AP Calculus AB
Unit 1
Limits and Continuity

Class Notes
Packet

Name __________________

Teacher _____________

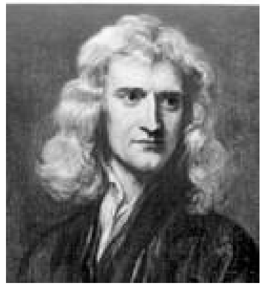
Calculus

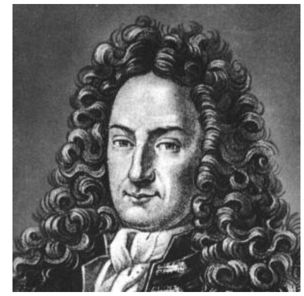
1.1 Can change occur at an instant?

Notes

Write your questions and thoughts here!

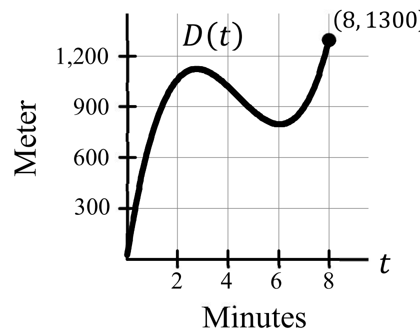
The Father(s) of Calculus





Can Change occur at an instant?

1. Mr. Brust's distance from his house is modeled by the function $D(t)$. While riding his bike to the store, he realizes he dropped his wallet and turns around to find it. After finding his wallet, he finishes his ride to the store.



   a. What is his average speed (rate of change) for his trip to the store if he arrives after 8 minutes?

   b. What was his average rate of change between 2 and 6 minutes?

   c. What was his average rate of change between 2 and 3 minutes?

   Is it possible to know how fast he was going at an instant?

   d. Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $t = 2$.

   e. Give a rough estimate of the instantaneous rate of change at $t = 2$.

2. $b(t)$ represents the buffalo population in the United States where t is measured in years since 1800.

   a. What does $b(90)$ represent?

   b. What does $\frac{b(50)-b(0)}{50-0}$ represent?

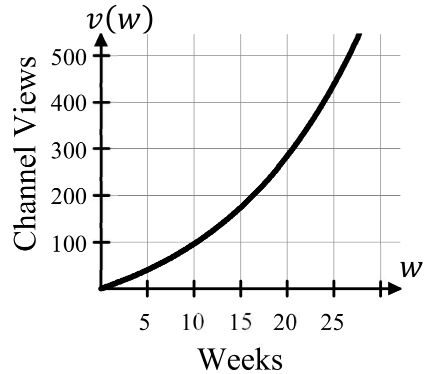
   c. What does $\frac{b(32)-b(31.999)}{32-31.999}$ represent?

1.1 Can change occur at an instant?

Calculus

Practice

1. Mr. Kelly has decided to quit his job as a teacher and be a social influencer. The number of views on his new channel is modeled by the function v, where $v(w)$ gives the number of views and w gives the number of weeks since he started the channel for $0 \leq w \leq 26$. The graph of the function v is shown to the right.



   a. Draw a tangent line at $w = 10$.

   b. Give a rough estimate of the instantaneous rate of change at $w = 10$.

   c. Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $w = 5$.

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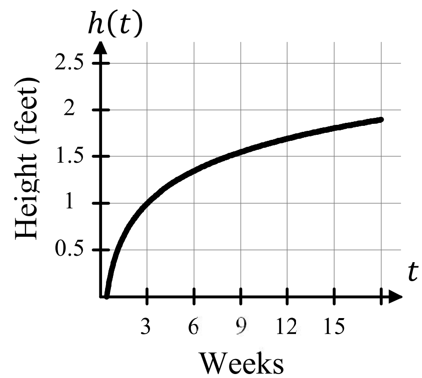
2. The height of a raspberry bush can be modeled by the function h, where $h(t)$ gives the height measured in feet and t gives the number of weeks it was planted for $0 \leq t \leq 12$. The graph of the function h is shown to the right.



   a. Draw a tangent line at $t = 9$.

   b. Give a rough estimate of the instantaneous rate of change at $t = 9$.

   c. Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $t = 12$.

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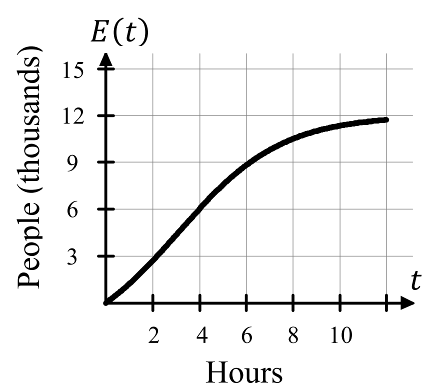
3. The number of people who have entered an amusement park is modeled by the function E, where $E(t)$ gives the number of people in thousands who have entered the park and t gives the number of hours since 10:00 a.m. for $0 \leq t \leq 11$. The graph of the function E is shown to the right.



   a. Draw a tangent line at $t = 3$.

   b. Give a rough estimate of the instantaneous rate of change at $t = 3$.

   c. Give an example of how to calculate a rate of change that would give a close estimate to the instantaneous rate of change at $t = 6$.

4. A basketball player's free throw attempts can be modeled by f, where $f(g)$ is the total number of made free throws during the season and g is the number of games for $0 \leq g \leq 82$.

a. What does $f(50)$ represent?	b. What does $\frac{f(50)-f(0)}{50-0}$ represent?	c. What does $\frac{f(50)-f(49.999)}{50-49.999}$ represent?

