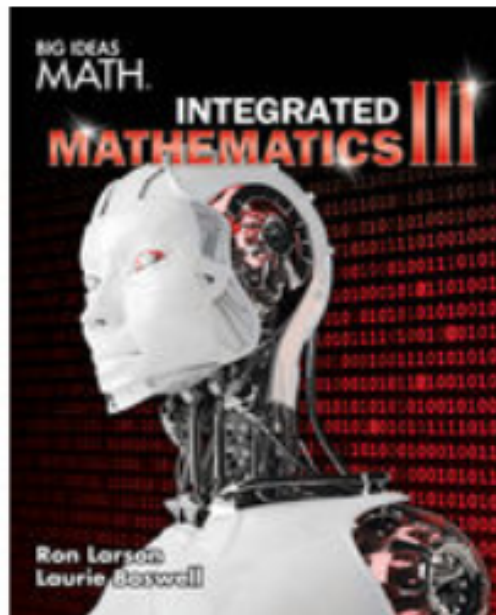


REQUIRED
Mathematics Summer Review Packet
For
ALL Students Entering
Integrated Math III HONORS

DUE DATE: SPECIFIED ON THE NEXT PAGE



Medford Public Schools
Department of Mathematics



Printed Copies Available:

If you prefer a paper copy, please don't hesitate to reach out before the school year ends. The Medford Public Library will have a limited number of paper copies available for students throughout the summer.

Medford Public Schools

489 Winthrop Street, Medford, Massachusetts 02155

Faiza I Khan

fkhan@medford.k12.ma.us

Director of Mathematics K - 12

781-393-2214

June 1, 2025

Dear Student/Parent/Caregiver:

Welcome to your Summer Mathematics Review Packet, which is **REQUIRED** for all incoming Integrated Math III Honors students. **You must complete this packet whether you completed an Honors or Standard Integrated Math II class and are scheduled to take Integrated Math III Honors.**

- If you are coming from an Integrated Math II Honors class, turn in your work on paper to your Integrated Math III Honors teacher on the 1st day of school.
- If you are coming from an Integrated Math II CP/Standard class, please send this completed packet scanned by Aug 15 to Director of Mathematics, Ms. Khan fkhan@medford.k12.ma.us. Please do not take pictures of the packet to be sent. You should scan all pages using a scanner.

To help students meet with success in Integrated Math III Honors, students are provided with this set of math problems. In today's environment of rigorous academic standards with the newly implemented Common Core, students need to understand core topics to progress in their mathematical learning. This **REQUIRED** packet is intended to help students retain some basic skills they have learned as well as help them gain some new skills.

Students, please note that all honors classes are fast-paced and not only cover more concepts but also cover concepts in greater depth than their College Prep (CP) counterparts. You will need to spend some time this summer preparing for the class. It is important to remember that maintaining academic integrity is incredibly important to your overall success in the course and life. You need to **LEARN** the material in this packet by watching the linked videos and completing the following practice problems. You are also expected to show your hand-written work completely in the space provided (or attach extra paper when needed). **Just providing answers is not sufficient.** You are expected to complete this packet **FULLY** by the first day of school, **DO NOT USE PHOTOMATH OR SIMILAR APPS** to complete this, as it will not prepare you conceptually and mentally for an honors-level math class. The expectations and rubric are on page 3 and 4 of this packet. If you do not understand any portion of the packet, please do not use any other means; plan to stay after school during the first two weeks with your teacher to learn and review.

Sincerely,

Faiza Khan

Director of Mathematics K - 12

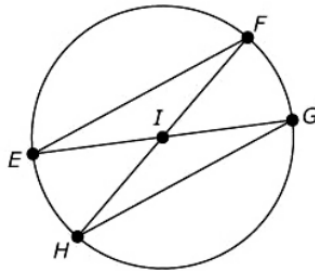
Medford Public Schools

Sample Question Solved

(Note the type of work shown in this problem. Make sure you show your work to this level)

In circle I , $\overline{EG}$ and $\overline{FH}$ are diameters, and $\overline{EF}$ and $\overline{GH}$ are parallel, as shown.

$$m\angle EIH = 42^\circ.$$



A) What is the measure, in degrees, of $\widehat{FG}$? Show or explain how you got your answer.

Sample Answer:

The measure is 42 degrees. I got my answer because the arc corresponding with a central angle is the same measure as the central angle, and $\angle FIG$ is 42 degrees (vertical angles) so its arc $\widehat{FG}$ is also 42 degrees.

B) What is the measure, in degrees, of $\widehat{EF}$? Show or explain how you got your answer.

