cos^2 x}=\sec x\tan x$

You should be able to use composite function notation and be able to find inverse functions.

84. If $(f\circ g)(x)=24x+7$ and $f(x)=6x+13$, find $g(x)$.

85. If $f(x)=x^2+2x-1$ and $g(x)=2x-3$, find each of the following functions:

a. $f\circ g$ b. $g\circ f$

You should be able to sketch functions without a graphing calculator. The functions you need to know include: $y=x^2$, $y=x^3$, $y=\sqrt{x}$, $y=2^x$, $y=e^x$, $y=\ln x$, $y=|x|$, $y=\frac{1}{x}$. You also need to be able to shift these graphs as shown in the examples below.

86A. Without using a calculator, make a rough sketch of each function below. Label all important features of each graph.

a. $y=x^3$ b. $y=(x+1)^3$ c. $y=(x-2)^3+3$ d. $y=4-x^2$ e. $y=\sqrt{x}$

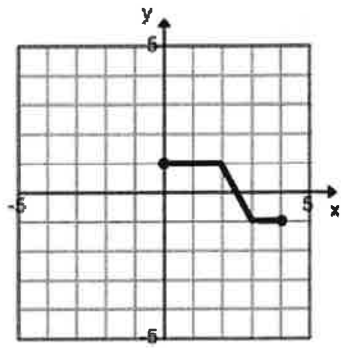
f. $y=2\sqrt{x}$ g. $y=-2^x$ h. $y=\frac{1}{x}+1$ i. $y=\ln x$ j. $y=\ln(x-2)$

86B. In addition to graphing without a calculator, you also need to be able to evaluate functions without a calculator. Specifically, you should know the following:

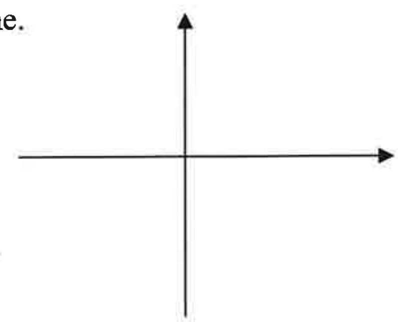
Given $f(x) = \ln x$, without a calculator, find:

a. $f(1)$ b. $f(e)$ c. $f(e^2)$

87. Given the graph of $f(x)$ below, sketch the graph of $f(x+4)+3$ on the same set of axes.



88. Sketch a graph of $f(x) = e^x$. Then on the same set of axes, sketch a graph of $f^{-1}(x)$ as a dashed line.



89. Sketch a graph of $y = 2\cos x$

You should be comfortable with all aspects of piecewise functions, including graphing and evaluating them.

90. Graph the piecewise function:

$$f(x) = \begin{cases} 3x^2, & x \le -1 \\ 3, & -1 < x \le 1 \\ 3x+1, & x > 1 \end{cases}$$

91. Let $f(x)=\begin{cases}1-x^2 & \text{if } x\le 0\\ 2x+1 & \text{if } x>0\end{cases}$

a. Evaluate $f(-2)$ and $f(1)$.

b. Sketch a graph of f.

You should be able to rationalize a binomial denominator by using a conjugate pair.

Rationalize the following (means get the radical out of the denominator):

92. $\dfrac{5}{2-\sqrt{x}}$

93. $\dfrac{3}{5-\sqrt{x+1}}$

94. $\dfrac{8}{6+\sqrt{2x+4}}$

MIXED REVIEW:

95. Find the inverse of $f(x)=5x^3-1$

96. The graph of a function f is given to the right.

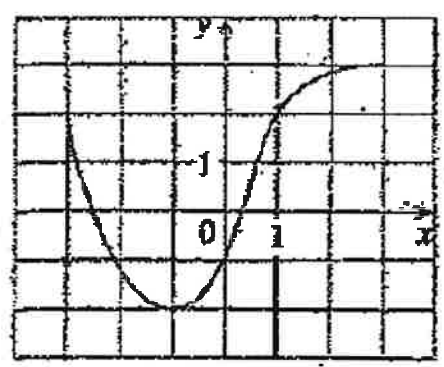
a. State the value of $f(-1)$.