5. A monthly electric bill charges for each kilowatt-hour (kWh) used. This can be modeled by k where $k(m)$ is the kWh used for the month and m is the month for $0 \leq m \leq 12$.

a. What does $k(8)$ represent?	b. What does $\frac{k(8)-k(5)}{8-5}$ represent?	c. What does $\frac{k(2)-k(1.999)}{2-1.999}$ represent?

6. In a country, the number of deaths in a year can be modeled by d, where $d(t)$ is the number of deaths and t is the number of years since 1950 for $0 \leq t \leq 50$.

a. What does $d(40)$ represent?	b. What does $\frac{d(20)-d(10)}{20-10}$ represent?	c. What does $\frac{d(49)-d(48.999)}{49-48.999}$ represent?

7. A dam has a "dam release" that releases water. The amount of water released can be modeled by V, where $V(t)$ is the volume of cubic liters of water and t is the seconds since opening the dam release for $0 \leq t \leq 3600$.

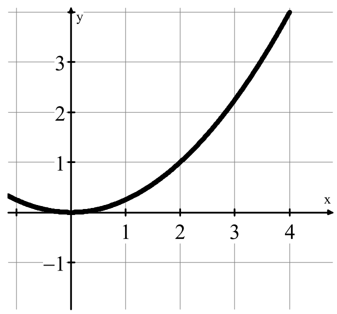
a. What does $V(100)$ represent?	b. What does $\frac{V(100)-V(0)}{100-0}$ represent?	c. What does $\frac{V(100)-V(99.999)}{100-99.999}$ represent?

1.2 Defining Limits

Write your questions and thoughts here!

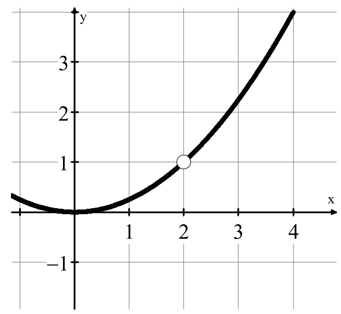
Limits

As x approaches ___, $f(x)$ approaches ___.



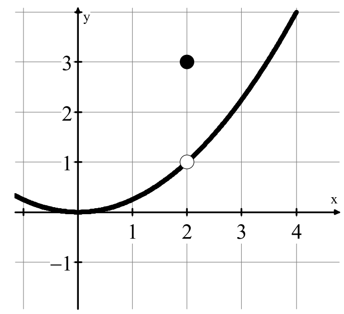
$\lim_{x \to 2} f(x) =$

$f(2) =$



$\lim_{x \to 2} f(x) =$

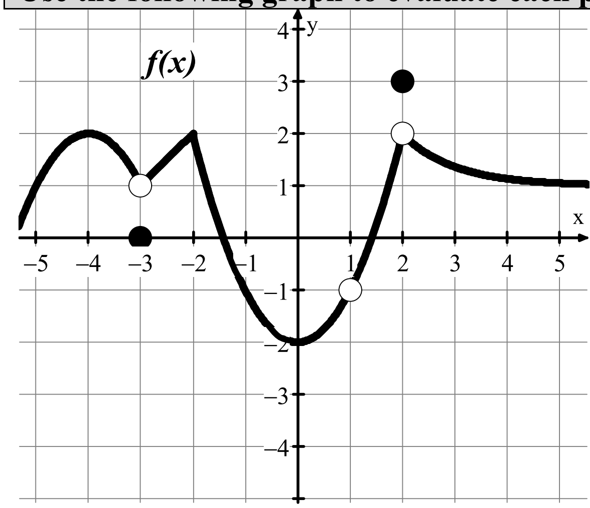
$f(2) =$



$\lim_{x \to 2} f(x) =$

$f(2) =$

Use the following graph to evaluate each problem.



1. $\lim_{x \to 1} f(x) =$
2. $f(-3) =$
3. $\lim_{x \to 2} f(x) =$
4. $f(2) =$
5. $f(1) =$
6. $f(-2) =$
7. $\lim_{x \to 0} f(x) =$
8. $\lim_{x \to -3} f(x) =$

9. Give an interpretation of the statement $\lim_{x \to 7} f(x) = 10$

A limit does NOT tell us the value of $f(x)$. It just tells us what the function approaches!

True or false? $f(1) = \lim_{x \to 1} f(x)$ in all cases.

True or false? $f(1) \neq \lim_{x \to 1} f(x)$ in all cases.

1.2 Defining Limits

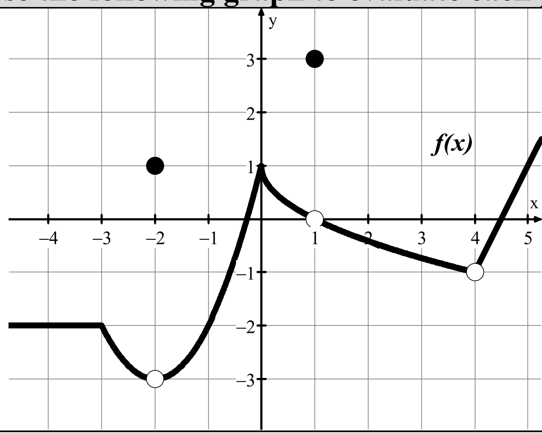
Calculus

Practice

Give an interpretation of each statement.

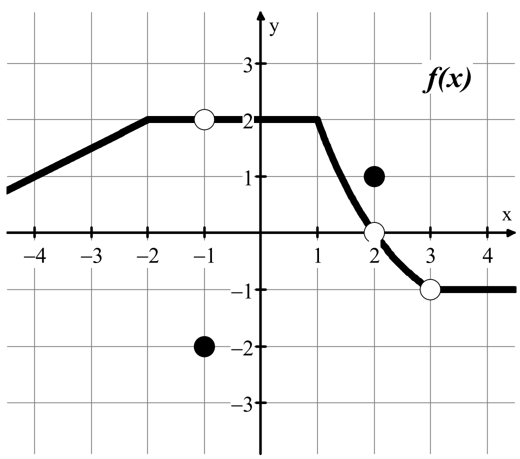
1. $\lim_{x\to 1} f(x) = 9$	2. $\lim_{x\to -2} f(x) = 3$	3. $\lim_{x\to 4} f(x) = -8$

Use the following graph to evaluate each problem.