Sample Answer:

$$\frac{(360 - 42 - 42)}{2} = 138, \text{ so } \widehat{EF} \text{ is } 138 \text{ degrees}$$

INTEGRATED MATH SUMMER ASSIGNMENT RUBRIC

	Conceptual Understanding	Mathematical Skills	Work Habits
4	- Shows complete conceptual understanding. - Demonstrate a strong ability to explain why and how different concepts work and apply in context.	- Includes relevant information effectively. - Answers correctly to all parts of the question.	- Work is very neat and well organized. - Support work is done thoroughly with proper mathematical notations, definitions, properties, and/or theorems. Unknown variables are clearly defined. Conclusions in context are very effective.
3	- Shows nearly complete conceptual understanding. - Demonstrate a moderate ability to explain why and how different concepts work and apply in context.	- Includes relevant information. - Answers correctly to most parts of the question.	- Work is neat and organized. - Support work is done with proper mathematical notations, definitions, properties, and/or theorems. Unknown variables are vaguely defined. Conclusions in context are somewhat effective.
2	- Shows some conceptual understanding. - Weak conceptual explanation and contextual response.	- Include some relevant information. - Answers correctly to some parts of the question.	- Work is barely readable and semi-organized. - Support work is partially done with little use of mathematical notations, definitions, properties, and/or theorems. Unknown variables are undefined. Conclusions in context are not effective.
1	- Shows little conceptual understanding. - Lack of explanation.	Answers incorrectly to all parts of the question.	- Work is sloppy and disorganized. - Support work is missing.
Score / Teacher's Comment			
Part 1			
Part 2			
Part 3			
Part 4			

Total Score: _____

Part 1: Number and Quantity - The Complex Number System

Part 1 **Videos to watch**



Sal explains what is the conjugate of a complex number, and shows how the product of a complex number and its conjugate is always a real number. Created by Sal Khan.

https://youtu.be/BZxZ_eEuJBM

Sal finds the conjugate of $(7-5i)$. Created by Sal Khan and Monterey Institute for Technology and Education.

<https://youtu.be/dbxJ6LD0344>

Sal divides $(6+3i)$ by $(7-5i)$. Created by Sal Khan and Monterey Institute for Technology and Education.

<https://youtu.be/Z8j5RDOibV4>

Sal finds the absolute value of $(3-4i)$. Created by Sal Khan.

<https://youtu.be/yvzyC4VBpUU>

Sal shows how to determine which members in a set of complex numbers have the same modulus (or absolute value). He also shows how to visualize all of the complex numbers with a given modulus as a circle centered at the origin on the complex plane, since all points on such a circle are the same distance from the origin. Created by Sal Khan.

<https://youtu.be/uB5QDraFefs>

Sal finds the modulus (which is the absolute value) and the argument (which is the angle) of $\sqrt{3}/2 + 1/2i$. Created by Sal Khan.

<https://youtu.be/FwuPXchH2rA>

Sal rewrites the complex number $-3+2i$ (which is given in rectangular form) in its polar form. Created by Sal Khan.

https://youtu.be/8RasCV_Lggg

Given a complex number in polar form, we can convert that number to rectangular form and plot it on the complex plane. Created by Sal Khan.

<https://youtu.be/auywa7dydAk>

We can multiply complex numbers graphically on the complex plane by rotating and scaling. Multiplying a complex number z by $-3i$ rotates and scales z . Created by Sal Khan.

<https://youtu.be/fqwR6RNPJgc>

We can multiply complex numbers graphically on the complex plane. We rotate an amount equal to the argument and scale by the modulus of the complex number by which we're multiplying. Created by Sal Khan.

<https://youtu.be/ebEwF4kb6pl>

We can multiply two complex numbers in polar form by multiplying their moduli and adding their arguments. Created by Sal Khan.

<https://youtu.be/VkdXztTFsvM>

We can divide two complex numbers in polar form by dividing their moduli and subtracting their arguments. Created by Sal Khan.

<https://youtu.be/lyWaNZ1ERMw>

Given a complex number in rectangular form, we can convert it to polar form to show how to visualize powers of the complex number by scaling and rotating it by its own modulus and argument. Created by Sal Khan.

<https://youtu.be/VZmnLQ3CMbE>

Sal finds all complex solutions to the equation $x^3=1$. Created by Sal Khan.

