Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Bits and Pieces II

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1 Introduction

This guide supports the Connected Mathematics™ student textbook *Bits and Pieces II.* This book is in the Number and Operations strand. Its primary topic is mathematical operations with fractions and decimals.

2 Goals/Objectives

This unit will help students:

- Continue to build understanding of fractions, decimals and percents and the relationships among these representations.
- Use strategies to quickly estimate sums and products, including use of 0, ½, 1, 1½, and 2 as benchmarks.
- Develop strategies for adding, subtracting, multiplying and dividing fractions and decimals.
- Become proficient at changing fractions to decimals.
- Understand how to use percent as an expression of frequency.
- Use percents to estimate or compute taxes, tips and discounts.

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary in the student text gives definitions for many of these words.

Essential Terms	Non Essential Terms
decimal	base ten number
denominator	system
equivalent fraction	benchmark
fraction	unit fraction
numerator	
percent	

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4 Summary of Investigations

4.1 Investigation 1 – Using Percents

This investigation builds on the last investigation of *Bits and Pieces I*. The typical situations of discounts, taxes and tips are used to help students find a percent of a number. Students begin by reviewing means to calculate 6% of one hundred and build on that and what they know about finding ten percent of a number to find other percents. Real life examples with discounts and taxes provide students an opportunity to apply discounts and taxes in proper sequence and calculate budgets and purchases.

6% of one hundred is 6. Remembering that percent means “out of 100”, students recall that 6 percent means 6 out of 100. Therefore 6% of a dollar (100 cents) is 6 cents. Students may learn that 6 cents is written $0.06.

6% of 100 is 6, then 6% of 200 is twice as much, or 12. Students use this knowledge to calculate 6% of different values.

Students also know that 10% means 10 out of 100. So, 10% of one dollar is 10 cents. 10% of $15.00 is $1.50. Once students have calculated 10% they come to understand that 5% (or 5 out of 100) is half of 10%, and that 15% can be found by calculating 10%, then 5%, then adding these two values together.

For example, to find 15% of $25.00 students could:

1. calculate 10% as $2.50
2. calculate 5% as $1.25
3. add $2.50 + $1.25 to find the sum $3.75.

Students have not yet determined the formula for finding the percent of a number through direct multiplication (15% of $25 can also be found by multiplying 0.15 by $25). This will come in a later investigation.

4.2 Investigation 2 – More About Percents

Students use percents in a variety of situations and in analysis of pie charts. They determine strategies for calculating percents of numbers greater than 100.

Once students calculate 5% of 100, they can multiply that value (i.e., 5) by 2 to get 5% of 200. Once having found 14% of 100 (i.e., 14), they can multiply that value by 6 to get 14% of 600 (i.e., 84).

Problem 2.4 Follow-up:

Step 1: Discount = $68.98 x 20% = $13.80
Step 2: Sale Price = $68.98 - $13.80 = $55.18
Step 3: Tax = $55.18 x 6% = $3.31
Step 4: Total Cost = $55.18 + $3.31 = $58.49

4.3 Investigation 3 – Estimating with Fractions and Decimals

Students estimate sums of fractions and decimals through the use of benchmark fractions and decimals. These benchmarks are 0, ¼ (0.25), ½ (0.5), ¾ (0.75), 1, 1¼ (1.25), 1½ (1.5), 1¾ (1.75), and 2. Students select a benchmark to which the given fractions/decimals are closest, and then add the benchmarks as a quick means to estimate the sum of the original two decimals/fractions.

Problem 3.2 Follow-up:

Fraction	Decimal	Percent
½	0.5	50%
1/3	0.333	33 1/3%
¼	0.25	25%
2/3	0.666	66 2/3%
¾	0.75	75%
1/6	0.1667	16 2/3%
1/5	0.2	20%

4.4 Investigation 4 – Adding and Subtracting Fractions

This investigation provides students an opportunity to figure out an algorithm for adding and subtracting fractions. In the course of working through the problems, students may invent ways of doing the addition/subtraction. Students will discuss this as a whole class and strategies will be refined and formalized.

One strategy with which the students are most familiar is to find a common denominator between the fractions, re-write the fractions as equivalent fractions with the common denominator and then add or subtract the numerators. The resulting sum or difference is then to be reduced to lowest terms.

For example, 1/3 + 5/7 can be calculated by re-writing 1/3 as 7/21 and 5/7 as 15/21. 7/21 + 15/21 = 22/21. 22/21 can be written as the mixed number 1 1/21.

4.5 Investigation 5 – Finding Areas and Other Products

This investigation provides students an opportunity to figure out an algorithm for multiplying fractions. In the course of working through the problems, students may invent ways of calculating the product. Students will discuss this as a whole class and strategies will be refined and formalized.

One strategy students develop for multiplying fractions is to draw a representation of the problem. For example, to calculate 1/3 of 1/3, students may divide a square in thirds and then each third in thirds again to determine that 1/3 of 1/3 is 1/9.