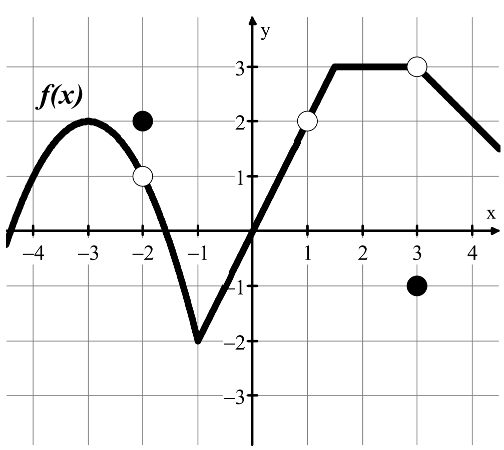
4. $f(-2) =$	5. $\lim_{x\to 1} f(x) =$
6. $\lim_{x\to -2} f(x) =$	7. $\lim_{x\to 0} f(x) =$
8. $f(4) =$	9. $\lim_{x\to 4} f(x) =$
10. $\lim_{x\to -4} f(x) =$	11. $f(1) =$

Use the following graph to evaluate each problem.



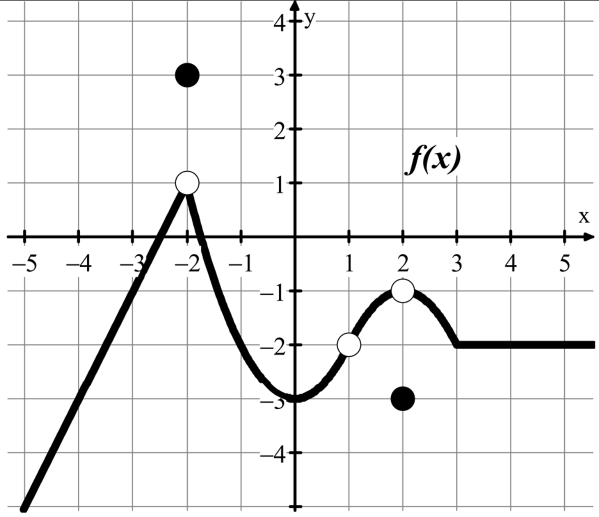
12. $\lim_{x\to -1} f(x) =$	13. $\lim_{x\to 3} f(x) =$
14. $f(2) =$	15. $\lim_{x\to -2} f(x) =$
16. $\lim_{x\to 1} f(x) =$	17. $f(3) =$
18. $f(-1) =$	19. $\lim_{x\to 2} f(x) =$

Use the following graph to evaluate each problem.



20. $\lim_{x\to 2} f(x) =$	21. $f(1) =$
22. $\lim_{x\to 3} f(x) =$	23. $\lim_{x\to -2} f(x) =$
24. $\lim_{x\to 1} f(x) =$	25. $f(-2) =$
26. $\lim_{x\to -3} f(x) =$	27. $f(3) =$

Use the following graph to evaluate each problem.



28. $\lim_{x \to -2} f(x) =$	29. $\lim_{x \to 1} f(x) =$
30. $\lim_{x \to 2} f(x) =$	31. $f(-2) =$
32. $f(1) =$	33. $\lim_{x \to 0} f(x) =$
34. $\lim_{x \to -4} f(x) =$	35. $f(2) =$

1.2 Defining Limits

Test Prep

36. Let f be a function that is defined for all real numbers x. Of the following, which is the best interpretation of the statement $\lim_{x \to 4} f(x) = 8$.

(A) The value of the function f at $x = 4$ is 8.

(B) The value of the function f at $x = 8$ is 4.

(C) As x approaches 4, the values of $f(x)$ approach 8.

(D) As x approaches 8, the values of $f(x)$ approach 4.

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37. Let f be a function that is defined for all real numbers x. Of the following, which is the best interpretation of the statement $\lim_{x \to -1} f(x) = 2$.

(A) As x approaches 2, the values of $f(x)$ approach -1

(B) The value of the function f at $x = -1$ is 2.

(C) The value of the function f at $x = 2$ is -1.

(D) As x approaches -1, the values of $f(x)$ approach 2.

Calculus

1.3 Finding Limits from Graphs

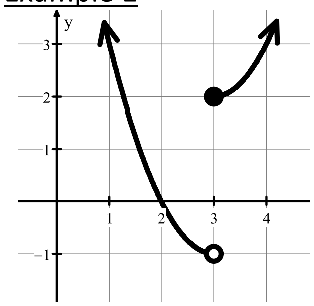
Notes

Write your questions and thoughts here!

What is a *one-sided* limit?

A ***one-sided limit*** is the _______ a function approaches as you approach a given __________ from either the _______ or _______ side.

Example 1



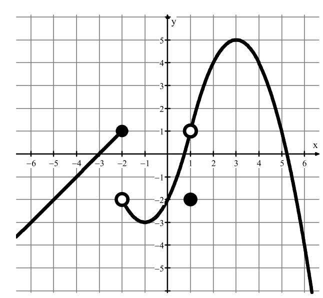
The limit of f as x approaches 3 from the left side is -1. $\lim\limits_{x\to} f(x) =$

The limit of f as x approaches 3 from the right side is 2. $\lim\limits_{x\to} f(x) =$

If the two sides are different? $\lim\limits_{x\to} f(x) =$

Example 2

a. $\lim\limits_{x\to -2^-} f(x) =$	b. $\lim\limits_{x\to -2^+} f(x) =$	c. $\lim\limits_{x\to -2} f(x) =$
d. $\lim\limits_{x\to 1} f(x) =$	e. $\lim\limits_{x\to 0} f(x) =$	f. $\lim\limits_{x\to 3^-} f(x) =$
g. $\lim\limits_{x\to -1} f(x) =$	h. $f(1) =$	i. $f(-2) =$



Example 3

Sketch a graph of a function g that satisfies all of the following conditions.