b. Estimate the value of $f(2)$.

c. For what values of x is $f(x)=2$?

d. Estimate the values of x such that $f(x)=0$.

e. State the domain and range of f



97. Simplify: $\dfrac{9}{\dfrac{9}{x}+\dfrac{2}{3x}}$

98. Simplify: $\dfrac{\dfrac{25}{12}+\dfrac{x+1}{4}}{\dfrac{1}{18}-\dfrac{x+1}{36}}$

99. Simplify: $\dfrac{\frac{a}{b}-\frac{b}{a}}{\frac{1}{a}+\frac{1}{b}}$

100. Given $f(x)=5x^2-4x+3$, evaluate and simplify $f(3-x)$

101. Given $f(x)=\dfrac{\frac{3}{x}+7}{\frac{x}{2}}$, evaluate and simplify $f(x+1)$

102. Given $f(x)=3x^2-6x+5$

a.) evaluate and simplify $f(x+h)$

b.) evaluate and simplify $f(x+h)-f(x)$

103. Prove the following identity: $\tan\theta+\cot\theta=\sec\theta\csc\theta$

104. a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ c) $\sin^{-1}\left(\sin\left(\frac{3\pi}{2}\right)\right)$

105. Sketch a graph of $y=\ln(x+5)$ then use your graph to give the domain and range.

106. Given $f(x)=\begin{cases} x^2+1, & x<2 \\ 5-x, & 2\le x\le 6 \\ -1, & x>6 \end{cases}$ a. find $f(0)$ b. find $f(2)$ c. sketch a graph of $f(x)$

107. Solve: $4e^{x-8}-5=10$

108. Solve: $\ln\left(\frac{5}{x}\right)+2\ln x=3$

109. Solve: $3x^{\frac{1}{3}}+9=3$

110. Solve the system: $\begin{aligned} y&=e^{x+4} \\ \ln y&=6+2x \end{aligned}$

ANSWERS:

1. $\frac{5-a}{a}$ 2. $\frac{2x}{5x+20}$ 3. $\frac{4x-12}{5x}$ 4. $\frac{x^2-x-1}{x^2+x+1}$ 5. $\frac{x-4}{3x^2-4x+32}$

6. $\frac{2-5x}{3x+4x^3}$ 7. $20x$ 8. $125x^{\frac{26}{15}}$ 9. $\frac{1}{r^{\frac{1}{5}}}=\frac{1}{\sqrt[5]{r}}$ 10. b^2 11. $y^{\frac{1}{2}}=\sqrt{y}$ 12. $\frac{2y^3}{3x^4}$

13. $\frac{1}{25}$ 14. $-\frac{1}{16}$ 15. $\frac{-h}{x^2+xh}$ 16. $\frac{15}{4}$ 17. $\frac{x}{5}$ 18. $x(2x+5)(3x+4)$

19. $7(2x+1)(x-4)$ 20. $3x\sqrt{x}$ 21. $8\sqrt{a^5}$ 22. $\frac{-1}{x+5}$ 23. $\frac{x^2-2x+12}{(x-4)(x+4)(x+1)}$

24. 5 25. 17 26. $2t+3$ 27. 15 28. $8m^2+40m+49$ 29. 2 30. 1

31. $\frac{\sqrt{3}}{2}$ 32. 15 33. $(2x+3)^2$ 34. $2x^6+3$ 35. 9 36. -2

37. $\left(\frac{5}{2},0\right)$ and $(0,-5)$ 38. $(1,0)$ and $(-2,0)$ and $(0,-2)$

39. $(0,0)$ and $(-4,0)$ and $(4,0)$ 40. $(0,0)$ and $(-2,0)$ and $(2,0)$

41. $(3,5)$ 42. $(-1,5)$ and $(2,2)$ 43. Domain: $(-\infty,\infty)$ Range: $[-5,\infty)$