To calculate 2/3 of a value, students may divide the value by 3 to calculate 1/3 and then multiply the result by 2 to calculate 2/3. For example, students may calculate 2/3 of $27 by recognizing that 1/3 of $27 is 9. Therefore two thirds (2/3) of $27 is $18.

Students ultimately develop the algorithm for multiplying fractions, which is to multiply the numerators and then multiply the denominators "straight across" as they would explain.

For example, ¾ x ½ = 3/8. 5/6 x 2/3 = 10/18.

To multiply mixed numbers, students often find it helpful to convert the mixed number first to an improper fraction, and then perform the appropriate calculation.

4.6 Investigation 6 – Computing with Decimals

Since students have already had some experience with computing with decimals, this investigation presents a game and real-world situations that use decimals. Addition, subtraction, multiplication and division methodology are developed.

Students are already familiar with adding decimals. They know that they are to "line up" the decimal points and add. They learn that to multiply decimals, they are to perform the multiplication as if there were no decimal point, and then place the decimal point in the product according to the number of places after the decimal in the numbers being multiplied.

For example:
2 x 4 = 8
2 x 0.4 = 0.8 The product has one digit after the decimal point, as does one of the numbers being multiplied.

0.2 x 0.4 = 0.08 The product has two digits after the decimal point, as do the numbers being multiplied.

4.7 Investigation 7 – Dividing Fractions

Students will develop the meaning of division with fractions and the strategies and algorithms for dividing them. Everyday situations are presented to help students make sense of when division is the appropriate operations. Other operations with fractions are reviewed and the relationship between division and multiplication is explored.

Students discover that to divide by a fraction, they can multiply by its reciprocal. The reciprocal of a fraction can be found by reversing its numerator and denominator. For example, the reciprocal of ¾ is 4/3. The reciprocal of 5/6 is 6/5. The reciprocal of ½ is 2/1 or 2.

Any whole number can be written as an equivalent fraction of itself over 1. For example 3 = 3/1; 5 = 5/1; 16 = 16/1. Therefore, the reciprocal of a whole number can also be found by interchanging its numerator with its denominator. The reciprocal of 3 is 1/3; the reciprocal of 14 is 1/14; the reciprocal of 5 is 1/5.

To calculate ¾ divided by ½, a student would write and solve the equation ¾ x 2/1 = 6/4. 6/4 can be reduced to 1½.

3/7 divided by 5/4 can be calculated by multiplying 3/7 by 4/5 to obtain 12/35.

It is important that students remember that they are to take the reciprocal of the number being **divided by** (the divisor), not the number it is being divided into (the dividend).

To divide mixed numbers, students often find it helpful to convert the mixed number first to an improper fraction, and then perform the appropriate calculations.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Questions, page 12:

2a. $0.69

2b. The bill will be $13.75 + 0.69 = $14.44. Since 10% of $14.44 is $1.44 and half of that is $0.72, the tip should be $1.44 + $0.72 = $2.16. They should leave about $2.15 or $2.20.

2c. The bill including tip will be $14.44 + $2.20 = $16.64. Since $16.64 - $2.75 = $13.89 and half of $13.89 is about $6.95, Tat Ming should pay $6.95 and Faaiz should pay the remaining portion which is $16.64 - $6.95 = $9.69

5a. 30%
5b. 70%

5.2 Investigation 2

ACE questions, page 24:

1. 43%
2. 9%
3. 60%

13. about 15%

5.3 Investigation 3

ACE Questions, page 35:
5. ½
6. ½
7. ½
8. ½
9. 1
10. ½

5.4 Investigation 4

ACE Questions, page 49.
1a. 1/8 + 1/16 = 3/16 of the page will be used for ads. The remainder is 13/16.
1b. 3 x ¼ + 4 x 1/8 + 10 x 1/16 = ¾ + ½ + 5/8 = 15/8 = 1 7/8

1c. The magazine could charge $160 x 1/32 = $5 for 1/32 of a page; $160 x 1/16 = $10 for 1/16 of a page; $160 x 1/8 = $20 for 1/8 page; $160 x ¼ = $40 for ¼ of a page; $160 x ½ = $80 for ½ page and $160 for a whole page.

5.5 Investigation 5

ACE Questions, page 61.

1. 2/3 x 4/5 = 8/15 of a section
2. a) ½ x 7/8 = 7/16 of a yard

11. One possible answer: 7/12. ½ = 6/12 and 2/3 = 8/12. Therefore 7/12 is in between the two.

5.6 Investigation 6

ACE Questions, page 72.

4a. The measures for the second week are larger because the plant is growing. For a given week, measures might be different because each player measured from a different starting point on the plant, or because they were using different measuring tools and there was slight error.

4b. First week mean = 3.36 cm (add the 5 values and divide by 5).
Second week mean = 8.02 cm (add the 5 values and divide by 5).

5.7 Investigation 7

ACE Questions, page 83.

20. 15
21. 5/37
22. 6/28 or 3/14
23. 3/20
24. 6/5 or 1 1/5