<https://youtu.be/N0Y8ia57C24>

Part 1
Problems to complete



For the next four questions, write the sum or difference in the standard form $a + bi$ without using a calculator.

$$(7 - 3i) + (6 - i)$$

$$(2 + i) - (9i - 3)$$

$$(2 - i) + (3 - \sqrt{-3})$$

$$(i^2 + 3) - (7 + i^3)$$

For the next four questions, write the product in standard form without using a calculator.

$$(1 - 4i)(3 - 2i)$$

$$(5i - 3)(2i + 1)$$

$$(7i - 3)(2 + 6i)$$

$$(\sqrt{-4} + i)(6 - 5i)$$

For the next four questions, solve the problem without using a calculator.

Multiple Choice Which of the following is the standard form for the product $(2 + 3i)(2 - 3i)$?

(A) $-5 + 12i$ (B) $4 - 9i$ (C) $13 - 3i$

(D) -5 (E) $13 + 0i$

Multiple Choice Which of the following is the standard form for the quotient $\frac{1}{i}$?

- (A) 1 (B) -1 (C) i (D) $-1/i$ (E) $0 - i$

Multiple Choice Assume that $2 - 3i$ is a solution of $ax^2 + bx + c = 0$, where a, b, c are real numbers. Which of the following is also a solution of the equation?

- (A) $2 + 3i$ (B) $-2 - 3i$ (C) $-2 + 3i$
(D) $3 + 2i$ (E) $\frac{1}{2 - 3i}$

Multiple Choice Which of the following is the standard form for the power $(1 - i)^3$?

- (A) $-4i$ (B) $-2 + 2i$ (C) $-2 - 2i$ (D) $2 + 2i$ (E) $2 - 2i$

Finding the Modulus of a Complex Number

Find the exact value of $|-8 + 2i|$.

Calculate $\left[2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)\right]^3$ and express the answer in simplest polar form with the argument from $0 \leq \theta < 2\pi$.

What is the conjugate of $-\frac{4}{5} + \frac{2}{5}i$?

Express as a complex number in simplest $a+bi$ form:

$$\frac{-26 - 14i}{3 + 10i}$$

Convert the complex number $0 + 6i$ into polar form. Express the angle using radians in terms of π over the interval $0 \leq \theta < 2\pi$.

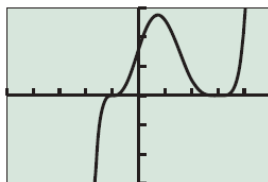
Convert the complex number $\sqrt{3}(\cos 180^\circ + i \sin 180^\circ)$ into rectangular (standard) form. Express your answer in simplest radical form.

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include those listed.

2, 3, and i

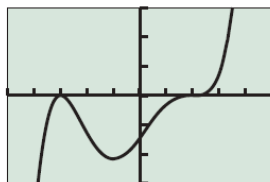
5 and $3 + 2i$

Match the polynomial function graph to the given zeros and multiplicities.



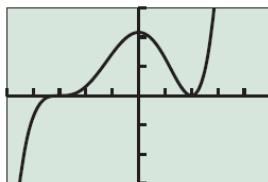
$[-5, 5]$ by $[-150, 150]$

(a)



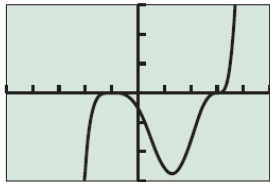
$[-5, 5]$ by $[-150, 150]$

(b)



$[-5, 5]$ by $[-150, 150]$

(c)



$[-5, 5]$ by $[-150, 150]$

(d)

-3 (multiplicity 2), 2 (multiplicity 3)

-3 (multiplicity 3), 2 (multiplicity 2)

-1 (multiplicity 4), 3 (multiplicity 3)

-1 (multiplicity 3), 3 (multiplicity 4)

Part 2: Algebra - Reasoning with Equations and Inequalities

Part 2 Videos to watch



Sal introduces the concept of an inverse matrix. Created by Sal Khan.

<https://youtu.be/iUQR0enP7RQ>

Sal shows why a matrix is invertible if and only if its determinant is not 0. Created by Sal Khan.

<https://youtu.be/UqyN7-tRS00>

Sal gives an example of how to find the inverse of a given 2x2 matrix. Created by Sal Khan.

<https://youtu.be/01c12NaUQDw>

Sal explains how we can find the inverse of a 3x3 matrix using Gaussian elimination. Created by Sal Khan.

https://youtu.be/obts_JDS6_Q

Sal finds the appropriate combination of two given vectors in order to obtain a third given vector. This is done by representing the problem with a single matrix equation and solving that equation. Created by Sal Khan.