44. Domain: $[-3,\infty)$ Range: $(-\infty,0]$ 45. Domain: $(-\infty,\infty)$ Range: $[-3,3]$

46. Domain: $(-\infty,1)\cup(1,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$ 47. $f^{-1}(x)=\frac{x-1}{2}$

48. $f^{-1}(x)=\sqrt[3]{2x}$ 49. $y+5=7(x-3)$

50. $y-2=-\frac{1}{3}(x-9)$ or $y-6=-\frac{1}{3}(x+3)$ (Note, if multiplied out, these two equations are equivalent to $y=-\frac{1}{3}x+5$)

51. $y-8=\frac{5}{6}(x-2)$ 52. $y=\frac{5}{7}x-\frac{40}{7}$ 53. $x=5$ 54. $y=2$

55. $y-3=-\frac{3}{2}x$ 56. $x=1\pm\sqrt{13}$ 57. $x=11$ 58. $x=-4\pm 3\sqrt{2}$

59. $x=\frac{1}{4},\frac{5}{4}$ 60. $x=-\frac{69}{4}$ 61. $x=\frac{\ln 9}{\ln 8.76}=\frac{\log 9}{\log 8.76}$ 62. 48 63. 10 64. 1/8

65. $2+e^5$ 66. $\ln 6$ 67. $\frac{\ln 4}{-2}=\ln\left(\frac{1}{2}\right)$ 68. $24\sqrt{3}$ 69. $a=-4,\ b=3$

70. $x=\ln 7$ 71. $-\frac{\sqrt{3}}{2}$ 72. $-\frac{\sqrt{2}}{2}$ 73. $\sqrt{3}$ 74. $-\frac{2\sqrt{3}}{3}$ 75. -2 76. $-\frac{\pi}{4}$

77. $\frac{\pi}{4}$ 78. $\frac{\pi}{6}$ 79. $\frac{2\pi}{3}$ 80. $-\frac{\pi}{6}$