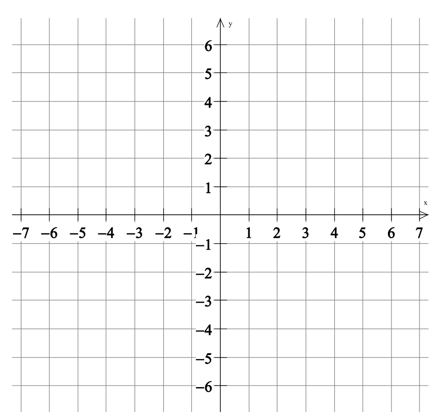
a. $g(3) = -1$

b. $\lim\limits_{x\to 3} g(x) = 4$

c. $\lim\limits_{x\to -2^+} g(x) = 1$

d. g is increasing on $-2 < x < 3$

e. $\lim\limits_{x\to -2^-} g(x) > \lim\limits_{x\to -2^+} g(x)$



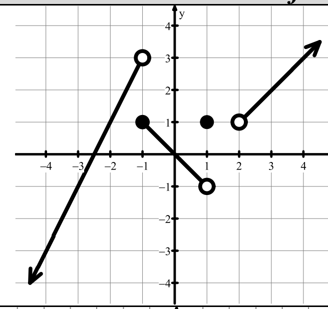
1.3 Finding Limits from Graphs

Calculus

Practice

For 1-3, give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined.*"

1.

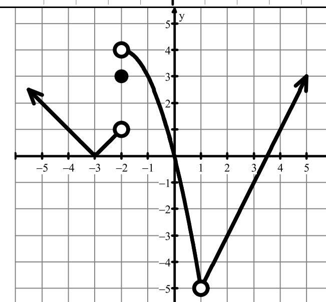


a. $\lim_{x\to -1^-} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 2^+} f(x) =$ e. $f(-1) =$ f. $f(2) =$

g. $\lim_{x\to -1^+} f(x) =$ h. $\lim_{x\to 1^-} f(x) =$ i. $\lim_{x\to 2} f(x) =$

2.

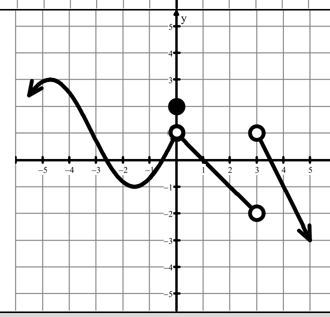


a. $\lim_{x\to -3} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 1} f(x) =$

d. $\lim_{x\to -2^+} f(x) =$ e. $f(3) =$ f. $\lim_{x\to -2^-} f(x) =$

g. $\lim_{x\to -2} f(x) =$ h. $f(-2) =$ i. $f(4) =$

3.

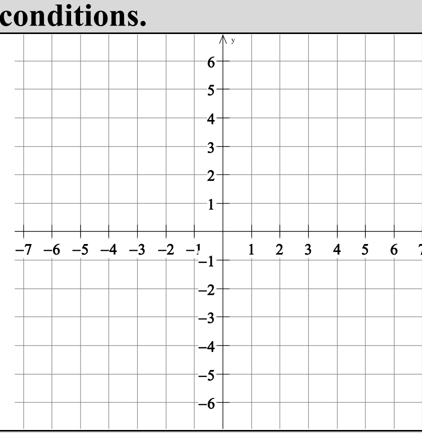


a. $\lim_{x\to 3^+} f(x) =$ b. $f(3) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 3} f(x) =$ e. $f(0) =$ f. $\lim_{x\to 3^-} f(x) =$

g. $\lim_{x\to 0^+} f(x) =$ h. $f(1) =$

4. Sketch a graph of a function f that satisfies all of the following conditions.



a. $f(-2) = 5$

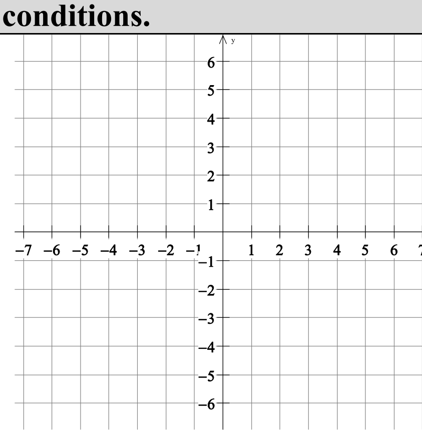
b. $\lim_{x\to -2} f(x) = 1$

c. $\lim_{x\to 4^+} f(x) = 3$

d. f is increasing on $x < -2$

e. $\lim_{x\to 4^-} f(x) < \lim_{x\to 4^+} f(x)$

5. Sketch a graph of a function g that satisfies all of the following conditions.



a. $g(1) = 3$

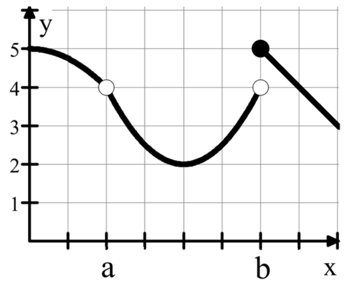
b. $\lim_{x\to 1} g(x) = -2$

c. $\lim_{x\to -3^+} g(x) = 5$

d. g is increasing only on $-5 < x < -3$ and $x > 1$

e. $\lim_{x\to -3^-} g(x) > \lim_{x\to -3^+} g(x)$

6. The graph of the function *f* is shown. Which of the following statements about *f* is true?



(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$ (B) $\lim_{x \to a} f(x) = 4$

(C) $\lim_{x \to b} f(x) = 4$ (D) $\lim_{x \to b} f(x) = 5$

(E) $\lim_{x \to a} f(x)$ does not exist.