<https://youtu.be/gsNgdVdAT1o>

Sal shows how matrices can be used to efficiently represent data about the prices of toilet paper and toothpaste in two different cities. Created by Sal Khan

<https://youtu.be/PPOiILhsT6s>

For the next two questions, write the augmented matrix corresponding to the system of equations.

$$\begin{aligned} 2x - 3y + z &= 1 \\ -x + y - 4z &= -3 \\ 3x - z &= 2 \end{aligned}$$

$$\begin{aligned} 2x - 5y + z - w &= -3 \\ x - 2z + w &= 4 \\ 2y - 3z - w &= 5 \end{aligned}$$

For the next two questions, write the system of equations corresponding to the augmented matrix.

$$\left[\begin{array}{ccc|c} 3 & 2 & -1 & \\ -4 & 5 & 2 & \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 2 & 0 & 1 & 3 & \\ -1 & 1 & 0 & 2 & \\ 0 & 2 & -3 & -1 & \end{array} \right]$$

For the next two questions, solve the system of equations by using an inverse matrix.

$$\begin{aligned} 2x - 3y &= -13 \\ 4x + y &= -5 \end{aligned}$$

$$\begin{aligned} 2x - y + z &= -6 \\ x + 2y - 3z &= 9 \\ 3x - 2y + z &= -3 \end{aligned}$$

Train Tickets

At the Pittsburgh zoo, children ride a train for 25 cents, adults pay \$1.00, and senior citizens 75 cents. On a given day, 1400 passengers paid a total of \$740 for the rides. There were 250 more children riders than all other riders. Find the number of children, adult, and senior riders.

Show work.

Investment

Monica receives an \$80,000 inheritance. She invests part of it in CDs (certificates of deposit) earning 6.7% APY (annual percentage yield), part in bonds earning 9.3% APY, and the remainder in a growth fund earning 15.6% APY. She invests three times as much in the growth fund as in the other two combined. How much does she have in each investment if she receives \$10,843 interest the first year?

Show work.

Part 3: Functions - Interpreting Functions and Building Functions

Part 3a **Videos to watch**



This algebra video tutorial explains how to identify the horizontal asymptotes and slant asymptotes of rational functions by comparing the degree of the numerator with the degree of the denominator of the rational expression. The equation of the slant asymptote can be determined using long division if the degree of the numerator exceeds the degree of the denominator by exactly 1. This algebra video tutorial contains plenty of examples and practice problems.

<https://youtu.be/kvhpu1TkSjl>

This series of videos will show you the different aspects of graphing a rational function.

<https://www.youtube.com/playlist?list=PL15323063DDE303DD>

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

<https://virtualnerd.com/common-core/hsf-functions/HSF-IF-interpreting-functions/C/7/7d/>

Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

<https://sites.google.com/a/gilmerschools.com/ghs---honors-advanced-algebra/home/unit-3-rational-equations-and-functions/lesson-4-8---graphing-rational-functions?tmpl=%2Fsystem%2Fapp%2Ftemplates%2Fprint%2F&showPrintDialog=1>

Part 3a
Problems to complete



Match the rational function with its graph. Identify the viewing window and the scale used on each axis.

$$f(x) = \frac{1}{x - 4}$$

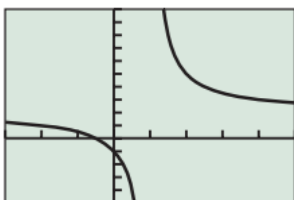
$$f(x) = -\frac{1}{x + 3}$$

$$f(x) = 2 + \frac{3}{x - 1}$$

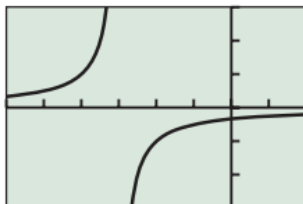
$$f(x) = 1 + \frac{1}{x + 3}$$

$$f(x) = -1 + \frac{1}{4 - x}$$

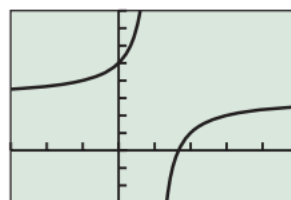
$$f(x) = 3 - \frac{2}{x - 1}$$



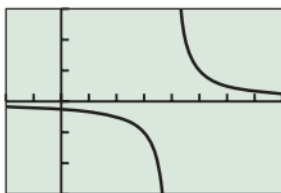
(a)



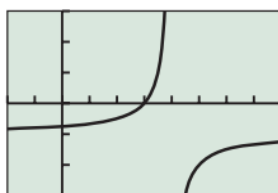
(b)



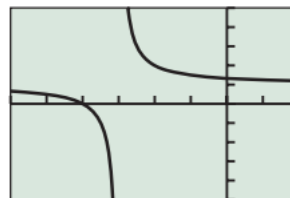
(c)



(d)



(e)



(f)

For the next two questions, find the intercepts, vertical asymptotes, and end behavior asymptote, and graph the function together with its end behavior asymptote.

$$h(x) = \frac{x^4 + 1}{x + 1}$$

$$h(x) = \frac{2x^3 - 3x + 2}{x^3 - 1}$$

Multiple Choice Let $f(x) = \frac{-2}{x^2 + 3x}$. What values of x have to be excluded from the domain of f ?