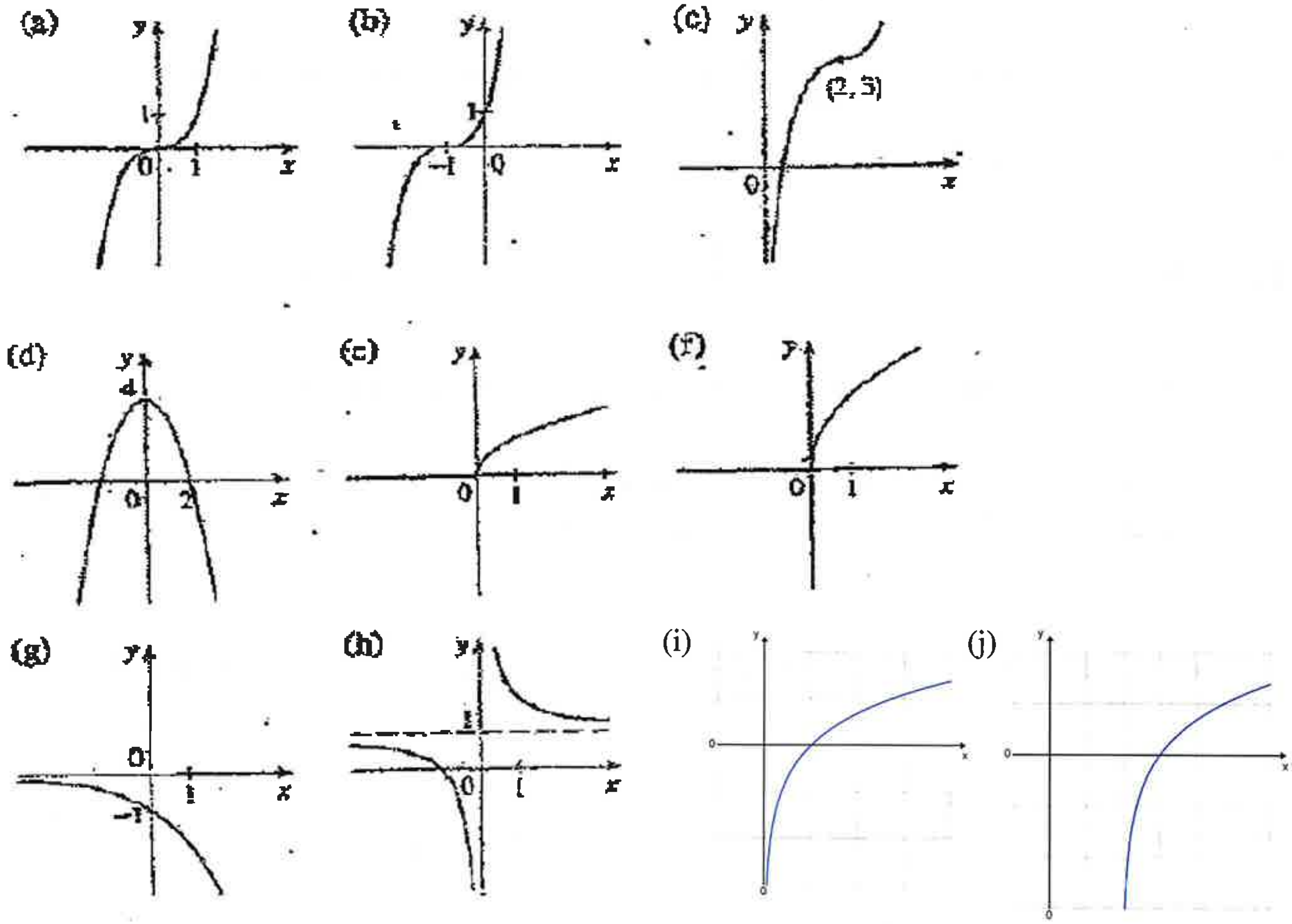
81. $\frac{\sin x}{1-\cos x}+\frac{\sin x}{1+\cos x}=\frac{\sin x(1+\cos x)+\sin x(1-\cos x)}{(1-\cos x)(1+\cos x)}=\frac{2\sin x}{1-\cos^2 x}=\frac{2\sin x}{\sin^2 x}=2\csc x$

82. $\cos^2 x-\sin^2 x=(1-\sin^2 x)-\sin^2 x=1-2\sin^2 x$

83. $\frac{\sin x}{\cos^2 x}=\frac{\sin x}{\cos x\cdot\cos x}=\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}=\sec x\tan x$

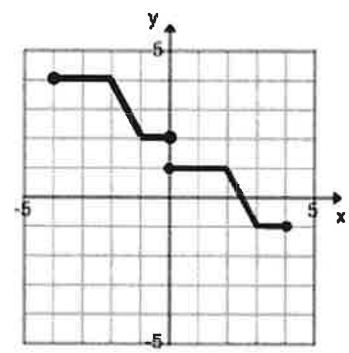
84. $g(x)=4x-1$ 85. (a) $(f\circ g)(x)=4x^2-8x+2$
(b) $(g\circ f)(x)=2x^2+4x-5$

86A.



86B. a. 0 b. 1 c. 2

87.



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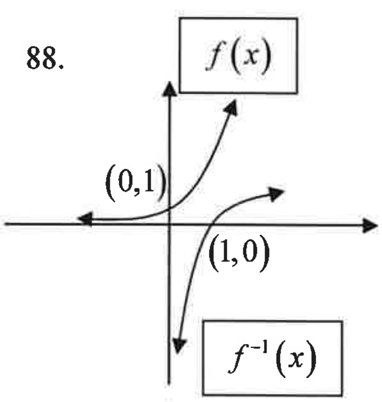