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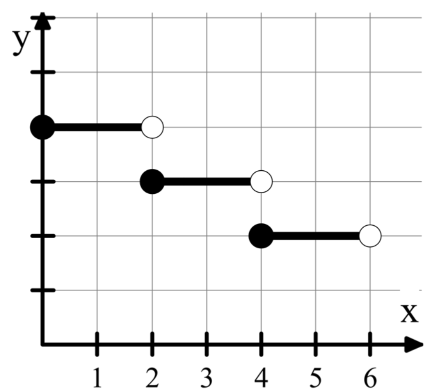
7. The figure below shows the graph of a function f with domain $0 \le x < 6$. Which of the following statements are true?

I. $\lim_{x \to 4^-} f(x)$ exists.
II. $\lim_{x \to 4^+} f(x)$ exists.
III. $\lim_{x \to 4} f(x)$ exists.



(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

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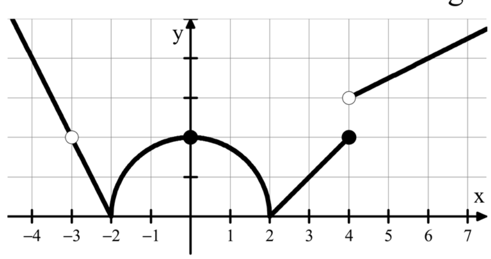
8. The graph of a function f is shown below. For which of the following values of c does $\lim_{x \to c} f(x) = 2$?



(A) 0 only (B) 0 and 4 only (C) -3 and 0 only

(D) -3 and 4 only (E) $-3, 0$, and 4

Calculus

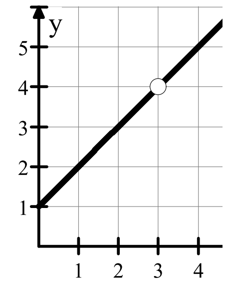
1.4 Finding Limits from Tables

Notes

Write your questions and thoughts here!

Calculator required for part of this lesson!

If we have the graph, it is easy to see the value of $\lim_{x \to 3} f(x) =$



Without the graph, we could use a table of values.

x	2.9	2.99	3.01	3.1
$f(x)$	3.9	3.99	4.01	4.1

1. According to the table, what is the value of $\lim_{x \to -4} f(x)$?

x	-4.4	-4.001	-3.999	-3.5
$f(x)$	2.43	2.499	2.501	2.68

2. If $f(x) = \frac{x^3 - 4x^2 - 7x + 10}{x + 2}$, create your own table of values to help you evaluate $\lim_{x \to -2} f(x)$.

x					
$f(x)$					

$$\lim_{x \to -2} f(x) =$$

Several ways to find values of a function on a calculator. Here are two:

- Table values (not as accurate, but fast)
- Function Notation

3. The function f is continuous and increasing for $x \geq 1$. The table gives values of f at selected values of x. Approximate the value of $\lim_{x \to 2} \cos\big(f(x)\big)$.

x	1.99	1.999	2.001	2.01
$f(x)$	4.85	4.999	5.001	5.15

$$\lim_{x \to 2} \cos\big(f(x)\big) =$$

1.4 Finding Limits from Tables

Calculus

Practice

Use the table for each problem to evaluate the limit.

1. $\lim_{x \to 9} f(x) =$

x	8.7	8.999	9.001	9.8
$f(x)$	-5.8	-5.001	-4.999	-4

2. $\lim_{x \to -7} f(x) =$

x	-7.5	-7.001	-6.999	-6.5
$f(x)$	3.8	3.501	3.499	3.2

3. $\lim_{x \to -2} f(x) =$

x	-2.1	-2.001	-1.999	-1.9
$f(x)$	-8.7	-8.999	-9.001	-9.4

4. $\lim_{x \to 11} f(x) =$

x	10.7	10.99	11.01	11.3
$f(x)$	10.3	10.001	9.999	9.6

For each function, create your own table of values to evaluate the limit.

5. $f(x) = \frac{x^2-2x-35}{x+5}$

x					
$f(x)$					

$\lim_{x \to -5} f(x) =$

6. $f(x) = \frac{x^2+5x+6}{x+2}$

x					
$f(x)$					

$\lim_{x \to -2} f(x) =$

7. $f(x) = \frac{x^2+4x-12}{x-2}$

x					
$f(x)$					

$\lim_{x \to 2} f(x) =$

8. $f(x) = \frac{5x^3+2x^2-13x+6}{x-1}$

x					
$f(x)$					

$\lim_{x \to 1} f(x) =$

Use the information given for each problem to evaluate the limit. Always round (or truncate) answers to three decimal places!

9. The function f is continuous and increasing $x \geq 0$. The table gives values of f at selected values of x.

x	6.9	6.999	7.001	7.1
$f(x)$	3.7	3.999	4.001	4.16

Approximate the value of $\lim_{x \to 7} 2\cos\big(f(x)\big)$.

10. The function f is continuous and decreasing for $x \geq 3$. The table gives values of f at selected values of x.

x	4.9	4.999	5.001	5.1
$f(x)$	2.2	2.001	1.999	1.75

Approximate the value of $\lim_{x \to 5} e^{3f(x)}$.