- (A) Only 0 (B) Only 3 (C) Only -3
(D) Only 0, 3 (E) Only 0, -3

Multiple Choice Let $g(x) = \frac{2}{x+3}$. Which of the transformations of $f(x) = \frac{2}{x}$ produce the graph of g ?

- (A) Translate the graph of f left 3 units.
(B) Translate the graph of f right 3 units.
(C) Translate the graph of f down 3 units.
(D) Translate the graph of f up 3 units.
(E) Vertically stretch the graph of f by a factor of 2.

Multiple Choice Let $f(x) = \frac{x^2}{x+5}$. Which of the following statements is true about the graph of f ?

- (A) There is no vertical asymptote.
(B) There is a horizontal asymptote but no vertical asymptote.
(C) There is a slant asymptote but no vertical asymptote.
(D) There is a vertical asymptote and a slant asymptote.
(E) There is a vertical and horizontal asymptote.

Part 3b
Videos to watch



This algebra video tutorial provides a basic introduction into composite functions. It explains how to evaluate composite functions.

<https://youtu.be/ZFPkQkURSxk>

This algebra video tutorial explains how to find the domain of a function that contains radicals, fractions, and square roots in the denominator using interval notation.

<https://youtu.be/djT6-YamHaA>

This video explains how to find the range of a function.

<https://youtu.be/Si2vmzUWfJE>

This precalculus video tutorial explains how to evaluate composite functions with three functions.

<https://youtu.be/Ote1KvW6BXw>

This precalculus video tutorial explains how to decompose a composite function into two functions $f(x)$ and $g(x)$.

<https://youtu.be/NmfaC7etZms>

Part 3b
Problems to complete



$f(x) = 2x - 3; g(x) = x + 1$ find $(f \circ g)(3)$ and $(g \circ f)(-2)$	$f(x) = x^2 + 4; g(x) = \sqrt{x + 1}$ find $(f \circ g)(3)$ and $(g \circ f)(-2)$
$f(x) = x^2 - 2; g(x) = \sqrt{x + 1}$ find $f(g(x))$ and $g(f(x))$. State the domain of each.	$f(x) = \frac{1}{x - 1}; g(x) = \sqrt{x}$ find $f(g(x))$ and $g(f(x))$. State the domain of each.

Weather Balloons

A high-altitude spherical weather balloon expands as it rises due to the drop in atmospheric pressure. Suppose that the radius r increases at the rate of 0.03 inch per second and that $r = 48$ inches at time $t = 0$. Determine an equation that models the volume V of the balloon at time t and find the volume when $t = 300$ seconds. Justify.





Part 4: Geometry - Expressing Geometric Properties with Equations;
Geometric Measurement and Dimension; Circles

Part 4a
Videos to watch



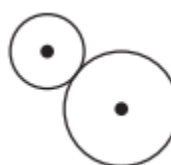
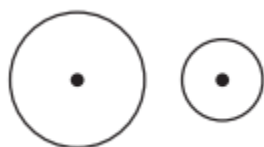
Construct a tangent line from a point outside a given circle to the circle.

<https://www.youtube.com/watch?v=YtGK1cA4EGI>

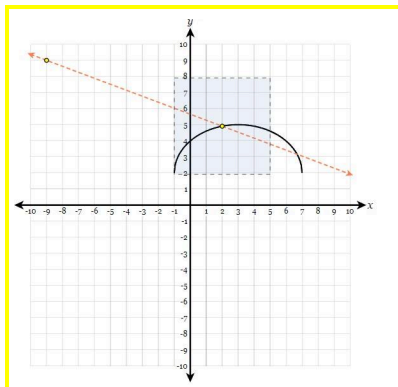
Part 4a
Problems to complete



For the next four questions, tell how many common tangents the circles have and draw them. State whether the tangents are external tangents or internal tangents.



Move the yellow dot on the upper left corner of the screen in order to make a line tangent to the semicircle when $x=2$. Construct the tangent line.



Part 4b

Videos to watch



Sal introduces the four conic sections and shows how they are derived by intersecting planes with cones in certain ways. Created by Sal Khan.