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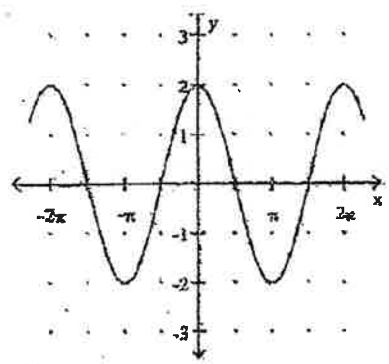

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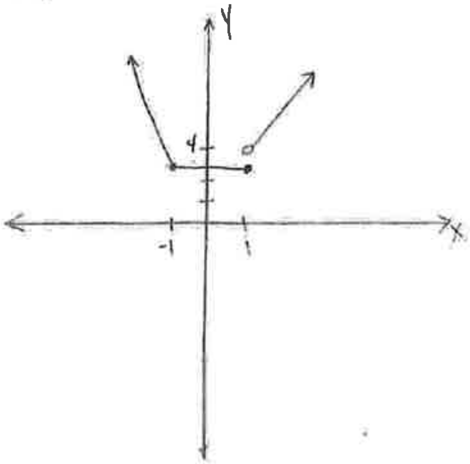
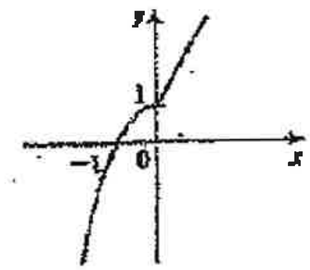


91. (a) $-3, 3$

(b)



92. $\dfrac{10+5\sqrt{x}}{4-x}$ 93. $\dfrac{15+3\sqrt{x+1}}{24-x}$ 94. $\dfrac{24-4\sqrt{2x+4}}{16-x}$

95. $f^{-1}(x)=\sqrt[3]{\dfrac{x+1}{5}}$

96. (a) -2 (b) 2.8
(c) $-3, 1$ (d) $-2.5, 0.3$
(e) $[-3, 3], [-2, 3]$

97. $\dfrac{27x}{29}$ 98. $\dfrac{84+9x}{1-x}$ 99. $a-b$

100. $5x^2-26x+36$ 101. $\dfrac{14x+20}{x^2+2x+1}$

102. a. $3x^2+6xh+3h^2-6x-6h+5$ b. $6xh+3h^2-6h$

104. a. $\dfrac{\pi}{3}$ b. $-\dfrac{\pi}{3}$ c. $-\dfrac{\pi}{2}$ 105. Domain: $(-5,\infty)$, Range: $(-\infty,\infty)$

106. a. 1 b. 3 107. $x=8+\ln\left(\dfrac{15}{4}\right)$ 108. $\dfrac{e^3}{5}$ 109. -8

110. $x=-2$ $y=e^2$

Limits

Section Art by HH Fisher

Circuit Training – Limits   Name ______________________________

Directions: Beginning in cell #1, use analytic techniques to evaluate the limit. Do not use technology! Hunt for your answer, call that problem #2, and advance in this manner until you complete the circuit.

Answer: 3 # 1 $\lim_{y\to 2}\frac{y^2+5y+6}{y+3}$	Answer: -1 # ____ $\lim_{x\to\infty} e^{-x}cosx$
Answer: 12 # ____ $\lim_{w\to 0}\frac{\frac{1}{2+w}-\frac{1}{2}}{w}$	Answer: $-\infty$ # ____ $\lim_{x\to 5^-}\frac{\lvert x-5\rvert}{3x-15}$
Answer: $+\infty$ # ____ Given $f(x)=\begin{cases}\lvert x\rvert, & x<0\\ x^2-3, & x\geq 0\end{cases}$, find $\lim_{x\to 0}f(x)$	Answer: 2a # ____ $\lim_{x\to -3}(x-5)^{\frac{4}{3}}$
Answer: 0 # ____ $\lim_{s\to 16}\frac{\sqrt{s}-4}{s-16}$	Answer: 1 # ____ $\lim_{x\to 0}\frac{sin2x}{x}$
Answer: 4 # ____ $\lim_{x\to -\frac{1}{3}}6x^2(1-3x)$	Answer: DNE (function is not defined from the left) # ____ $\lim_{w\to\infty}\frac{3w^4-2}{(w^2-1)(2w^2+7)}$