11. The function f is continuous and decreasing for $x \geq -5$. The table gives values of f at selected values of x.

x	-3.1	-3.01	-2.99	-2.8
$f(x)$	-3.4	-3.499	-3.501	-3.8

Approximate the value of $\lim_{x \to -3} \ln(-f(x))$.

12. The function f is continuous and increasing for $x \geq -7$. The table gives values of f at selected values of x.

x	-5.1	-5.001	-4.999	-4.8
$f(x)$	3.7	3.999	4.001	4.2

Approximate the value of $\lim_{x \to -5} \sqrt[5]{f(x)}$.

1.4 Finding Limits from Tables

Test Prep

13. The table below shows values of the function f at selected values of x. Which of the following is true based on the data from the table?

x	8.9	8.99	8.999	9.001	9.01	9.1
$f(x)$	0.7	0.8	0.999	2.001	2.01	2.3

(A) $\lim_{x \to 9} f(x) = 1$

(B) $\lim_{x \to 9} f(x) = 2$

(C) $\lim_{x \to 9^-} f(x) = 2$ and $\lim_{x \to 9^+} f(x) = 1$

(D) $\lim_{x \to 9^-} f(x) = 1$ and $\lim_{x \to 9^+} f(x) = 2$

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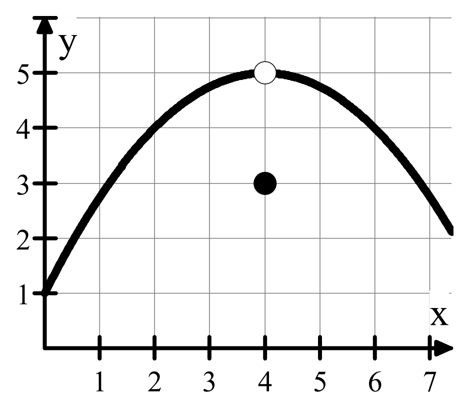
14. The graph of the function f is shown to the right. The value of $\lim_{x \to 4} 2\cos(f(x))$ is



(A) 0.567

(B) -1.307

(C) -1.979

(D) Does not exist

---

15. If $[x]$ represents the greatest integer that is less than or equal to x, then $\lim_{x \to 0^-} \frac{2}{[x]} =$

(A) -2 (B) -1 (C) 0 (D) 2 (E) the limit does not exist

Calculus

1.5 Algebraic Properties of Limits and Piecewise Functions

Notes

Write your questions and thoughts here!

$$x + x =$$

$$\lim_{x \to c}[f(x) + f(x)] =$$

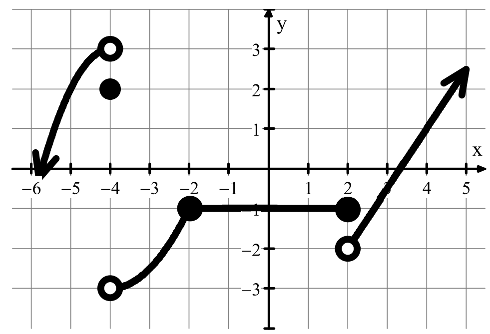
Example 1:

$\lim_{x \to -1} f(x) = 2$	$\lim_{x \to 1} f(x) = 4$	$\lim_{x \to 1} g(x) = 6$

The table above gives selected limits of the functions f and g. What is $\lim_{x \to 1}\left(f(-x) + \frac{g(x)}{2}\right)$?

Example 2:

The graph of the function f is shown on the right. What is $\lim_{x \to 4} f\big(f(x)\big)$?



Graph of f

Example 3:

$f(5) = 1$	$\lim_{x \to 5} f(x) = 6$
$g(5) = 2$	$\lim_{x \to 5} g(x) = -1$
$h(5) = 3$	$\lim_{x \to 5} h(x) = 5$

The table above gives selected values and limits of the functions f, g, and h.
What is $\lim_{x \to 5}\Big(h(x)\big(f(x) + 2g(x)\big)\Big) - h(5)$?

1.5 Algebraic Properties of Limits
Calculus

Notes

Example 4: Piecewise Functions

Piecewise defined functions and limits

$$f(x) = \begin{cases} \sqrt{11 - x}, & x < -5 \\ \dfrac{x + 3}{5 - x^2}, & x \geq -5 \end{cases}$$

a. $\lim_{x \to -5^-} f(x) =$

b. $\lim_{x \to -5^+} f(x) =$

c. $\lim_{x \to -5} f(x) =$

$$g(x) = \begin{cases} \sqrt{10 - x^2}, & x < -1 \\ \dfrac{26 - 5x^2}{7} & -1 < x \leq e \\ \ln x^3, & x > e \end{cases}$$

a. $\lim_{x \to -1} g(x) =$

b. $\lim_{x \to e^+} g(x) =$

c. $\lim_{x \to e} g(x) =$

Practice

Use the table for each problem to find the given limits.

1.

$\lim_{x \to 3} f(x) = 4$	$\lim_{x \to -3} f(x) = 2$	$\lim_{x \to 3} g(x) = 1$	$\lim_{x \to -3} g(x) = 5$

a. $\lim_{x \to 3} \left(2f(x) + g(-x)\right)$

b. $\lim_{x \to -3} \left(\frac{g(x)}{f(-x)}\right)$

2.

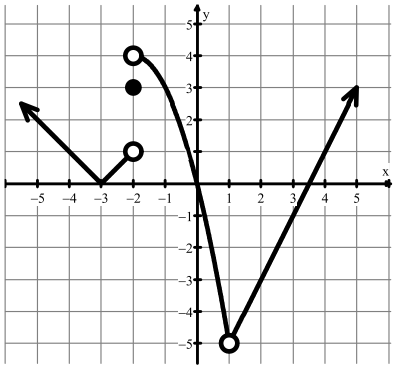
$\lim_{x \to 2} f(x) = -1$	$\lim_{x \to 1} f(x) = 6$	$\lim_{x \to 4} f(x) = 2$	$\lim_{x \to -2} f(x) = -3$

a. $\lim_{x \to 2} \left(f(2x) - f\left(\frac{x}{2}\right)\right)$

b. $\lim_{x \to 2} \left(\frac{f\left(\frac{x}{2}\right)}{f(-x)}\right)$

Use the graph for each problem to find the given limits.