<https://youtu.be/0A7RR0oy2ho>

Sal introduces ellipses and shows how their standard equation relates to their center and radii (ellipses have two radii: major and minor). Created by Sal Khan and NASA.

<https://youtu.be/lvAYFUIEpFI>

Given an ellipse on the coordinate plane, Sal finds its standard equation, which is an equation in the form $(x-h)^2/a^2 + (y-k)^2/b^2 = 1$.

https://youtu.be/_JrQF8Rkaio

Given the standard equation of an ellipse, Sal finds the graph of the ellipse.

<https://youtu.be/h5dIVNjVjXg>

Sal explains how the radii and the foci of an ellipse relate to each other, and how we can use this relationship in order to find the foci from the equation of an ellipse. Created by Sal Khan.

<https://youtu.be/QR2vxfwiHAU>

Sal introduces the standard equation for hyperbolas, and how it can be used in order to determine the direction of the hyperbola and its vertices. Created by Sal Khan.

<https://youtu.be/pzSyOTkAsY4>

Sal picks the graph of $y^2/9 - x^2/4 = 1$ based on the hyperbola's center, direction, & vertices.
<https://youtu.be/oO3nWnJppqg>

Sal matches an equation to a given graph of a hyperbola, based on the hyperbola's direction & vertices.
<https://youtu.be/hnVFThmLW5Q>

Sal discusses the foci of hyperbolas and shows how they relate to hyperbola equations. Created by Sal Khan.
<https://youtu.be/S0Fd2Tg2v7M>

Sal proves why, for the general hyperbola equation $x^2/a^2 - y^2/b^2 = 1$, the focal length f forms the equation $f^2 = a^2 + b^2$ with the parameters a and b . Created by Sal Khan.
<https://youtu.be/HPRFmu7JsKU>

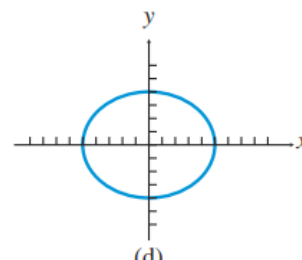
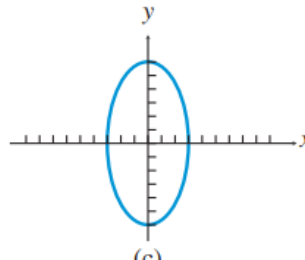
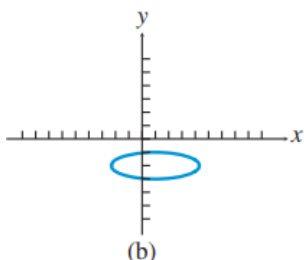
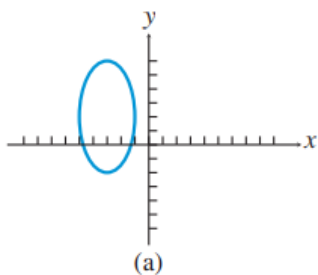
Sal analyzes the hyperbola whose equation is $(x-1)^2/16 - (y+1)^2/4 = 1$, and graphs it. Created by Sal Khan.
<https://youtu.be/IGQw-W1PxBE>

Part 4b

Problems to complete



For the next four questions, match the graph with its equation, given that the tick marks on all axes are 1 unit apart.



$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$\frac{y^2}{36} + \frac{x^2}{9} = 1$$

$$\frac{(y-2)^2}{16} + \frac{(x+3)^2}{4} = 1$$

$$\frac{(x-1)^2}{11} + (y+2)^2 = 1$$

For the next four questions, find an equation in standard form for the ellipse that satisfies the given conditions. Show work.

Foci $(\pm 2, 0)$, major axis length 10	Endpoints of axes are $(\pm 4, 0)$ and $(0, \pm 5)$
Major axis endpoints $(1, -4)$ and $(1, 8)$, minor axis length 8	Foci $(1, -4)$ and $(5, -4)$, major axis endpoints $(0, -4)$ and $(6, -4)$

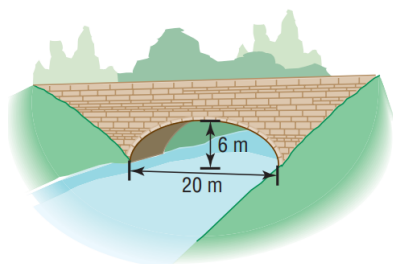
A Whispering Gallery

The whispering gallery in the Museum of Science and Industry in Chicago is 47.3 feet long. The distance from the center of the room to the foci is 20.3 feet. Find an equation that describes the shape of the room. How high is the room at its center? Show work.