Answer: $\frac{\sqrt{3}}{2}$ # ____ $\lim_{\theta\to 3}\sqrt{\theta-3}$	Answer: $-\frac{1}{4}$ # ____ $\lim_{t\to 3}\frac{t^2-2t-3}{t^2-9}$
Answer: $\frac{1}{2}$ # ____ $\lim_{h\to 0}\frac{(3+h)^2-9}{h}$	Answer: -3 # ____ Given $f(x)=\begin{cases}\lvert x\rvert, & x<0\\ x^2-3, & x\geq 0\end{cases}$, find $\lim_{x\to 2}f(x)$
Answer: $\frac{1}{8}$ # ____ $\lim_{n\to -\infty}\frac{5n^3-2n^2+7}{2n^3+3n+1}$	Answer: $\frac{3}{2}$ # ____ $\lim_{x\to 0}\frac{9tanx}{3sinx}$
Answer: $\frac{2}{3}$ # ____ Given $f(x)=\begin{cases}\lvert x\rvert, & x<0\\ x^2-3, & x\geq 0\end{cases}$, find $\lim_{x\to o^+}f(x)$.	Answer: $\frac{4}{3}$ # ____ $\lim_{x\to a}\sqrt{3a^2+x^2}, a\geq 0$
Answer: 16 # ____ $\lim_{x\to -1}\frac{x+1}{1-x^2}$	Answer: 6 # ____ $\lim_{a\to -2}\frac{a^3+8}{a+2}$
Answer: 2 # ____ $\lim_{x\to 3^-}\frac{x}{3-x}$	Answer: $\frac{5}{2}$ # ____ $\lim_{n\to 10^-}\ln(100-n^2)$
Answer: $-\frac{1}{3}$ # ____ $\lim_{h\to 0}\frac{\sin(\frac{\pi}{6}+h)-\sin(\frac{\pi}{6})}{h}$	Answer: DNE (limit from the left does not equal the limit from the right) # ____ $\lim_{x\to 0}\frac{sinx}{3x^2-x}$

Circuit Training – Limits and Continuity REVIEW Name ______________________

Beginning in cell #1, work the problem. Circle your answer. Hunt for your answer and make that problem #2. Continue in this manner until you complete the circuit. ☺

NOTE! Use separate paper and turn in any additional sheets used!

# 1 $\lim_{x \to 7} \frac{x-7}{x^2-49}$	Answer: DNE # ______ The function $f(x) = \tan x$ is (a) continuous on its domain. (b) not continuous on its domain.
Answer: -10 # ______ $\lim_{x \to 0}(e^x \cos x)$	Answer: 0 # ______ The function $g(x) = \frac{2-3x^2}{2-2x^2}$ has one horizontal asymptote. It is y = ______.
Answer: $y = 6x + 39$ # ______ Write an equation of a function with a removable discontinuity at x=4, a vertical asymptote at x=5, and a horizontal asymptote of y = 0.	Answer: $\frac{1}{14}$ # ______ $\lim_{h \to 0} \frac{2-\sqrt{4+h}}{h}$

Answer: $-\frac{1}{4}$ # ______ The function $f(x) = \frac{x-7}{x^2-49}$ has only one vertical asymptote. It is at x = ______.	Answer: $\frac{1}{2}$ # ______ $\lim_{x\to 0} \frac{\lvert x\rvert}{x}$
Answer: 1 # ______ $\lim_{w\to\sqrt{5}}(5-\sqrt{9-w^2})$	Answer: (b) # ______ $\lim_{x\to -3^-} \frac{\lvert x+3\rvert}{x+3}$
Answer: -7 # ______ The function $p(x) = \frac{2x+3}{4x^2-9}$ has a removable discontinuity when $x =$ ____.	Answer: $-\frac{1}{6}$ # ______ $\lim_{x\to\infty}\left(\frac{3}{x^2}+\frac{2}{x}+4\right)$