3.

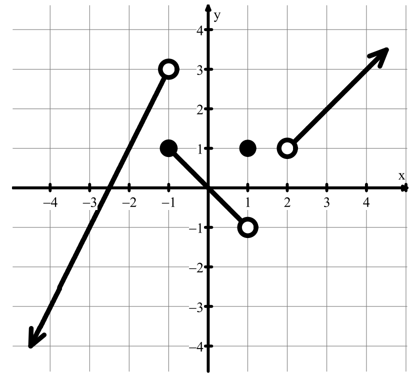


Graph of f

a. $\lim\limits_{x \to 3} f(f(x)) =$

b. $\lim\limits_{x \to 1} f(f(x)) =$

4.



Graph of f

a. $\lim\limits_{x \to -2} f(f(x)) =$

b. $\lim\limits_{x \to 4} f(f(x)) =$

Use the table for each problem to find the given limits.

5.

$f(1) = 4$	$\lim\limits_{x \to 1} f(x) = -1$
$g(1) = -2$	$\lim\limits_{x \to 1} g(x) = 3$
$h(1) = -3$	$\lim\limits_{x \to 1} h(x) = 6$

a. $\lim\limits_{x \to 1} \left(\left(f(x) \right)^2 - h(x) \right) - g(1)$

b. $f(1) + \lim\limits_{x \to 1} \left(-g(x) \right)$

6.

$f(-2) = 7$	$\lim_{x \to -2} f(x) = 2$
$g(-2) = 1$	$\lim_{x \to -2} g(x) = -1$
$h(-2) = -4$	$\lim_{x \to -2} h(x) = -3$

a. $\lim_{x \to -2} \left(h(x)\big(2f(x)\big)\right) + h(-2)$

b. $f(-2) \lim_{x \to -2} \big(g(x) - h(x)\big)$

Use the piecewise functions to find the given values.

7. $g(x) = \begin{cases} \sqrt{5 - x}, & x < -4 \\ x^2 - 5, & -4 \le x < 2 \\ x - 3, & x \ge 2 \end{cases}$

a. $\lim_{x \to 2^-} g(x) =$

b. $\lim_{x \to -4^+} g(x) =$

c. $g(2) =$

d. $\lim_{x \to -4^-} g(x) =$

e. $\lim_{x \to 2^+} g(x) =$

f. $\lim_{x \to 2} g(x) =$

g. $\lim_{x \to -4} g(x) =$

h. $g(-4) =$

8. $h(x) = \begin{cases} -|x|, & x \le -5 \\ 20 - x^2, & -5 < x \le 3 \\ 4x - 1, & x > 3 \end{cases}$

a. $\lim_{x \to -5^+} h(x) =$

b. $\lim_{x \to -5} h(x) =$

c. $h(3) =$

d. $\lim_{x \to -5^-} h(x) =$

e. $\lim_{x \to 3^+} h(x) =$

f. $\lim_{x \to 3} h(x) =$

g. $h(-5) =$

h. $\lim_{x \to 3^-} h(x) =$

9. $w(\theta) = \begin{cases} \sin\theta, & \theta \leq \pi \\ \cos\theta, & \pi < \theta < 2\pi \\ \tan\theta, & \theta > 2\pi \end{cases}$

a. $\lim_{x\to\pi^-} w(\theta) =$

b. $w(\pi) =$

c. $\lim_{x\to\pi^+} w(\theta) =$

d. $\lim_{x\to 2\pi^-} w(\theta) =$

e. $\lim_{x\to\pi} w(\theta) =$

f. $\lim_{x\to 2\pi^+} w(\theta) =$

g. $\lim_{x\to 2\pi} w(\theta) =$

h. $w(2\pi) =$

10. $f(x) = \begin{cases} \frac{1}{x+6}, & x < -2 \\ 2^x, & -2 \leq x < 0 \\ x^2 - 4, & x \geq 0 \end{cases}$

a. $\lim_{x\to -2} f(x) =$

b. $\lim_{x\to -2^-} f(x) =$

c. $\lim_{x\to -2^+} f(x) =$

d. $\lim_{x\to 0} f(x) =$

e. $\lim_{x\to 0^-} f(x) =$

f. $\lim_{x\to 0^+} f(x) =$

g. $f(-2) =$

h. $f(0) =$

11. If f is a continuous function such that $f(3) = 7$, which of the following statements must be true?

(A) $\lim_{x\to 3} f(3x) = 9$ (B) $\lim_{x\to 3} f(2x) = 14$ (C) $\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = 7$

(D) $\lim_{x\to 3} f(x^2) = 49$ (E) $\lim_{x\to 3} \left(f(x)\right)^2 = 49$

---

12. If $f(x) = \begin{cases} \ln 3x, & 0 < x \le 3 \\ x\ln 3, & 3 < x \le 4 \end{cases}$, then $\lim_{x\to 3} f(x)$ is

(A) $\ln 9$ (B) $\ln 27$ (C) $3\ln 3$ (D) $3 + \ln 3$ (E) nonexistent

---

13. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

Calculus

1.6 Algebraic Manipulation (Limits)

Notes

Write your questions and thoughts here!

