Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Clever Counting

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1 Introduction

Clever Counting is part of the Number and Operations strand. The primary goal of this unit is to help students learn some new and very useful strategies for reasoning in situations that involve counting possibilities. Students develop and apply strategies for counting combinations of whole numbers, letters and other denumerable items.

From a diverse set of experiences in this unit, students will develop a sense of the structure of problems involving counting. They will also learn to consider the special circumstances of various counting situations, such as whether or not order is important.

Students are expected to have ready access to calculators to perform arithmetic operations, so they can focus on the problem-analysis and pattern-search aspects of mathematics. They will demonstrate that they know how to model a problem successfully, using an organized list or some other representation that captures the essence of the number of choices.

2 Goals/Objectives

This unit will help students:

- Recognize situations in which counting techniques apply
- Construct organized lists of outcomes for complex processes and uncover patterns that help in counting the outcomes of those processes
- Use diagrams, tables, and symbolic expressions to organize examples in listing and counting tasks
- Analyze the usefulness of counting trees and use counting trees
- Use mental arithmetic to make estimates in multiplication and division calculations
- Invent strategies for solving problems that involve counting
- Analyze counting problems involving choices in various contexts
- Differentiate among situations in which order does and does not matter and in which repeats are and are not allowed
- Analyze the number of paths through a network
- Compare the structure of networks with problems involving combinations
- Create networks that satisfy different constraints
- Apply thinking and reasoning skills to an open-ended situation in which assumptions must be made and create a persuasive argument to support a conjecture

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary (page 72) in the student text gives definitions for many of these words.

Essential Terms	Terms developed in previous units	Nonessential terms
counting tree edge network node	Organized list Probability	

4 Summary of Investigations

4.1 Investigation 1 – Counting Possibilities (pp 5-14)

- The main theme of the unit, counting by using multiplication, is introduced.
- The basic problem form is a sequence constructed by making a choice for each element in the sequence.
- If there are a ways to fill the position, b ways to fill the second, c ways to fill the third, and so on, there will be a x b x c ... possible sequences.
- Consider a problem that has only a few outcomes and can be easily modeled with an organized list or a counting tree.
- Search for solutions by reasoning rather than listing all possible outcomes.

4.2 Investigation 2 – Opening Locks (pp 15-26)

- Become aware of some fundamental properties of counting by investigating the number of possible combinations for two kinds of locks. The mathematical analyses of the two situations are similar: characters cannot be repeated in a combination, and the counting of possible combinations can be expedited by multiplication.

4.3 Investigation 3 – Networks (pp 27-36)

- Find the number of paths through a network. The basic idea of counting paths is that the number of getting from point A to point C via point B is the product of the number of paths from A to B and the number of paths from B to C.
- Create networks that satisfy given constraints.

4.4 Investigation 4 – Deciding Whether Order is Important (pp 37-46)

- Students discover that, in each counting situation, they must consider whether order is important.

4.5 Investigation 5 – Wrapping Things Up (pp 47-56)

- In this open ended investigation, students are encouraged to ask and answer more sophisticated questions and to have fun speculating about the many ways they can apply their newfound reasoning.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Question 1 page 9

ANSWER

1a. 2
1b. 2 (For each choice of shoes, he can choose from 2 pairs of pants.)
1c. 3 (For each choice of pants he can choose from 3 shirts.)
1d. There are 12 combinations.
1e. Count the number of entries in the top row, or compute 2 x 2 x 3.

5.2 Investigation 2

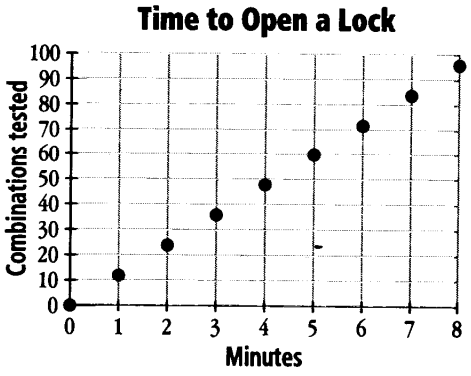
ACE question 1 page 31.

ANSWER

1a. 100,000 x 5 = 500,00 seconds
1b. 500,000 / 60 = about 8333 minutes = about 139 hours / 24 = about 5.8 days.

1c. Tina can try 12 combinations every minute. The table shows some data that fit this relationship.

Minutes	Combinations tested
0	0
1	12
2	24
3	36
4	48
5	60
6	72
7	84
8	96



1d. Possible answer: Lock designers could add more buttons to increase the number of combinations or design locks that shut down for a period of time if a certain number of incorrect sequences are entered in a row.

5.3 Investigation 3

ACE Question 1 page 32

ANSWER

1a. 2 x 5 x 3 = 30 paths

1b. Six paths (2 x 1 x 3) pass the marked location

5.4 Investigation 4

ACE Question 1 page 41

ANSWER

1. Solution methods will vary. As there are 9 dominoes with at least one blank, 8 other dominoes with at least one half with one pip, and so on, a set would contain 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45 dominoes.

5.5 Investigation 5

ACE Question 2 page 50.

ANSWER

2. 10 x 10 x10 = 1000 passwords