Section 5: Math Review

The APES exam has a significant amount of math. Most students find that with a little practice, the math is not difficult, but as many of us have not had practice with setting up and solving problems with dimensional analysis in a long time, in the beginning it can be daunting. It is encouraged that you attend ONE lunch in the first week of the course to receive additional help, if needed.

In this class, it will be assumed that you are able to solve math problems using the following skills.

Scientific Notation

Scientific notation is the way that scientists easily handle very large numbers or very small numbers. For example, instead of writing 0.0000000056, we write 5.6×10^{-9}. So, how does this work?

We can think of 5.6×10^{-9} as the product of two numbers: 5.6 (the digit term) and 10^{-9} (the exponential term).

Here are some examples of scientific notation

$10000 = 1 \times 10^4$	$24327 = 2.4327 \times 10^4$
$1000 = 1 \times 10^3$	$7354 = 7.354 \times 10^3$
$100 = 1 \times 10^2$	$482 = 4.82 \times 10^2$
$10 = 1 \times 10^1$	$89 = 8.9 \times 10^1$ (not usually done)
$1 = 10^0$	
$1/10 = 0.1 = 1 \times 10^{-1}$	$0.32 = 3.2 \times 10^{-1}$ (not usually done)
$1/100 = 0.01 = 1 \times 10^{-2}$	$0.053 = 5.3 \times 10^{-2}$
$1/1000 = 0.001 = 1 \times 10^{-3}$	$0.0078 = 7.8 \times 10^{-3}$
$1/10000 = 0.0001 = 1 \times 10^{-4}$	$0.00044 = 4.4 \times 10^{-4}$

As you can see, the exponent of 10 is the number of places the decimal point must be shifted to give the number in long form. A **positive** exponent shows that the decimal point is shifted that number of places to the right. A **negative** exponent shows that the decimal point is shifted that number of places to the left.

In scientific notation, the digit term indicates the number of significant figures in the number. The exponential term only places the decimal point. As an example,

$$46600000 = 4.66 \times 10^7$$

This number only has 3 significant figures. The zeros are not significant; they are only holding a place.

As another example, $0.00053 = 5.3 \times 10^{-4}$

This number has 2 significant figures. The zeros are only place holders.

How to do calculations with scientific notation:

Addition and Subtraction:

- All numbers are converted to the same power of 10, and the digit terms are added or subtracted.
- Example: $(4.215 \times 10^{-2}) + (3.2 \times 10^{-4}) = (4.215 \times 10^{-2}) + (0.032 \times 10^{-2}) = 4.247 \times 10^{-2}$
- Example: $(8.97 \times 10^4) - (2.62 \times 10^3) = (8.97 \times 10^4) - (0.262 \times 10^4) = 8.71 \times 10^4$

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Multiplication:

- The digit terms are multiplied in the normal way and the exponents are added. The end result is changed so that there is only one nonzero digit to the left of the decimal.
- Example: $(3.4 \times 10^{6})(4.2 \times 10^{3}) = (3.4)(4.2) \times 10^{(6+3)} = 14.28 \times 10^{9} = 1.4 \times 10^{10}$ (to 2 significant figures)
- Example: $(6.73 \times 10^{-5})(2.91 \times 10^{2}) = (6.73)(2.91) \times 10^{(-5+2)} = 19.58 \times 10^{-3} = 1.96 \times 10^{-2}$ (to 3 significant figures)

Division:

- The digit terms are divided in the normal way and the exponents are subtracted. The quotient is changed (if necessary) so that there is only one nonzero digit to the left of the decimal.
- Example: $(6.4 \times 10^{6})/(8.9 \times 10^{2}) = (6.4)/(8.9) \times 10^{(6-2)} = 0.719 \times 10^{4} = 7.2 \times 10^{3}$ (to 2 significant figures)
- Example: $(3.2 \times 10^{3})/(5.7 \times 10^{-2}) = (3.2)/(5.7) \times 10^{3-(-2)} = 0.561 \times 10^{5} = 5.6 \times 10^{4}$ (to 2 significant figures)

Powers of Exponentials:

- The digit term is raised to the indicated power and the exponent is multiplied by the number that indicates the power.
- Example: $(2.4 \times 10^{4})^{3} = (2.4)^{3} \times 10^{(4\times3)} = 13.824 \times 10^{12} = 1.4 \times 10^{13}$ (to 2 significant figures)
- Example: $(6.53 \times 10^{-3})^{2} = (6.53)^{2} \times 10^{(-3)\times2} = 42.64 \times 10^{-6} = 4.26 \times 10^{-5}$ (to 3 significant figures)

Percentage

- 17% = 17/100 = 0.17
- Remember that “percent” literally means divided by 100.
- Percentage is a measure of the part of the whole. Or part divided by whole.
- What is 20% of this $15 bill so that I can give a good tip? $15 x .20 = $15 x 20/100 = $3

Rates

- Percent change = (final-initial)/initial
- Rates will often be written using the word “per” followed by a unit of time, such as cases per year, grams per minute or miles per hour. The word per means to divide, so miles per gallon is the number of miles divided by one gallon.
- Rates are calculating how much an amount changes in a given amount of time.

