Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Bits and Pieces I

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School math department
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1 Introduction

This guide supports the Connected Mathematics™ student textbook *Bits and Pieces I.* This book is in the Number and Operations strand. Its primary topic is understanding the concepts of fractions, decimals and percents.

2 Goals/Objectives

This unit will help students:

- Understand fractions, decimals and percents and the relationships between them.
- Develop strategies for modeling situations involving fractions, decimals and percents.
- Compare and order fractions.
- Use equivalent fractions to solve problems.
- Move easily between fraction, decimal and percent representations of a given quantity.
- Use benchmark fractions to help estimate the size of a number or sum.
- Recognize benchmark quantities represented as fractions, decimals or percents.

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary in the student text gives definitions for many of these words.

Essential Terms	**Non Essential Terms**
decimal	base ten number system
denominator	benchmark
equivalent fraction	unit fraction
fraction	
numerator	
percent	

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4 Summary of Investigations

4.1 Investigation 1 – Fund-Raising Fractions

Students explore three representations of fractions: a visual model (fraction strips), word names, and symbols. The part-whole interpretation of fractions is developed.

Students learn in Problem 1.1 that 25% or ¼ of the sales goal is reached, so students have raised $75 of the $300 needed. Students still need to raise 75% or ¾, which is $225.

This same strategy is applied to Problem 1.2 – 1.3. In Problem 1.3 Follow-up, student answers may vary. A student may say that the 8th grade is closer to their goal though they have raised less money than the other grades. Other students may pick the seventh grade because they have raised the most money, although they are not the closest to their goal.

4.2 Investigation 2 – Comparing Fractions

Students develop an understanding of equivalent fractions by comparing their fraction strips. They make a number line that contains all the fractions identified on each fraction strip, and can identify which are equivalent. Benchmarks are used to estimate the size of a fraction and to make comparisons between fractions.

Fractions are equivalent if they represent the same value. When written in lowest terms, equivalent fractions will be identical. For example, 2/4, 20/40, 40/80, 3/6, 12/24 are all equivalent fractions. When written in lowest terms, each fraction can be expressed as ½. 3/9, 12/36, 4/12, 5/15, 7/21 are all equivalent fractions and can be written in simplest form (lowest terms) as 1/3.

Students can find fractions equivalent to a given fraction by multiplying (or dividing) the numerator and denominator by the same number. For example, to find a fraction equivalent to 4/5, a student can multiply the numerator (4) and the denominator (5) by any whole number (e.g., 3) and produce the fraction 12/15. It is also possible in some cases to find equivalent fractions by reducing the given fraction by dividing the numerator and denominator by a common factor. For example, 20/30 can be simplified (“reduced” or “written in lowest terms”) by dividing numerator and denominator by 10 to get 2/3.

Students use their fraction strips to compare fractions with benchmarks. Benchmark fractions which the students should be able to easily recognize and vision as a quantity are 0, ¼, ½, ¾, 1.

4.3 Investigation 3 – Cooking with Fractions

Students are introduced to using fractions to label an area or portion of an area that's being discussed. They use a recipe and need to make multiples of it to expand their understanding. Squares and rectangles are used for the ease with which they can be divided and shaded. A circle is used due to its use in data analysis and probability charts and graphs.

4.4 Investigation 4 – From Fractions to Decimals

Students are introduced to decimal representations of fractions and explore the place-value of decimals. They investigate a 100-square grid and divide to make 1000 parts and 10,000 parts. This process of subdividing is critical in their understanding of fractions and the equivalence of decimals as well as the relationship between fractions and decimals.

For problem 4.1 Follow-up: Potatoes = 27/100 of the garden; tomatoes = 9/100; corn = 54/100; the remaining vegetables make up 10/100 of the garden.

In the decimal 0.3456, the digit 3 is in the tenths place; 4 is in the hundredths place; 5 is in the thousandths place; 6 is in the ten thousandths place.

Decimal benchmarks with which all students must become familiar are 0, 0.25 (1/4), 0.5 and 0.50 (1/2), 0.75 (3/4) and 1.

4.5 Investigation 5 – Moving Between Fractions and Decimals

Students find decimal estimates for fractions being compared. They are encouraged to consider whether fractions or decimals are easier to compare. The division interpretation of fractions is discussed (for example, ½ is equivalent to 1 divided by 2). A strategy for converting a fraction to a decimal is discovered.

Problem 5.1:
Angela's success rate = 68/100, Emily's is 75/100, Carma's is 70/100. Emily is the most successful shooter.

Problem 5.1 Follow-up: Naomi averages 76 out of 100 free throws. Bobbie averages 80 out of 100, Kate averages 72 out of 100, Olympia averages 80 out of 100. Most students will select Olympia or Bobbie to make the free throws, as they have the same rate.

In Problem 5.3 students need to use division to divide the quantity of each food among the 24 boxes.

4.6 Investigation 6 – Out of One Hundred

Students should feel comfortable with fractions and decimals and their meanings and be able to move back and forth between the two. Percents are now introduced as another representation. Students are engaged in activities requiring them to move among fractions, decimals and percents. Students learn that percent means "out of 100". Therefore it is said that if 15 out of 100 books are red, 15% of those books are red.

To convert a fraction to a decimal, divide the numerator by the denominator. Students may use a calculator for this exercise. For example, 3/8 can be written as a decimal by performing the calculation 3 divided by 8. The decimal equivalent (the result of the division problem) is 0.375. To convert a decimal to a percent, multiply the decimal by 100. 3/8 = 0.375 as a decimal. 0.375 x 100 = 37.5%. This procedure can be followed for any fraction.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Questions, page 14:

8a. about 2/3
8b. about 80 cups
8c. about 1/3
8d. about 40 cups

10a. about 1/3
10b. about 1/12
10c. about ½.

5.2 Investigation 2

ACE questions, page 26:

1. true; 2. true; 3. false; 4. true
17. 7/6 is larger. 7/6 = 14/12 which is greater than 13/12.
38. 1/5, 2/10, 6/30, 9/45, 4/20, 5/25, 7/35, 8/40

5.3 Investigation 3

ACE Questions, page 34:

10. 1/8
11. 5/16
12. 9/28

5.4 Investigation 4

ACE Questions, page 46.

1. 30/100, 3/10, 0.30, 0.3
3. 53/100, 0.53
5. 90/100, 45/50, 9/10, 0.90, 0.9
10. >
38. 7/5 = 1 2/5.
42. 599.5, 599.504, 599.5044

5.5 Investigation 5

ACE Questions, page 58.
9. 1.45
10. 2.80 or 2.8
11. 0.75
12. 0.67
15. 9/10
17. =
18. <
33a. 8/10 or 0.8 of a pizza

5.6 Investigation 6

ACE Questions, page 77.
14. 875/1000, 0.875, 87.5%
15. 14/49 = 2/7 which is about 0.286 or 28.6%