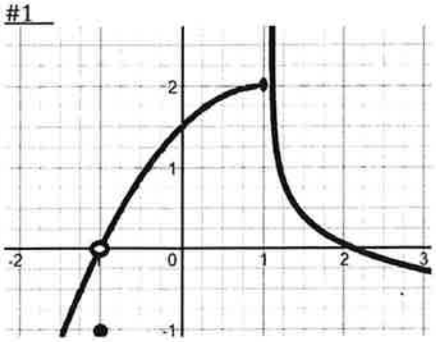
Answer: (a) # _____ Calculate the instantaneous rate of change for the function $f(x) = \frac{2}{3}x + 5$ when $x = -7$.	Answer: $-\frac{3}{2}$ # ______ The function $w(x) = \sqrt{2x+3}$ has domain: ______________
Answer: $\frac{3}{2}$ # ______ The function $f(x) = 2 + e^x$ has an asymptote at ________.	Answer: (c) # ______ $\lim_{x \to 4^+}\left(\frac{x+3}{16-x^2}\right)$
Answer: 4 # ______ Write the equation of the tangent line to $f(x) = \sqrt{3-x}$ at $x = -6$.	Answer: $[-\frac{3}{2}, \infty)$ # ______ $\lim_{x \to \infty} \frac{x-7}{x^2-49} =$
Answer: $y = -\frac{1}{6}x + 2$ # _____ Write the equation of the normal line to $f(x) = \sqrt{3-x}$ at $x = -6$.	Answer: $-\frac{1}{9}$ # _____ Find b to make the function continuous over its domain. $f(x) = \begin{cases} x+5, & x < 3 \\ 2x^2 + b, & x \geq 3 \end{cases}$

Answer: 3 # _____ Sketch a graph of the function: $\lim_{x\to 3} f(x) = 2$ $f(3) = 4$ $\lim_{x\to\infty} f(x) = -\infty$ $\lim_{x\to-\infty} f(x) = -\infty$ Which is true? (a) The function is continuous at x = 3. (b) The function has a non-removable discontinuity at x=3. (c) The function has a removable discontinuity at x=3.	Answer: 6 # ______ What value for a will make $f(x) = \begin{cases} \frac{\sin x}{2x} & , x \neq 0 \\ a & , x = 0 \end{cases}$ a continuous function?
Answer: $y = 2$ # ______ The greatest integer function, $g(x) = [\![x]\!]$ is (choose one)... (a) continuous on its domain. (b) not continuous on its domain.	Answer: 11 # ______ Find the instantaneous rate of change for the function $f(x) = \sqrt{3-x}$ at $x = -6$.
Answer: -2 # ______ What is the slope of the tangent line (i.e. instantaneous rate of change) for $g(x) = x^2$ at $x = 3$?	Answer: $-\infty$ # ______ What value should k be to make $g(x) = \begin{cases} \frac{3x^2-x-10}{x-2} & , x \neq 2 \\ k , & x = 2 \end{cases}$ continuous?
Answer: $\frac{2}{3}$ # _____ Calculate the instantaneous rate of change for the function $f(x) = \frac{1}{x}$ when $x = 3$.	Answer: -1 # _____ Find the average rate of change for $f(x) = -x^2$ over the closed interval [-1, 3].

Circuit Training – Limits Name________________________________

Directions: Beginning in the first cell marked #1, find the requested information. To advance in the circuit, hunt for your answer and mark that cell #2. Continue working in this manner until you complete the circuit.

Ans: ∞
#1__



Find $f(-1)$.