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Percent Change

When a quantity grows (gets bigger), then we can compute it's **PERCENT INCREASE:**

$$\text{PERCENT INCREASE} = \frac{\text{new amount} - \text{original amount}}{\text{original amount}} \cdot 100\%$$

When a quantity shrinks (gets smaller), then we can compute it's **PERCENT DECREASE:**

$$\text{PERCENT DECREASE} = \frac{\text{original amount} - \text{new amount}}{\text{original amount}} \cdot 100\%$$

Note that when you compute percent increase or decrease, you always compare how much a quantity has changed to the **original** amount.

Dimensional Analysis

- You should be able to convert any unit into any other unit accurately if given the conversion factor.
  Example: 24 miles/gallon = how many kilometers/liter?

24 mi	1.6093 km	3.7854 gal	= <u>150 km</u> = 150 km/L
1 gal	1 mi	1 L	1L

- Online dimensional analysis tutorials are available:
  - https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:working-units/x2f8bb11595b61c86:rate-conversion/v/dimensional-analysis-units-algebraically
  - http://www.chem.tamu.edu/class/fyp/mathrev/mr-da.html
  - https://youtu.be/jrPSOrKUs6Q?si=eliCNB6tZM3ScVZb

Common Prefixes to Remember

m (milli)	= 1/1000	= 10^{-3}
c (centi)	= 1/100	= 10^{-2}
k (kilo)	= 1,000	= 10^{3}
M (mega)	= 1,000,000	= 10^{6}
G (giga)	= 1,000,000,000	= 10^{9}
T (tera)	= 1,000,000,000,000	= 10^{12}

***<u>DO NOT</u>* place this part of the section in your APES envelope. Keep this as a math reference sheet.**

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Complete each of the following problems including a detailed set up with labeled units and proper scientific notation. You must show all work to get credit. Must be handwritten.

***All problems should be expressed in scientific notation (do not write out large numbers with multiple zeros as place holders). If you need assistance with this, please refer to the provided reference materials listed above.*

Put the following numbers in scientific notation.

1) 0.003 = ________________
2) 1,530,000 = ________________
3) 0.00005 = ________________
4) 142 = ________________
5) 2020 = ________________

Write the following in standard notation *(convert from sn)*.

6) 1×10^{6} = _________________
7) 3.5×10^{2} = _________________
8) 4.5×10^{0} = _________________
9) 5.1×10^{-3} = _________________
10) 2×10^{1} =_________________

Solve the following.

11) $10^{3} \times 10^{4}$ = _________________
12) $10^{-1} \times 10^{5}$ = _________________
13) $10^{-2} \times 10^{-2}$ = _________________
14) $10^{-4} / 10^{-2}$ = _________________
15) $10^{2} / 10^{1}$ = _________________

Solve the following using scientific notation.

16) 0.003 x 0.0005

17) 0.000005 x 0.000006

18) 150,000,000 x 0.00005

19) 15 / 0.00015

20) 0.001 / .0000001

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Percentages: solve the following percentage problems and remember to show all work.

21) 10 is what percent of 1,000?

22) What is 30% of 3,000,000?

23) You start with 100 units and end with 150 units. What is the percent increase?

24) You start with 200 units. How many units would you have after a 75% decrease?

25) You use 1.000 kilowatts of power. You increase your usage by 30%. How many total kilowatts are you using?

26) A population starts the year with 1,000 residents. By the end of the year, 100 new babies are born. What is the percent increase for this population?

27) A fluorescent bulb uses 22 watts and gives off the same amount of light as a 100-watt regular bulb. What is the percentage in energy savings by switching to a fluorescent bulb?

Demensional Analysis Problems: Set up and solve the following problems using all units and showing all work. Conversion factors are included. Use scientific notation when appropriate.

28) There are 2.2 in 1 kilogram. How many pounds are found in 120 kilograms?

29) If there are 2.53 cm in 1 inch and there are 36 inches in 1 yard, how many centimeters are in one yard?

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30) There are 100 cm in 1 meter. How many yards are in 3 meters?

31) A coal-fired electric power plant produces 14 million kilowatt-hours (kWh) of electricity each day. Assume that an input of 10,000 BTUs of heat is required to produce an output of one kilowatt-hour of electricity. Calculate the number of BTUs of heat needed to generate the electricity produced by the power plant each day.

32) (Using the information in #31) Calculate the pounds of coal consumed by the power plant each day if one pound of coal yields 5,000 BTUs of heat.

33) There are 145 blades of grass in a square cm of lawn. Assuming the grass stand is even, how many blades of grass would be found in a lawn measuring 7 meters by 8 meters?

34) Your car gets 24 miles to the gallon and your friend's car gets 32 miles to the gallon. You decide to go on a road trip to Virginia Tech, which is 300 miles away. If gas costs $3 per gallon and you decide to split the gas money, how much money will you save by driving your friend's car?

Place this section in your APES envelope and mark completed on your checklist.