Semielliptical Arch Bridge

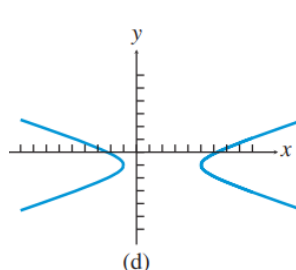
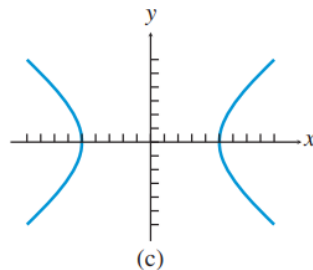
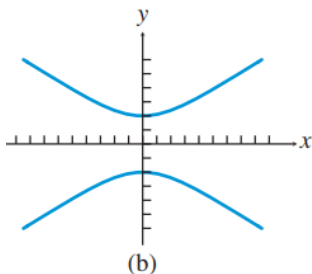
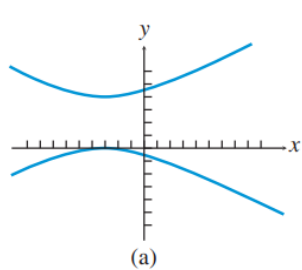
An arch in the shape of the upper half of an ellipse is used to support a bridge that is to span a river 20 meters wide. The center of the arch is 6 meters above the center of the river. See the figure. Write an equation for the ellipse in which the x-axis coincides with the water level and the y-axis passes through the center of the arch. Show work.



For the next three questions, you may use a graphing calculator to solve the problem.

Multiple Choice One focus of $x^2 + 4y^2 = 4$ is (A) $(4, 0)$. (B) $(2, 0)$. (C) $(\sqrt{3}, 0)$. (D) $(\sqrt{2}, 0)$. (E) $(1, 0)$.	Justify your answer.
Multiple Choice The focal axis of $\frac{(x - 2)^2}{25} + \frac{(y - 3)^2}{16} = 1$ is (A) $y = 1$. (B) $y = 2$. (C) $y = 3$. (D) $y = 4$. (E) $y = 5$.	Justify your answer.
Multiple Choice The center of $9x^2 + 4y^2 - 72x - 24y + 144 = 0$ is (A) $(4, 2)$. (B) $(4, 3)$. (C) $(4, 4)$. (D) $(4, 5)$. (E) $(4, 6)$.	Justify your answer.

For the next four questions, match the graph with its equation



$$\frac{x^2}{25} - \frac{y^2}{16} = 1$$

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

$$\frac{(x-2)^2}{9} - (y+1)^2 = 1$$

$$\frac{(y-2)^2}{4} - \frac{(x+3)^2}{16} = 1$$

For the next four questions, find an equation in standard form for the hyperbola that satisfies the given conditions. Show work.

Foci $(\pm 3, 0)$, transverse axis length 4

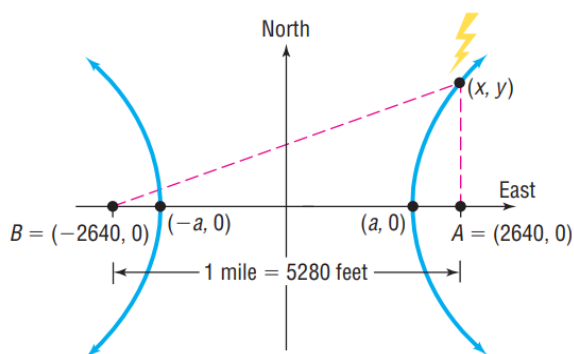
Foci $(0, \pm 15)$, transverse axis length 8

Transverse axis endpoints $(2, 3)$ and $(2, -1)$, conjugate axis length 6

Foci $(-4, 2)$ and $(2, 2)$, transverse axis endpoints $(-3, 2)$ and $(1, 2)$

Lightning Strikes

Suppose that two people standing 1 mile apart both see a flash of lightning. After a period of time, the person standing at point A hears the thunder. One second later, the person standing at point B hears the thunder. If the person at B is due west of the person at A and the lightning strike is known to occur due north of the person standing at point A, where did the lightning strike occur? Justify.

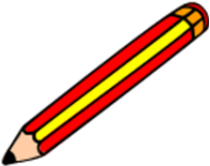


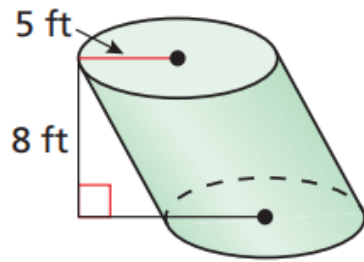
Fireworks Display

Suppose that two people standing 2 miles apart both see the burst from a fireworks display. After a period of time, the first person standing at point A hears the burst. One second later, the second person standing at point B hears the burst. If the person at point B is due west of the person at point A and if the display is known to occur due north of the person at point A, where did the fireworks display occur? Justify your answer.