Ans: 0
#___

$$\lim_{x\to 0}\frac{\frac{1}{2+x}-\frac{1}{2}}{x}$$

Ans: DNE (and not ∞ or $-\infty$)
#___

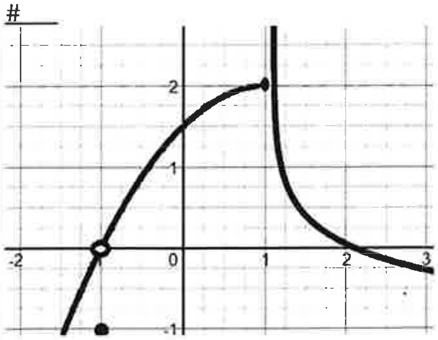
$$f(x)=\frac{x^3-4x^2+3x-12}{x^2-6x+8}$$

$f(x)$ has a hole at $x = ?$.

Ans: 0.249
#___

$$\lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{3}}{x}$$

Ans: 3
#___



Find $\lim_{x\to 1} f(x)$

Ans: -1
#___

$$\lim_{x\to 1}\frac{x^3-1}{x^2+x-2}$$

Ans: 2
#___

$$\lim_{x\to 1^-}\frac{x}{x^2-1}$$

Ans: -2
#___

$$\lim_{h\to 0}\frac{(4+h)^2-3(4+h)-4}{h}$$

Ans: -¼

#___

$$f(x) = \frac{x^2 - 5x + 6}{x^2 + 2x - 15}$$

$f(x)$ has a vertical asymptote at x = ?.

Ans: 4

#___

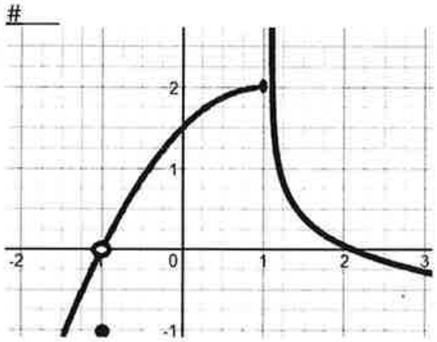
Is $f(x) = \begin{cases} \cos x, x < 0 \\ x^2 + 1, x \geq 0 \end{cases}$ continuous at $x = 0$?

If yes, it is continuous, then go find the Ans: -2.

If no, it is not continuous, then go find the Ans: 5.

Ans: -∞

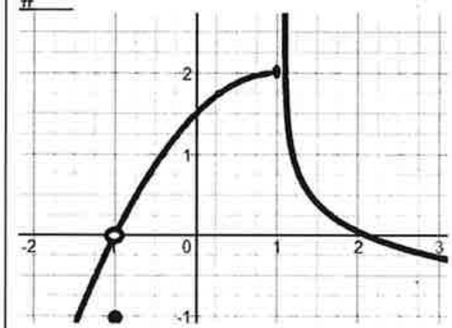
#___



Find $\lim_{x \to 1^+} f(x)$

Ans: 5

#___



Find $f(1)$.

Ans: 0.289

#___

Is $f(x) = \begin{cases} x, x \leq 1 \\ 2x - 3, x > 1 \end{cases}$ continuous at $x = 1$?

If yes, it is continuous, then go find the Ans: 3.

If no, it is not continuous, then go find the Ans: -4.

Ans: -5

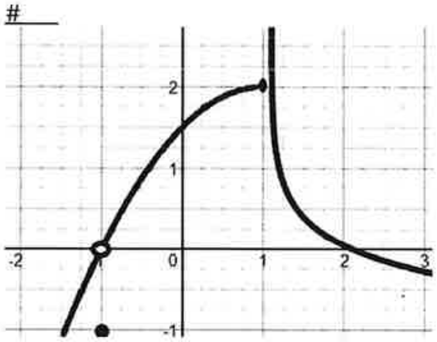
#___

$$f(x) = \frac{x^2 - 5x + 6}{x^2 + 2x - 15}$$

$f(x)$ has a removable discontinuity at x = ?.

Ans: -4

#___



Find $\lim_{x \to -1} f(x)$

Ans: 1

#___

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	A	B	C	-	D	E	F

$\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$ Filling in the table above, what value would take the place of E? (Round to three places.)