Direct Substitution		Factor and Cancel	
1. $\lim\limits_{x \to -1} (x^2 + 2x - 4)$	2. $\lim\limits_{x \to 2} 6$	3. $\lim\limits_{x \to 0} \frac{4x^2 - 5x}{x}$	4. $\lim\limits_{x \to -7} \frac{2x^2 + 13x - 7}{x + 7}$

Limit Does Not Exist

5. $\lim\limits_{x \to -6} \frac{x^2 + 4x + 3}{x + 6}$

Special Trig Limits:

$$\lim_{x \to 0} \frac{\sin x}{x} = \quad \text{or} \quad \lim_{x \to 0} \frac{x}{\sin x} =$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = \quad \text{or} \quad \lim_{x \to 0} \frac{\cos x - 1}{x} =$$

6. $\lim\limits_{x \to 0} \frac{\sin 3x}{x}$	7. $\lim\limits_{x \to 0} \frac{\sin 7x}{\sin 9x}$	8. $\lim\limits_{x \to 0} \frac{\cos^2 x - 1}{x(\cos x + 1)}$

1.6 Algebraic Manipulation of Limits

Calculus

Practice

Evaluate each limit.

1. $\lim_{x\to 2}(x - x^2)$	2. $\lim_{x\to 5}(x+1)^2$	3. $\lim_{x\to 1}\frac{x^2-5x}{x-1}$	4. $\lim_{x\to 1}\frac{x^2+x-30}{x-1}$
5. $\lim_{x\to 0}\frac{3x}{\sin x}$	6. $\lim_{x\to 0}\frac{\sin(2x)}{3x}$	7. $\lim_{x\to -2}(3x^2 - x + 1)$	8. $\lim_{x\to 3}(2x^2 + 5x - 6)$
9. $\lim_{x\to -7}\frac{2x^3+11x^2-21x}{x^2+7x}$	10. $\lim_{x\to 8}\frac{x^2+2x-80}{x-8}$	11. $\lim_{x\to \frac{1}{3}}\frac{6x^2+13x-5}{3x-1}$	12. $\lim_{x\to 0}\frac{7x^2+x}{x}$
13. $\lim_{x\to -3} 14$	14. $\lim_{x\to 0}\frac{x^2+2x-8}{x-4}$	15. $\lim_{x\to -2}\frac{x^2-4x-10}{x}$	16. $\lim_{x\to 0}\frac{3x^2+5x}{x}$

17. $\lim_{x\to 4}\frac{5x^2-21x+4}{x-4}$

18. $\lim_{x\to \frac{1}{2}}\frac{1-x-2x^2}{2x-1}$

19. $\lim_{x\to \pi}\cos x$

20. $\lim_{x\to \frac{\pi}{8}}\sin(4x)$

21. $\lim_{x\to 2}\frac{x^2+6x-16}{2-x}$

22. $\lim_{x\to 5}\frac{2x^2-17x+35}{5-x}$

23. $\lim_{x\to 0}\frac{(1-\cos^2 x)\sin x}{x^2}$

1.6 Algebraic Manipulation of Limits

Test Prep

24. Evaluate $\lim_{x\to 1}\frac{\ln x}{3x}$ is

(A) 0 (B) $\frac{3}{e}$ (C) e (D) 3 (E) The limit does not exist.

25. $\lim_{x\to 0}\frac{\sin x\cos x-\sin x}{x^2}$ is

(A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefined

26. $\lim_{x\to a}\frac{x^2-2ax+a^2}{x-a}=$

(A) $-\infty$ (B) a (C) 0 (D) ∞ (E) The limit does not exist.

27. $\lim_{x\to 0}\left(\frac{3x^2+5\cos x-5}{2x}\right)=$

(A) 0 (B) $\frac{5}{2}$ (C) 3 (D) 4 (E) Does not exist

Calculus

1.7 Selecting Procedures (Limits)

Notes

Write your questions and thoughts here!

Rationalize Fractions with Radicals

1. $\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5}$

2. $\lim_{x\to 10} \frac{x-10}{3-\sqrt{x-1}}$

Complex Fractions

3. $\lim_{x\to 0} \frac{x}{\frac{1}{x-4}+\frac{1}{4}}$

4. $\lim_{x\to 0} \frac{\frac{1}{(x+3)^2}-\frac{1}{9}}{x}$

1.7 Selecting Procedures for Determining Limits

Calculus

Practice

Evaluate each limit.

1. $\lim_{x \to 0} \frac{\sqrt{x+7}-\sqrt{7}}{x}$

2. $\lim_{x \to 1} \sqrt{x+4}$

3. $\lim_{x \to 0} \frac{\frac{1}{x+3}-\frac{1}{3}}{x}$

4. $\lim_{x \to -3} \frac{x-2}{x^2-3x+2}$

5. $\lim_{x \to 7} \frac{\sqrt{x+9}-4}{x-7}$

6. $\lim_{x \to 0} \frac{2x^5+3x^4}{x^4}$

7. $\lim_{x \to 6} \frac{x}{\frac{1}{x+6} - \frac{1}{6}}$

8. $\lim_{x \to 5} \frac{x^2 - 5x}{x - 5}$

9. $\lim_{x \to 0} \frac{\sqrt{x+9} - 3}{x}$

10. $\lim_{x \to 1} \frac{\frac{1}{3} - \frac{1}{3x}}{x - 1}$

11. $\lim_{x \to 0} \frac{\sqrt{x+11} - \sqrt{11}}{x}$

12. $\lim_{x \to 3} \frac{\sqrt{2x-6}}{x}$

13. $\lim_{x \to 0} \frac{\frac{1}{(x+2)^2} - \frac{1}{4}}{x}$

Algebraic Manipulation of Limits

14. $\lim_{x \to b} \frac{b-x}{\sqrt{x}-\sqrt{b}}$ is

(A) $-2\sqrt{b}$ (B) $-\sqrt{b}$ (C) $2b$ (D) $\sqrt{b}$ (E) $2\sqrt{b}$