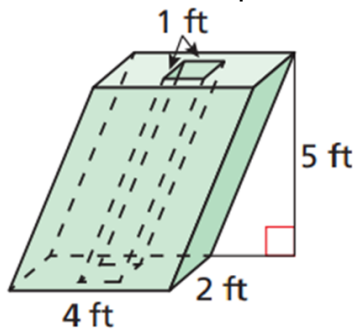
For the next four questions, you may use a graphing calculator to solve the problem.	
Multiple Choice One focus of $x^2 - 4y^2 = 4$ is (A) (4, 0). (B) $(\sqrt{5}, 0)$. (C) (2, 0). (D) $(\sqrt{3}, 0)$. (E) (1, 0).	Justify your answer.
Multiple Choice The focal axis of $\frac{(x + 5)^2}{9} - \frac{(y - 6)^2}{16} = 1$ is (A) $y = 2$. (B) $y = 3$. (C) $y = 4$. (D) $y = 5$. (E) $y = 6$.	Justify your answer.
Multiple Choice The center of $4x^2 - 12y^2 - 16x - 72y - 44 = 0$ is (A) (2, -2). (B) (2, -3). (C) (2, -4). (D) (2, -5). (E) (2, -6).	Justify your answer.
Multiple Choice The slopes of the asymptotes of the hyperbola $\frac{x^2}{4} - \frac{y^2}{3} = 1$ are (A) ± 1. (B) $\pm 3/2$. (C) $\pm \sqrt{3}/2$. (D) $\pm 2/3$. (E) $\pm 4/3$.	Justify your answer.

Part 4c Videos to watch	
Cavalieri's principle in 2D (Reading) https://www.khanacademy.org/math/geometry/hs-geo-solids/xff63fac4:hs-geo-cavalieri-s-principle/a/cavalieri-s-principle-in-2d	
Cavalieri's principle in 3D (Reading) https://www.khanacademy.org/math/geometry/hs-geo-solids/xff63fac4:hs-geo-cavalieri-s-principle/a/cavalieri-s-principle-in-3d	
Cavalieri's principle (Video with Explanation) https://www.khanacademy.org/math/geometry/hs-geo-solids/xff63fac4:hs-geo-cavalieri-s-principle/a/cavalieri-s-principle-in-2d	
The volume of a pyramid is a fraction of the volume of the rectangular prism that encloses it. We can find out what that fraction is by cutting a prism into several pyramids. Created by Sal Khan. https://youtu.be/fD52yzLF8DA	
Use Cavalieri's Principle to establish that the formula for the volume of a cone is the same as the formula for the volume of a pyramid ($\frac{1}{3} \times \text{base area} \times \text{height}$). Created by Sal Khan. https://youtu.be/LmBVIMoeYXg	
Use related volumes (using the volume of one figure to determine the volume of another). Created by Sal Khan. https://youtu.be/knNLHsjlcl8	

Part 4c Problems to complete	
Find the volume of the cylinder.	Show work here:

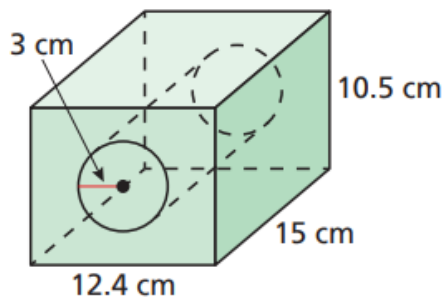


Find the volume of the composite solid.



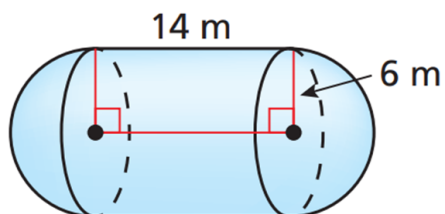
Show work here:

A cylindrical hole is drilled through a wooden block that is in the shape of a rectangular prism. Find the volume of the resulting solid.



Show work here:

Find the volume of the composite solid.



Show work here:

MODELING WITH MATHEMATICS

Three tennis balls are stored in a cylindrical container with a height of 8 inches and a radius of 1.43 inches. The circumference of a tennis ball is 8 inches.



a. Find the volume of a tennis ball.

Show work.

b. Find the amount of space within the cylinder not taken up by the tennis balls.

Show work.